Seminar at RCNP, Osaka University
7- 9March, 2019

How are hadrons formed from quarks?

Hiroyuki Noumi
RCNP, Osaka University
Institute of Particle and Nuclear Studies, KEK

Part I: Introduction

Part II: Experimental Study of Hadrons

II-1: Λ(1405) and $K^{bar}N$ Interaction

II-2: Charmed Baryons

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I-1: Standing Point of this Lecture

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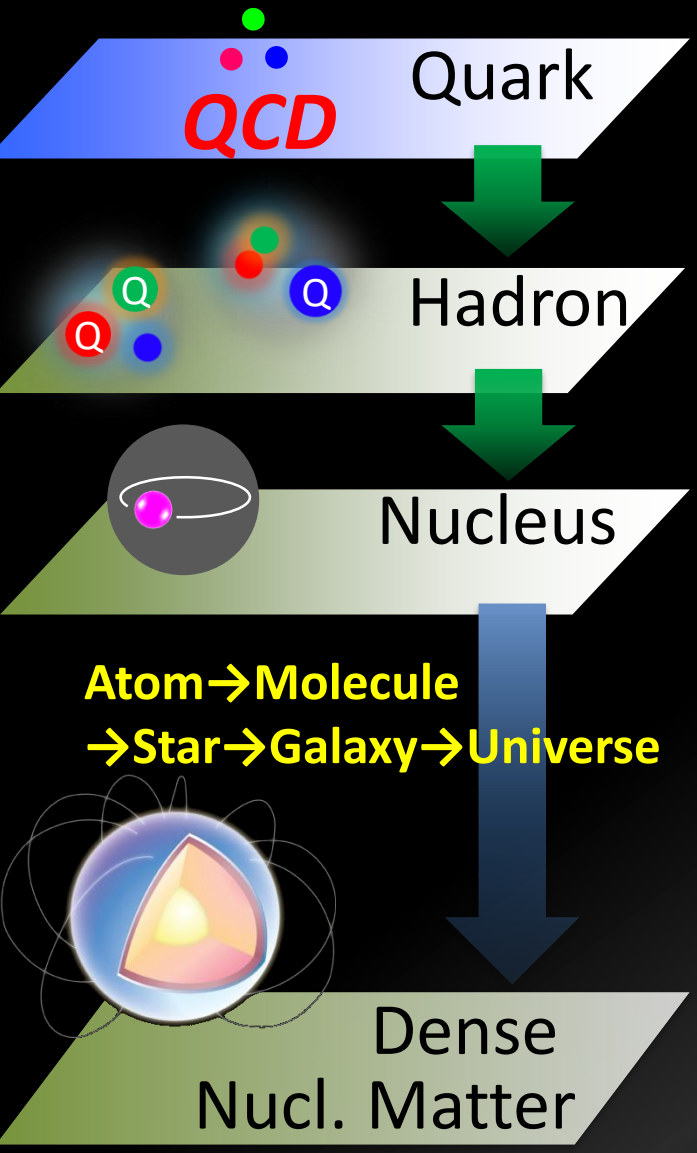
How does Matter Evolve in the Universe?

- Roles of the Strong Int. -



QCD dynamics

QGP at High-T, High-rho, Phase Border

Form of Hadron

Hadron Spectroscopy, Tomography

Hadron at High-T, High-rho

BB Int.（2BF, 3BF, …）

BB scattering, Nuclear Structure

Nuclear Matter: EoS

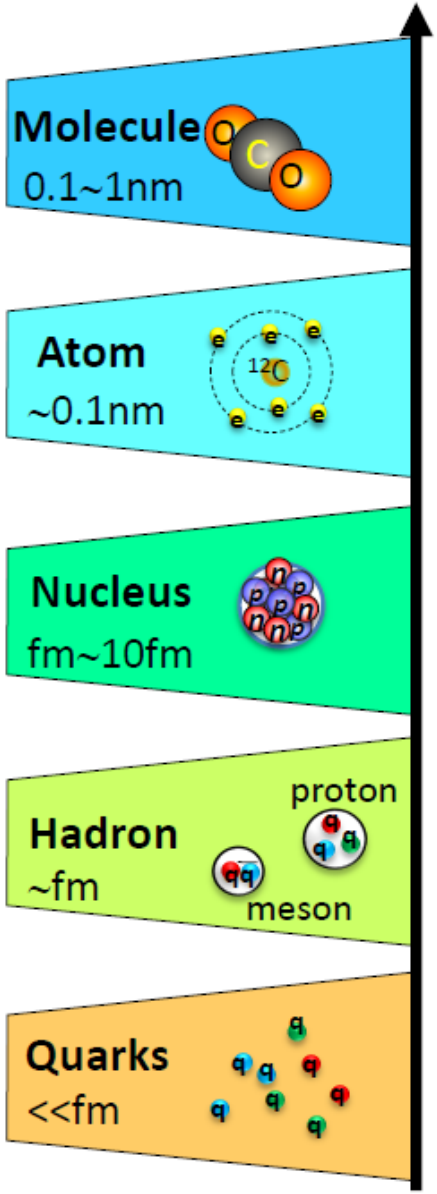
Nuclear Response, Single Particle Energy

Nuclear Synthesis

Magic No., Drip-line, S.H.E.,…

Mystery of Heavy NS:
NS Merger as a Factory of H.E.

Why hierarchy is formed in quantum particles？



Science in 20th Century

More Elementary

Diverse Universe

History of Universe

微視的粒子の世界に階層構造がどうして形成されたのか

Fundamental Question But Unresolved

→*Not necessarily investigated*

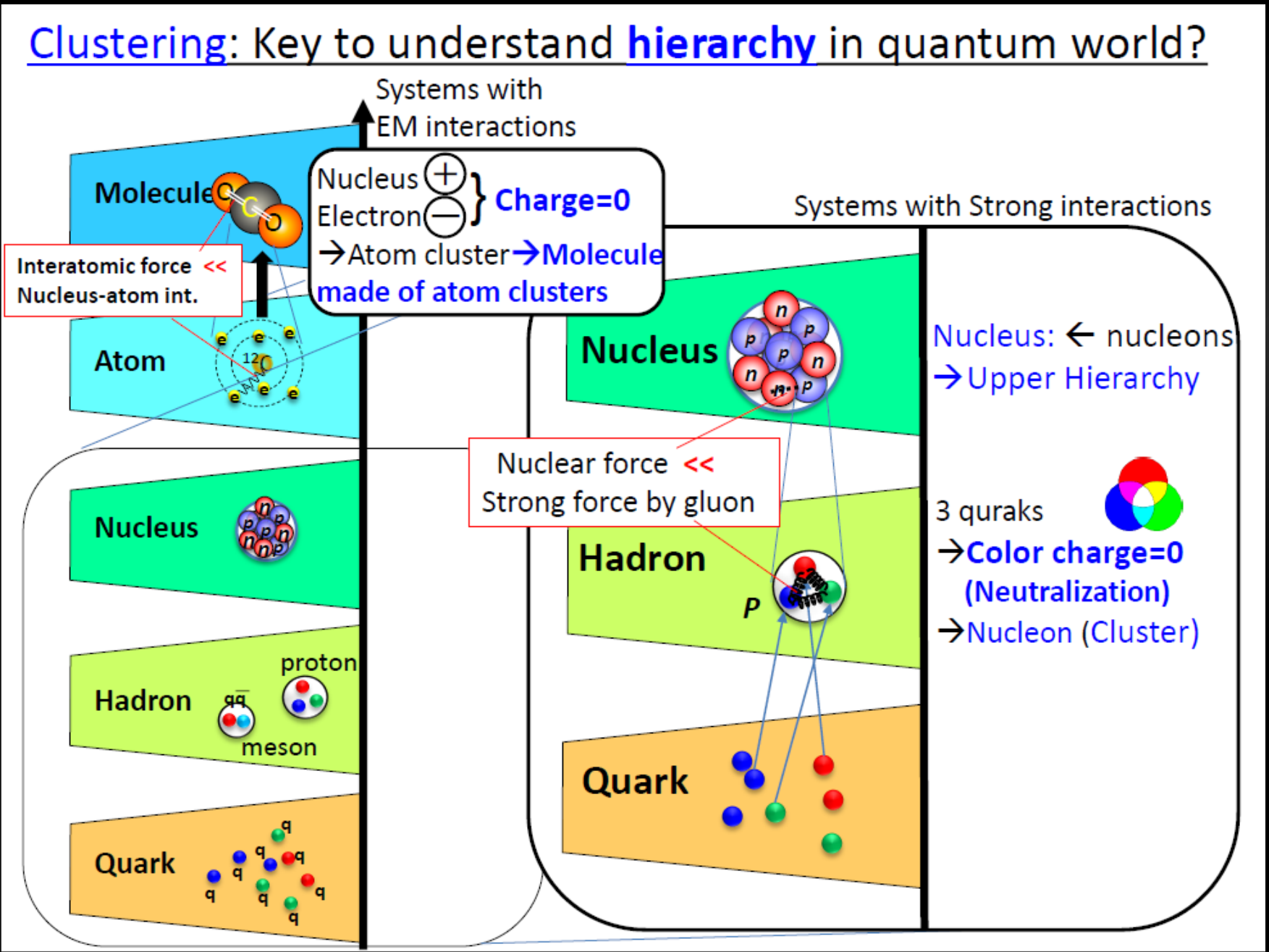
- *Research Areas : Independently evolved*
- *Connection between hierarchies: Unclear*

~380kY : Clear up of the Universe 宇宙の晴上がり
Nucleus+electrons →Atom→Molecule

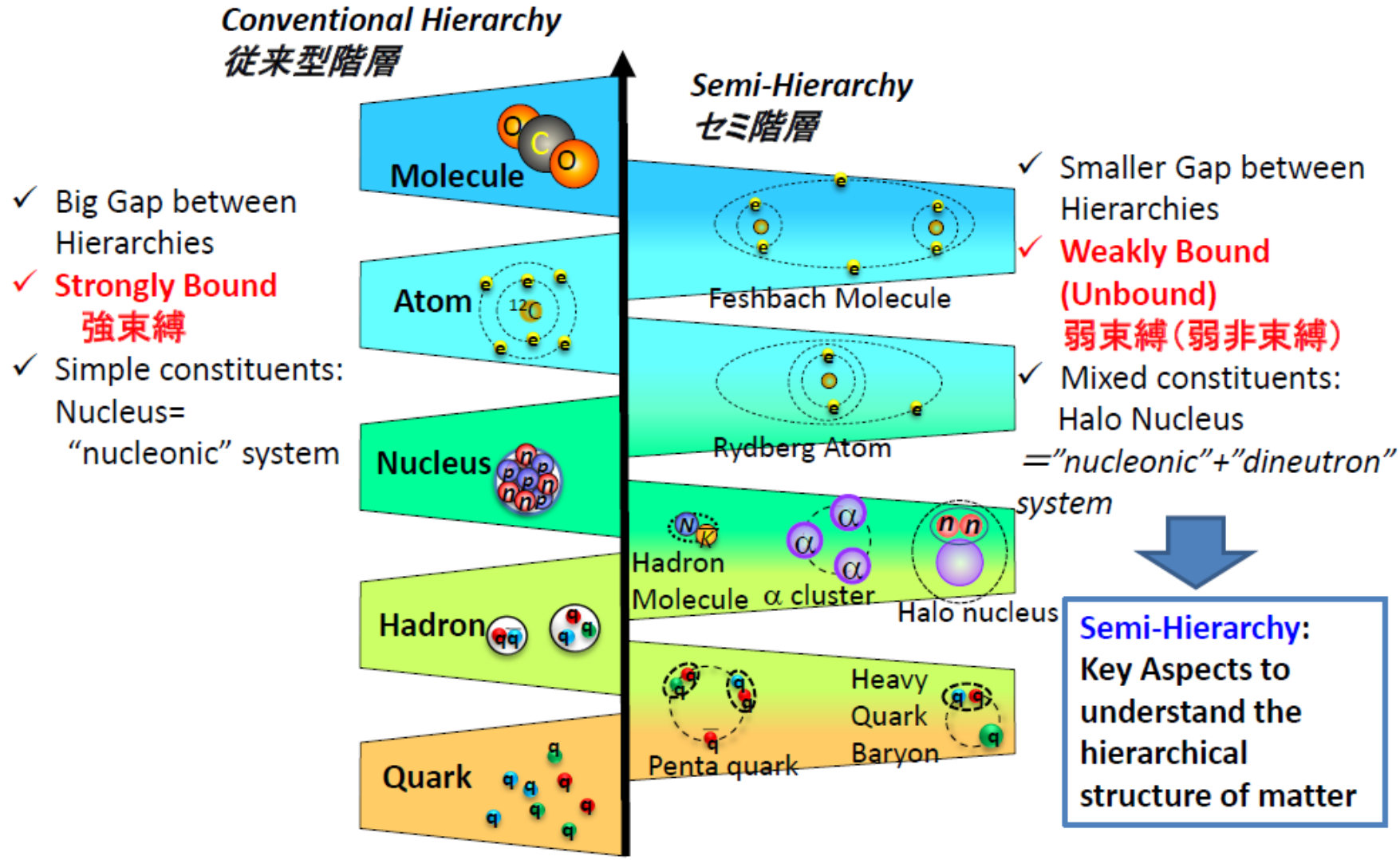
~3min : Big Bang Nucleosynthesis ビックバン元素合成
nucleon（p, n) →^{4}He nucleus

~0.01ms : QCD Phase Transition QCD相転移

Big Bang


Clustering: Key to understand hierarchy in quantum world?
Systems with
EM interactions
Molecule
Nucleus
Electron
Charge=0
→Atom cluster→Molecule
made of atom clusters
Interatomic force <<
Nucleus-atom int.
Atom
Systems with Strong interactions
Nucleus
Nucleus: ← nucleons
→Upper Hierarchy
Nuclear force <<
Strong force by gluon
Nucleus
Hadron
P
3 quraks
→Color charge=0
(Neutralization)
→Nucleon (Cluster)
proton
Hadron
meson
Quark
Quark

Clusters and Semi-Hierarchy

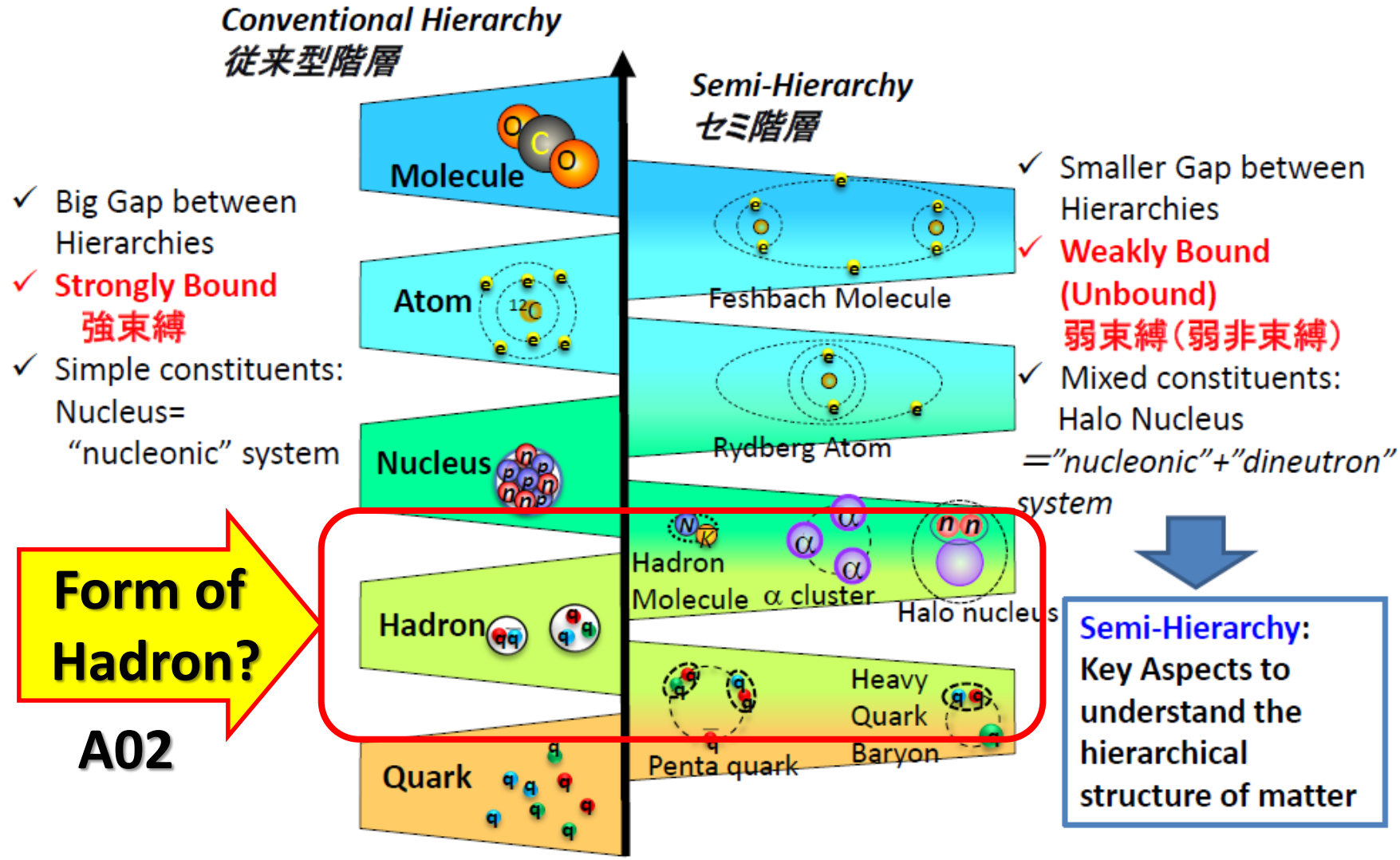


- ✓ Big Gap between Hierarchies
- ✓ **Strongly Bound 強束縛**
- ✓ Simple constituents: Nucleus= "nucleonic" system

- ✓ Smaller Gap between Hierarchies
- ✓ **Weakly Bound (Unbound) 弱束縛(弱非束縛)**
- ✓ Mixed constituents: Halo Nucleus *="nucleonic"+"dineutron" system*

Semi-Hierarchy: Key Aspects to understand the hierarchical structure of matter

Clusters and Semi-Hierarchy



- ✓ Big Gap between Hierarchies
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  強束縛
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- ✓ Smaller Gap between Hierarchies
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  弱束縛(弱非束縛)
- ✓ Mixed constituents: Halo Nucleus
  ="nucleonic"+"dineutron" system

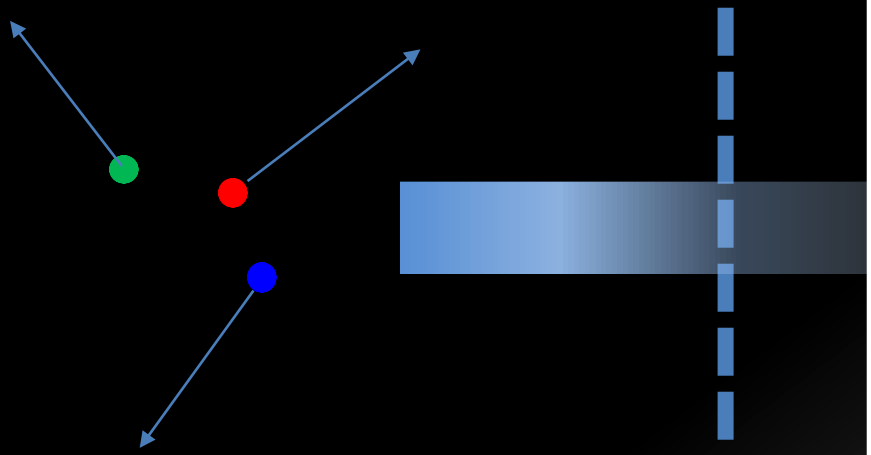
Form of Hadron?

A02

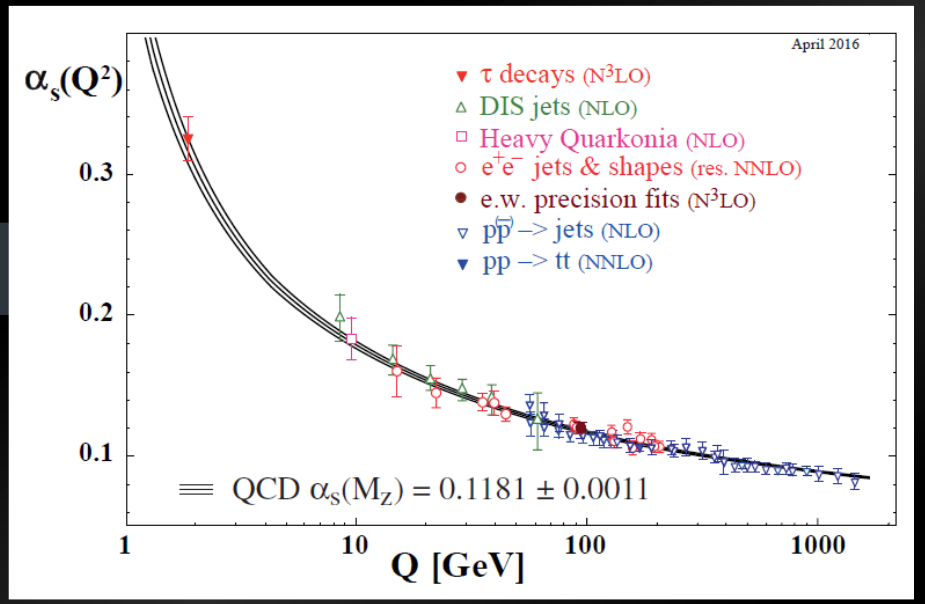
Semi-Hierarchy: Key Aspects to understand the hierarchical structure of matter

How Hadrons are formed?

High E → Low E

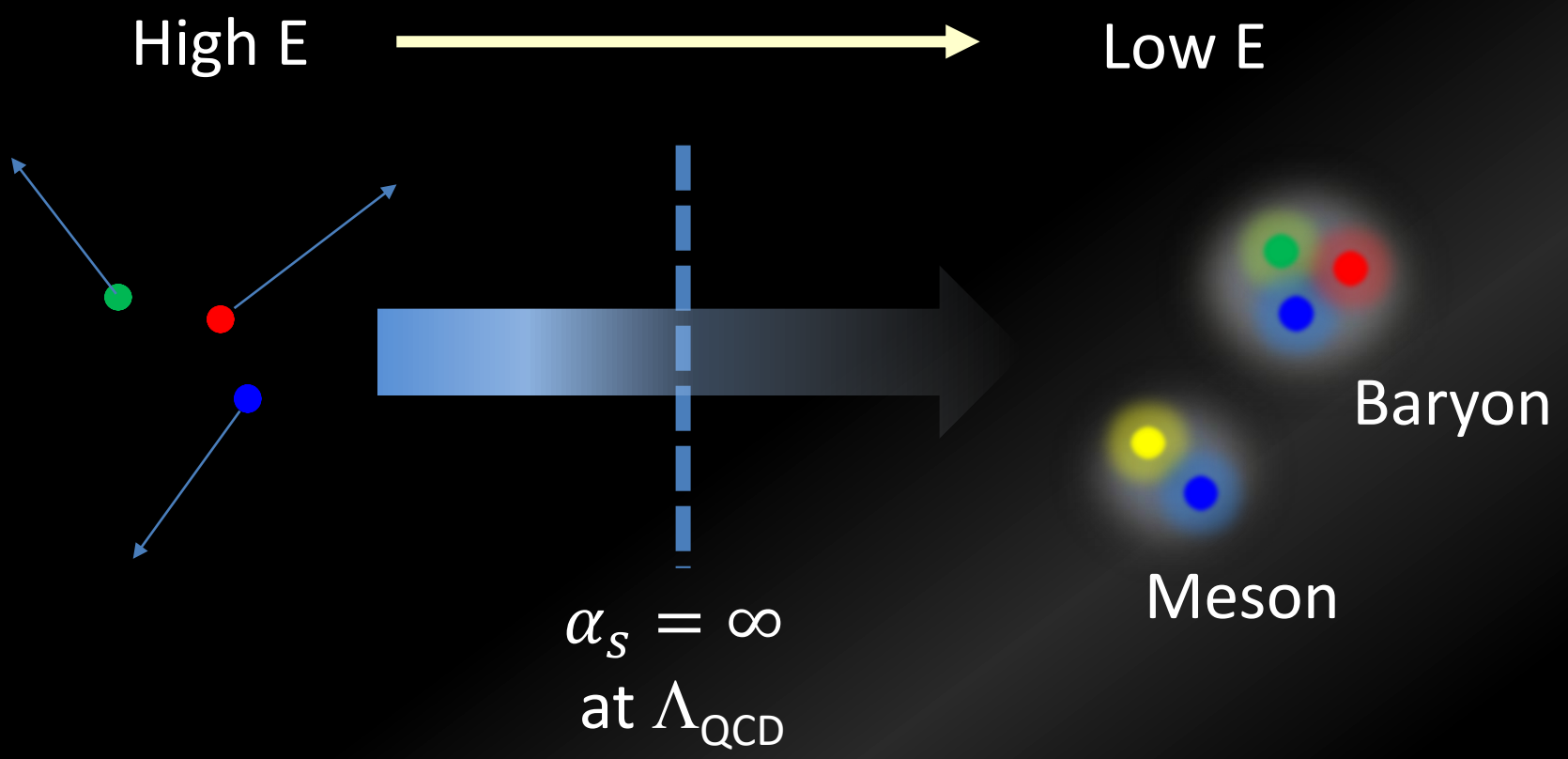


$\alpha_s = \infty$
at Λ_{QCD}



Quarks drastically change themselves below Λ_{QCD}.

How Hadrons are formed?



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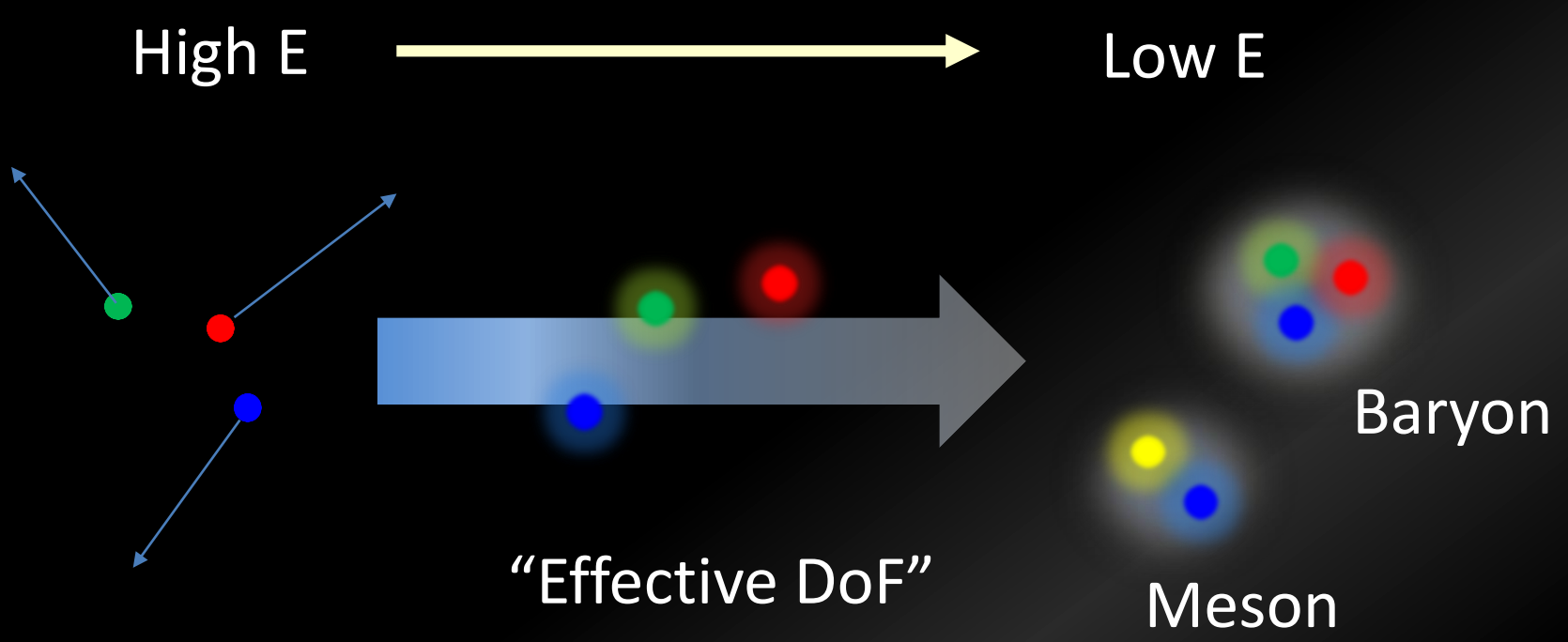
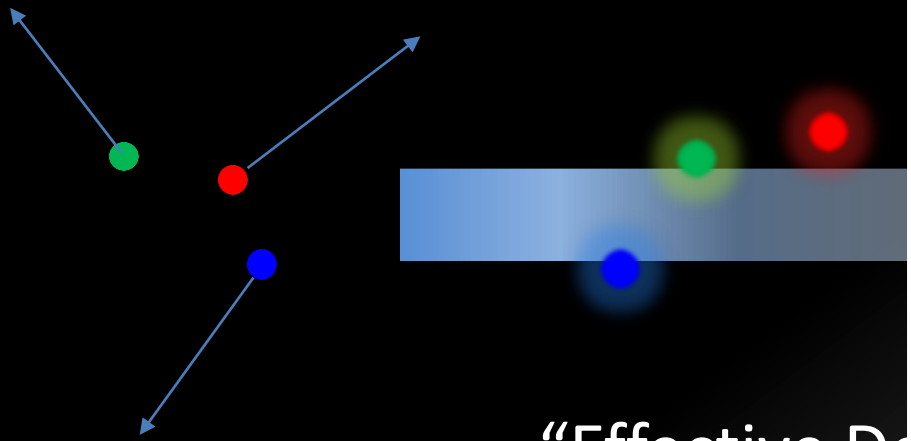


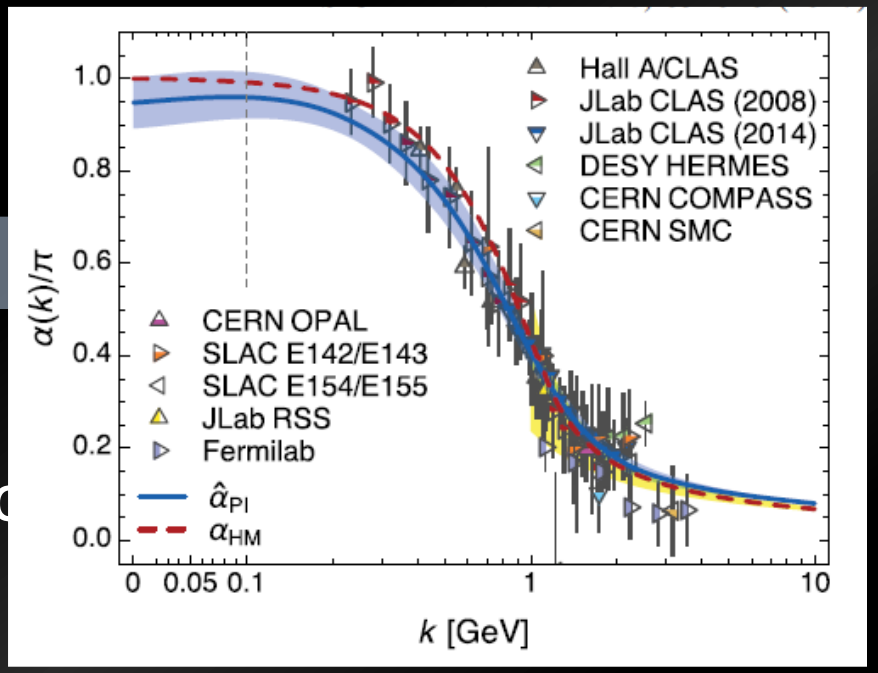
"Constituent Quarks" seem to work rather well
as good building blocks of hadrons…

How Hadrons are formed?

High E → Low E



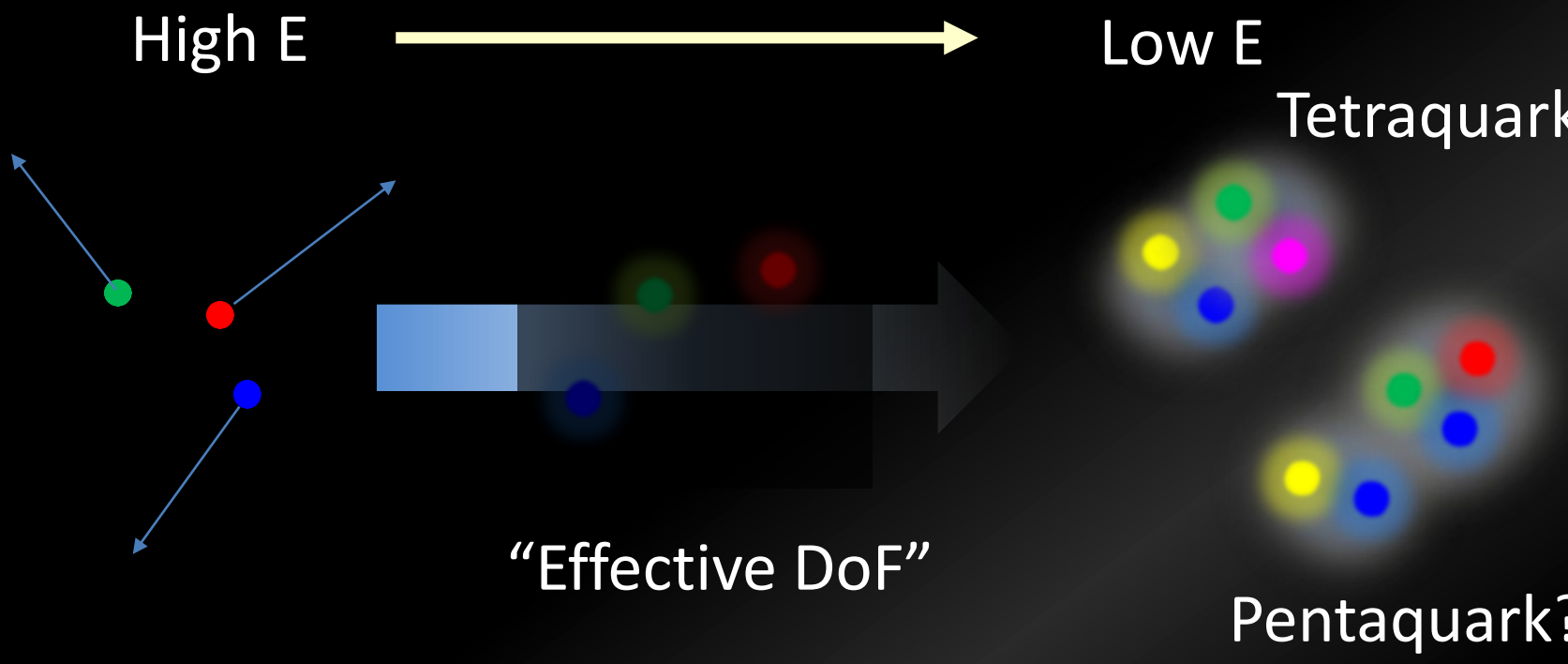
"Effective Dc



"Constituent Quarks" seem to work rather well as good building blocks of hadrons…

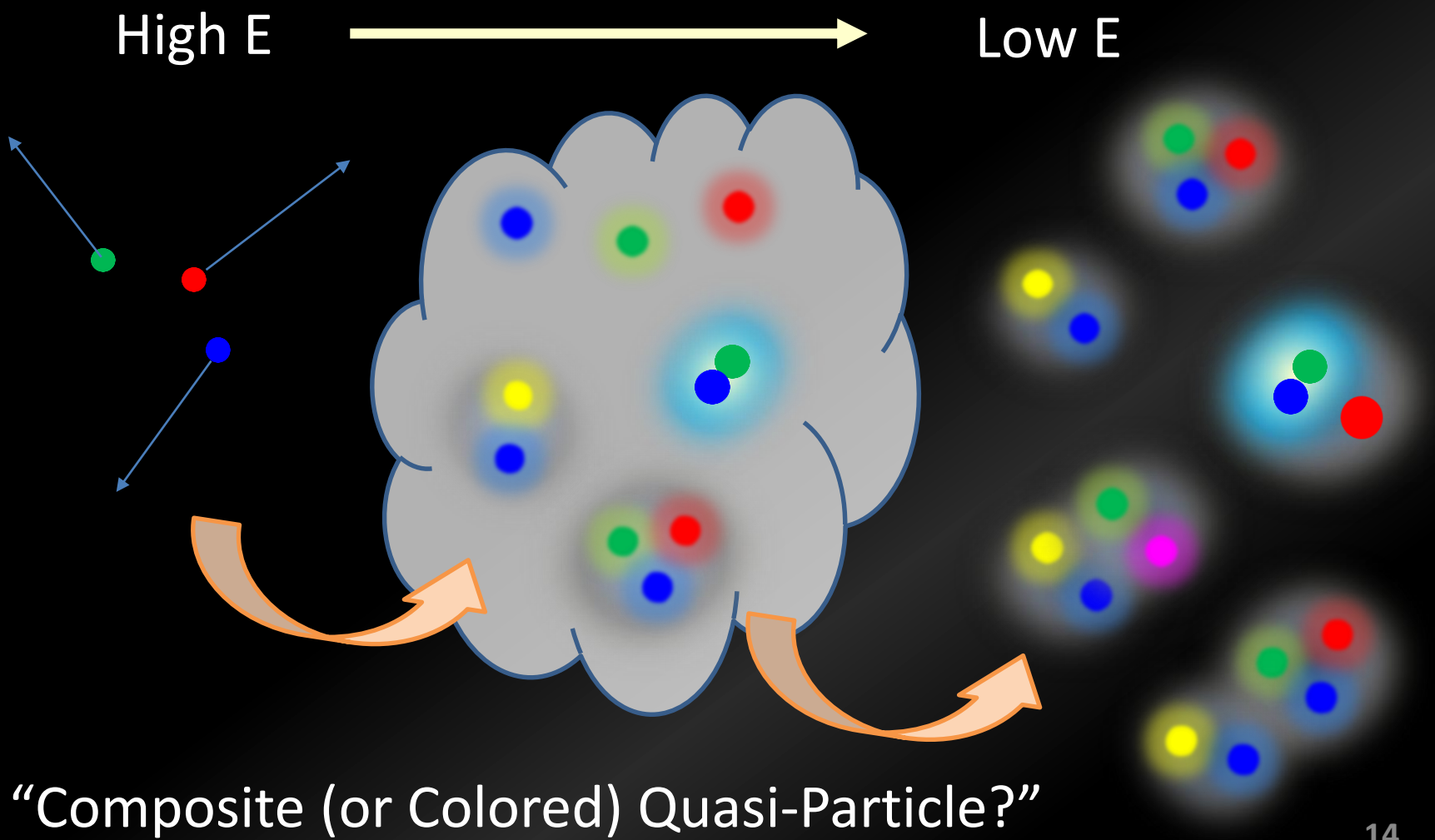
Saturation of "Effective Charge" at low k. (PRD96,054026(2017)

How Hadrons are formed?



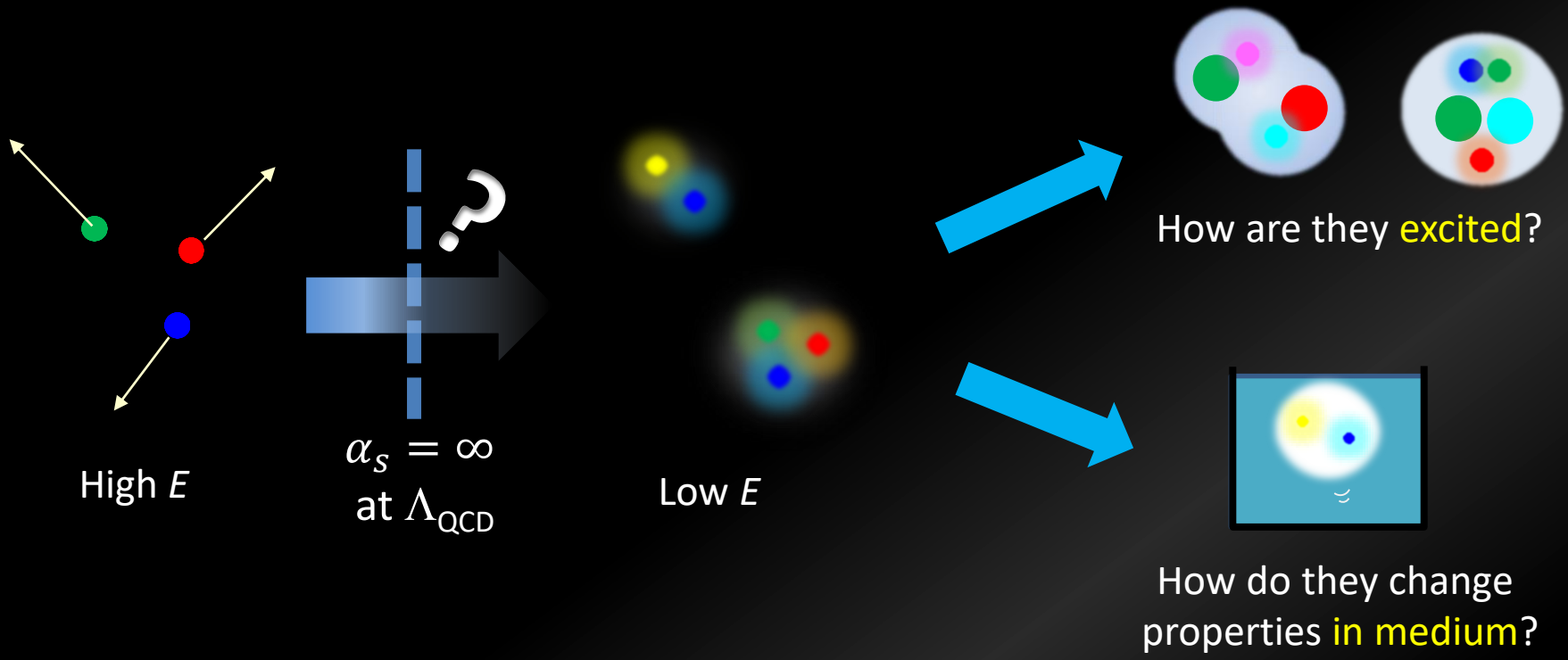
"Exotic hadrons" require a new aspect
in describing hadrons beyond the "standard picture".

How Hadrons are formed?



"Composite (or Colored) Quasi-Particle?"

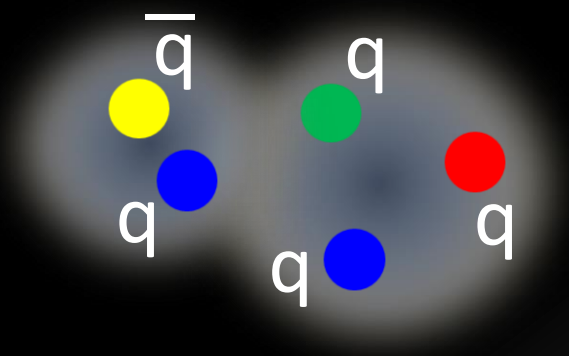
Hadron Physics at J-PARC



Quasi-Particles (= Effective DoF) emerging at Low E describe hadron properties effectively.

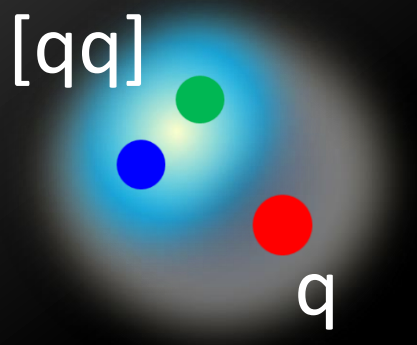
What are good building blocks of Hadrons?

Constituent Quark



hadron (colorless cluster)

Diquark?
(Colored cluster)



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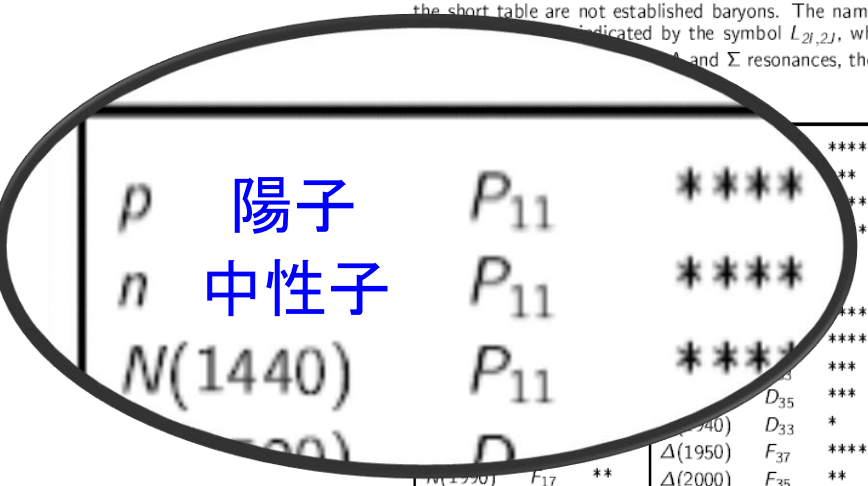
Classification of Hadrons

Observable	Relevant Physics Quantity	What we learn	
Mass Spectrum	Mass, Width (pole:$M_R - i\Gamma/2$)	Particle state Resonant state	Classification
Angular Correl. (decay)	Spin, Parity		

Baryon Family qqq

Baryon Summary Table

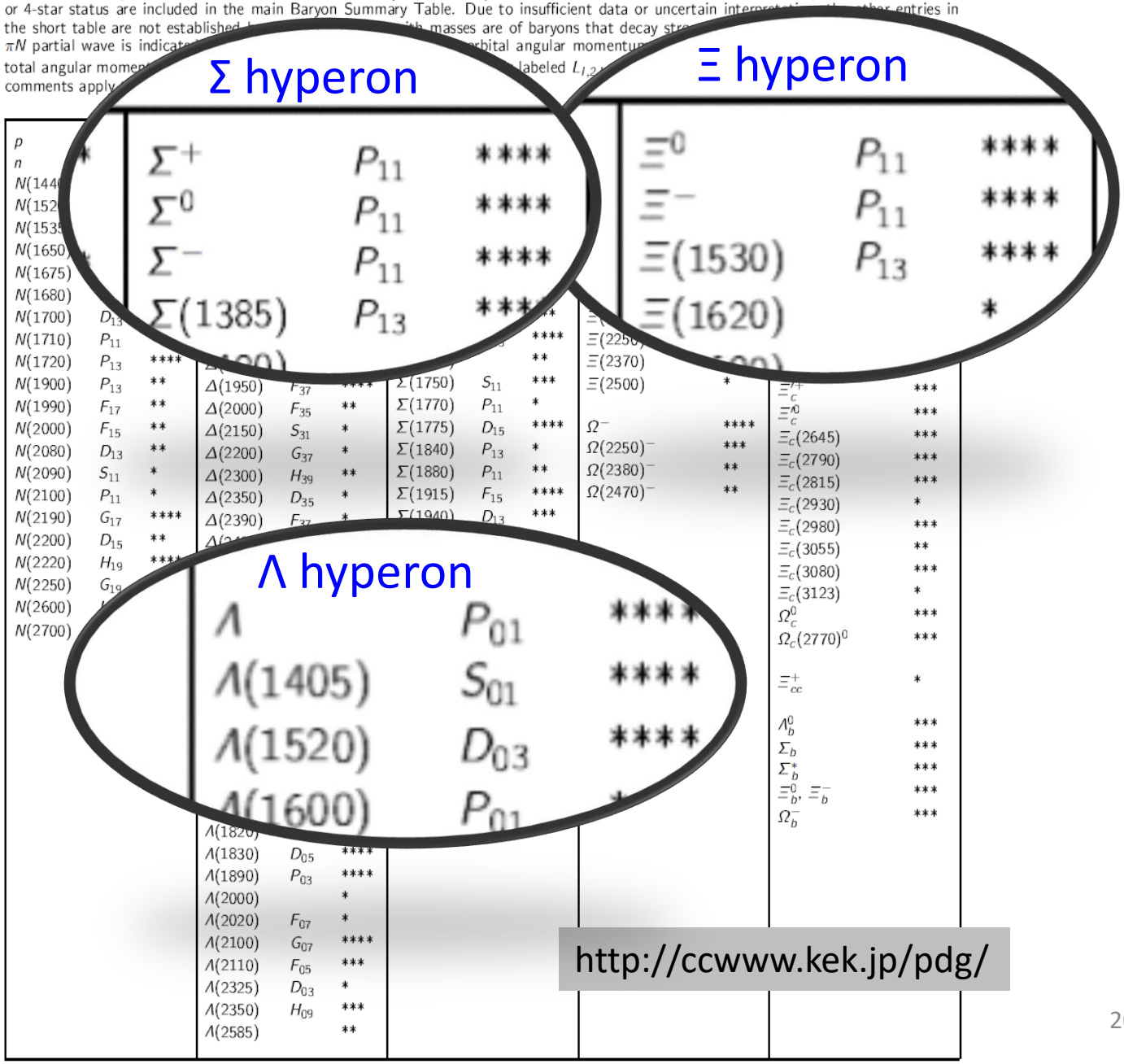
This short table gives the name, the quantum numbers (where known), and the status of baryons in the Review. Only the baryons with 3- or 4-star status are included in the main Baryon Summary Table. Due to insufficient data or uncertain interpretation, the other entries in the short table are not established baryons. The names with masses are of baryons that decay strongly. For N, Δ, and Ξ resonances, the ... indicated by the symbol $L_{2I,2J}$, where L is the orbital angular momentum (S, P, D, ...), I is the isospin, and J is the ... and Σ resonances, the $\overline{K}N$ partial wave is labeled $L_{I,2J}$. The nucleon is a pole in the P_{11} wave, and similar



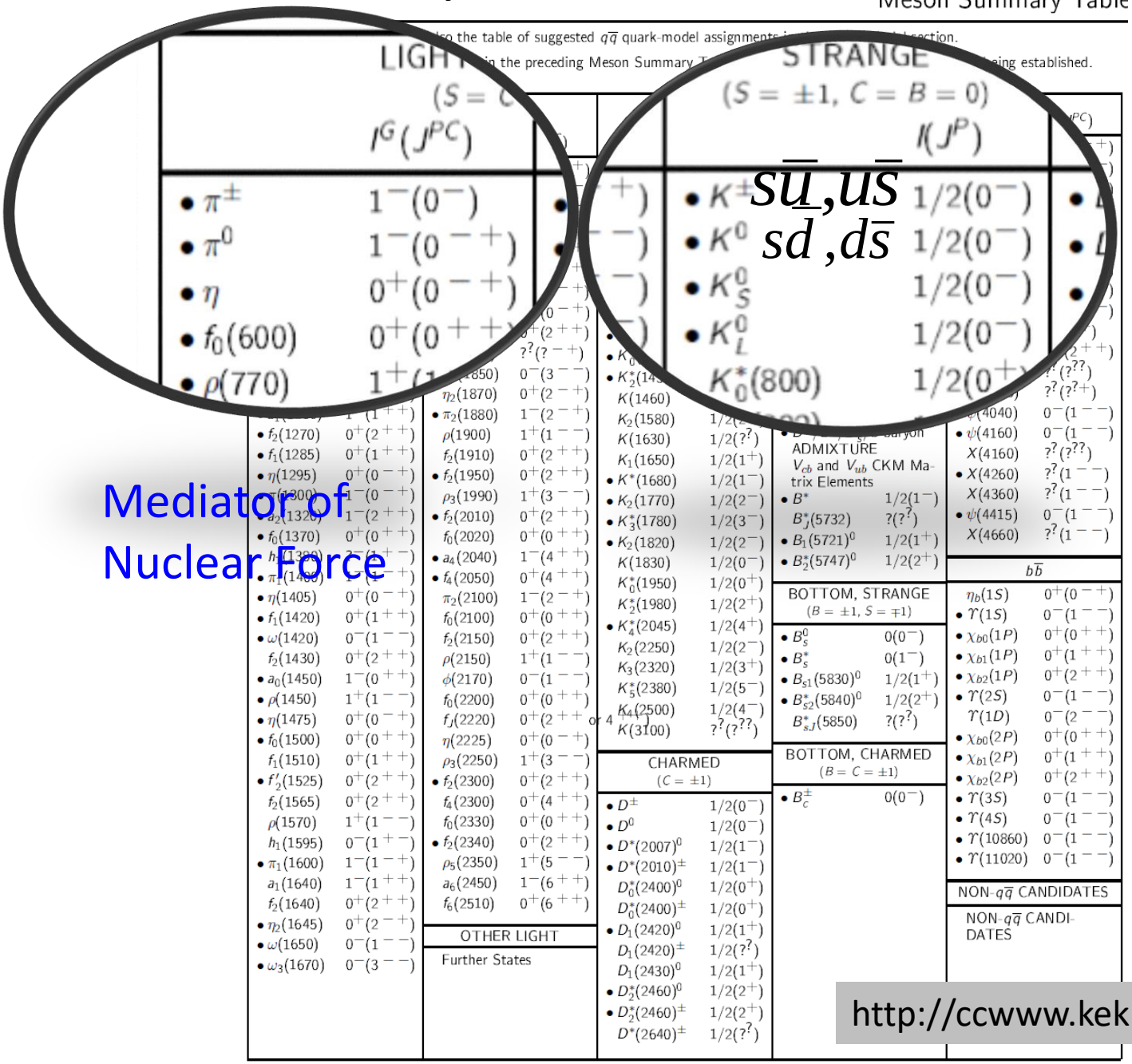
					****	Σ^+	P_{11}	****	Ξ^0	P_{11}	****	Λ_c^+	****
					**	Σ^0	P_{11}	****	Ξ^-	P_{11}	****	$\Lambda_c(2595)^+$	***
					**	Σ^-	P_{11}	****	$\Xi(1530)$	P_{13}	****	$\Lambda_c(2625)^+$	***
					*	$\Sigma(1385)$	P_{13}	****	$\Xi(1620)$		*	$\Lambda_c(2765)^+$	*
						$\Sigma(1480)$		*	$\Xi(1690)$		***	$\Lambda_c(2880)^+$	***
						$\Sigma(1560)$		**	$\Xi(1820)$	D_{13}	***	$\Lambda_c(2940)^+$	***
					***	$\Sigma(1580)$	D_{13}	*	$\Xi(1950)$		***	$\Sigma_c(2455)$	****
					****	$\Sigma(1620)$	S_{11}	**	$\Xi(2030)$		***	$\Sigma_c(2520)$	***
					***	$\Sigma(1660)$	P_{11}	***	$\Xi(2120)$		*	$\Sigma_c(2800)$	***
					***	$\Sigma(1670)$	D_{13}	****	$\Xi(2250)$		**	Ξ_c^+	***
					*	$\Sigma(1690)$		**	$\Xi(2370)$		**	Ξ_c^0	***
					****	$\Sigma(1750)$	S_{11}	***	$\Xi(2500)$		*	$\Xi_c'^+$	***
					**	$\Sigma(1770)$	P_{11}	*				$\Xi_c'^0$	***
					*	$\Sigma(1775)$	D_{15}	****	Ω^-		****	$\Xi_c(2645)$	***
					*	$\Sigma(1840)$	P_{13}	*	$\Omega(2250)^-$		***	$\Xi_c(2790)$	***
					**	$\Sigma(1880)$	P_{11}	**	$\Omega(2380)^-$		**	$\Xi_c(2815)$	***
					*	$\Sigma(1915)$	F_{15}	****	$\Omega(2470)^-$		**	$\Xi_c(2930)$	*
					**	$\Sigma(1940)$	D_{13}	***				$\Xi_c(2980)$	***
					*	$\Sigma(2000)$	S_{11}	*				$\Xi_c(3055)$	**
					**	$\Sigma(2030)$	F_{17}	****				$\Xi_c(3080)$	***
					**	$\Sigma(2070)$	F_{15}	*				$\Xi_c(3123)$	*
					**	$\Sigma(2080)$	P_{13}	**				Ω_c^0	***
					**	$\Sigma(2100)$	G_{17}	*				$\Omega_c(2770)^0$	***
						$\Sigma(2250)$		***					
						$\Sigma(2455)$		**				Ξ_{cc}^+	*
						$\Sigma(2620)$		**					
						$\Sigma(3000)$		*				Λ_b^0	***
						$\Sigma(3170)$		*				Σ_b	***
												Σ_b^*	***
												Ξ_b^0, Ξ_b^-	***
												Ω_b^-	***
$N(1990)$	F_{17}	**	$\Delta(1950)$	F_{37}	****								
$N(2000)$	F_{15}	**	$\Delta(2000)$	F_{35}	**								
$N(2080)$	D_{13}	**	$\Delta(2150)$	S_{31}	*								
$N(2090)$	S_{11}	*	$\Delta(2200)$	G_{37}	*								
$N(2100)$	P_{11}	*	$\Delta(2300)$	H_{39}	**								
$N(2190)$	G_{17}	****	$\Delta(2350)$	D_{35}	*								
$N(2200)$	D_{15}	**	$\Delta(2390)$	F_{37}	*								
$N(2220)$	H_{19}	****	$\Delta(2400)$	G_{39}	**								
$N(2250)$	G_{19}	****	$\Delta(2420)$	$H_{3,11}$	****								
$N(2600)$	$I_{1,11}$	***	$\Delta(2750)$	$I_{3,13}$	**								
$N(2700)$	$K_{1,13}$	**	$\Delta(2950)$	$K_{3,15}$	**								
			Λ	P_{01}	****								
			$\Lambda(1405)$	S_{01}	****								
			$\Lambda(1520)$	D_{03}	****								
			$\Lambda(1600)$	P_{01}	***								
			$\Lambda(1670)$	S_{01}	****								
			$\Lambda(1690)$	D_{03}	****								
			$\Lambda(1800)$	S_{01}	***								
			$\Lambda(1810)$	P_{01}	***								
			$\Lambda(1820)$	F_{05}	****								
			$\Lambda(1830)$	D_{05}	****								
			$\Lambda(1890)$	P_{03}	****								
			$\Lambda(2000)$		*								
			$\Lambda(2020)$	F_{07}	*								
			$\Lambda(2100)$	G_{07}	****								
			$\Lambda(2110)$	F_{05}	***								
			$\Lambda(2325)$	D_{03}	*								
			$\Lambda(2350)$	H_{09}	***								
			$\Lambda(2585)$		**								

http://ccwww.kek.jp/pdg/

Baryon Family qqq



Meson Family $\bar{q}q$



Mediator of Nuclear Force

http://ccwww.kek.jp/pdg/

In 1964 Quark Model

M. Gell-mann（Nobel Prize in 1969）

G. Zweig

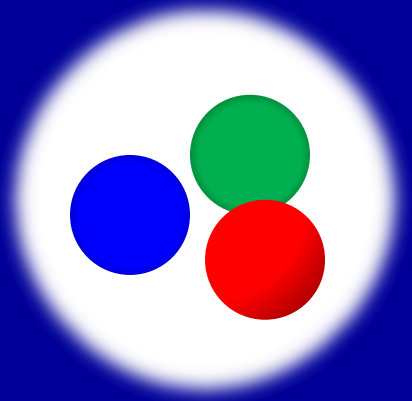
Introduced

Fractional electric charge

Color charge

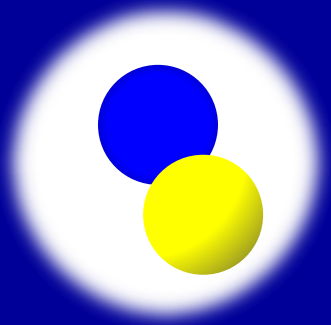
Quantum
Chromo
Dynamics

$q + q + q$



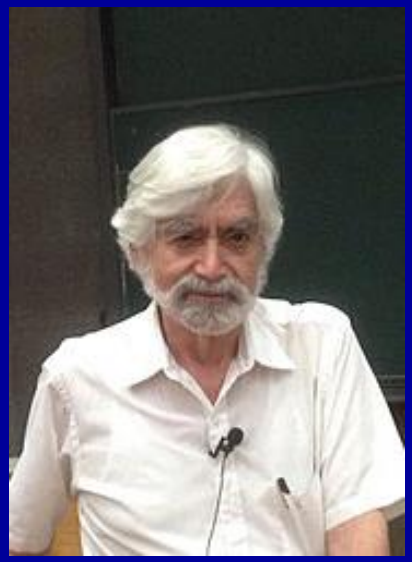
Baryon Family

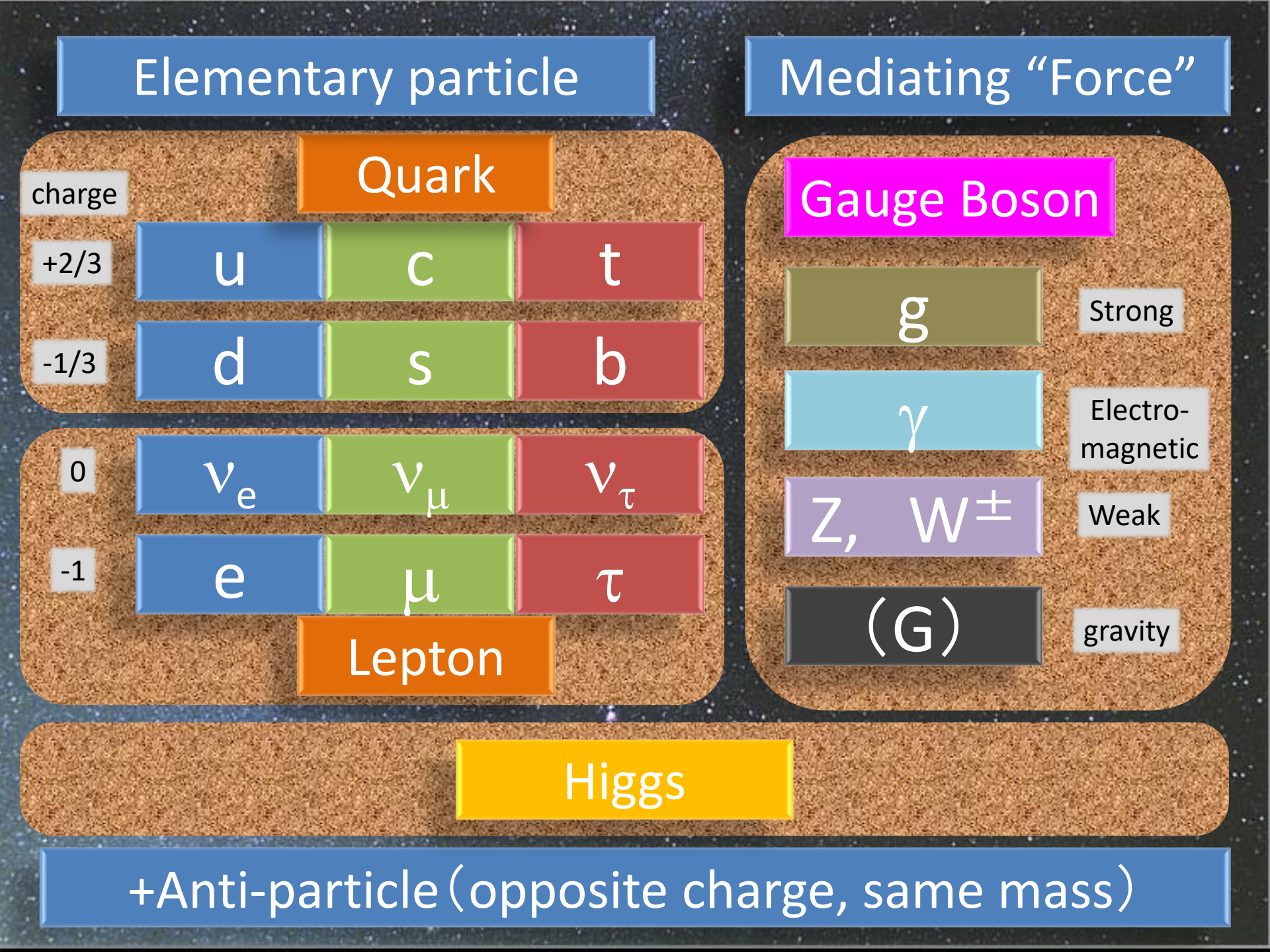
$q + \bar{q}$



Meson Family

Hadron




Elementary particle
Mediating “Force”
Quark
charge
+2/3
u
c
t
-1/3
d
s
b
0
ν_e
ν_μ
ν_τ
-1
e
μ
τ
Lepton
Gauge Boson
g
Strong
γ
Electro-magnetic
Z, W±
Weak
(G)
gravity
Higgs
+Anti-particle（opposite charge, same mass）

Quark Mass

http://www-pdg.lbl.gov/

u

$I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$

$m_u = 2.3^{+0.7}_{-0.5}$ MeV Charge $= \frac{2}{3}\,e$ $I_z = +\frac{1}{2}$
$m_u/m_d = 0.38$–0.58

d

$I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$

$m_d = 4.8^{+0.5}_{-0.3}$ MeV Charge $= -\frac{1}{3}\,e$ $I_z = -\frac{1}{2}$
$m_s/m_d = 17$–22
$\overline{m} = (m_u+m_d)/2 = 3.5^{+0.7}_{-0.2}$ MeV

s

$I(J^P) = 0(\frac{1}{2}^+)$

$m_s = 95 \pm 5$ MeV Charge $= -\frac{1}{3}\,e$ Strangeness $= -1$
$m_s\ /\ ((m_u + m_d)/2) = 27.5 \pm 1.0$

c

$I(J^P) = 0(\frac{1}{2}^+)$

$m_c = 1.275 \pm 0.025$ GeV Charge $= \frac{2}{3}\,e$ Charm $= +1$

b

$I(J^P) = 0(\frac{1}{2}^+)$

Charge $= -\frac{1}{3}\,e$ Bottom $= -1$

$m_b(\overline{\text{MS}}) = 4.18 \pm 0.03$ GeV
$m_b(1S) = 4.66 \pm 0.03$ GeV

Quarks forming proton and neutron

t

$I(J^P) = 0(\frac{1}{2}^+)$

Charge $= \frac{2}{3}\,e$ Top $= +1$

Mass (direct measurements) $m = 173.21 \pm 0.51 \pm 0.71$ GeV [a,b]
Mass ($\overline{\text{MS}}$ from cross-section measurements) $m = 160^{+5}_{-4}$ GeV [a]
Mass (Pole from cross-section measurements) $m = 176.7^{+4.0}_{-3.4}$ GeV
$m_t - m_{\overline{t}} = -0.2 \pm 0.5$ GeV (S = 1.1)
Full width $\Gamma = 2.0 \pm 0.5$ GeV
$\Gamma(Wb)/\Gamma(Wq(q = b, s, d)) = 0.91 \pm 0.04$

PDGのクォークのページの最後

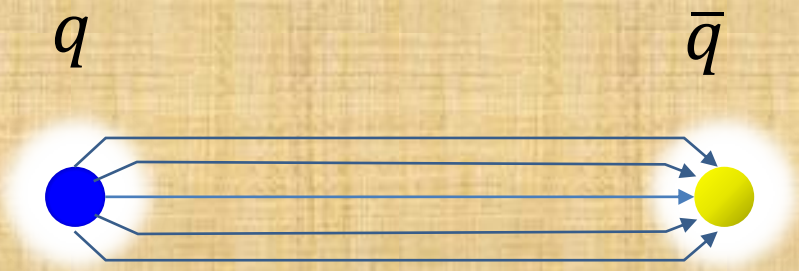
Free Quark Searches

All searches since 1977 have had negative results.

- Quarks are confined in hadron

Why are quarks confined ?

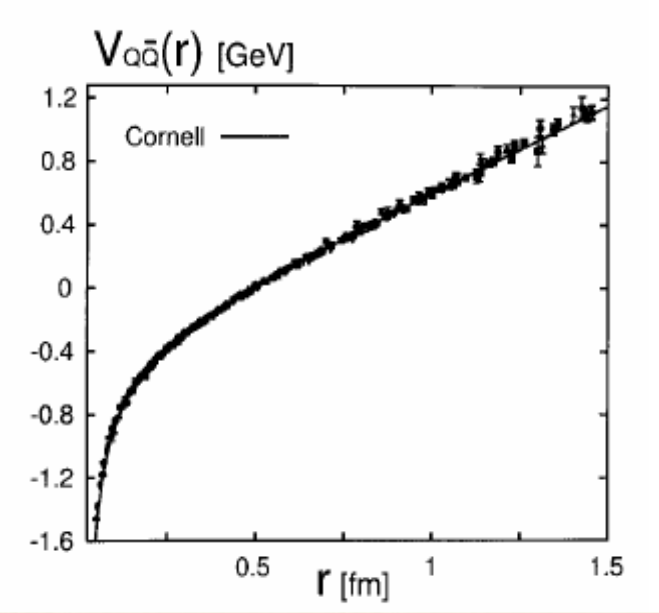
- Force independent in a distance (Linear Potential)
- Coulomb-like in a close distance (1/r Potential)

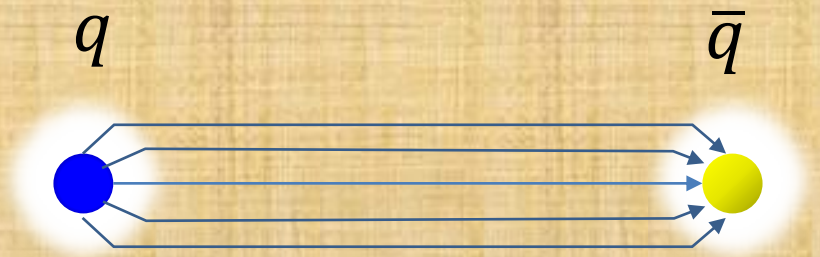


- Cornel Potential: V=ar-b/r

How are quarks confined ?

- Force independent in a distance (Linear Potential)
- Coulomb-like in a close distance (1/r Potential)

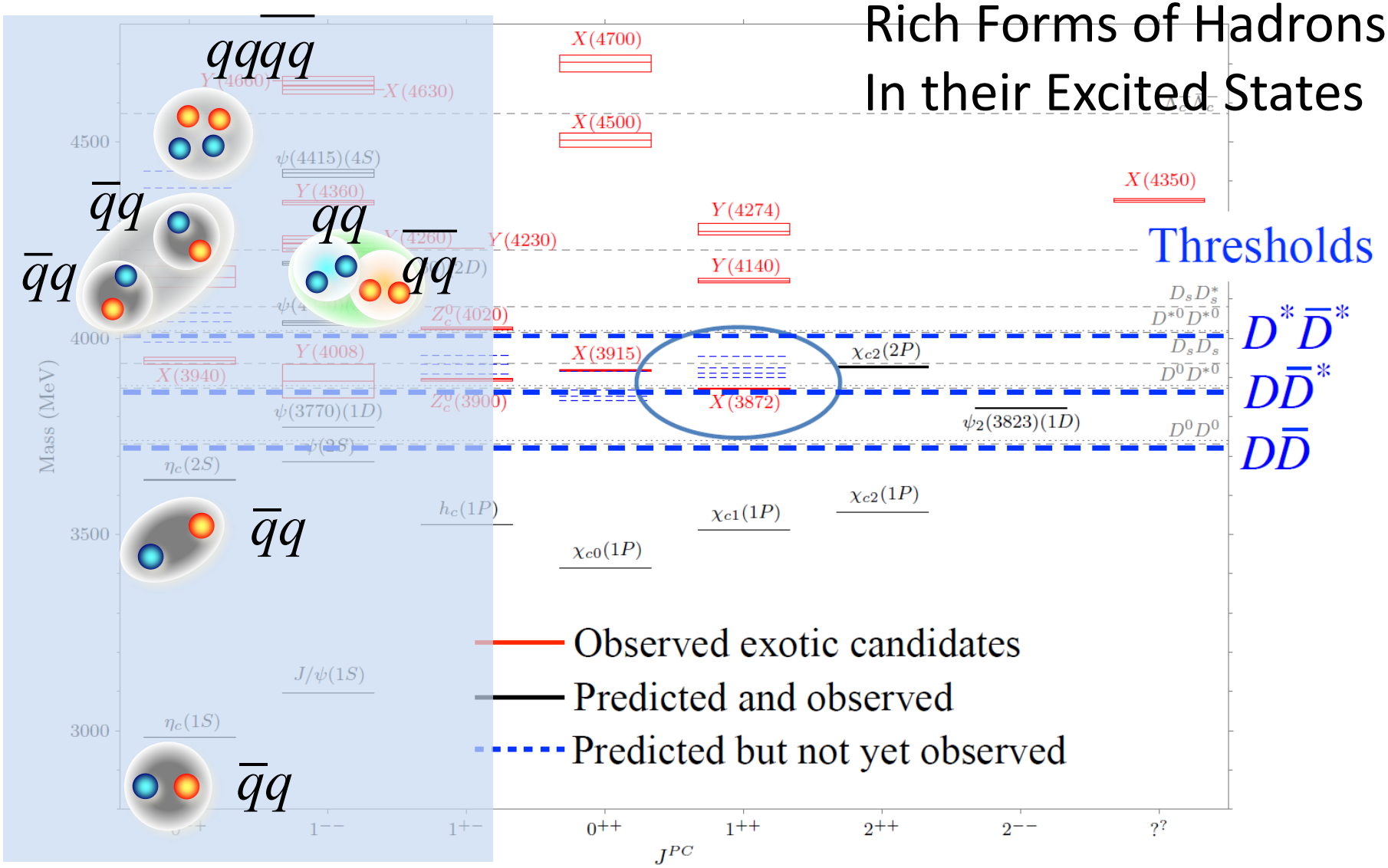




- Cornell Potential: V=ar-b/r

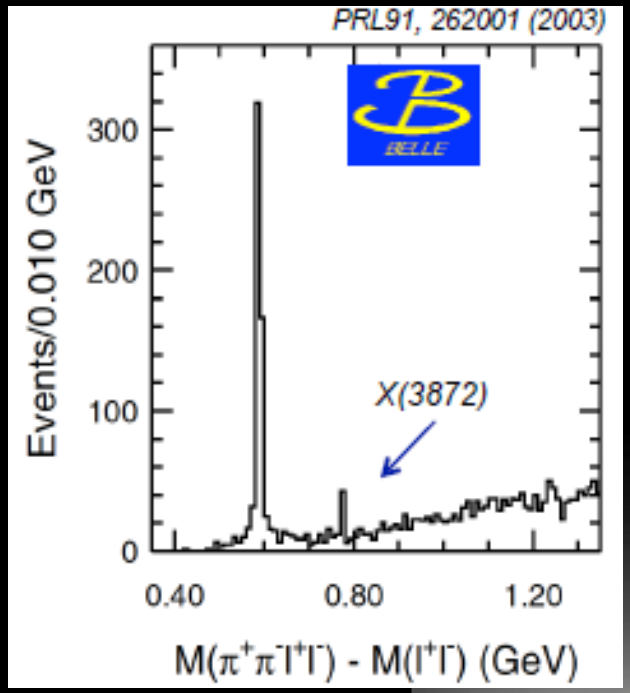
Neutral X, Y, Z_0 states

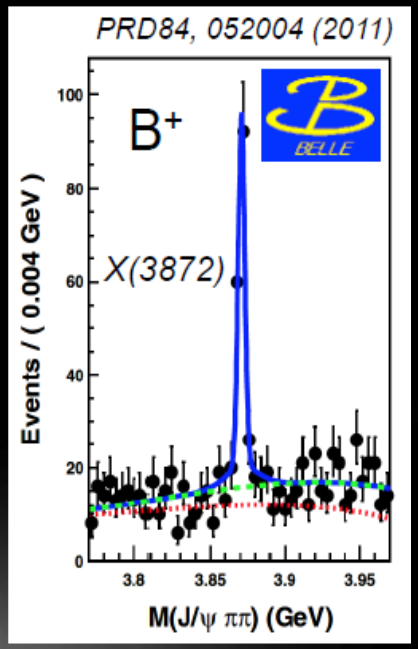
Rich Forms of Hadrons
In their Excited States

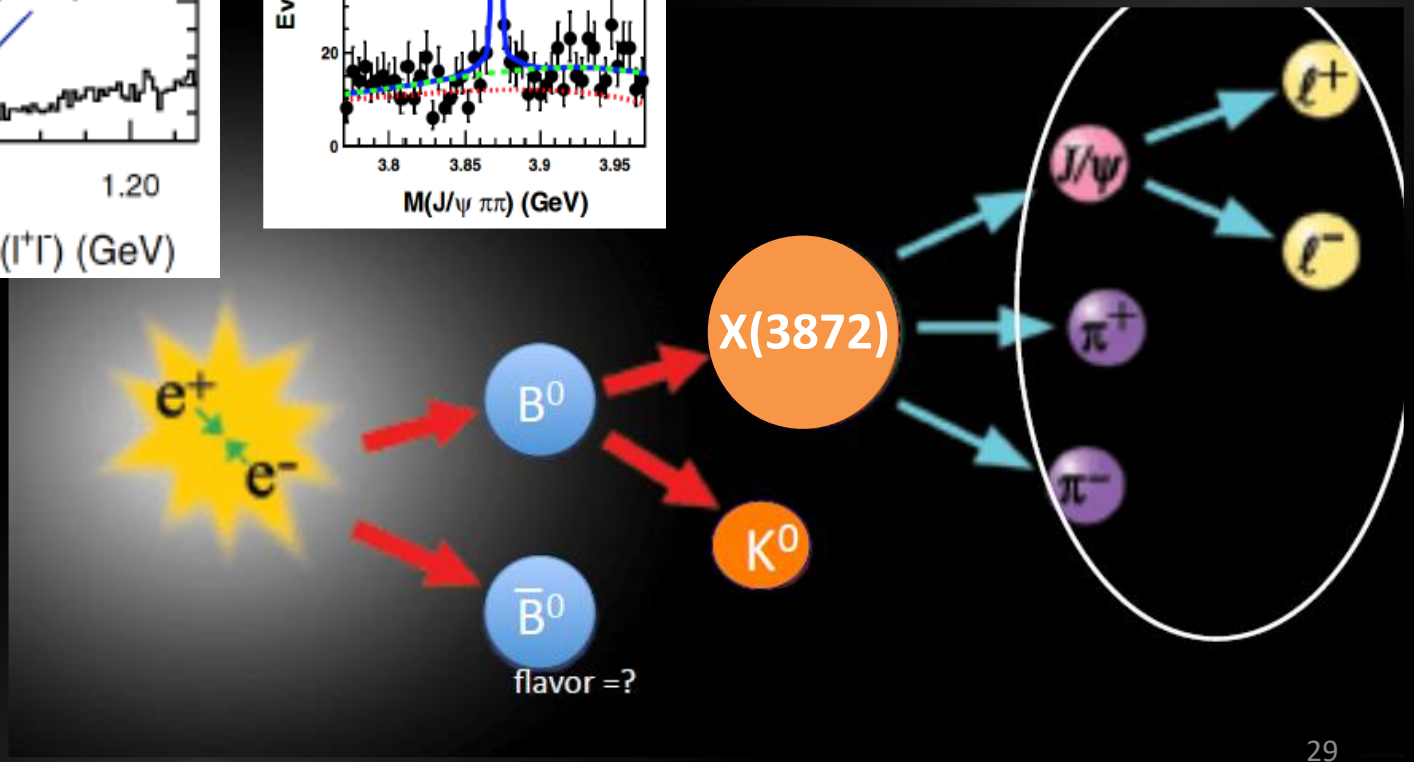


Slides from Prof. Hosaka

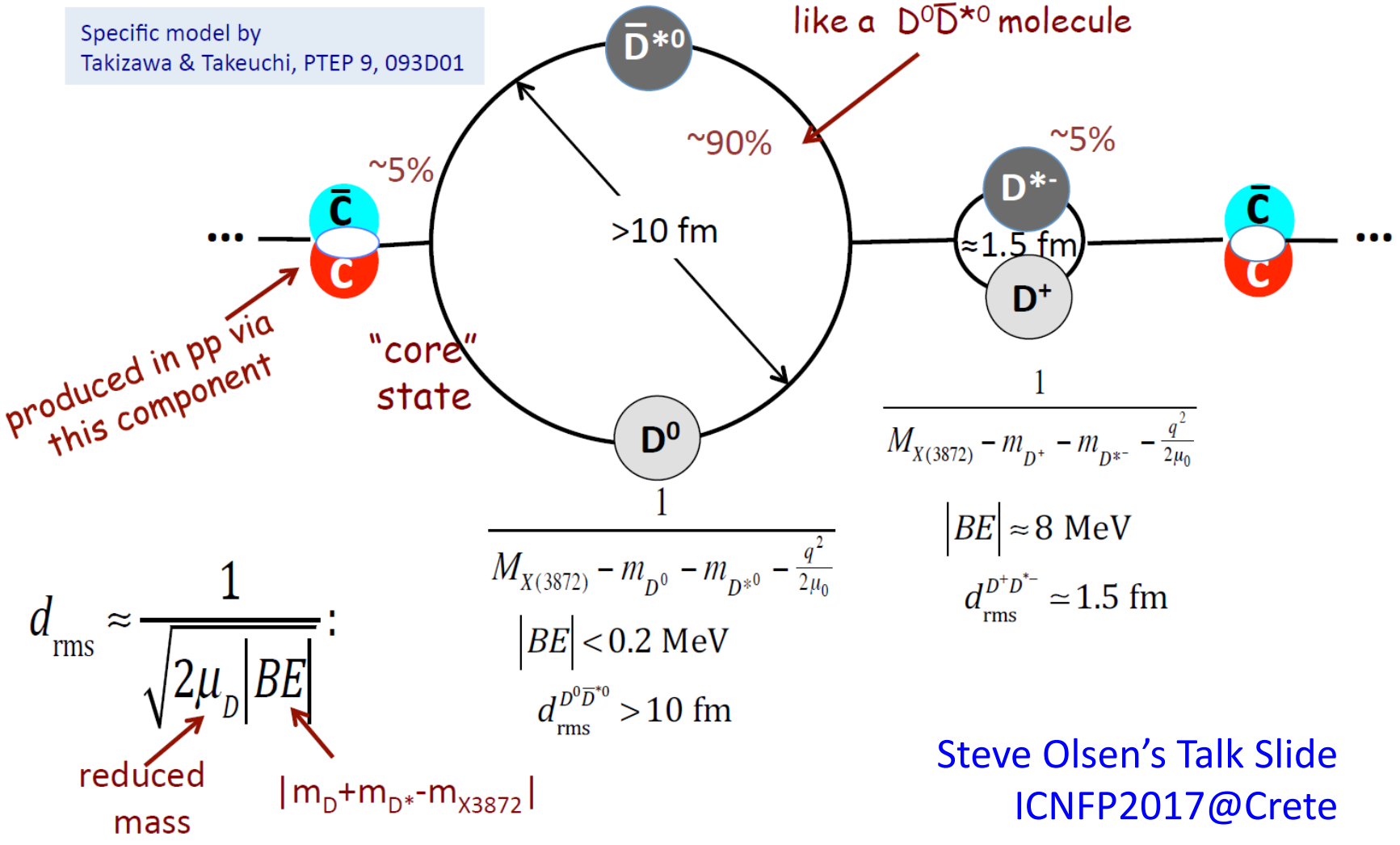
X(3872)


PRL91, 262001 (2003)
Events/0.010 GeV
300
200
100
0
X(3872)
0.40
0.80
1.20
M(π⁺π⁻l⁺l⁻) - M(l⁺l⁻) (GeV)


PRD84, 052004 (2011)
B⁺
X(3872)
Events / (0.004 GeV)
100
80
60
40
20
0
3.8
3.85
3.9
3.95
M(J/ψ ππ) (GeV)


e+
e−
B0
B̄0
flavor =?
X(3872)
K0
J/ψ
ℓ+
ℓ−
π+
π−

X(3872): Consensus?



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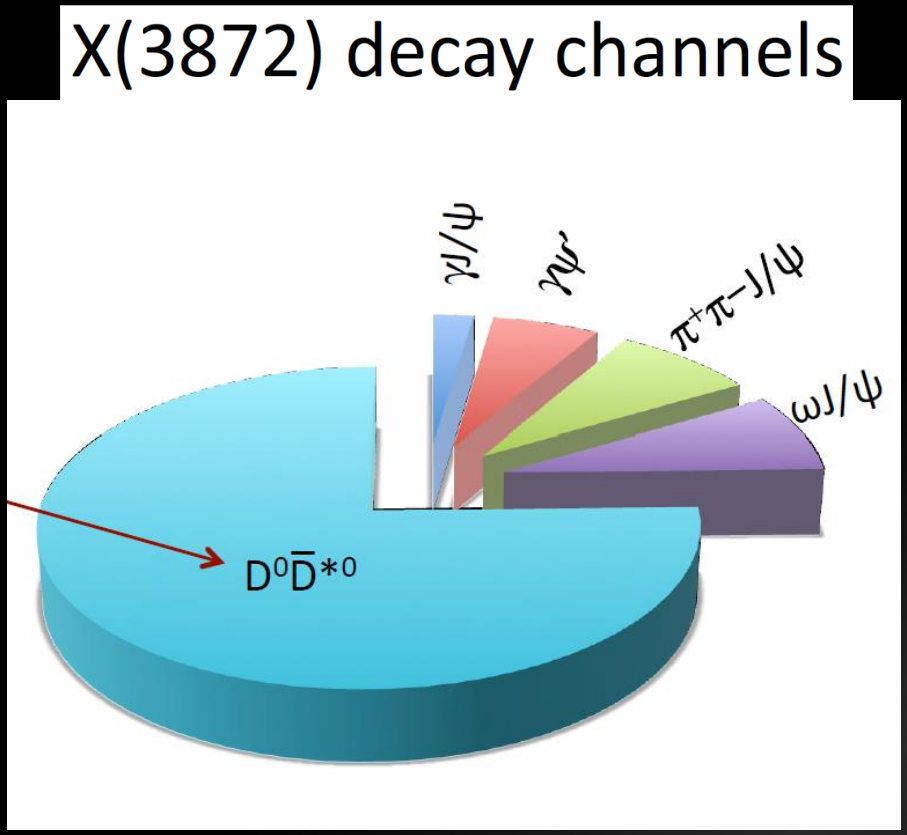
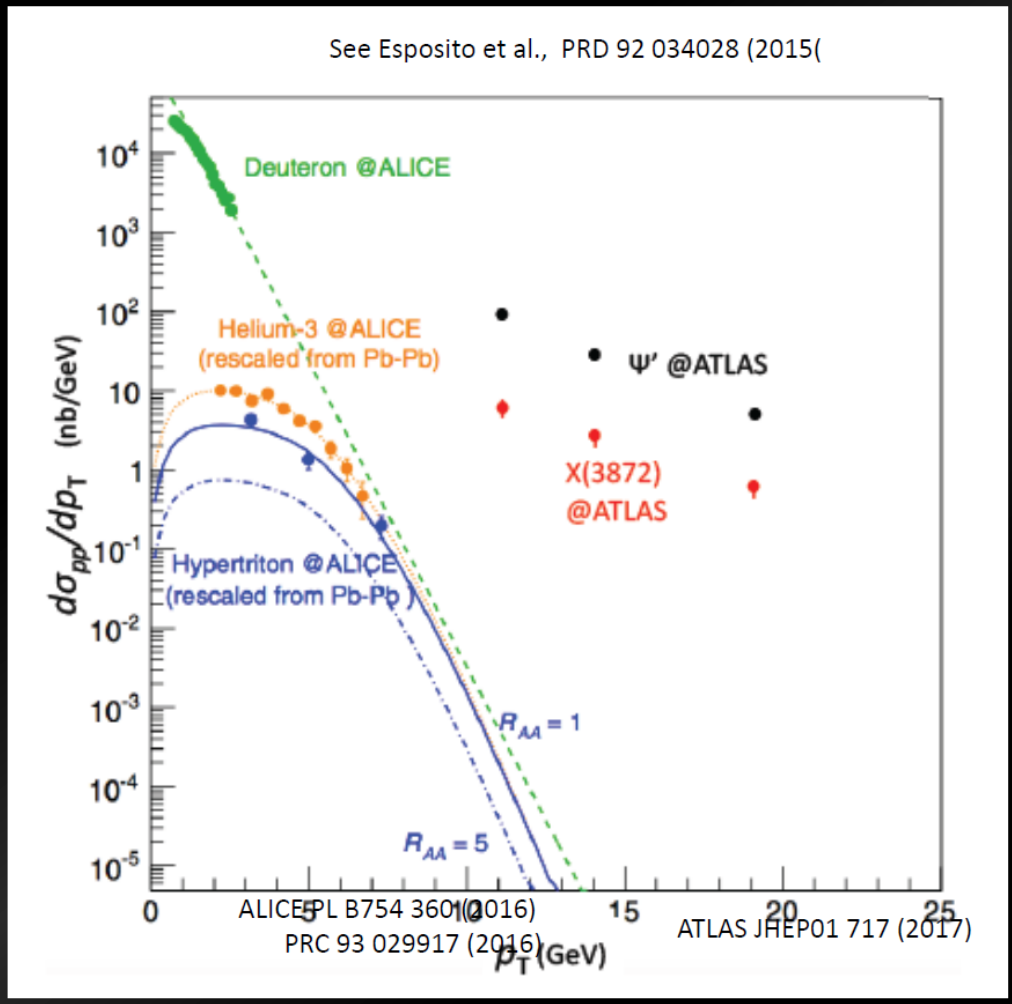
Coupled to DD*, Produced like Y’





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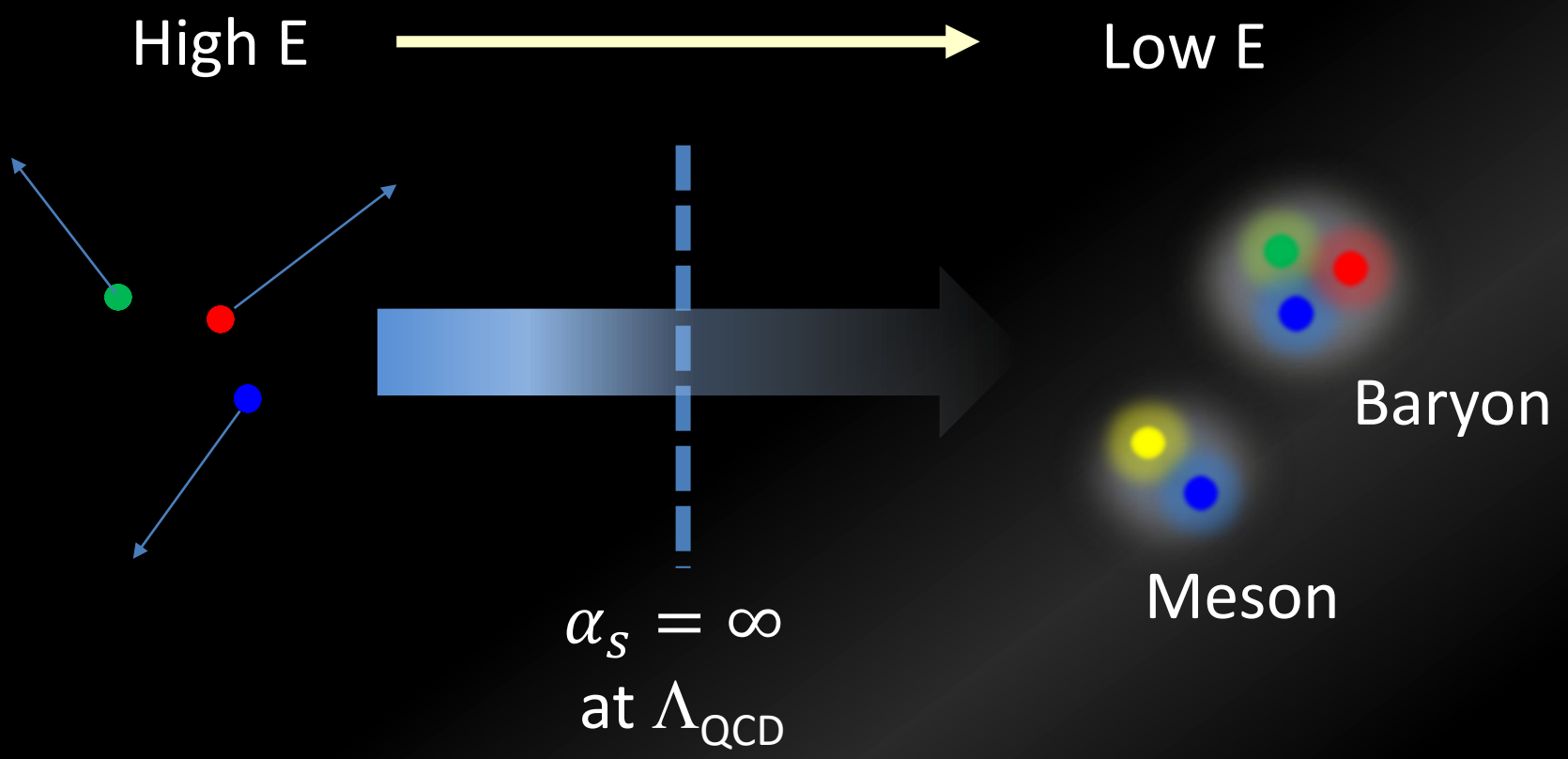
Form of Hadrons

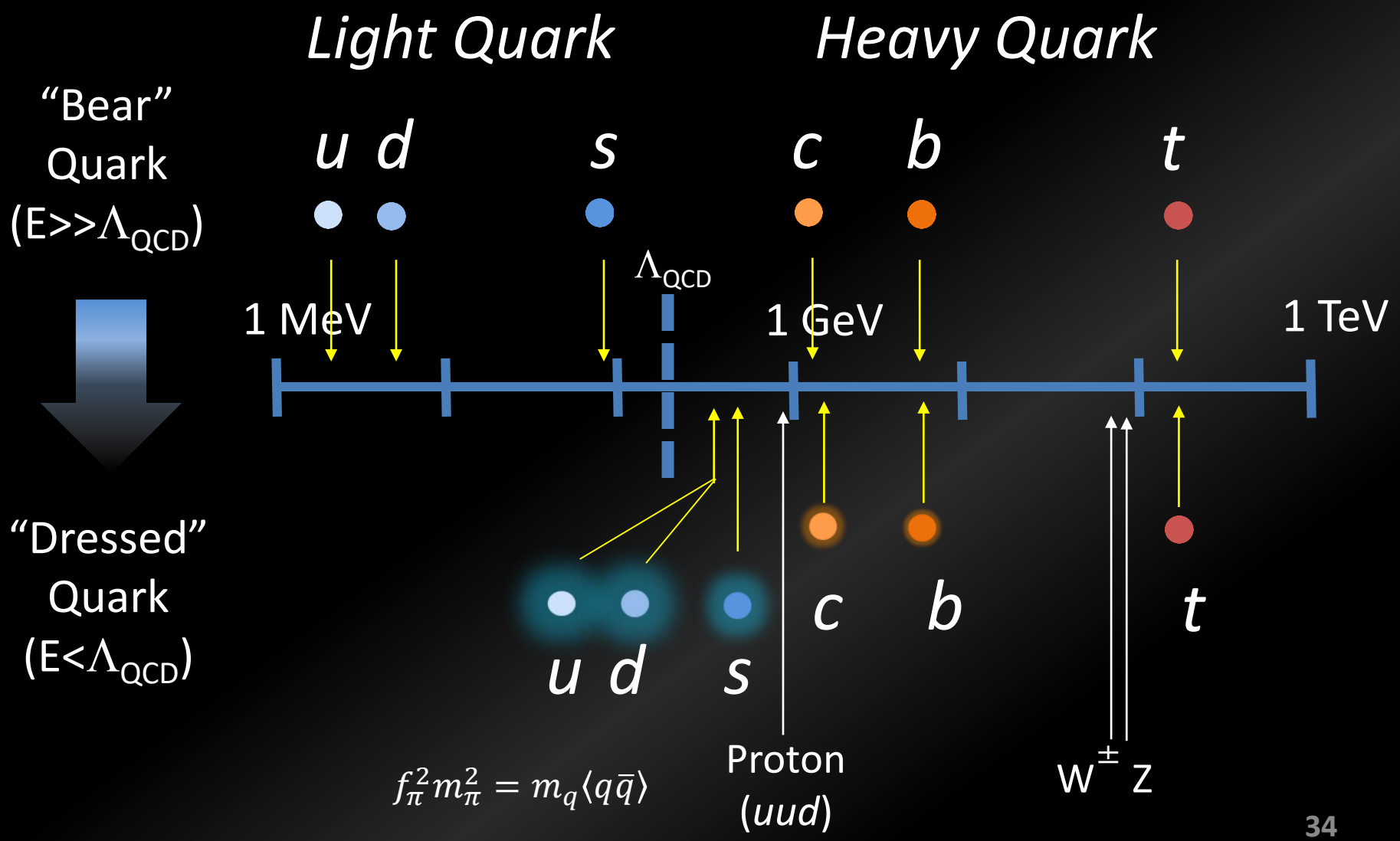
Observable	Relevant Physics Quantity	What we learn	
Mass Spectrum	Mass, Width (pole:$M_R - i\Gamma/2$)	Particle state Resonant state	Classification
Angular Correl. (decay)	Spin, Parity		
Level structure		Internal (effective) DoF	Form (Dynamics of effective DoF in Hadron)
Production Rate (Diff. Cross Sect.)	Response Function (Transition) Form Factor	Reaction Mechanism Internal Motion/Corr.	
Partial Width	Internal Correlation (Wave function)	Decay Mechanism Internal Motion/Corr.	

How Hadrons are formed?



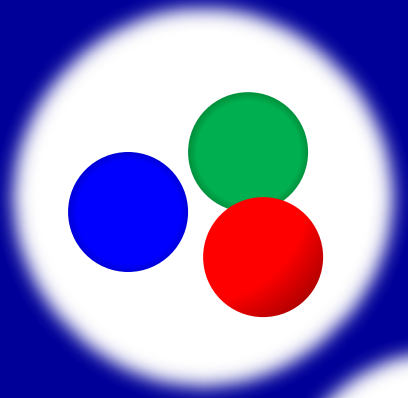
Quarks drastically change themselves below Λ_{QCD}.

Quark Mass

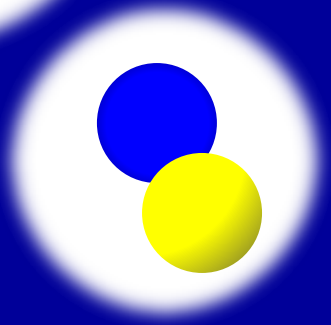

Light Quark
Heavy Quark
"Bear" Quark (E>>Λ_{QCD})
u d
s
c
b
t
Λ_{QCD}
1 MeV
1 GeV
1 TeV
"Dressed" Quark (E<Λ_{QCD})
u d
s
c
b
t
Proton (uud)
W± Z
$f_\pi^2 m_\pi^2 = m_q \langle q\bar{q} \rangle$

物質の質量の起源のなぞ

1965年 「カイラル対称性の自発的破れ」提唱
南部陽一郎による。
ある対称性が破れることによってもともと
質量がなかった粒子に質量が生じる機構を
解明。粒子に働く相互作用によって対称性
が破れ、同時に質量が生じる。



ハドロン



ハドロン（陽子や中性子など）形成のなぞ：
クォーク間の強い相互作用によって、
真空中にqと反qが凝縮して質量を獲
得。同時に、自身の相互作用でクォーク
はハドロン内部に閉じ込められる。
クォークが単独で取り出せない理由。

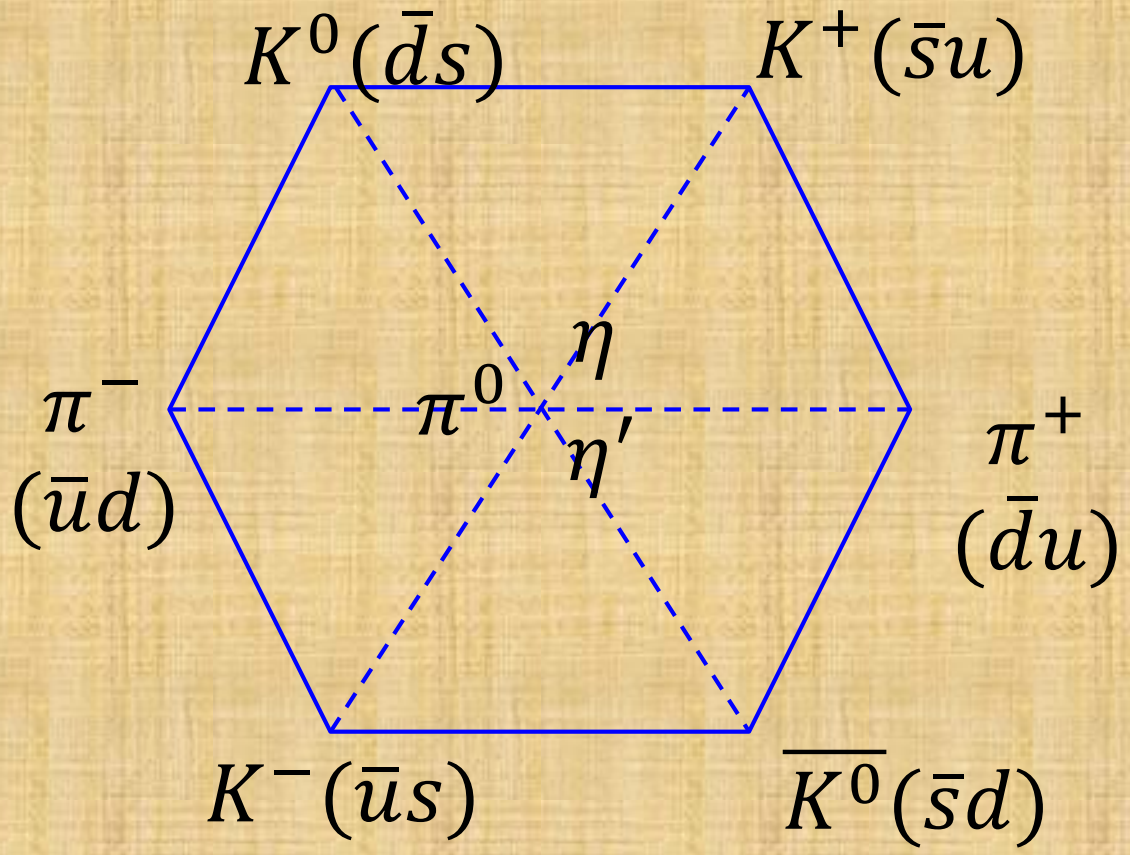
メカニズムの検証はまだ不十分

Constituent Quark Model

- Successful to describe nature of hadrons in the ground state
  - Mass
  - Spin-Isospin(flavor) classification
  - Magnetic moment of baryons
- Sometimes fails in excited state
  - Missing resonance problem
  - Exotic states
- Provides a Good Guide Line to insight Hadrons

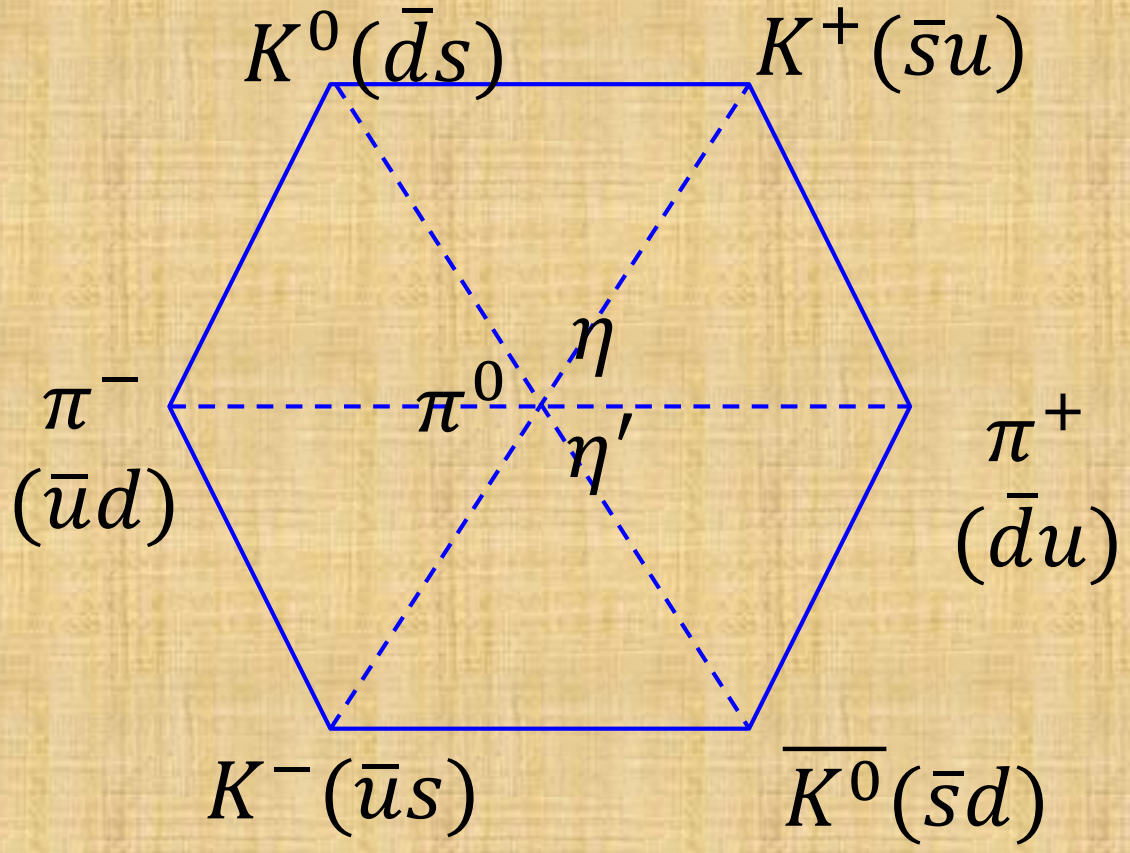
Classification of Light Meson $SU_F(3)$

- $\bar{q}q \sim (\bar{u}, \bar{d}, \bar{s}) \otimes (u, d, s)$



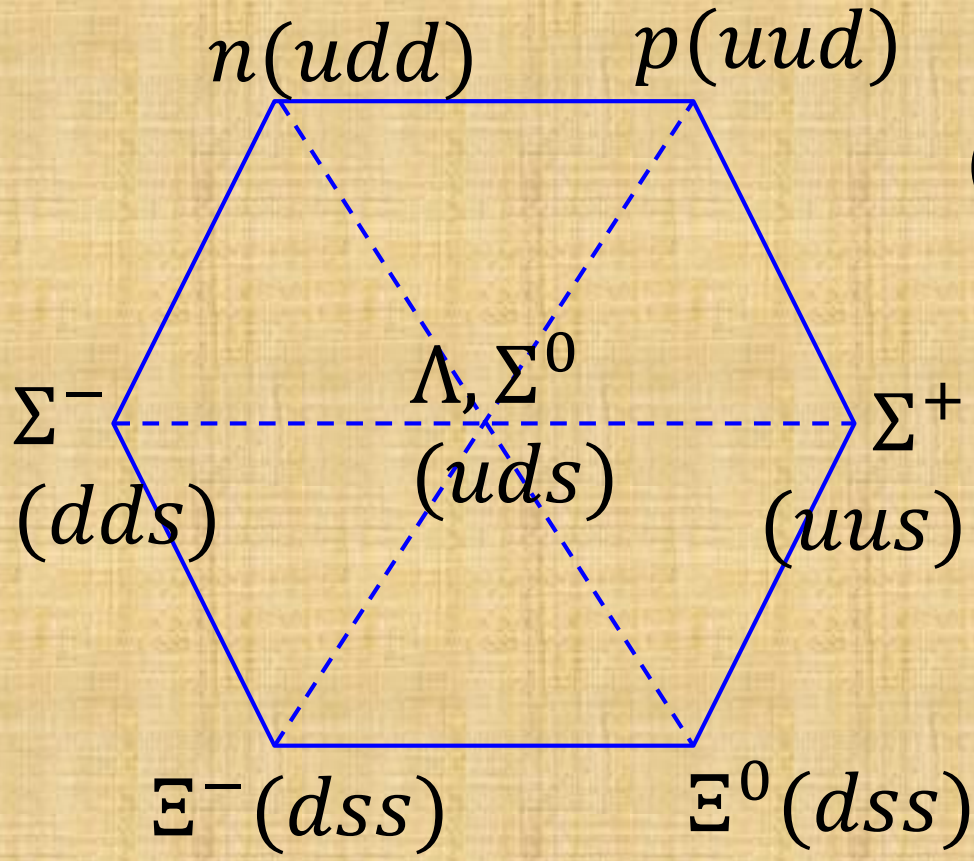
Classification of Light Meson $SU_F(3)$

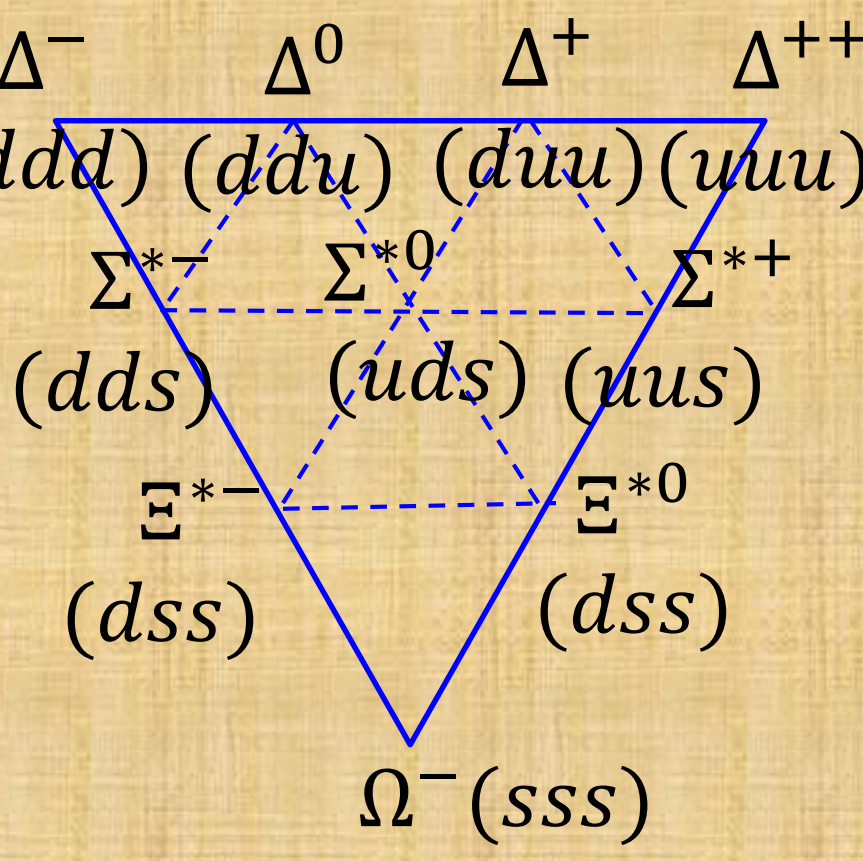
- $\bar{3} \otimes 3 = 1 \oplus 8$



Classification of Light Baryon $SU_F(3)$

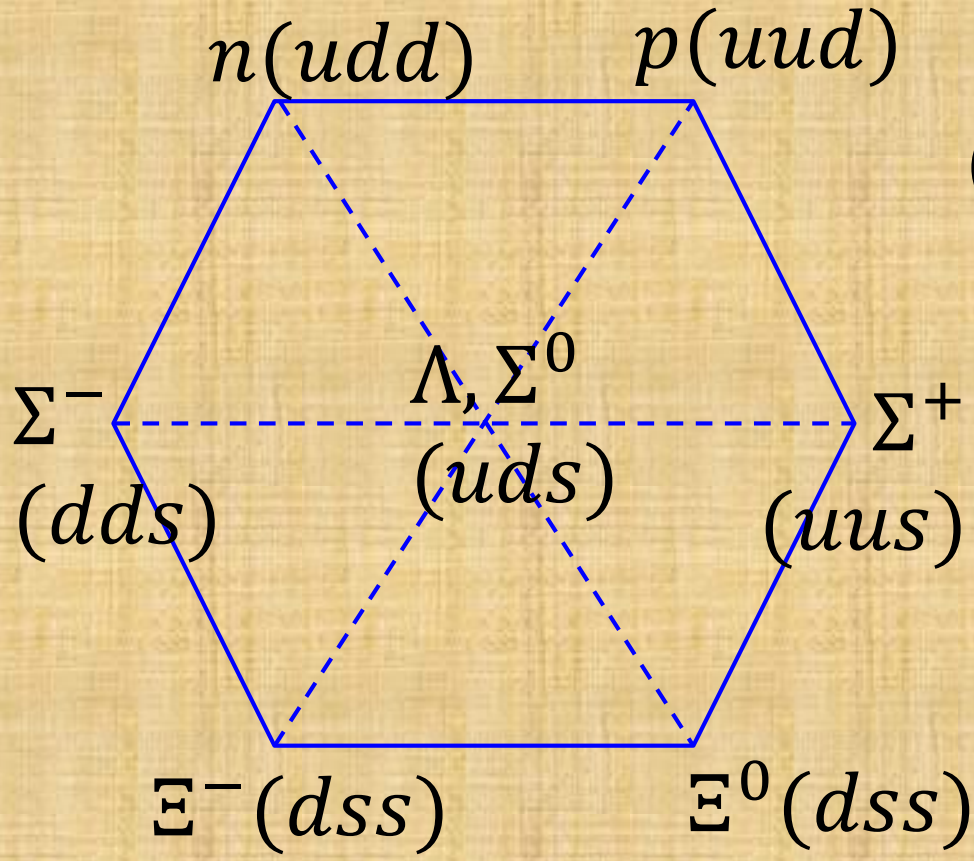
- $qqq \sim (u,d,s) \otimes (u,d,s) \otimes (u,d,s)$

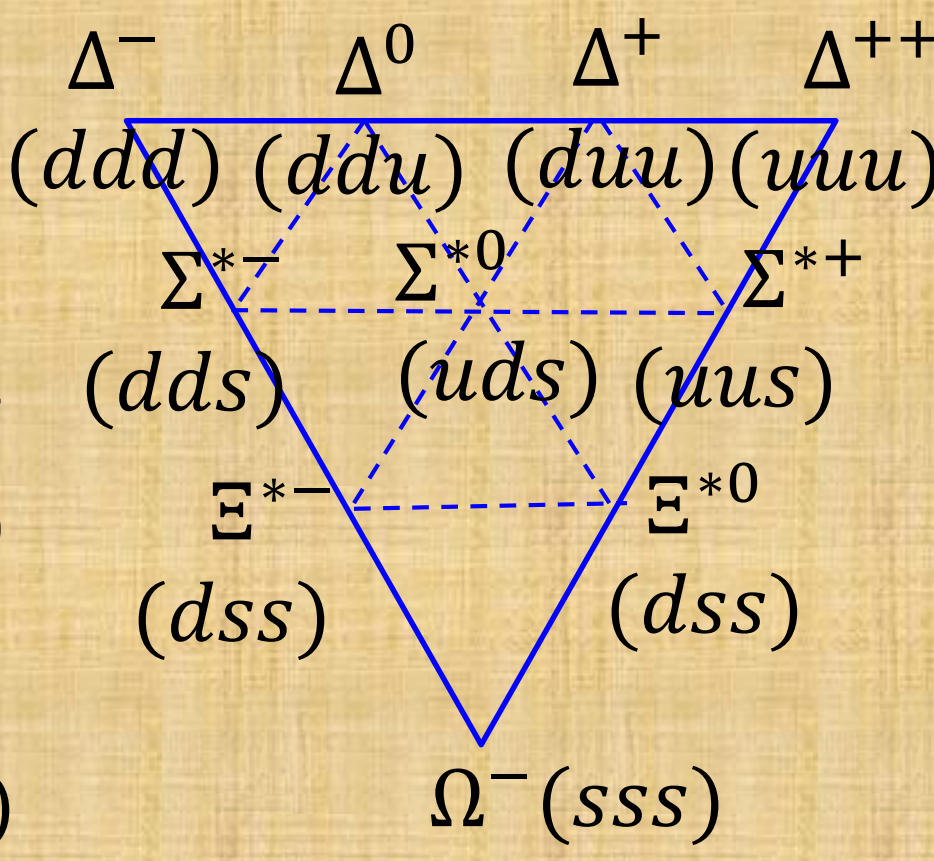




Classification of Light Baryon $SU_F(3)$

- $3 \otimes 3 \otimes 3 = 10 \oplus 8 \oplus 8 \oplus 1$



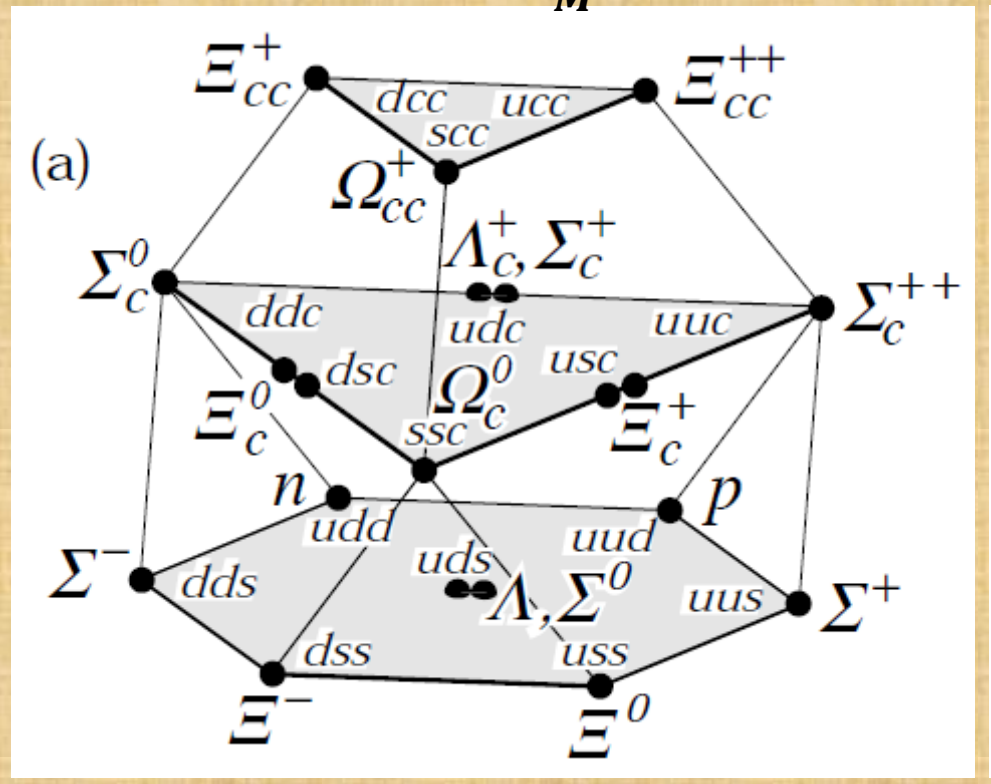


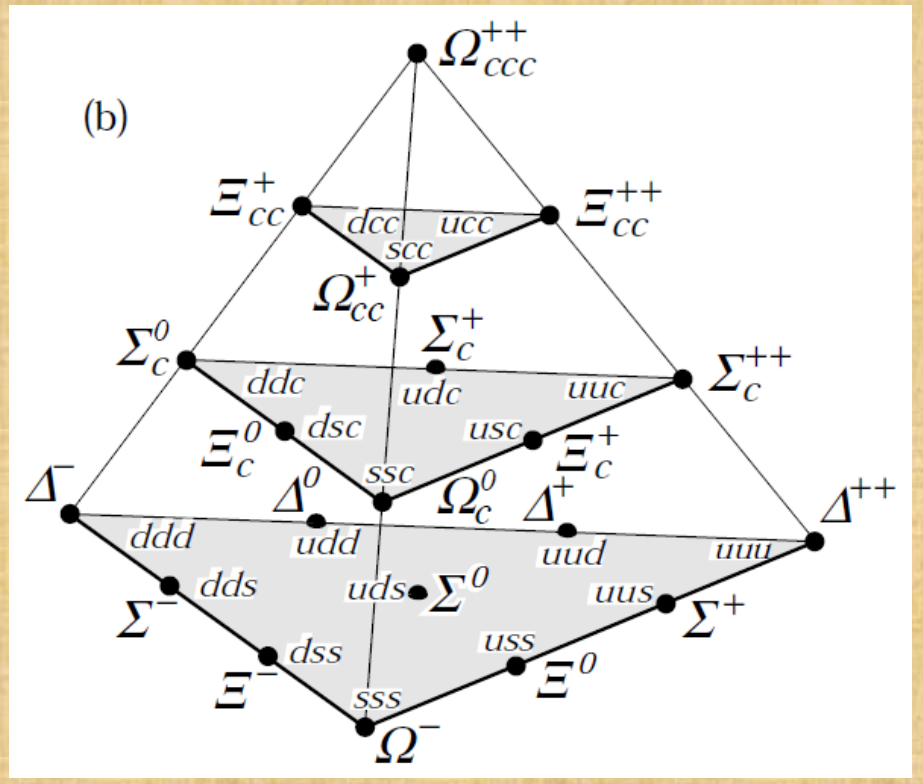
Classification of Light Baryon

$SU_F(4)$: (heavily broken due to $m_c \gg m_{u,d,s}$)

- $qqq \sim (u,d,s,c) \otimes (u,d,s,c) \otimes (u,d,s,c)$

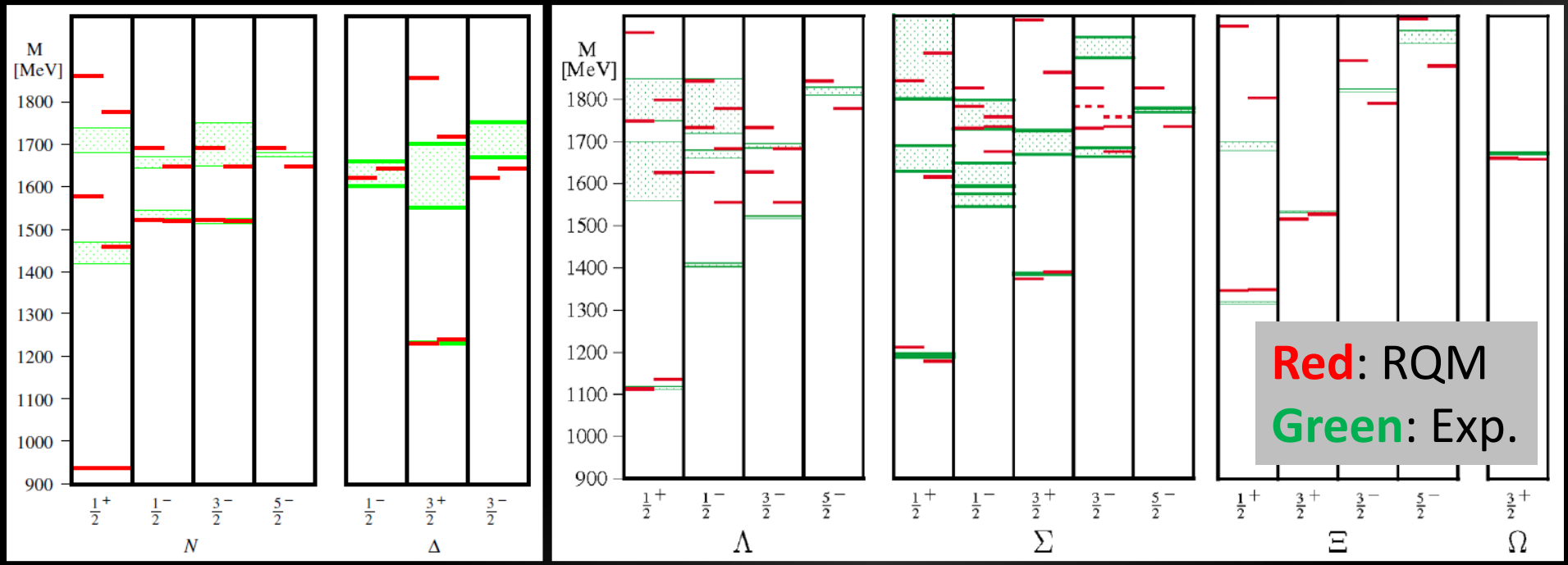
$\mathbf{20_M}$ $\mathbf{20_S}$





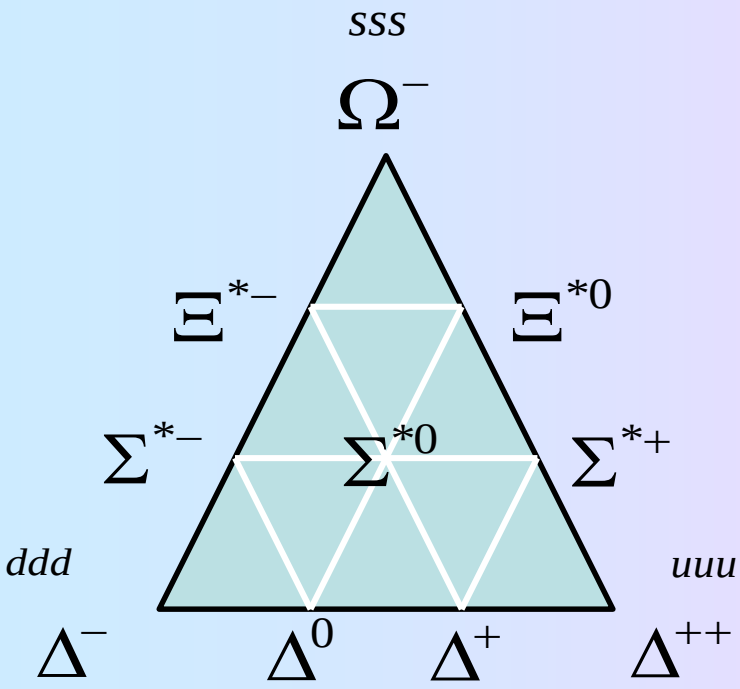
Relativistic Quark Model

T. Melde, W. Plessas, and B. Sengl
Phys. Rev. D77, 114002(2008)



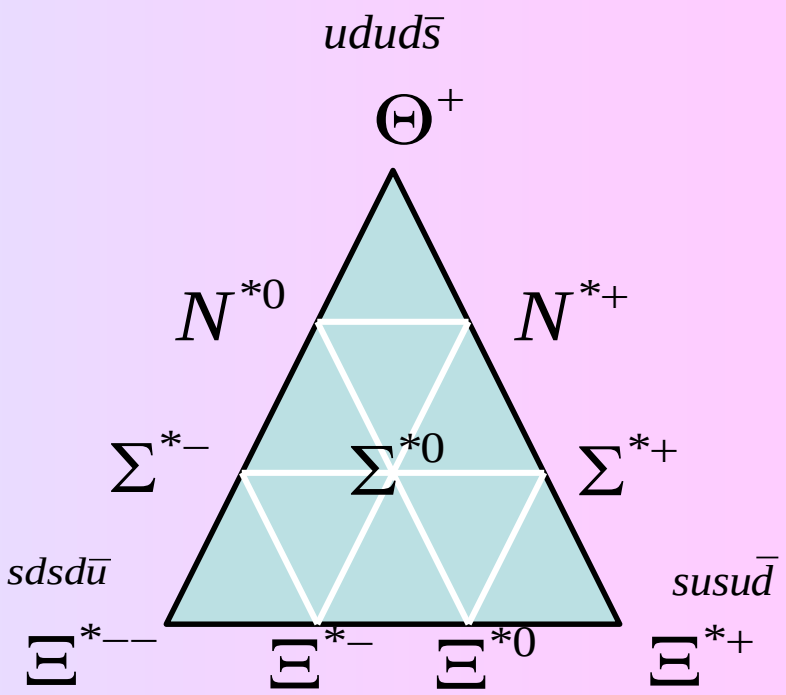
- unobserved states
- unexpected states

Exotic Baryon: Θ^+



$$10$$

qqq



$$\overline{10}$$

$qqqq\bar{q}$

Magnetic Moment of Octet Baryons

✓ Constituent Quark Model Picture works well!

In the framework of spin-flavor SU(6)

$$\mu_B = \langle B | \hat{\mu}_z | B \rangle$$

$$\hat{\mu}_z = \sum_q q_q s_z^q \hbar / m_q$$

Only 3 parameters: m_u, m_d, m_s

✓ Spontaneous χ Symmetry Breaking

Current quark:m_q~0 -> Constituent quark:m_q~M_N/3

Partial restoration of χ Sym. in finite density?

Magnetic Moment

- μ of Dirac particle ∝ spin × charge/mass

proton

μ_N

$$\mu_N = \frac{e\hbar}{2M_N c}$$

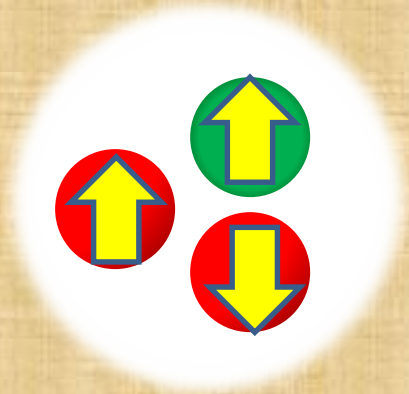
neutron

0

Magnetic Moment

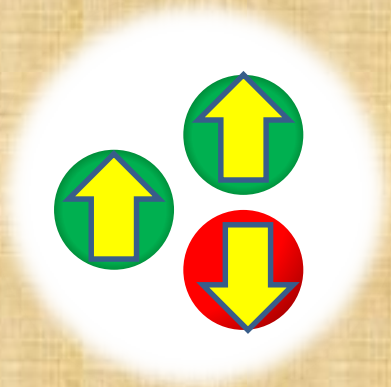
- μ of Dirac particle $\propto$ spin $\times$ charge/mass

proton



$2.79\mu_N$

neutron

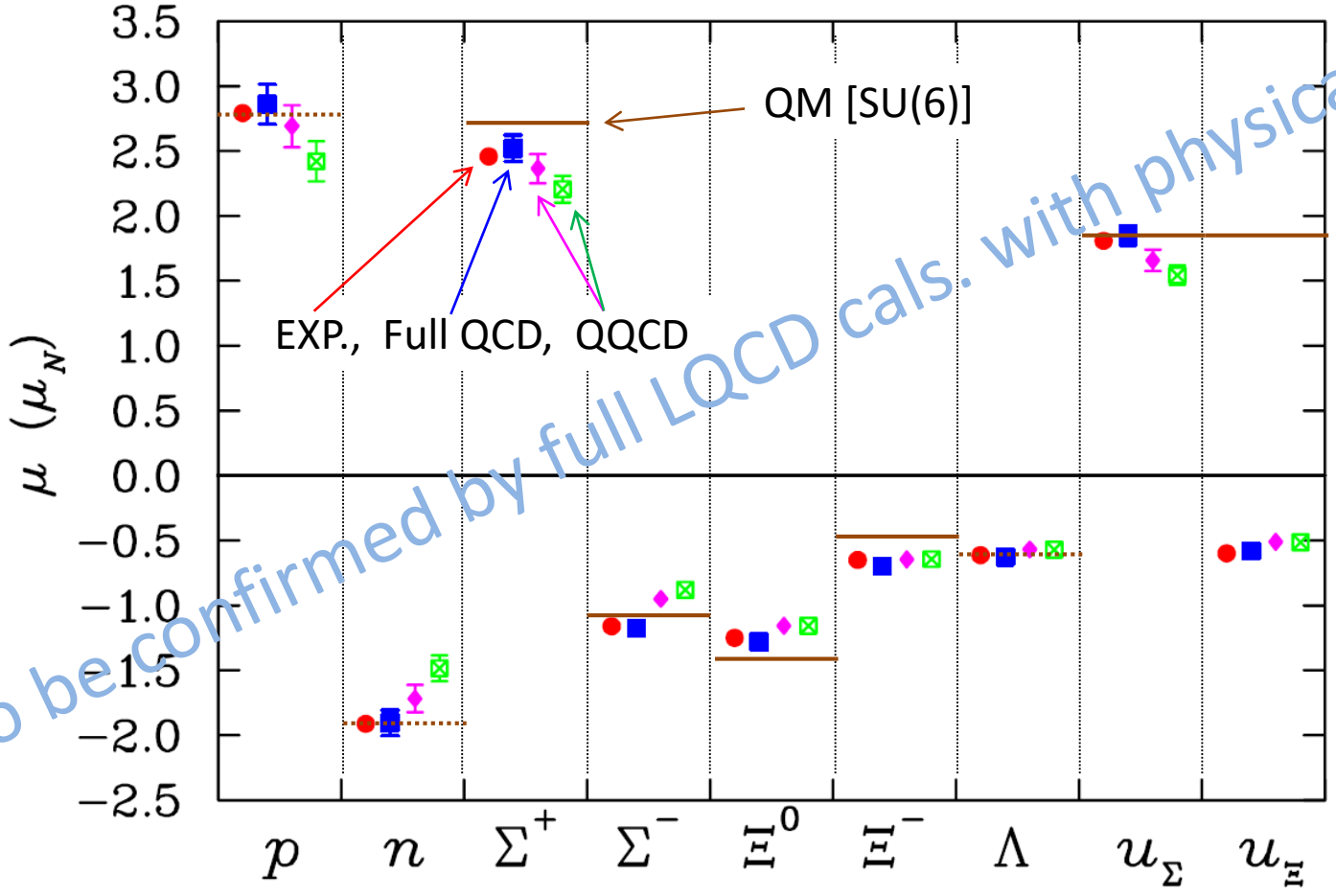


$-1.91\mu_N$

- CQM explains the measured values of nucleons

Magnetic Moment of Octet Baryons

Analytically deduced, based on Lattice QCD data
In cooperation with ChPT



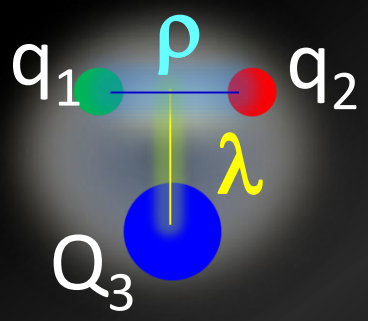
To be confirmed by full LQCD cals. with physical m_π

D. B. Leinweber et al., PRL94, 212001(2005).

Essence of
Non-relativistic Formulation in CQM

$$H = H_0 + V_c + V_{ss} + V_{SO} + V_T \ldots$$

- H_0: kinematic term
- V_c: confinement potential
- V_{ss}: spin-spin interaction
- V_{SO}: spin-orbit interaction (LS force)
- V_T: Tensor interaction



Coordinate: r_1, r_2, r3

$\rho = (r_1 - r_2)/\sqrt{2}$

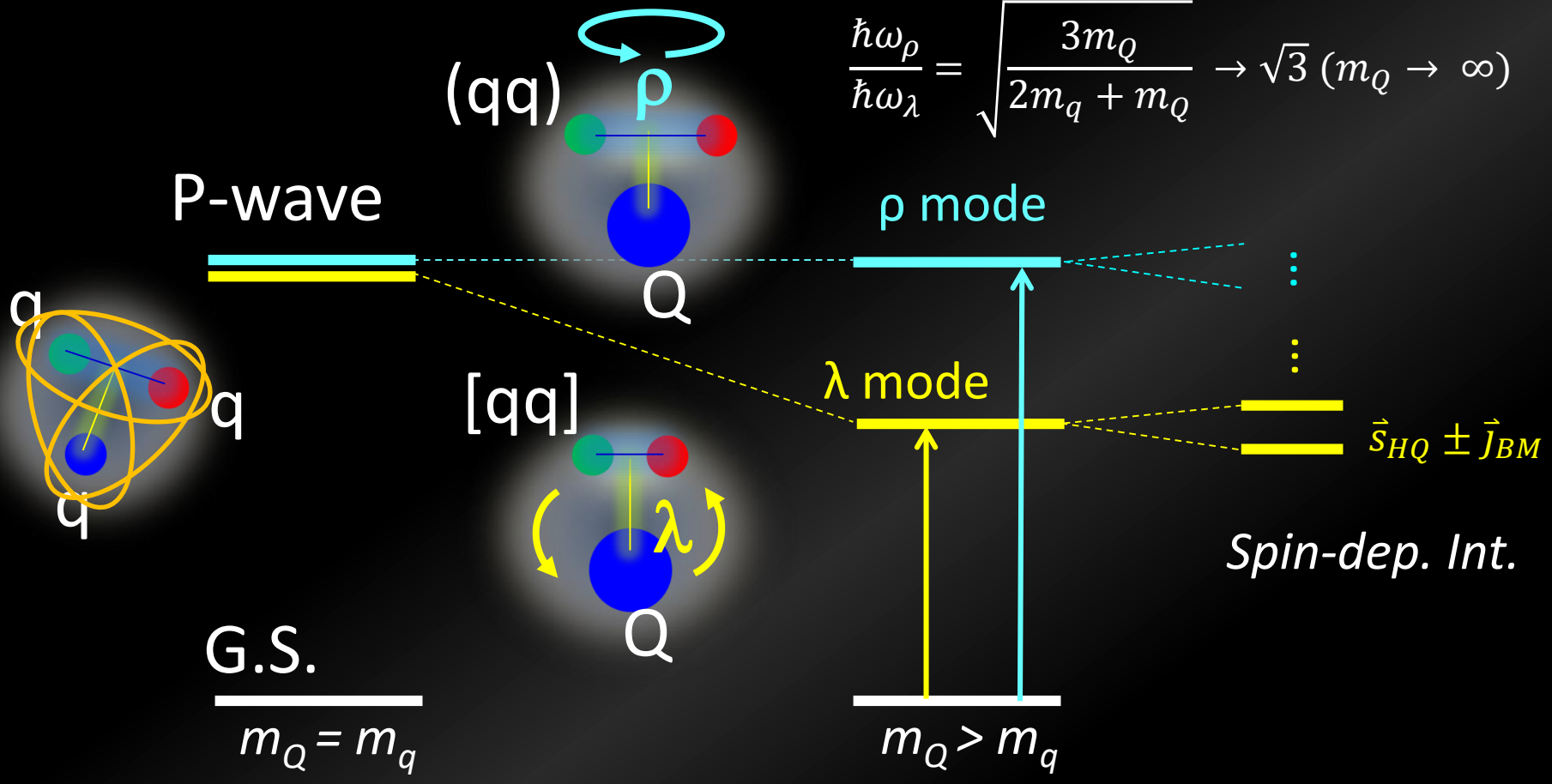
$\lambda = (r_1 + r_2 - 2r_3)/\sqrt{6}$

$\mu_\rho = m_q$

$\mu_\lambda = 3m_q m_Q/(2m_q + m_Q)$

Schematic Level Structure of Heavy Baryons

- λ and ρ motions split (Isotope Shift)
- HQ spin multiplet（$\vec{s}_{HQ} \pm \vec{J}_{Brown\ Muck}$）



Confinement

- $H = H_0 + V_c + V_{ss} + \cdots$

  $\Psi \sim \psi_\ell \chi_S \phi_I(color)$ → symmetrize (anti-symm.)

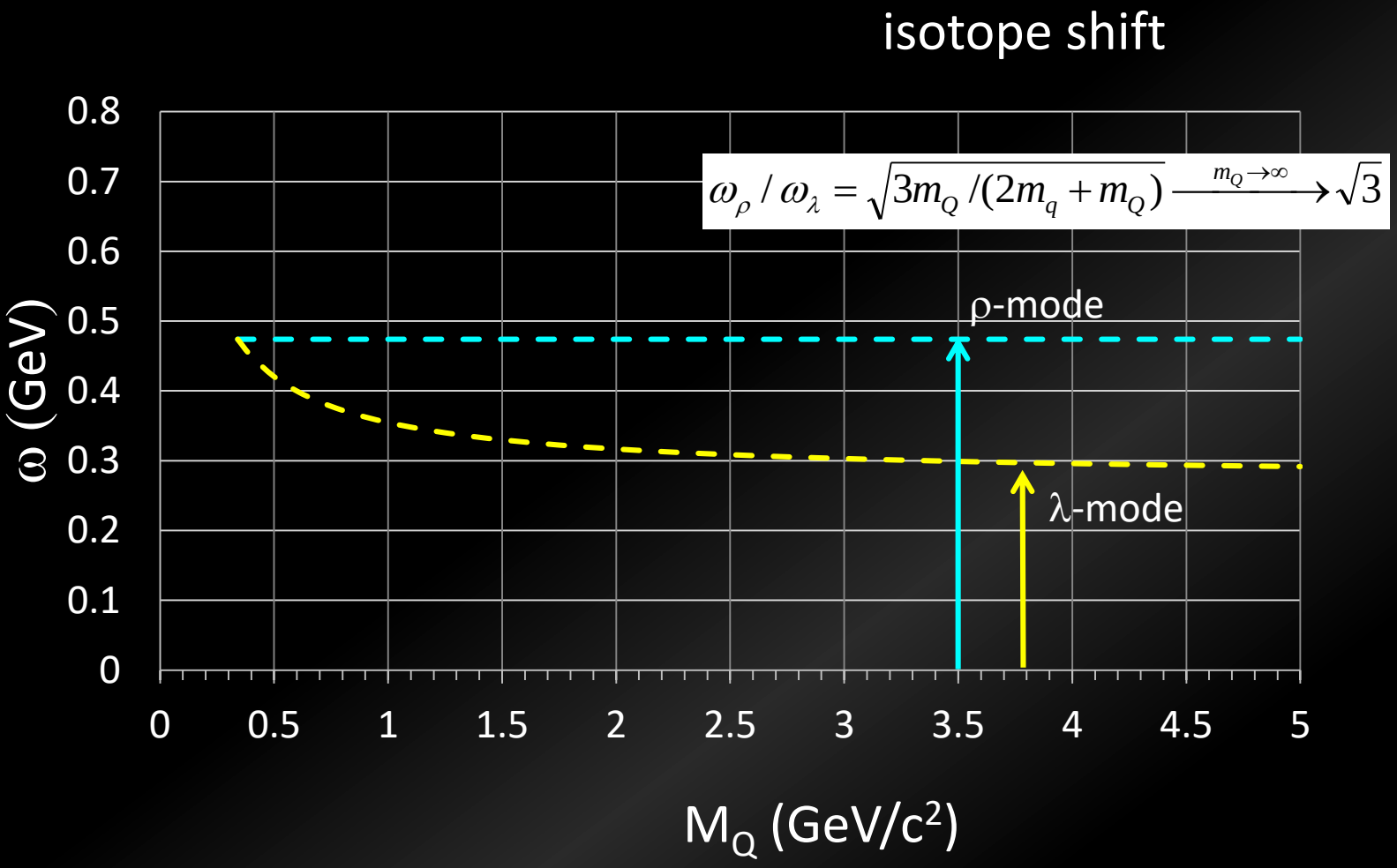
- $V_c = k/2 \sum r_{ij}^2$ → analytic (<-> cornell potential $\sum(\frac{a}{r_{ij}} + br_{ij})$)

$$\omega_{\lambda,\rho} = \sqrt{3k/m_{\lambda,\rho}}, \qquad \left(m_\lambda = \frac{3m_q m_Q}{2m_q + m_Q}, m_\rho = m_q\right)$$

$$k = 0.33^2 m_\lambda/3, \ at\ m_Q = 1.5\ \mathrm{GeV}/c^2$$

$$c.f.\ 1\hbar\omega_\lambda \sim \frac{\Lambda_c\left(\frac{1}{2}^-\right) + 2\Lambda_c\left(\frac{3}{2}^-\right)}{3} - \Lambda_c\left(\frac{1}{2}^+\right) \sim 0.33\ \mathrm{GeV}/c^2$$

P-wave (ρ, λ-mode excitations)



Spin-spin Interaction

- $H = H_0 + V_c + V_{ss} + \cdots$

  $\Psi \sim \psi_\ell \chi_S \phi_I(color) \rightarrow$ symmetrize (anti-symm.)

- $V_c = k/2 \sum r_{ij}^2$

  $$\omega_{\lambda,\rho} = \sqrt{3k/m_{\lambda,\rho}}, \qquad \left(m_\lambda = \frac{3m_q m_Q}{2m_q + m_Q}, m_\rho = m_q \right)$$

- $V_{ss} = c_s \sum \frac{\sigma_i \cdot \sigma_j}{m_i m_j} \delta(r_{ij})$ $\quad \langle \chi_S | V_{ss} | \chi_S \rangle$:

  $\Lambda\left(\frac{1}{2}^+\right) = \omega_0 - 3c_s/m_q^2$ $\quad (S, \chi^\rho)$: "qq"-spin anti-symm.

  $\Sigma\left(\frac{1}{2}^+\right) = \omega_0 + c_s\left(\frac{1}{m_q^2} - \frac{4}{m_q m_Q}\right)$ $\quad (S, \chi^\lambda)$: "qq"-spin symm., $[qqQ]^{1/2}$

  $\Sigma^*\left(\frac{3}{2}^+\right) = \omega_0 + c_s\left(\frac{1}{m_q^2} + \frac{2}{m_q m_Q}\right)$ $\quad (S, \chi^s)$: "qqQ" spin symm.

Spin-spin Interaction

- $H = H_0 + V_c + V_{ss} + \cdots$

  $\Psi \sim \psi_\ell \chi_S \phi_I(color) \rightarrow$ symmetrize (anti-symm.)

- $V_c = k/2 \sum r_{ij}^2$

  $$\omega_{\lambda,\rho} = \sqrt{3k/m_{\lambda,\rho}}, \qquad \left(m_\lambda = \frac{3m_q m_Q}{2m_q + m_Q}, m_\rho = m_q \right)$$

- $V_{ss} = c_s \sum \frac{\sigma_i \cdot \sigma_j}{m_i m_j} \delta(r_{ij})$ $\quad \langle \chi_S | V_{ss} | \chi_S \rangle$:

$$\Lambda\left(\frac{1}{2}^+\right) = \omega_0 - 3c_s/m_q^2$$

$$\Sigma\left(\frac{1}{2}^+\right) = \omega_0 + c_s\left(\frac{1}{m_q^2} - \frac{4}{m_q m_Q}\right)$$

$$\Sigma^*\left(\frac{3}{2}^+\right) = \omega_0 + c_s\left(\frac{1}{m_q^2} + \frac{2}{m_q m_Q}\right)$$

$$\Big\} \rightarrow \frac{\Sigma + 2\Sigma^*}{3} - \Lambda = c_s \frac{4}{m_q^2}$$

$\sim$0.2 GeV/c^2

Spin-spin Interaction

- $H = H_0 + V_c + V_{ss} + \cdots$

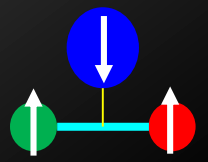
  $\Psi \sim \psi_\ell \chi_S \phi_I (color) \rightarrow$ symmetrize (anti-symm.)

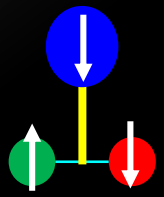
- $V_c = k/2 \sum r_{ij}^2$

$$\omega_{\lambda,\rho} = \sqrt{3k/m_{\lambda,\rho}}, \qquad \left(m_\lambda = \frac{3m_q m_Q}{2m_q + m_Q}, m_\rho = m_q \right)$$

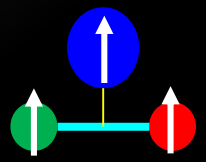
- $V_{ss} = c_s \sum \frac{\sigma_i \cdot \sigma_j}{m_i m_j} \delta(r_{ij})$ $\quad \langle \chi_S | V_{ss} | \chi_S \rangle$:

$$\Lambda\left(\frac{1}{2}^-, \frac{3}{2}^-\right) = \begin{cases} \omega_\rho + c_s \left(\frac{1}{m_q^2} - \frac{4}{m_q m_Q} \right) & (\ell_\rho = 1, \chi^\lambda) \\ \omega_\lambda - 3c_s/m_q^2 & (\ell_\lambda = 1, \chi^\rho) \end{cases}$$

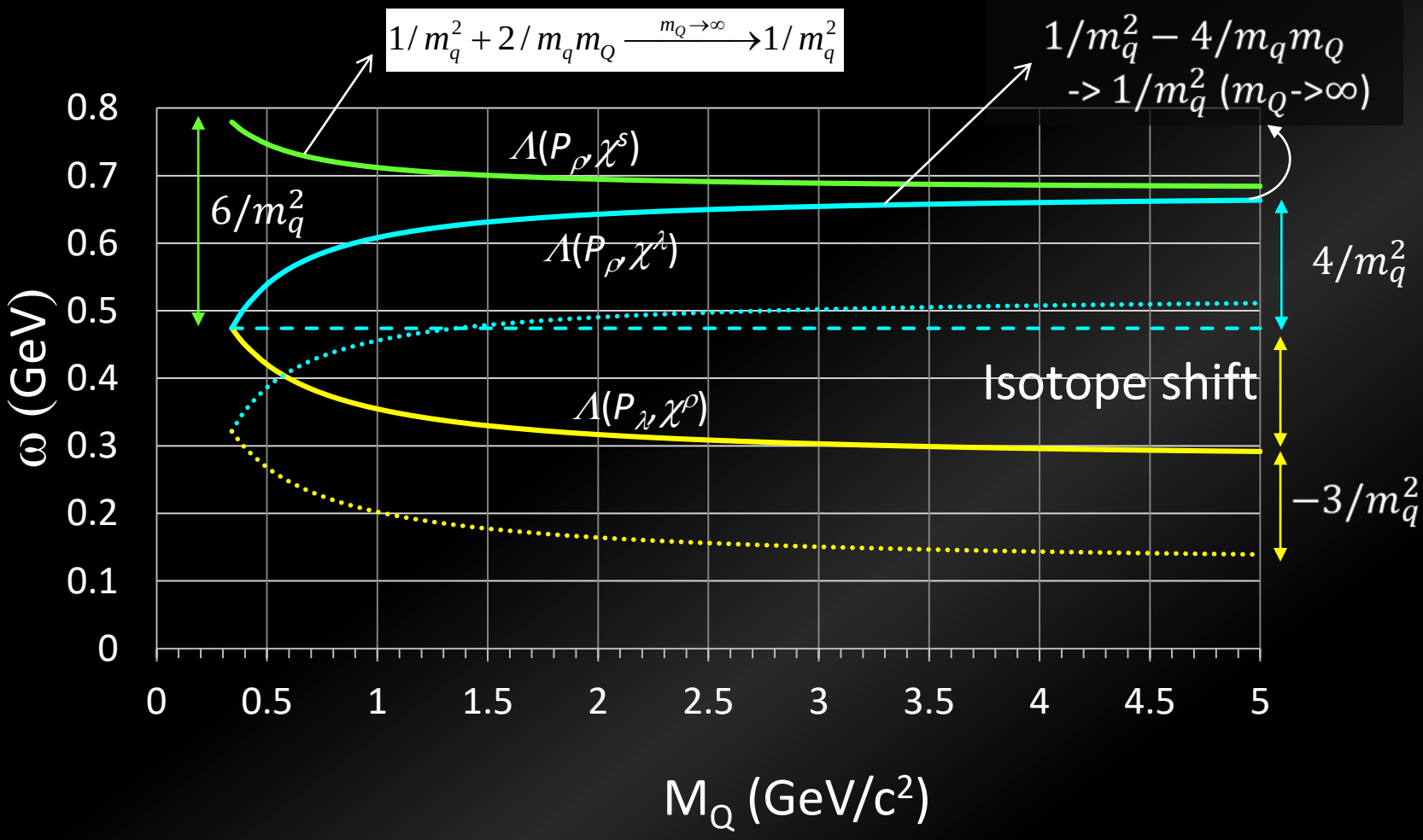




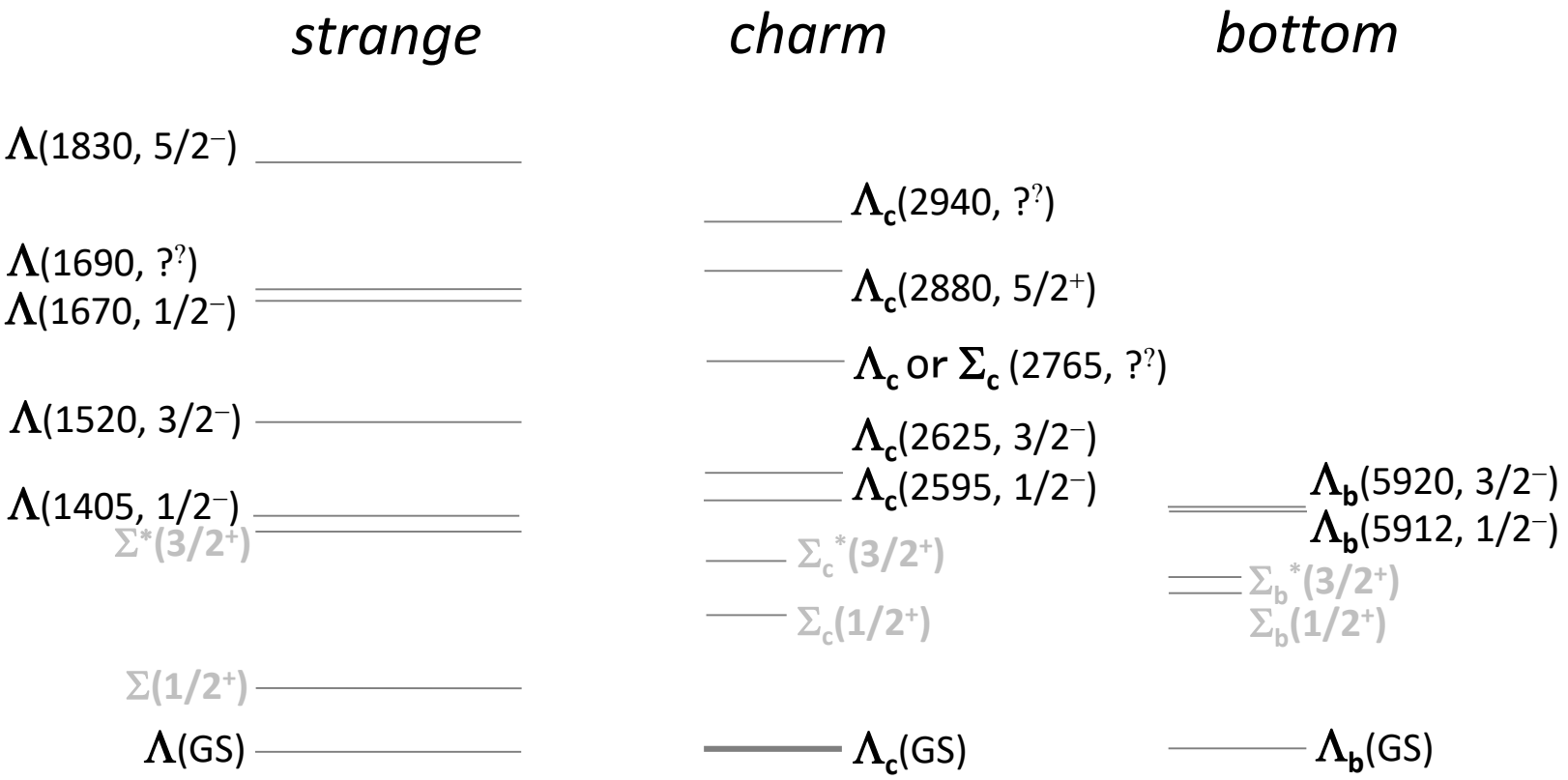
$$\Lambda\left(\frac{1}{2}^-, \frac{3}{2}^-, \frac{5}{2}^-\right) = \omega_\rho + c_s \left(\frac{1}{m_q^2} + \frac{2}{m_q m_Q} \right) (\ell_\rho = 1, \chi^s)$$



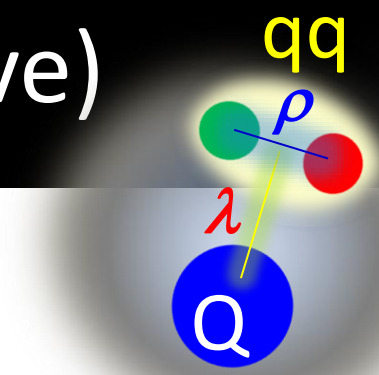
$\Lambda(P,\chi)$ (ρ, λ-mode excitations w/ V_{ss})

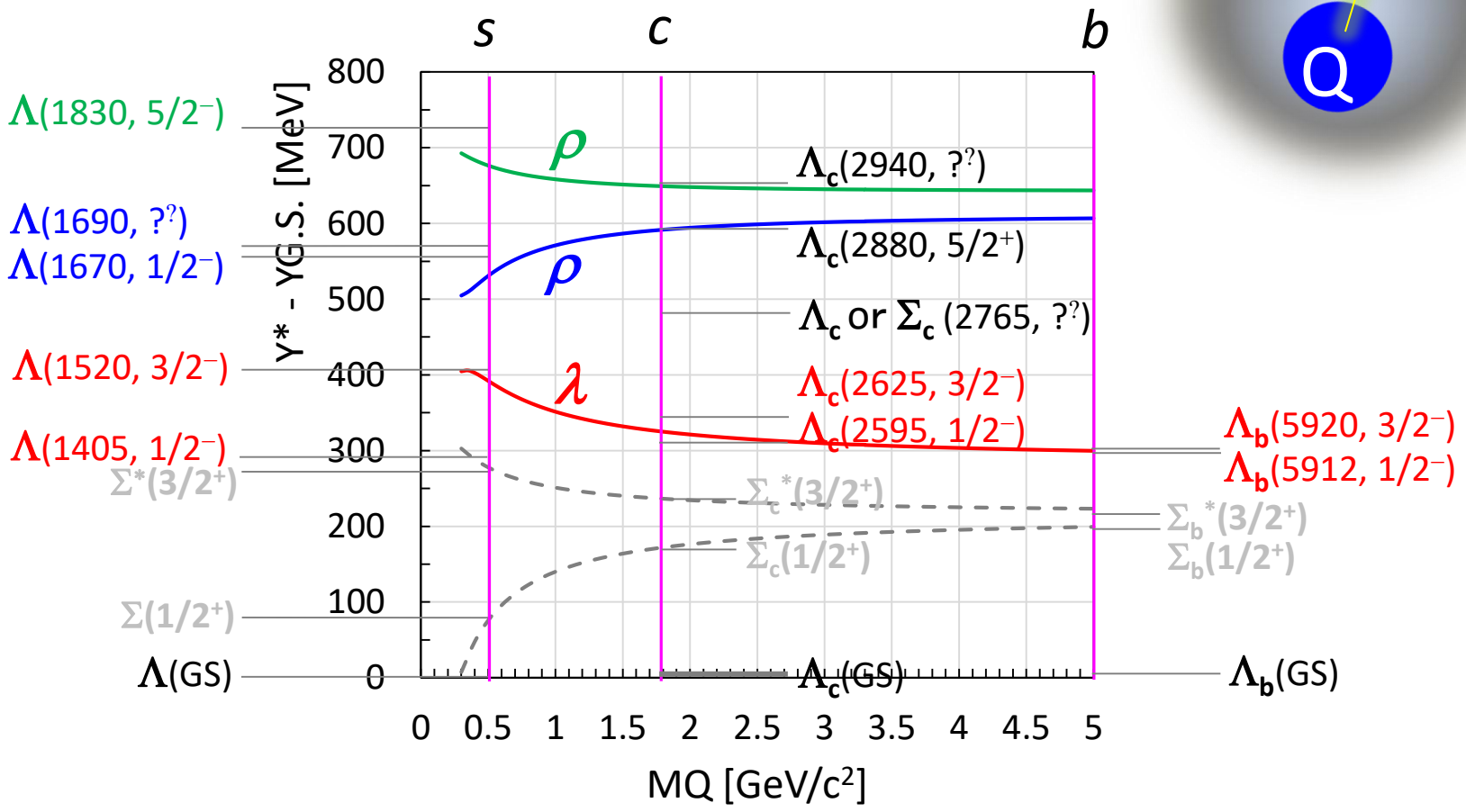


Lambda Baryons (P-wave)



Lambda Baryons (P-wave)

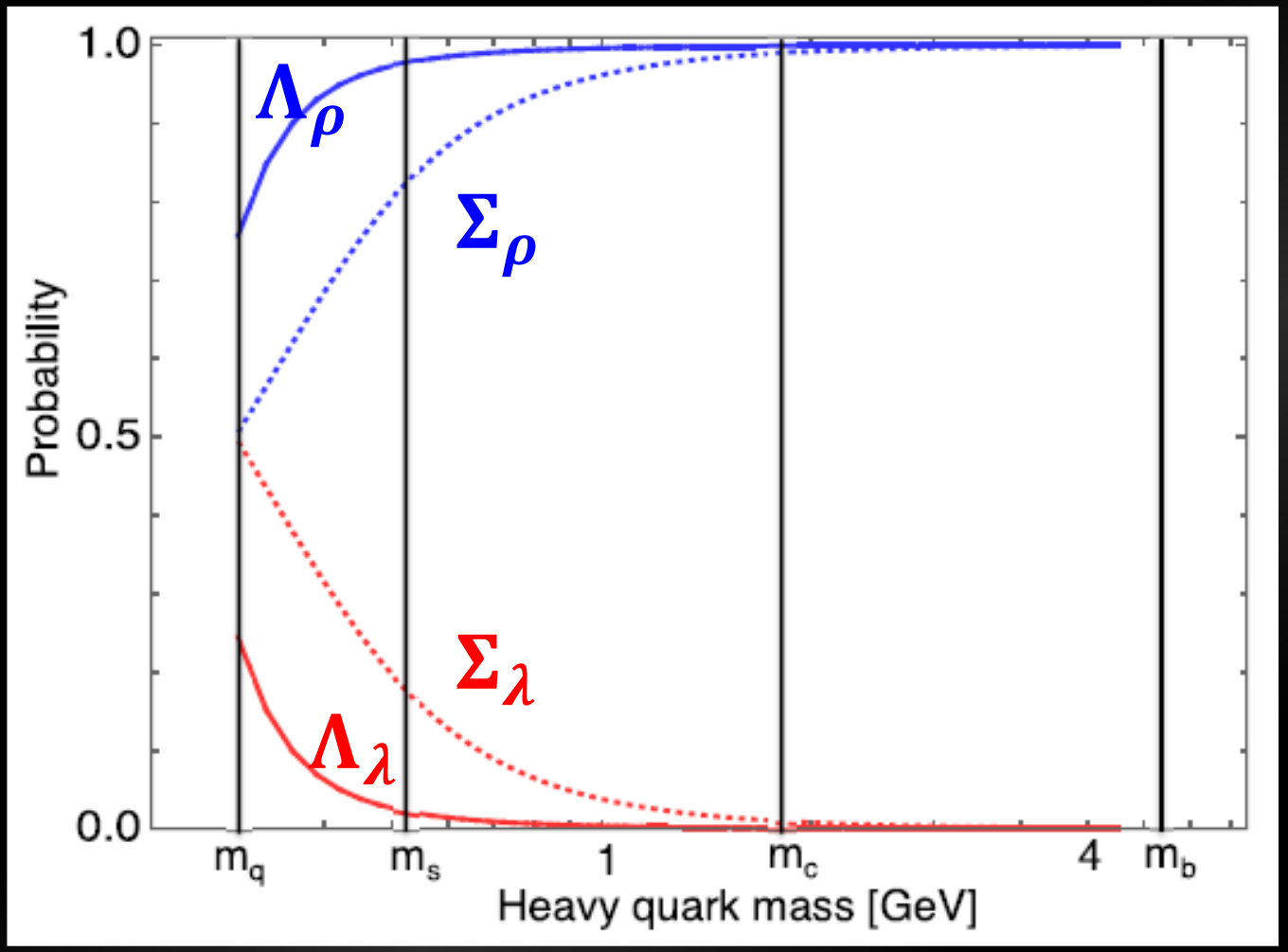




non-rel. QM: $H=H_0+V_{conf}+V_{SS}+V_{LS}+V_T$
ρ–λ mixing (cal. By T. Yoshida)

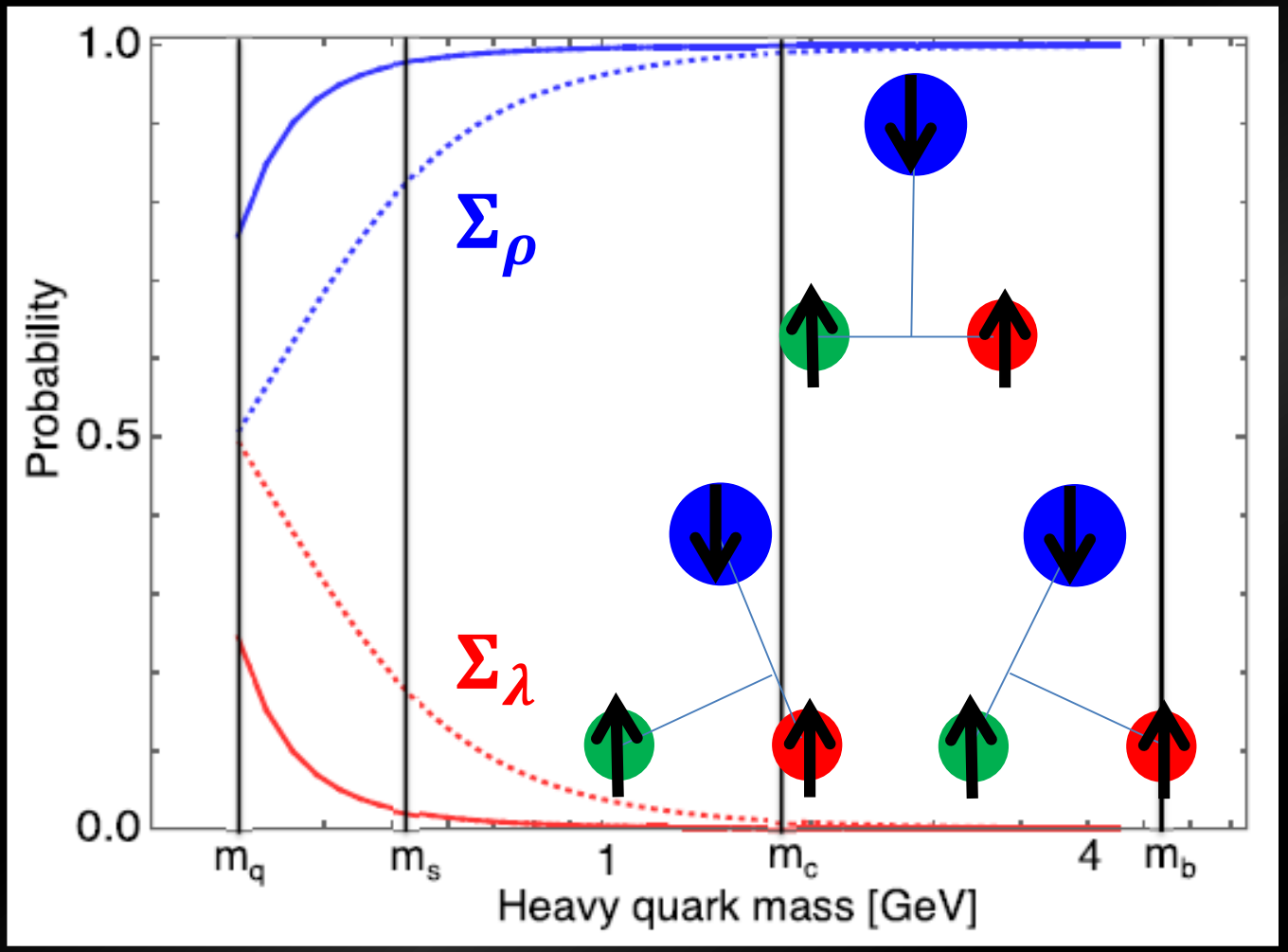
T. Yoshida et al.,
Phys. Rev. D**92**, 114029(2015)

$\Lambda,\Sigma(P,\chi)$ (ρ, λ-mode mixing w/ V_{ss})



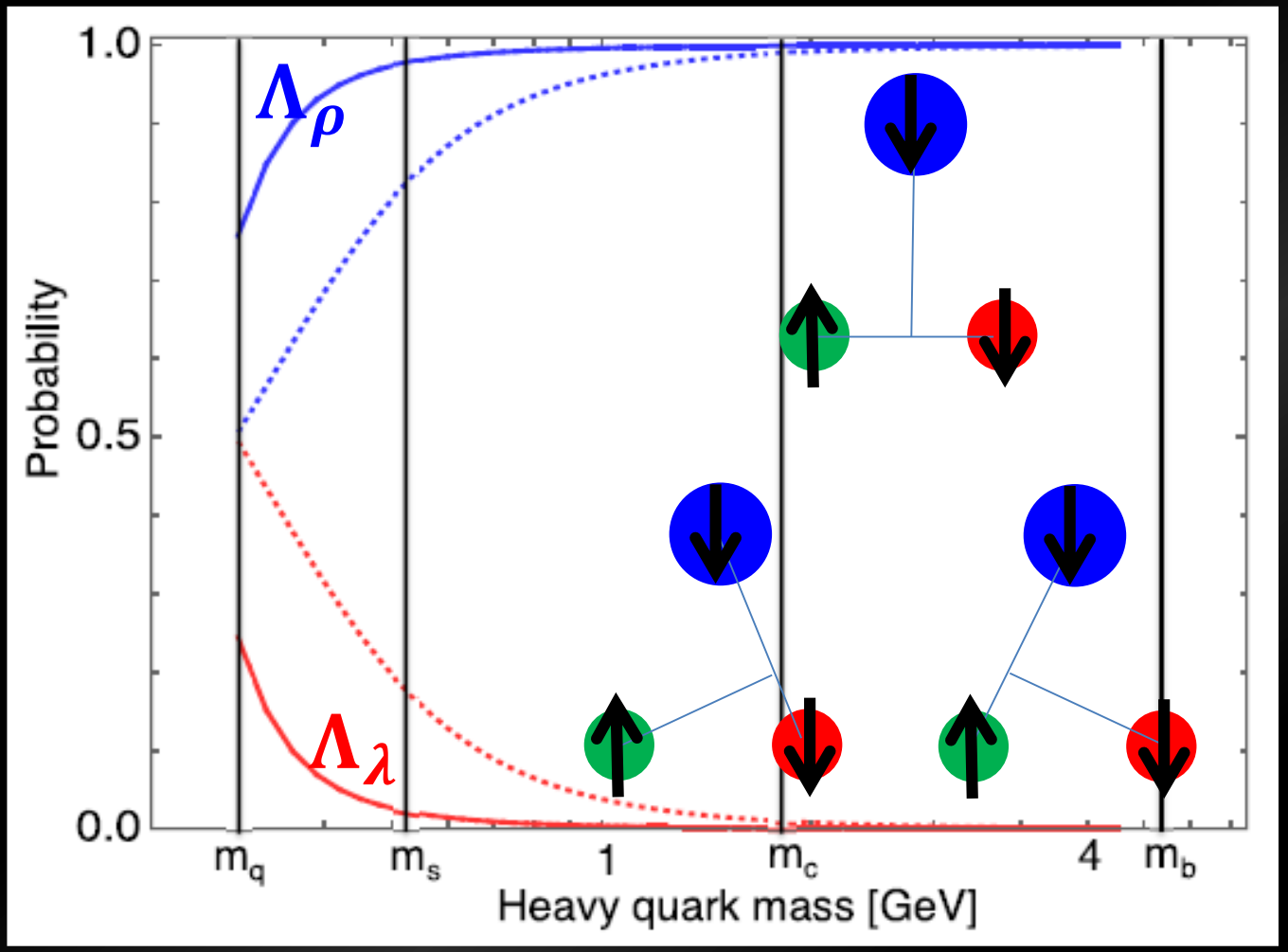
T. Yoshida et al., Phys. Rev. D**92**, 114029(2015)

$\Lambda,\Sigma(P,\chi)$ (ρ, λ-mode mixing w/ V_{ss})



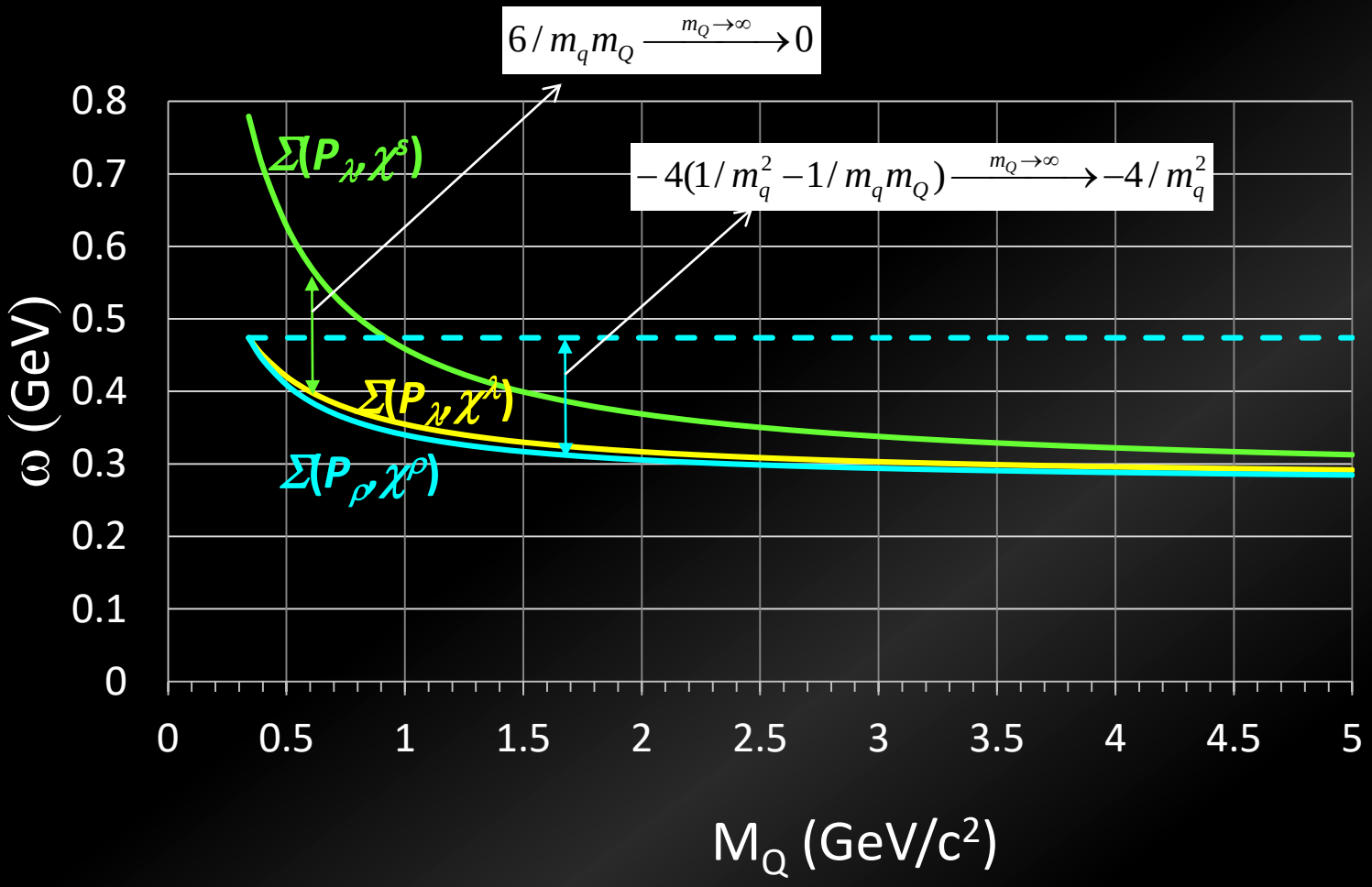
T. Yoshida et al., Phys. Rev. D**92**, 114029(2015)

$\Lambda,\Sigma(P,\chi)$ (ρ, λ-mode mixing w/ V_{ss})

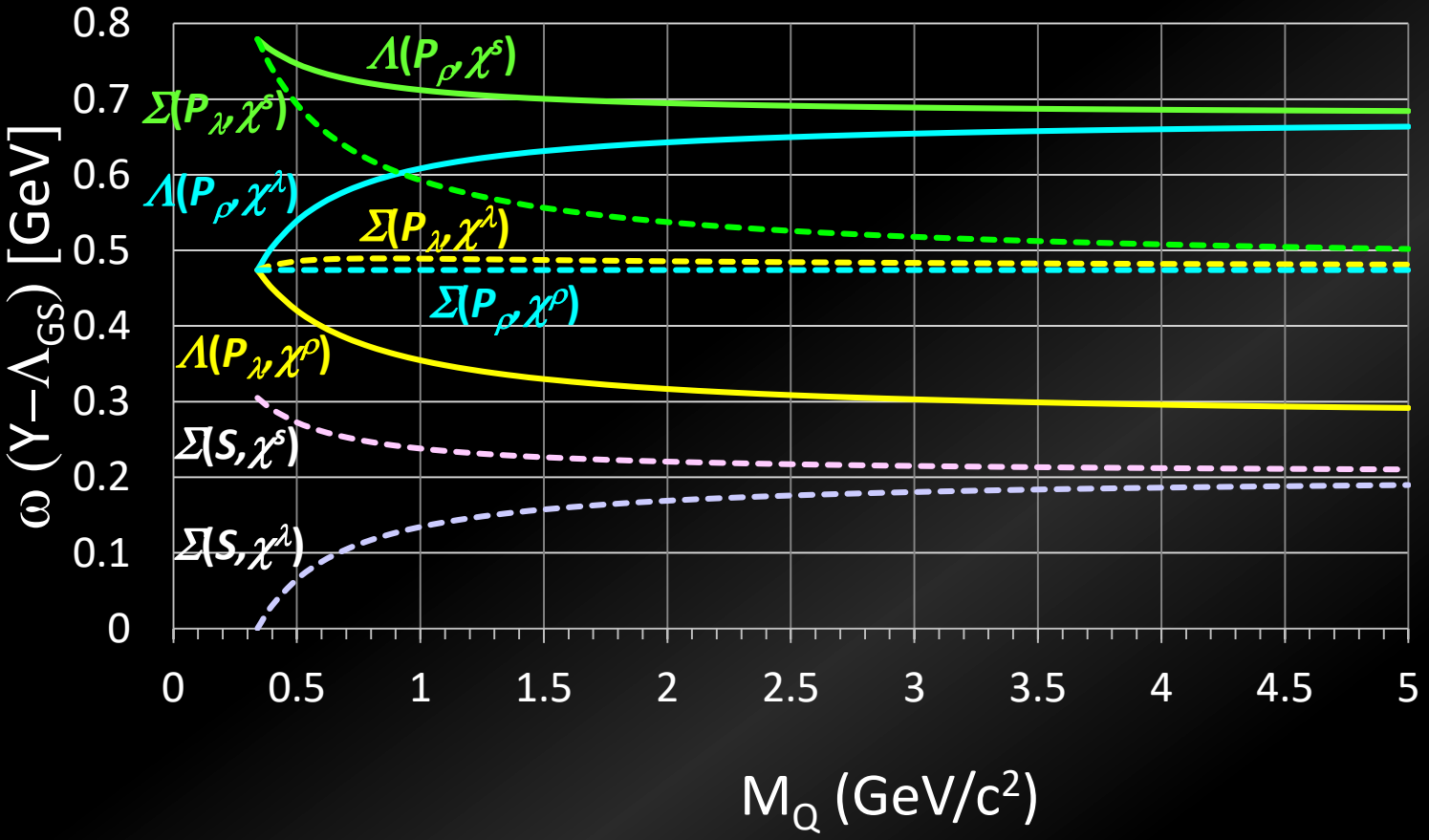


T. Yoshida et al., Phys. Rev. D**92**, 114029(2015)

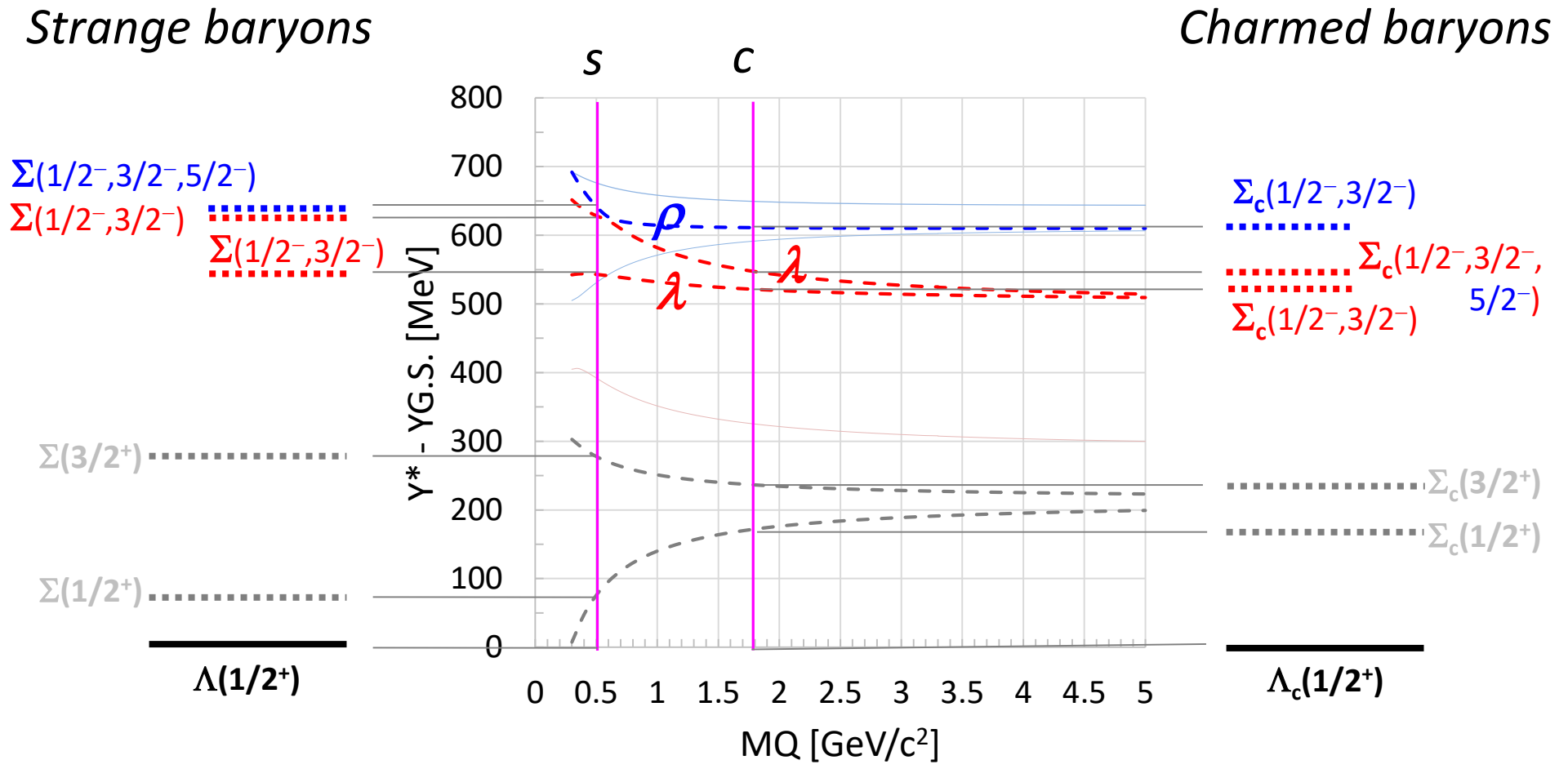
$\Sigma(P,\chi)$ (ρ, λ-mode excitations w/ V_{ss})



$\Lambda,\Sigma(1/2\text{-})$ (ρ, λ-mode excitations w/ V_{ss})



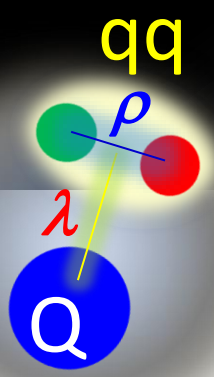
CQM calculation (P-wave Sigma)

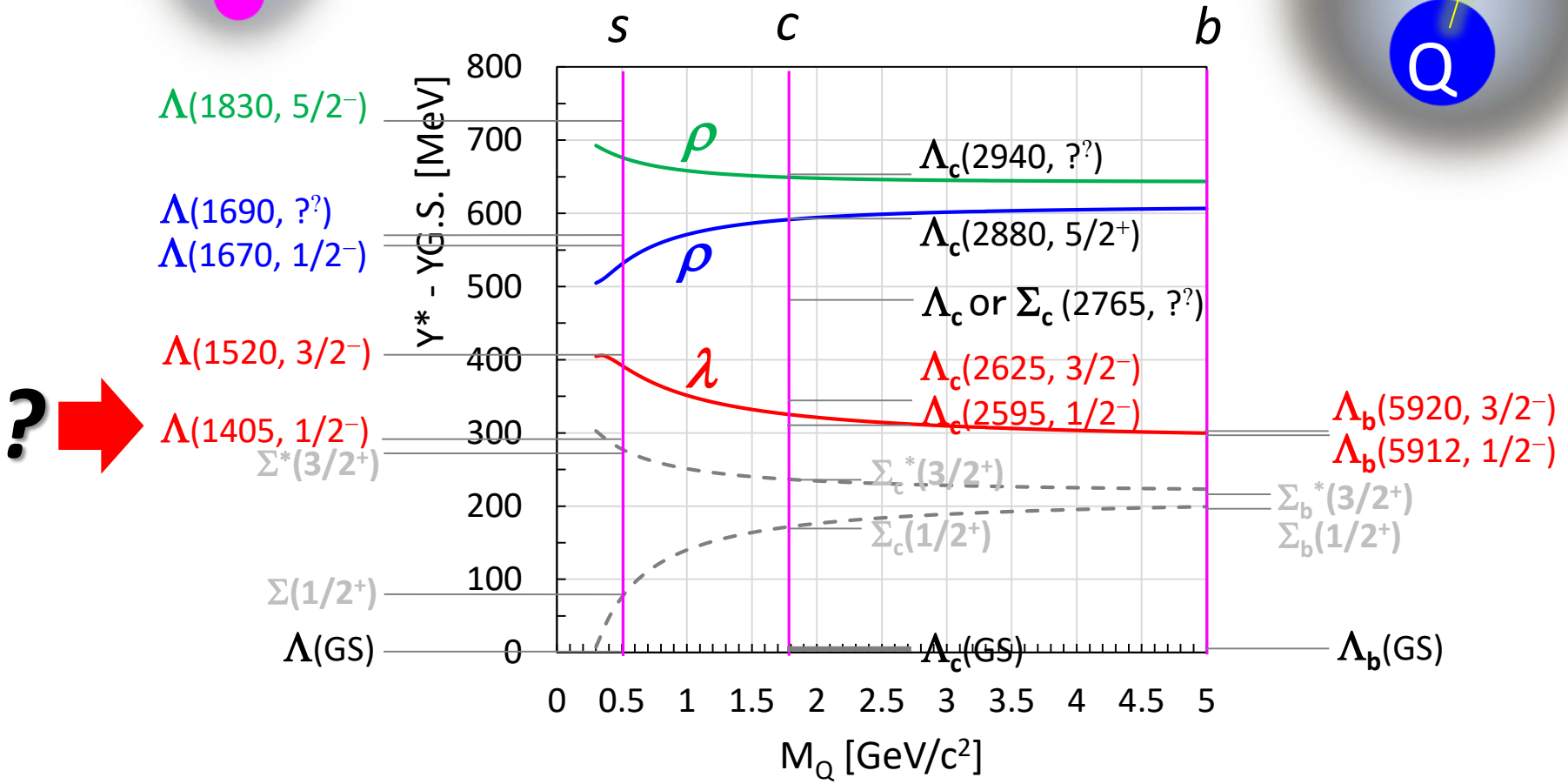


non-rel. QM: $H=H_0+V_{conf}+V_{SS}+V_{LS}+V_T$
ρ–λ mixing (cal. By T. Yoshida)

T. Yoshida et al.,
Phys. Rev. D**92**, 114029(2015)

Lambda Baryons (P-wave)





?

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ρ–λ mixing (cal. By T. Yoshida)

T. Yoshida et al.,
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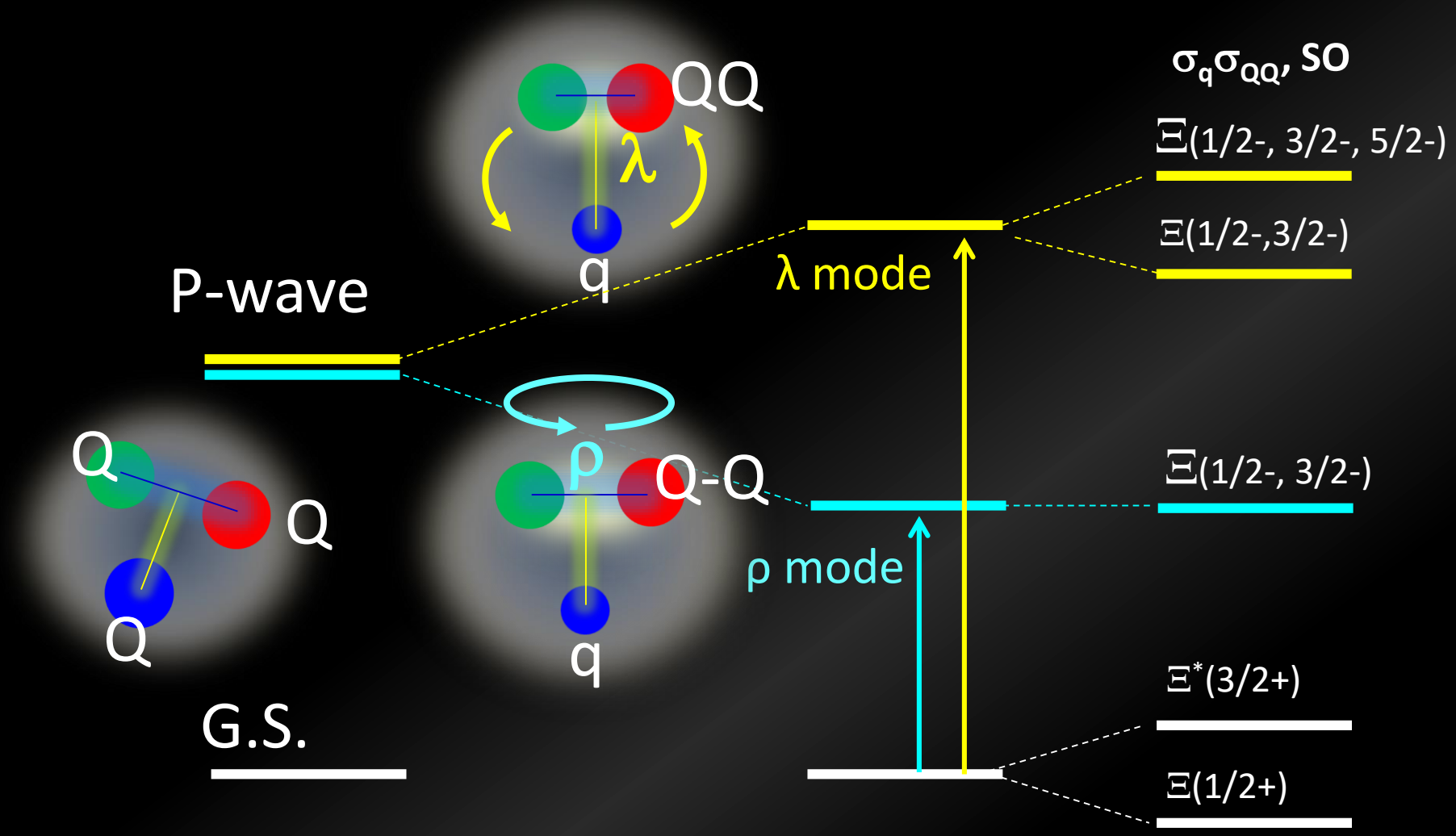
QQq

Level Structure of double-strange baryons

- λ and ρ mode excitations interchange



Confinement

- $H = H_0 + V_c + V_{ss} + \cdots$

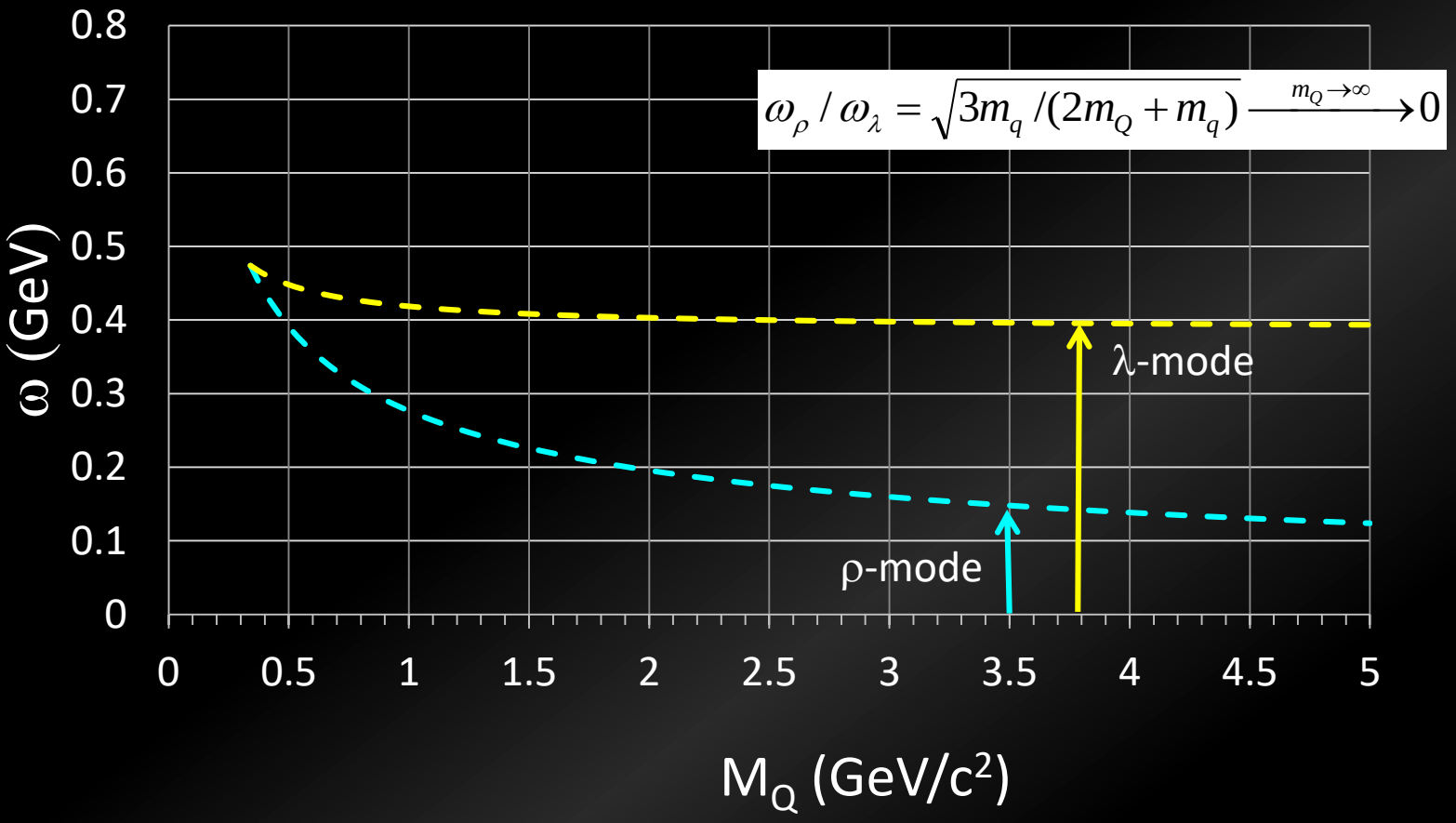
  $\Psi \sim \psi_\ell \chi_S (Isospin * color)$ → symmetrize (anti-symm.)

- $V_c = k/2 \sum r_{ij}^2$

$$\omega_{\lambda,\rho} = \sqrt{3k/m_{\lambda,\rho}}, \qquad \left(m_\lambda = \frac{3m_Q m_q}{2m_Q + m_q}, m_\rho = m_Q \right)$$

P-wave (ρ, λ-mode excitations)

isotope shift



Spin-spin Interaction

- $H = H_0 + V_c + V_{ss} + \cdots$

  $\Psi \sim \psi_\ell \chi_S (Isospin * color) \rightarrow$ symmetrize (anti-symm.)

- $V_c = k/2 \sum r_{ij}^2$

$$\omega_{\lambda,\rho} = \sqrt{3k/m_{\lambda,\rho}}, \qquad \left(m_\lambda = \frac{3m_Q m_q}{2m_Q + m_q}, m_\rho = m_Q \right)$$

- $V_{ss} = c_s \sum \frac{\sigma_i \cdot \sigma_j}{m_i m_j} \delta(r_{ij})$ $\quad \langle \chi_S | V_{ss} | \chi_S \rangle$:

$$\Xi\left(\frac{1}{2}^+\right) = \omega_0 + c_s \left(\frac{1}{m_Q^2} - \frac{4}{m_q m_Q} \right) \qquad (S, \chi^\lambda) : \text{“QQ”-spin symm., } [\text{QQq}]^{1/2}$$

$$\Xi^*\left(\frac{3}{2}^+\right) = \omega_0 + c_s \left(\frac{1}{m_Q^2} + \frac{2}{m_q m_Q} \right) \qquad (S, \chi^S) : \text{“QQq” spin symm.}$$

Spin-spin Interaction

- $H = H_0 + V_c + V_{ss} + \cdots$

  $\Psi \sim \psi_\ell \chi_S (Isospin * color) \rightarrow$ symmetrize (anti-symm.)

- $V_c = k/2 \sum r_{ij}^2$

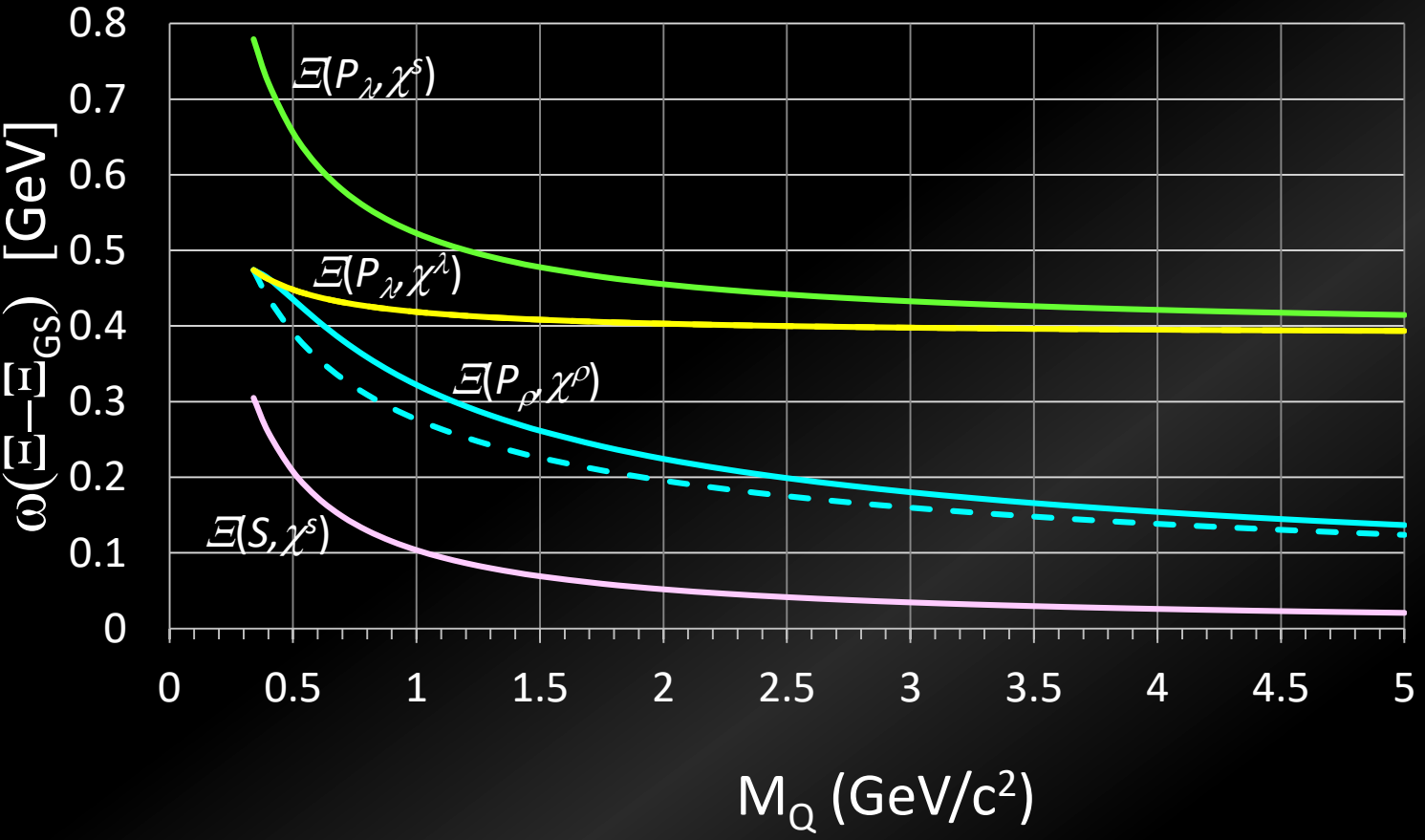
$$\omega_{\lambda,\rho} = \sqrt{3k/m_{\lambda,\rho}}, \qquad \left(m_\lambda = \frac{3m_Q m_q}{2m_Q + m_q}, m_\rho = m_Q \right)$$

- $V_{ss} \sim c_s \sum \frac{\sigma_i \cdot \sigma_j}{m_i m_j} \delta(r_{ij})$ $\qquad \langle \chi_S | V_{ss} | \chi_S \rangle$:

$$\Xi\left(\frac{1}{2}^-, \frac{3}{2}^-\right) = \begin{cases} \omega_\rho - 3c_s/m_Q^2 & (\ell_\rho = 1, \chi^\rho) \\ \omega_\lambda + c_s \left(\frac{1}{m_Q^2} - \frac{4}{m_q m_Q} \right) & (\ell_\lambda = 1, \chi^\lambda) \end{cases}$$

$$\Xi\left(\frac{1}{2}^-, \frac{3}{2}^-, \frac{5}{2}^-\right) = \omega_\lambda + c_s \left(\frac{1}{m_Q^2} + \frac{2}{m_q m_Q} \right) (\ell_\lambda = 1, \chi^s)$$

Ξ (ρ, λ-mode excitations w/ V_{ss})



Level Structure of double-strange baryons

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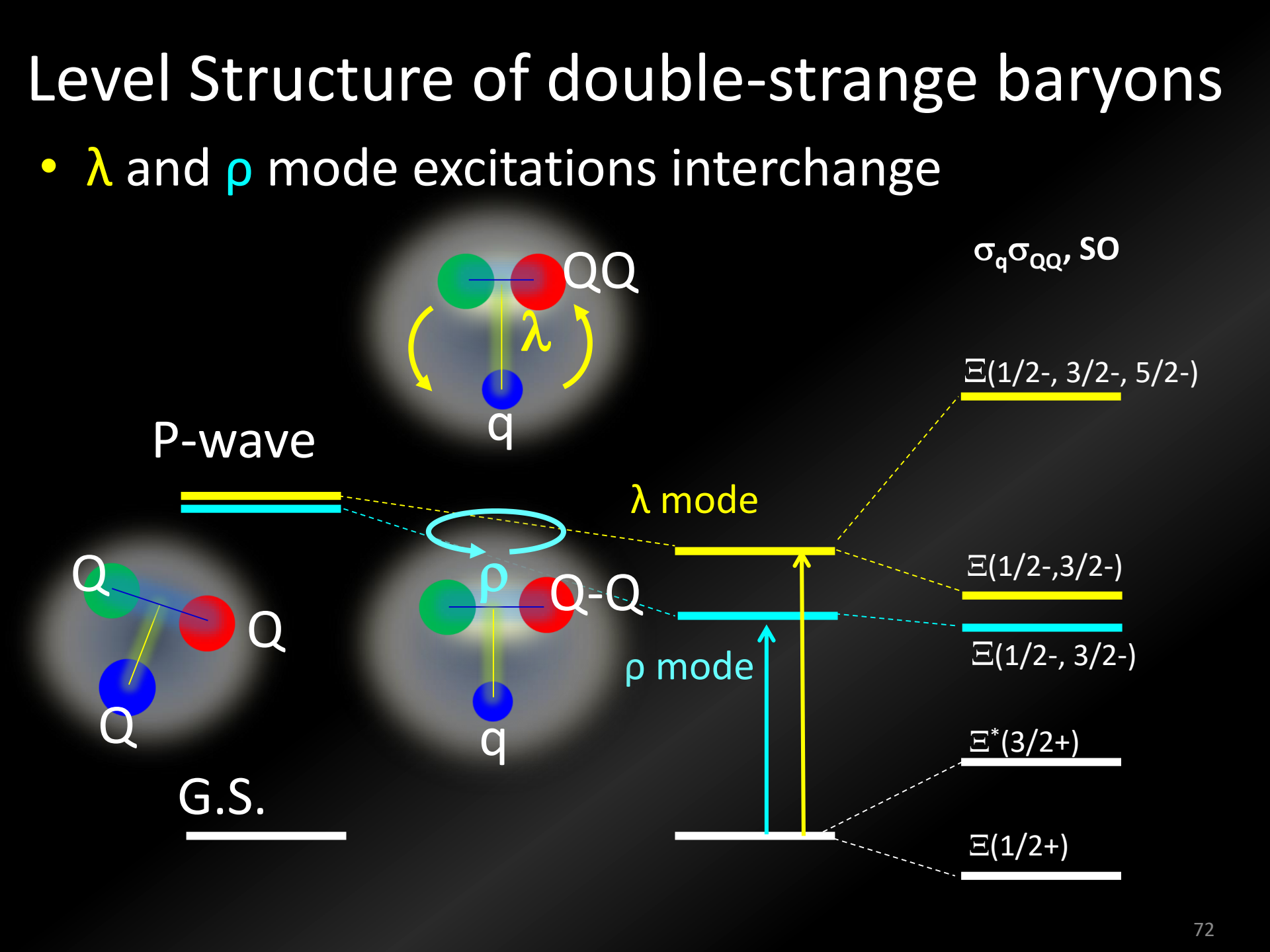
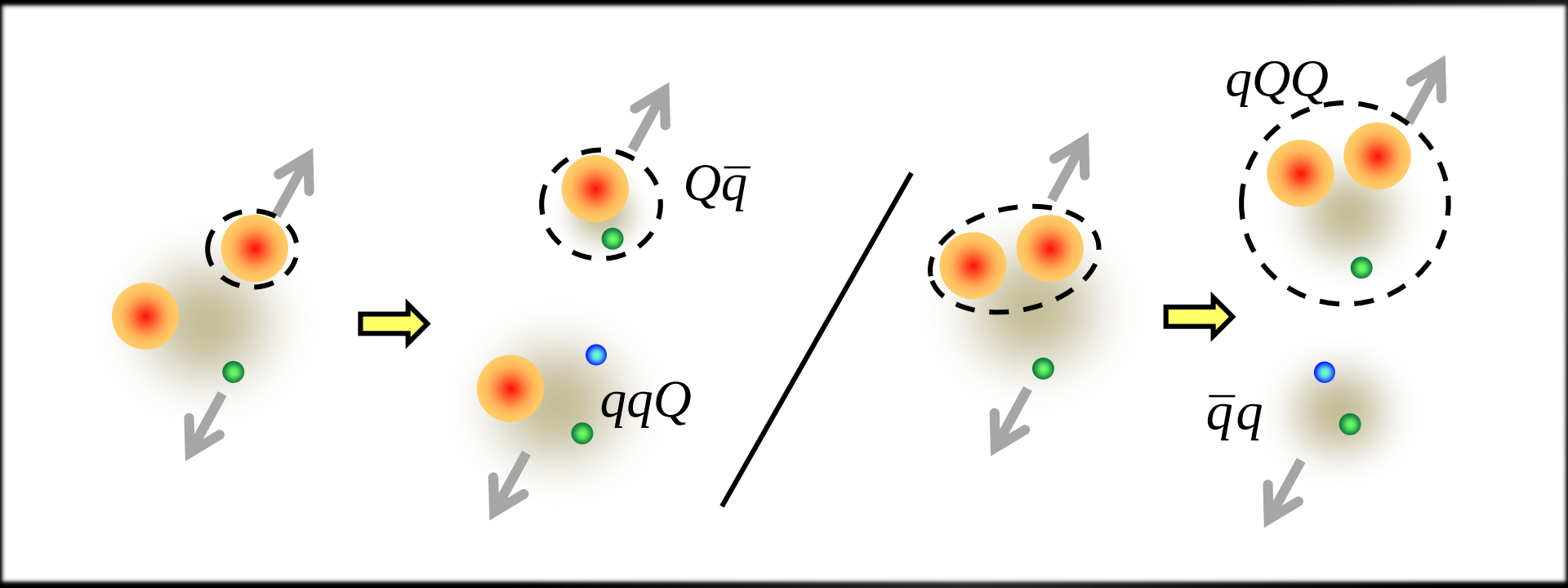


Structure and Decay Partial Width



ρ mode (QQ)

λ mode [QQ]

Form of Hadrons

Observable	Relevant Physics Quantity	What we learn	
Mass Spectrum	Mass, Width (pole:$M_R - i\Gamma/2$)	Particle state Resonant state	Classification
Angular Correl. (decay)	Spin, Parity		
Level structure		Internal (effective) DoF	Form (Dynamics of effective DoF in Hadron)
Production Rate (Diff. Cross Sect.)	Response Function (Transition) Form Factor	Reaction Mechanism Internal Motion/Corr.	
Partial Width	Internal Correlation (Wave function)	Decay Mechanism Internal Motion/Corr.	