

Cluster phenomena in nuclear systems

Y. Kanada-En'yo (Kyoto Univ.)

Lesson1: Introduction (overview)

Lesson2: what is correlation?

Lesson3: pn correlation

Lesson4: clustering in neutron-rich Be

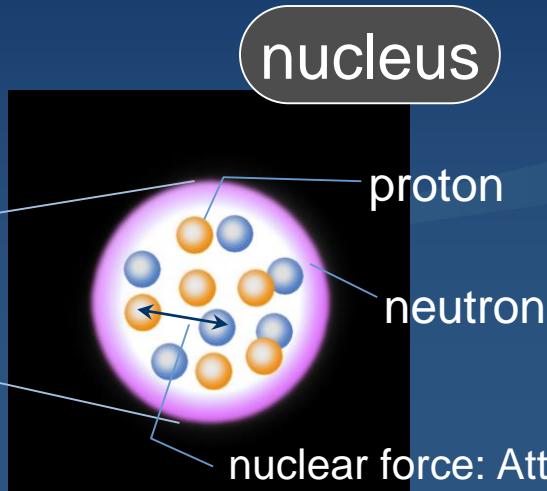
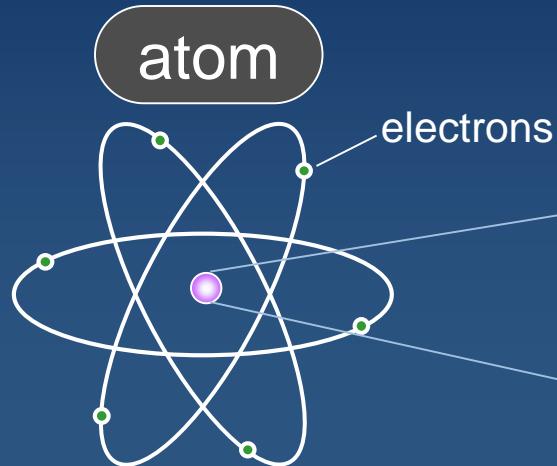
Lesson5: Monopole & Dipole excitations in light nuclei

Lesson6: Analogy to other systems

Lesson 1. Overview

cluster phenomena
in nuclear systems
(Personal opinion)

Nuclear system



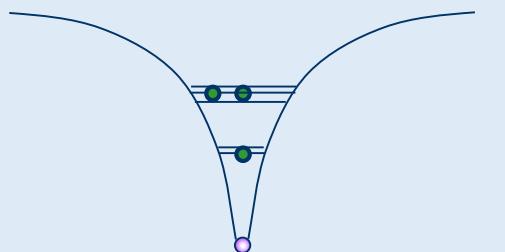
A finite quantum many-fermion system of protons and neutrons

Isospin \times spin : SU(4)

Analogy & Differences

Electron motion

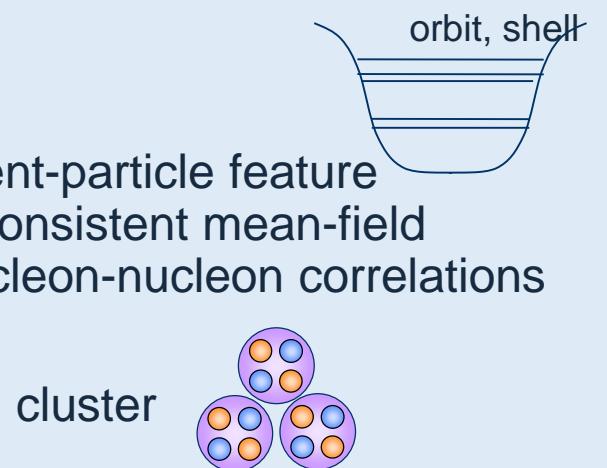
Confined by the external field



Nucleon motion

Self-bound

1. Independent-particle feature in self-consistent mean-field
2. Strong nucleon-nucleon correlations



correlations

- general phenomena in interacting many-particle systems
- cluster=strongly correlated subsystems
- cluster limit: internal-strong & external-weak

by H. Tanaka, K. Ikeda

Internal strong

energy & length scale



$$E_x(\text{Internal}) \gg E_r(\text{inter-cluster})$$

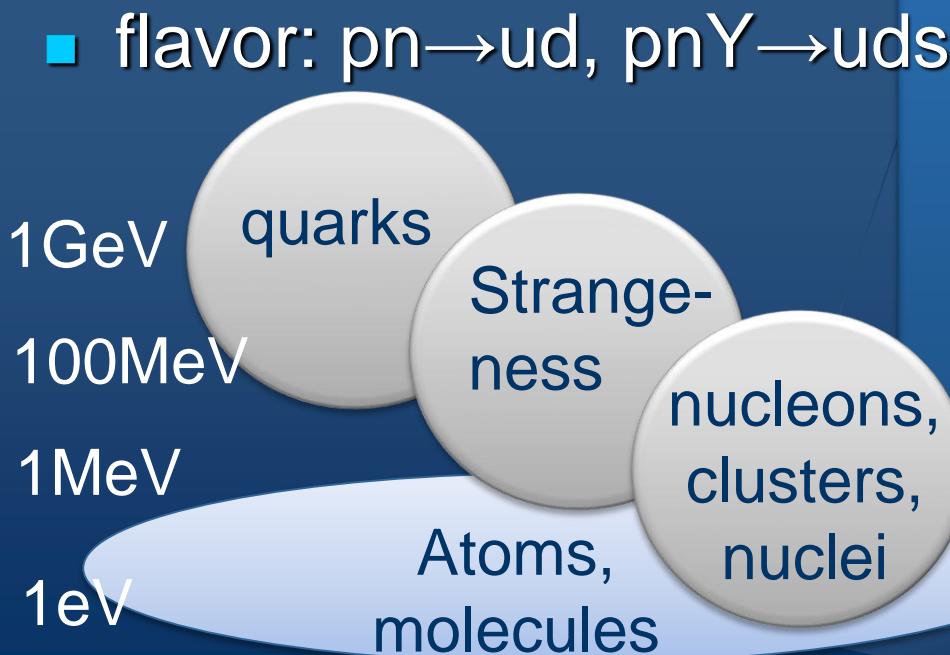
$$\xi \ll \rho^{-1/3}$$

→decoupling of scale(DOF) → Hierarchy

- Dependences on density and energy

analogy to other systems

- $[n\uparrow n\downarrow]$, $[n\uparrow n\downarrow p\uparrow p\downarrow]$, $[qqq]$, FF
 $SU(2)$, $SU(4)$, $SU(3)$ singlet \rightarrow screening
robust cluster unit



"Internal strong & external weak"



energy & length scale

$E_x(\text{Internal}) \gg E_r(\text{inter-cluster})$

$$\xi \ll \rho^{-1/3}$$

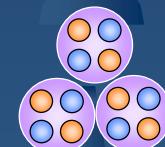
Cluster & Mean field

Mean field, shell structure
Independent single-particle

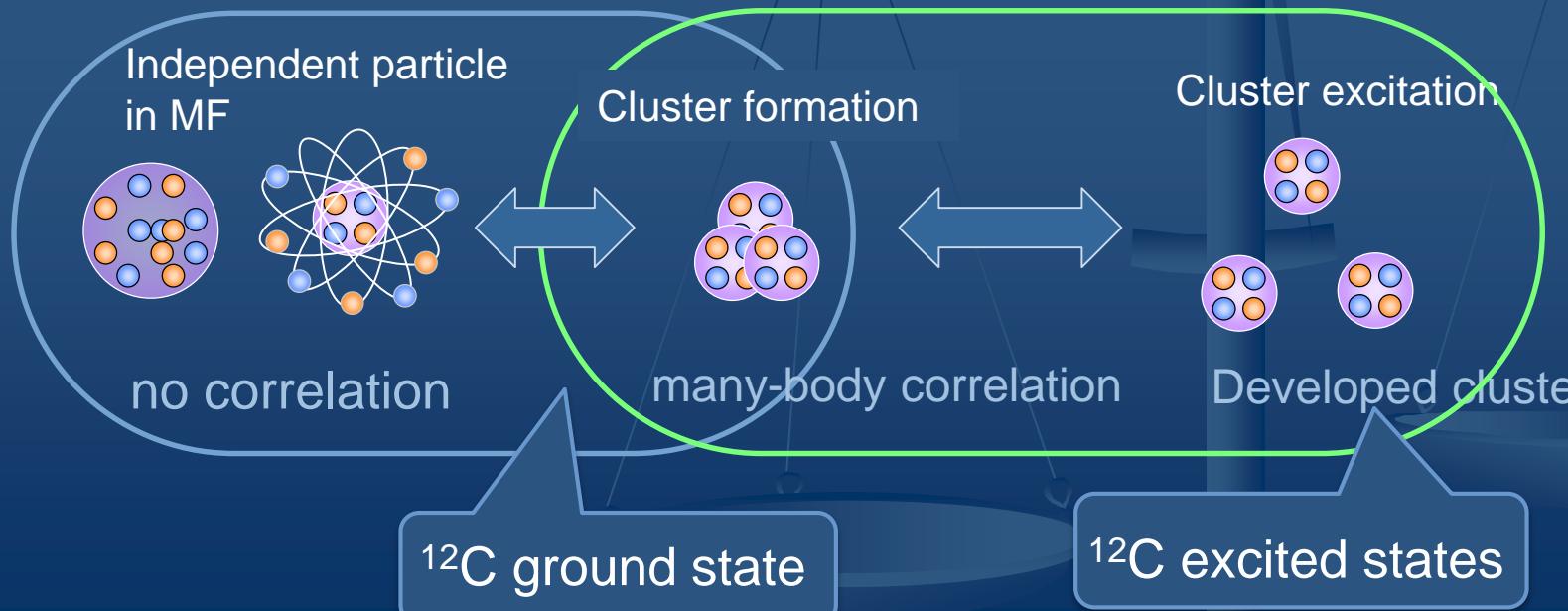


Shell structure · MF

Cluster:
Many-body correlation



Cluster



Cluster structures

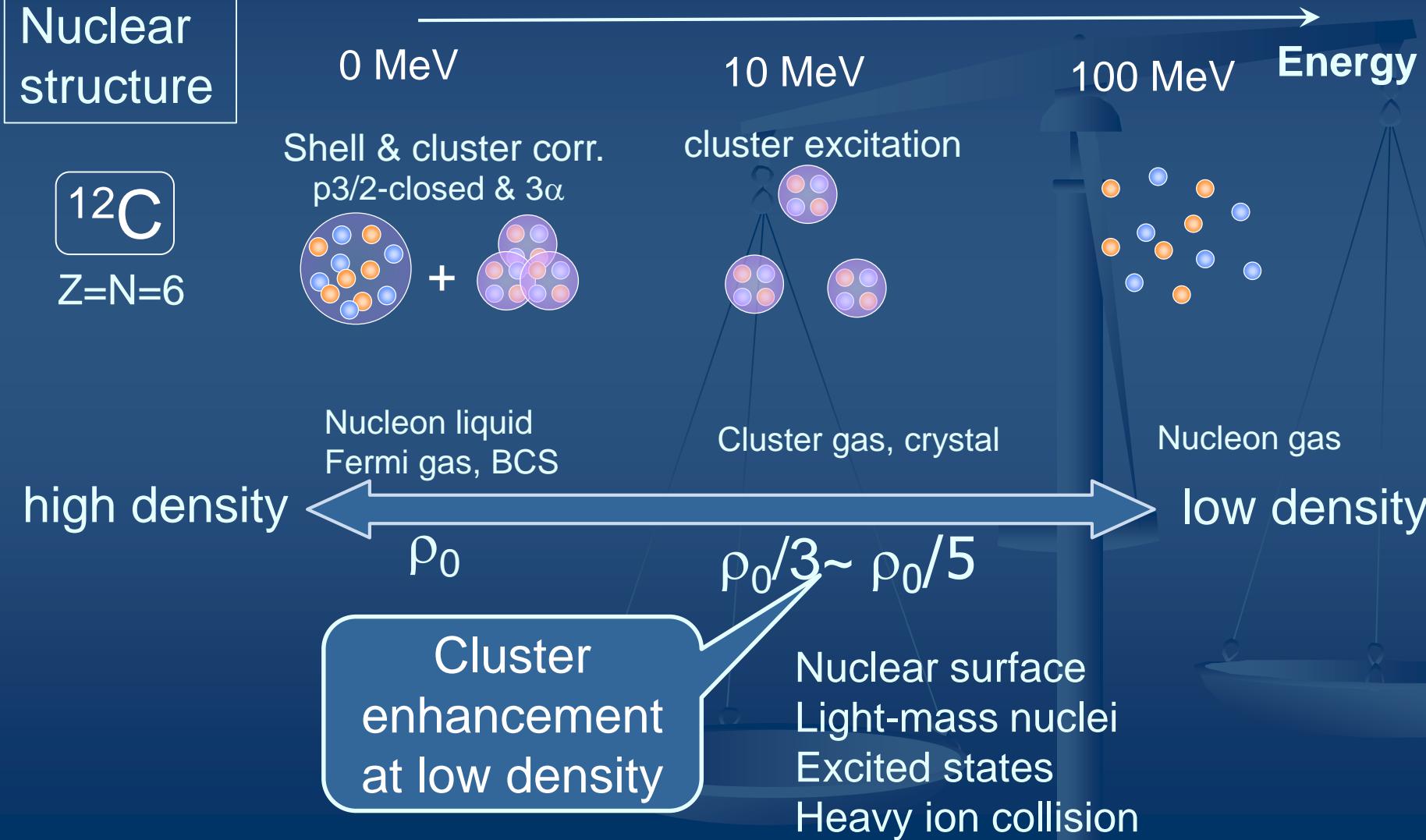
Nuclear
structure

^{12}C
 $Z=N=6$

high density

ρ_0

Cluster
enhancement
at low density



Typical examples of cluster structures in stable and unstable nuclei

Typical cluster structures known in stable nuclei

^7Li



$\alpha + t$

^8Be



$\alpha + \alpha$

^{12}C



3α

^{20}Ne



$^{16}\text{O} + \alpha$

$^{16}\text{O}^*$



$^{12}\text{C} + \alpha$

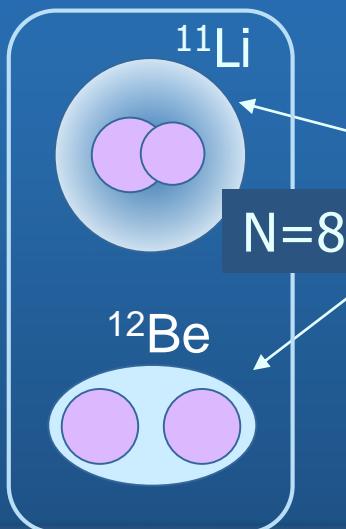
$Z=3$



$Z=4$

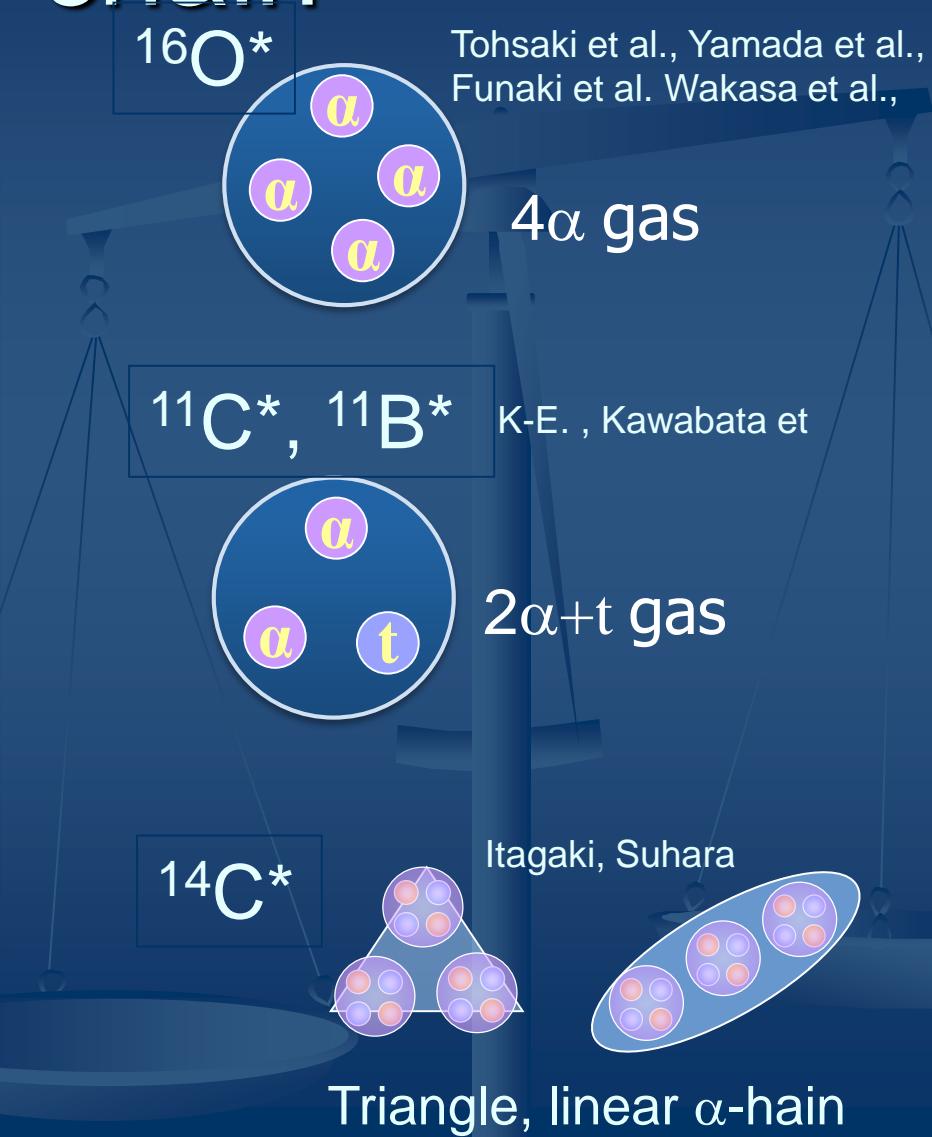
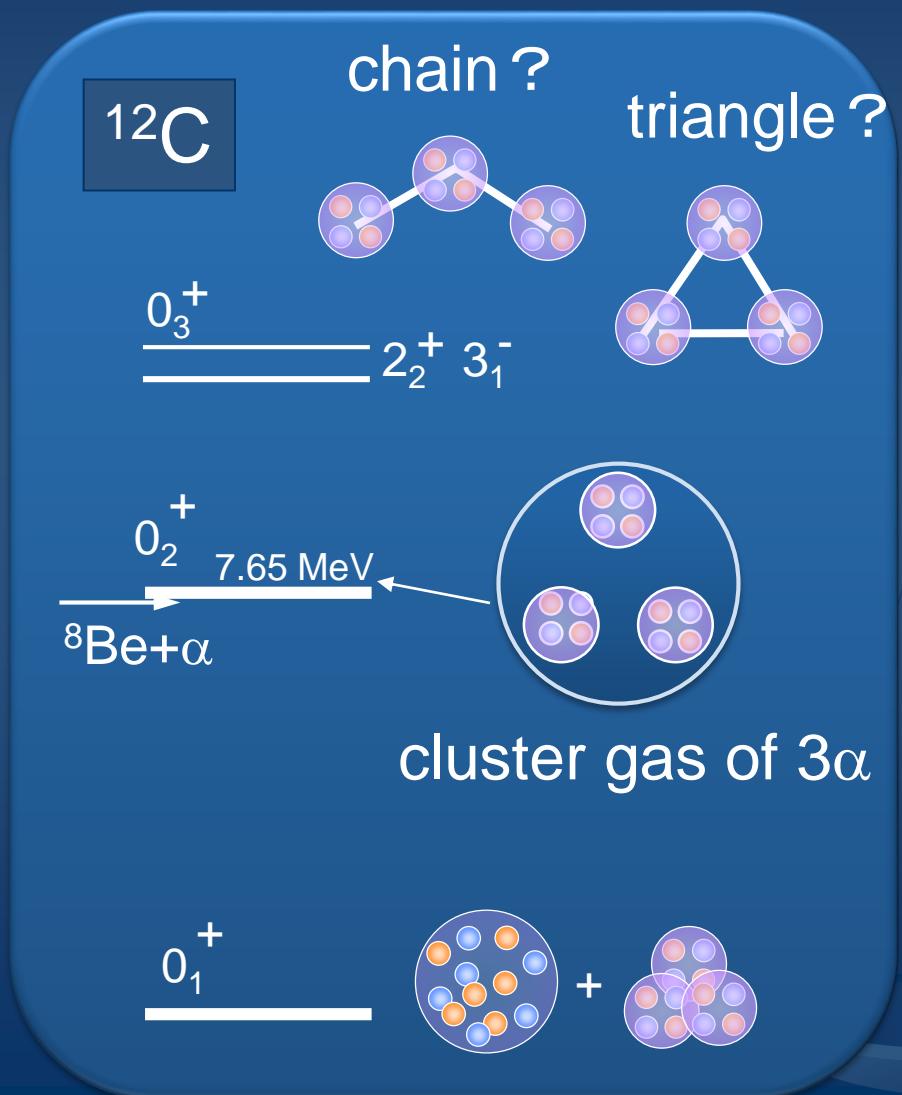


Cluster

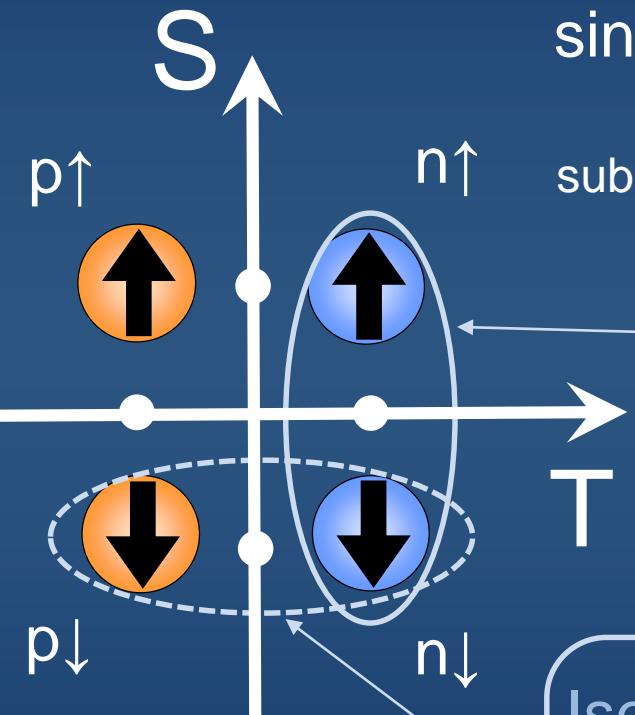


Vanishing of Magic number

multi-cluster: gas, triangle, linear chain



α cluster: 4 species of fermions



SU(4): $SU(2) \times SU(2)$ 4 species of fermions
singlet: $T=0, S=0$ ${}^4\text{He}$, α clusters $\text{ffff}=\text{b}$

subspace $SU(2)$ NN Pairs: $\text{ff}=\text{b}$

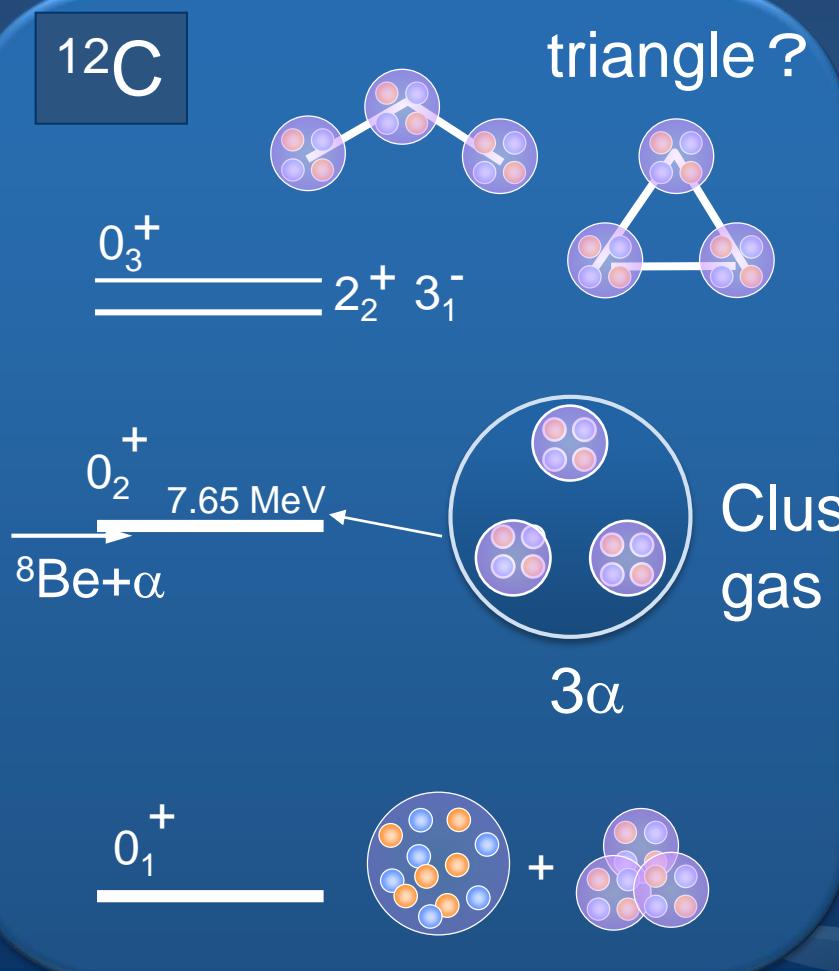
Iso-vector (""), spin-scalar ($S=0$)
Isospin active \rightarrow unstable nuclei
weak(β) -decay

Iso-scalar ($T=0$), spin-vector ("S=1")
spin active \rightarrow nuclear spintoronics
non-central (Is, tensor), EM (M1), β (GT)

1. $S=0, T=0$ world $Z=N=\text{even}$ nuclei

Excitations in Coordinates(r, L) space

multi-cluster: gas, triangle, linear chain

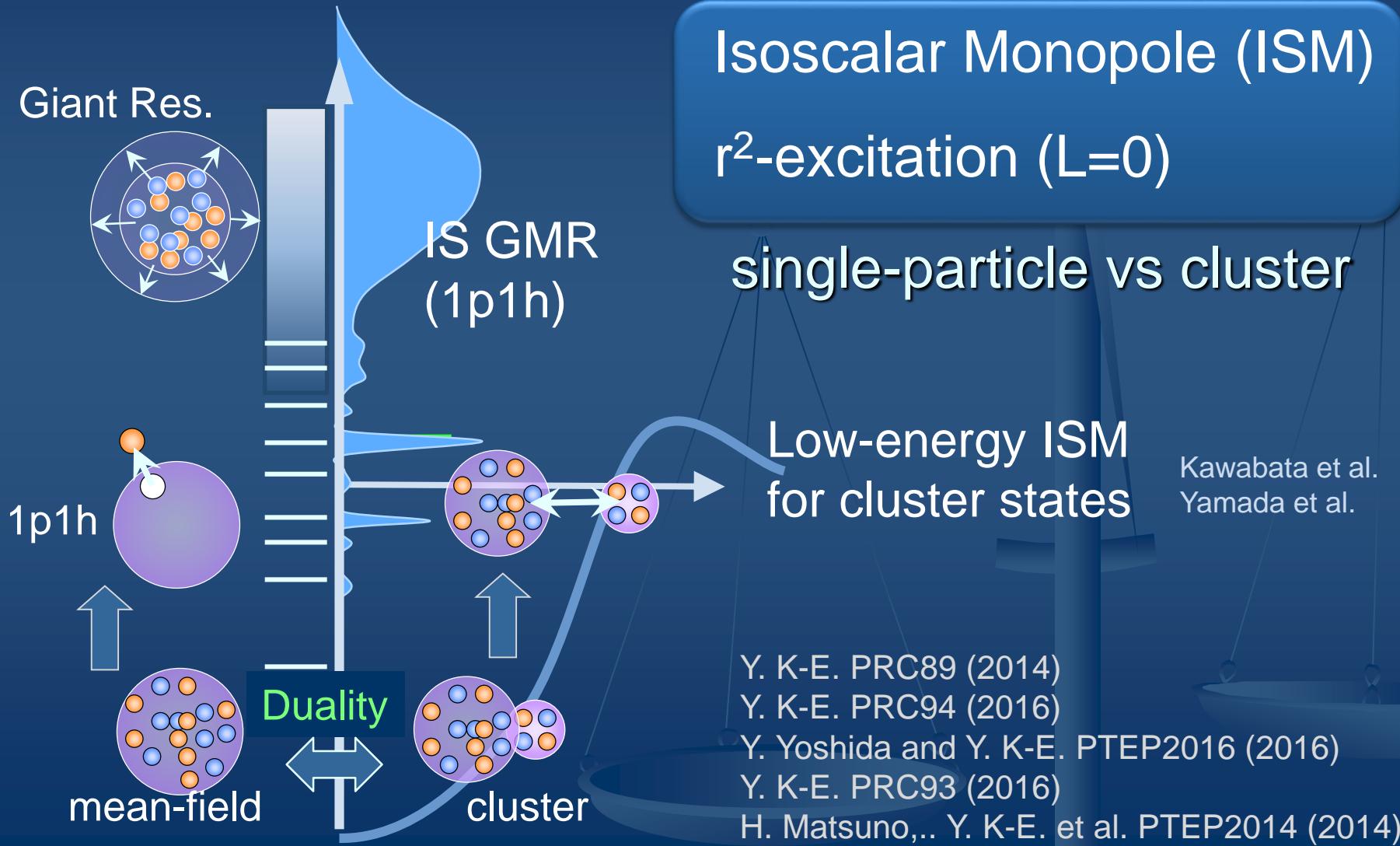


Analogy to
BEC in nuclear matter

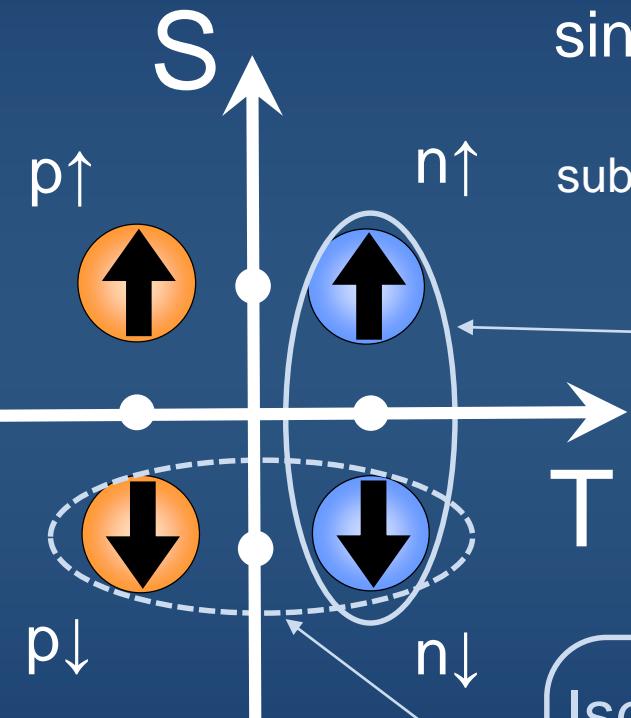


1. $S=0, T=0$ world $z=N=\text{even nuclei}$

Excitations in Coordinates(r, L) space



Species of particles



SU(4): $SU(2) \times SU(2)$ 4 species of fermions
singlet: $T=0, S=0$ ${}^4\text{He}$, α clusters $\text{ffff}=\text{b}$

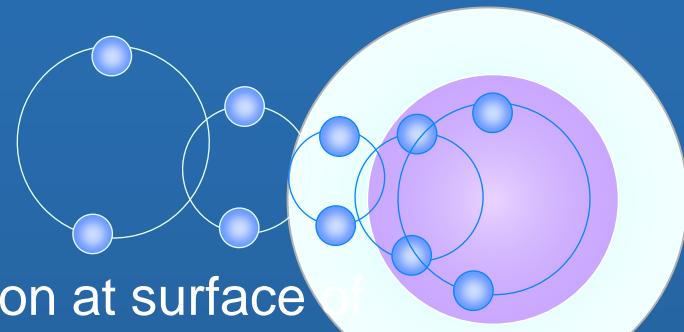
subspace $SU(2)$ NN Pairs: $\text{ff}=\text{b}$

Iso-vector (“ $T=1$ ”), spin-scalar ($S=0$)
Isospin active \rightarrow unstable nuclei
weak(β) -decay

Iso-scalar ($T=0$), spin-vector (“ $S=1$ ”)
spin active \rightarrow nuclear spintorronics
non-central (Is, tensor), EM (M1), $\beta(\text{GT})$

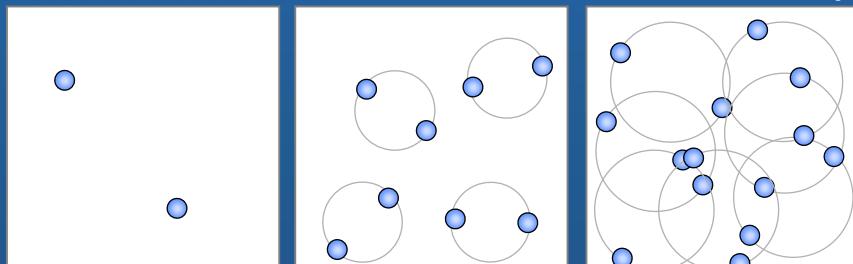
Isospin $T \neq 0$ world $\text{core}_{T=0} + \text{nn..}$ dynamics

Neutron skin, halo, matter



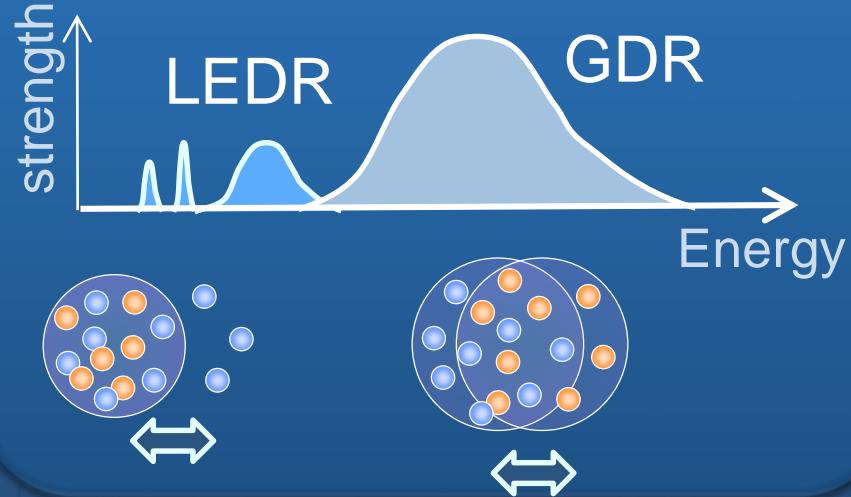
Dineutron at surface of
neutron-rich nuclei

Neutron matter Matsuo et al. PRC73 044309 ('06)



Different from alpha matter !

Isovector(E1) excitations



$^{14}\text{C}^*$

Itagaki, Suhara



Triangle, linear α -hain

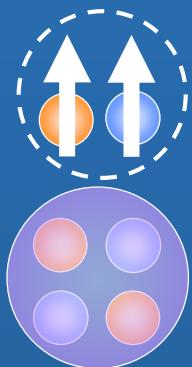
Spin $S \neq 0$ world pn pair in $Z=N=$ odd nuclei

$nn \rightarrow pn \quad GT(\sigma\tau)$

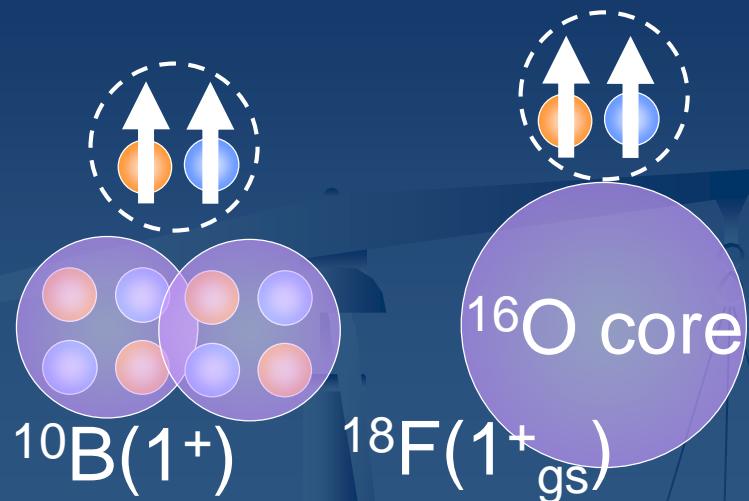
Gamov-Teller transition
Rotation in $SU(2) \times SU(2)$ space
Strong GT in $SU(4)$ symmetry

Super-allowed GT
by $SU(4)$ symmetry

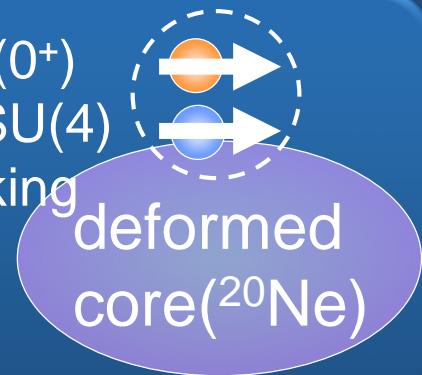
$S=1, T=0$
pn pair



${}^6Li(1^+_{gs})$



${}^{22}Na(1^+) \leftarrow {}^{22}Ne(0^+)$
GT splitting by $SU(4)$
Symmetry breaking
in spin space
(defo. Is force)

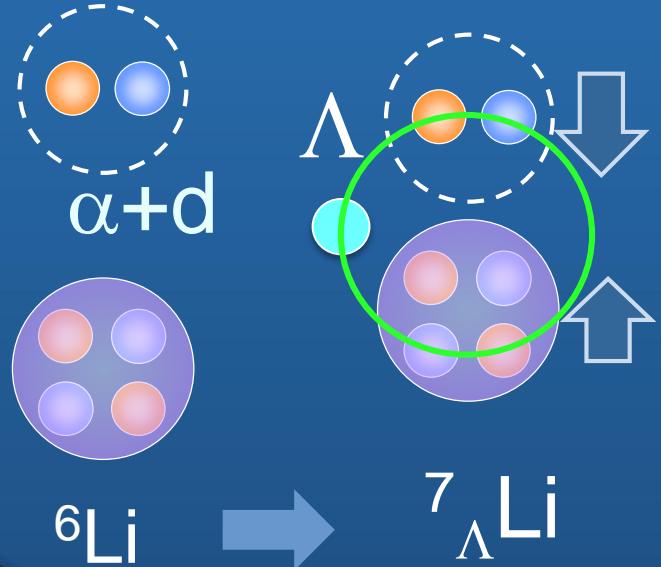


Isospin-Strangeness world SU(6)

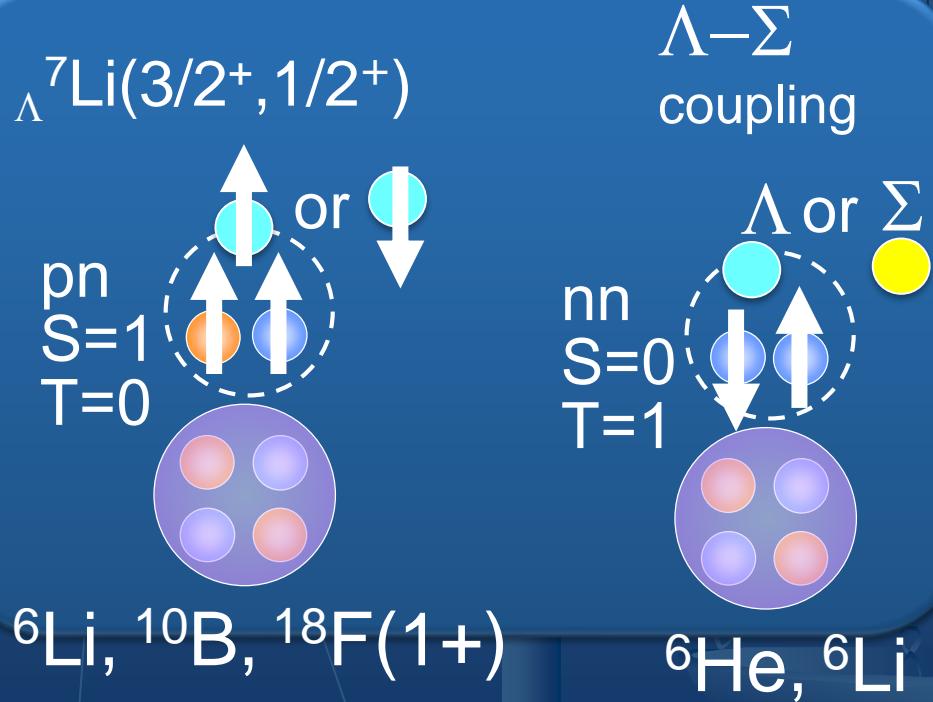
■ Λ T=0, Σ T=1

spin-independent Λ -N
Dynamical effect in r-space

Glue-like role ${}^7_{\Lambda}\text{Li}$, ${}^{13}_{\Lambda}\text{C}$
Motoba et al., Hiyama et al.,
Tanida et al.



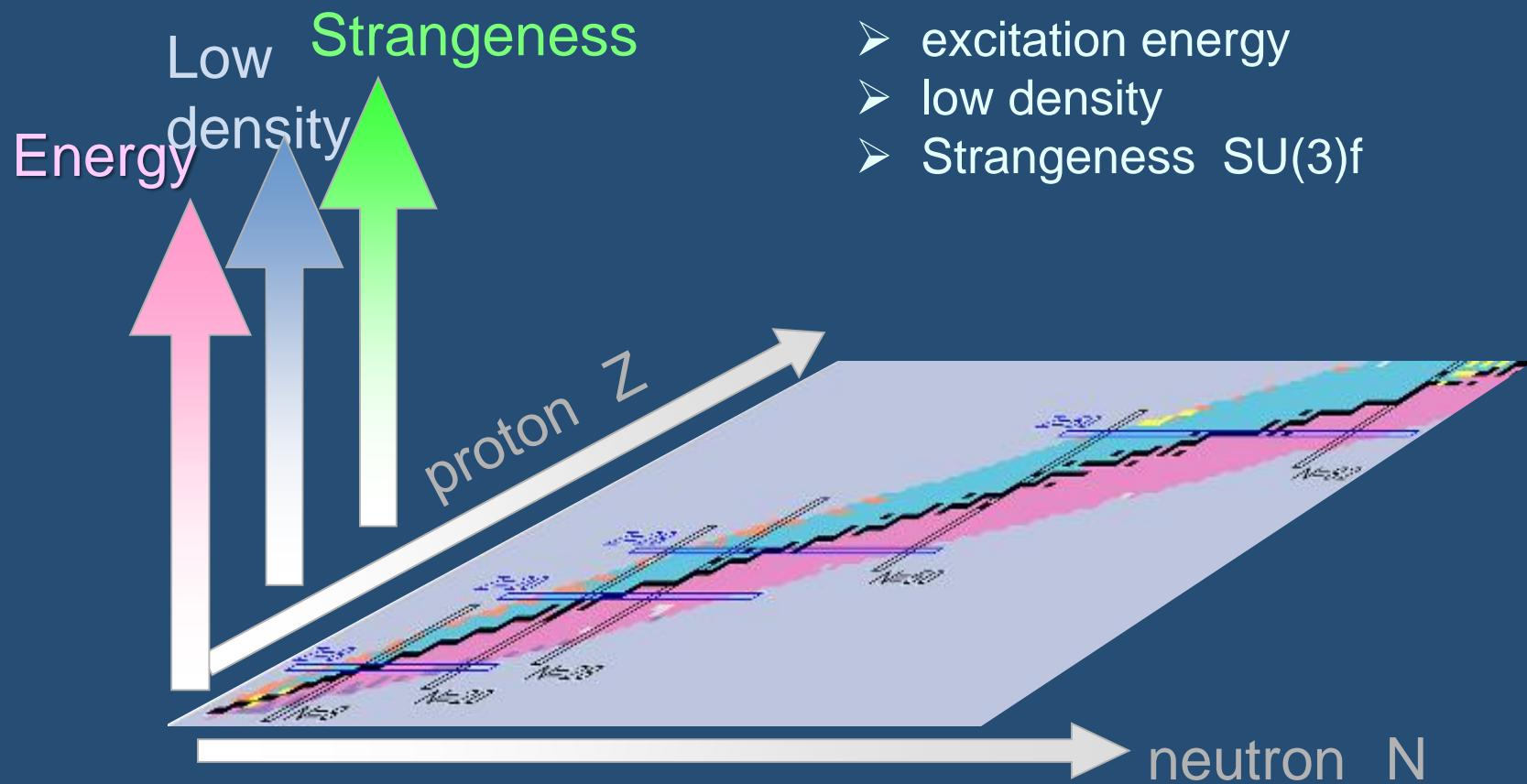
spin- and isospin-dependence: Λ -N
spin $S \neq 0$ nuclei & isospin $T \neq 0$ nuclei



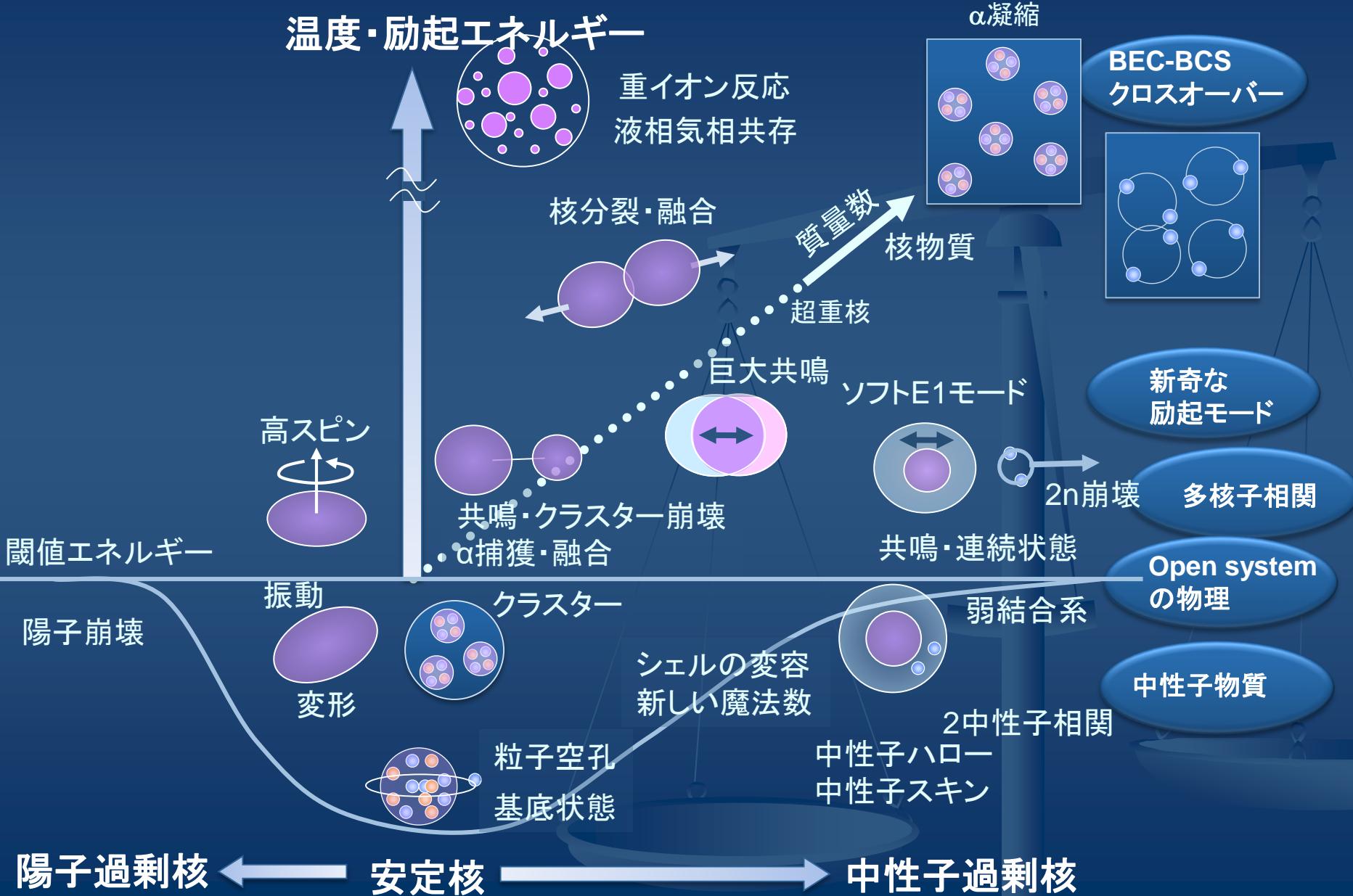
Y. K-E. PRC97 (2018)
Y. K-E. PRC97 (2018)
Y. K-E. M. Isaka, T. Motoba, arXiv:1803.04089

Active flavor(isospin) world

■ New Facilities



核子多体系の多様な現象



近年の進展、課題と展望

新奇な
現象発見
予言

従来の常識・模型の破綻

新たな理論的枠組みの発展
基礎(核力)から構築する理論

(近年の大発見の例)

★魔法数の消滅

★異常に大きな半径
(中性子ハロー・スキュー)

★核融合・分裂

★奇妙な励起モード
(変形、E1励起、GT)

★対相関・クラスター

★ ^{12}C 励起状態の謎



第一原理計算の発展

大規模シェル模型計算
テンソル力、3体力

少数系計算

弱結合系・連続状態

反応理論の進展

平均場理論(DFT)の発展
二核子(対)相関

クラスター現象記述の発展
アルファ凝縮

新しい物理の発展

原子核の統一的理解

新魔法数、新モード、多体相関
核物質の状態方程式

対称エネルギー、圧縮率



元素合成・中性子星の
性質などの解明

→ 天体核物理への貢献

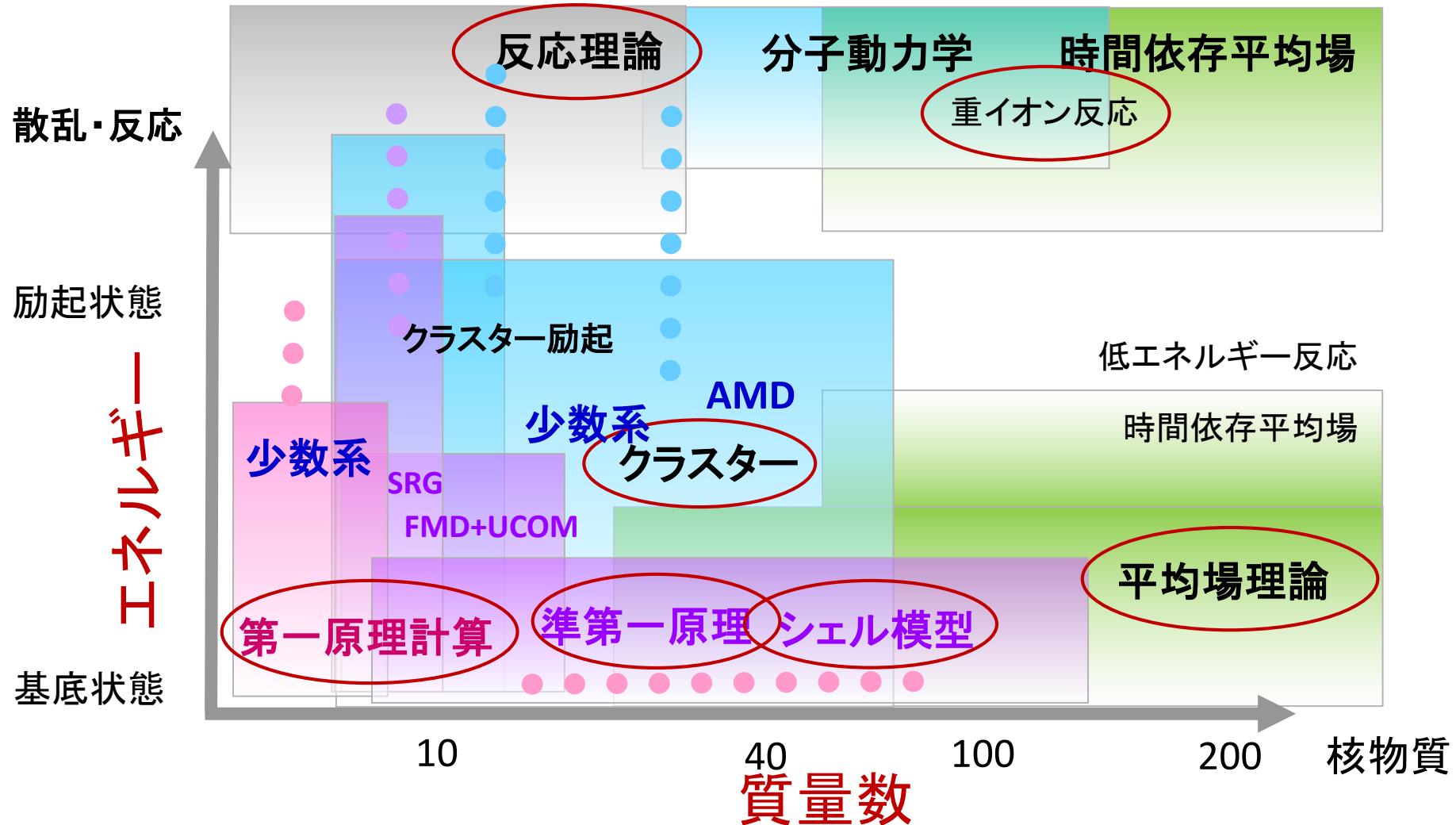
多体系の新しい存在様式

量子多体系の新現象、
普遍性、新理論

→ 物性物理との関連

系統的研究: 広範な質量数領域、エネルギー領域をカバー
予言力: 核力に基づく理論(テンソル、3体力)

広範な領域をカバーした系統的研究、核力に基づく予言力のある研究

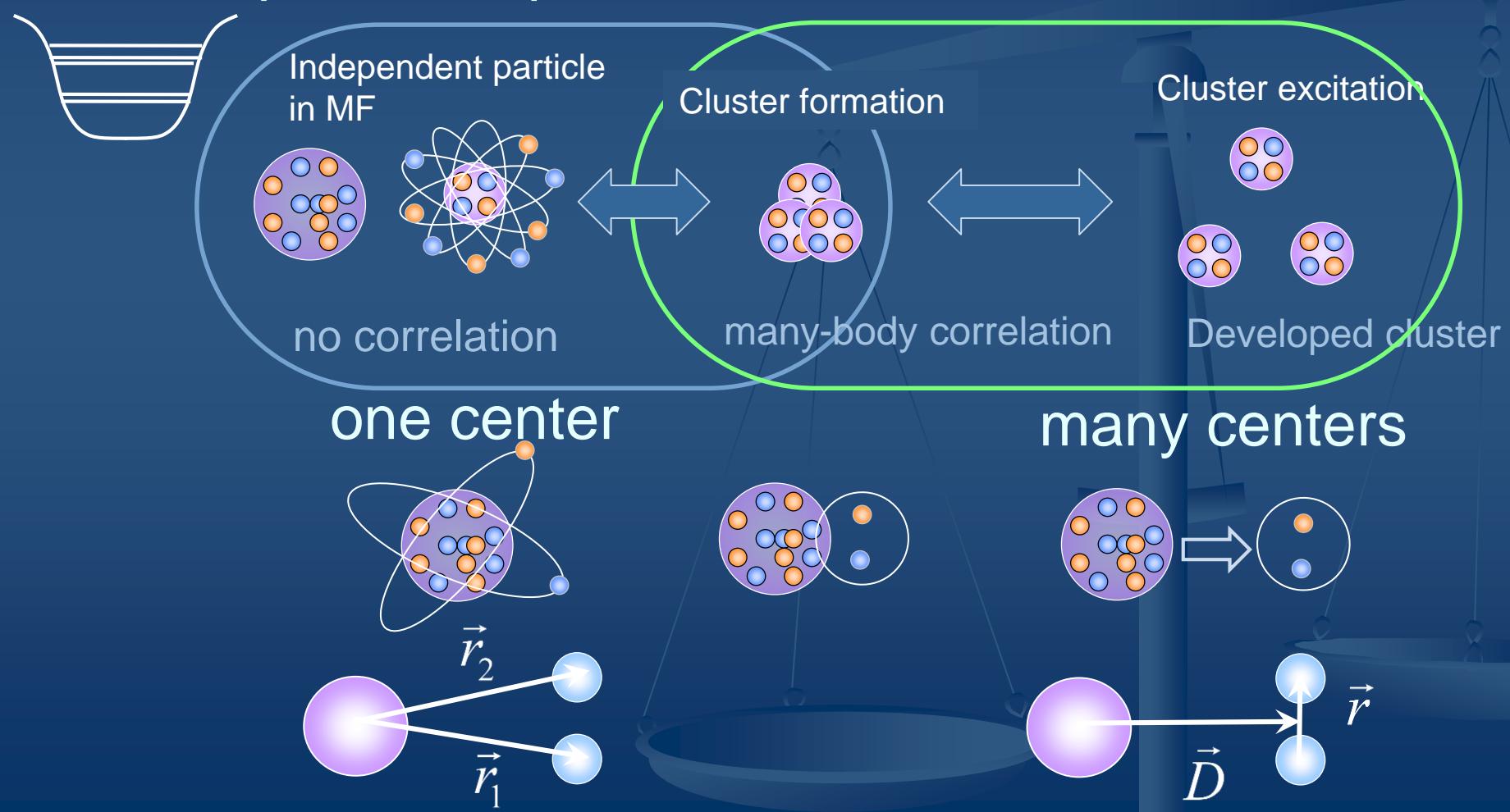


Lesson 2. What is cluster? (case of spatial correlations)

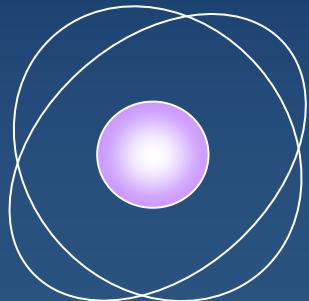
theoretical description

Cluster formation & development one center & many centers

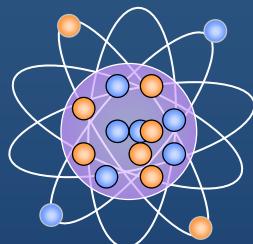
Independent sp in MF v.s. Cluster



How to measure correlations?



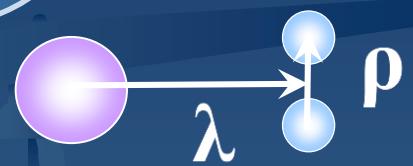
cluster size
≈ system size



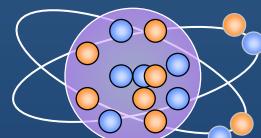
核子“非”局在



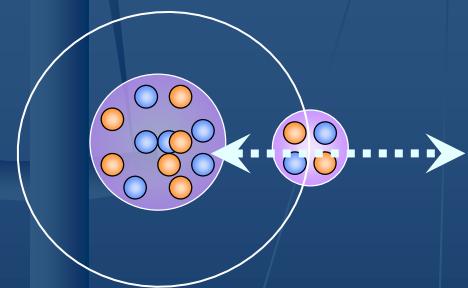
cluster size
> system size
duality



cluster size
>> system size



核子局在
クラスター局在

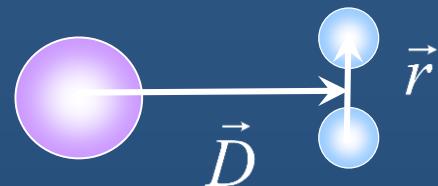
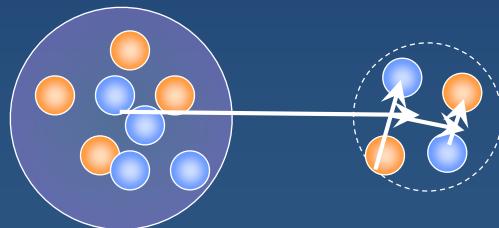


核子局在
クラスター“非”局在

1体密度(演算子)では区別できない→2体以上の相関

Cluster: strongly correlated state

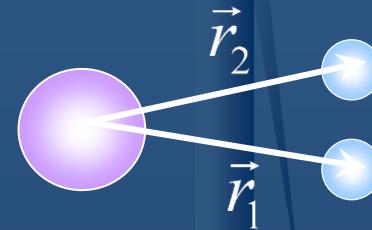
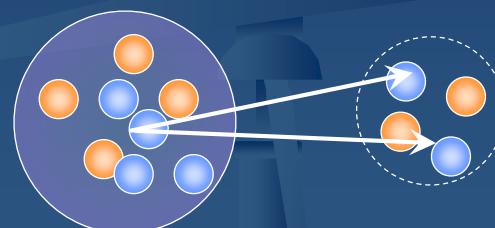
Strongly correlated state



$$\Delta r \Delta p \geq \hbar$$
$$l \approx D \times \frac{\Delta p}{\hbar} \geq \frac{D}{\Delta r}$$



Basis expansion from a center



Mixing of high angular momentum components

Large l

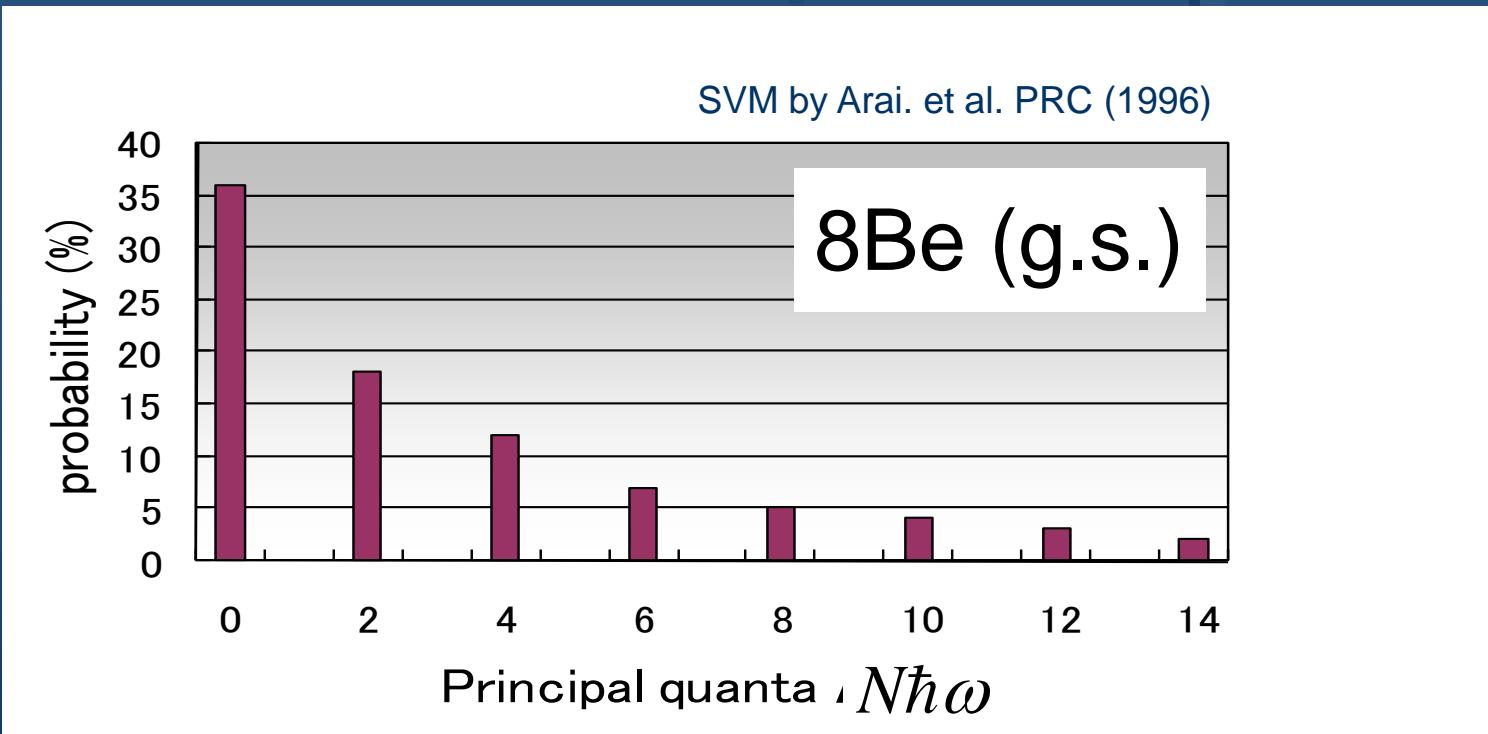
$l_1 \otimes l_2$

8Be (g.s.)

Developed 2α

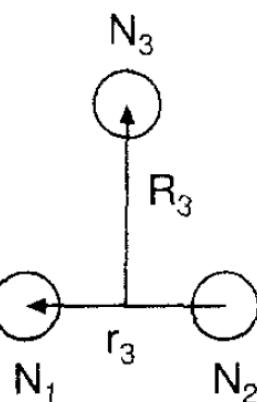
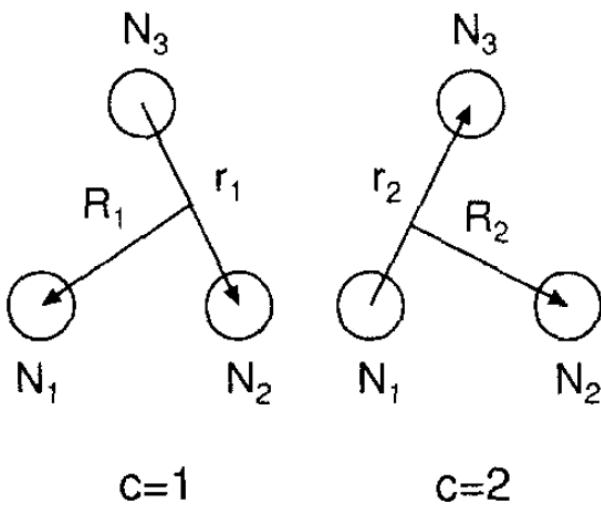


Strong spatial correlation involves higher-shell components.

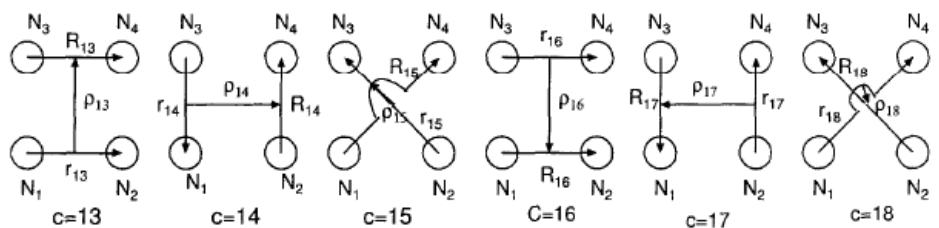
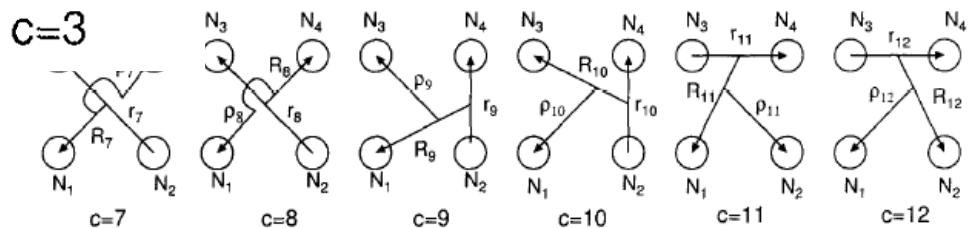
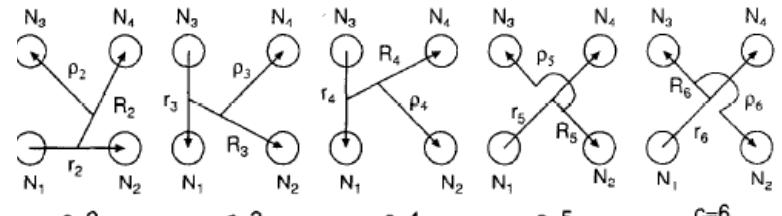


Developed cluster states are usually beyond mean-field

Jacobi coordinates



Hiyama, Kamimura
PPNP, 2003



Jacobi coordinates

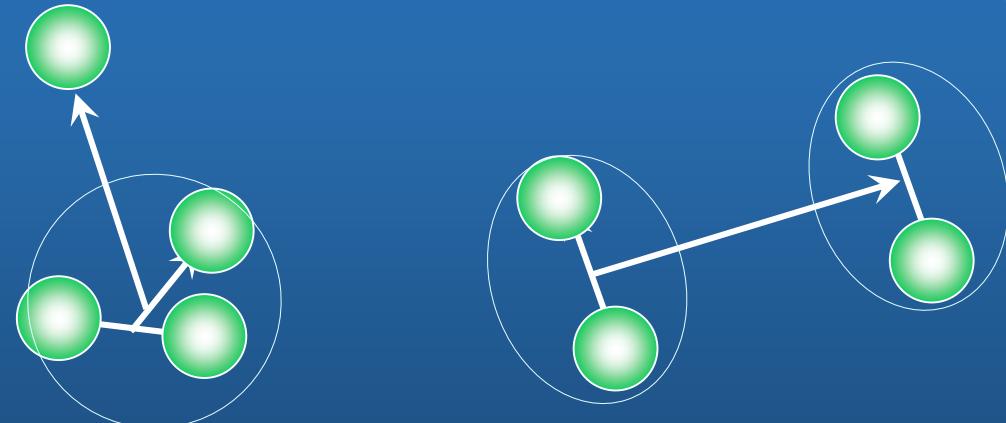
~coupling and decoupling of scale(DOF)

散乱問題なら当たり前

3-body



4-body



A model: AMD

Antisymmetrized molecular dynamics

AMD wave fn.

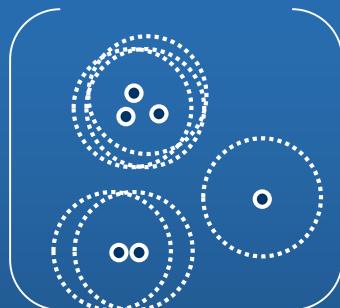
Similar to FMD wave fn.

$$\Phi = c\Phi_{\text{AMD}} + c'\Phi'_{\text{AMD}} + c''\Phi''_{\text{AMD}} + \dots$$

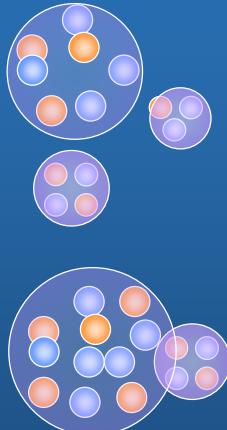
$$\Phi_{\text{AMD}} = \det \{\varphi_1, \varphi_2, \dots, \varphi_A\}$$

$$\varphi_i = \phi_{Z_i} \chi_i \begin{cases} \phi_{Z_i}(r_j) \propto \exp \left[-\nu \left(\mathbf{r} - \frac{\mathbf{Z}_i}{\sqrt{\nu}} \right)^2 \right] \\ \chi_i = \begin{pmatrix} \frac{1}{2} + \xi_i \\ \frac{1}{2} - \xi_i \end{pmatrix} \times \begin{array}{l} p \text{ or } n \\ \text{isospin} \end{array} \end{cases}$$

det



A variety of cluster st.



Cluster and MF formation/breaking

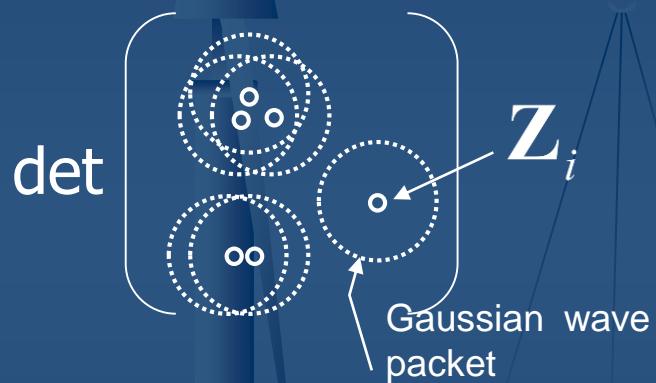


$$\Phi_{\text{AMD}}(\mathbf{Z})$$

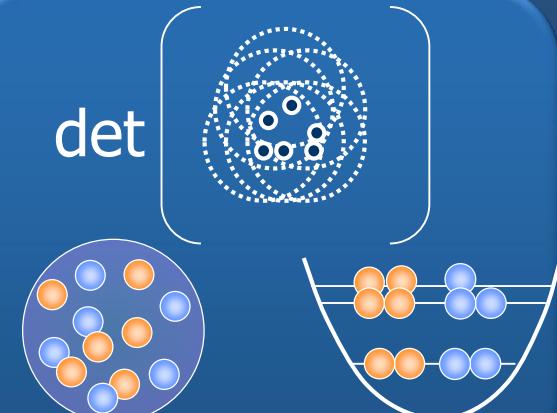
Variational parameters:

$$\mathbf{Z} = \{\mathbf{Z}_1, \mathbf{Z}_2, \dots, \mathbf{Z}_A, \xi_1, \dots, \xi_A\}$$

Gauss centers, spin orientations



det



Shell-model states

Energy variation

Model wave fn. $\Phi(\mathbf{Z})$

$$\delta \frac{\langle \Phi(\mathbf{Z}) | H | \Phi(\mathbf{Z}) \rangle}{\langle \Phi(\mathbf{Z}) | \Phi(\mathbf{Z}) \rangle} = 0$$

For $\delta\mathbf{Z}$

Phenomenological effective nuclear force

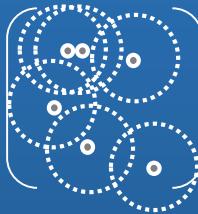
$$H^{\text{eff}} = \sum_{i=1} t_i + \sum_{i < j} v_{ij}^{\text{eff}} + \sum_{i < j < k} v_{ijk}^{\text{eff}}$$

Volkov, MV1, Gogny etc.

Energy surface

$$\frac{d\mathbf{Z}}{dt} = (\lambda + i\mu) \frac{1}{i\hbar} \frac{\partial E}{\partial \mathbf{Z}^*}$$

Randomly chosen
Initial states



Model space (Z plane)

$\Phi(\mathbf{Z}^0)$

Energy minimum

Variation, J^π -projection

- Variation after/before J^π -projection (VAP/VBP)
- Constraint AMD+GCM

Cluster limit v.s. Shell-model limit of AMD functions

Identical 2 fermions

$$\phi_1(r_1) \quad \phi_2(r_2)$$

Anti-Symm.



$$\det \begin{bmatrix} \phi_1(r_1) & \phi_2(r_1) \\ \phi_1(r_2) & \phi_2(r_2) \end{bmatrix} = \det \begin{bmatrix} \phi_1'(r_1) & \phi_2'(r_1) \\ \phi_1'(r_2) & \phi_2'(r_2) \end{bmatrix}$$

with

$$\phi_1' \propto \phi_1 + \phi_2$$

$$\phi_2' \propto \phi_1 - \phi_2$$

2 Gaussians at short distance

$$\phi_1 \quad \phi_2$$

$$\phi_1' = \text{Gaussian} + \text{Gaussian} = \text{s-orbit}$$

$$\phi_2' = \text{Gaussian} + \text{V-shape} = \text{p-orbit}$$

(0s)(0p)
config.

2α system



large d

Developed 2α

Strongly correlated



$d \rightarrow 0$ limit



$(0s)^4 (0p)^4$



$s^4 p^4$



Lesson 3.

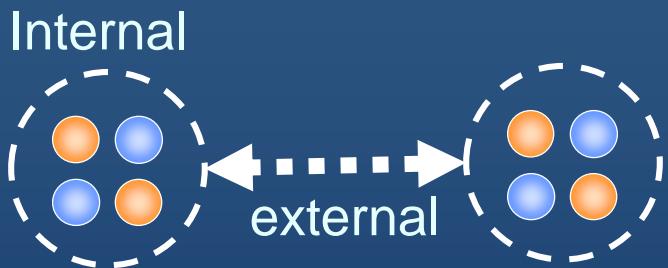
NN(pn,nn) correlation probed by Gamov-Tellar transition

clustering~correlations

- cluster=strongly correlated "sub"systems
- decoupling of "DOF" (scale)
- cluster limit: "internal"-strong & "external"-weak

by H. Tanaka, K. Ikeda

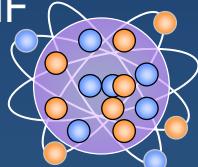
case 1)
spatial correlation



$$\psi(r_i, \sigma_i, \tau_i)$$
$$SO(3) \otimes SU(2) \otimes SU(2)$$

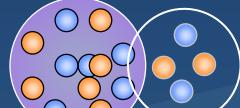
Cluster formation & development

Independent particle
in MF



no correlation

Cluster formation



correlation in
spin-isospin space

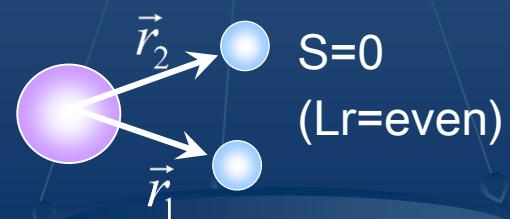
Cluster development



correlation in
coordinate space

Core+ α

S=0, T=0 object

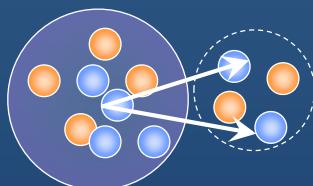


internal
coordinates



4N, 2N correlations at nuclear surface

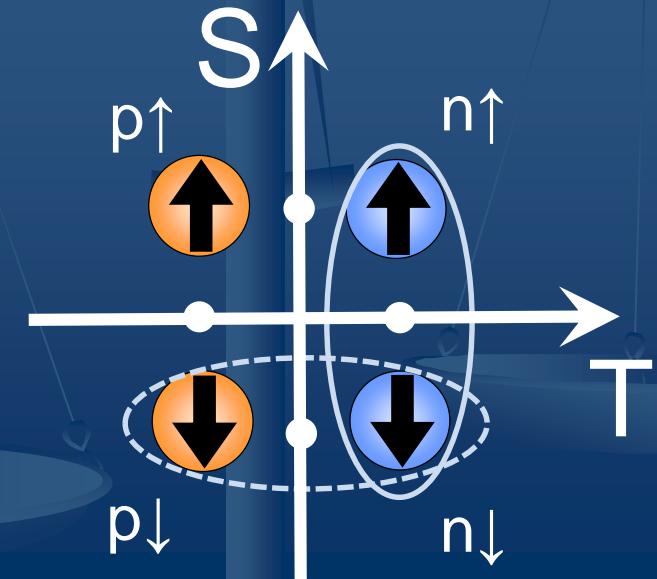
- Formation of S=0 (and T=0) subsystem
- decoupling of spin-isospin space
- Core+ $\alpha(s)$, core+NN-pair(s)
- with and without spatial development



(ST)=(00) 4N: α -cluster

(ST)=(01) 2N: nn (dineutron)

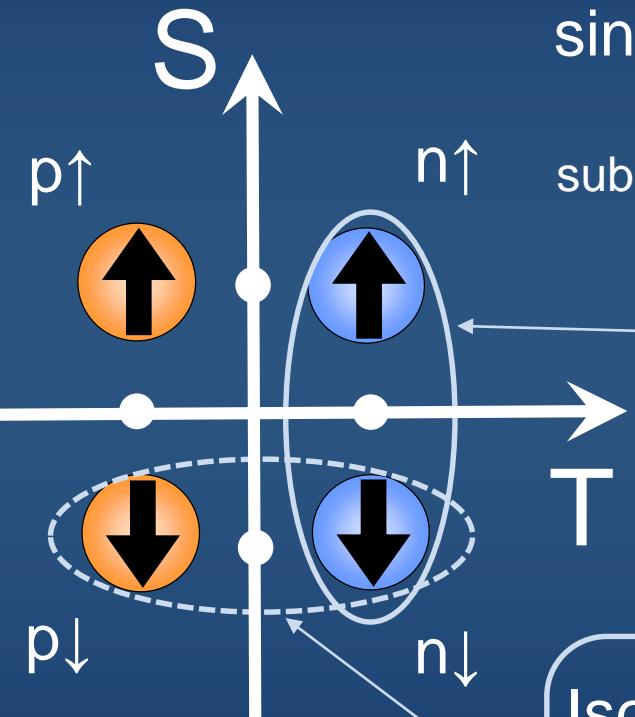
(ST)=(10) 2N: pn (deuteron)



Cluster: decoupling in spin-isospin(flavor) and/or coordinate space

- 2n-halo nuclei: nn-cluster
 - decoupling in spin-SU(2)&coordinate space
- heavy-quark symmetry in hadrons:qq-cluster
 - decoupling in spin-flavor space
- quarkyonic phase -> hadronization
 - decoupling in color-SU(3) space
 - but not in coordinate space (like BCS->BEC)
- hadron molecules: qqq(qqbar)-clusters
 - decoupling in color&coordinate space

4 species of particles



SU(4): $SU(2) \times SU(2)$ 4 species of fermions
singlet: $T=0, S=0$ ${}^4\text{He}$, α clusters $\text{ffff}=\text{b}$

subspace $SU(2)$ NN Pairs: $\text{ff}=\text{b}$

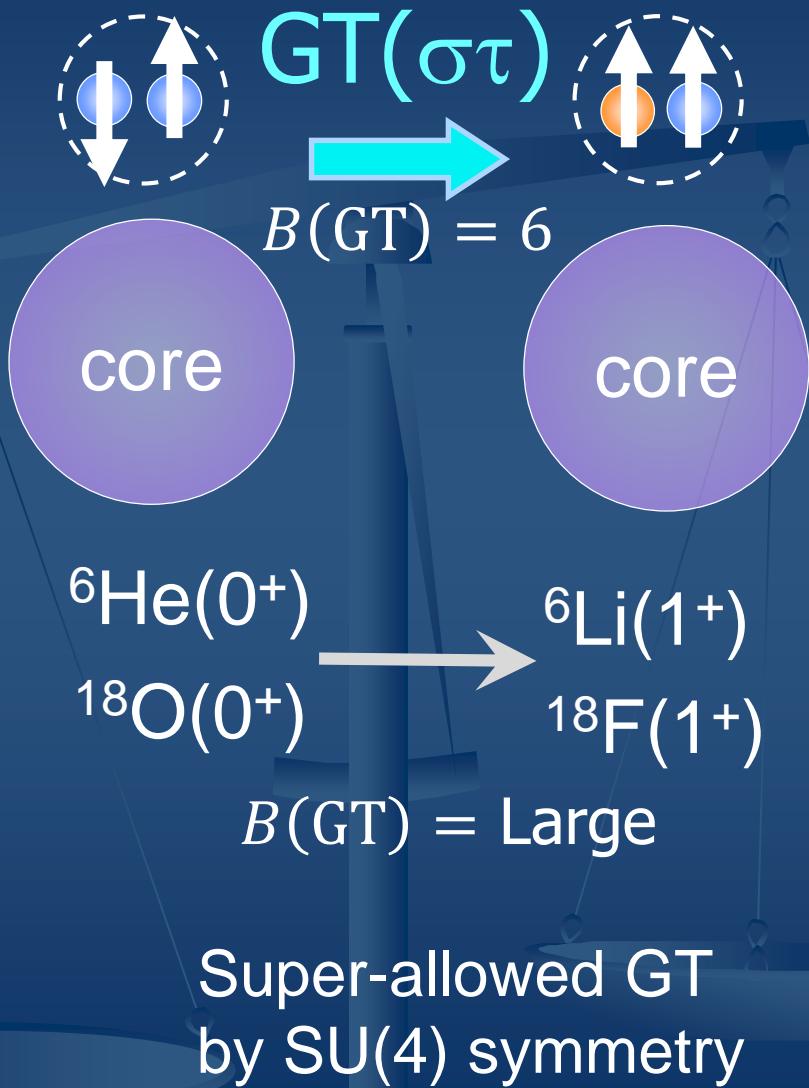
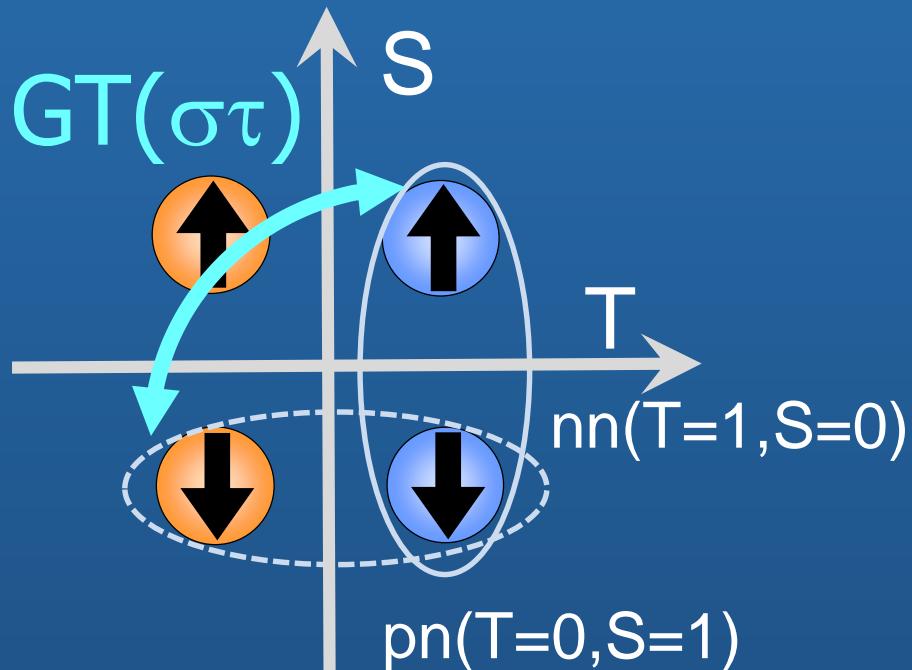
Iso-vector ("T=1"), spin-scalar (S=0)
Isospin active \rightarrow unstable nuclei
weak(β) -decay

Iso-scalar (T=0), spin-vector ("S=1")
spin active \rightarrow nuclear spintoronics
non-central (Is, tensor), EM (M1), $\beta(\text{GT})$

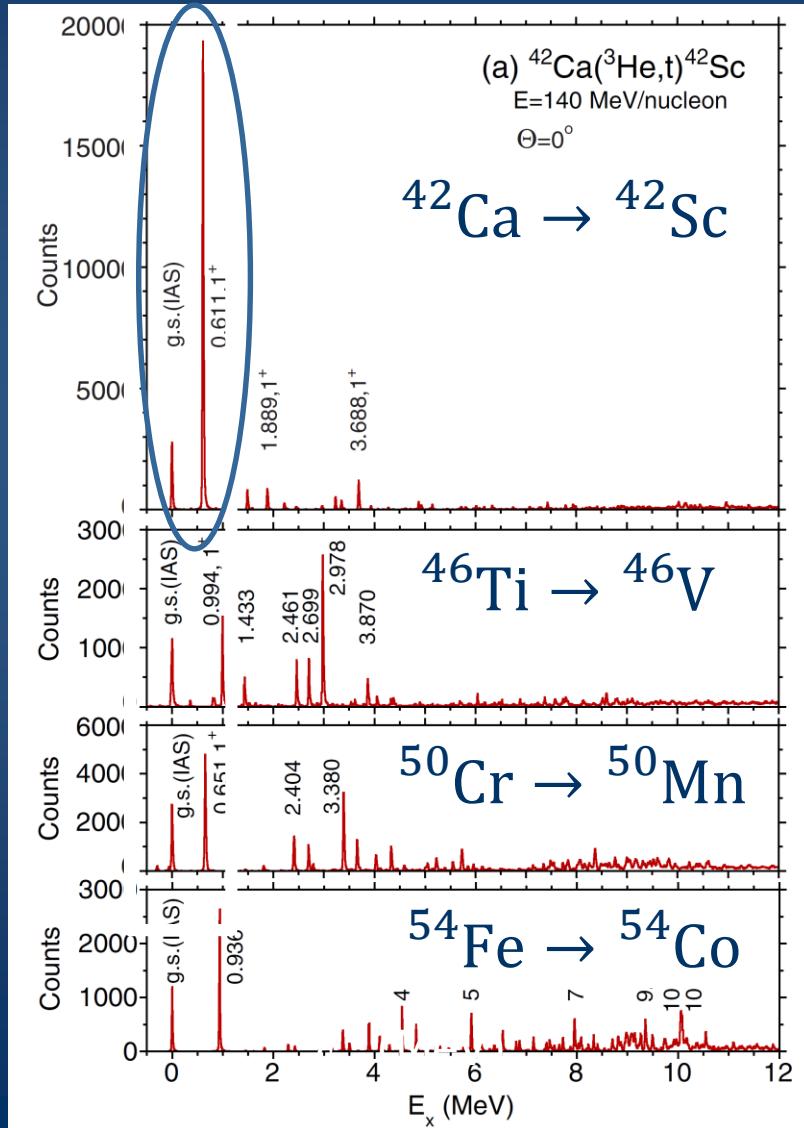
GT transition of pn pair in Z=N=odd nuclei

$nn \rightarrow pn \quad GT(\sigma\tau)$

Gamov-Teller = spin-isospin flip transition
Rotation in $SU(2) \times SU(2)$ space
Strong GT in $SU(4)$ symmetry



GT strength: N=Z+2 to N=Z=odd



$(^{40}\text{Ca} + \text{NN})$

▼ Super-allowed GT by
SU(4) symmetry

$(^{40}\text{Ca} + 6\text{N})$

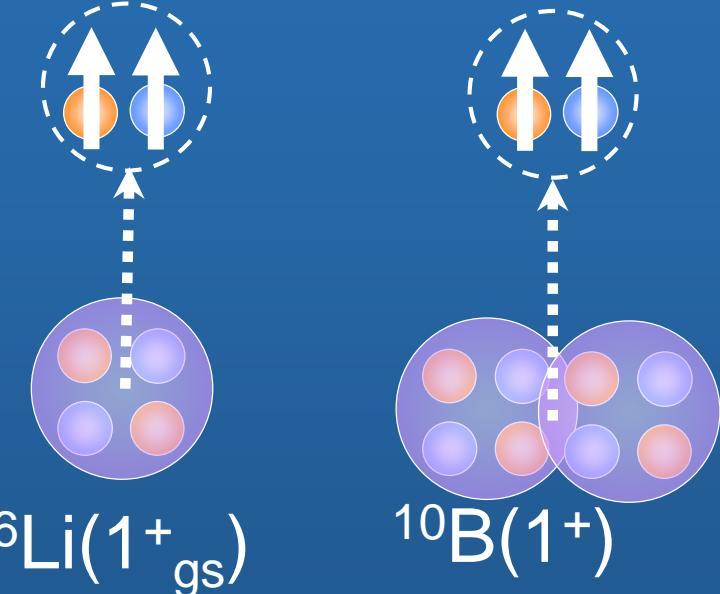
GT splitting
(partially broken
SU(4) symmetry)

$(^{40}\text{Ca} + 10\text{N})$

GT fragmented
symmetry broken
by spin-orbit int.

GT transition: pn pairs in p-, sd-shell

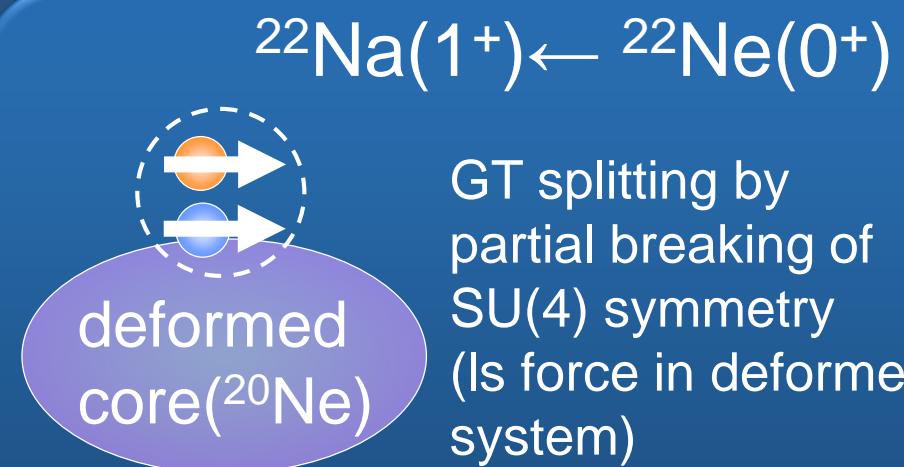
$S=1, T=0$ pn pair
with spatial development



Super-allowed GT

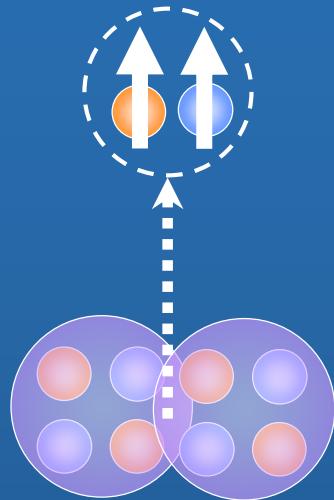


Super-allowed GT
by SU(4) symmetry



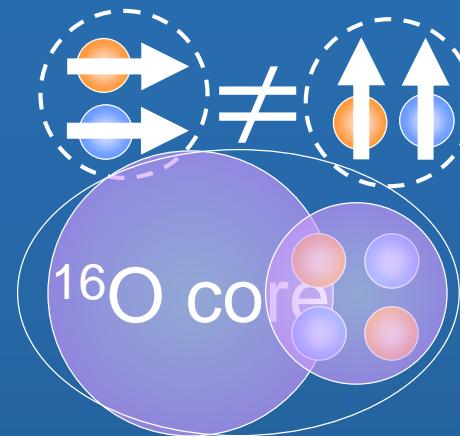
GT splitting by
partial breaking of
SU(4) symmetry
(ls force in deformed
system)

GT transition: pn pair in deformed nuclei



S=1,T=0 pn pair
with spatial development
free from
spin-orbit potentials

Super-allowed GT



ST=10 pair



ST=0 core

deformed core(^{20}Ne)

partial breaking
of SU(4) symmetry
by spin-orbit potential
in deformation

decoupling of
core+(NNNN)+NN
in SU(2)xSU(2)

coupling of
spin-SU(2)
x coordinate

GT splitting into two 1+ states

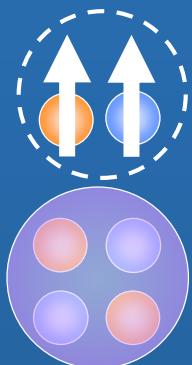
Spin $S \neq 0$ world pn pair in $Z=N=$ odd nuclei

$nn \rightarrow pn \quad GT(\sigma\tau)$

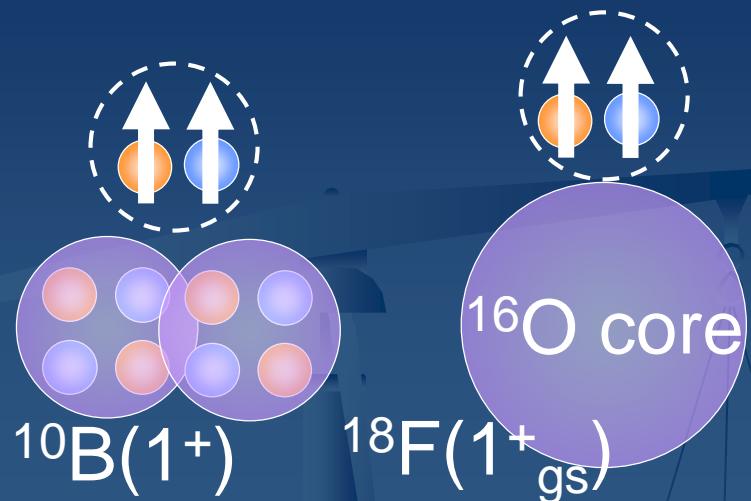
Gamov-Teller transition
Rotation in $SU(2) \times SU(2)$ space
Strong GT in $SU(4)$ symmetry

Super-allowed GT
by $SU(4)$ symmetry

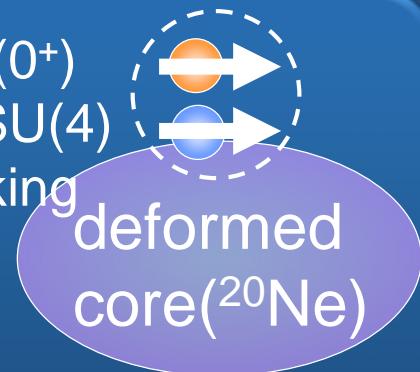
$S=1, T=0$
pn pair



${}^6Li(1^+_{gs})$



${}^{22}Na(1^+) \leftarrow {}^{22}Ne(0^+)$
GT splitting by $SU(4)$
Symmetry breaking
in spin space
(defo. Is force)



Lesson 4. cluster phenomena in neutron-rich Be isotopes

Model setting: AMD wave function

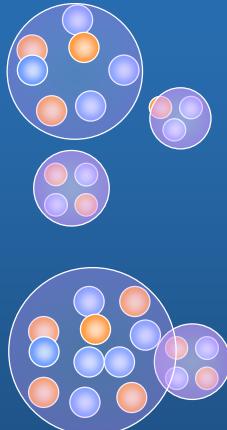
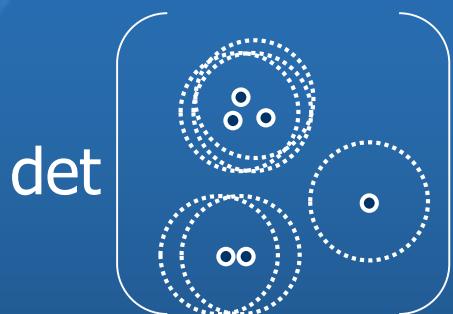
AMD wave fn.

Similar to FMD wave fn.

$$\Phi = c\Phi_{\text{AMD}} + c'\Phi'_{\text{AMD}} + c''\Phi''_{\text{AMD}} + \dots$$

$$\Phi_{\text{AMD}} = \det \{\varphi_1, \varphi_2, \dots, \varphi_A\}$$

$$\varphi_i = \phi_{Z_i} \chi_i \begin{cases} \phi_{Z_i}(r_j) \propto \exp \left[-\nu \left(\mathbf{r} - \frac{\mathbf{Z}_i}{\sqrt{\nu}} \right)^2 \right] \\ \chi_i = \begin{pmatrix} \frac{1}{2} + \xi_i \\ \frac{1}{2} - \xi_i \end{pmatrix} \times \begin{array}{l} p \text{ or } n \\ \text{isospin} \end{array} \end{cases}$$



Cluster and MF formation/breaking

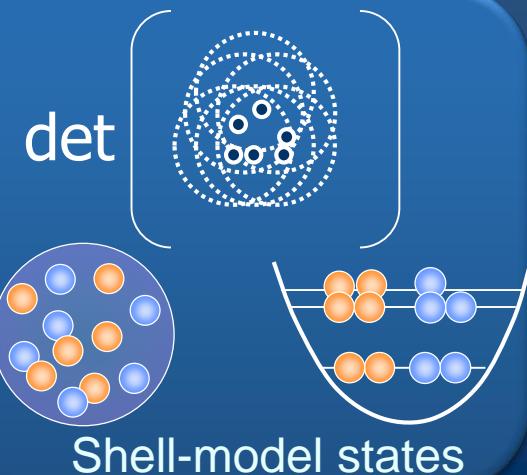
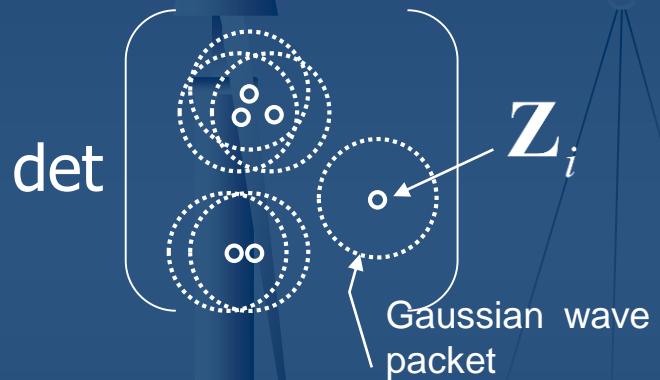


$$\Phi_{\text{AMD}}(\mathbf{Z})$$

Variational parameters:

$$\mathbf{Z} = \{\mathbf{Z}_1, \mathbf{Z}_2, \dots, \mathbf{Z}_A, \xi_1, \dots, \xi_A\}$$

Gauss centers, spin orientations



Energy variation

Model wave fn. $\Phi(\mathbf{Z})$

$$\delta \frac{\langle \Phi(\mathbf{Z}) | H | \Phi(\mathbf{Z}) \rangle}{\langle \Phi(\mathbf{Z}) | \Phi(\mathbf{Z}) \rangle} = 0$$

For $\delta\mathbf{Z}$

Phenomenological effective nuclear force

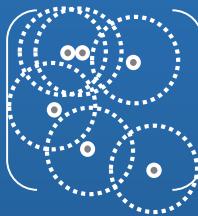
$$H^{\text{eff}} = \sum_{i=1} t_i + \sum_{i < j} v_{ij}^{\text{eff}} + \sum_{i < j < k} v_{ijk}^{\text{eff}}$$

Volkov, MV1, Gogny etc.

Energy surface

$$\frac{d\mathbf{Z}}{dt} = (\lambda + i\mu) \frac{1}{i\hbar} \frac{\partial E}{\partial \mathbf{Z}^*}$$

Randomly chosen
Initial states



Model space (Z plane)

$\Phi(\mathbf{Z}^0)$

Energy minimum

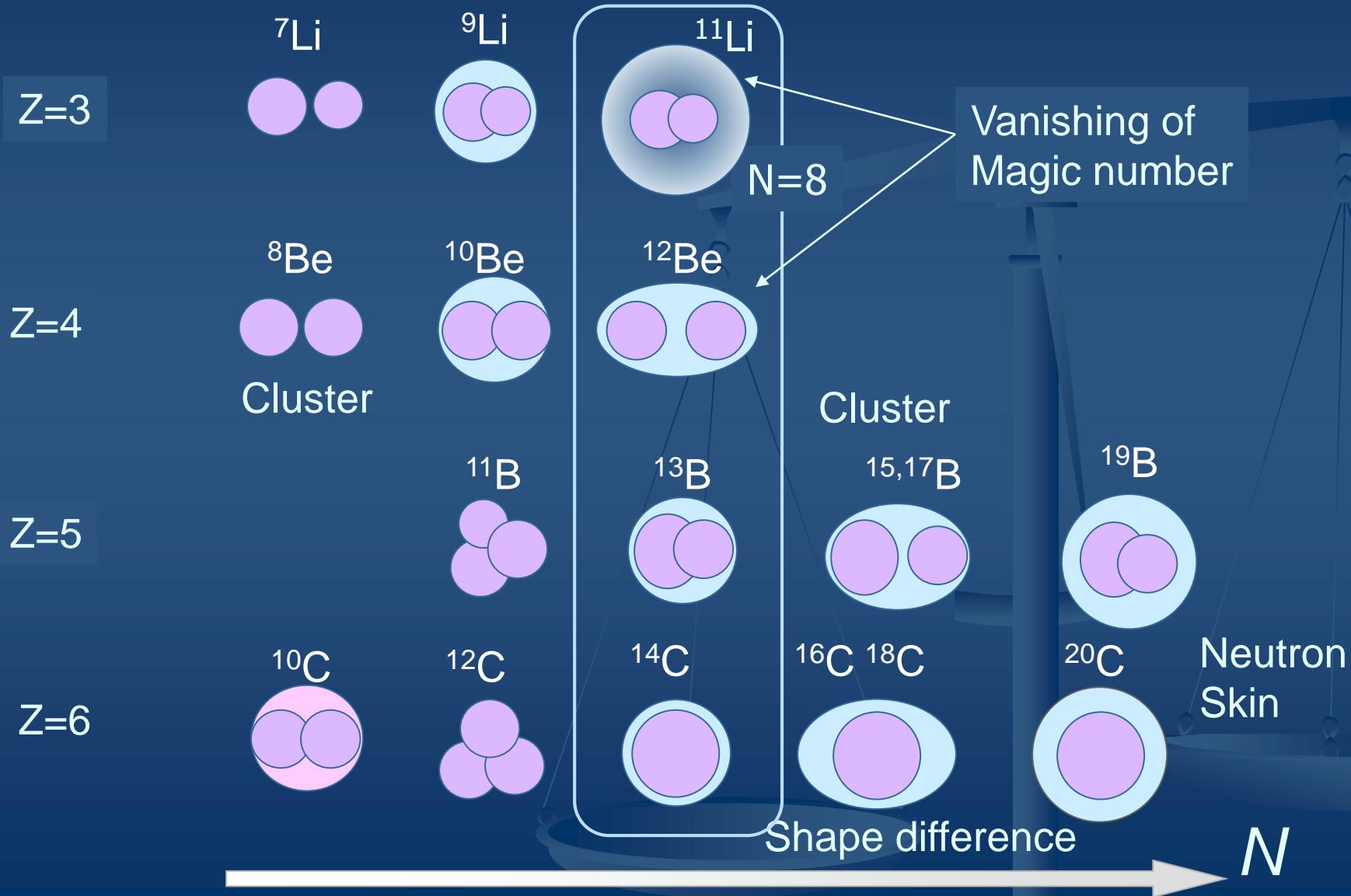
Variation, J^π -projection

- Variation after/before J^π -projection (VAP/VBP)
- Constraint AMD+GCM

- Z-,N-dependence of g.s. structure
- MO & magic number breaking
- Cluster resonances

- Z-,N-dependence of g.s. structure
- MO & magic number breaking
- Cluster resonances

Z -, N -dependence of ground states



How to experimentally probe N-dependence of clustering

◆ Clustering in g.s.



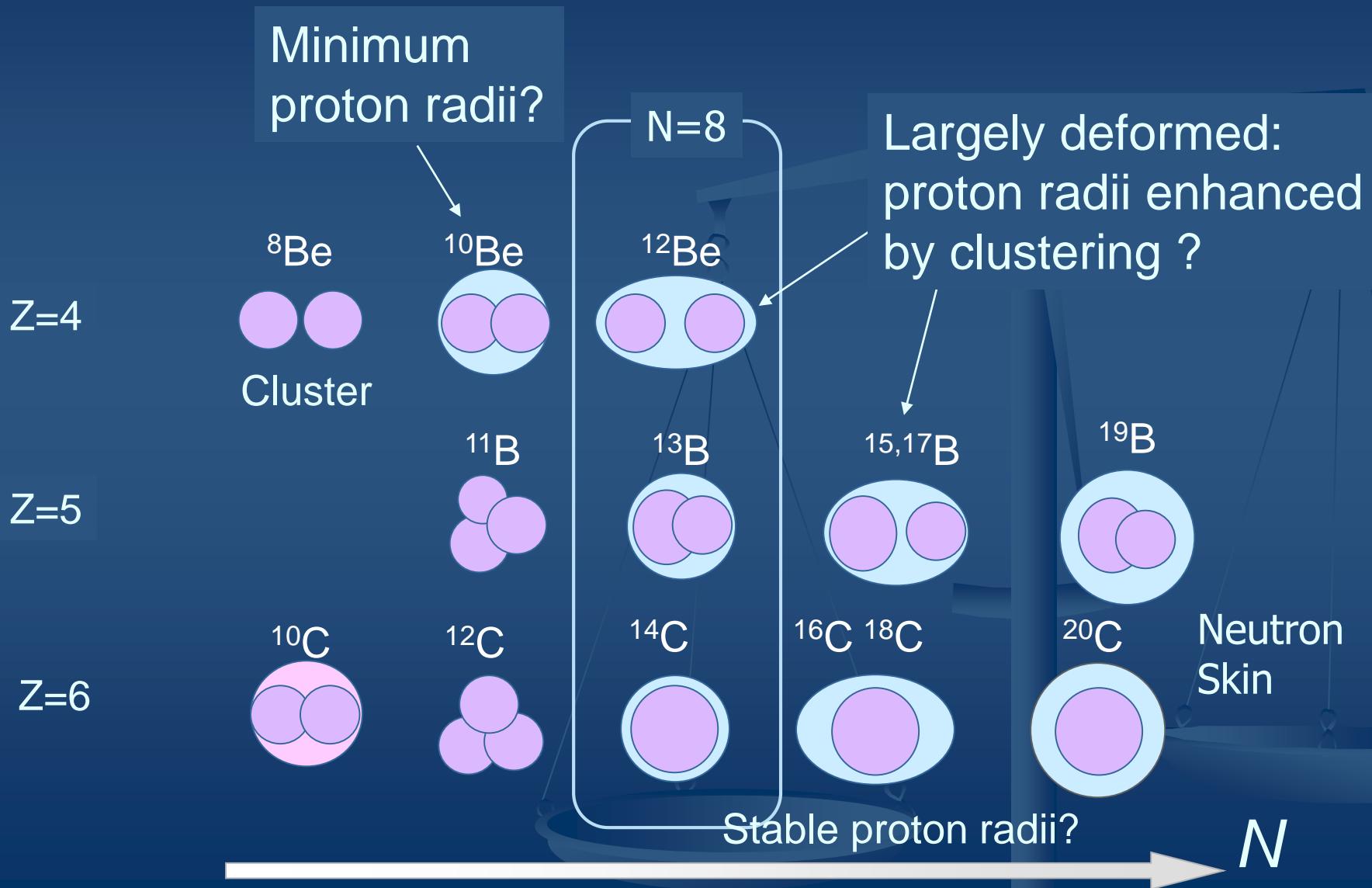
β_p, r_p
enhanced

- Deformation $Q, B(E2)$
- Charge radii
 - isotope shift: Be
 - charge changing σ : B, C

◆ Clustering in excited states

- α -transfer, α knock-out ?

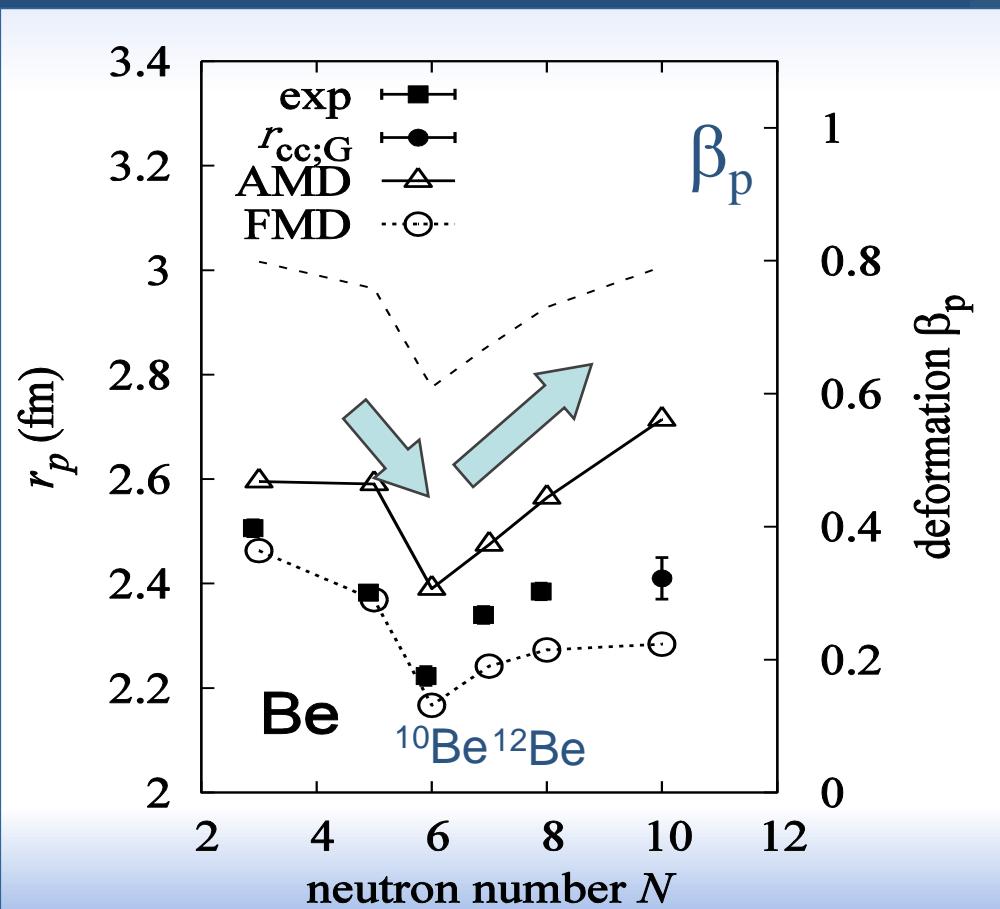
Z -, N -dependence of g.s. structure



proton radii along isotope chain

Exp data:

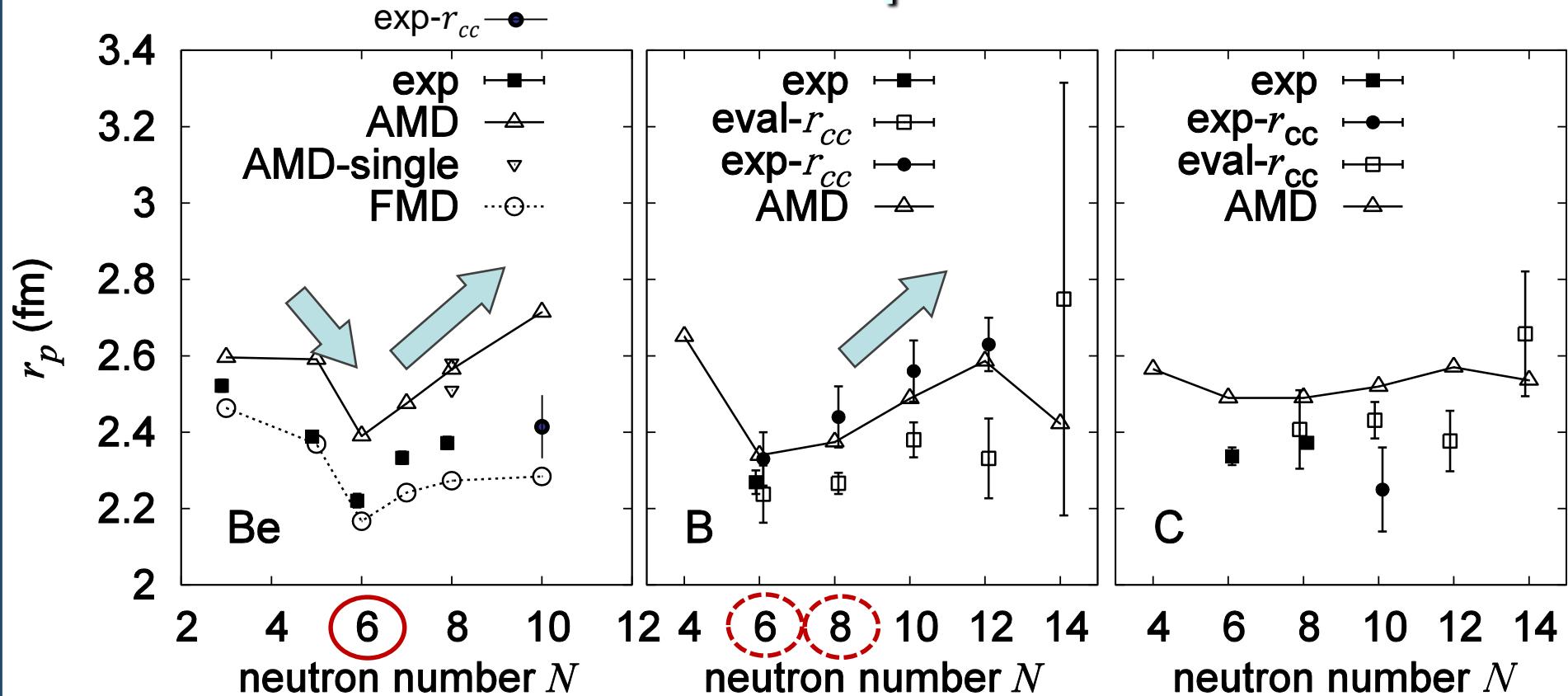
Isotope shift W. Nortershauser et al. PRL102, 062503 (2009); A. Krieger, PRL108, 142501 (2012).
CC :Terashima et al.



- minimum at $N=6$ (^{10}Be)
new magic?
- Increasing in $N>6$.
- proton radii(charge radii) ref
- N dependence of clustering.
- Good probe for cluster structure

N dependence of r_p in Be,B,C

AMD & exp. data



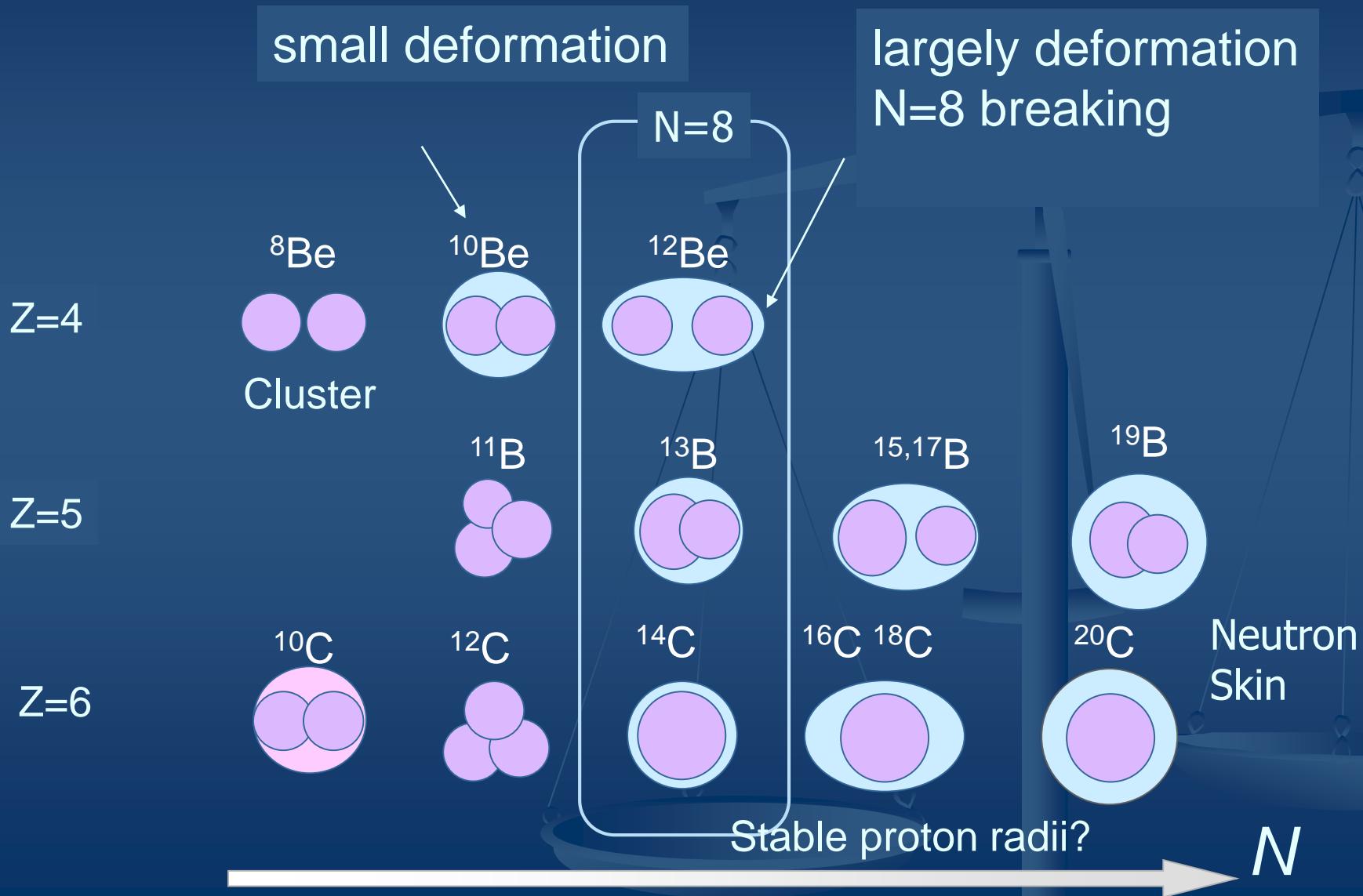
exp :determined by isotope shift

exp- r_{cc} :determined by charge changing(cc) cross section

eval- r_{cc} : evaluated by relation $\sigma_{cc} \propto \pi(R_p(P) + R_m(T))^2$ using cc cross section

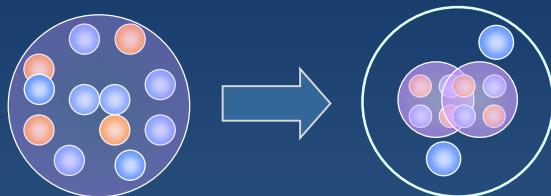
- Z-,N-dependence of g.s. structure
- MO & magic number breaking
- Cluster resonances

Z -, N -dependence of g.s. structure



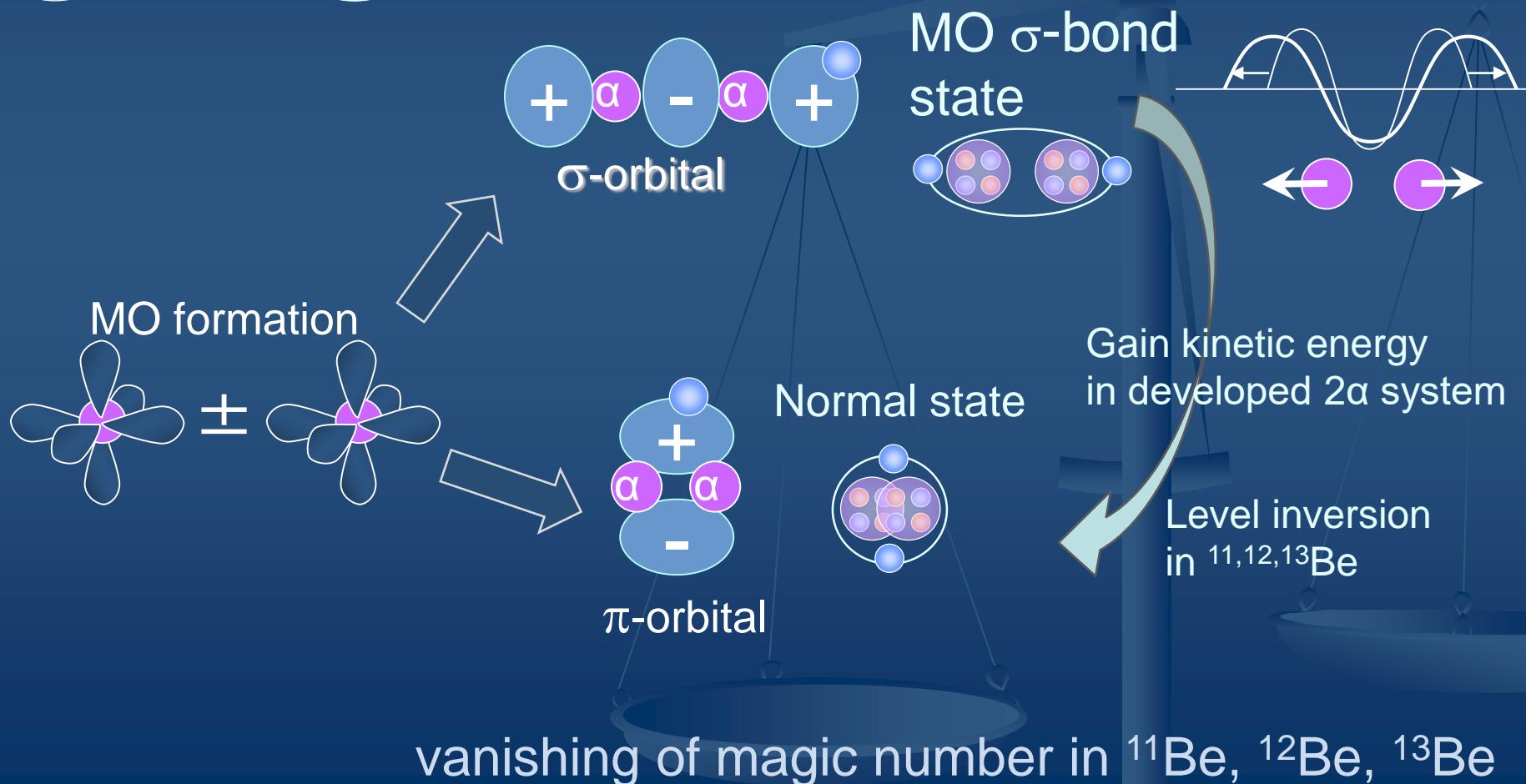
Molecular orbital(MO) structure in 2α

2α -core formation

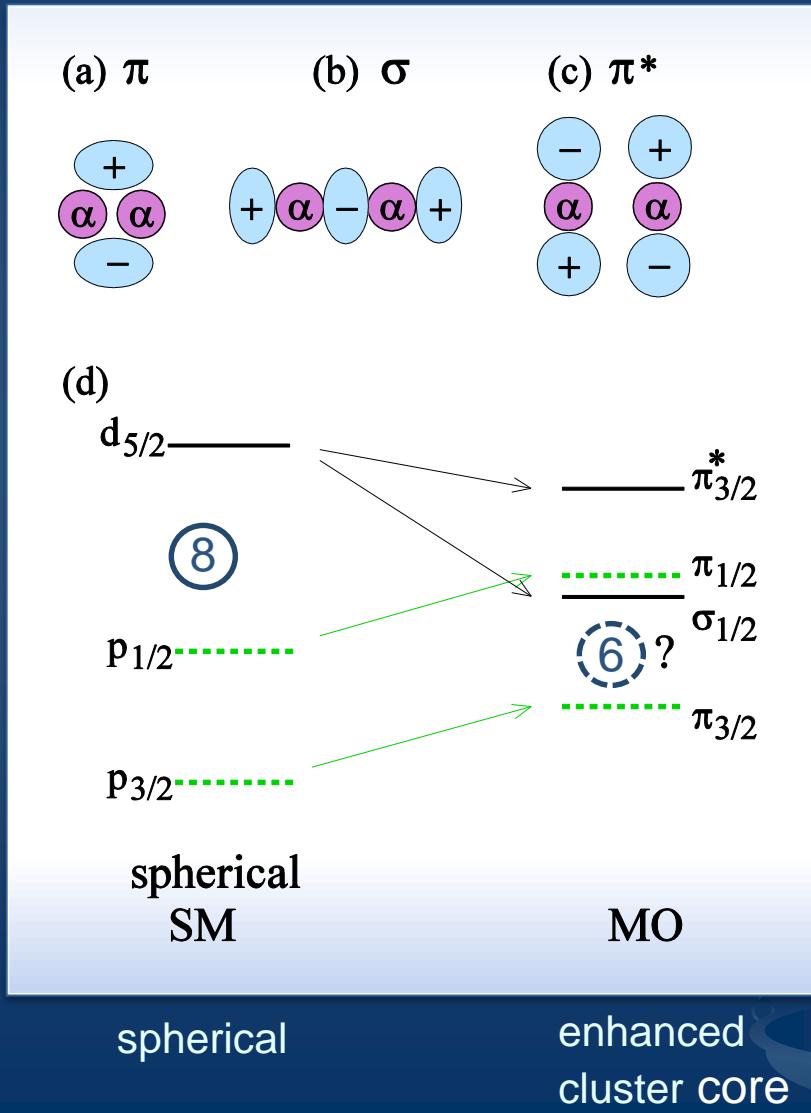


MO formation

Seya PTP65(81), von Oertzen ZPA354(96)
N. Itagaki PRC61(00), Y. K-E.. Ito PLB588(04)

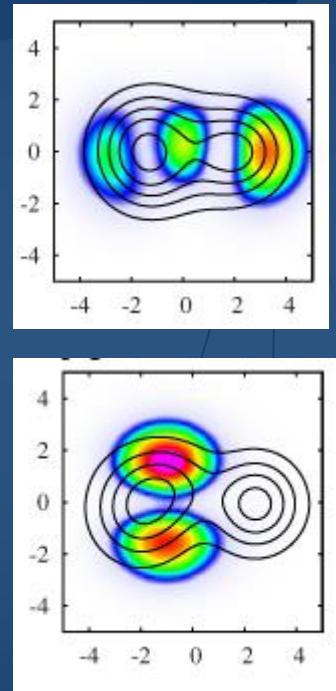
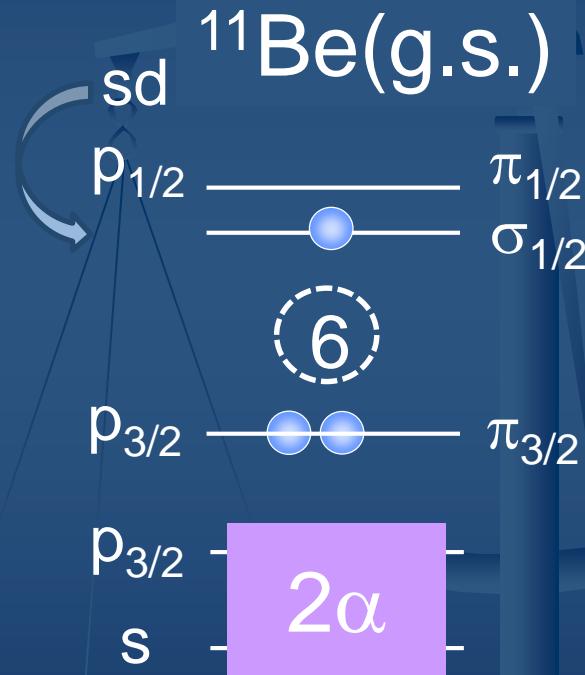
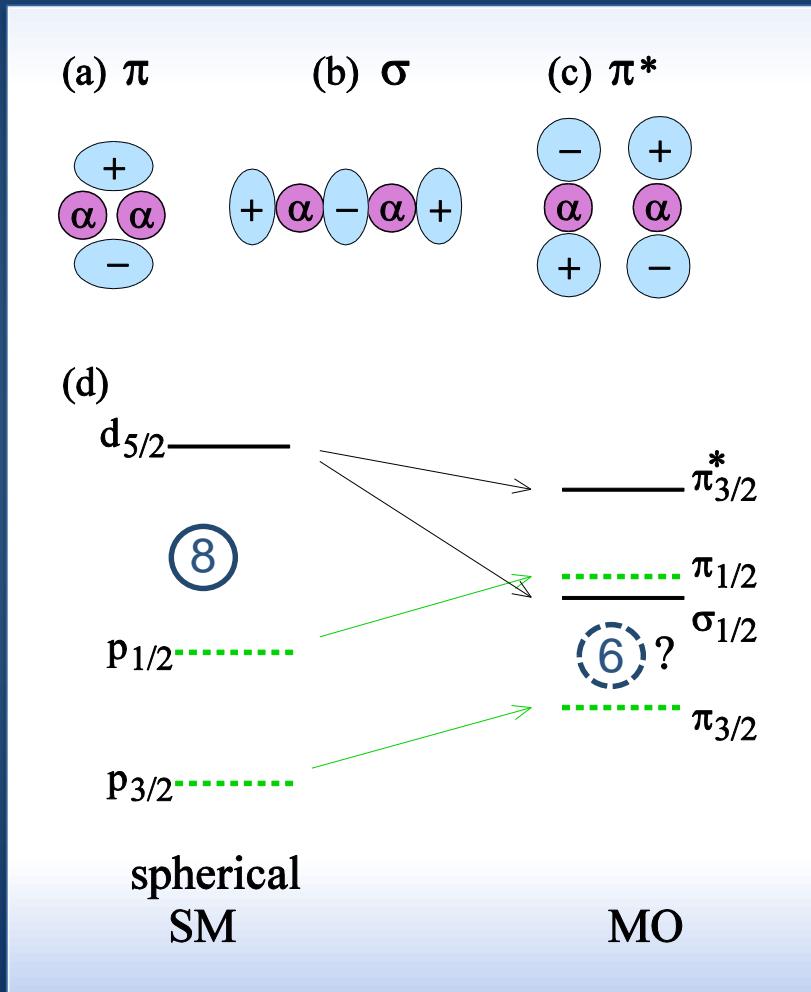


$N=8$ magic number breaking

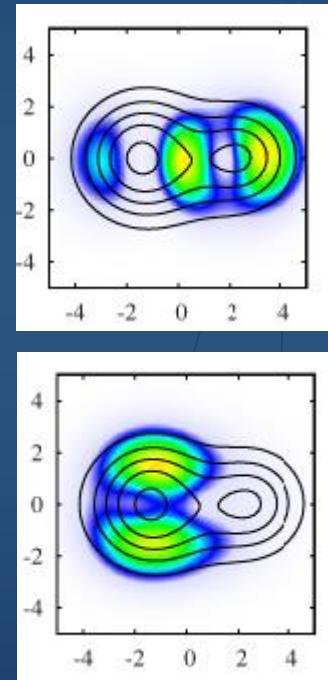
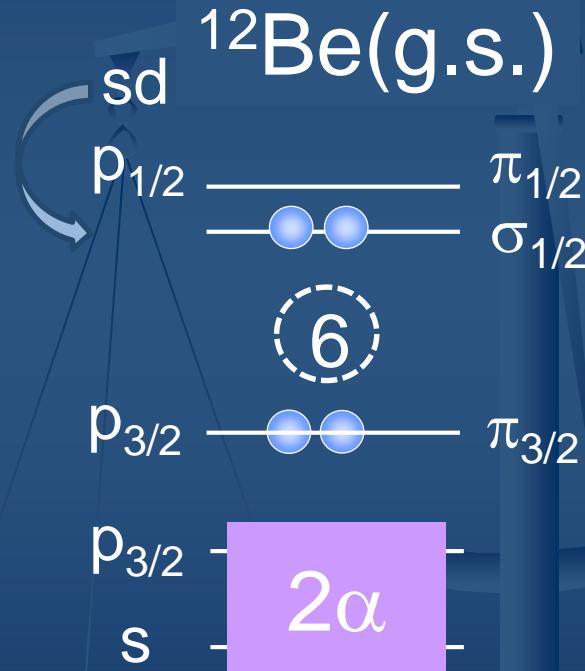
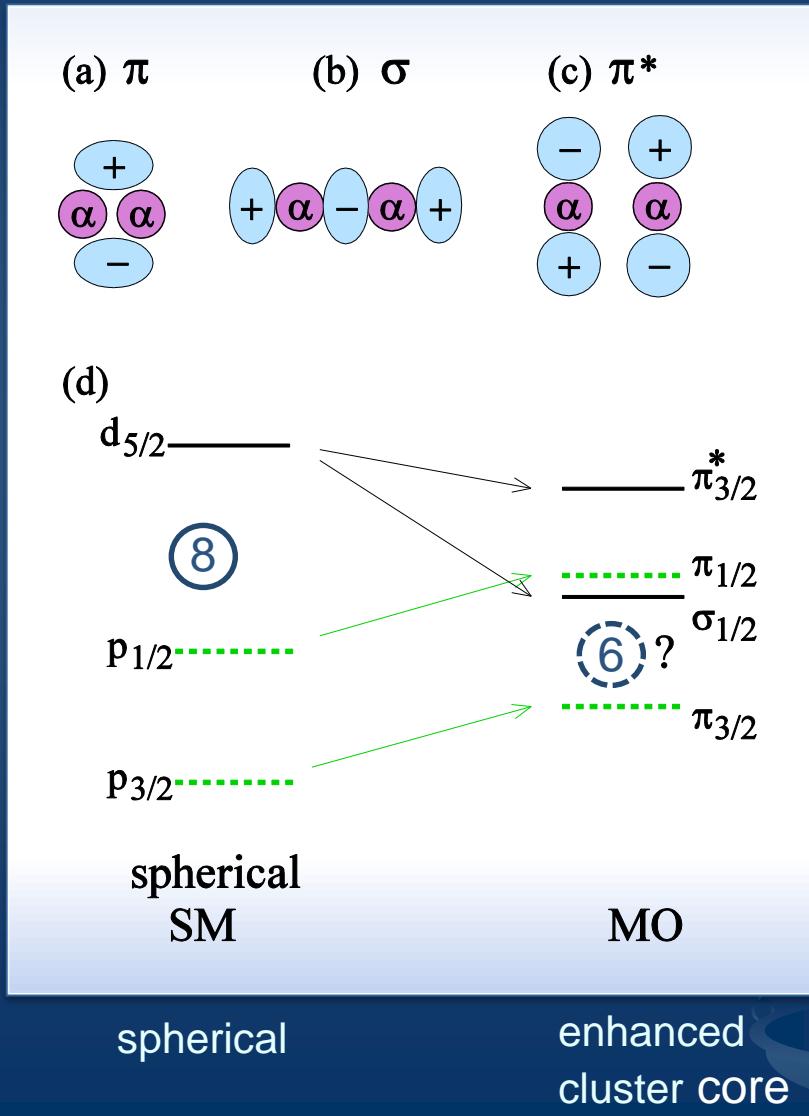


- Level inversion in developed cluster system
- $N=8$ shell vanishes at $N=7,8$ ($^{11}\text{Be}, ^{12}\text{Be}$)
- Positive- and negative-parity states degenerate
- New magic $N=6$ appears?
- σ -orbital neutron enhances clustering

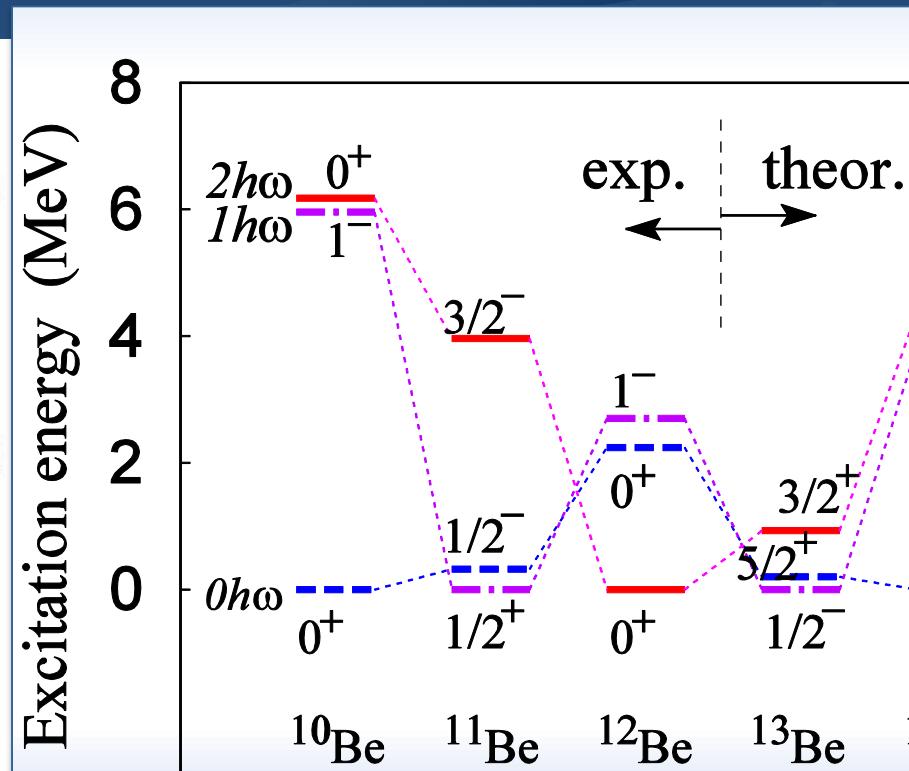
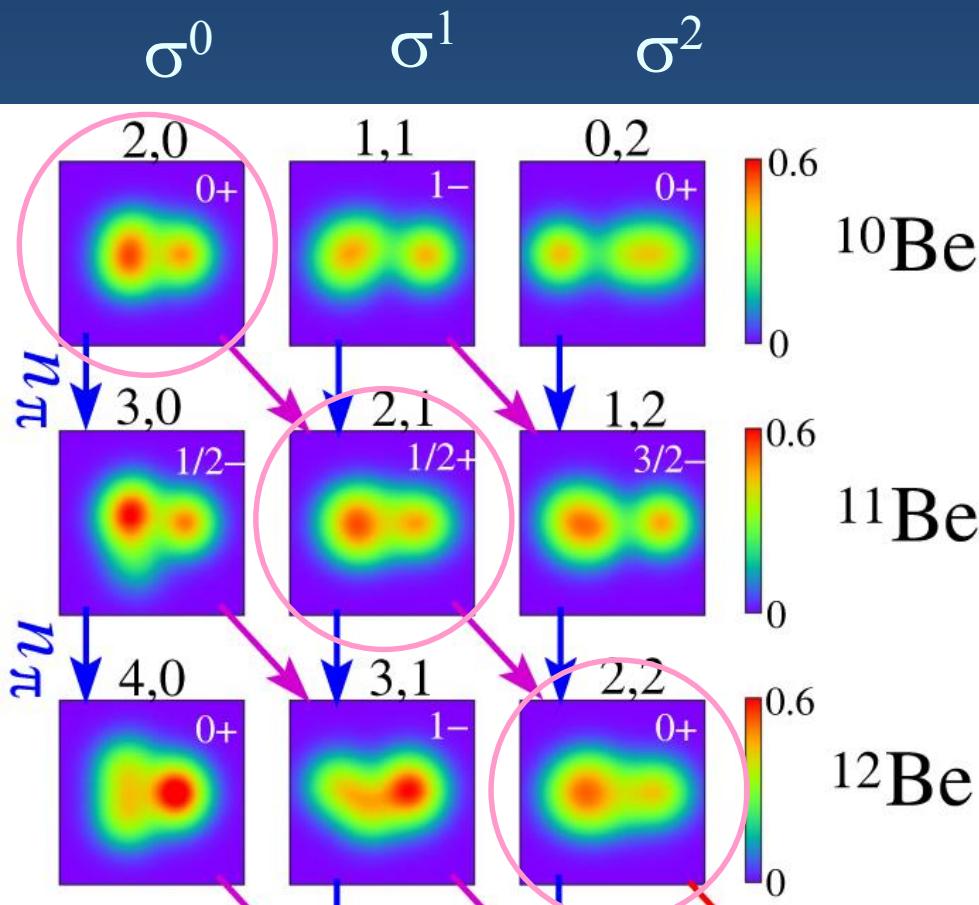
$N=8$ magic number breaking



$N=8$ magic number breaking



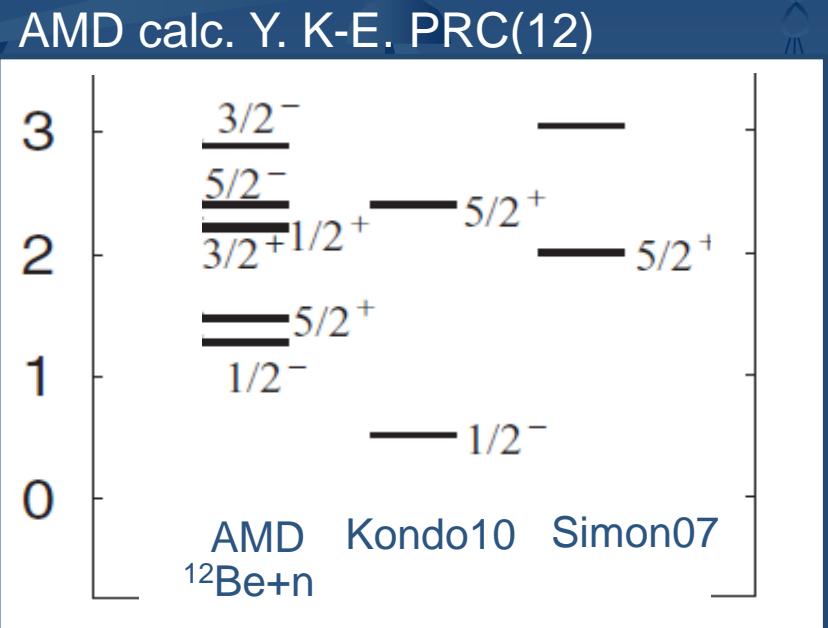
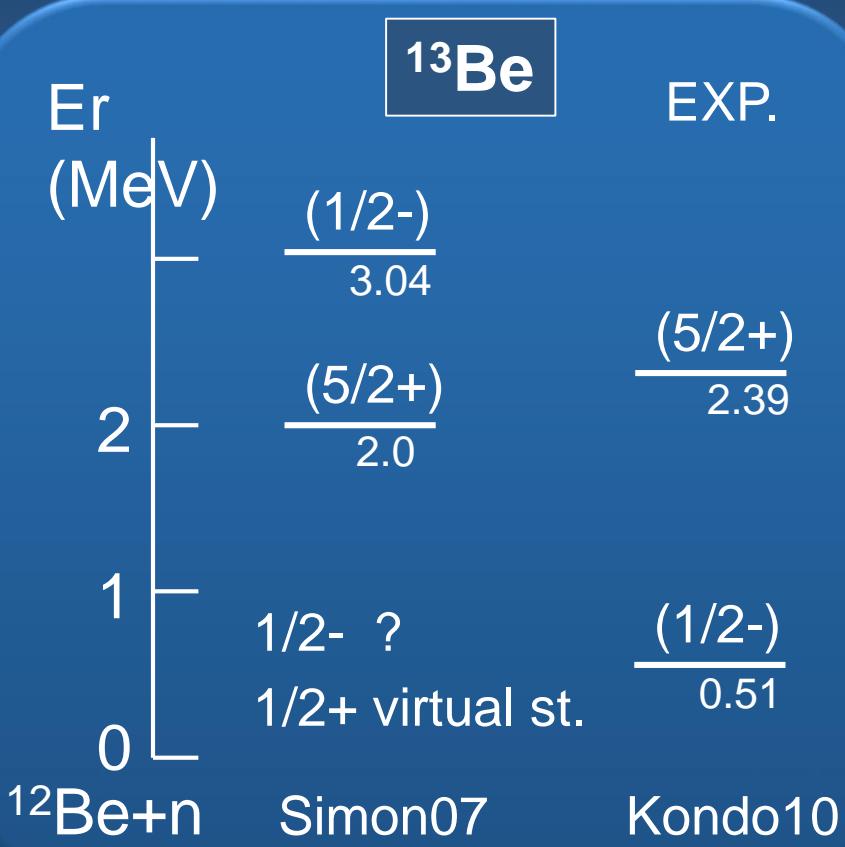
Cluster enhanced by σ -orbital neutrons



Breaking of magic number in ^{13}Be

^{13}Be : unbound

^{13}Be spectra measured by 1n knock-out reactions
at GSI(Simon et al. 2007) and RIKEN(Kondo et al.,2010)



intruder

$d_{5/2}$

$p_{1/2}$

$s_{1/2}$

$p_{3/2}$

$s_{1/2}$

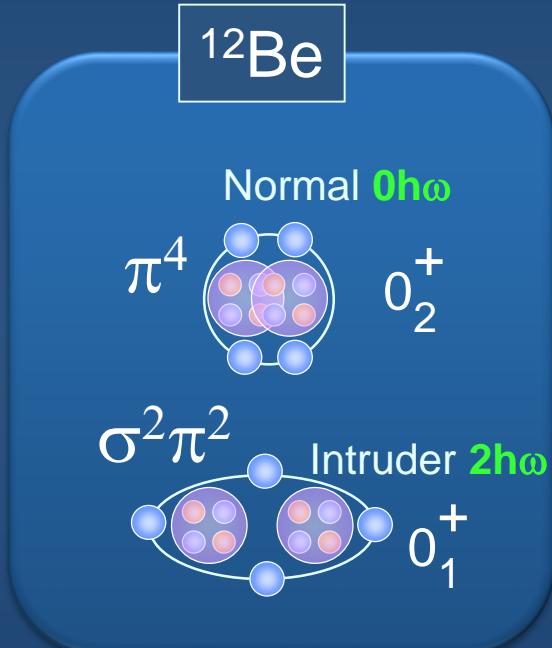
1/2- is lowest in ^{13}Be

Experimental evidence of N=8 shell vanishing in ^{12}Be

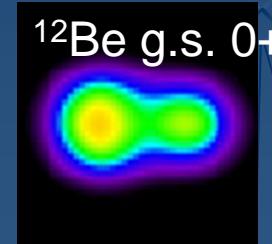
Y.K-E.PRC (03),(12) , Ito PRL(08) Dufour NPA(10)

Fortune PRC(06), Blanchon PRC(10)

Energy



- deformation in $^{12}\text{Be(gs)}$
Inelastic scat. life time:
Iwasaki PLB481(00),
Imai PLB673(09)
- intruder config. in $^{12}\text{Be(gs)}$
1n-knockout reac.:
Navin PRL85(00),
Pain PRL96(06)
- $^{12}\text{Be}(0_2^+)$ with p-shell config.
Shimoura PLB654 (07)



B(GT) with charge ex.:
Meharchand PRL108 (12)

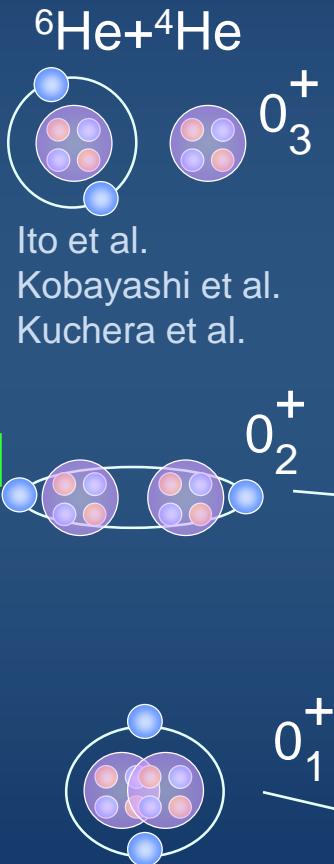
- Z-,N-dependence of g.s. structure
- MO & magic number breaking
- Cluster resonances

Spectra of ^{10}Be

cluster
res.

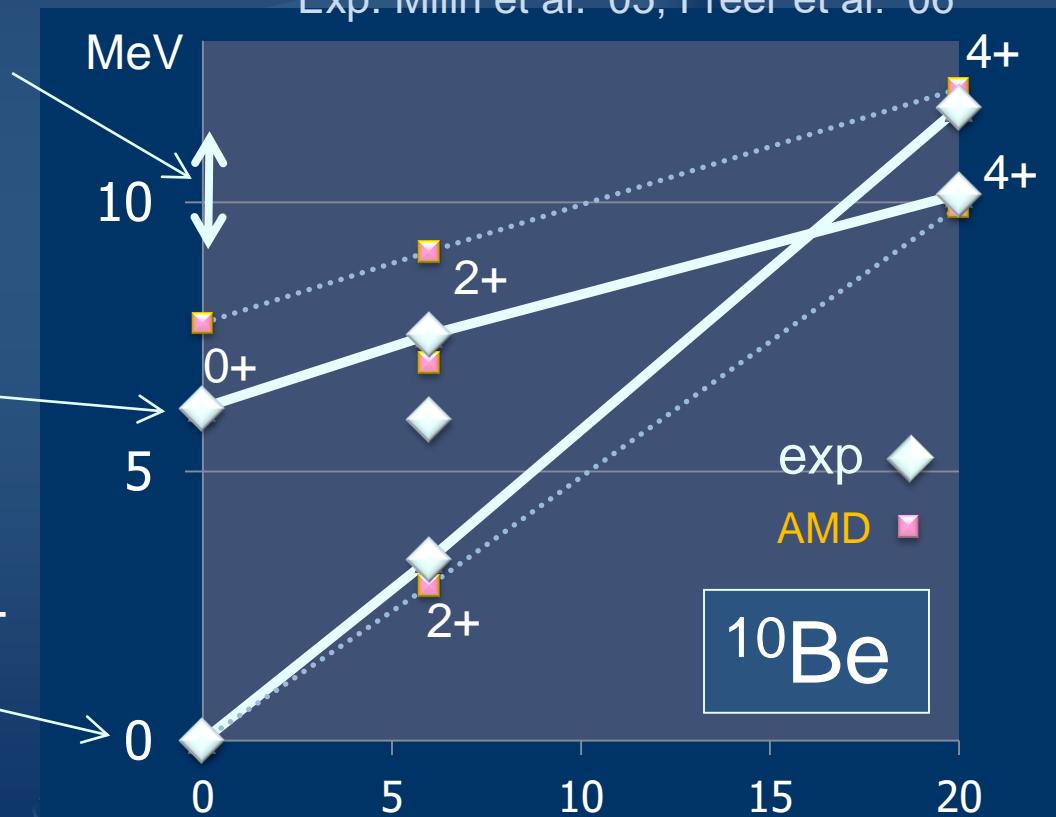
MO σ -bond

Normal



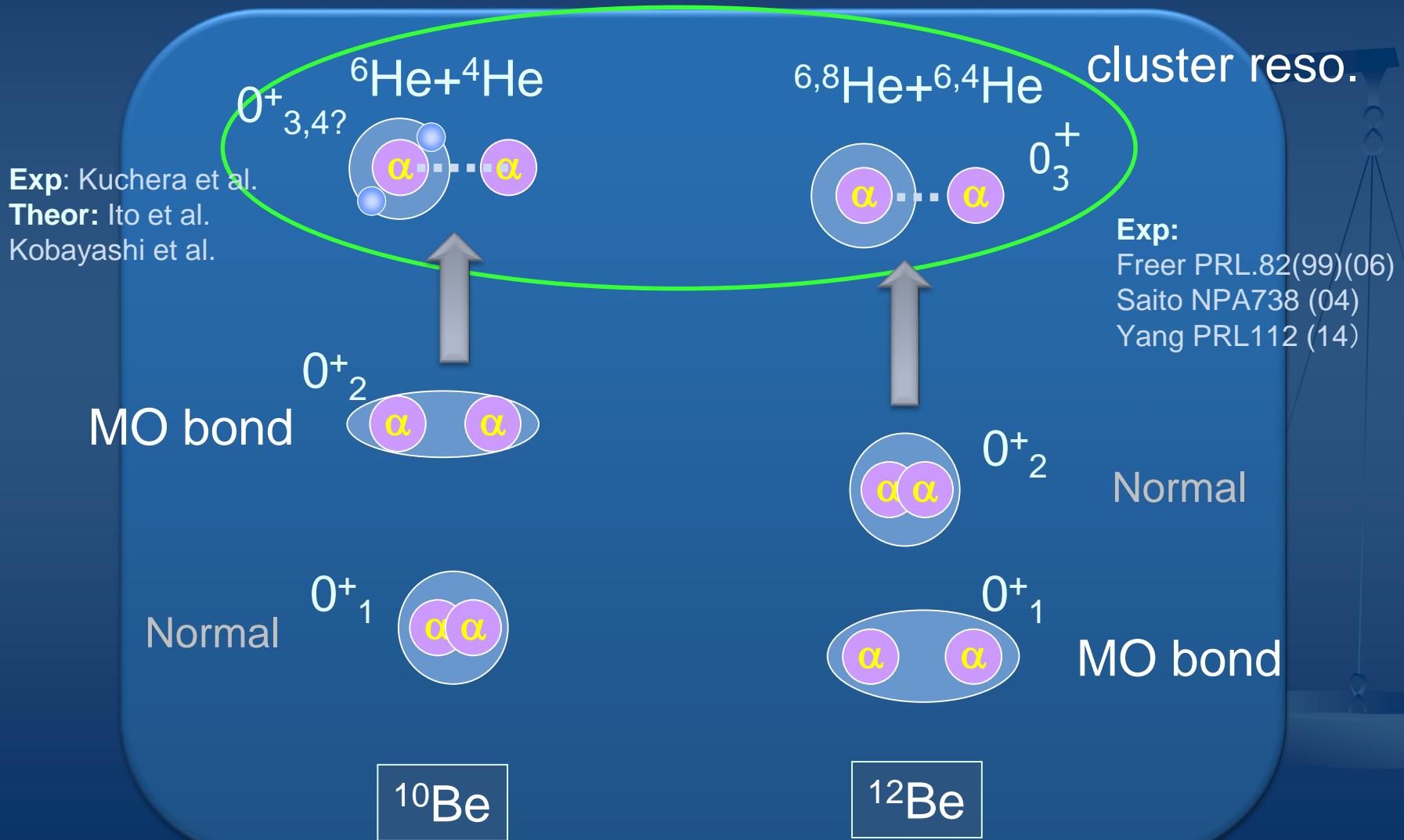
^{10}Be : energy levels

AMD calc. Y. K-E, et al. PRC (98)
Exp: Milin et al. '05, Freer et al. '06



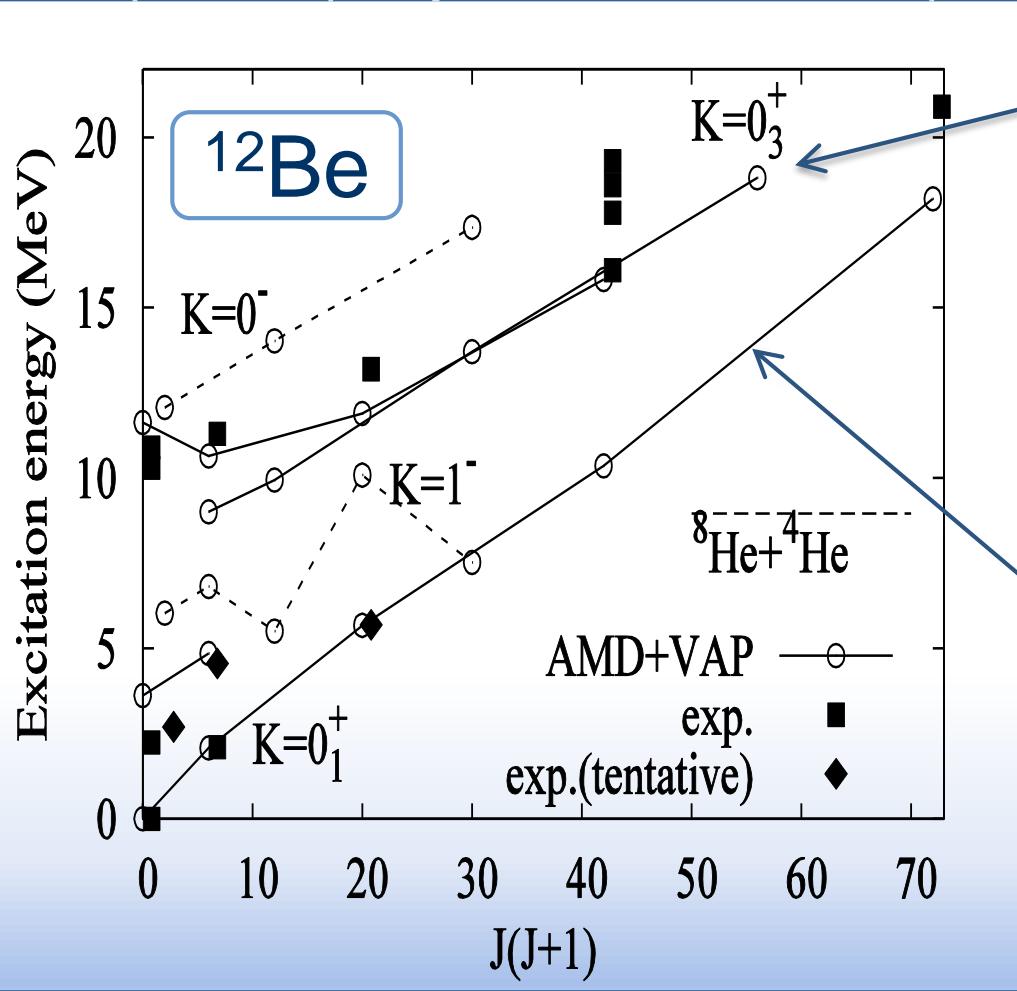
$J(J+1)$

Cluster resonances

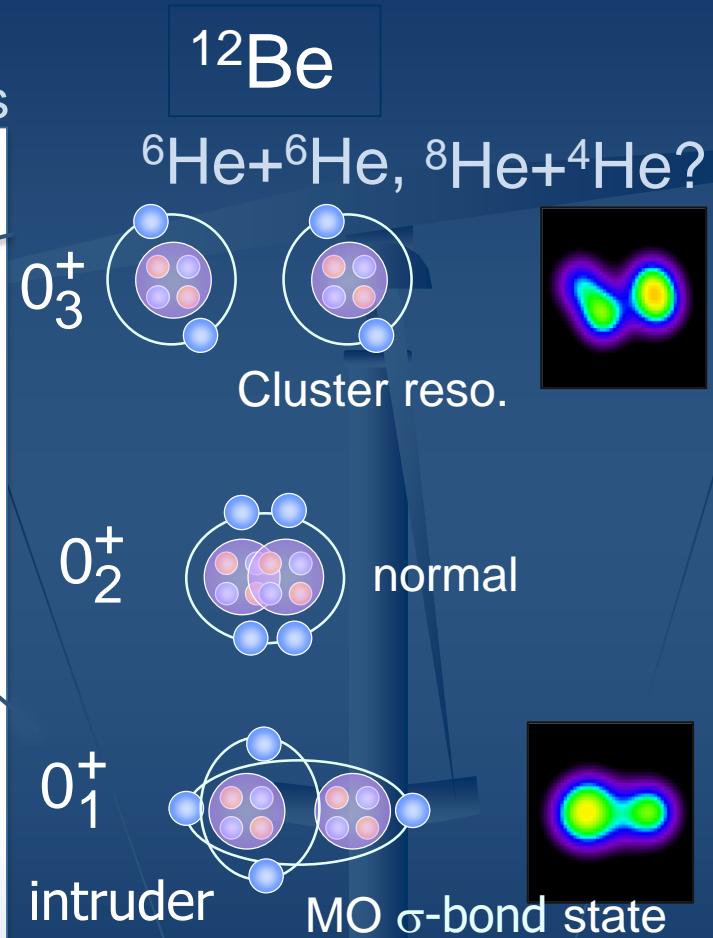


Energy spectra of ^{12}Be

VAP calculation with AMD method
positive parity states with normal spins



Y.K et al., PRC 68, 014319 (2003)



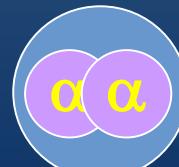
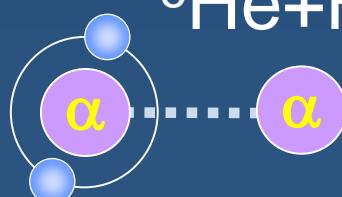
Breaking of N=8 magicity
Formation of 2a+molecular orbitals

MO v.s. Cluster resonance in other n-rich systems

^{10}Be

Soic et al., Freer et al., Saito et al.,
Curtis et al., Milin et al., Bohlen et al.,
Seya, Von Oerzten, Descouvemont et al.,
Itagaki et al., Dote et al., K-E et al.
Arai et al., M. Ito et al.

$^6\text{He} + \text{He}$



Be isotopes

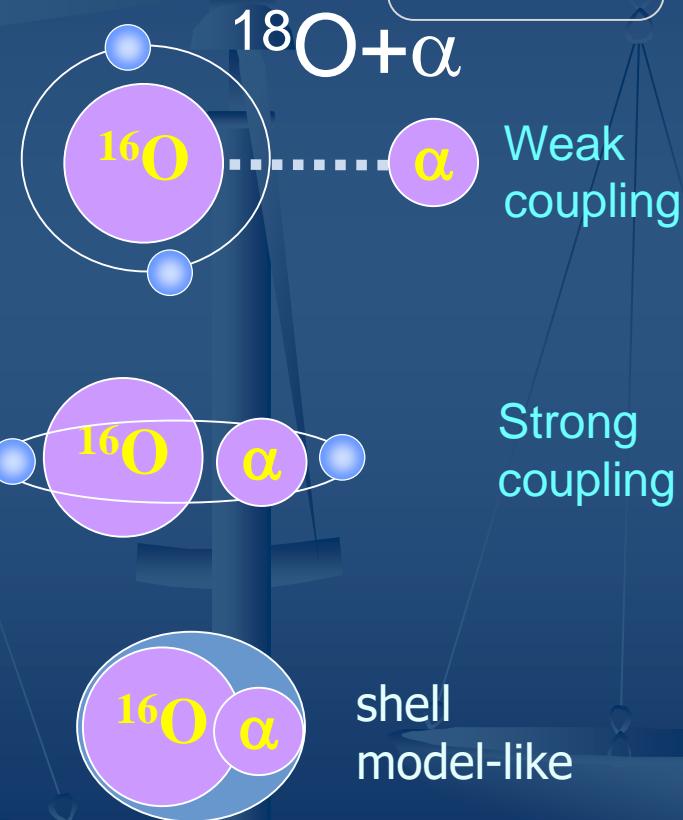
Atomic:
Cluster resonance

Molecular Orbital:
 σ -bond structure

Normal states

Scholz et al., Rogachev et al., Goldberg et al.,
Ashwood et al., Yildiz et al.,
Descouvemont, Kimura,

^{22}Ne



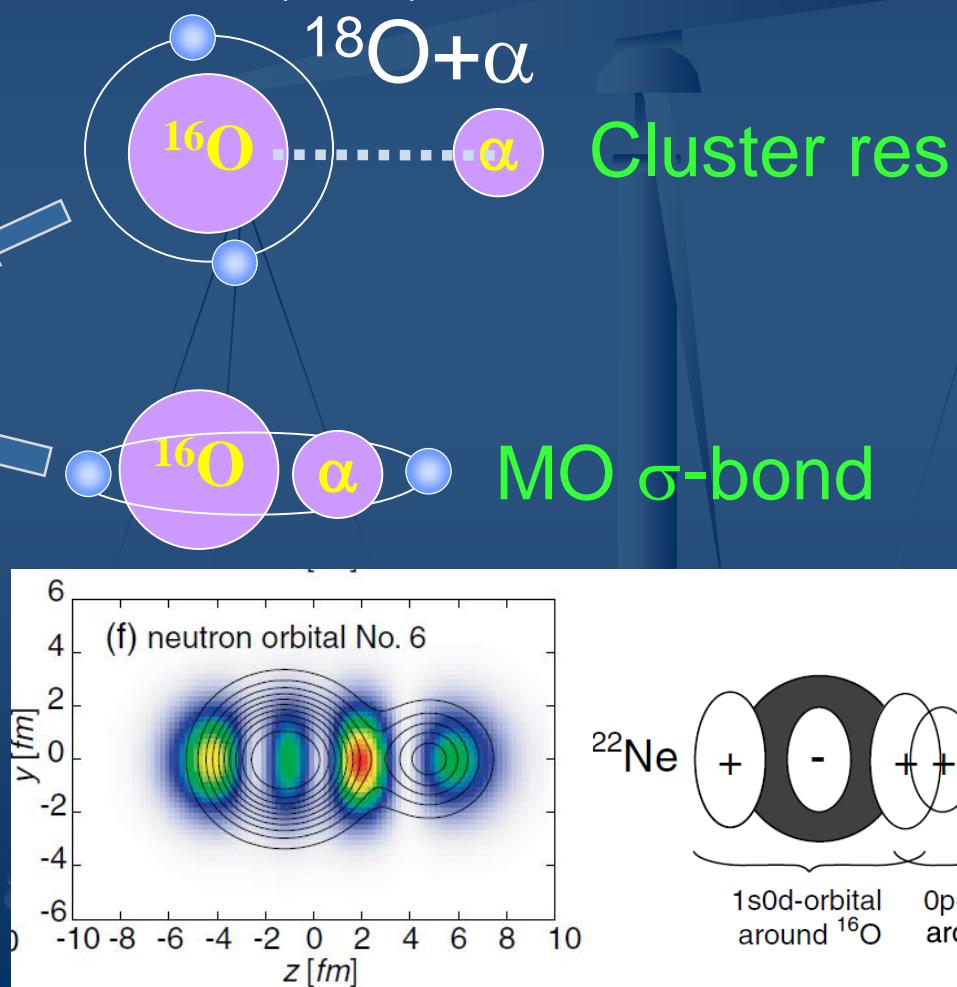
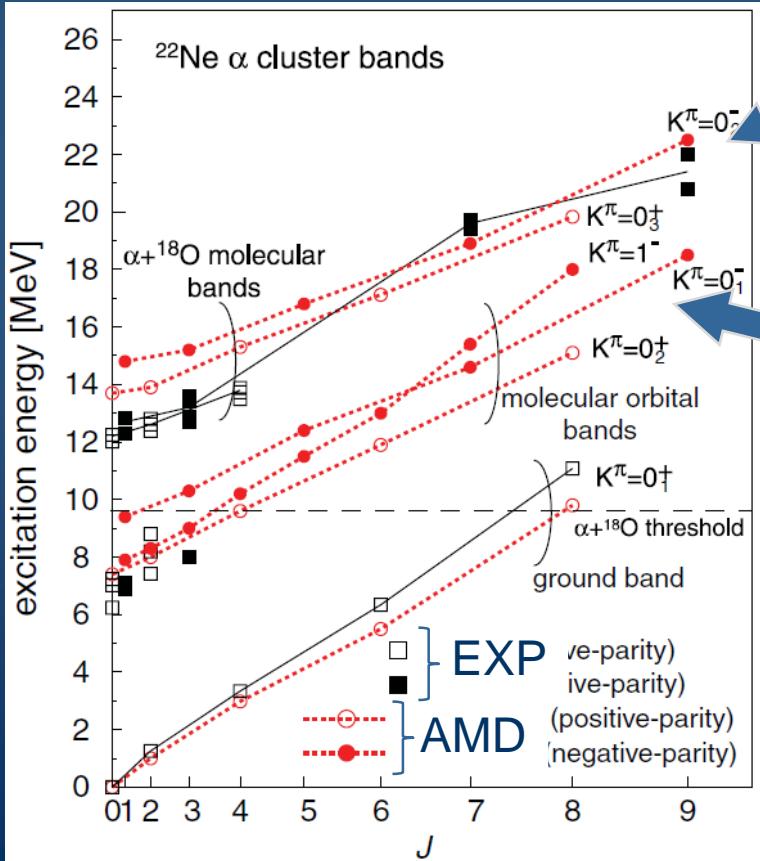
Ne, F, O isotopes

MO bond and Cluster res. in ^{22}Ne

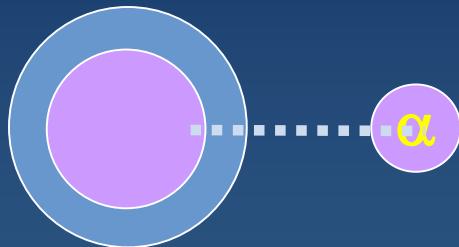
Exp Scholz et al., Rogachev et al., Goldberg et al., Ashwood et al., Yildiz et al.,
Theor: Descouvemont, Kimura,

^{22}Ne

AMD study by Kimura, PRC75 (2007)



α -cluster states in n-rich nuclei



Cluster resonances

New states discovered and suggested at
 $E_x = \text{several} \sim 20 \text{ MeV}$
in α -decay, α -transfer, α -scattering

$^{6,8}\text{He} + \alpha$ in Be^*

Exp: Soic et al., Freer et al., Saito et al., Curtis et al., Milin et al., Bohlen et al.,
Theor: Seya, von Oertzen, Descouvemont et al., Itagaki et al., K-E et al.,
Arai et al., M. Ito et al.

$^{10}\text{Be} + \alpha$ in $^{14}\text{C}^*$

Exp Soic 04, von Oertzen '04, Price 07, Haigh 08, Fritsch '16, Yamaguchi
Theor: Suhara '10

$^{14}\text{C} + \alpha$ in $^{18}\text{O}^*$

Exp Scholz et al., Rogachev et al., Goldberg et al., Ashwood et al., Yildiz et al.,
Theor: Descouvemont, Kimura,

$^{18}\text{O} + \alpha$ in $^{22}\text{Ne}^*$

Exp Scholz '72, Rogachev '01, Goldberg '04, Ashwood '06, Yildiz et al.,
Theor: Descouvemont '88, Kimura '07

Universal phenomena?

Further experimental and theoretical studies are requested.

Lesson 5. Nuclear excitations

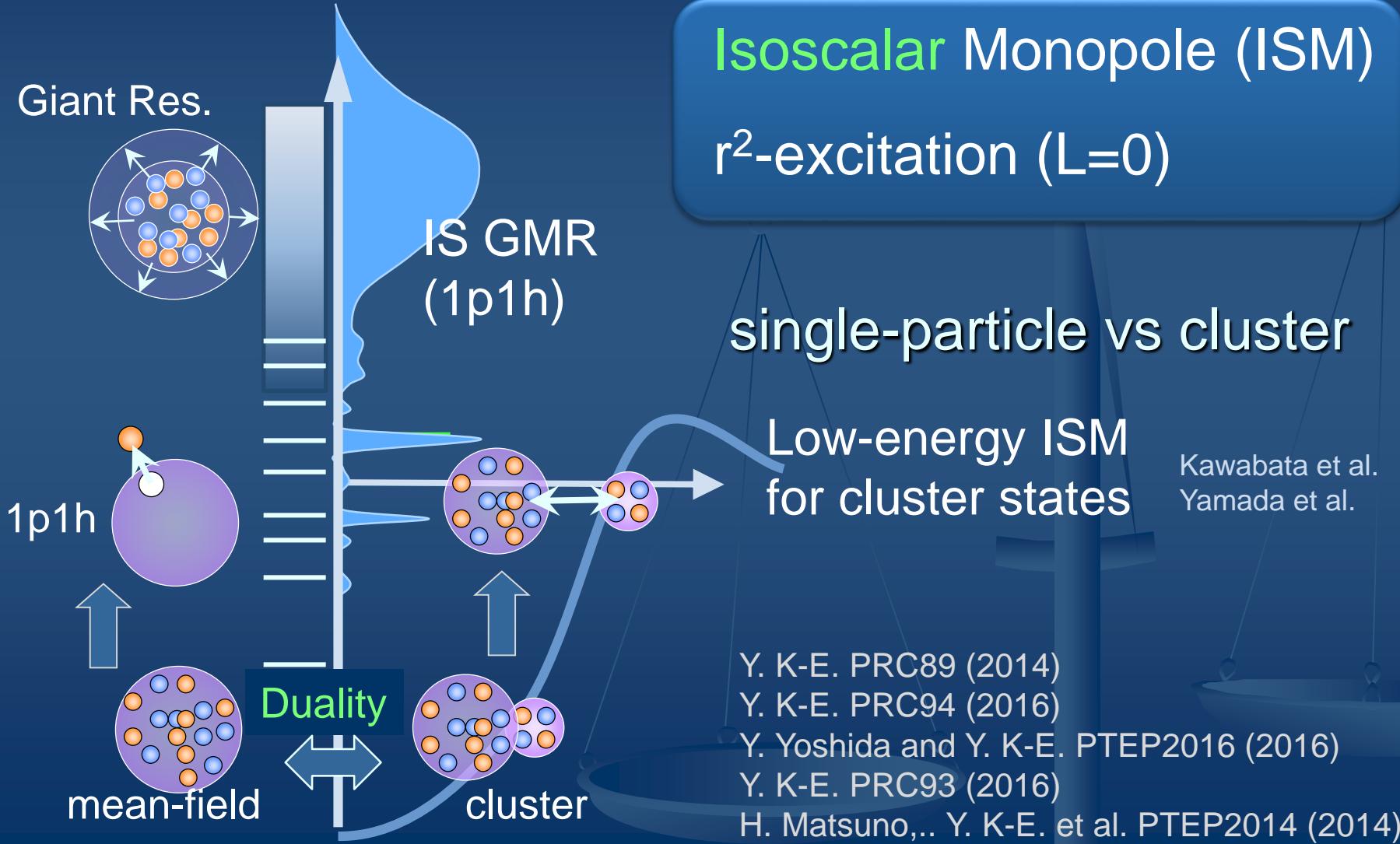
Isoscaler monopole and dipole excitations

how to probe clustering in nuclei with reactions

- charge ex (${}^3\text{He}, t$) reactions: GT transitions
- α -transfer, α -knockout reactions
- (α, α') scattering:
 - isoscalar monopole & dipole excitations
- $(p, p') \dots$

Isoscalar monopole excitations

$T=0, S=0$ excitations in coordinates(r, L) space



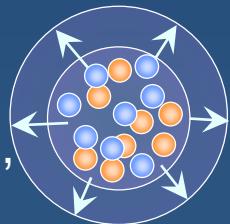
Giant monopole and dipole resonances

IS monopole (IS0):

$$\sum_i r_i^2 Y_{00}(\hat{\mathbf{r}}_i) \sqrt{4\pi}$$

GMR

Compressive,
breathing



IV dipole (E1):

$$\sum_{i=\text{proton}} r_i Y_{1\mu}(\hat{\mathbf{r}}_i)$$

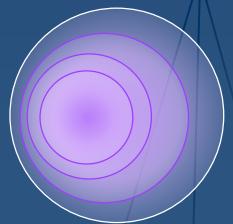


translational \leftrightarrow

IS dipole (IS1):

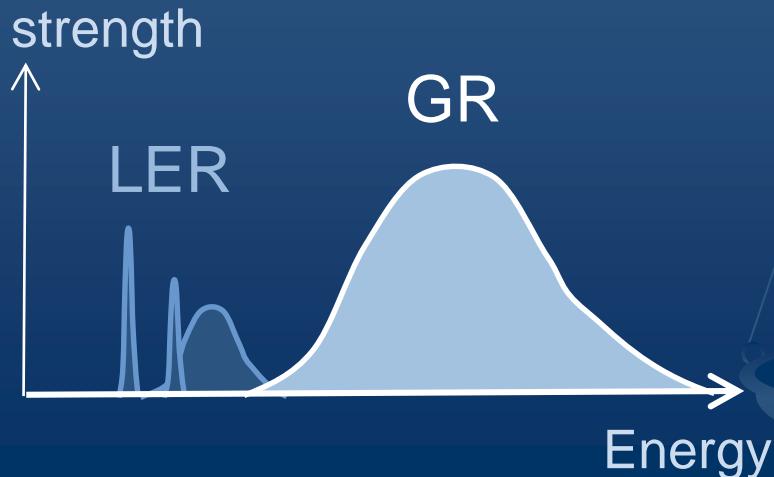
$$\sum_i r_i^3 Y_{1\mu}(\hat{\mathbf{r}}_i)$$

ISGDR



compressive \leftrightarrow

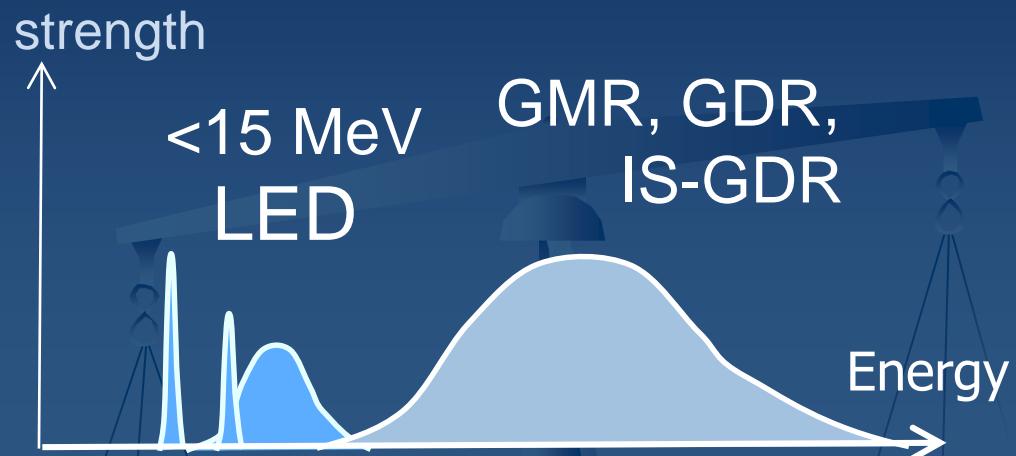
Response to external perturbation



- GR: broad bump in HE region
ISM:10-20 MeV, IVGDR:10-30MeV
- Collective oscillation of system
coherent 1p-1h excitation
- Physical aspects:matter properties
ISGMR: Incompressibility,
IVGDR:Symmetry energy

LED v.s. GDR

Low-energy (<15 MeV)
strengths below GRs
observed



80's: ISM, ISD, IVD(E1) in stable nuclei
90's: IVD(E1) in neutron-rich nuclei

Separation of LE strengths from GRs

→ New excitation modes decoupled from GRs.

Origins of LE strengths have not clarified yet.
Various origins?

LE-ISM, LE-ISD for cluster states

Yamada et al. PRC85, 034315 (2012)

Chiba et al. PRC93 (2016) no.3, 034319

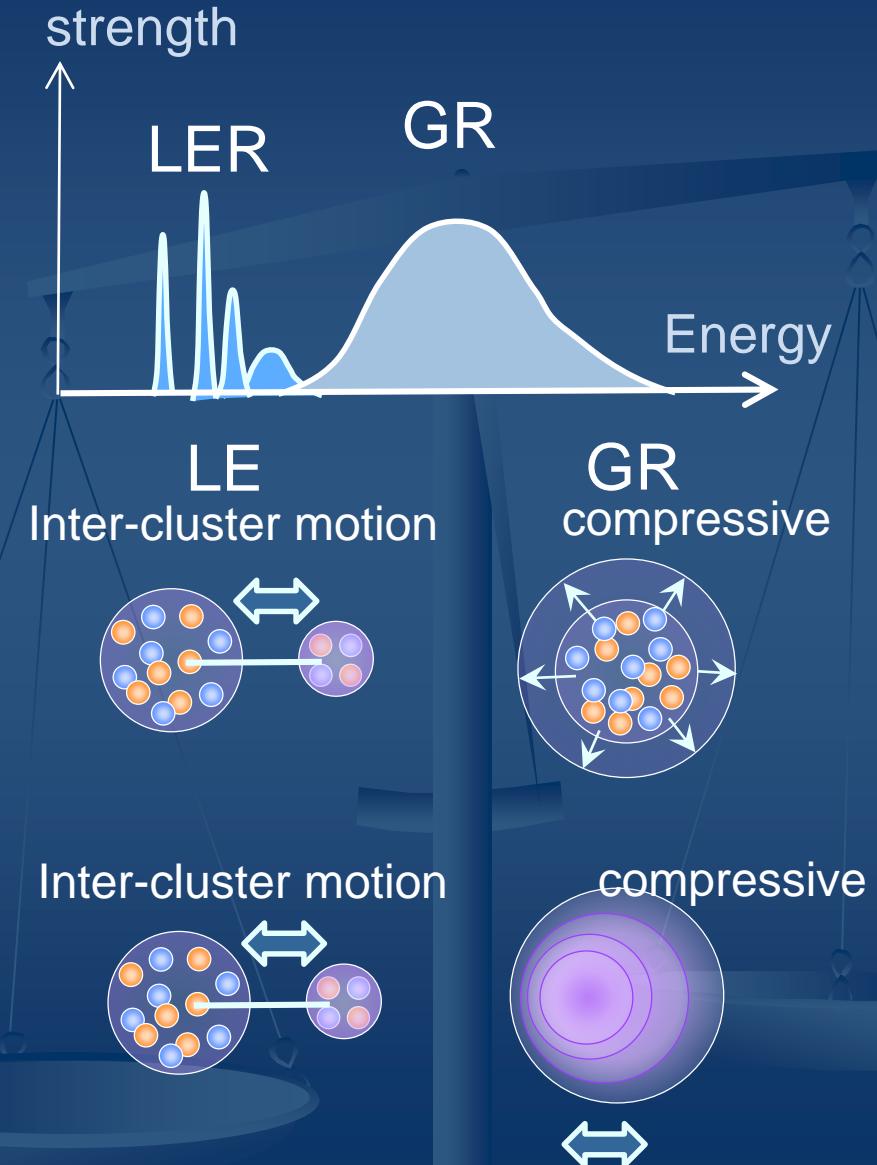
Isoscalar monopole (IS0):

$$M(IS0) = \sum_i r_i^2 Y_{00}(\hat{\mathbf{r}}_i) \sqrt{4\pi}$$

sensitive to radial motion

Isoscalar dipole (ISD=CD):

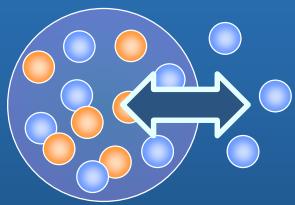
$$M(ISD; \mu) = \sum_i r_i^3 Y_{1\mu}(\hat{\mathbf{r}}_i)$$



What are natures of LED?

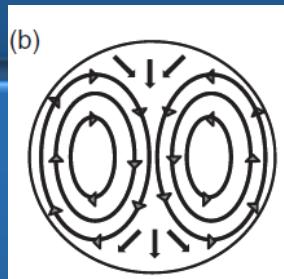
LED

IVD
(E1)

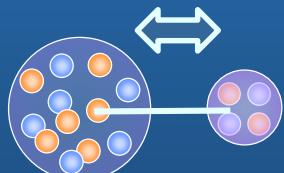


Neutron skin
only in $N > Z$ nuclei
not in $N = Z$ nuclei

ISD



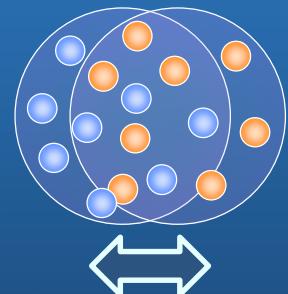
Toroidal



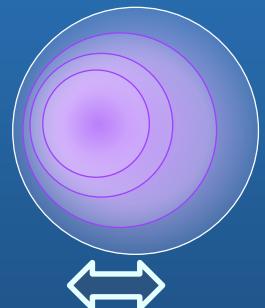
Cluster

HED

IVGDR

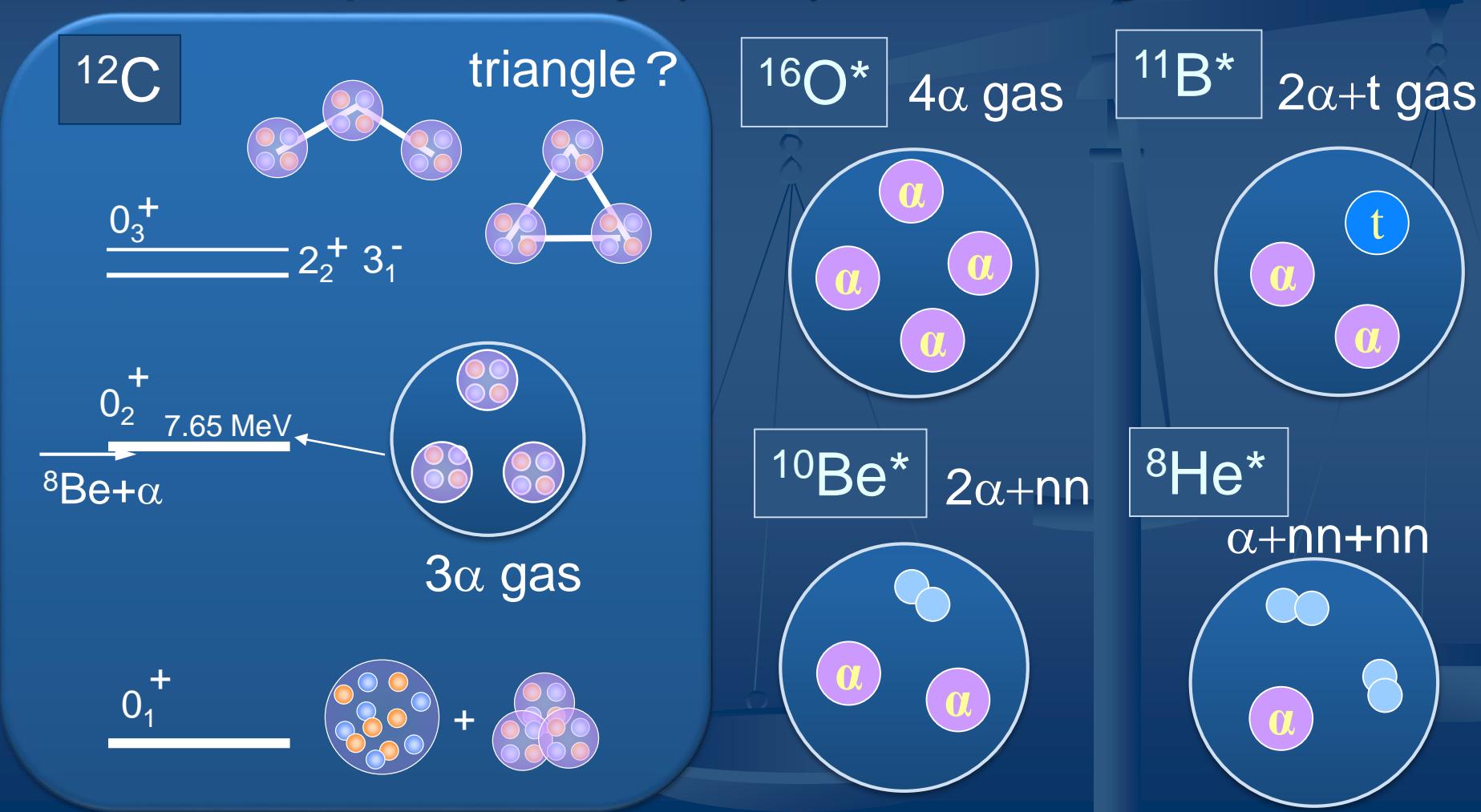


ISGDR

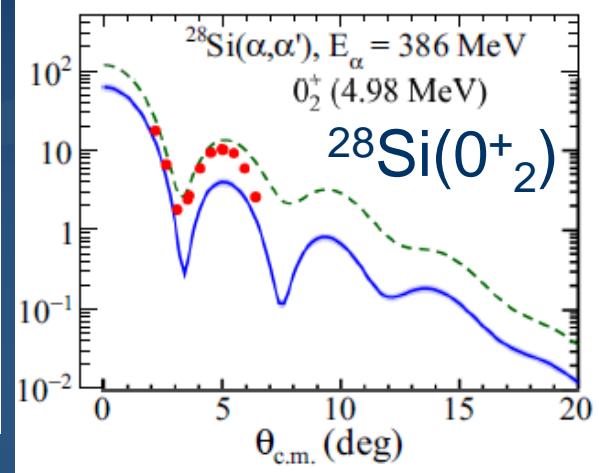
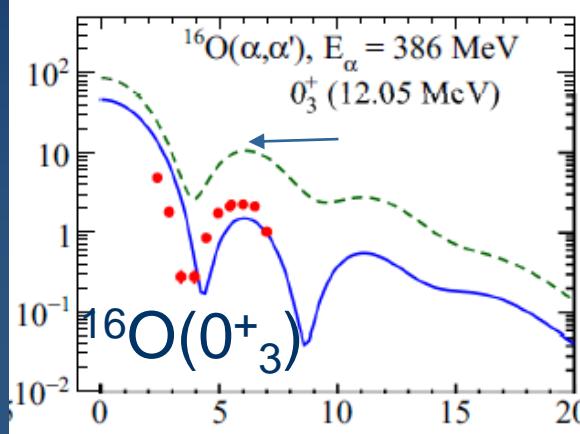
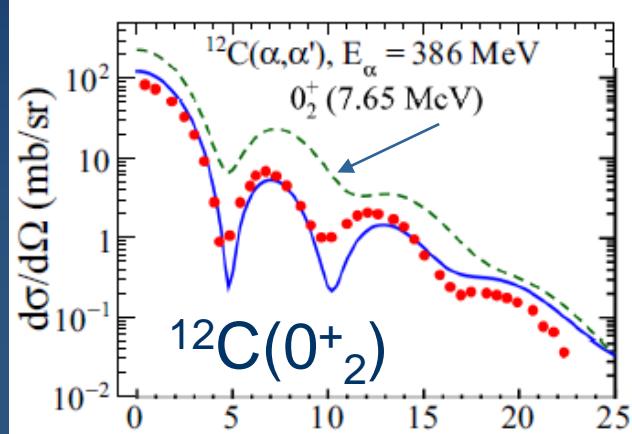


Maybe, not a single but various modes contribute to the LE strengths.

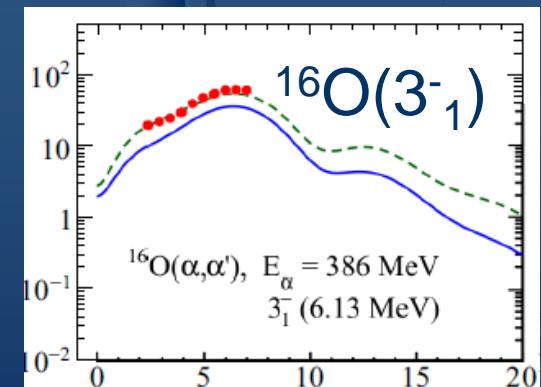
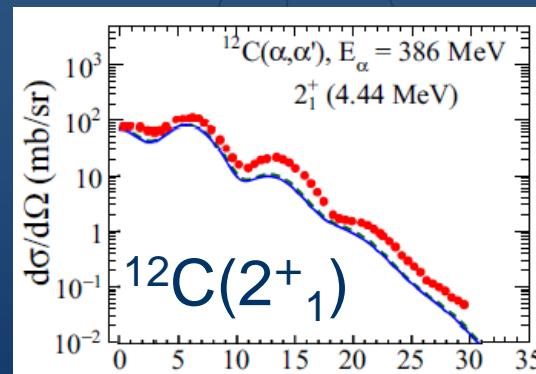
multi-cluster gas: 3-,4-body efimov states ? strong ISM(E0) transitions probed by (α, α') scattering



A puzzle: missing monopole strengths

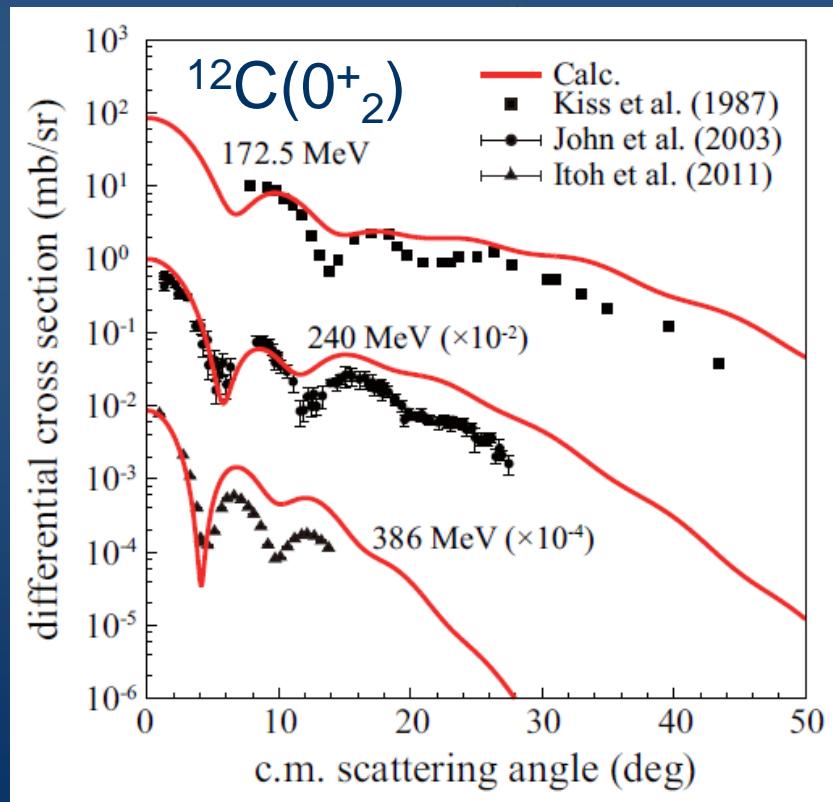


traditional reaction model
no problem in 2+, 3-,
but overshoot
0+ cross section
by a factor of 3-5



microscopic reaction calculations with g-matrix folding model

- The puzzle is solved: no problem in 0^+ cross sections *Minomo et al. C 93, 051601(R) (2016)*



Lesson 6. Advanced course analogy to other systems

What we learn and where we
are going in this project?

discretized symmetry decoupling in SU(N)

without spatial correlation

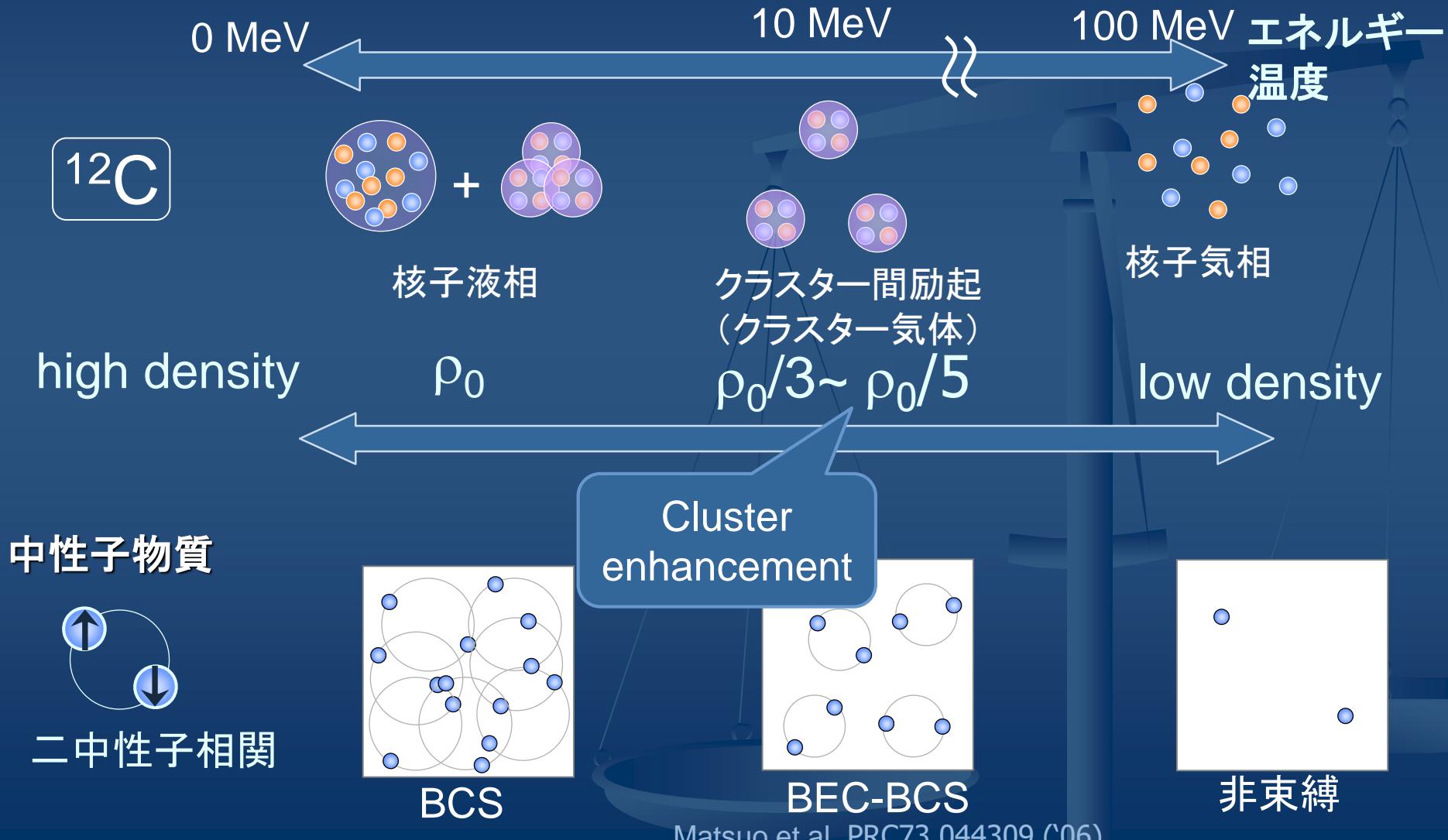
- nucleon systems:
BCS nn pairing, GT in pn pair
- quark hadron system:
quarkyonic, heavy quark symmetry
- cold atom
BEC-BCS

symmetry breaking and restoration inhomogeneous matter

- discrete symmetry: multi-alpha systems
triangle, tetrahedral, linear, planer
- symmetry in uniform matter: BEC-BCS
- symmetry in inhomogeneous matter
 - density wave in nuclei (cluster states)
 - pion condensation in nuclear matter
 - chiral DW(spiral) in quark matter
 - LOFF states ...

環境依存性: cluster enhancement

エネルギー、密度の関数



階層化・対称性: breaking & restoration

Y. K-E., Hidaka

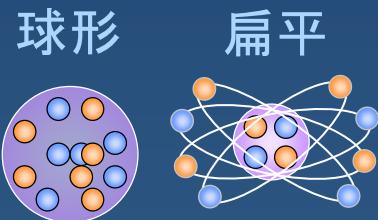
PRC84 (2011); PRC96 (2017)

with 日高

階層化=自由度(スケール)の分離:

クラスター相関(形成)と発達→対称性の破れと回復を誘発

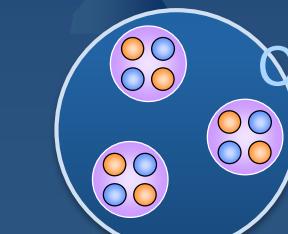
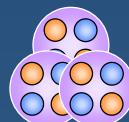
二面性: 微視的ダイナミクス(核子レベルからのクラスター形成)が本質!



扁平



相関: クラスター形成



Cluster 気体

相関なし $O(3) \rightarrow O(2) \rightarrow D3h$ (DW)

回転対称

並進対称

対称
核物質

Fermi gas

破れ
非一様密度
Density wave(DW)
or BCS?

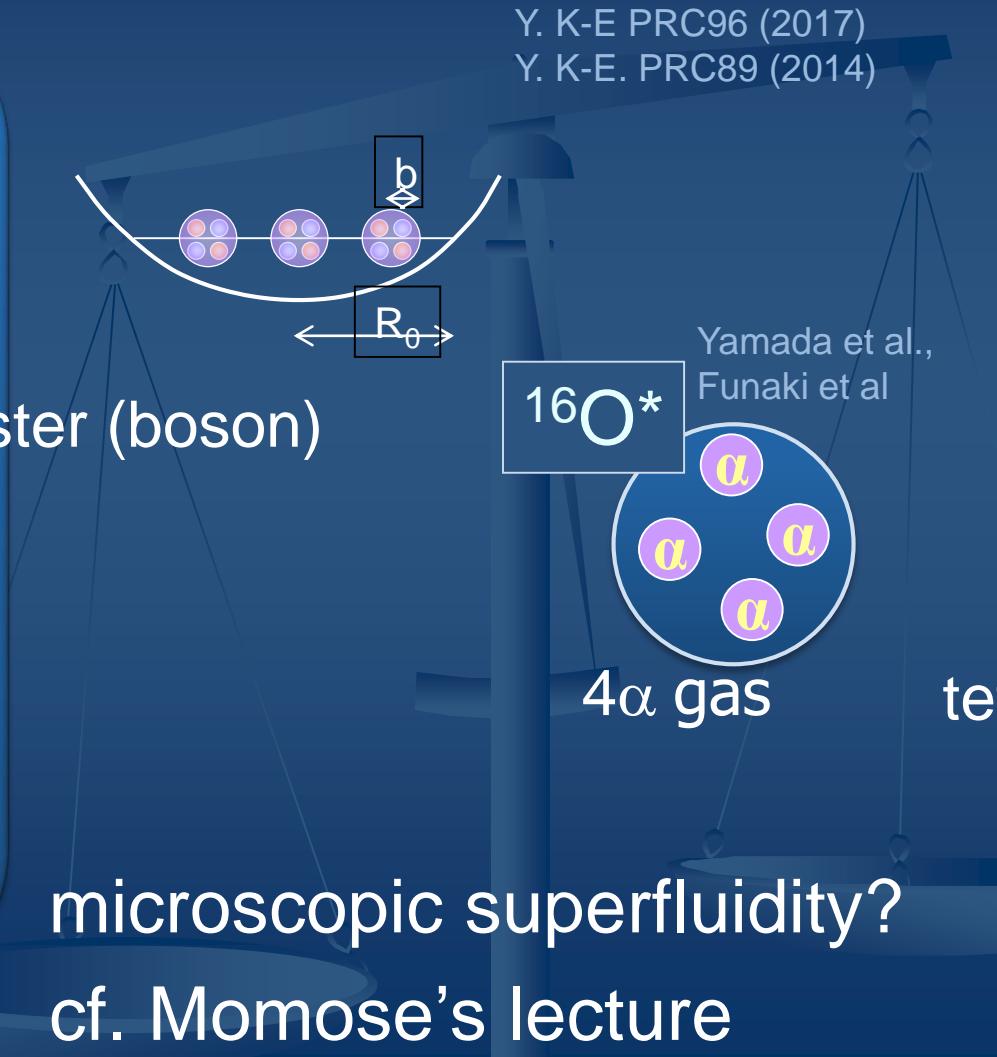
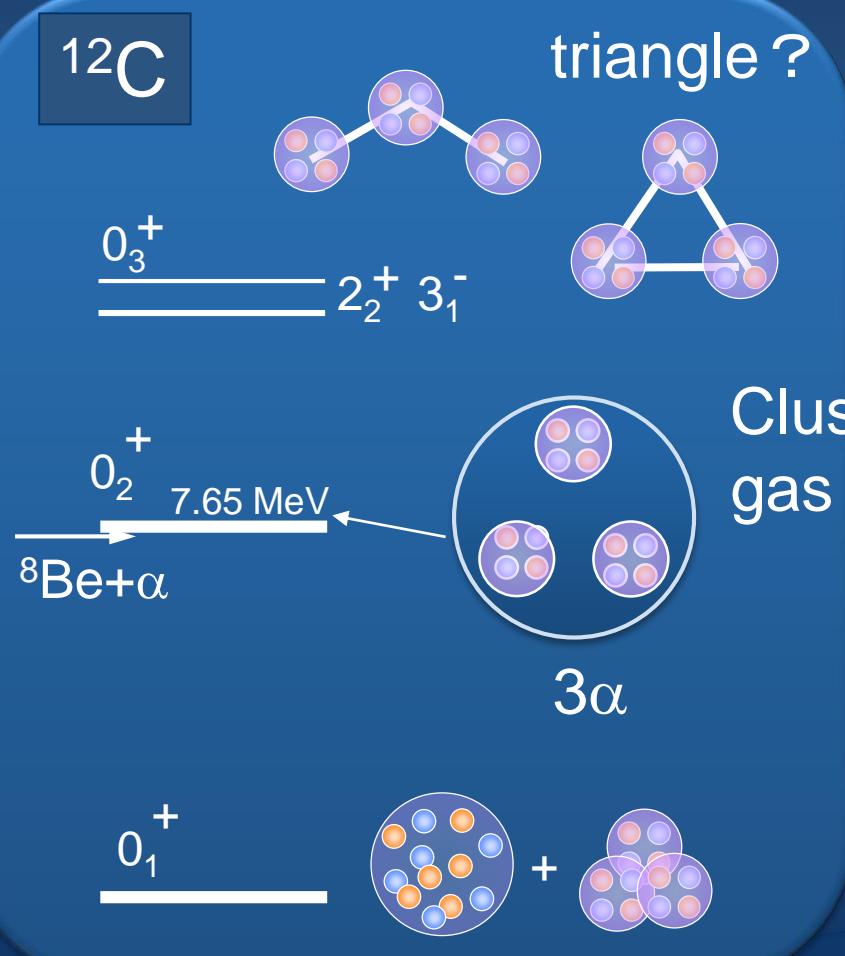
→ 回転対称性回復 $\rightarrow O(3)$ 強相関

回転対称性回復

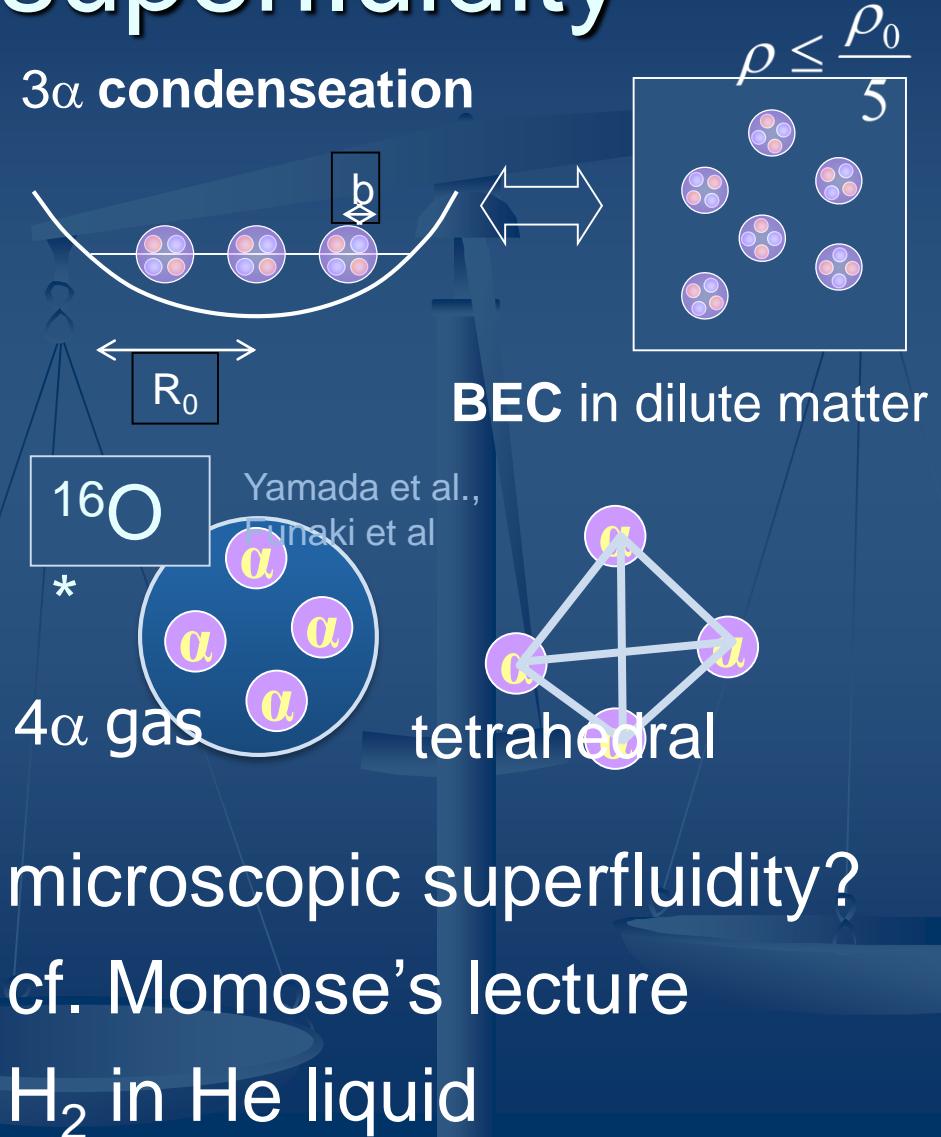
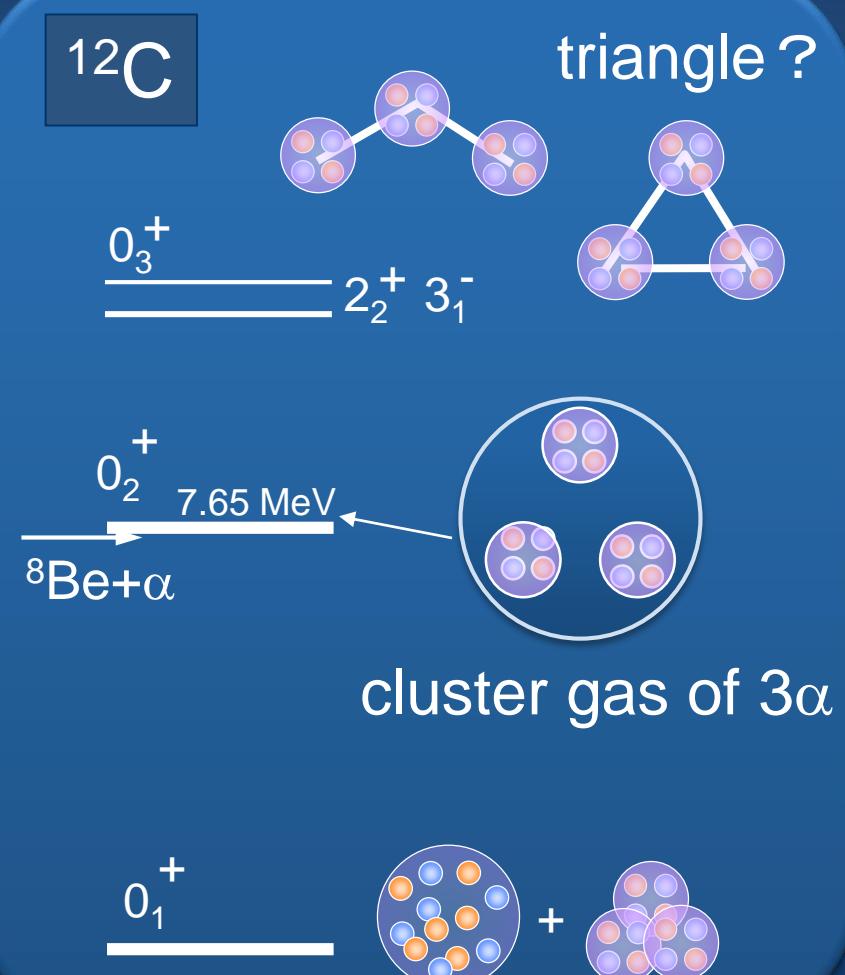
並進対称性回復

BEC: strong corr.
 $k=0$

α condensation v.s. microscopic superfluidity



α condensation v.s. microscopic superfluidity



cluster gas, halo v.s. Efimov states

To be Efimov or not?

- 2n-halo: Borromean
- Hoyle states (3a gas) in $^{12}\text{C}^*$

What is efimov?

- universality of weakly-bound 3-body systems
- independent from interactions

Naidon&Endo's : Not Efimov

Wiki, Suno et al. : Yes an efimov analogue