Viscosity and contact correlations in ultracold Fermi gases

Tilman Enss (Heidelberg University)

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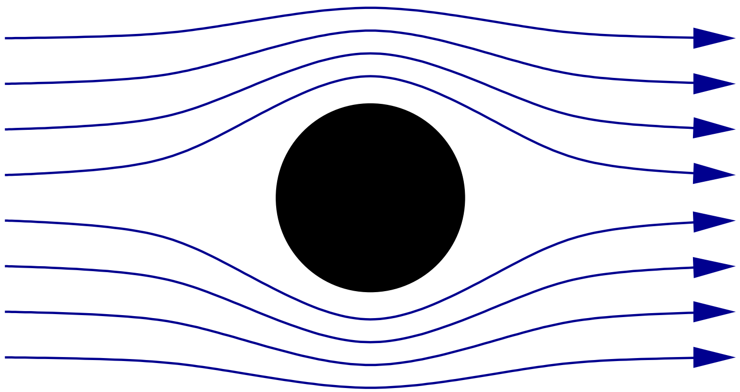
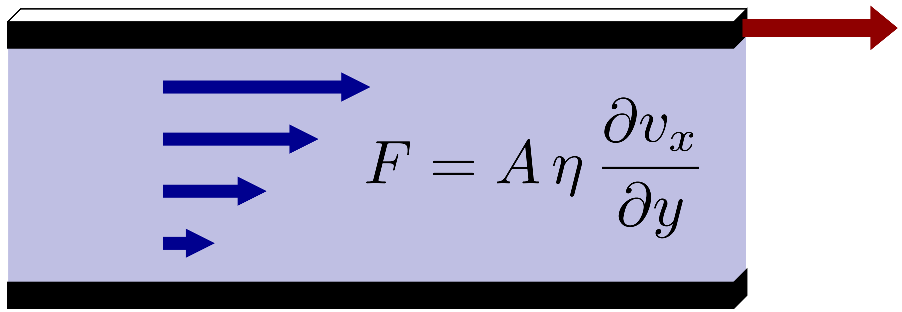
UNIVERSITÄT HEIDELBERG ZUKUNFT SEIT 1386

Is there an ideal fluid in nature?

- flow around an obstacle



shear viscosity η:

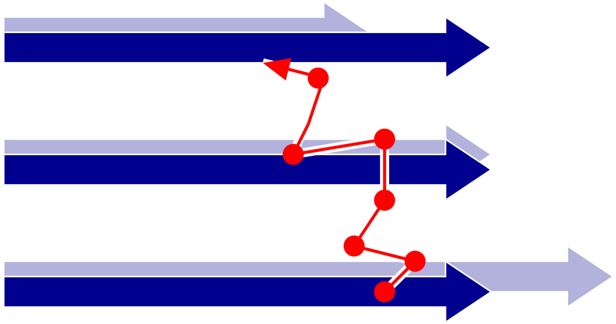


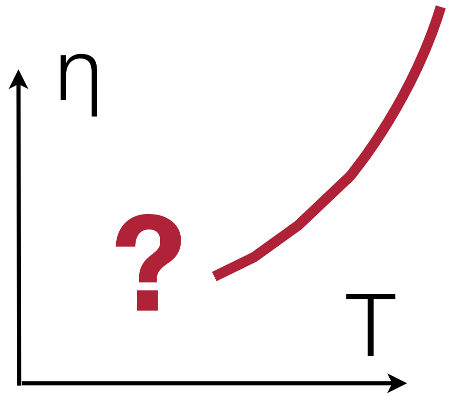
- ideal hydrodynamics (Euler equation): η=0

- real fluids: is flow without friction (η=0) possible?

How does friction arise?

- shear viscosity measures momentum transport:





- kinetic theory for dilute gas:

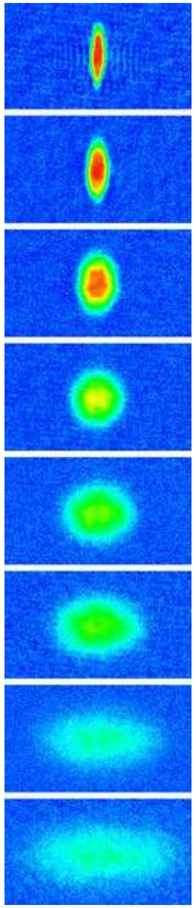
$$\eta = \tfrac{1}{3} n\,\bar{p}\,\ell_{\rm mfp}\,, \qquad \ell_{\rm mfp} = \frac{1}{n\sigma} \;\;:\qquad \eta = \frac{\bar{p}}{3\sigma}$$

- degenerate Fermi gas:

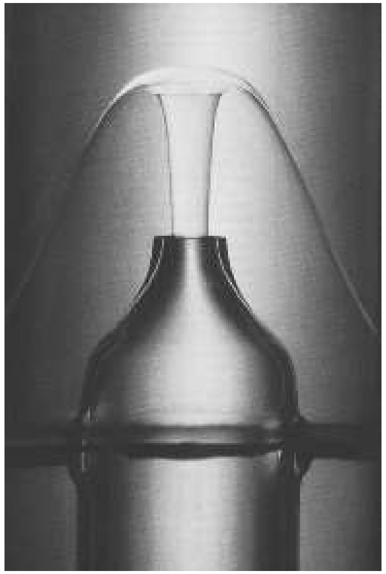
$$\frac{\eta}{s} \sim \frac{n\bar{p}\ell_{\rm mfp}}{nk_B} \sim \frac{\ell_{\rm mfp}}{\ell}\,\frac{\hbar}{k_B} \gtrsim \mathcal{O}(1)\,\frac{\hbar}{k_B} \qquad \text{(KSS bound: } 1/4\pi\text{)}$$

Shear viscosity of real fluids

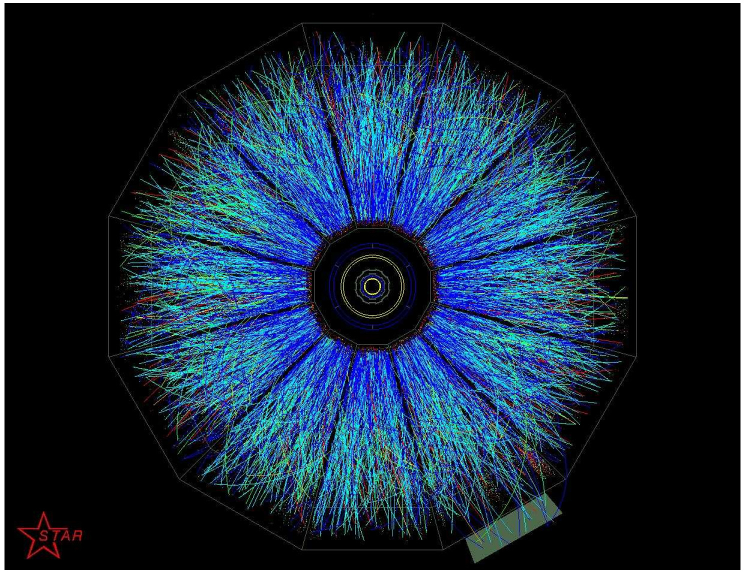
Schäfer & Teaney 2009



Trapped Atoms
(T=0.1 neV)
$\eta = 1.7 \cdot 10^{-15}\,\mathrm{Pa\cdot s}$



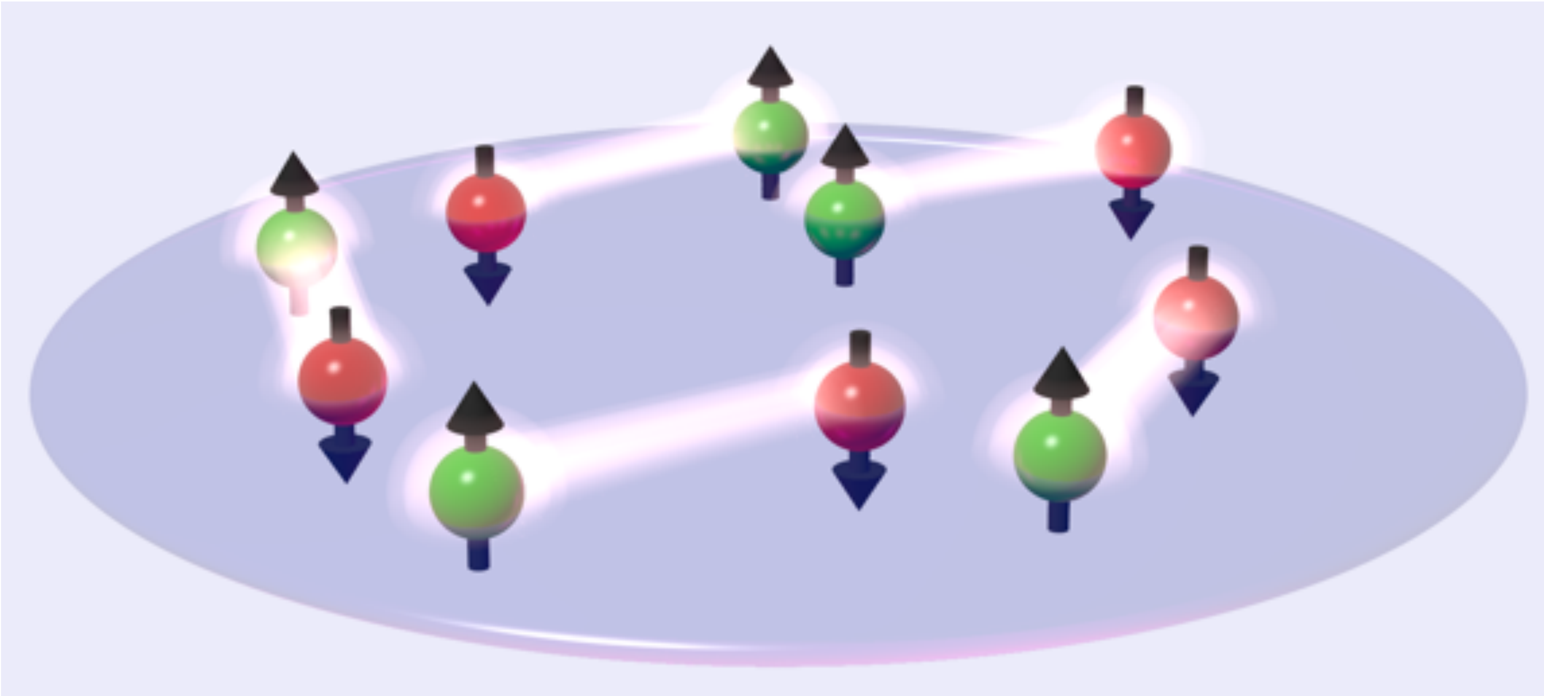
Liquid Helium
(T=0.1 meV)
$\eta = 1.7 \cdot 10^{-6}\,\mathrm{Pa\cdot s}$



Quark-Gluon Plasma
(T=180 MeV)
$\eta = 5 \cdot 10^{11}\,\mathrm{Pa\cdot s}$

consider ratio η/s:
min=0.5; 0.8; 0.4 $\hbar/k_B$

Strongly interacting Fermi gas

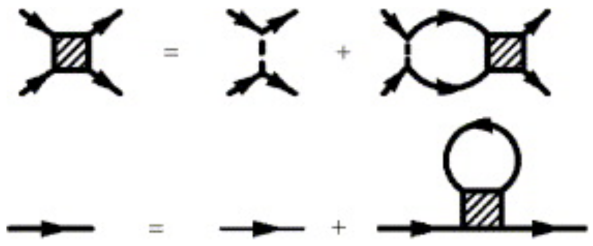


dilute gas of ⬆ and ⬇ fermions with contact interaction:

$$\mathcal{H} = \int d\mathbf{x} \sum_{\sigma=\uparrow,\downarrow} \psi_\sigma^\dagger \Big(-\frac{\hbar^2\nabla^2}{2m} - \mu_\sigma\Big)\psi_\sigma + g_0 \psi_\uparrow^\dagger \psi_\downarrow^\dagger \psi_\downarrow \psi_\uparrow$$

Luttinger-Ward theory

- repeated particle-particle scattering dominant in dilute gas:

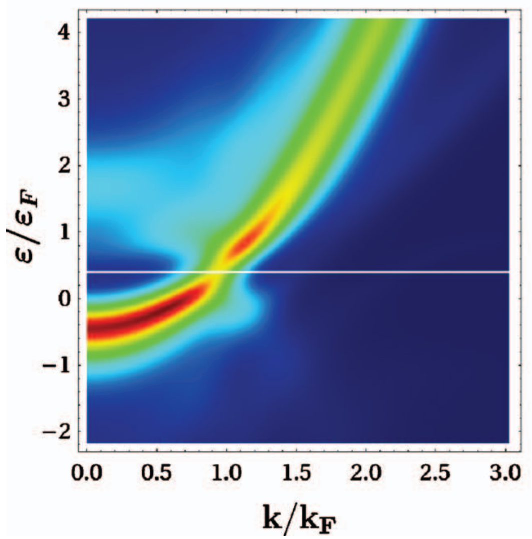


self-consistent T-matrix

Haussmann 1993, 1994;
Haussmann et al. 2007

self-consistent fermion propagator
(300 momenta / 300 Matsubara frequencies)

- spectral function A(k,ε) at Tc



Haussmann et al. 2009

works above and below Tc;
directly in continuum limit

Tc=0.16(1) and ξ=0.36(1)
agree with experiment

conserving: exactly fulfills scale invariance and Tan relations
Enss PRA 2012

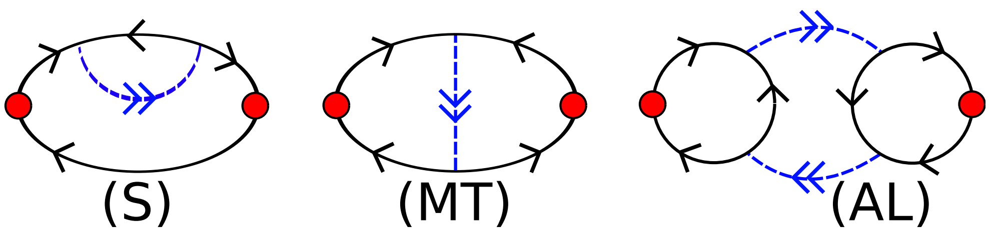
Transport in linear response

- shear viscosity from stress correlations (cf. hydrodynamics),

$$\eta(\omega) = \frac{1}{\omega}\,\mathrm{Re}\int_0^\infty dt\, e^{i\omega t}\int d^3x\,\left\langle\left[\hat{\Pi}_{xy}(\boldsymbol{x},t),\hat{\Pi}_{xy}(0,0)\right]\right\rangle$$

with stress tensor $\hat{\Pi}_{xy} = \sum_{\mathbf{p},\sigma}\frac{p_x p_y}{m}\,c^\dagger_{\mathbf{p}\sigma}c_{\mathbf{p}\sigma}$ (cf. Newton $\frac{\partial v_x}{\partial y}$)

- correlation function (Kubo formula): Enss, Haussmann & Zwerger, Annals Physics 2011

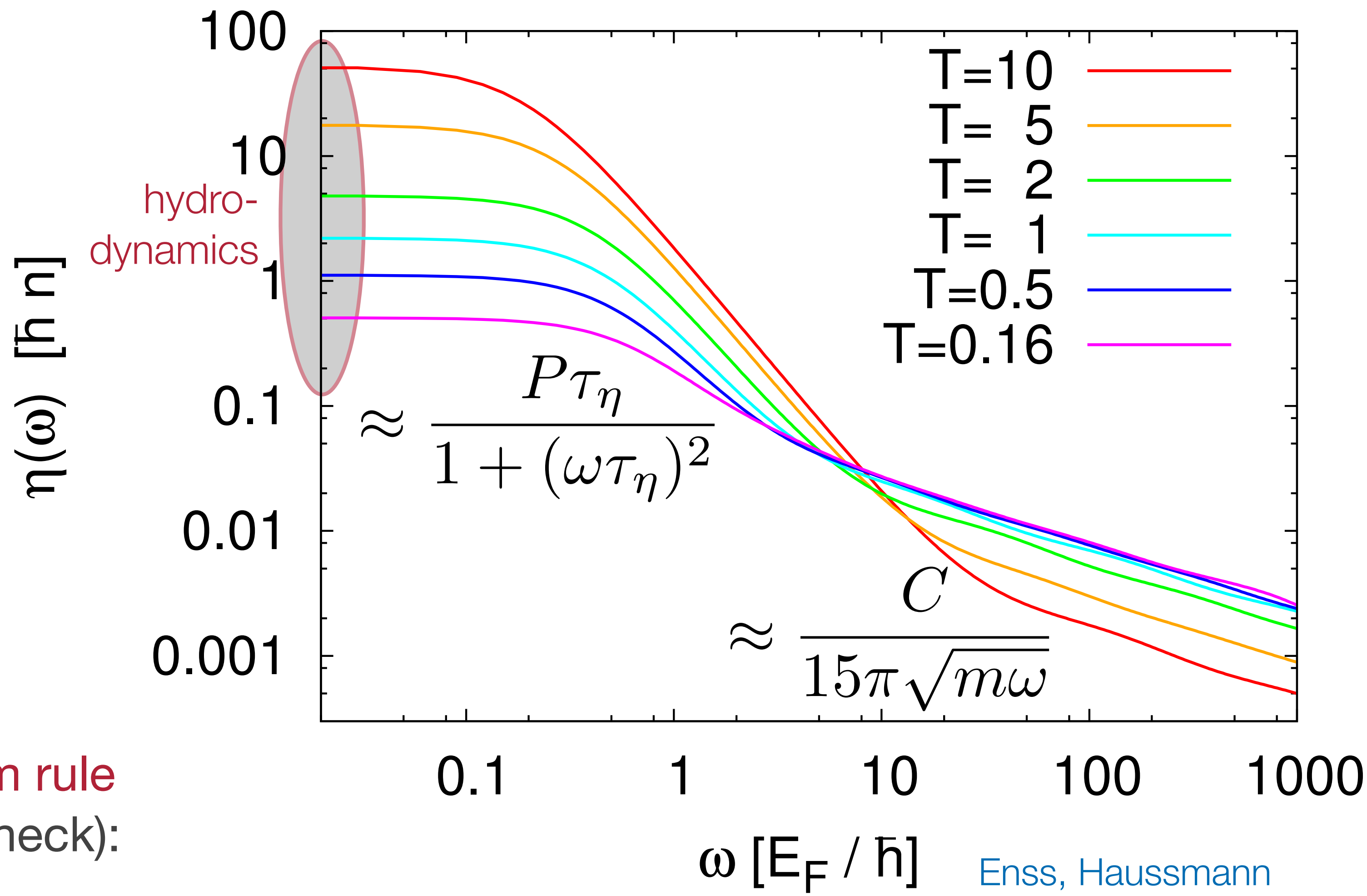
η(ω) = 

(resummed to infinite order)

- transport via fermions and bosonic molecules: very efficient description, satisfies conservation laws, scale invariance and Tan relations Enss PRA 2012

- assumes no quasiparticles: beyond Boltzmann kinetic theory, works near Tc; includes pseudogap **and** vertex corrections

Dynamical stress correlations (shear viscosity)



Enss, Haussmann & Zwerger 2011

exact viscosity sum rule
(nonperturbative check):

$$\frac{2}{\pi}\int_0^\infty d\omega\ [\eta(\omega) - \mathrm{tail}] = P - \frac{C}{4\pi m a}$$

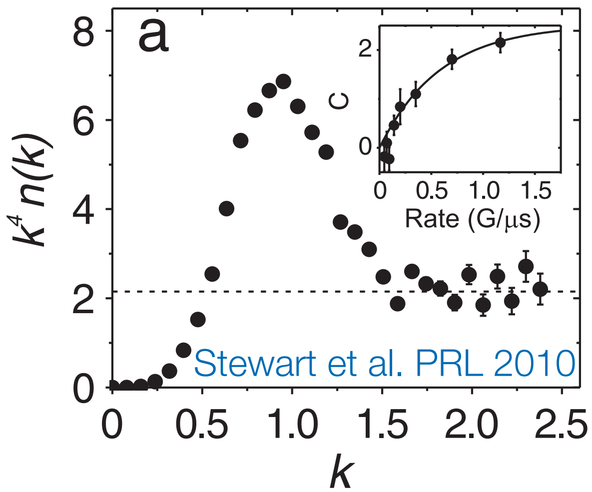
Taylor & Randeria 2010; Enss, Haussmann & Zwerger 2011; Enss 2013

Contact density

- dilute gas: **universal short-distance behavior** for $r_0 \lesssim r \lesssim \ell$

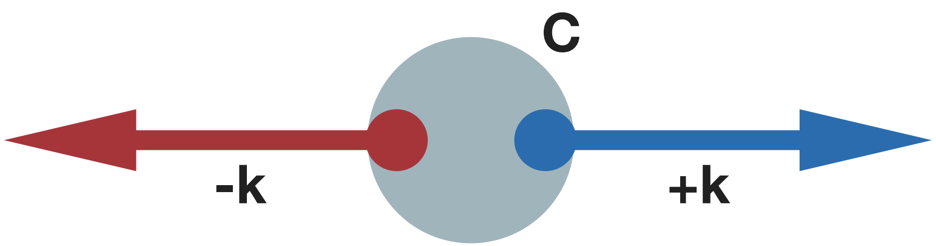
many-body **few-body**

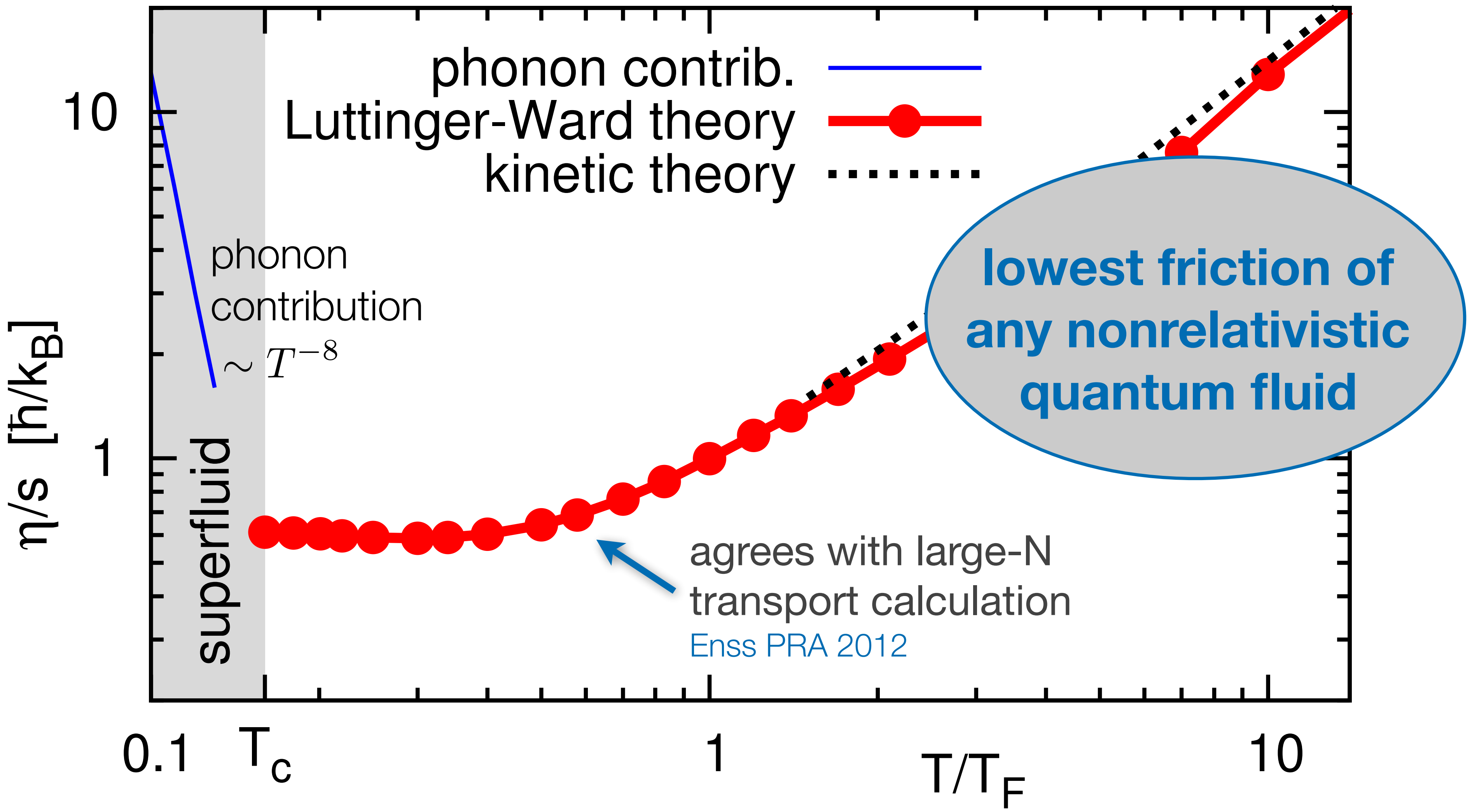
$$\langle \hat{n}_\uparrow(r)\hat{n}_\downarrow(0)\rangle \simeq C\left(\frac{1}{r}-\frac{1}{a}\right)^2 \qquad n(k) \simeq \frac{C}{k^4}$$



- **Tan contact density C:** probability of finding ↑ and ↓ close together (property of medium) Tan 2008; Braaten and Platter 2008

- universal high-energy tails in correlation functions:





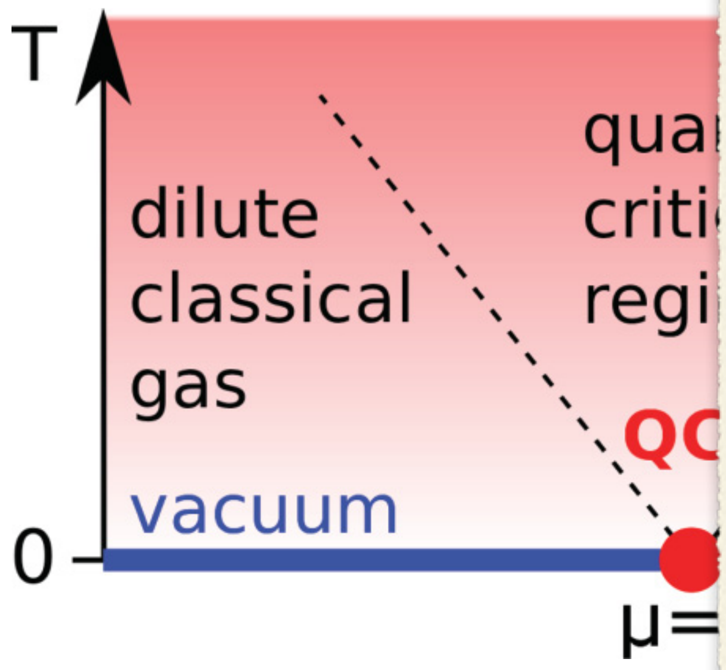
experiment: John Thomas
recent theory: Kagamihara et al. 2019

Shear viscosity/entropy of the unitary Fermi gas

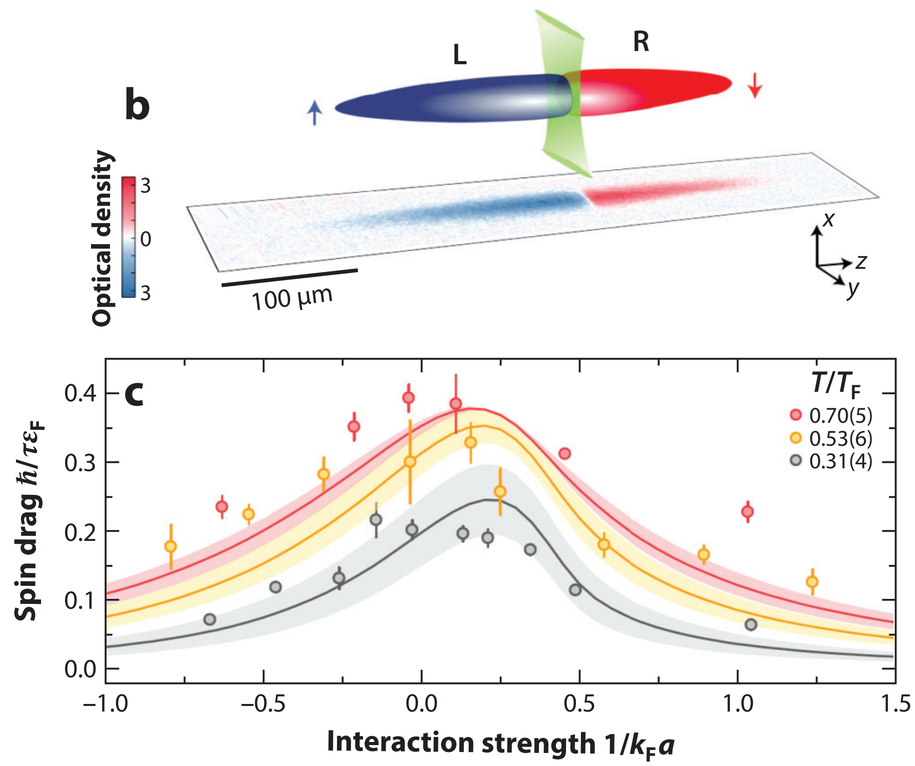
Enss, Haussmann & Zwerger 2011

Universal phase diagran

- **Quantum Critical Point** (QCP) at T
  Nikolic & Sachdev 2007



Universal spin diffusion:



Valtolina et al. Nature Phys. 2017
Enss & Thywissen, Annu. Rev. Cond. Mat. Phys. 2019

- change of ground state at QCP, der

- **quantum critical regime:** incoherent relaxation rate $\tau^{-1} \sim k_B T/\hbar$

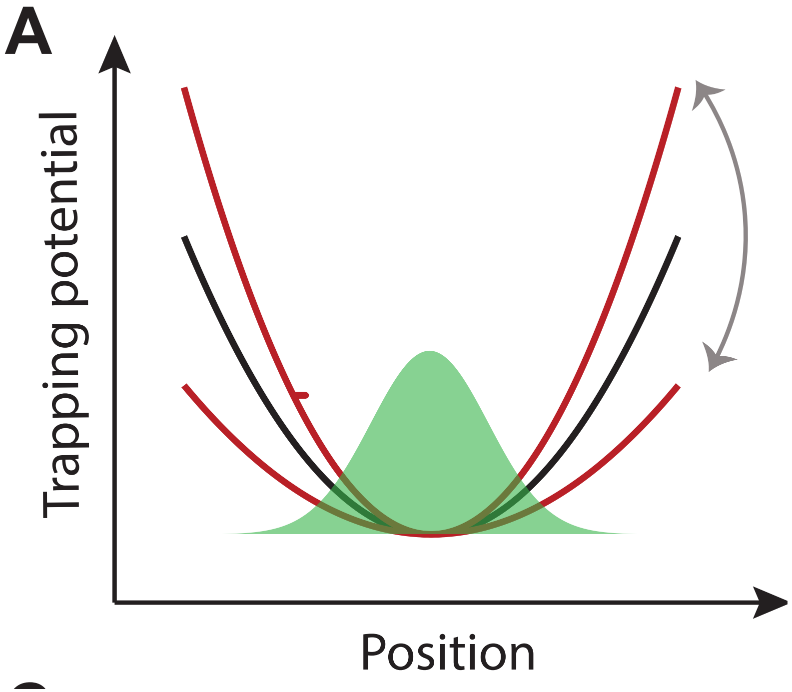
Bulk viscosity

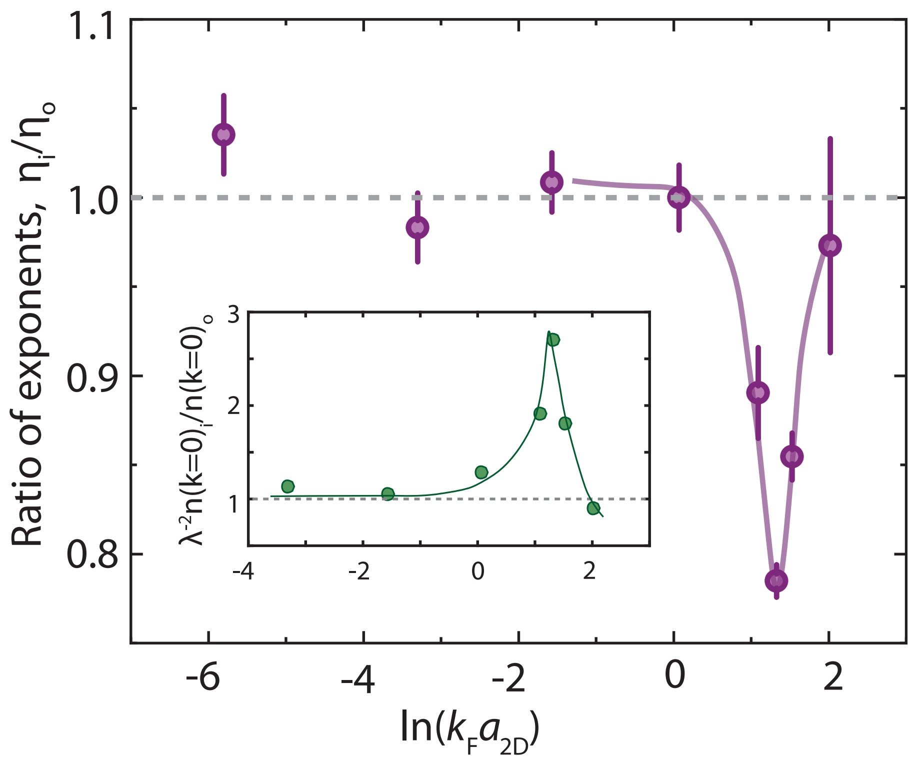
Bulk viscosity

- definition $\text{bulk } F/A = \zeta \nabla \cdot \overrightarrow{v}$ $\text{shear } F/A = \eta \frac{\partial v_x}{\partial y}$

- determines
  - damping of breathing motion / free expansion
  - sound attenuation
  - **breaking of scale invariance**

Quantum scale anomaly in 2D Fermi gas





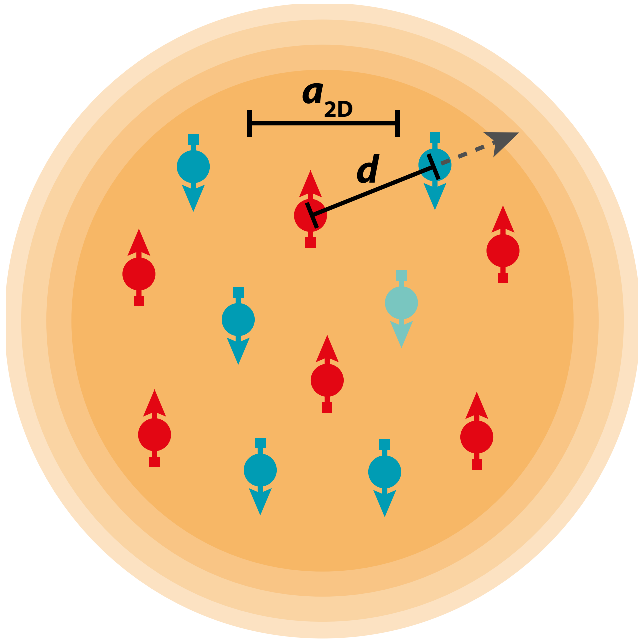
Murthy, Defenu, Bayha, Holten, Preiss, TE & Jochim, Science **365,** 268 (2019)

breaking of scale invariance (log. running coupling)
accompanied by **damping of breathing motion**

Scale transformation

- Hamiltonian

$$H = \int_x \sum_\sigma \psi_\sigma^\dagger \Big(-\frac{\nabla^2}{2m}\Big)\psi_\sigma + g_0 \int_x n_\uparrow(x) n_\downarrow(x)$$



- expansion $\boldsymbol{x} \mapsto \lambda \boldsymbol{x}$:

  kinetic term $H_{\text{kin}} \mapsto \dfrac{1}{\lambda^2} H_{\text{kin}}$

  at resonance also $H_{\text{int}} \mapsto \dfrac{1}{\lambda^2} H_{\text{int}}$

  unitary Fermi gas is scale invariant

  entropy per particle unchanged: no dissipation, $\zeta \equiv 0$ Son 2007

Correlation function

- Kubo formula: correlation of stress tensor (pressure)

$$\zeta(\omega) = -\frac{1}{id^2\omega}\int_0^\infty dt\, e^{i\omega t}\int d\boldsymbol{x}\,\langle[\Pi_{ii}(\boldsymbol{x},t),\Pi_{jj}(0,0)]\rangle$$

- dilute quantum gas

$$\Pi_{ii} = 3P = [H, iD] = 2H + \boxed{\frac{\mathscr{C}}{4\pi m a}} \qquad (\beta \text{ function } \frac{\partial H_{\text{int}}}{\partial \ln|a|})$$

- **scale invariant:** no contact term, $\Pi_{ii} = 2H$, and $\zeta(\omega) \equiv 0$:

  bulk viscosity vanishes for Unitary gas and ideal Fermi/Bose gas

  Werner, Castin 2006; Son 2007; Enss, Haussmann, Zwerger 2011; Hou, Pitaevskii, Stringari 2013

Contact correlations

- bulk viscosity arises purely from contact term (scaling violation):

$$\zeta(\omega) = -\frac{1}{i\omega d^2(4\pi m a)^2}\int_0^\infty dt\, e^{i\omega t}\int d\boldsymbol{x}\,\langle[\mathscr{C}(\boldsymbol{x},t),\mathscr{C}(0,0)]\rangle$$

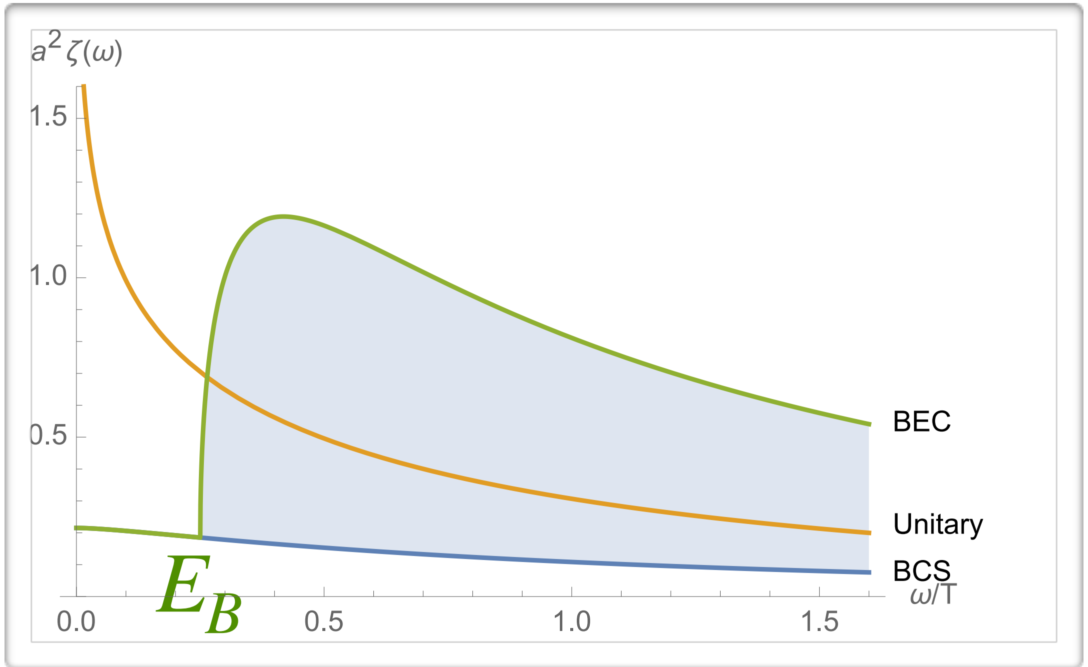
- **scale anomaly arises purely from pair fluctuations:**

$\mathscr{C} = g_0^2\,\psi_\uparrow^\dagger\psi_\downarrow^\dagger\,\psi_\downarrow\psi_\uparrow = \Delta^\dagger\Delta$ density of local pairs with

$\Delta(x) = g_0\,\psi_\downarrow(x)\,\psi_\uparrow(x)$ pair operator

Fujii & Nishida PRA 2018; Enss PRL 2019

High-temperature limit: ζ vs. frequency



Enss PRL 2019; Nishida Ann. Phys. 2019; Hofmann PRA 2020

prediction

BEC side: bulk viscosity from breaking pairs

probe with frequency

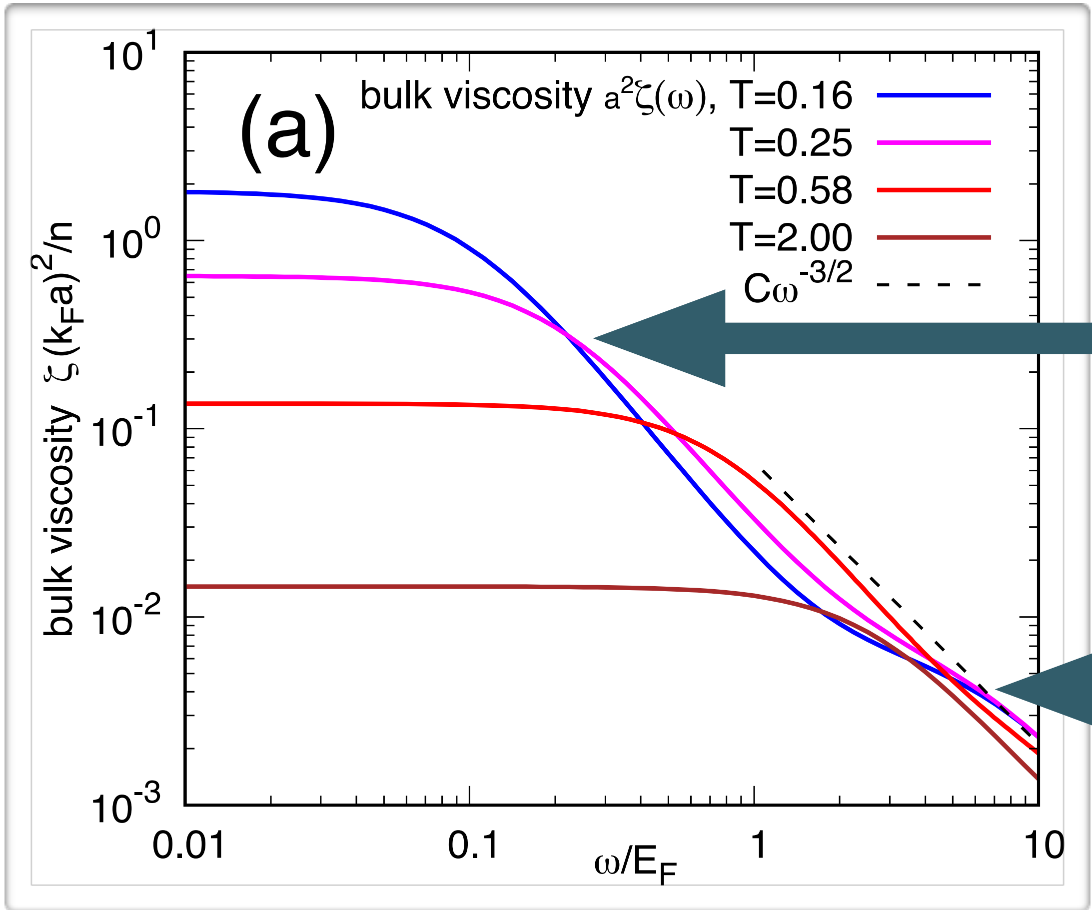
$\omega < E_B$: small damping

$\omega > E_B$: large damping

dynamical viscosity (unitarity) at leading order in $z = e^{\beta\mu} \ll 1$:

$$\zeta(\omega) = \frac{2\sqrt{2}}{18\pi^2}\,\frac{z^2}{\lambda a^2}\cdot\frac{\sinh(\omega/2T)}{\omega/2T}K_0(\omega/2T) \qquad \sim \frac{\ln(T/\omega)}{a^2}z^2$$

Quantum degenerate regime (Luttinger-Ward theory)



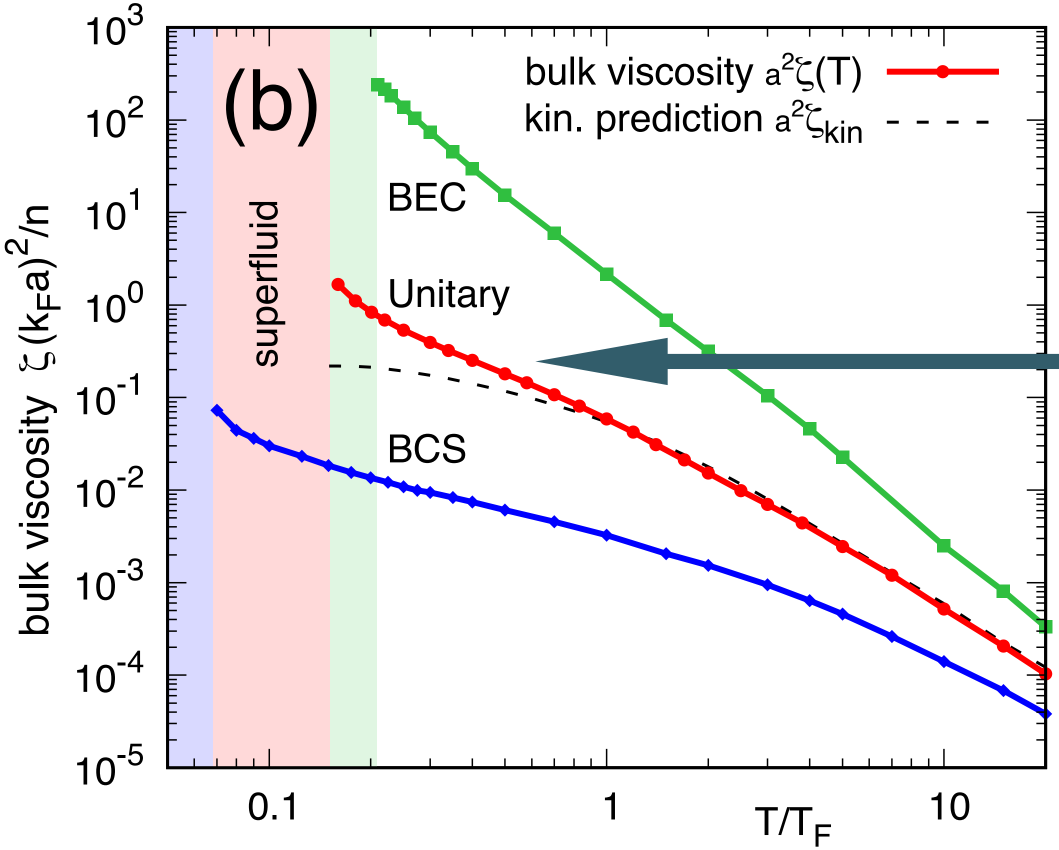
transport peak of width
$\tau^{-1} \simeq k_B T/\hbar$:
Quantum Critical Regime

contact tail $C/\omega^{3/2}$
at high frequency

Enss PRL 2019

Quantum degenerate regime (Luttinger-Ward theory)

strong enhancement in quantum degenerate regime ($\zeta > \eta$)



Enss PRL 2019

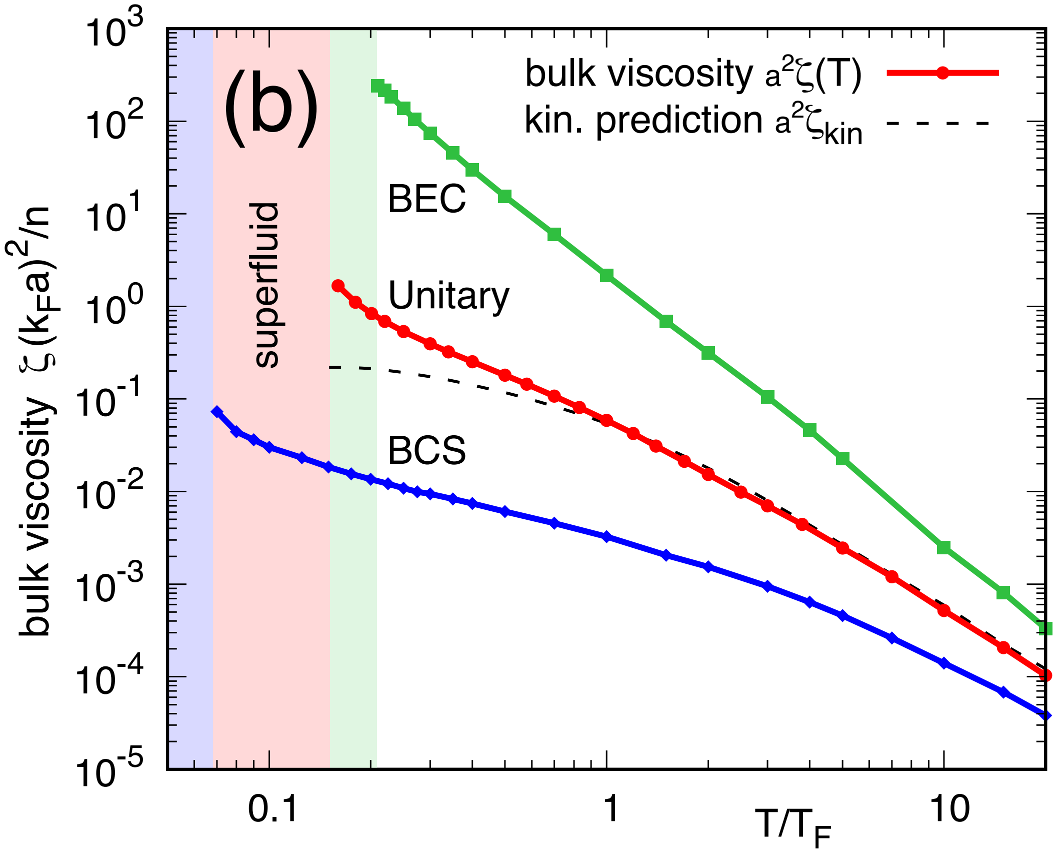
larger than kinetic theory prediction for $T < T_F$:

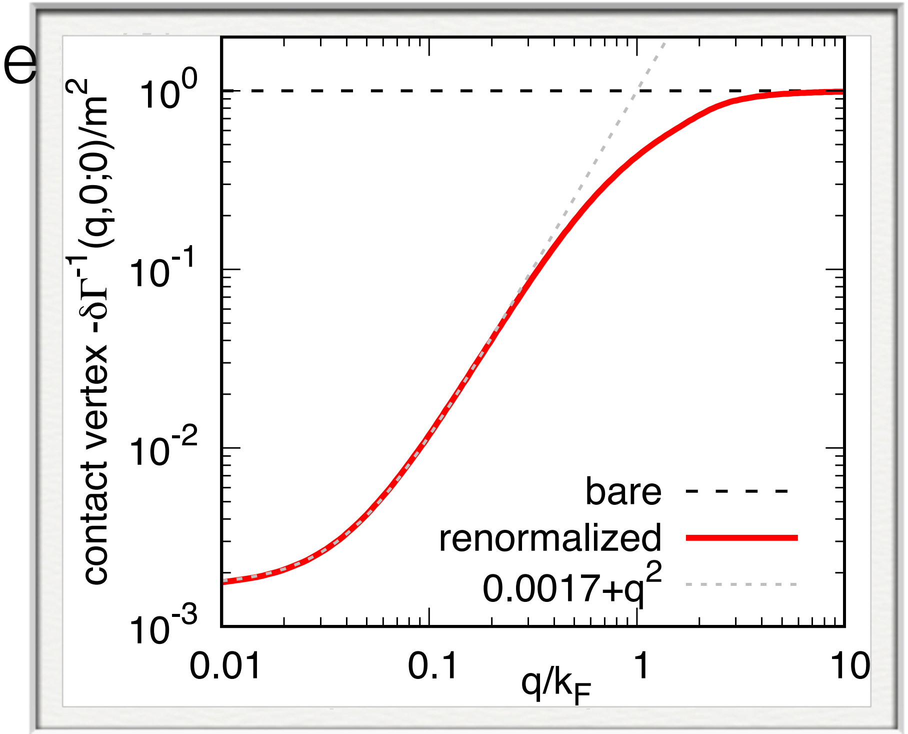
$$\frac{a^2\zeta}{\eta} \simeq a^2\left(\frac{P - 2\mathscr{E}/3}{P}\right)^2 \simeq \left(\frac{\mathscr{C}}{P}\right)^2$$

Critical fluctuations

$$\zeta \simeq \left\langle [\Delta^\dagger \Delta(x,t), \Delta^\dagger \Delta(0,0)] \right\rangle$$ pair density correlation:

enhanced by pairing fluctuations ne





contact **vertex corrections** strongly suppress fluctuations

Enss PRL 2019

Measure contact correlations

- linear response of contact to change in scattering length

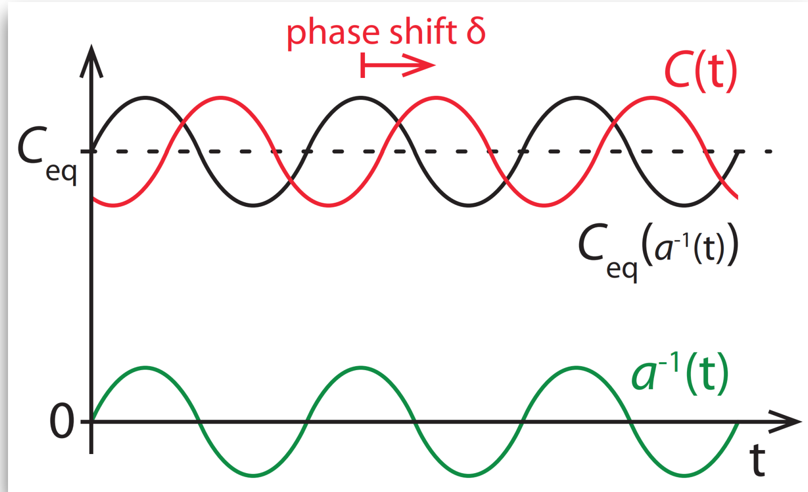
$$\int d\boldsymbol{x}\,\langle[\mathscr{C}(\boldsymbol{x},t),\mathscr{C}(0,0)]\rangle = -4\pi m\left(\frac{\partial\langle C(t)\rangle}{\partial a^{-1}(0)}\right)_{S/N}$$

- dissipation yields phase shift Fujii & Nishida PRA 2018

$$C(t) = C_{\text{eq}}[a(t)] - 36\pi m\cdot a^2\zeta\cdot\frac{da^{-1}}{dt}$$

- dissipative heating rate Horikoshi 2019

$$\frac{dE}{dt} = \frac{9}{2}A^2\omega^2\cdot a^2\zeta(\omega)$$



Outlook

- **contact correlations** in new observables, e.g. sound diffusion
- **critical fluctuations** near phase transition and behavior below T_c
- local transport measurements in **homogeneous systems,** novel techniques
- how does hydrodynamics emerge in **small systems?**