Low-energy neutron scattering on Light Nuclei and $^{19}\text{B}$ isotope as a $^{17}\text{B-n-n}$ three-body cluster in the unitary limit

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In Nuclear Physics, few things are more interesting than the very low energy (S-wave) scattering of n’s

**On heavy nuclei** it gives rise to the fantastic forest of « resonances » (see the scale!)

and can be even **very dangerous**…

In any case, free from Coulomb, partial waves, centrifugal barrier, spins-orbit, tensor, … it makes the delicious of theorists and it is very sensitive to the interaction

**On light nuclei** is certainly less spectacular, **ALTHOUGH** ….
INTRODUCTION

The S-wave neutron-Nucleon (n,p) interaction is **attractive in all spin and isospin channels**.

The S=1 \( np \) state is the more attractive one, enough to **bind** the deuteron by \( B = 2.22 \text{ MeV} \).

The S=0 \( np \) and \( nn \) states are not bound... but almost: have a “virtual state” close to threshold.

This spin-dependence accounts for a 20% difference in the attractive strength of NN interaction.

Despite **all** \( V_{nN} \) are attractive, a low energy \( n \) scattering on a light nucleus soon (\(^2\text{H}\)) behaves as if the \( V_{nA} \) was repulsive...

A \( n \) approaching a nucleus "feels" the others \( n \)’s in the target and **it doesn’t like it!** (Pauli)
INTRODUCTION

A dramatic consequence happens in $3n$ and $4n$ systems: $H_{3n}$ has a (ground) bound state at about 1 MeV (5 MeV for $H_{4n}$) … but in nature neither $3n$ nor $4n$ are bound.

The lowest state of $H_{3n}$ and $H_{4n}$ is symmetric. The first antisymmetric state is much higher in spectrum. Everything happens as if there was a repulsion among n’s: the “Pauli repulsion”

An interesting quantity to measure the repulsive/attractive character of $V_{nA}$ is the scatt length $a_{nA} = - f_{nA}(E=0)$

For purely repulsive $V$, $a > 0$

For purely attractive $V$, $a < 0$ … until a bound state appears.

For a realistic interaction – mixing repulsive core with attractive parts – it will result as a balance of both tendencies.
**INTRODUCTION**

The evolution of $a_{nA}$ when increasing $N$ is summarized below

<table>
<thead>
<tr>
<th>Z</th>
<th>N</th>
<th>A</th>
<th>Sym</th>
<th>J</th>
<th>$a_-$</th>
<th>$a_+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>p</td>
<td>$\frac{1}{2}^+$</td>
<td>-23.71</td>
<td>+5.41*</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>n</td>
<td>$\frac{1}{2}^+$</td>
<td>-18.59</td>
<td>/</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>$^2$H</td>
<td>1-</td>
<td>+0.65*</td>
<td>+6.35</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>3</td>
<td>$^3$He</td>
<td>$\frac{1}{2}^+$</td>
<td>+6.6-3.7i</td>
<td>+3.5</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>$^3$H</td>
<td>$\frac{1}{2}^+$</td>
<td>+3.9</td>
<td>+3.6</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>4</td>
<td>$^4$He</td>
<td>0+</td>
<td>+2.61</td>
<td>/</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>6</td>
<td>$^6$Li</td>
<td>1+</td>
<td>+4.0</td>
<td>+0.57</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>7</td>
<td>$^7$Li</td>
<td>$\frac{3}{2}^-$</td>
<td>+0.87</td>
<td>-3.63</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>8</td>
<td>$^8$He</td>
<td>0+</td>
<td>-3.17</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>9</td>
<td>$^9$Li</td>
<td>$\frac{3}{2}^-$</td>
<td>-14</td>
<td></td>
</tr>
</tbody>
</table>

For $A=n,p$ all channels are attractive, as expected (despite its sign, like for +5.41*)

With $A=2$, the quartet state ($S=3/2$) starts being repulsive: Pauli repulsion dominates over $nN$ attraction

In $A=7$ an attractive channel appears again: $^7$Li ($J=3/2^-$)

P-wave $n$'s decrease the Pauli repulsion: 2 $p_{3/2}$ n's enough to balance into an “attractive” $V_{nA}$

Rm: previous repulsion were only in S-wave: P-wave were attractive, even resonant (n-$^3$H,n-$^4$He)

The “attraction” persists in $^{12}$Be,$^{15}$B... **until something spectacular occurs**...
ONE OF THE MOST FASCINATING SYSTEMS IN NUCLEAR PHYSICS

$^{17}$B is a (strong) stable nucleus with $J^\pi = 3/2^-$ consisting on a sea of $^{12}n$ sourrounding $^{5}p$

The balance between attractive $\pi$-exchange between $n$ and $^{17}$Nucleon and “Pauli repulsion” with $^{12}n$’s of $^{17}$B is so fine-tuned that the scattering length is $a_{n-^{17}B} \sim -100$ fm (max $\chi^2$)

A low energy $n$ scattering on $^{17}$B will “feel” a monster of geometrical size $D \sim 400$ fm
Not yet a virus but we are getting close ! (“visible” ?)

The « low energy region » where $n$ feels the monster is « very low » …

\[ \sigma_L(k) = (2L + 1)4\pi \frac{\sin^2 \delta_L(k)}{k^2} \]

\[ \sigma(0) = 4\pi a^2 \]

Nevertheless the effect is huge, even with respect to what was considered huge untill now !
EXPERIMENTAL

How do we know that this history is true?

I. A first MSU measurement Spyrou et al. PLB683(2010)129 claimed the existence of a $^{18}$B “virtual” (unbound) state and a n-$^{17}$B $a_s < -50$ fm (max $\chi^2$ at $a_s = -100$ fm)

Claims for $J^\pi=2^-$

II. A recent RIKEN result (SAMURAI Collaboration) this state was observed in other channels

The precise value of $a_s$ it is not (yet) known, most probably $<-100$ fm
THEORY

The large value of $a_s$ indicates the existence of a “18B virtual state” very close to threshold.
It corresponds to a pole in the $n$-$17B$ scattering amplitude $f(k)$ at $\text{Im}(k)<0$, as in $nn$ case.

One of the most interesting virtual states in Nucl Physics:
- the scattering length $a_s$ is the « nuclear chart record » … waiting for a final result!
- much larger than the highly celebrated $a_{NN}=-24$ fm, which, « controls the nuclear chart »
  
  We argue that many features of the structure of nuclei emerge from a strictly perturbative expansion
  around the unitarity limit, where the two-nucleon S waves have bound states at zero energy
- It is even comparable to atomic physics cases! and a candidate to Efimov martyrology

But this not all….

- $19B$ is bound with a binding energy $B$ in $[0,0.53]$ MeV
- $19B$ has several resonant states
- A series of $20B, 21B$ resonances were recently discovered S. Leblond et al, PRL121, 262502(2018)

All that gave a strong motivation to model $19B$ as a $17B$-$n$-$n$ 3-body cluster
- built wit 2 resonant scattering lengths (exemple of Borromean state)
- with possible extensions to $17B$-$n$-$n$-$n$ and $17B$-$n$-$n$-$n$-$n$

First results in E. Hiyama, R. Lazauskas, M. Marqués, J. Carbonell, PRC100, 011603R(2019)
MODELING THE n-17B SYSTEM

Ingredients:

- Repulsive+Attractive part : $V_r, V_a, \mu$

- Hard core radius : $n$ cannot penetrate at $r < R = \text{size parameter}$
  $R$ can be (matter radius, experimentally known $R_m = 299$)

- Pion exchange (dominant at large $r$) $\mu = 0.70$ fm$^{-1}$

Simplest ansatz

$$V(r) = V_r \frac{\exp(-2\mu r)}{r} - V_a \frac{\exp(-\mu r)}{r}$$

Equivalent to

$$V(r) = V_r \left( e^{-\mu r} - e^{-\mu R} \right) \frac{e^{-\mu r}}{r}$$

$\mu$ and $R$ being fixed, there is one single parameter $V_r$

$V_r$ is adjusted to reproduce the experimental value of $a_s$
Since we are still waiting for it, we parametrize all in terms of $a_s$
Determining $a_s = f(V_r)$

Dashed lines correspond to $a_s = -50$ (3864 MeV), -100 (4030), -150 (4090) fm with $R=3.0$ fm

Singularity on right would corresponds to the (unphysical) bound $^{18}$B state

Corresponding potentials saturates for $a_s \sim -100$ fm
MODELING $^{19}$B as $^{17}$B-n-n CLUSTER

Solve the 3-body problem (Faddeev+Gaussian) with $V_{n-^{17}B}$ and some realistic $V_{nn}$ $^{19}$B appears to be bound for $a_s < -50$ (the only parameter!) in a $J^\pi=3/2^-$ state $(L=0,S=0)$

We used 2 different $nn$ interactions and let $V_{n-^{17}B}$ act in S-wave (s. blue) or in all PW (s. black)
The energy is always compatible with the experimental value $E = -0.14 +/- 0.39$ MeV

In the S-wave case we consider the unitary limit: $a_s = a_{nn} \to -\infty$ (blue dashed)
The result is still compatible with experimental value and constitutes a first illustration of this interesting limit in Nuclear Physics.
Spatial probability amplitude $| \Psi(r, R) |^2$ fixing $a_s=-100$ fm

Compared with a similar system $^6\text{He}=^4\text{He}+n+n$

$\text{RMS}_{nC}(^{19}\text{B})=12.0$ fm

$\text{RMS}_{nC}(^{6}\text{He})=4.5$ fm
We also found two $^{19}\text{B}$ resonances: fixing $a_s=-150$ and using the S-wave model

\begin{align*}
L &= 1 & E_1 &= 0.24-0.31i \text{ MeV} \\
L &= 2 & E_2 &= 1.02-1.22i \text{ MeV}
\end{align*}

Their existence is in agreement with experimental findings

J. Gibelin et al., Contribution to FB22, Caen July 2018, Springer Proc in Press

Very simple and successful model:

- local S-wave potential
- no 3-body force
- one single parameter

The key of the « succes » is the double resonant character
**Some refinements : the spin-spin dependence**

$^{17}\text{B}$ being $J^\pi=3/2^-$, there are two different scattering lengths $a_s$ corresponding to $S=1,2$. Assuming that the virtual state we adjusted was $a_2$ there is no reason that $a_1= a_2$.

Introduced a spin-spin dependence with different $V_n^{17\text{B}}$ for each $S$, keeping the same form

$$V_n^{17\text{B}}(S) (r) = V_r^{(S)} (e^{-\mu r} - e^{-\mu R}) \frac{e^{-\mu r}}{r} \quad S = 1,2$$

There exists a critical value $a_1^c$ above which $^{17}\text{B}$ binding disappears but this requires unphysical SS beaking $V_r^{(1)}/V_r^{(2)}=2$: results are stable even when varying $R$.
CONCLUSIONS

We present a local S-wave potential to describe the n-\(^{17}\)B interaction and its virtual state. It depends on 1 parameter, adjusted to reproduce the huge n-\(^{17}\)B scattering length (\(a_s \approx -100\) fm).

Supplemented with the nn interaction we describe well the \(^{19}\)B as a 3-body \(^{17}\)B-n-n cluster:
- Its ground state (E=-0.14 +/- 0.40) MeV
- Two (L=1, and L=2) resonances all in agreement with experimental findings.

The \(^{19}\)B ground state is a « double resonant » state compatible with the unitary limit in both nn and n-\(^{17}\)B interactions.

MSU/RIKEN finding on \(^{18}\)B virtual state was quite fortuitous. The possibility of finding similar resonant structures, bound (\(a_s >0\)) instead of virtual, in a systematic scanning of the nuclear chart cannot be excluded. This will correspond to an extremely large and fragile (A+1) nuclear structure involving sizes still smaller but close to atomic sizes – and only accessible via scattering experiments. They could offer a unique possibility to "visualize" a nucleus using microscopic techniques as it is currently done with atoms.

Resonant \(a_s\) leads to new clusterization mechanism: model extends to describe new B isotopes:

\(^{19}\)B=\(^{17}\)B-n-n
\(^{20}\)B=\(^{17}\)B-n-n-n
\(^{21}\)B=\(^{17}\)B-n-n-n-n

with the methods used in computing \(^4\)H and \(^5\)H \(\quad\) (L.H.C., PLB 791, 335 (2019))
CONCLUSIONS

Despite the large values of the scattering length in both $n^{-17B}$ and $nn$ channels, we found that the appearance of the first Efimow excitation is excluded (would require $a_s \sim$ few thousands fm).

To fix the model parameter $V_r$ it is mandatory to determine $a_2$ and $a_1$ and obtain a more accurate value of $E(^{19}B)$.
$E_{^{19}B}(\text{MeV})$ vs $1/a_s$ (fm$^{-1}$)

- Black dots: $V$ all PW + nn Bonn A
- Blue dots: $V$ S-wave + nn CD-MT
The three-body problem was solved independently by using experimental values. In order to cross-check the results, lines correspond, respectively, to model versions (i) and (ii).

In view of these results, the following remarks are in order:

1. The quantum numbers of the total wave function is given by the intrinsic parity of the system, which slightly depends on the model version:

   \[ S = \pm 3 \text{ or } \pm 1, \]

   \[ S_0 < 50 \text{ fm} \]

2. In the range of \(-0.25 \text{ MeV} \leq E < -0.05 \text{ MeV}\), the centrifugal term varies with the spin of the cluster.

3. The adjusts the strength of the attractive term to its value in the spin-symmetric scattering.

4. The exact values of the scattering length are 77 fm (solid blue line), 93 fm (blue) and 150 fm (black).

5. The main difference between versions (i) and (ii) in the spin-symmetric wave function is explained in Ref. [21]. It was claimed that the gross features of the wave function are essentially the same in all partial waves. For the case of \( a = 77 \text{ fm} \), the theoretical result is in close agreement with the experimental result. The background energy is the same in all partial waves. It is clear that the Bonn A model is charge independent and has a remarkable property, though at the level of cluster description its structure is not well understood.

In the sake of comparison, we have computed the same amplitude of \((3)\) the Bonn A model is charge independent and has a remarkable property, though at the level of cluster description its structure is not well understood.

The full unitary result, where both interactions are taken into account, is obtained with energies close to the unitary limit. As expected from the EFT arguments, the interaction is smaller, and mainly given by the higher partial waves in the spin-symmetric wave function. The energy is increased.

The Bonn A model is charge independent and has a remarkable property, though at the level of cluster description its structure is not well understood.