Energy shifts of a three-alpha system in thermal plasmas

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Introduction

- Triple-alpha reactions in a normal star
  - A key reaction to produce ^{12}C
    - ^{12}C production occurs via $^{8}Be+\alpha \rightarrow {}^{12}C^{*}$ (Hoyle state)
  - Presence of plasma environment in stable helium burning
    - Coulomb interaction between alpha particles are screened off by helium ions and electrons
- Purpose of this study
  - Precise three-alpha model calculations for the Hoyle state with Coulomb screening
    - $1/r$ is replaced with Yukawa form e^{-Cr}/r (C: screening factor)

      How do the energy (Q-value) shifts develop with respect to C?
    - Interaction between alpha particles is changed depending on the plasma environment
      - Relevance to the Efimov physics

        Possibility of the realization of the unitary limit ($a \rightarrow \infty$)

Coulomb screening in thermal plasmas

Debye-Hückel (DH) approximation

Charged particles behave as an ideal, thermal gas

Debye screening length $\lambda_D = \left[\frac{k_B T}{4\pi e^2 (n_e + \sum_i n_i Z_i^2)}\right]^{1/2}$

n_e: The number of electrons
n_i: The number of ions (Charge Z_i)

Plasma composed of electron and helium ion (Z=2) (T～10^8 K, ρ～10^3-10^6 gcm^{-3})

Coulomb correction: 1/r → Yukawa form

$$\Delta V_C = \sum_{j<k} \frac{4e^2}{r_{jk}} e^{-r_{jk}/\lambda_D} - \sum_{j<k} \frac{4e^2}{r_{jk}},$$

Energy shift

$$\Delta E_C = -\frac{12e^2}{\lambda_D} + O(\langle r_{jk} \rangle)$$

Also, consistent with the shift derived from corrections to the chemical potentials within the chemical equilibrium condition of 3 alpha and C*.

$$3\mu_\alpha = \mu_{C^*} \qquad \mu_i = m_i c^2 - k_B T \ln\left[\frac{g_i}{n_i}\left(\frac{m_i k_B T}{2\pi\hbar^2}\right)^{3/2}\right] - \frac{Z_i^2 e^2}{2\lambda_D}$$

Three-alpha wave functions

- Hamiltonian $$H = \sum_{i=1}^{3} T_i - T_{\rm cm} + \sum_{i<j} [V_{ij}^{2\alpha} + V_{ij}^{\rm Coul}(C)] + V_{123}^{3\alpha},$$
- Two-alpha interactions
  - Modified Ali-Bodmer potential
    - L-independent
    - Reproduce the resonant energy of ^{8}Be

$$V_{ij}^{2\alpha} = 125 \exp\left(-\frac{r_{ij}^2}{1.53^2}\right) - 30.18 \exp\left(-\frac{r_{ij}^2}{2.85^2}\right)$$

S. Ali, A.R. Bodmer, NPA80, 99 (1966)
D.V. Fedorov, A.S. Jensen, PLB389, 631 (1996)

  - Screened Coulomb potential within the DH approx.

$$V_{ij}^{\rm Coul} = \frac{4e^2}{r_{ij}} \exp\left(-C r_{ij}\right) \longrightarrow V_{ij}^{\rm Coul} = \frac{4e^2}{r_{ij}} {\rm erf}(\kappa r_{ij}) \exp\left(-C r_{ij}\right)$$

Finite size of alpha particle (Gaussian)

- Three-alpha interaction

Set 1: purely attractive v_r=0

H. Suno, Y. Suzuki, P. Descouvemont, PRC94, 054607 (2016)

Set 2: short range repulsion

$$V_{123}^{3\alpha} = v_r \exp\left(-\frac{R^2}{b_r^2}\right) - v_a \exp\left(-\frac{R^2}{b_a^2}\right)$$

$$R^2 \equiv \sqrt{3} \sum_{i=1}^{3} (\boldsymbol{r}_i - \boldsymbol{x}_3)^2 = \frac{\sqrt{3}}{2} x_1^2 + \frac{2}{\sqrt{3}} x_2^2.$$

Variational calculations for three-alpha systems

Three-body wave function is expanded in symmetrized ($\mathcal{S}$) correlated Gaussians

$$\Psi^{(n)} = \sum_{k=1}^{K} c_k^{(n)} \, \mathcal{S} \, G(A_k, \boldsymbol{x}).$$

<u>Correlated Gaussian basis</u> K. Varga and Y. Suzuki, Phys. Rev. C52, 2885 (1995).

Invariant wrt coordinate transformation

Matrix elements are analytical

$$G(A, \boldsymbol{x}) = \exp\left(-\frac{1}{2}A_{11}x_1^2 - \frac{1}{2}A_{22}x_2^2 - \underline{A_{12}\boldsymbol{x}_1 \cdot \boldsymbol{x}_2}\right)$$

<u>Generalized eigenvalue problem</u>

$$\sum_{j=1}^{K} H_{ij} c_j^{(n)} = E^{(n)} \sum_{j=1}^{K} B_{ij} c_j^{(n)},$$

$$H_{ij} = \langle \mathcal{S}G(A_i, \boldsymbol{x}) | H | \mathcal{S}G(A_j, \boldsymbol{x}) \rangle$$

$$B_{ij} = \langle \mathcal{S}G(A_i, \boldsymbol{x}) | \mathcal{S}G(A_j, \boldsymbol{x}) \rangle$$

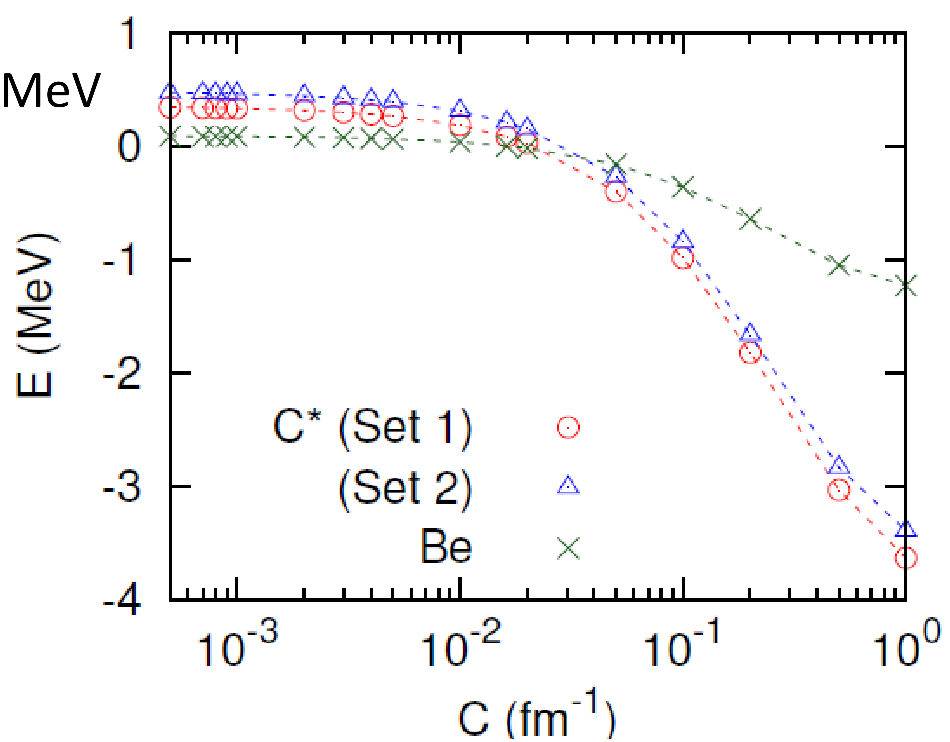
Optimization of the basis: Stochastic variational method (SVM)

Y. Suzuki and K. Varga, Stochastic variational approach to quantum-mechanical few-body problems, LNP 54 (Springer, 1998).
K. Varga and Y. Suzuki, Phys. Rev. C52, 2885 (1995).

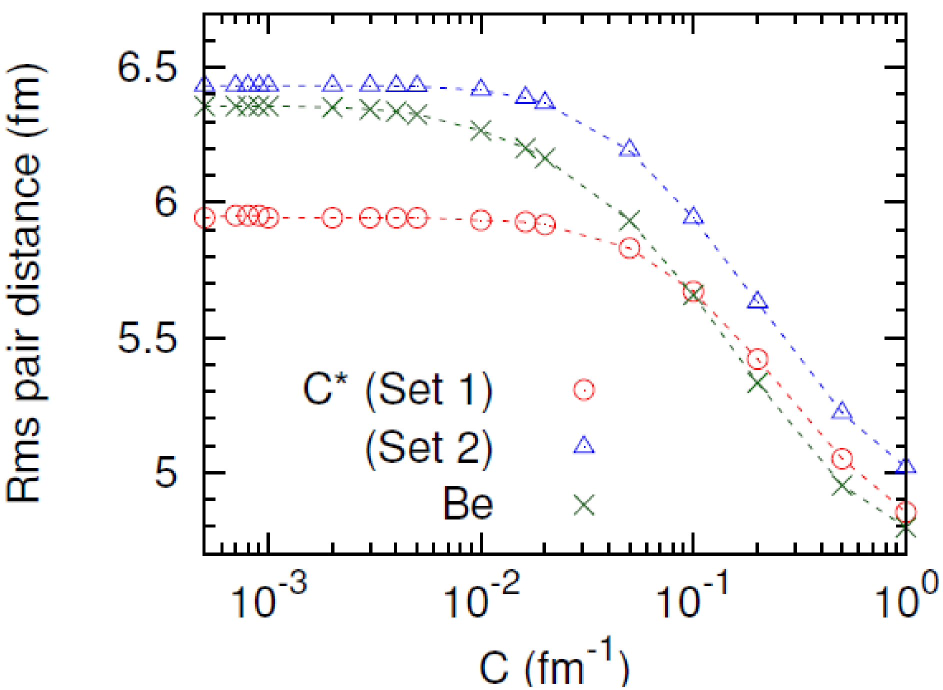
- Several sets of variational parameters $A_{11,k}$, $A_{22,k}$, $A_{12,k}$ are generated randomly, reaching up to ～20 fm
- Take one basis that gives the lowest energy and increase the number of basis until the energy is converged

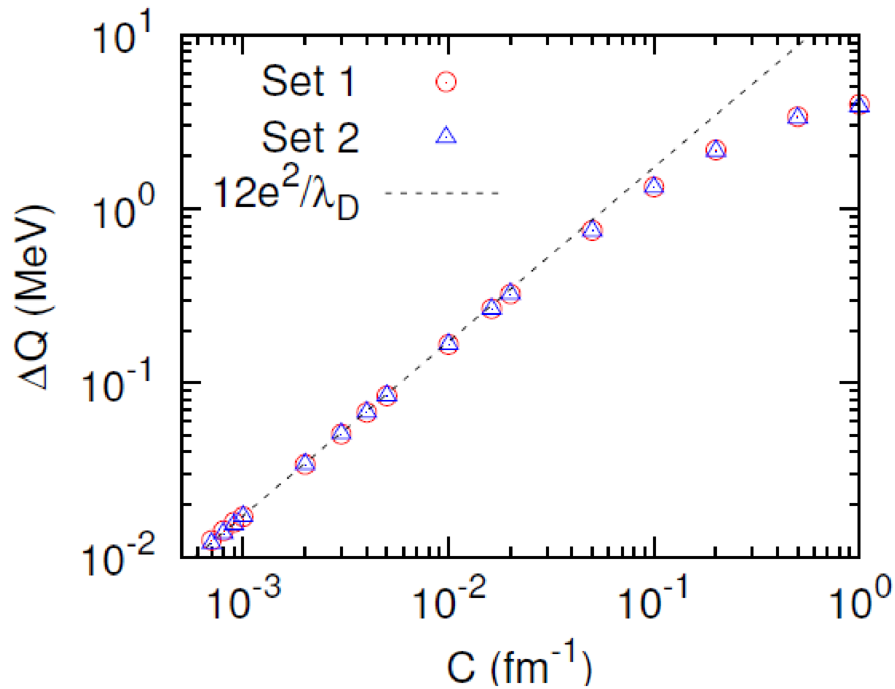
Screening effects on 2α and 3α energies

- Hoyle state energy as a function of C
  - Hoyle state becomes bound with C > 0.05 fm^{-1}
  - Typical astrophysical environment, stable burning in a normal star (T～10^8 K, ρ～10^3-10^6 gcm^{-3}), C～10^{-4}-10^{-3} fm^{-1}
  - C=0.0162 fm^{-1} $E_{Be}=10^{-5}$ MeV
    - E_{C*}=0.082 (Set 1), 0.210 (Set 2) MeV
    - Exclude the Efimov hypothesis



Energy shifts of the Hoyle states





- Rms pair distances are quite different with Sets 1 and 2
  → Repulsive components in the three-alpha interaction of Set 2
- Energy shifts $\Delta Q(C)$ appear to be the same
  - With small C, consistent with the conventional prediction obtained with point charges, $12e^2/\lambda_D$

Conclusions: Alpha particles in thermal plasmas

- Precise three-body calculations for the Hoyle state in thermal plasmas
  - Two- and three-alpha interactions consistent with the empirical values of ^{8}Be and the Hoyle state
- Energy shifts of a three-alpha system induced by the Coulomb screening in thermal plasmas
  - Finite size effect of the Hoyle state is explicitly incorporated
  - Consistent with the conventional result using the point-charge approximation

arXiv: 2003.08060