Chiral Effective Theory of Diquark and Heavy Baryon Spectroscopy

Makoto Oka

Advanced Science Research Center, JAEA

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M. Harada (Nagoya), Y.R. Liu (Shandong), MO, K. Suzuki (JAEA), Phys. Rev. D101, 054038 (2020), "Chiral effective theory of diquarks and $U_A(1)$ anomaly"
Y. Kim (Kyushu), E. Hiyama (Kyushu), MO, K. Suzuki, arXiv.2003.03525 "Spectrum of singly heavy baryons from a chiral effective theory of diquarks"

Introduction

Diquark

as open (colored) clusters in hadrons

Diquark

- The simplest *colorful cluster* in hadrons is the **diquark**.

  "bound" qq state

  color $3 \otimes 3 = \bar{3} \oplus 6$ spin : $\frac{1}{2} \otimes \frac{1}{2} = 0 \oplus 1$

- Two color anti-triplet S-wave diquarks:

  Scalar diquark S(0^+) in color anti-triplet

  Axialvector diquark A(1^+) in color anti-triplet

- *Color magnetic interaction* (CMI) via magnetic gluon exchange predicts a strong attraction in S(0^+). A significant attraction also comes from the *instanton induced interaction* (III).

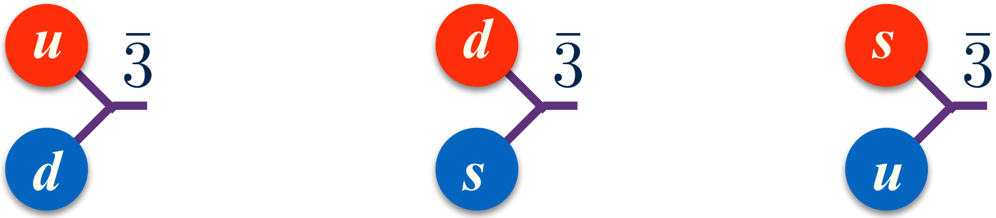
Diquark

- Scalar diquark S(0^+)

  L=0 (S), S=0 (A), color $\bar{3}$ (A) strongly attractive

  Flavor $SU(3)_f$ $\bar{3}$(A) :

  [ud]=(ud-du), [ds]=(ds-sd), [sd]=(sd-ds)



- Axial vector diquark A(1^+)

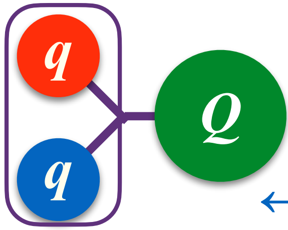
  L=0 (S), S=1 (S), color $\bar{3}$ (A)

  $SU(3)_f$ 6 (S): uu, {ud}, dd, {us}, {ds}, ss

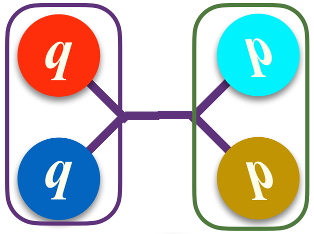
Diquark as a building block

- **Light diquark D_q (=qq) in hadrons**

  D_q Q = qq Q = HQ Baryon



↔Di-neutron+alpha

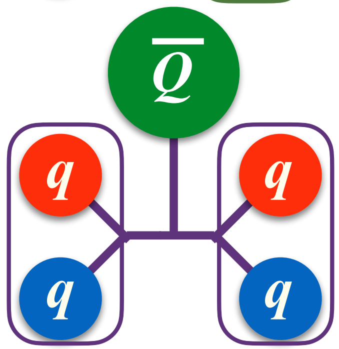


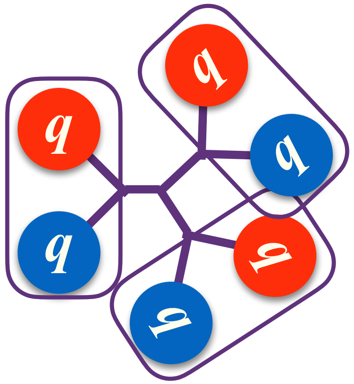
  Exotic hadrons

  $D_q D_q^{bar}$ = qq $q^{bar}q^{bar}$ = Tetraquark

  $D_q D_q Q^{bar}$ = qq qq Q^{bar} = Pentaquark

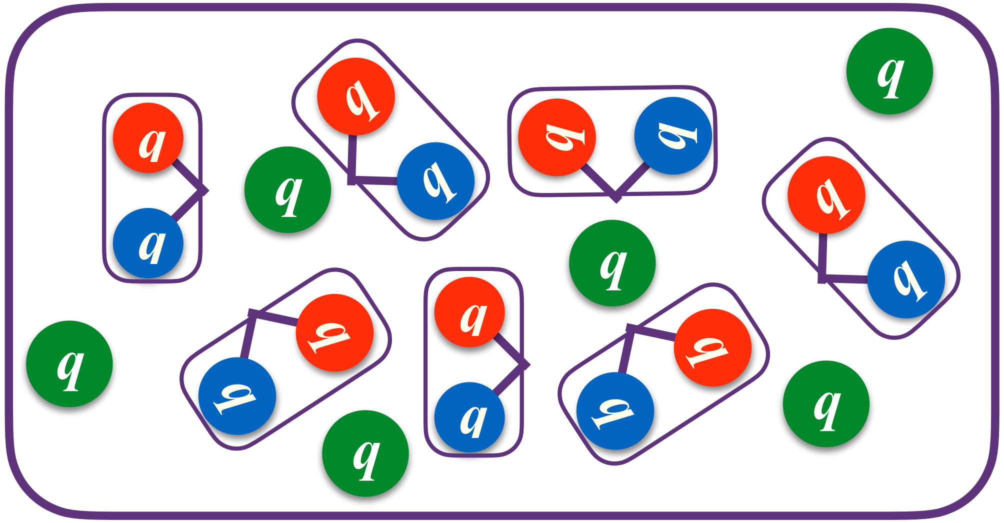
  $D_q D_q D_q$ = Hexaquark (Dibaryon)





- **BE condensate in dense hadronic matter**

  => color-superconducting phase



Chiral diquark effective theory

Diquark Effective Theory

Strategy

- Consider d*iquarks* as "colorful" clusters in hadrons and hadronic matter. In order to describe their dynamics, write down the d*iquark* effective Lagrangian.
- Chiral symmetry $SU(3)_R$ x $SU(3)_L$
  Write down general forms of the chiral invariant Lagrangian for the chiral diquarks with a background chiral meson field.
- Lattice QCD helps us to fix the parameters of the effective Lagrangian.

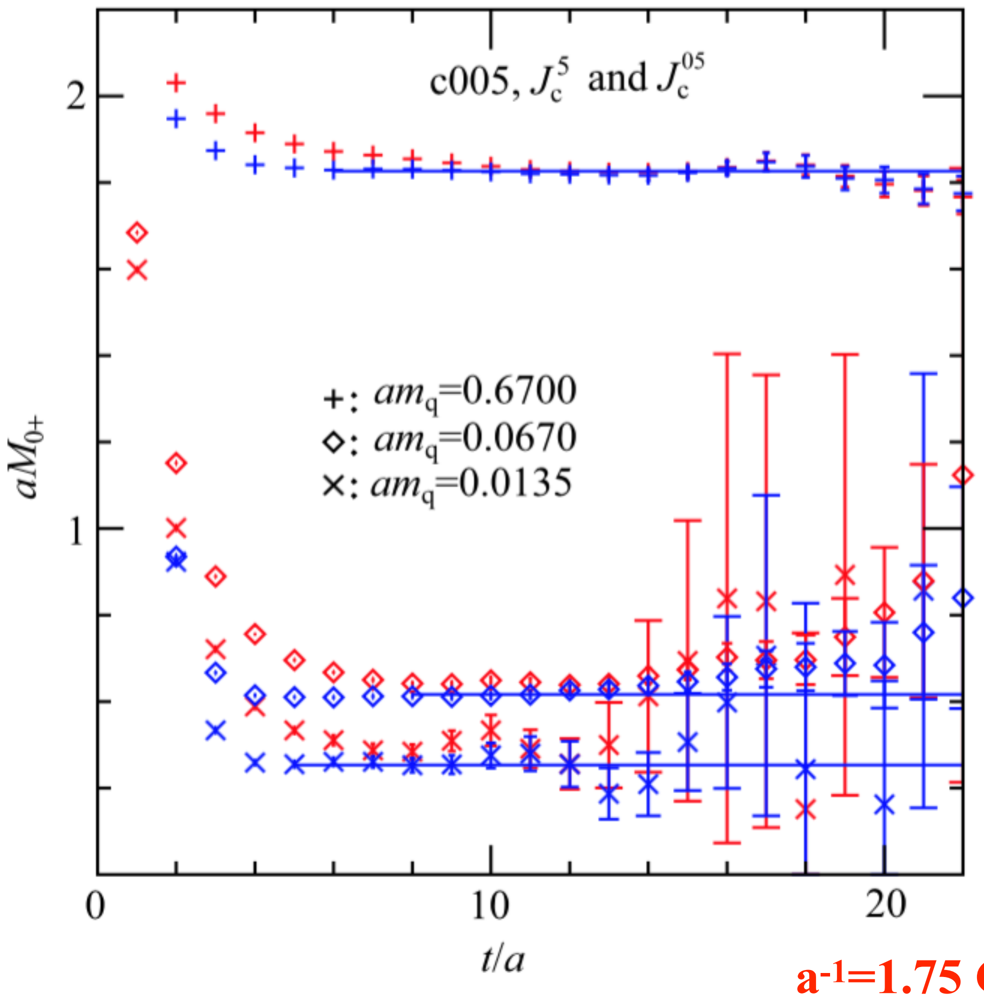
Diquark Effective Theory

- D.K.Hong, Y.J. Sohn, I. Zahed, PL B596 (2004) 191.
  D.K. Hong, C. Song, IJMP A27 (2012) 1250051.
  Non-linear chiral Diquark effective theory (for pentaquark/ tetraquarks)
- T. Hatsuda, M. Tachibana, N. Yamamoto, G. Baym, PRL 97, 122001 (2006), PR D76, 074001 (2007).
  Chiral/Diquark effective theory and the axial anomaly in dense QCD
- Y. Kawakami, M. Harada, PR D97 (2018) 114024, PR D99 (2019) 094016.
  Chiral effective theory of Single Heavy Baryons (HQ symmetry)

Chinese Physics C Vol. 40, No. 7 (2016) 073106

Diquark mass differences from unquenched lattice QCD

Yujiang Bi(毕玉江)[1;1)] Hao Cai(蔡浩)[1;2)] Ying Chen(陈莹)[2] Ming Gong(宫明)[2]
Zhaofeng Liu(刘朝峰)[2;3)] Hao-Xue Qiao(乔豪学)[1] Yi-Bo Yang(杨一玻)[3]



M(1$^+$)-M(0$^+$) ~ 290 MeV

M(0$^-$)- M(0$^+$) ~ 540 MeV

M(1$^-$)- M(1$^+$) ~ 510 MeV

M(0$^+$) ~ 720 MeV

M(0$^-$) ~ 1260 MeV

M(1$^+$) ~1010 MeV

M(1$^-$) ~ 1520 MeV

a^{-1}=1.75 GeV

Chiral Diquarks

- **Chiral symmetry SU(3)$_R$ x SU(3)$_L$**

$q^a_{\alpha i}$ $\quad a$ (color), α (Dirac), i (flavor)

$$q^a_{iR} = P_R\, q^a_i, \quad q^a_{iL} = P_L\, q^a_i \qquad P_{R,L} \equiv \frac{1 \pm \gamma_5}{2}$$

$$q_R \to U_R\, q_R = (U_R)_{ij} q_{jR}, \quad U_R \in SU(3)_R$$

$$q_L \to U_L\, q_L = (U_L)_{ij} q_{jL}, \quad U_L \in SU(3)_L$$

- **Scalar chiral diquarks (color 3$^{\text{bar}}$)**

$d^a_{iR} \equiv \epsilon_{ijk}(q^T_{jR}\, C\, q_{kR})^{\bar{3}}$ Right scalar diquark, chiral $(\bar{3},1)$, color $\bar{3}$

$d^a_{iL} \equiv \epsilon_{ijk}(q^T_{jL}\, C\, q_{kL})^{\bar{3}}$ Left scalar diquark, chiral $(1, \bar{3})$, color $\bar{3}$

- **Parity eigenstates: 0$^+$, 0$^-$ diquarks**

$$S^a_i = d^a_{iR} - d^a_{iL} = \epsilon_{ijk}(q^T_j\, C\gamma_5\, q_k)^{\bar{3}}$$

$$P^a_i = d^a_{iR} + d^a_{iL} = \epsilon_{ijk}(q^T_j\, C\, q_k)^{\bar{3}}$$

$$(\bar{3}, 1) + (1, \bar{3})$$

Chiral Diquark Effective Theory

- **Chiral meson field: Σ (scalar nonet + pseudo-scalar nonet)**

$$\Sigma_{ij} \equiv \sigma_{ij} + i\pi_{ij} \to U_{L,ik}\Sigma_{km}U^{\dagger}_{R,mj} \quad (\bar{3},3)$$

$$\Sigma_{ij} \equiv (\lambda_p)_{ij}(\sigma_p + i\pi_p)$$

Spontaneous symmetry breaking $\langle \Sigma_{ij} \rangle = \langle \sigma_{ij} \rangle = f\delta_{ij},$

- **The effective Lagrangian à la linear sigma model**

$$\mathcal{L} = \mathcal{D}_\mu d_{R,i} \left(\mathcal{D}^\mu d_{R,i}\right)^\dagger + \mathcal{D}_\mu d_{L,i} \left(\mathcal{D}^\mu d_{L,i}\right)^\dagger \qquad \mathcal{D}_\mu \equiv \partial_\mu + igT^\alpha G^{\alpha\,\mathrm{ext}}_\mu$$

$$\boxed{-m_0^2(d_{R,i}d^\dagger_{R,i} + d_{L,i}d^\dagger_{L,i}) \quad \boxed{-\frac{m_1^2}{f}(d_{R,i}\Sigma^\dagger_{ij}d^\dagger_{L,j} + d_{L,i}\Sigma_{ij}d^\dagger_{R,j})} \quad -\frac{m_2^2}{2f^2}\epsilon_{ijk}\epsilon_{\ell mn}(d_{R,k}\Sigma_{\ell i}\Sigma_{mj}d^\dagger_{L,n} + d_{L,k}\Sigma^\dagger_{\ell i}\Sigma^\dagger_{mj}d^\dagger_{R,n})}$$

$$+\frac{1}{4}\mathrm{Tr}\left[\partial^\mu\Sigma^\dagger\partial_\mu\Sigma\right] + V(\Sigma).$$

mass terms

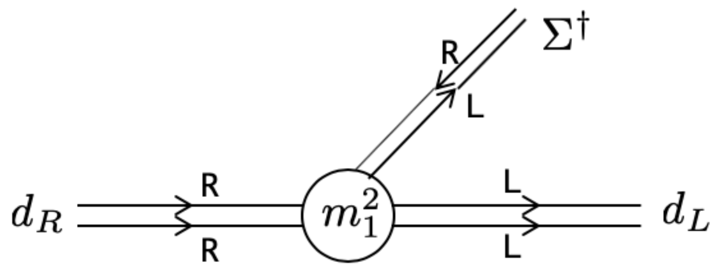
←U(1)$_A$ anomaly

$U_A(1)$ anomaly

- **$U_A(1)$ anomaly in the diquark effective theory**

$$-\frac{m_1^2}{f}(\underline{d_{R,i}\Sigma^\dagger_{ij}d^\dagger_{L,j}} + d_{L,i}\Sigma_{ij}d^\dagger_{R,j})$$

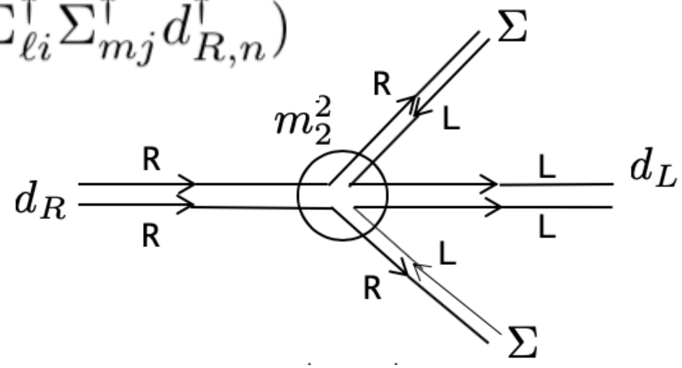
3 left quarks and 3 right antiquarks
flavor antisymmetric
induces anomalous singlet current



$$\partial_\mu J_A^{\mu 0} = \frac{3m_1^2}{2}(S\lambda_0 P^\dagger - P\lambda_0 S^\dagger)$$

- **non-anomalous term**

$$-\frac{m_2^2}{2f^2}\epsilon_{ijk}\epsilon_{\ell mn}(d_{R,k}\Sigma_{\ell i}\Sigma_{mj}d^\dagger_{L,n} + d_{L,k}\Sigma^\dagger_{\ell i}\Sigma^\dagger_{mj}d^\dagger_{R,n})$$



Chiral Diquark Effective Theory

- **The mass eigenstates are given by**

$$
\begin{array}{ccc}
 & d^a_{iR} & d^a_{iL} \\
M^2 = & \left(\begin{array}{c} m_0^2 \\ m_1^2 + m_2^2 \end{array} \right. & \left. \begin{array}{c} m_1^2 + m_2^2 \\ m_0^2 \end{array} \right)
\end{array}
$$

Scalar diquark

$$S_i^a = \frac{1}{\sqrt{2}}(d^a_{R,i} - d^a_{L,i})$$

$$\longrightarrow M(0^+) = \sqrt{m_0^2 - m_1^2 - m_2^2},$$

Pseudo-scalar diquark

$$P_i^a = \frac{1}{\sqrt{2}}(d^a_{R,i} + d^a_{L,i})$$

$$\longrightarrow M(0^-) = \sqrt{m_0^2 + m_1^2 + m_2^2},$$

SU(3) symmetry breaking

- **Explicit chiral symmetry breaking and SU(3) breaking**

$$\mathcal{M}_{\rm eff} = \mathcal{M} + g_s\langle\Sigma\rangle \simeq (g_s f_\pi)\ \mathrm{diag}\{1, 1, A\}, \qquad m_u \sim m_d \sim 0$$

$$\Sigma \longrightarrow \tilde{\Sigma} \equiv \Sigma + \mathcal{M}/g_s \qquad A \equiv \frac{f_s}{f_\pi}\left(1 + \frac{m_s}{g_s f_s}\right) \sim \frac{5}{3}$$

- **Mass of the diquarks**

***i*=3 (ud)**

$$M_3(0^+) = \sqrt{m_0^2 - Am_1^2 - m_2^2}, \qquad M_3(0^-) = \sqrt{m_0^2 + Am_1^2 + m_2^2}.$$

***i*=1,2 (ds), (us)**

$$M_1(0^+) = M_2(0^+) = \sqrt{m_0^2 - m_1^2 - Am_2^2}, \qquad M_1(0^-) = M_2(0^-) = \sqrt{m_0^2 + m_1^2 + Am_2^2},$$

$$\left[M_{1,2}(0^+)\right]^2 - \left[M_3(0^+)\right]^2 = \left[M_3(0^-)\right]^2 - \left[M_{1,2}(0^-)\right] = (A-1)(m_1^2 - m_2^2). > 0$$

$$\boxed{M_1(0^-) < M_3(0^-)}$$

(ds), (us) **(ud)**

Inverse mass hierarchy

Effective theory for vector diquarks

- **Vector (3,3) Diquarks**

$$d^{\mu a}_{ij} \equiv \epsilon_{abc}(q^{bT}_{iL}\, C\gamma^\mu\, q^c_{jR}) = \epsilon_{abc}(q^{bT}_{jR}\, C\gamma^\mu\, q^c_{iL})$$ chiral (3,3) vector diquark

$$d^{\mu a}_{V[ij]} = d^{\mu a}_{ij} - d^{\mu a}_{ji} = \epsilon_{abc}(q^{bT}_i\, C\gamma^\mu\gamma^5\, q^c_j)$$ Vector 1^- diquark, flavor $\bar{3}$

$$d^{\mu a}_{A\{ij\}} = d^{\mu a}_{ij} + d^{\mu a}_{ji} = \epsilon_{abc}(q^{bT}_i\, C\gamma^\mu\, q^c_j)$$ Axial-vector 1^+ diquark, flavor 6

$$\boxed{\begin{aligned}\mathcal{L} = -\frac{1}{2}\mathrm{Tr}[F^{\mu\nu}F_{\mu\nu}] &+ m^2_{V0}\mathrm{Tr}[d^\mu d^\dagger_\mu] \\ &+ \frac{m^2_{V1}}{f^2_\pi}\mathrm{Tr}[\Sigma^\dagger d^\mu \Sigma^T d^{\dagger T}_\mu] + \frac{2m^2_{V2}}{f^2_\pi}\mathrm{Tr}[\Sigma^\dagger \Sigma d^{\mu T} d^{\dagger T}_\mu]\end{aligned}}$$

$$F^{\mu\nu} = D^\mu d^\nu - D^\nu d^\mu$$

Effective theory for vector diquarks

- **Masses of the Axial-vector and Vector diquarks**

$$\mathcal{L}_{\rm mass} = m_{V0}^2 \mathrm{Tr}[d^\mu d_\mu^\dagger] + \frac{m_{V1}^2}{f_\pi^2}\mathrm{Tr}[\Sigma^\dagger d^\mu \Sigma^T d_\mu^{\dagger T}] + \frac{2m_{V2}^2}{f_\pi^2}\mathrm{Tr}[\Sigma^\dagger \Sigma d^{\mu T} d_\mu^{\dagger T}]$$

$$\sim m_{V0}^2 \mathrm{Tr}[d^\mu d_\mu^\dagger] + m_{V1}^2 \mathrm{Tr}[X d^\mu X d_\mu^{\dagger T}] + 2m_{V2}^2 \mathrm{Tr}[X^2 d^{\mu T} d_\mu^\dagger]$$

$$X = \mathrm{diag}(1,1,A) \qquad A \equiv \frac{f_s}{f_\pi}\left(1 + \frac{m_s}{g_s f_s}\right) \sim \frac{5}{3}$$

A(1+)

$$M_{\{qq\}}(1^+) = \sqrt{m_{V0}^2 + m_{V1}^2 + 2m_{V2}^2}$$

$$M_{\{qs\}}(1^+) = \sqrt{m_{V0}^2 + A m_{V1}^2 + (1+A^2) m_{V2}^2}$$

$$M_{\{ss\}}(1^+) = \sqrt{m_{V0}^2 + A^2 m_{V1}^2 + 2A^2 m_{V2}^2}$$

V(1-)

$$M_{[qq]}(1^-) = \sqrt{m_{V0}^2 - m_{V1}^2 + 2m_{V2}^2}$$

$$M_{[qs]}(1^-) = \sqrt{m_{V0}^2 - A m_{V1}^2 + (1+A^2) m_{V2}^2}$$

Diquark-meson couplings

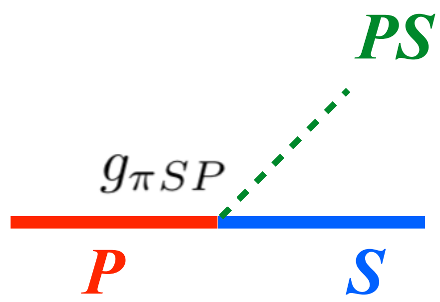
- **diquark - pseudo-scalar nonet π_p (p=0…8) couplings**

$$\mathcal{L}_{\pi SP} = \frac{i(m_1^2+m_2^2)}{f}\pi_p(S\lambda_p P^\dagger - P\lambda_p S^\dagger) \qquad \textit{octet + singlet}$$

$$-\frac{3im_2^2}{f}\pi_0(S\lambda_0 P^\dagger - P\lambda_0 S^\dagger) \qquad \textit{singlet}$$

- **The "Goldberger-Treiman" relation**

$$g_{\pi SP} \equiv \frac{m_1^2+m_2^2}{f} = \frac{M^2(0^-)-M^2(0^+)}{2f} \qquad \textit{for octet mesons}$$



$$\Lambda_c^* \to \Lambda_c + \eta_8$$

$$\Xi_c^* \to \Lambda_c + \bar{K}$$

Diquark-meson couplings

- **Scalar - Vector couplings**
  P-wave coupling

  A(1⁺, 6) + π_8 (0⁻) → S(0⁺, 3bar)
  V(1⁻, 3bar) + π_8 (0⁻) → P(0⁻, 3bar)
  V(1⁻, 3bar) + η_1 (0⁻) → P(0⁻, 3bar)

$$\mathcal{L}_{\rm int} = -\boxed{g_N}\epsilon_{ijk}\left[d^{\mu}_{(n,i)}(d^{\dagger}_{R})_j\partial_\mu\Sigma^{\dagger}_{kn} + d^{\mu}_{(i,n)}\partial_\mu\Sigma_{jn}(d^{\dagger}_{L})_k\right]$$

Normal

$$-\boxed{g_A}\epsilon_{ijk}\left[d^{\mu}_{(i,j)}\partial_\mu\Sigma_{kn}(d^{\dagger}_{R})_n + d^{\mu}_{(j,i)}\partial_\mu\Sigma^{\dagger}_{kn}(d^{\dagger}_{L})_n - d^{\mu}_{(i,n)}\partial_\mu\Sigma_{jn}(d^{\dagger}_{R})_k - d^{\mu}_{(n,i)}\partial_\mu\Sigma^{\dagger}_{jn}(d^{\dagger}_{L})_k\right]$$

Anomalous

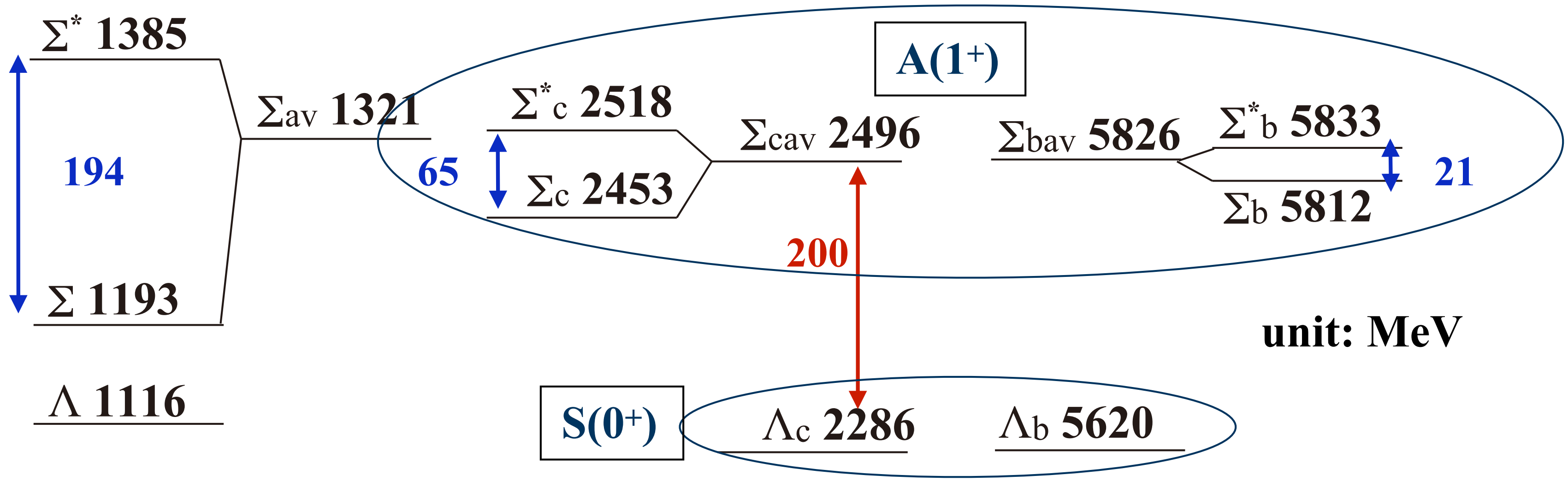
$$= (g_N + g_A)\epsilon_{ijk}A^{\mu}_{\{in\}}i\partial_\mu\pi^{(8)}_{jn}S^{\dagger}_{k}$$

$$-(g_N + g_A)\epsilon_{ijk}\sqrt{\frac{2}{3}}V^{\mu}_{[ij]}i\partial_\mu\pi^{(1)}P^{\dagger}_{k} + \frac{g_N - 3g_A}{2}\epsilon_{ijk}V^{\mu}_{[ij]}i\partial_\mu\pi^{(8)}_{kn}P^{\dagger}_{n}$$

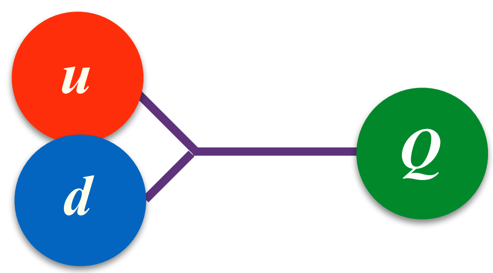
Diquarks in heavy baryons

Diquarks in Heavy Baryons

- **From Heavy baryon spectroscopy**
  Λ_Q/Σ_Q with $S(0^+)/A(1^+)$ diquarks

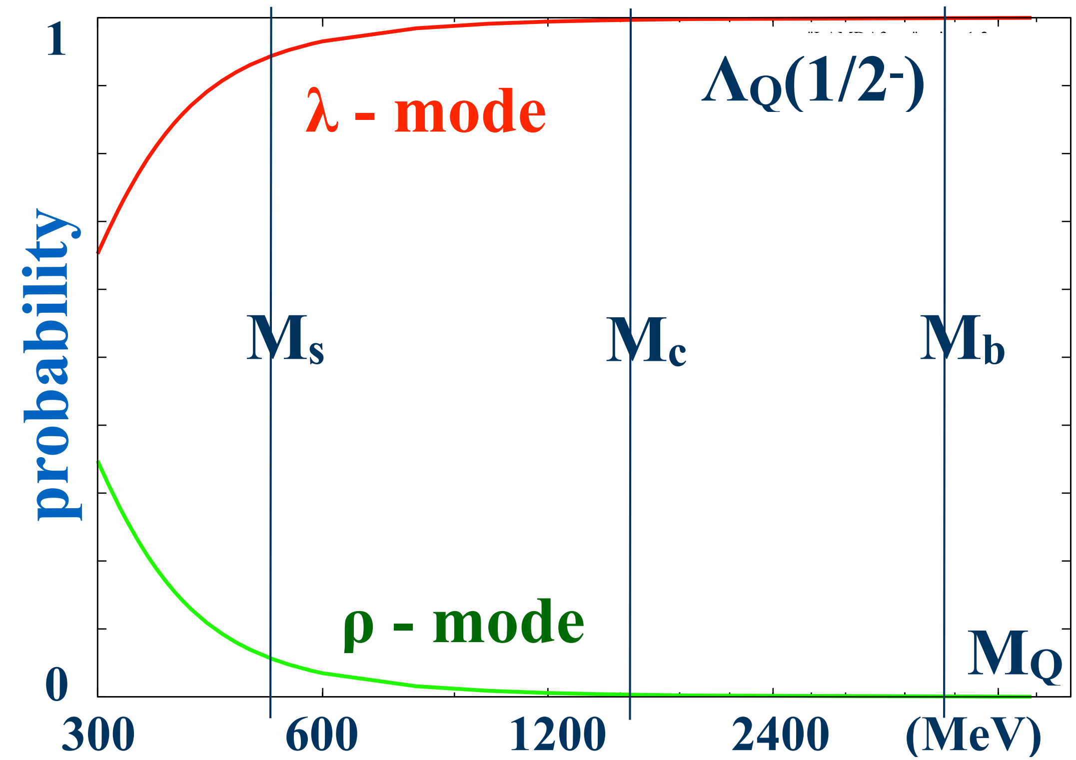


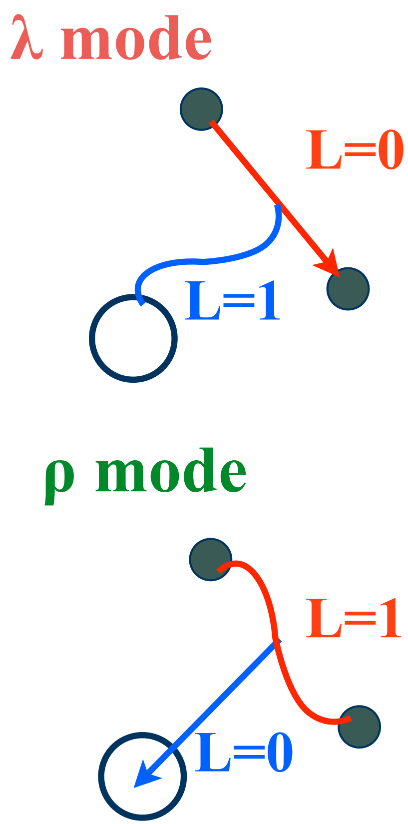
Diquarks
$S(0^+)$ *ud (S=0, I=0)*
$A(1^+)$ *(uu,ud,dd) (S=1, I=1)*



Diquarks in Heavy Baryons (P-wave)

- Two P-wave separation modes are well separated.





T. Yoshida, E. Hiyama, A. Hosaka, M. Oka and K. Sadato, Phys. Rev. D 92, 114029 (2015)

Fitting to the heavy baryon masses

- **Heavy baryon system as a bound state of Q + d**
- **Inputs for an estimate of the heavy baryon masses**

Observed masses of the ground state baryons

* $M(\Lambda_c, 1/2^+) = 2286.46$ MeV,
* $M(\Xi_c, 1/2^+) = \frac{1}{2}(M(\Xi_c^+) + M(\Xi_c^0)) = 2469.42$ MeV.

0^+ diquark (ud) mass from lattice QCD

* $M_3(0^+) = 725$ MeV,

0^- diquark (ud) mass from lattice

* $M_3(0^-) = 1265$ MeV,

<u>or</u> a model prediction of the ρ mode baryon mass

* $M(\Lambda_c, 1/2^-) = 2890$ MeV.

T. Yoshida, et al., Phys. Rev. D 92, 114029 (2015)

Fitting to the heavy baryon masses

- **A potential model estimate**

  Y. Kim, E. Hiyama, M. Oka, K. Suzuki, arXiv.2003.03525

  A linear + Coulomb potential between Q and d

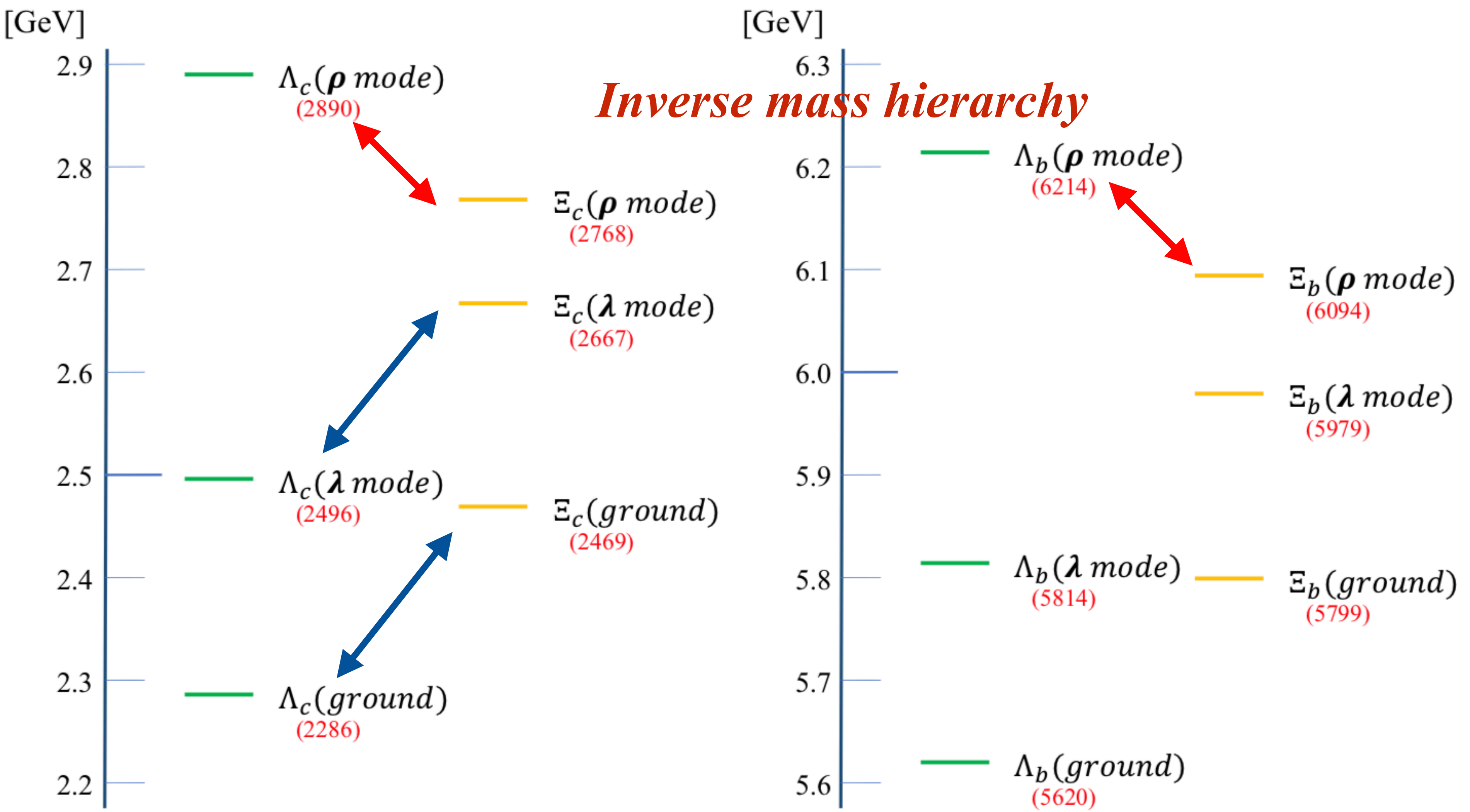
$$V(r) = -\frac{\alpha}{r} + \lambda r + C,$$

	α	$\lambda(\mathrm{GeV}^2)$	$C_c(\mathrm{GeV})$	$C_b(\mathrm{GeV})$	$M_c(\mathrm{GeV})$	$M_b(\mathrm{GeV})$
Yoshida	$(2/3)\times 90/\mu$	0.165	-0.58418362	-0.58829590	1.750	5.112
Silvestre	0.5069	0.1653	-0.70694965	-0.69555101	1.836	5.227
Barnes	$(4/3)\times 0.5461$	0.1425	-0.19063965	-	1.4794	-

T. Yoshida, E. Hiyama, A. Hosaka, M. Oka and K. Sadato, Phys. Rev. D 92, 114029 (2015)

Fitting to the heavy baryon masses

Y. Kim, E. Hiyama, M. Oka, K. Suzuki, arXiv.2003.03525



Conclusion

- We construct a chiral effective theory of Diquarks.
  Scalar and Pseudo-Scalar Diquarks are paired in $(\bar{3},1)+(1,\bar{3})$.
  Vector and Axial-Vector Diquarks are in (3,3) representation.
- Masses of the positive- and negative-parity diquarks are parametrized. There contributes the $U_A(1)$ anomaly couplings. With the help of heavy baryon masses and lattice QCD, we may determine the parameters of the chiral effective Lagrangian.
- We have found an inverse mass hierarchy of the 0^- diquark. As a result, we predict a low mass ρ-mode $\Xi_c(1/2^-)$ state.
- The goals are to apply the effective theory to tetra-quarks, heavy penta-quarks, diquarks in finite T/ρ matter and so on.