Conformality, bulk viscosity, and contact correlation

Yusuke Nishida (Tokyo Tech)

第４回クラスター階層領域研究会

2020年5月28日 @ zoom

Plan of this talk

1. Introduction
   - Scale and conformal symmetries in nonrelativistic systems
2. Physical consequences
   - Breathing mode & bulk viscosity
3. Bulk viscosity
   - Contact correlation from Kubo formula
   - Quantum virial expansion

Y. Nishida & D. T. Son, PRD (2007); arXiv:1004.3597

K. Fujii & Y. Nishida, Phys. Rev. A 98, 063634 (2018)

Y. Nishida, Ann. Phys. 410 (2019) 167949

Nonrelativistic CFT

Maximal spacetime symmetries of

U. Niederer, HPA (1972)
C. R. Hagen, PRD (1972)

$$S_{\text{free}} = \int dt\, d^d\vec{x}\ \psi^\dagger \left(i\partial_t + \frac{\vec{\nabla}^2}{2m} \right) \psi$$

- Translations in spacetime (4)
- Spatial rotations (3)
- Galilean boosts (3)
- Phase rotation (1)
- Scale transformation (1)

$$\vec{x} \to e^{-s}\vec{x}, \quad t \to e^{-2s}t, \quad \psi \to e^{(d/2)s}\psi$$

- Conformal transformation (1)

$$\vec{x} \to \frac{\vec{x}}{1-ct}, \quad t \to \frac{t}{1-ct},$$

$$\psi \to (1-ct)^{d/2} \exp\left(i\frac{c}{1-ct}\frac{m}{2}\vec{x}^2 \right) \psi$$

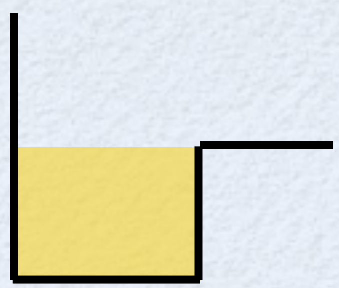
Nonrelativistic CFT

Generators (D, C, H) obey SO(2,1) Lie algebra

$$[D,H]=2iH,\quad [C,H]=iD,\quad [D,C]=-2iC$$

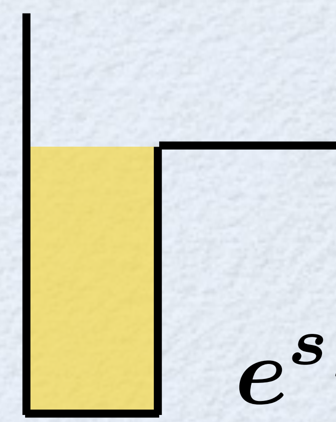
scale invariance $\qquad \dot{n}=-\vec{\nabla}\cdot\vec{j} \qquad$ always true

$$H=H_0+V(r) \quad \Rightarrow \quad H'=H_0+e^{-2s}V(e^{-s}r)$$



r_0 & a

$$e^{-isD}He^{isD}=e^{2s}H'$$



$e^s r_0$ & $e^s a$

H=H' for zero-range ($r_0=0$)
& infinite scattering length ($a=\infty$) interaction
(relevant to cold atom experiments)

Breathing mode

Arbitrary time-evolving state $|\Psi_t\rangle = e^{-iH_\omega t}|\Psi_0\rangle$

$$\langle C\rangle = \langle\Psi_0|e^{iH_\omega t}\,\frac{2H_\omega - L_+ - L_-}{4\omega^2}\,e^{-iH_\omega t}|\Psi_0\rangle$$

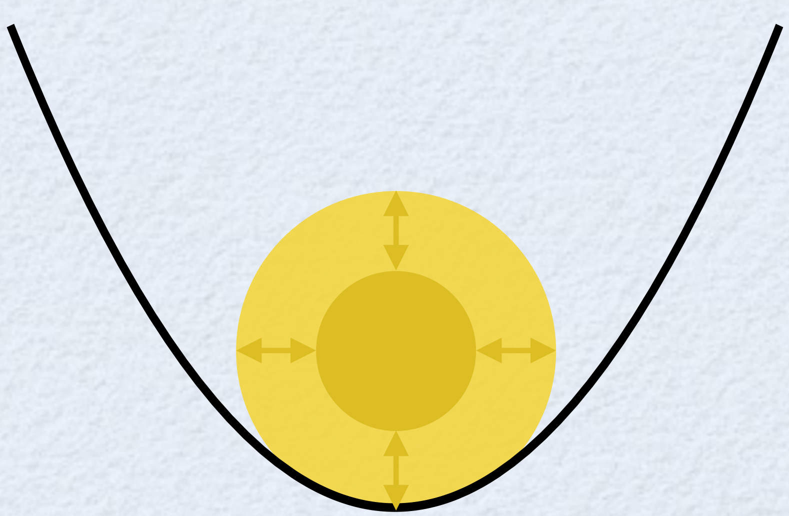
$$= \langle\Psi_0|\frac{2H_\omega - e^{i2\omega t}L_+ - e^{-i2\omega t}L_-}{4\omega^2}|\Psi_0\rangle$$

$$= \frac{\langle\Psi_0|H_\omega|\Psi_0\rangle - \cos(2\omega t+\varphi)|\langle\Psi_0|L_+|\Psi_0\rangle|}{2\omega^2}$$

$$\left(C \equiv \frac{m}{2}\int d^d\vec{x}\;\vec{x}^2\psi^\dagger\psi\right)$$

Mean square radius

$$\langle\vec{x}^2\rangle = A + B\cos\left(2\omega t+\varphi\right)$$



Undamped "breathing mode"

with frequency right at 2ω

L. P. Pitaevskii & A. Rosch, PRA (1997)

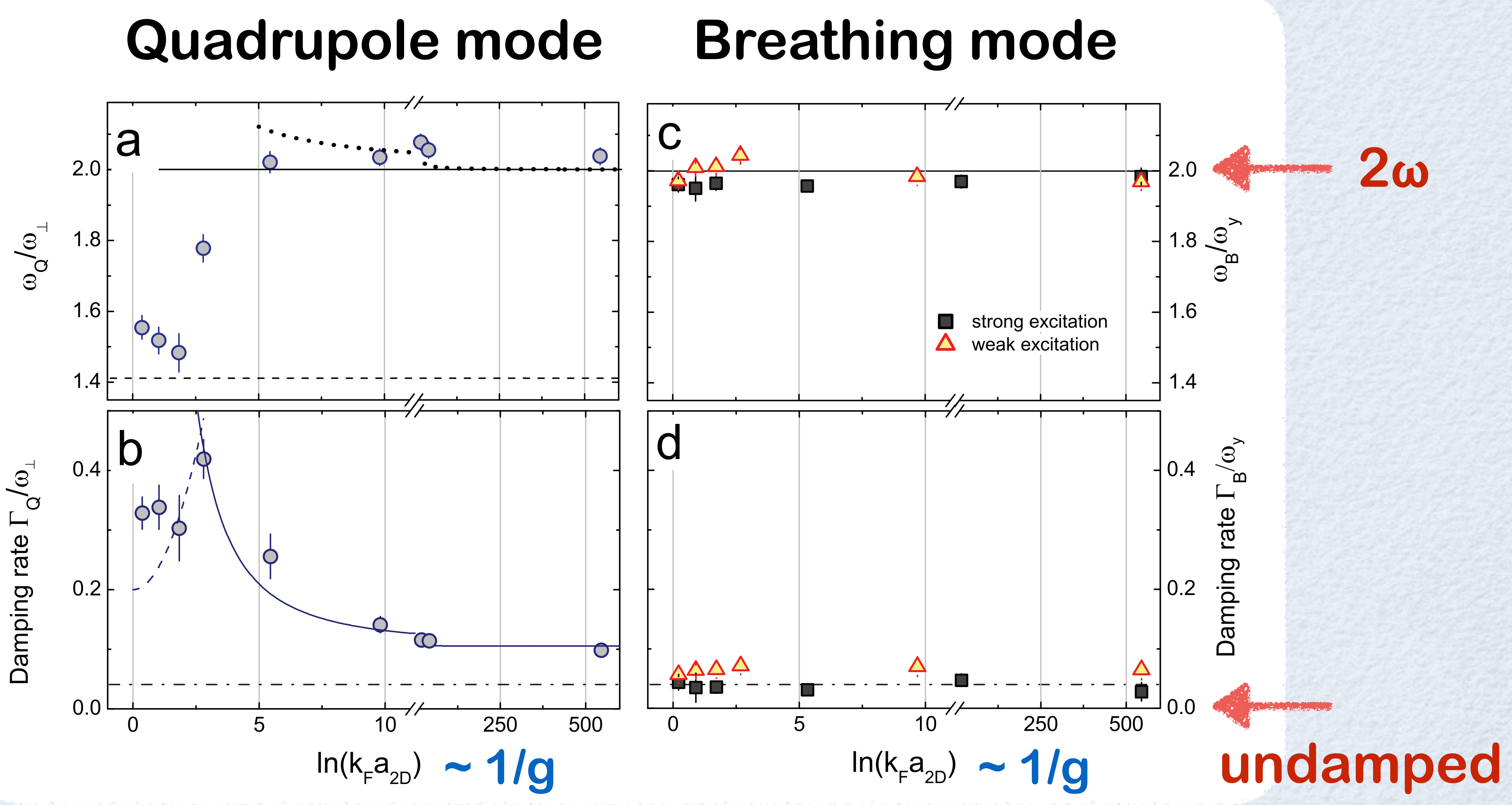
Breathing mode

H is scale invariant for $V = -g\,\delta^2(\vec{r})$ in 2D ??

Tunable via Feshbach resonance with ultracold atoms

T ~ 0.4 T_F

M. Köhl's group, PRL (2012)

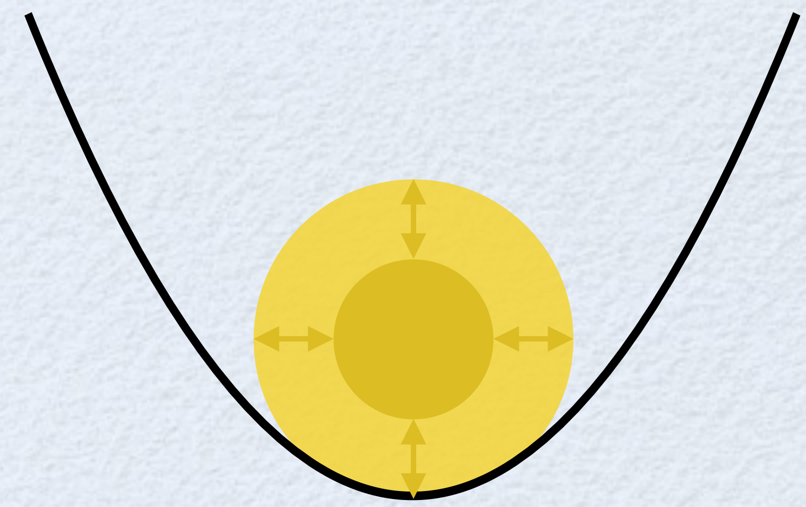


Bulk viscosity @ a=infinite

D. T. Son, PRL (2007)

Undamped "breathing mode"

for any scale invariant systems
confined by harmonic potential



⇒ Vanishing bulk viscosity !?

When coupled with external gauge field & metric

$$S = \int dt\, d^d\vec{x} \sqrt{g} \left(i\psi^\dagger \overleftrightarrow{D}_t \psi - \frac{g^{ij}}{2m} \vec{D}_i \psi^\dagger \vec{D}_j \psi + \mathcal{L}_{\text{int}} \right)$$

is invariant under

D. T. Son & M. Wingate, Ann Phys (2006)

- Gauge transformation $\psi \to e^{i\chi(\vec{x},t)}\psi$
- General coordinate transformation $\vec{x} \to \vec{x}'(\vec{x},t)$
- Conformal transformation $t \to t'(t)$

Bulk viscosity @ a=infinite D. T. Son, PRL (2007)

Microscopic symmetries must be inherited by hydrodynamics

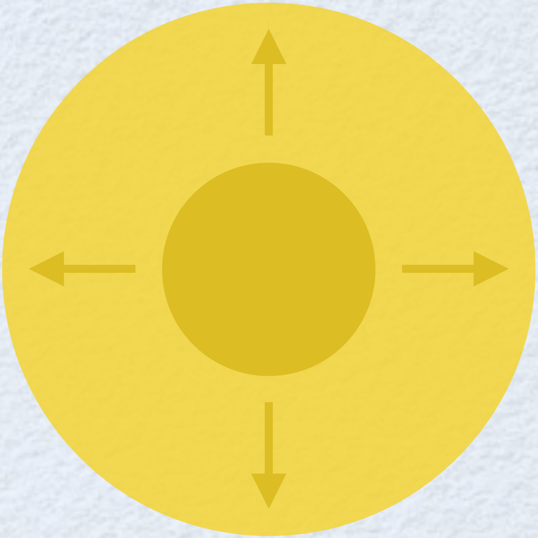
Viscous stress tensor coupled with metric

$$\pi_{ij} = \zeta\,\delta_{ij}\,\partial_k v^k + \text{shear}$$

fluid expansion

$$\pi_{ij} = \cancel{\zeta}\, g_{ij}\left(\nabla_k v^k + \partial_t \ln\sqrt{g}\right) + \text{shear}$$

volume expansion

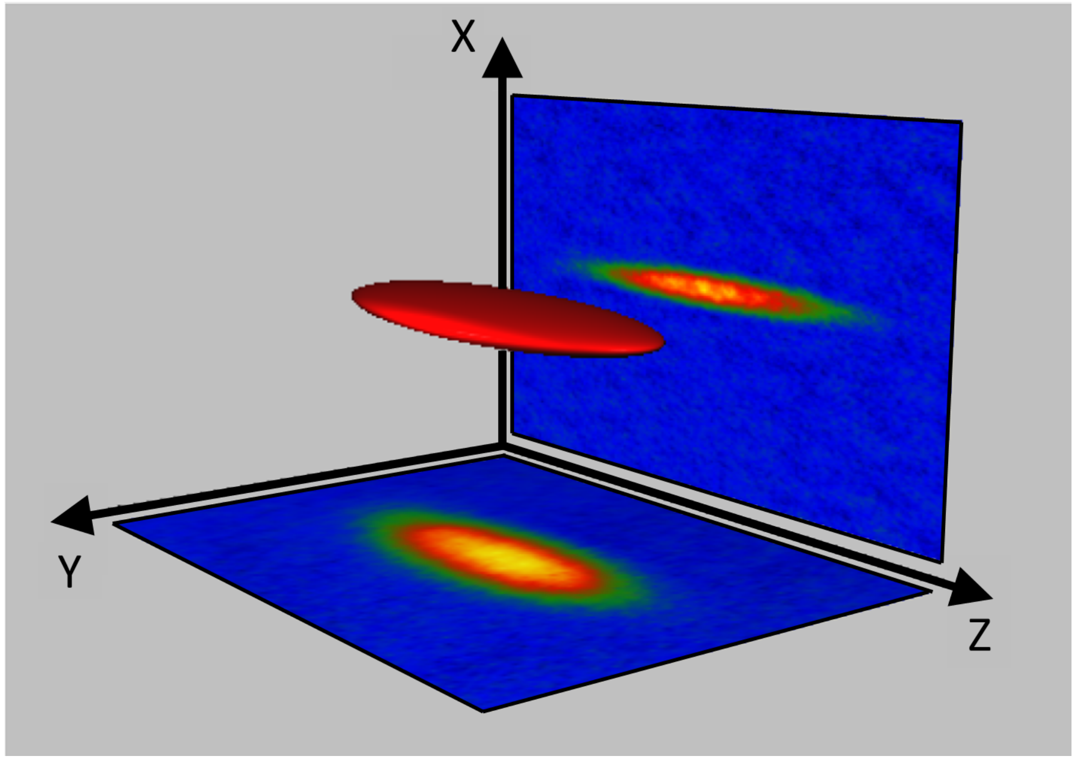


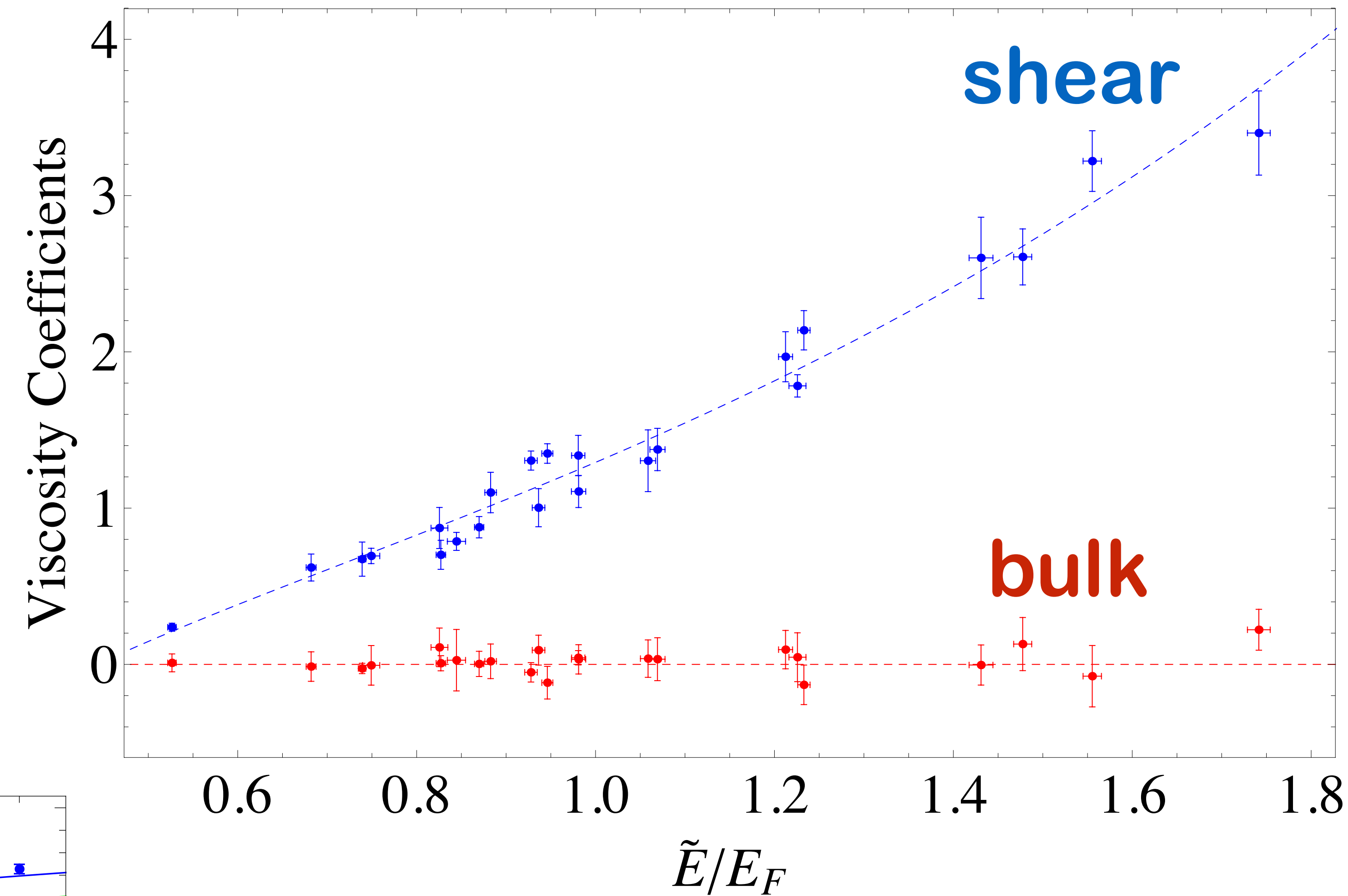
is invariant under general coordinate transformation

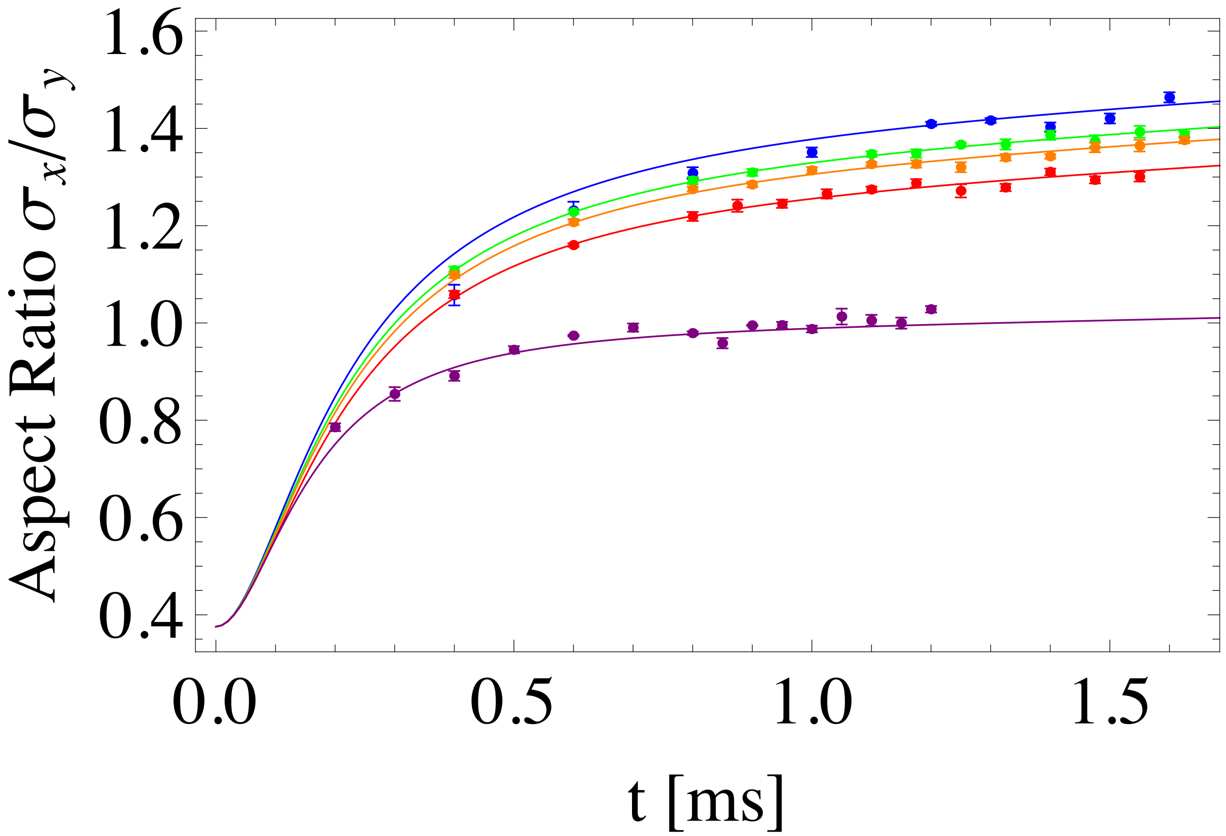
but is NOT under conformal transformation

Vanishing bulk viscosity @ a=infinite

Bulk viscosity @ a=infinite







$$\frac{1}{N}\int d^3\vec{x}\,(\eta,\zeta)$$

$$T/T_F = 0.2 \sim 0.6$$

J. E. Thomas's group, PRL (2014)

Bulk viscosity @ a=finite

Scattering length explicitly breaks scale invariance

because $S(a) \to S(e^s a)$

$$\left(\vec{x} \to e^{-s}\vec{x}, \quad t \to e^{-2s}t, \quad \psi \to e^{(d/2)s}\psi\right)$$

Scale invariance is "formally" recovered if

$$a(\vec{x}, t) \to a'(\vec{x}', t') = e^{-s}a(x, t)$$

spurion field (spacetime-dependent)

Microscopic symmetries must be

inherited by hydrodynamics

$$\pi_{ij}^{\text{bulk}} = \zeta\, g_{ij}\left[\left(\nabla_k v^k + \partial_t \ln\sqrt{g}\right)\right]$$

is NOT invariant under conformal transformation

Bulk viscosity @ a=finite

Scattering length explicitly breaks scale invariance

because $S(a) \to S(e^s a)$

$$\left(\vec{x} \to e^{-s}\vec{x}, \quad t \to e^{-2s}t, \quad \psi \to e^{(d/2)s}\psi\right)$$

Scale invariance is "formally" recovered if

$$a(\vec{x},t) \to a'(\vec{x}',t') = e^{-s}a(x,t)$$

spurion field (spacetime-dependent)

Microscopic symmetries must be

inherited by hydrodynamics

K. Fujii & Y. Nishida, PRA (2018)

$$\pi_{ij}^{\text{bulk}} = \zeta\, g_{ij}\left[\left(\nabla_k v^k + \partial_t \ln\sqrt{g}\right) - d\left(\partial_t \ln a + v^k \partial_k \ln a\right)\right]$$

is ~~NOT~~ invariant under conformal transformation

Bulk viscosity @ a=finite

$$\pi_{ij}^{\mathrm{bulk}} = \zeta\, g_{ij}\left[\left(\nabla_k v^k + \partial_t \ln\sqrt{g}\right) - d\left(\partial_t \ln a + v^k \partial_k \ln a\right)\right]$$

expansion of fluid ⟺ contraction of scattering length

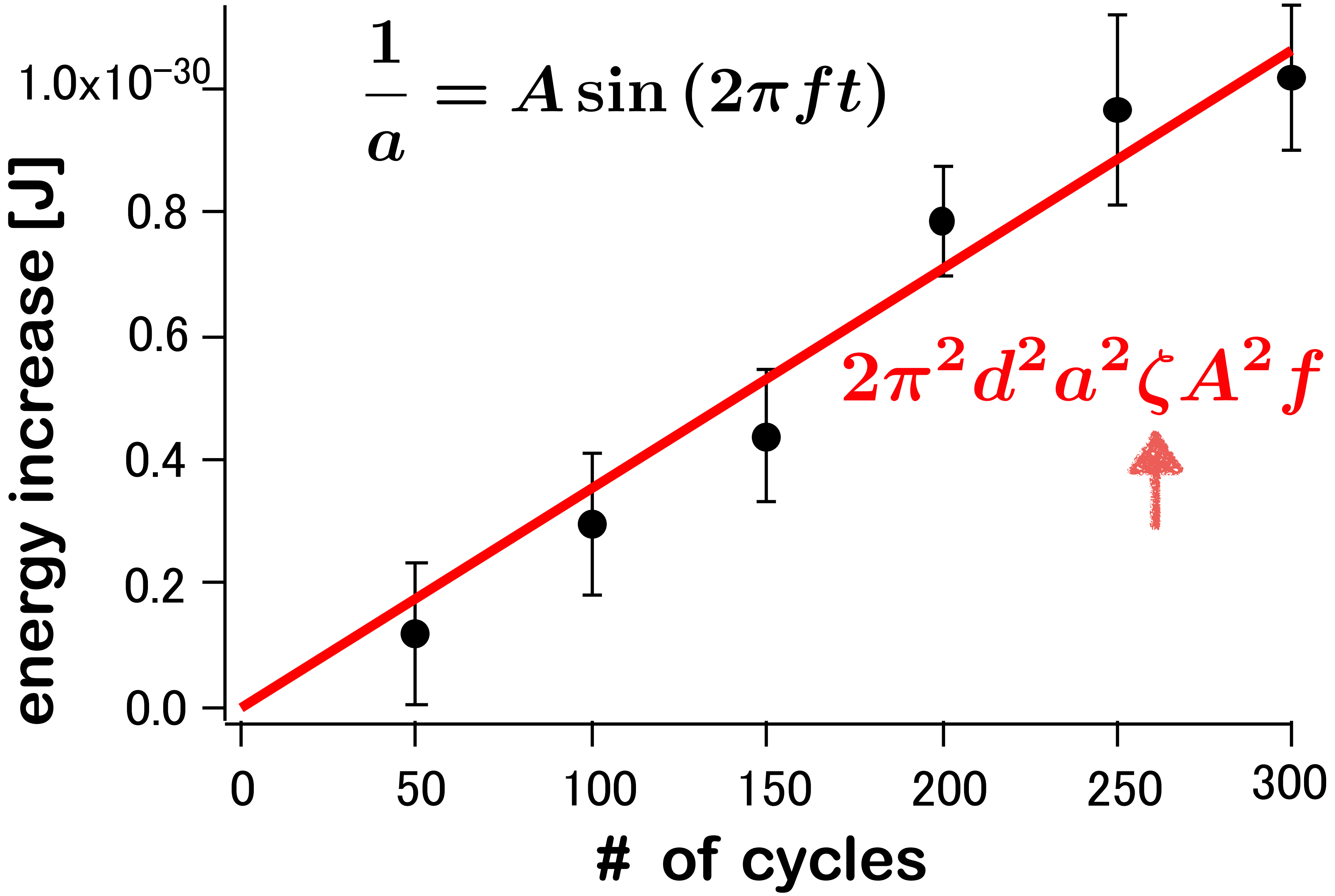
Entropy & energy production even in stationary systems

K. Fujii & Y. Nishida, PRA (2018)

$$T\dot{\mathcal{S}} = \frac{d^2\zeta}{a^2}\dot{a}^2 + O(\dot{a}^3)$$

$$\dot{\mathcal{E}} = \frac{\mathcal{C}_{\mathrm{eq}}}{m\Omega_{d-1}a^{d-1}}\dot{a} + \frac{d^2\zeta}{a^2}\dot{a}^2 + O(\dot{a}^3)$$

Bulk viscosity @ a=finite



Ongoing experiment
toward extraction of bulk viscosity

$$\dot{\mathcal{E}} = \frac{\mathcal{C}_{\mathrm{eq}}}{m\Omega_{d-1} a^{d-1}} \dot{a} + \frac{d^2 \zeta}{a^2} \dot{a}^2 + O(\dot{a}^3)$$

Tsumori, Horikoshi, Gonokami (U. Tokyo)

3. Contact correlation

K. Fujii & Y. Nishida, Phys. Rev. A 98, 063634 (2018)

Y. Nishida, Ann. Phys. 410 (2019) 167949

Microscopic derivation

Kubo formula for dynamic bulk viscosity

$$\zeta(\omega) = \mathrm{Im}\,\frac{i}{ZV}\int_0^\infty dt\,\frac{e^{i(\omega+i0^+)t}}{\omega}\,\frac{\langle[\hat{\Pi}_{ii}(t),\hat{\Pi}_{jj}(0)]\rangle}{d^2}$$

Trace of stress tensor

Contact $\hat{C} \equiv \frac{(mg)^2}{2}\hat{\psi}_\sigma^\dagger\hat{\psi}_\tau^\dagger\hat{\psi}_\tau\hat{\psi}_\sigma$

$$\hat{\Pi}_{ii} = 2\hat{H} + \frac{\hat{C}}{\Omega_{d-1}ma^{d-2}}$$

conformality breaking

$\Omega_{0,1,2} = 2, 2\pi, 4\pi$

$$\Rightarrow \quad \zeta(\omega) = \mathrm{Im}\,\frac{i}{ZV}\int_0^\infty dt\,\frac{e^{i(\omega+i0^+)t}}{\omega}\,\frac{\langle[\hat{C}(t),\hat{C}(0)]\rangle}{(d\,\Omega_{d-1}ma^{d-2})^2}$$

$$\equiv \frac{\mathrm{Im}\,\chi_{CC}(\omega)}{(d\,\Omega_{d-1}ma^{d-2})^2}$$

Contact-contact correlation function

Quantum virial expansion

$$\zeta(\omega) = \operatorname{Im} \frac{i}{ZV} \int_0^\infty dt \, \frac{e^{i(\omega + i0^+)t}}{\omega} \frac{\langle [\hat{C}(t), \hat{C}(0)] \rangle}{(d\,\Omega_{d-1} m a^{d-2})^2}$$

Systematic expansion in powers of fugacity $z \equiv e^{\beta\mu}$

$$\langle \cdots \rangle \equiv \operatorname{Tr}[\cdots e^{-\beta(\hat{H} - \mu\hat{N})}] = \sum_{N=0}^{\infty} \operatorname{tr}_N[\cdots e^{-\beta\hat{H}}] \times z^N$$

grand canonical trace

canonical trace
for particle number N

$z \sim n/(mT)^{d/2} \ll 1$ in the high-temperature limit

Quantum virial expansion

$$\zeta(\omega) = \mathrm{Im}\,\frac{i}{ZV}\int_0^\infty dt\,\frac{e^{i(\omega+i0^+)t}}{\omega}\,\frac{\langle[\hat{C}(t),\hat{C}(0)]\rangle}{(d\,\Omega_{d-1}ma^{d-2})^2}$$

Systematic expansion in powers of **fugacity** $z \equiv e^{\beta\mu}$

→ static bulk viscosity $\zeta \equiv \lim_{\omega\to 0}\zeta(\omega)$

non-analytic

$$\zeta_{2\mathrm{D}} \to \frac{2\pi z^2}{\lambda_T^2 \ln^4 a^2} \qquad \zeta_{3\mathrm{D}} \to \frac{\sqrt{2}\,z^2 \ln a^2}{9\pi^2 a^2 \lambda_T} \qquad (a\to\infty)$$

disagree with predictions from kinetic theory !?

$$\zeta_{2\mathrm{D}} \to \frac{z^2}{2\pi\lambda_T^2 \ln^4 a^2} \qquad \zeta_{3\mathrm{D}} \to \frac{\sqrt{2}\,z^2}{48\pi a^2 \lambda_T} \qquad (a\to\infty)$$

C. Chafin and T. Schäfer, PRA (2013); K. Dusling and T. Schäfer, PRL (2013)

Summary of this talk

- Conformal symmetry is useful to constrain how hydrodynamic equations are modified by spacetime-dependent scattering length
- Spacetime-dependent scattering length naturally couples with bulk viscosity (~ fluid expansion)

→ New experimental probe for bulk viscosity

- The same result can be derived microscopically from Kubo formula and linear response theory by expressing it as contact correlation function
- Quantum virial expansion for bulk viscosity disagrees with that from kinetic theory

→ Origin of discrepancy needs to be elucidated