Microscopic collective inertial masses in nuclear reaction

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Low-energy nuclear reaction

– Macroscopic model

$$\left\{-\frac{1}{2\mu_R}\frac{d^2}{dR^2}+\frac{L(L+1)}{2\mu_R R^2}+V(R)\right\}\psi_L(R)=E_L\psi_L(R)$$

–How good/bad is this?

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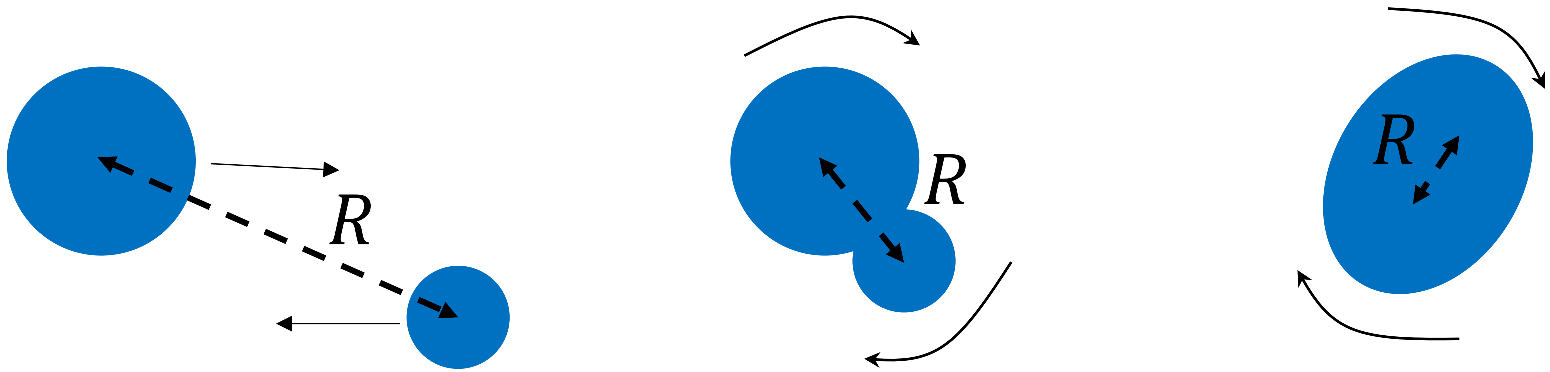
Nuclear reaction with shape evolution

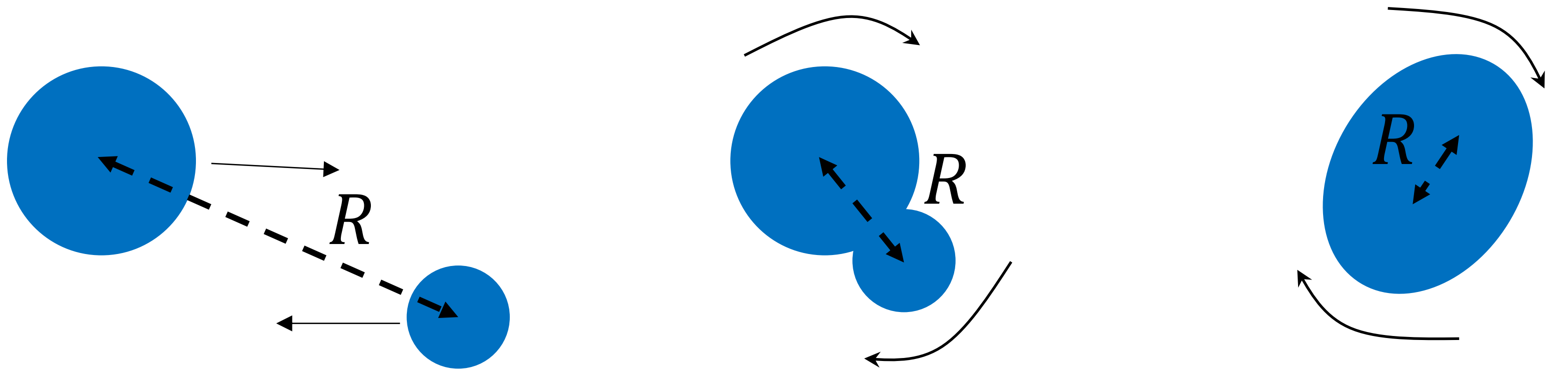
- Low-energy nuclear reaction
  - Relative motion between two nuclei
  - Shape change



Is the relative coordinate R meaningful?

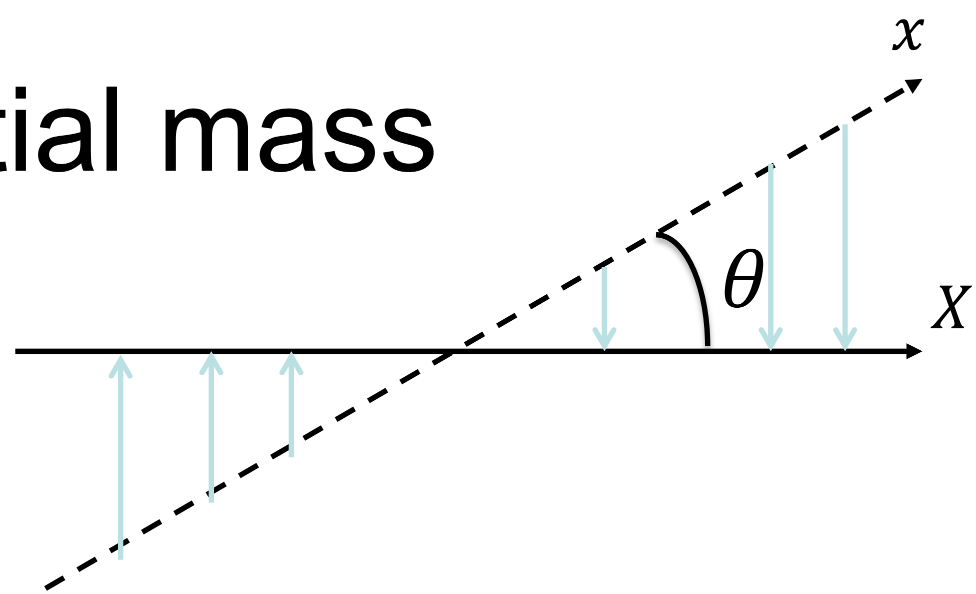
Nuclear reaction with shape evolution

- One-to-one correspondence
  - R can be a choice to define the scale of the coordinate.
  - R affects the inertial masses



Relative distance: $M(R) \sim \mu_R \quad (R \to \infty)$
Orientation: $I(R) \sim 2\mu_R R^2 \quad (R \to \infty)$

Inertial mass



- A particle moving along the x axis
  - $L = \frac{1}{2} m \dot{x}^2$
- Assuming the motion along the X axis
  - $L = \frac{1}{2} m \dot{X}^2$ (Wrong)
- Representing in the X axis ($x = f(X)$)
  - $L = \frac{1}{2} m_{eff} \dot{X}^2$ (Correct)
  - $m_{eff} = \frac{m}{(\cos\theta)^2}$

Effect of “effective mass”

- Velocity-dependent potential
- Nucleonic effective mass
  - $\frac{m^*}{m} \sim 0.7 - 0.8$
- Does this affect the inertial mass of nuclear reaction?
  - $\left(M(R), I(R)\right) \to (\mu_R, 2\mu_R R^2) \times \frac{m^*}{m}?,\ \text{at}\ R \to \infty$

Construction of macroscopic model

Model Hamiltonian

$$\left\{-\frac{d}{dR}\frac{1}{2M(R)}\frac{d}{dR}+\frac{L(L+1)}{2I(R)}+V(R)\right\}\psi_L(R)=E_L\psi_L(R)$$

Microscopically calculating $V(R), M(R), I(R)$

Can we reproduce the following asymptotic values at $R \to \infty$?
How good is the usage of these values?

$$M(R)=\mu_R, \qquad I(R)=\mu_R R^2$$

ASCC method

- Optimal reaction path based on TDHF dynamics
- Inertial masses with residual effect beyond mean fields (cf. Thouless-Valatin)
  - Neglecting the residual effect

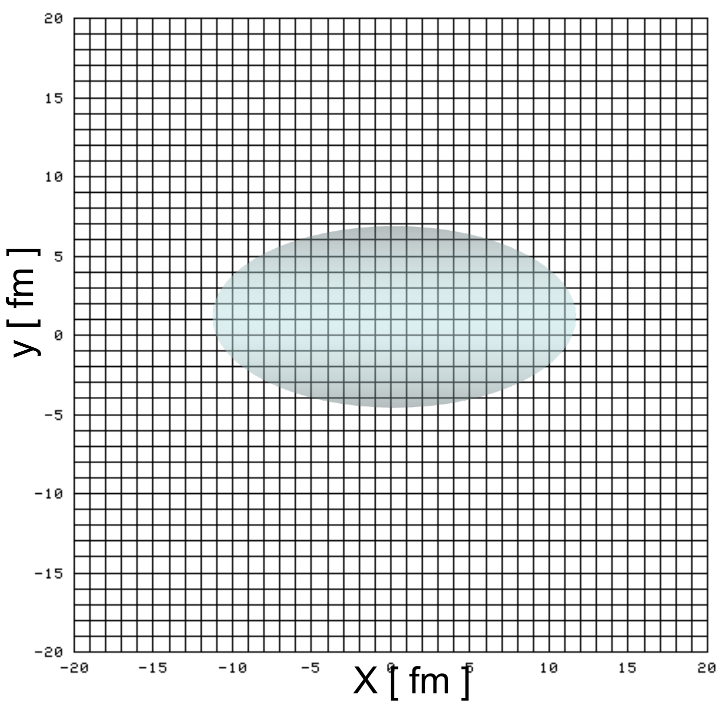
↓

  - Cranking formula for collective masses

$$M_{\rm cr}^{\rm NP}(R) = 2 \sum_{n\in p, j\in h} \frac{|\langle \varphi_n(R)|\partial/\partial R|\varphi_j(R)\rangle|^2}{e_n(R) - e_j(R)},$$

3D real space representation

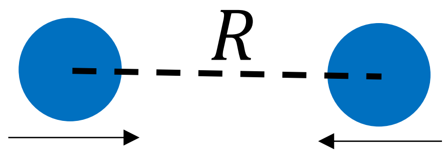
- 3D space discretized in lattice
- BKN functional: $E_{\mathrm{BKN}}[\rho,\tau]$ (*rather schematic*)
- Moving mean-field eq.: Imaginary-time method
- Moving RPA eq.: Finite amplitude method (PRC 76, 024318 (2007))

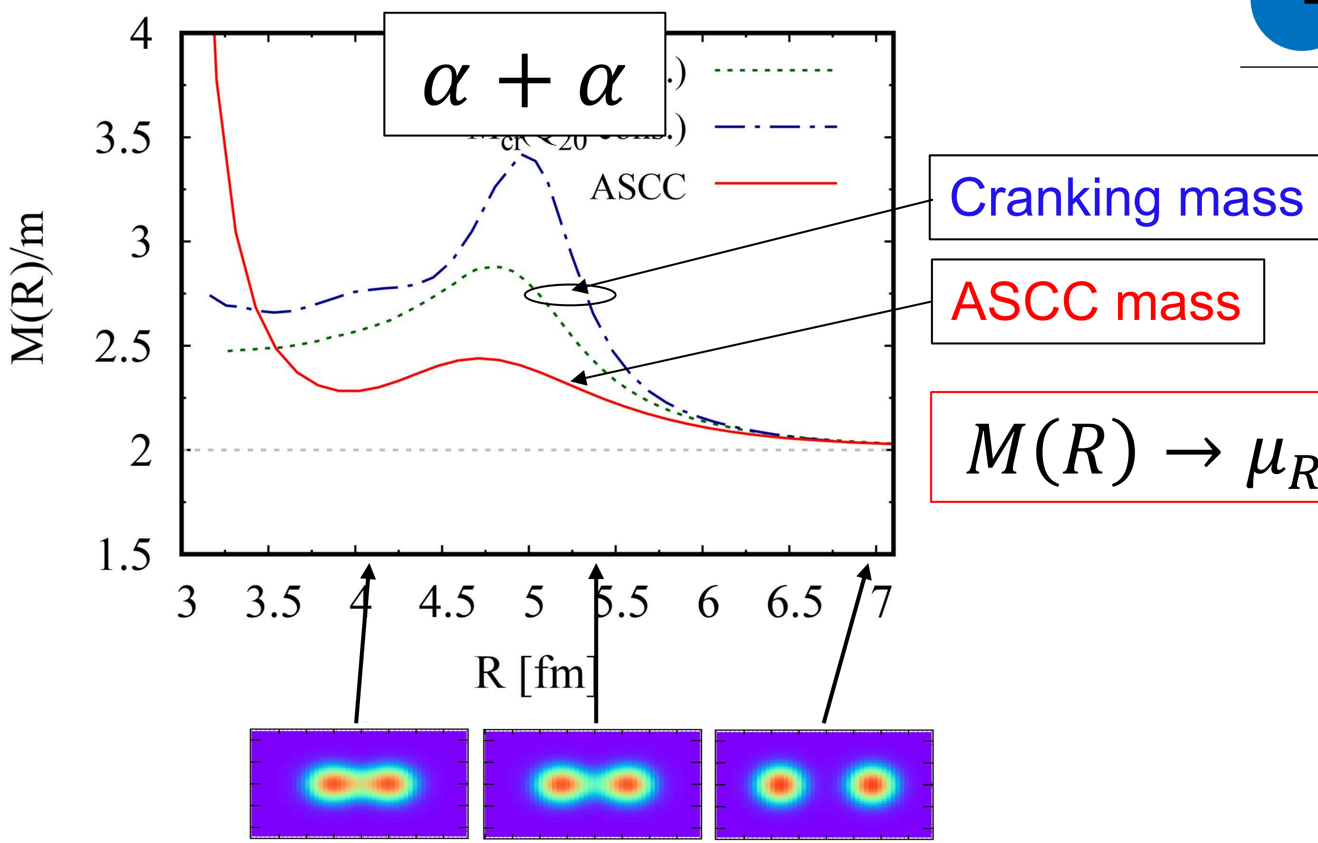


At a moment, no pairing

1-dimensional reaction path extracted from the Hilbert space of dimension of $10^4 \sim 10^5$.

$M(R) \quad (m^*/m = 1)$



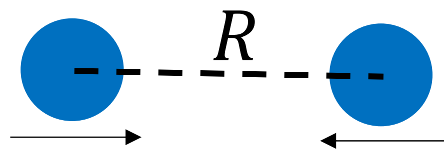


$$M(R) \to \mu_R \quad (R \to \infty)$$

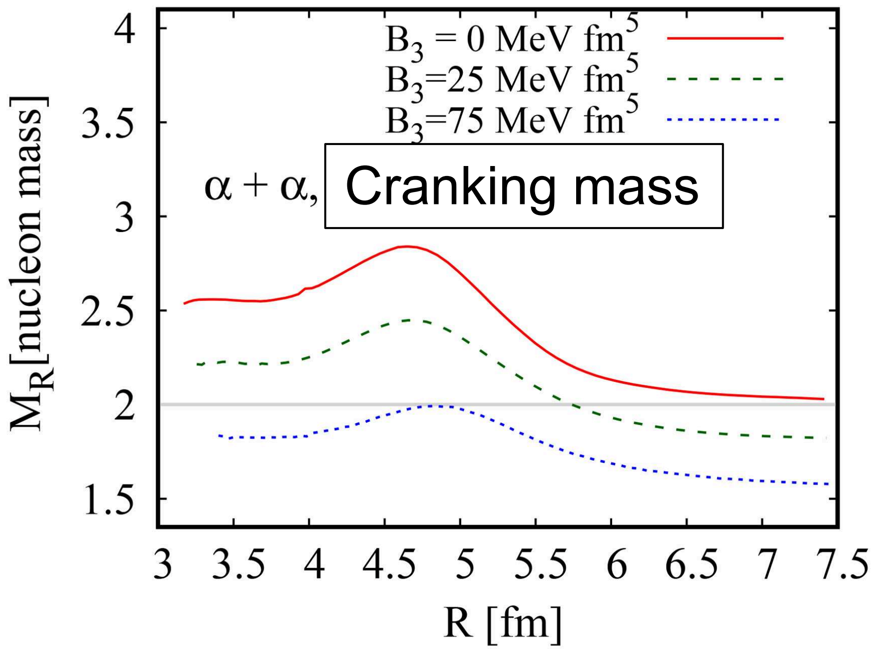
Different masses between ASCC & cranking after touching

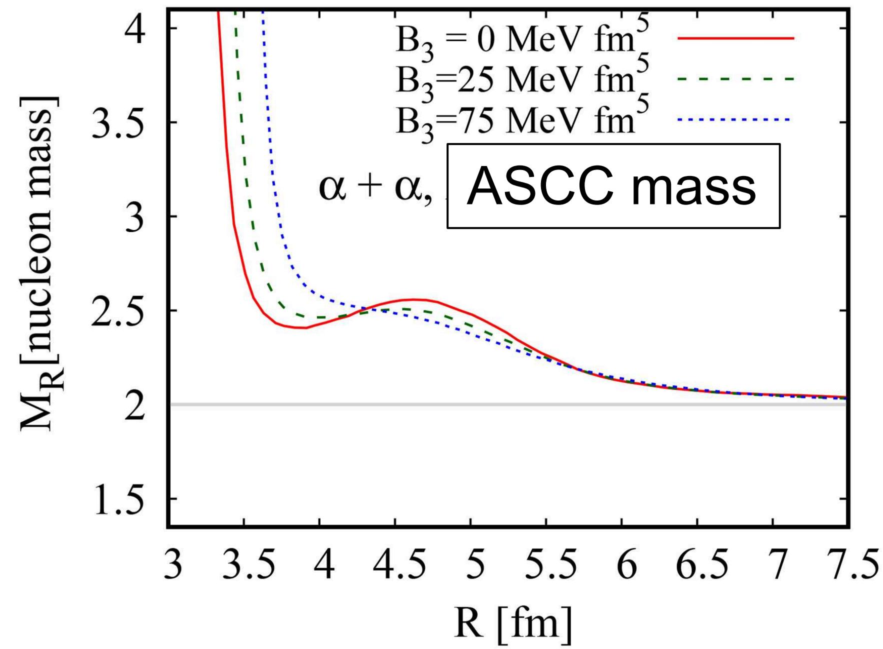
⇨ Reaction path deviates from R

$M(R) \quad (m^*/m \leq 1)$



$$m^* < m^* < m^* = m$$

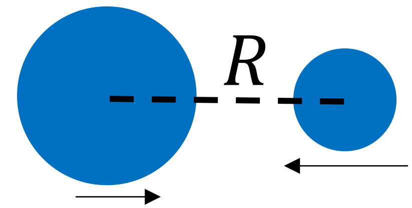


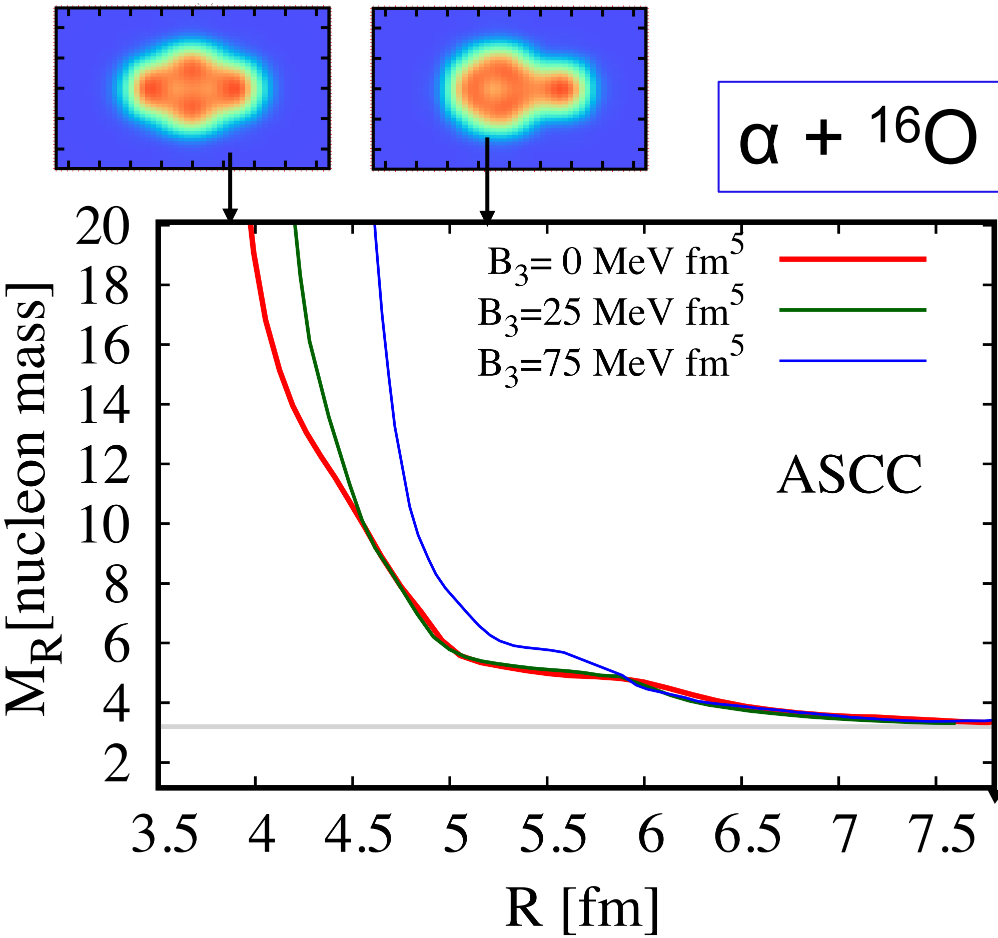


"Cranking mass" decreases with decreasing m^*

$$M(R) \neq \mu_R \quad (R \to \infty)$$

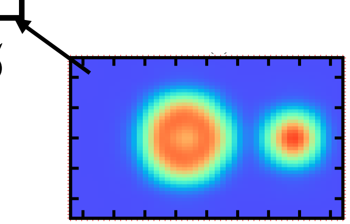
$M(R)$ $\quad (m^*/m \leq 1)$





$m^* < m^* < m^* = m$

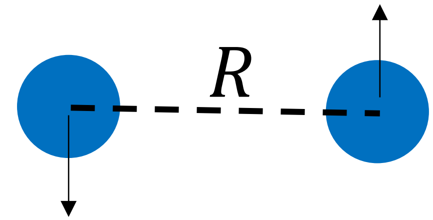
$M(R) = \mu_R \quad (R \to \infty)$

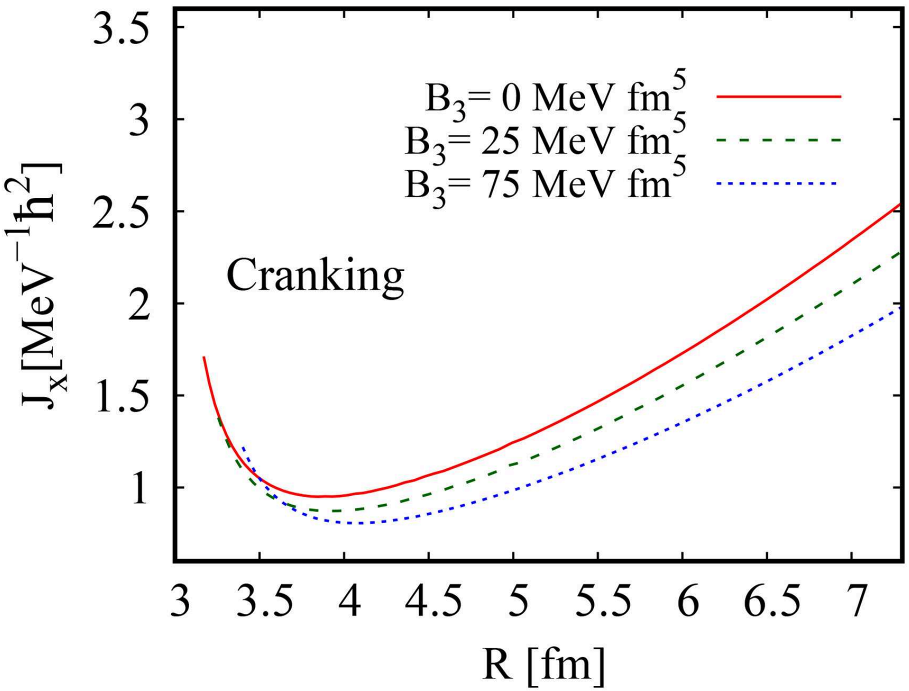


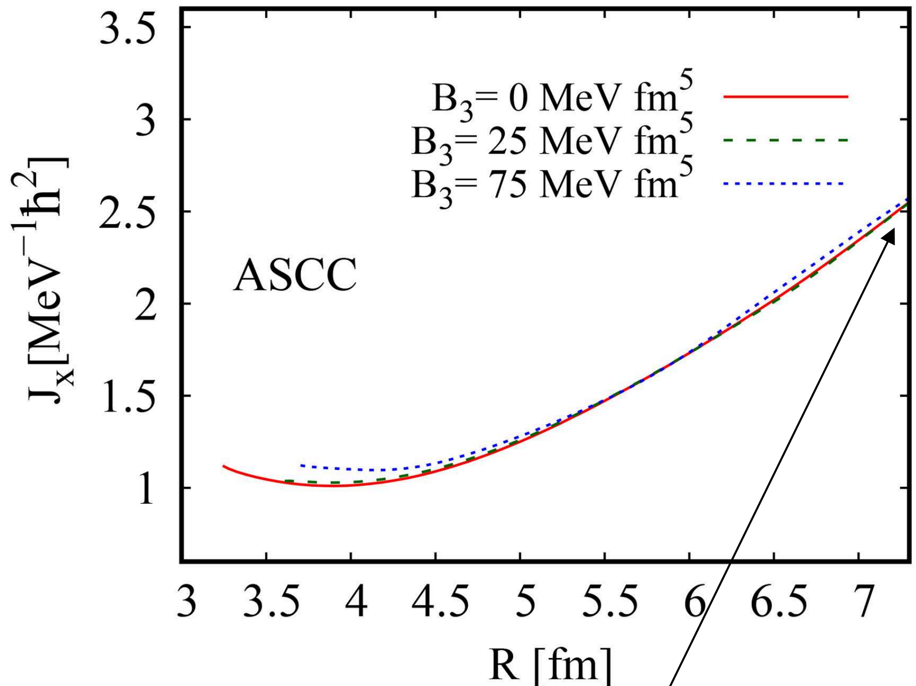
Moment of inertia: $I(R)$

$m^* < m^* < m^* = m$

$\alpha + \alpha$



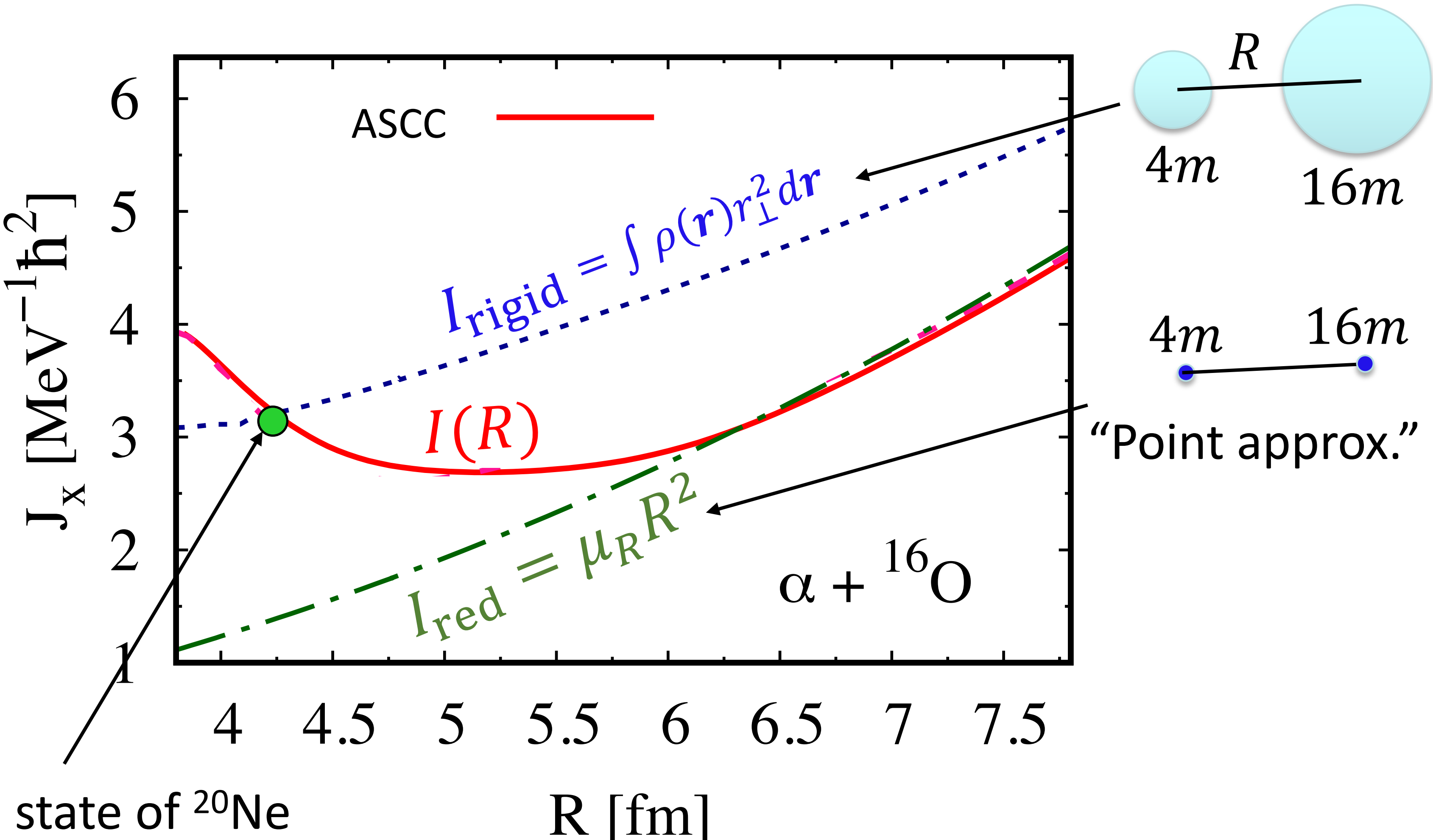




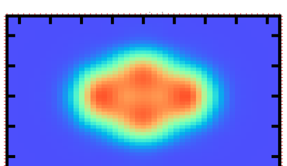
$I(R)$ decreases with reducing m^*

$I_{\mathrm{red}} = \mu_R R^2$

Moment of inertia



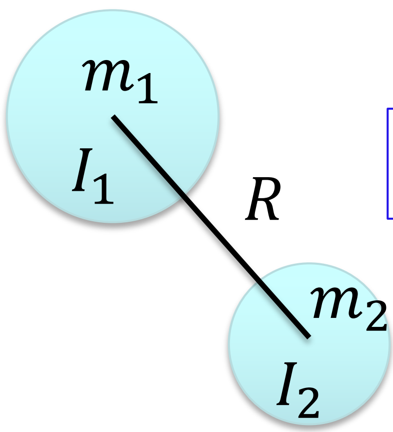
Ground state of ^{20}Ne



Smooth transition from $I_{\rm rigid}$ to $I_{\rm red}$.

Quantum effect

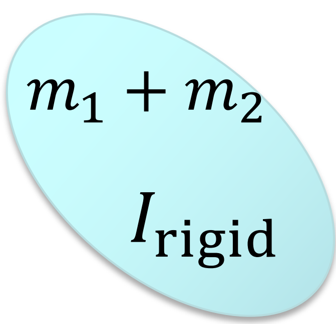
Quantum spherical systems: $I = 0$ (no rotation)



$$I(R) = I_{\mathrm{red}}(R) + I_1 + I_2$$

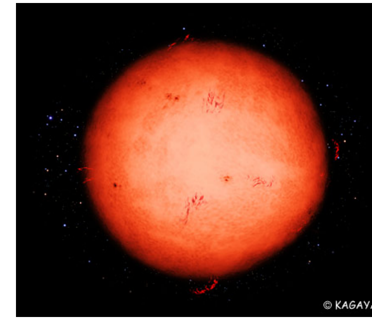
$$I_{\mathrm{red}}(R) = \mu_R R^2 = \frac{m_1 m_2}{m_1 + m_2} R^2$$

I_i: MoI of nucleus i w.r.t. its CoM



$$I(R) = I_{\mathrm{red}}(R) + I_{\mathrm{rigid}}^{(1)} + I_{\mathrm{rigid}}^{(2)}$$

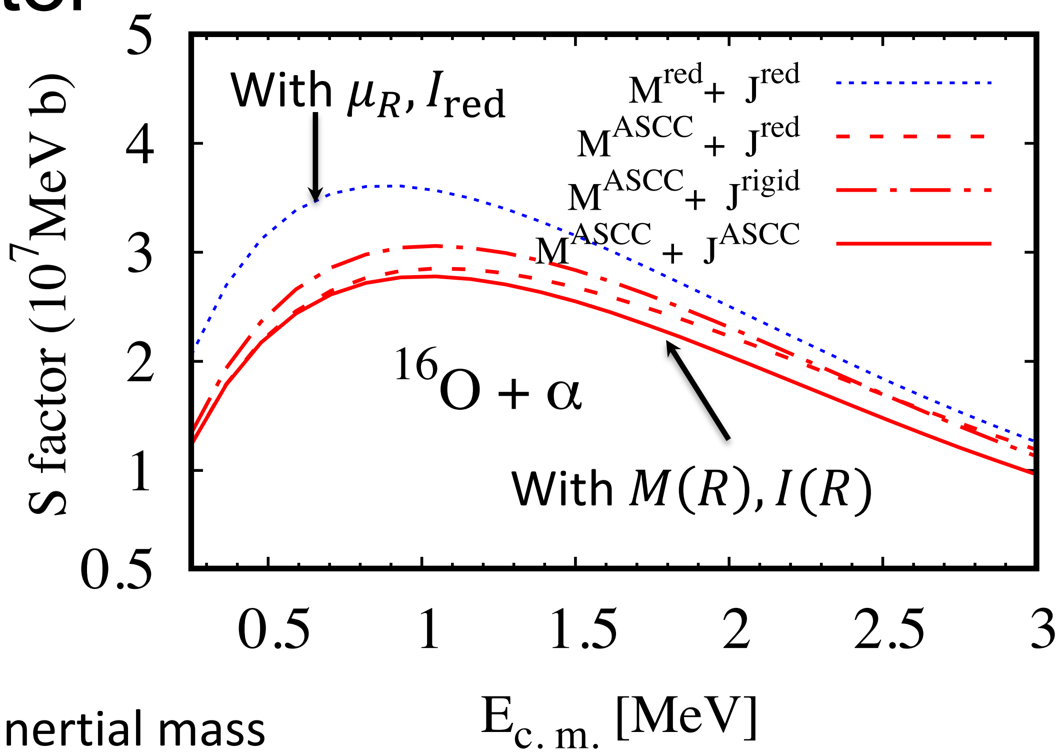
Alpha reaction : $^{16}O + \alpha$



Synthesis of ^{20}Ne

Fusion reaction:
Astrophysical S-factor

$$\sigma(E) = \frac{1}{E} P(E) \times S(E)$$



Effect of dynamical change of the inertial mass

Summary

- Self-consistent description of nuclear reaction path and dynamics
  - Reaction path affects the inertial mass
  - Effective mass is canceled by the residual effect in the asymptotic region (the cranking formula fails)
  - Vanishing MoI in spherical nuclei (quantum effect) is properly taken into account
  - Reduction of astrophysical S-factor

K. Wen, T. N., Front. Phys. 8 (2020) 16; Phys. Rev. C 96 (2017) 014610