新学術「量子クラスターで読み解く物質の階層構造」第4回研究会（Zoom）
May 28 (2020)

Shear viscosity and Kovtun-Son-Starinets conjecture in the BCS-BEC crossover regime of an ultracold Fermi

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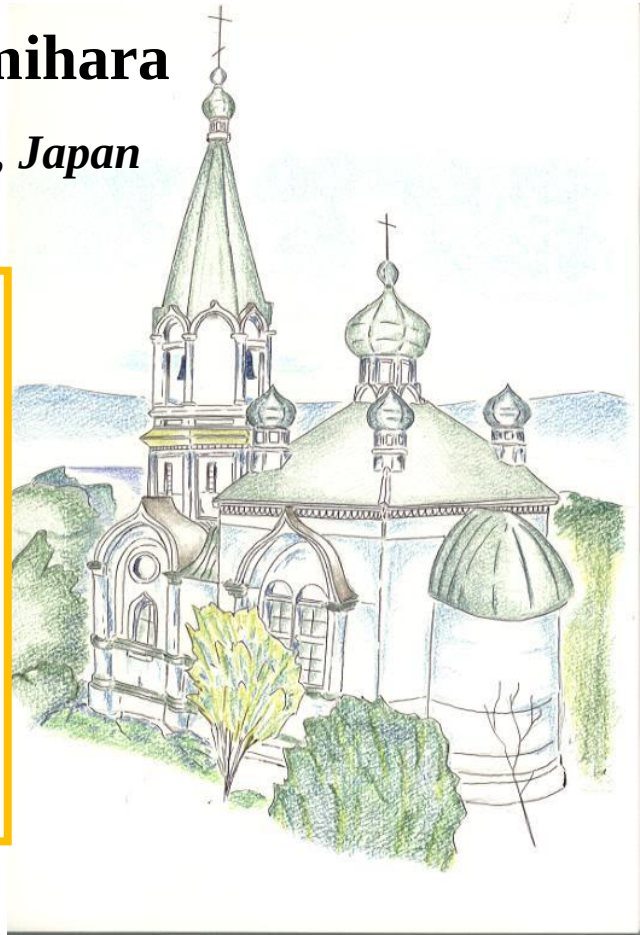
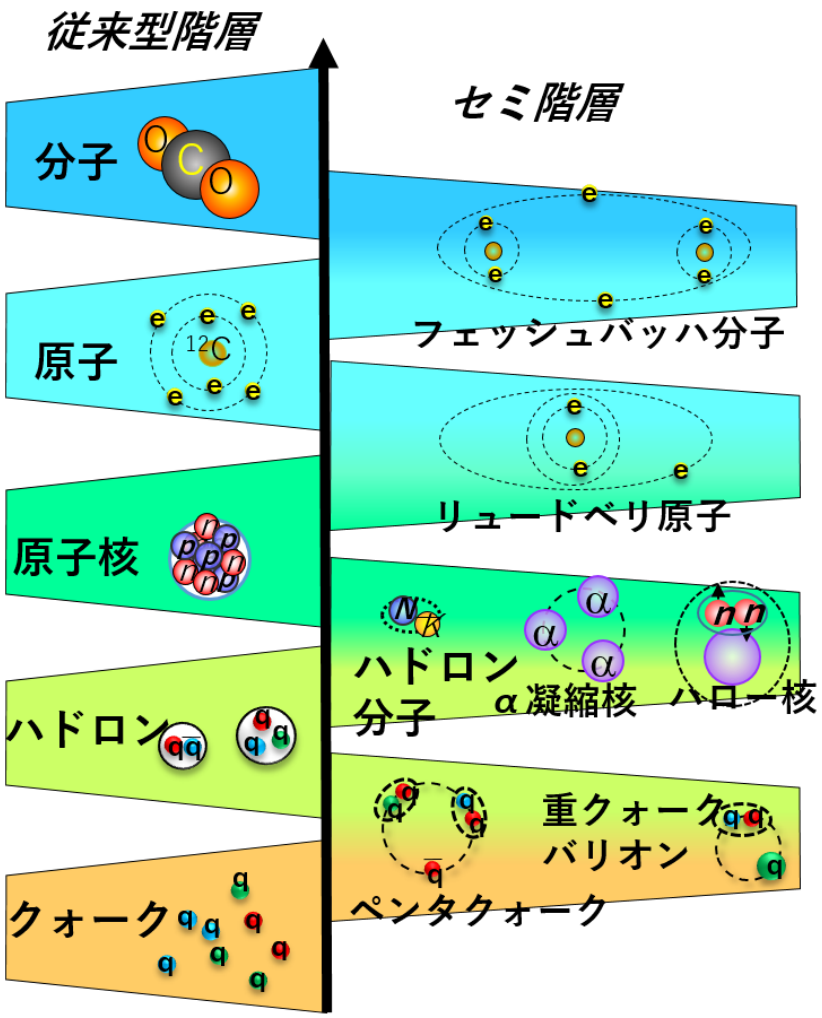
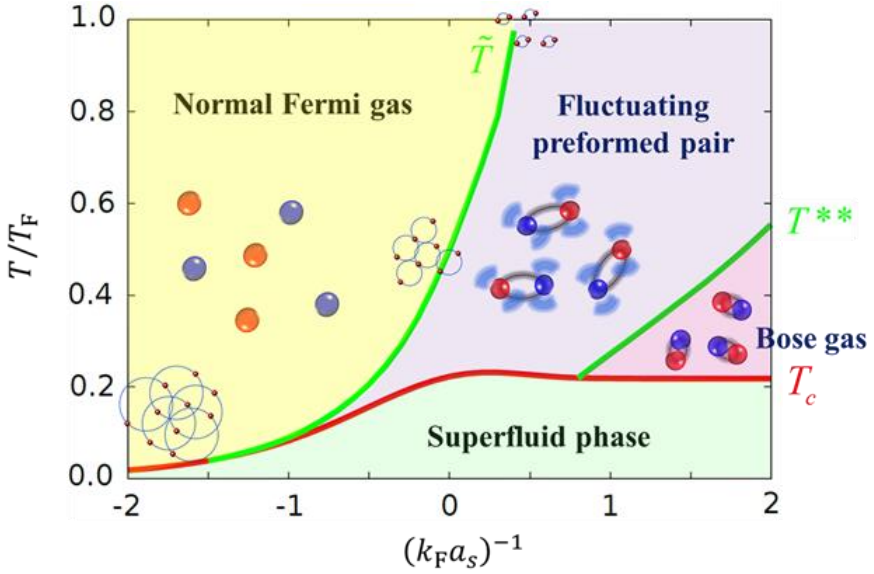
- Introduction: shear viscosity and KSS conjecture
- Strong-coupling theory
  - SCTMA Self-energy
  - Shear viscosity
  - entropy density
- Results
- Summary



Clustering as a window on the hierarchical structure of quantum systems



cold atom physics



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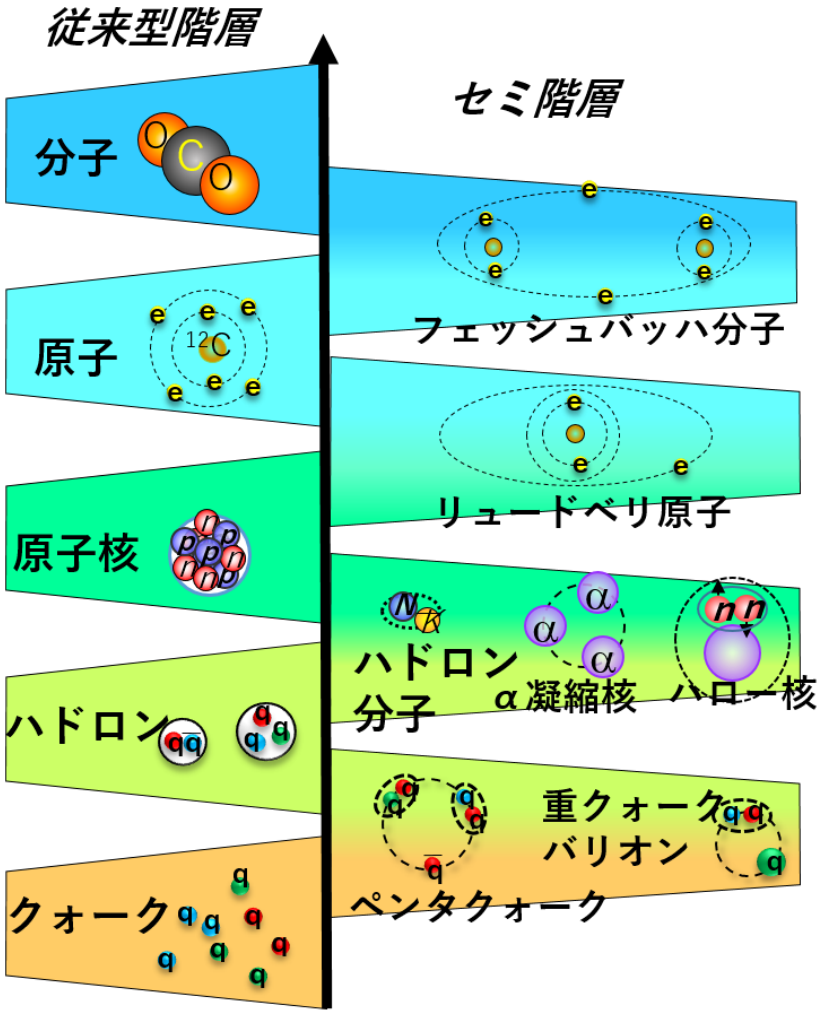
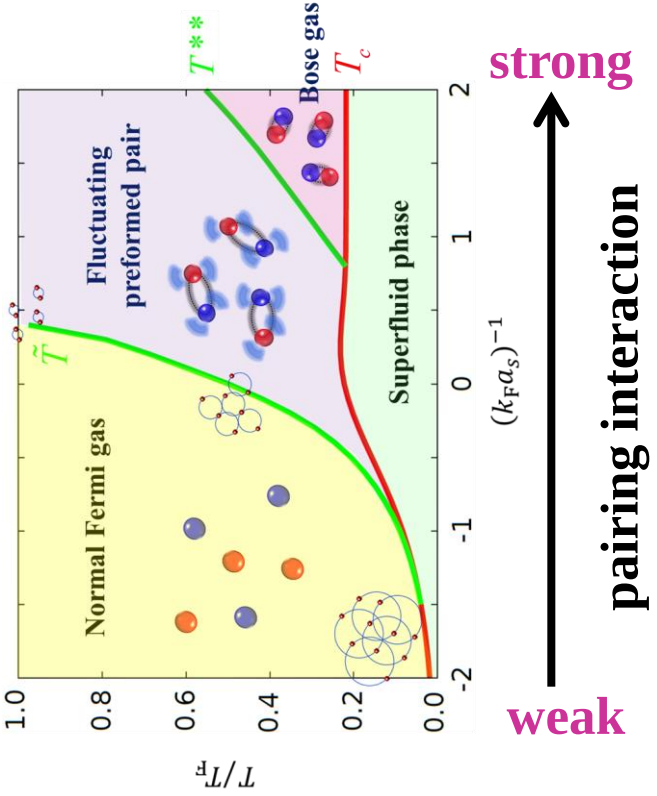


cold atom physics

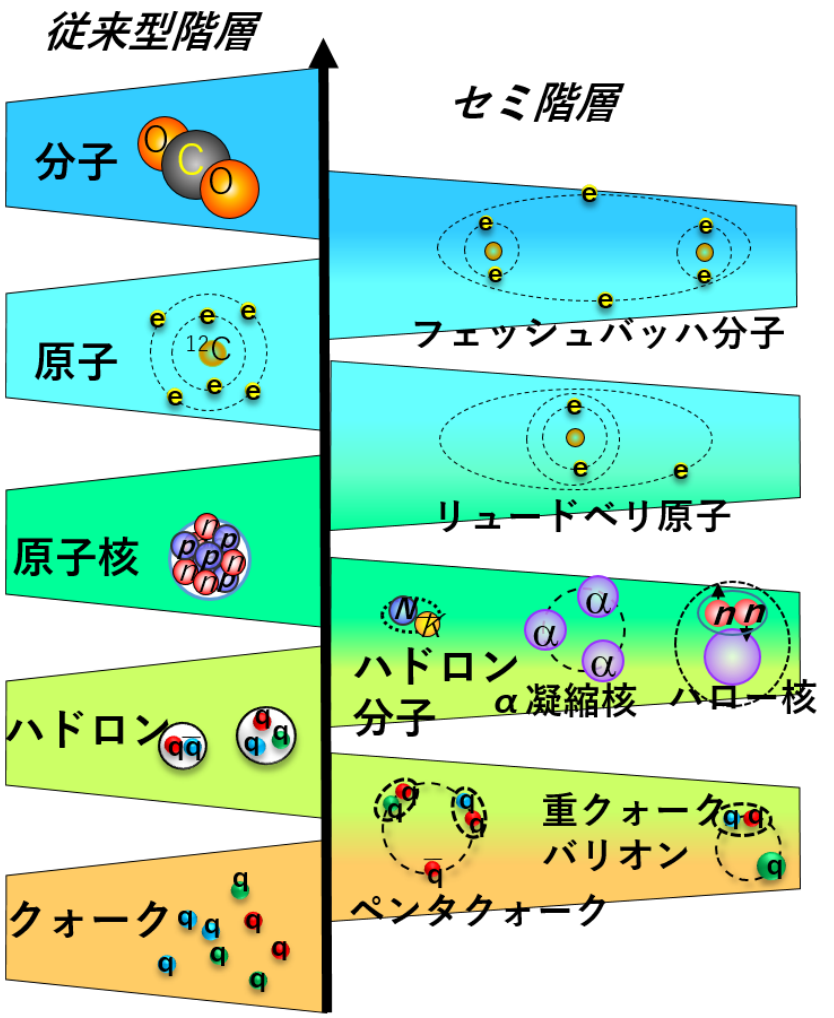


Fermi atomic gas

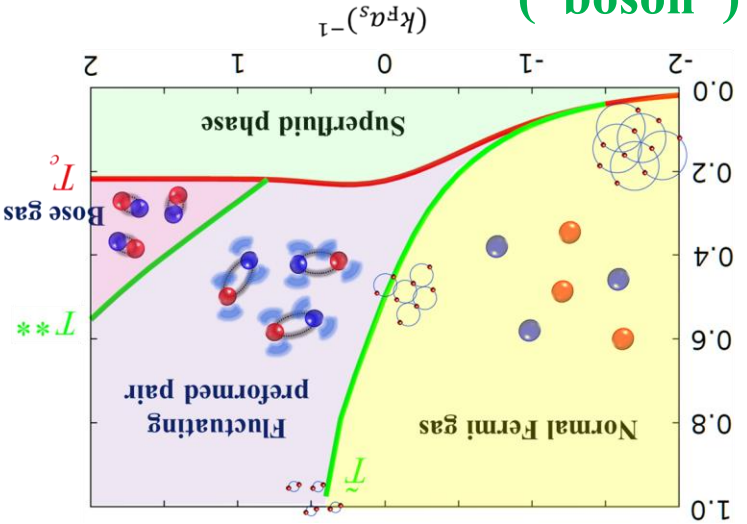
+

Feshbach resonance

Clustering as a window on the hierarchical structure of quantum systems



cold atom physics



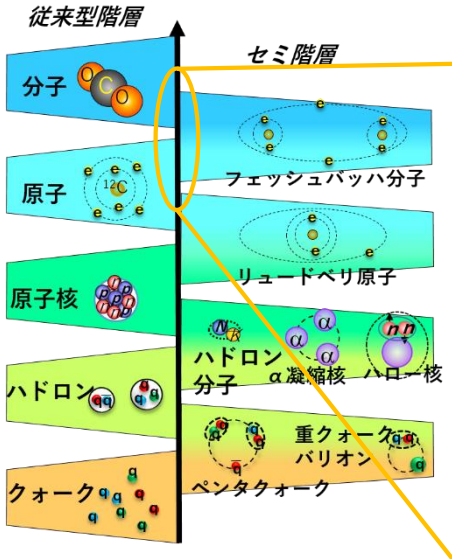
Fermi atomic gas

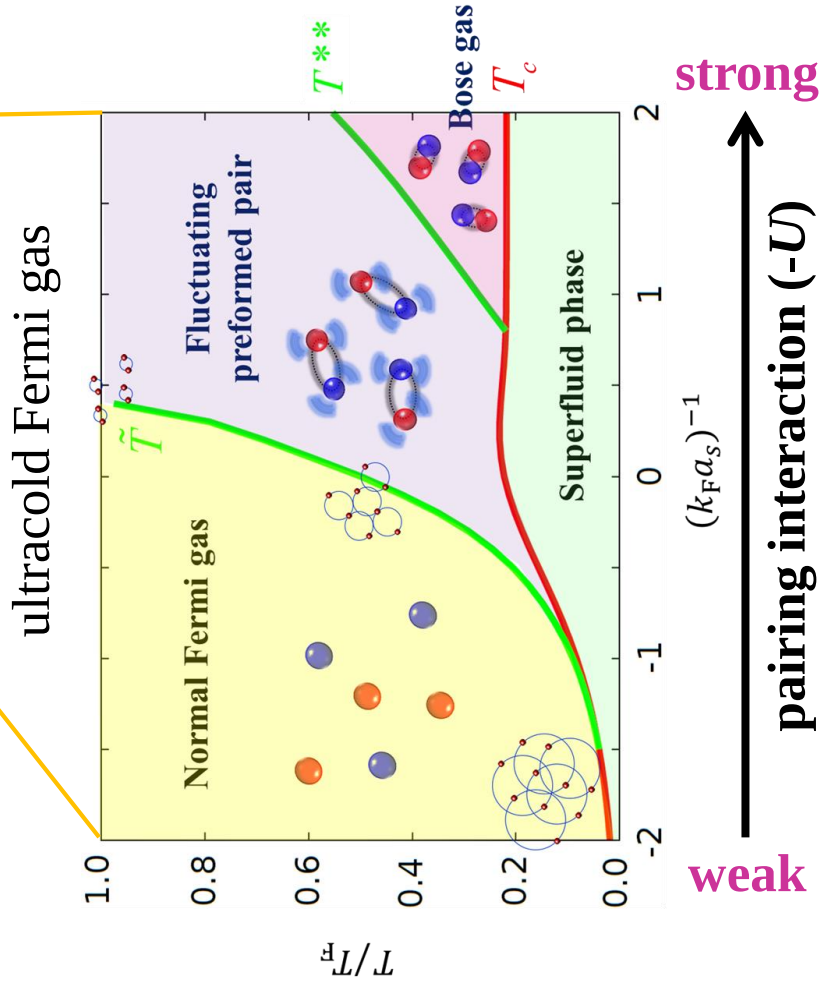
+

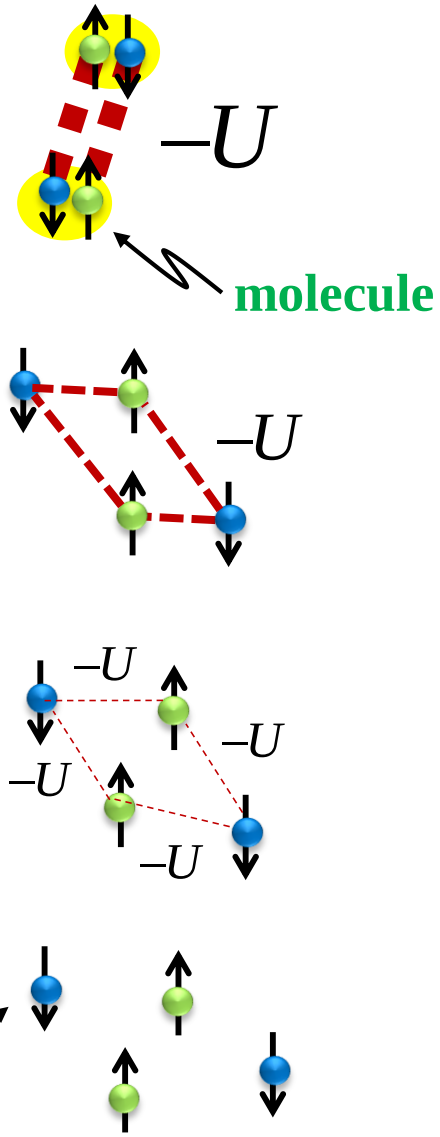
thermal effect

Clustering as a window on the hierarchical structure of quantum systems

strongly interacting Bose gas?

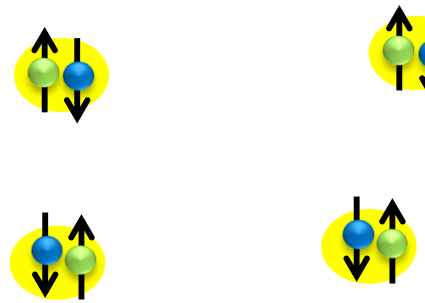




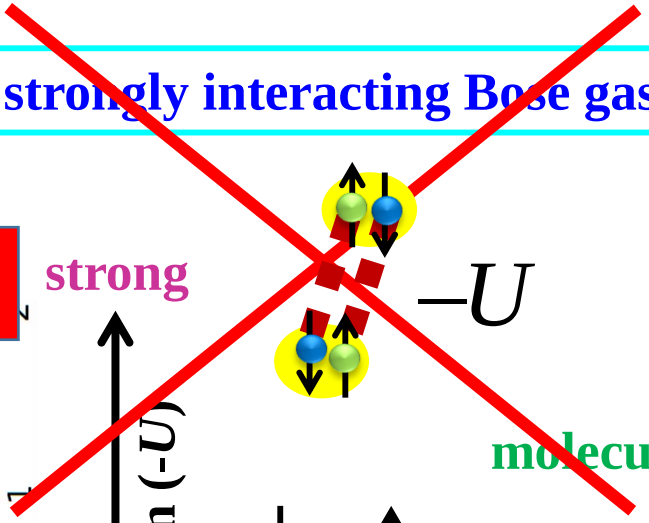


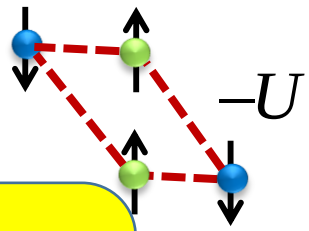
Clustering as a window on the hierarchical structure of quantum systems

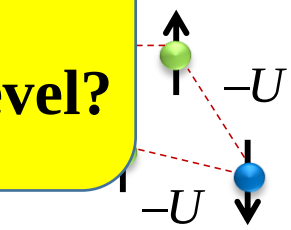
strong-coupling limit
→ ideal Bose gas



strongly interacting Bose gas?

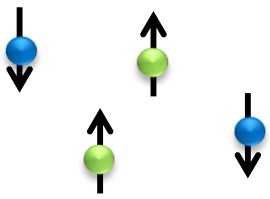






weak

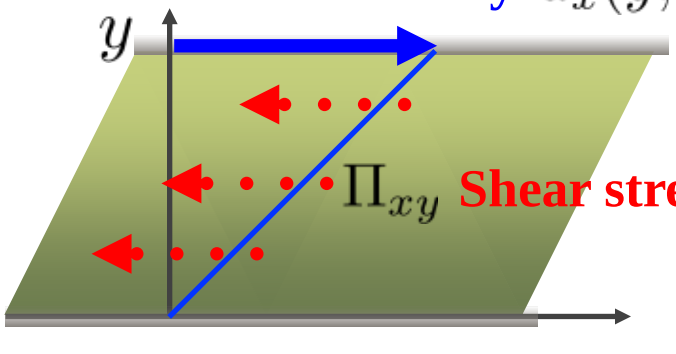
Fermi atom



- **How is this hierarchy formed?**
- **How can we observe the change of hierarchy level?**

Shear viscosity: A possible route to examine the hierarchy problem

fluid velocity $u_x(y,t) = e^{i\omega t} u_x(y)$

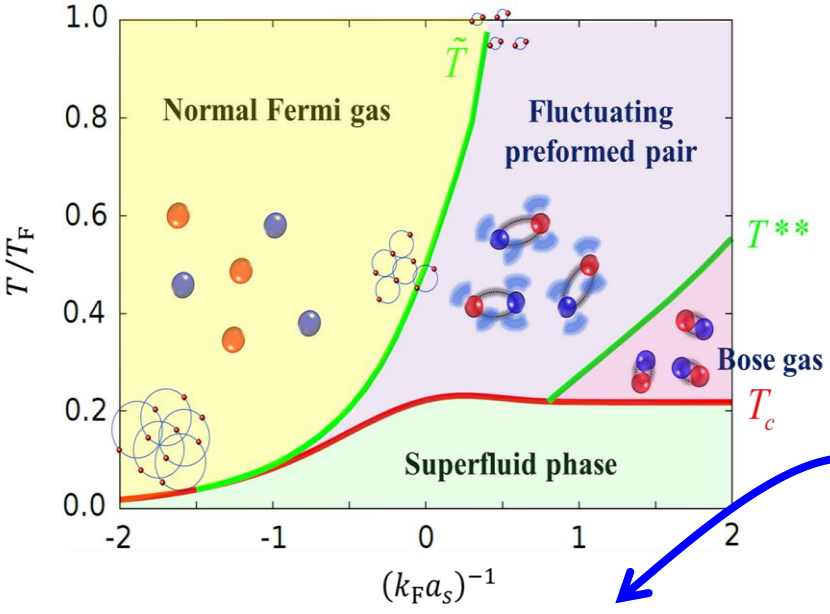


Shear stress → $\Pi_{xy} = -\eta \dfrac{\partial u_x}{\partial y}$

Shear viscosity

$\eta \sim n \langle p \rangle l_{\text{mfp}}$ **(classical picture)**

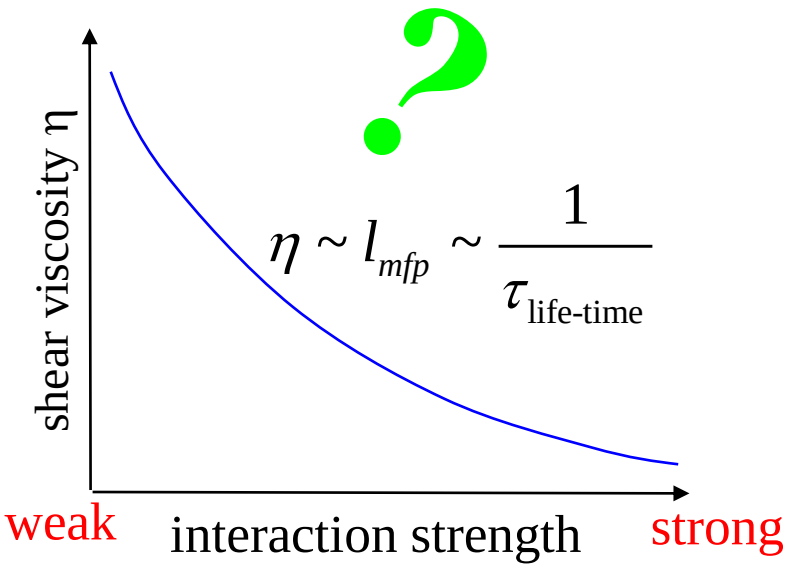
Mean-free path $\sim \dfrac{1}{\tau}$



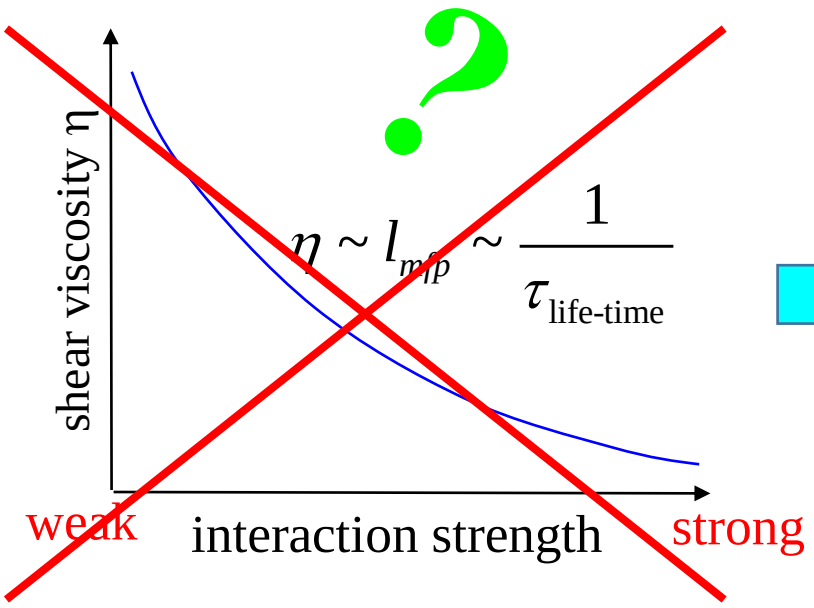
weak **pairing interaction** strong

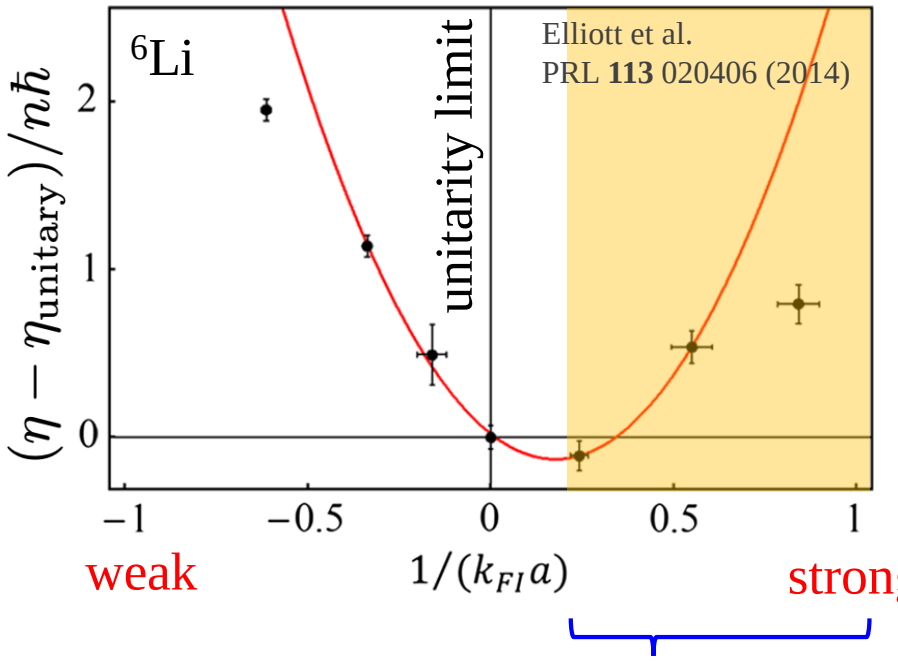
η simply *decreases* with increasing the atomic interaction strength?

Shear viscosity: A possible route to examine the hierarchy problem



Shear viscosity: A possible route to examine the hierarchy problem





We need to go to the NEXT hierarchy level…
(atom level → molecule level)

The shear viscosity η is a useful quantity to see the boundary between the two hierarchy levels.

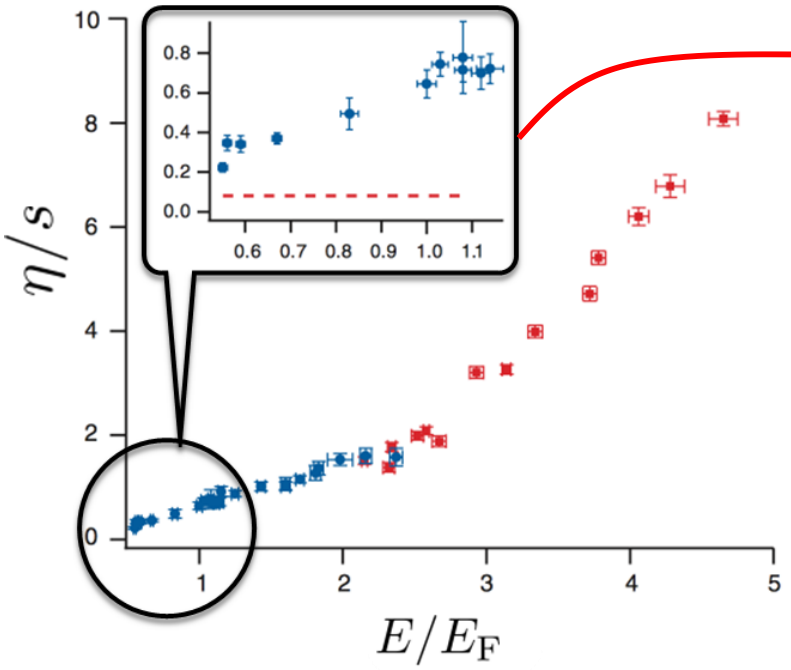
Another importance of η: Kovtun-Son-Starinets (KSS) conjecture

$$\frac{\text{shear viscosity}}{\text{entropy density}} = \frac{\eta}{s} \geq \frac{\hbar}{4\pi k_B}$$

Quantum!

KSS PRL 95 111601 (2005)

^{6}Li unitary Fermi gas



C. Cao, *et al.*, Science **331** 58 (2011)

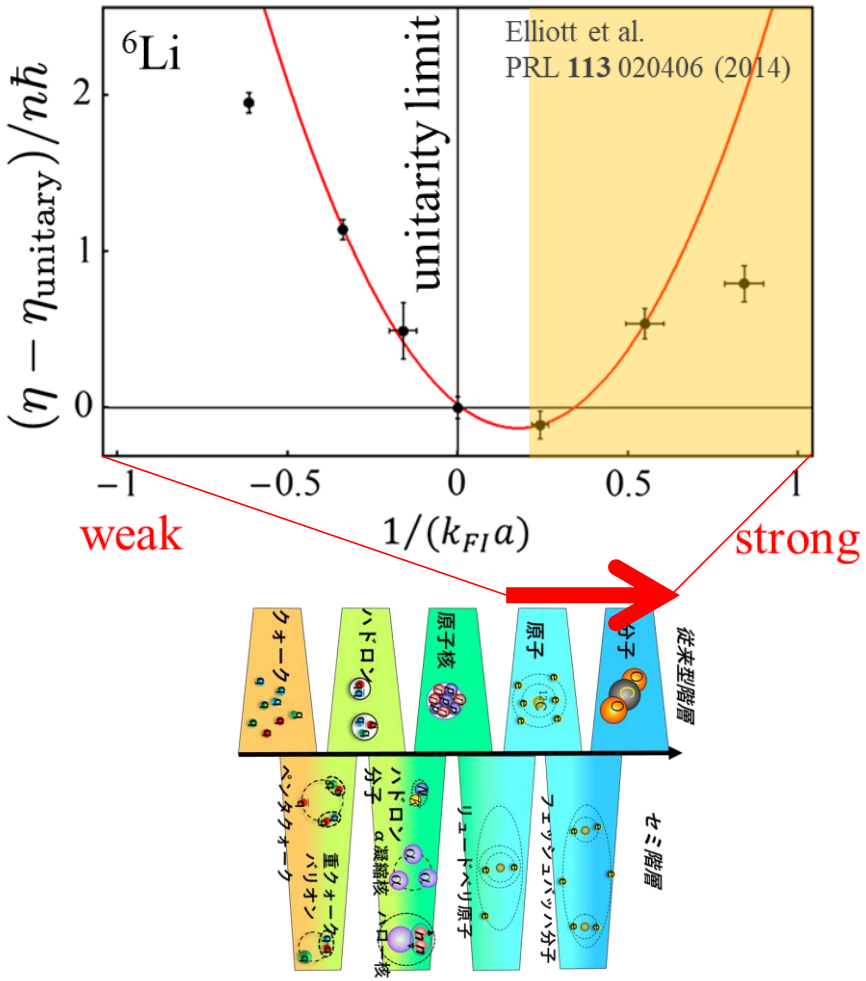
$$(\eta/s)_{\min} \sim 2.7\hbar/4\pi k_B$$

- Quark Gluon Plasma
  $(\eta/s)_{\min} \sim 2\hbar/4\pi k_B$
- Superfluid ^{4}He
  $(\eta/s)_{\min} \sim 5\hbar/4\pi k_B$
- N_2 (room temp.,1atm)
  $(\eta/s) \sim 3.3 \times 10^3 \hbar/4\pi k_B$

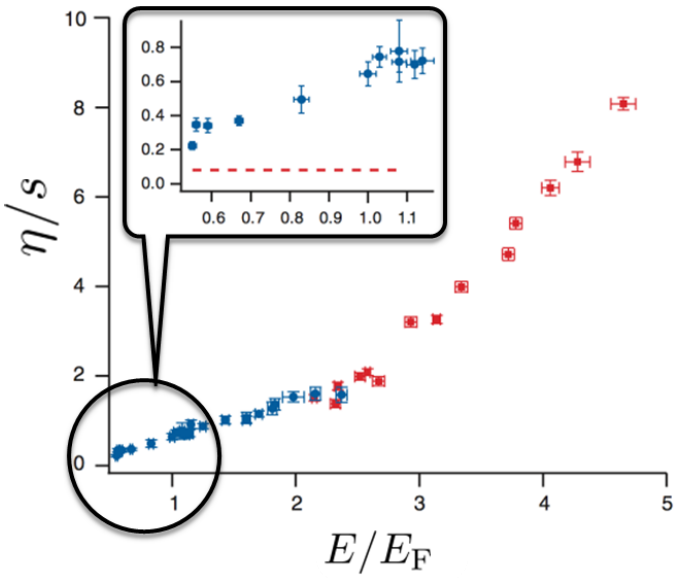
KSS conjecture has been discussed in various hierarchy levels, ranging from high-energy physics to low-energy physics. So, this topic may be useful for this 新学術.

Today's talk

We theoretically investigate the shear viscosity in the normal state of an ultracold Fermi atomic gas. Including effects of a tunable pairing interaction within the framework of SCTMA, we clarity how this transport coefficient in the BCS-BEC crossover region. We also assess the KSS conjecture.



^{6}Li unitary Fermi gas



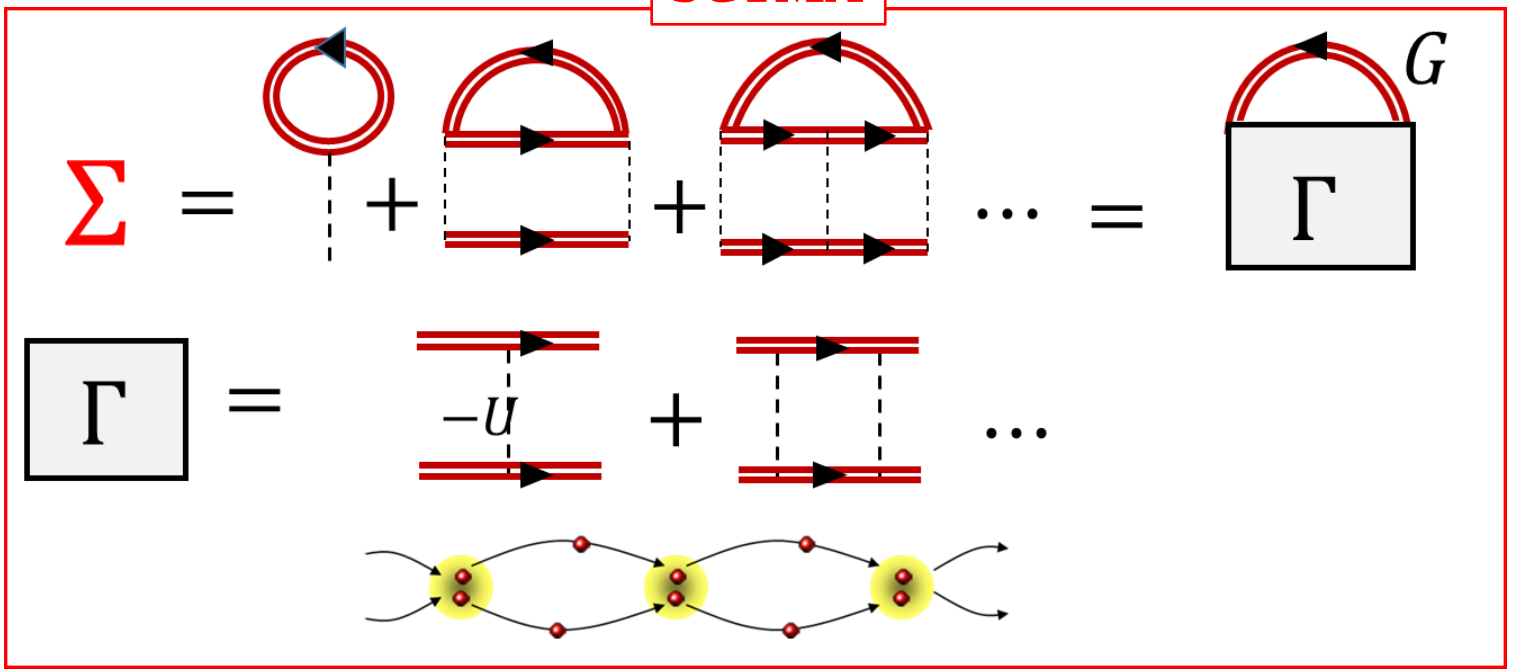
C. Cao, *et al.*, Science **331** 58 (2011)

Formulation: self-consistent T-matrix approximation (SCTMA)

$$H = \sum_{\mathbf{p},\sigma} (\varepsilon_{\mathbf{p}} - \mu) c^{\dagger}_{\mathbf{p}\sigma} c_{\mathbf{p}\sigma} - U \sum_{\mathbf{p},\mathbf{p}',\mathbf{q}} c^{\dagger}_{\mathbf{p}+\mathbf{q}/2\uparrow} c^{\dagger}_{-\mathbf{p}+\mathbf{q}/2\downarrow} c_{-\mathbf{p}'+\mathbf{q}/2\downarrow} c_{\mathbf{p}'+\mathbf{q}/2\uparrow}$$

$$G(\boldsymbol{p}, i\omega_n) = \frac{1}{i\omega_n - \varepsilon_p + \mu - \Sigma(p, i\omega_n)}$$

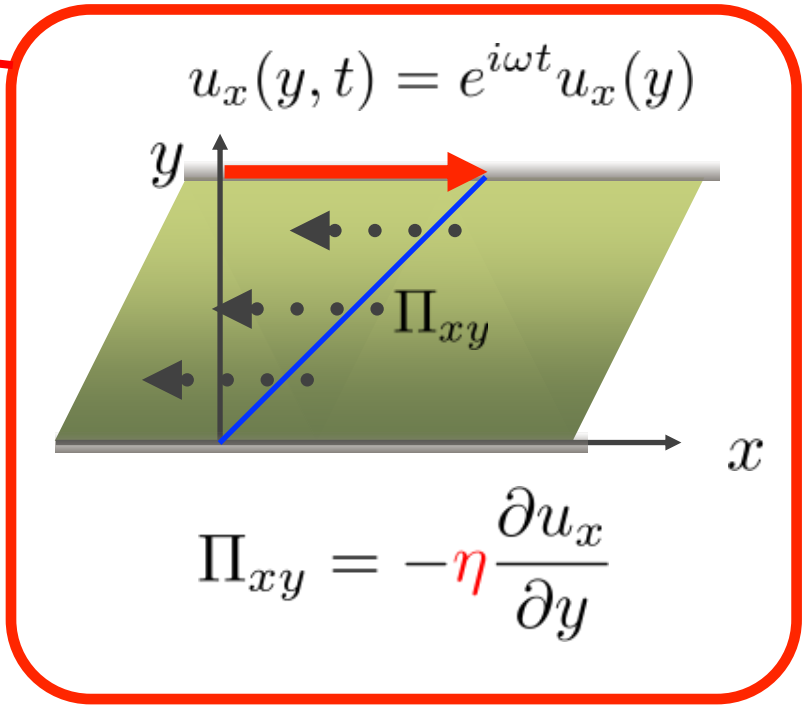
SCTMA



Shear viscosity η

- perturbation

$$H' = \frac{1}{i\omega}\frac{\partial u_x}{\partial y}\int d^3\boldsymbol{r}\Pi_{xy}$$



- Stress-tensor operator

$$\hat{\Pi}_{xy} = \sum_{\boldsymbol{p},\sigma}\frac{p_x p_y}{m}c^\dagger_{\boldsymbol{p},\sigma}c_{\boldsymbol{p},\sigma}$$

- shear viscosity

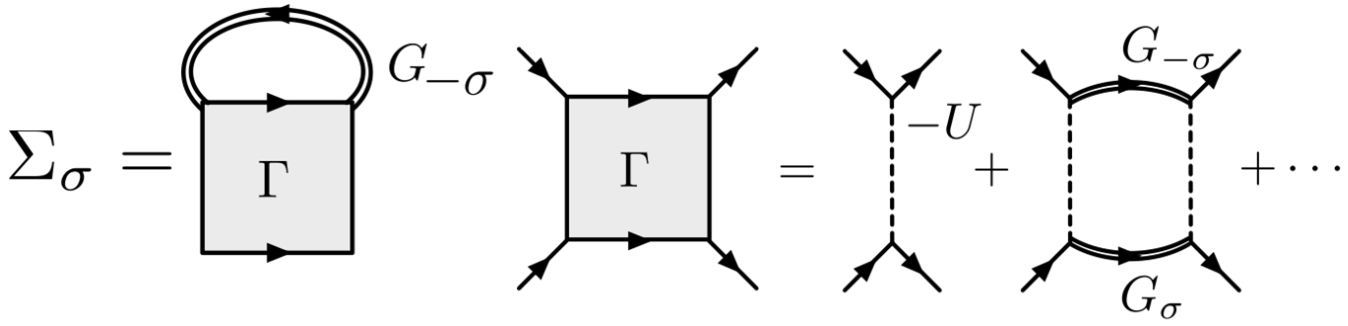
$$\eta = -\lim_{\omega\to 0}\frac{\mathrm{Im}\Xi(\omega)}{\omega}$$

$$\Xi(\omega) = -i\int d\boldsymbol{r}dte^{\imath\omega t}\theta(t)\left\langle[\hat{\Pi}^{xy}(\boldsymbol{r},t),\hat{\Pi}^{xy}(\boldsymbol{0},0)]\right\rangle$$

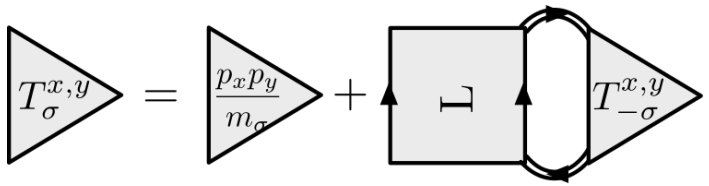
We consistently evaluate the shear viscosity within SCTMA.

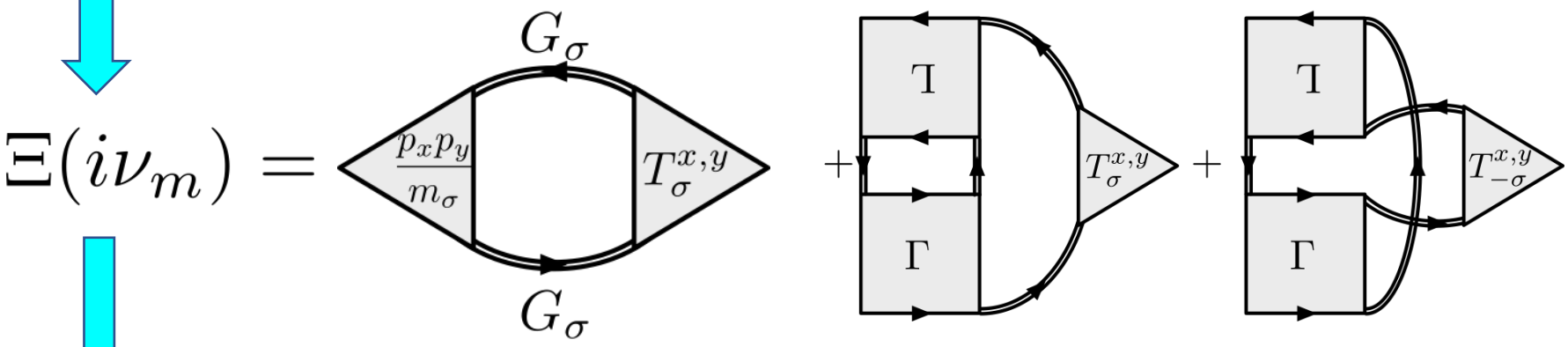
Shear viscosity η, being consistent with SCTMA

SCTMA self-energy



consistent vertex correction





(numerical) analytic continuation

shear viscosity

$$\eta = -\lim_{\omega\to 0}\frac{\mathrm{Im}\Xi(i\nu_m \to \omega + i\delta)}{\omega}$$

Entropy density s, being consistent with SCTMA

Tan's pressure relation

$$pV = \frac{2}{3}E + \frac{C}{12\pi m a_s}$$

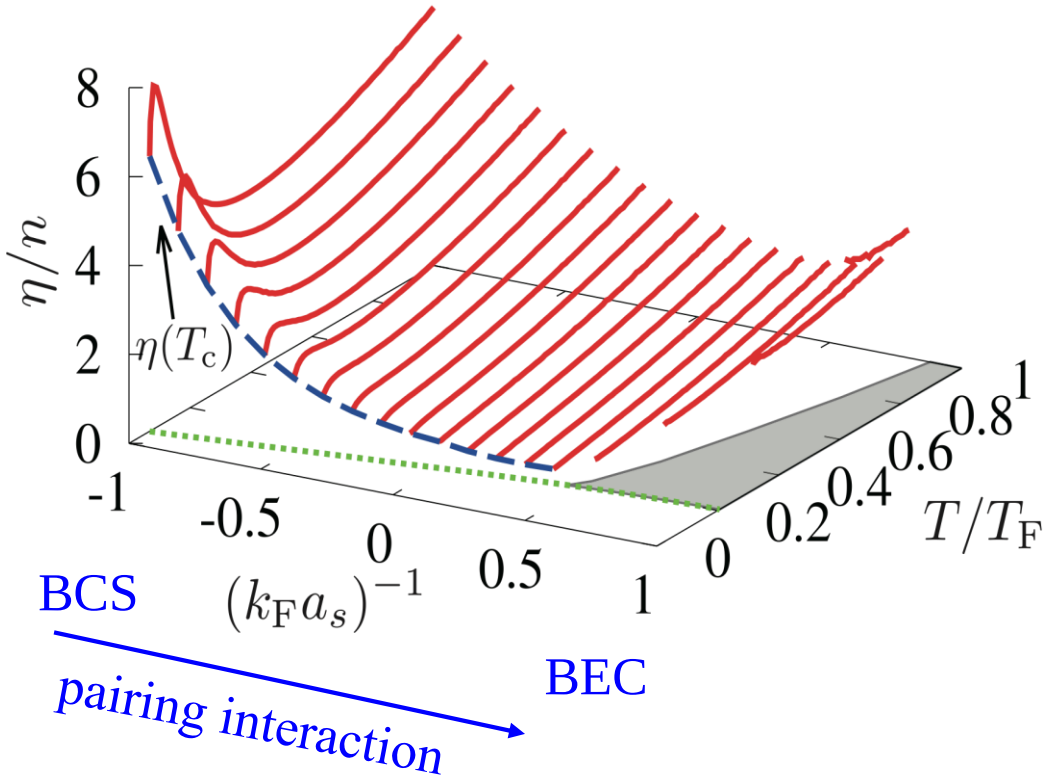
Tan's contact

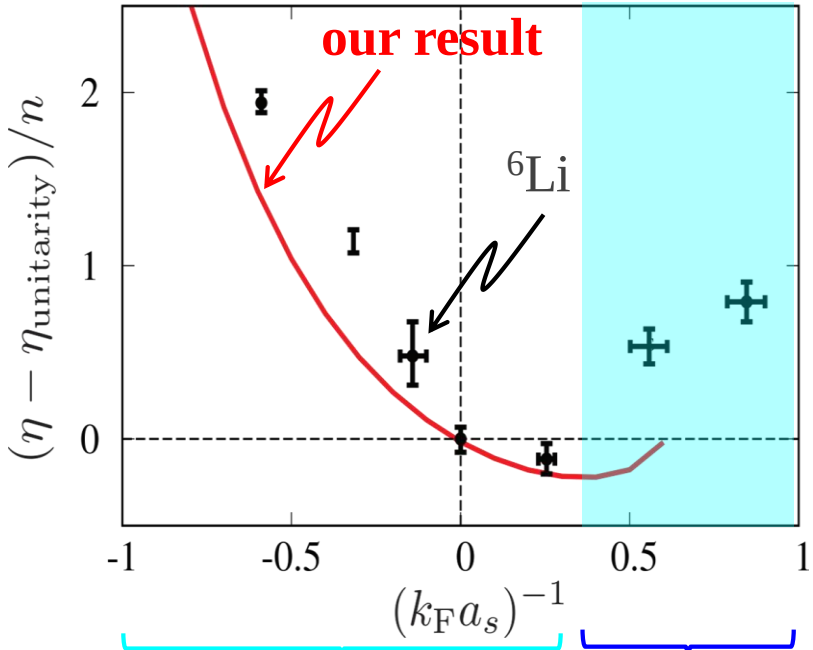
$$C = -m^2 T \sum_{\mathbf{q},\nu_n} \Gamma_{\text{SCTMA}}(\mathbf{q}, i\nu_n)$$

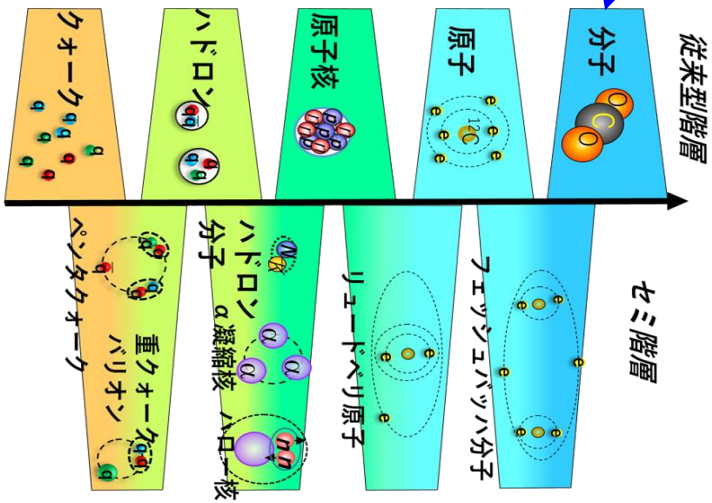
$$E = \langle H \rangle_{\text{SCTMA}} + \mu N$$

$$S = \frac{pV + E - \mu N}{T}$$

Shear viscosity in the BCS-BEC crossover region

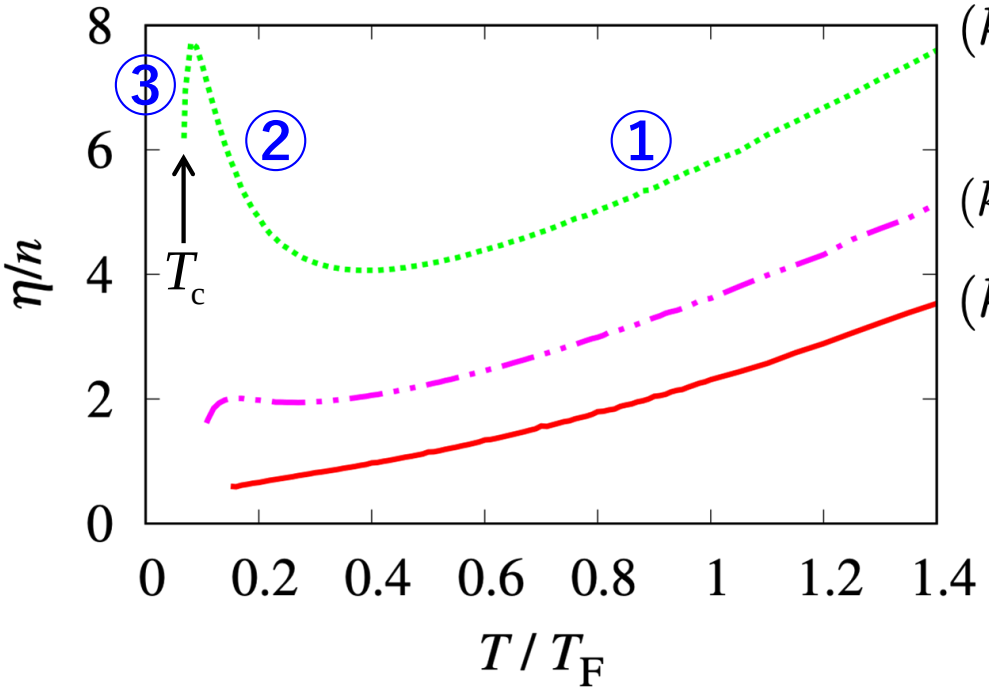






Kagamihara, Inotani, Ohashi, JPSJ 88 (2019), 114001

Shear viscosity in the WEAK-coupling side $(k_F a_s)^{-1}<0$



strong interaction

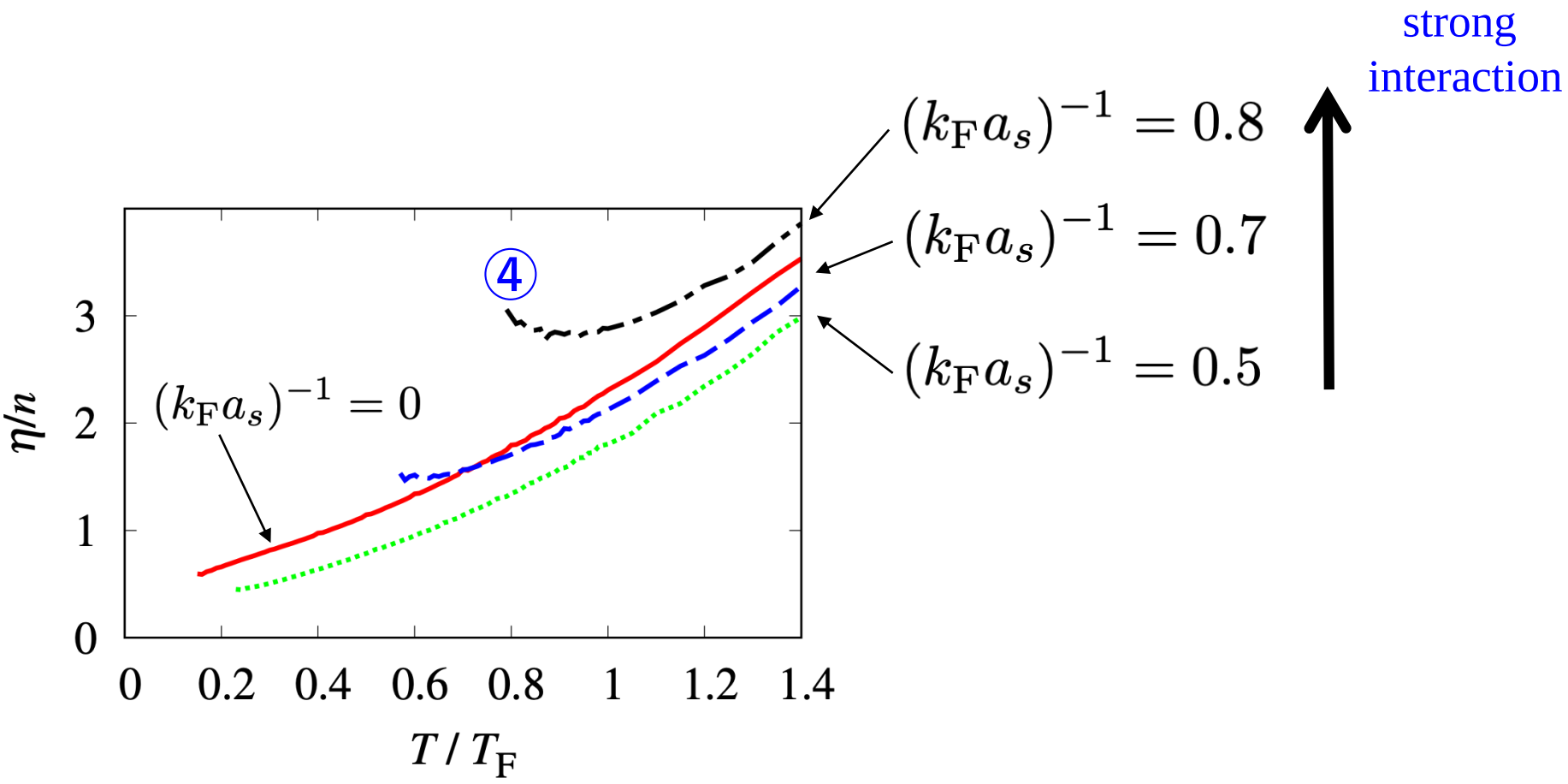
① **Classical Boltzmann gas** $\eta \sim T^{1/2\sim 3/2}$

② **Quantum Fermi liquid** $\eta \sim n\langle p\rangle l_{\mathrm{mfp}} \sim np_F \dfrac{1}{T^2}$

Pauli blocking effect

③ **Pseudogap regime (pairing fluctuations)**

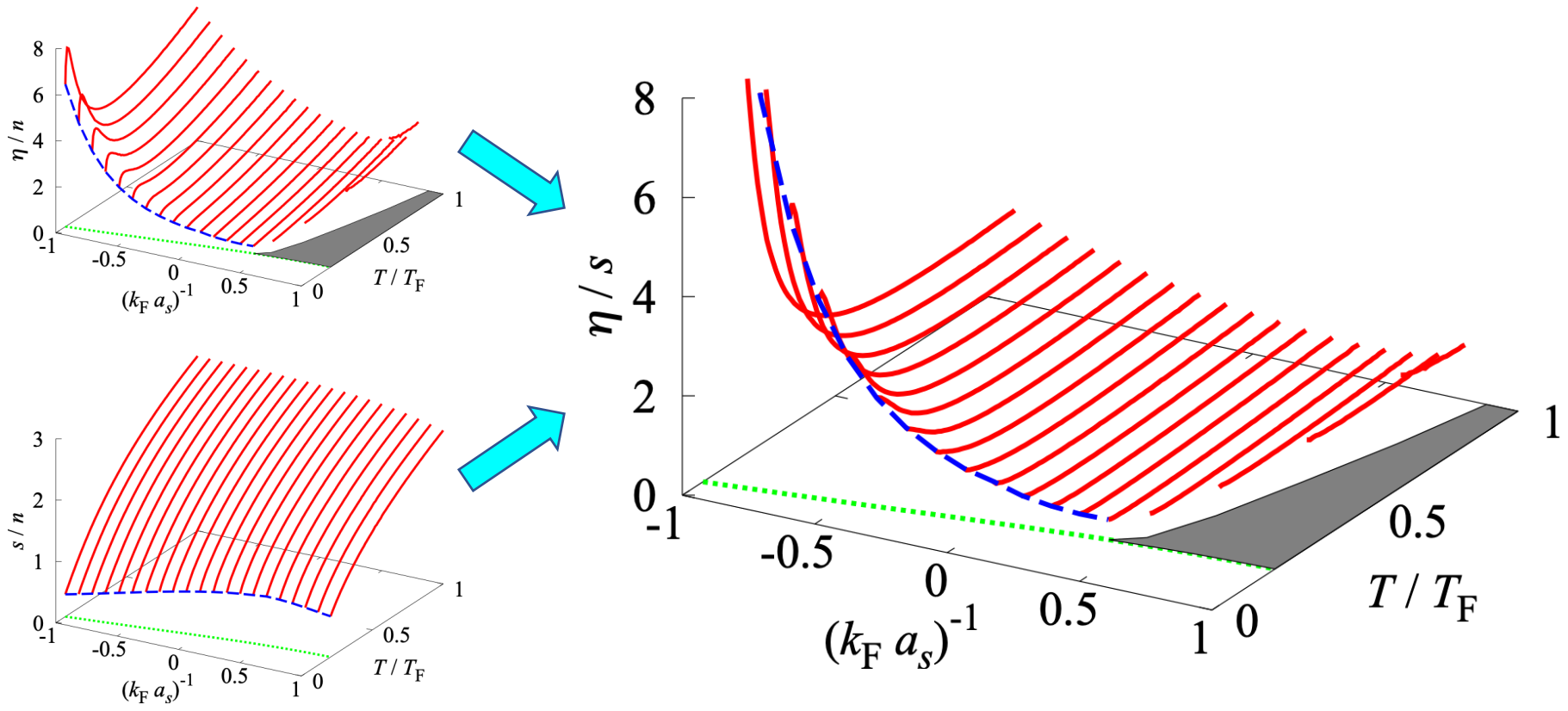
Shear viscosity in the STRONG-coupling side $(k_F a_s)^{-1}>0$



④ Molecular lifetime effect

$$\eta \sim n^{B} p_{av}^{B} l_{mfp}^{B} \qquad \tau_B \propto e^{-E_{bind}/2T} \Rightarrow \eta \propto e^{E_{bind}/2T}$$

Assessment of KSS conjecture in cold Fermi gas physics



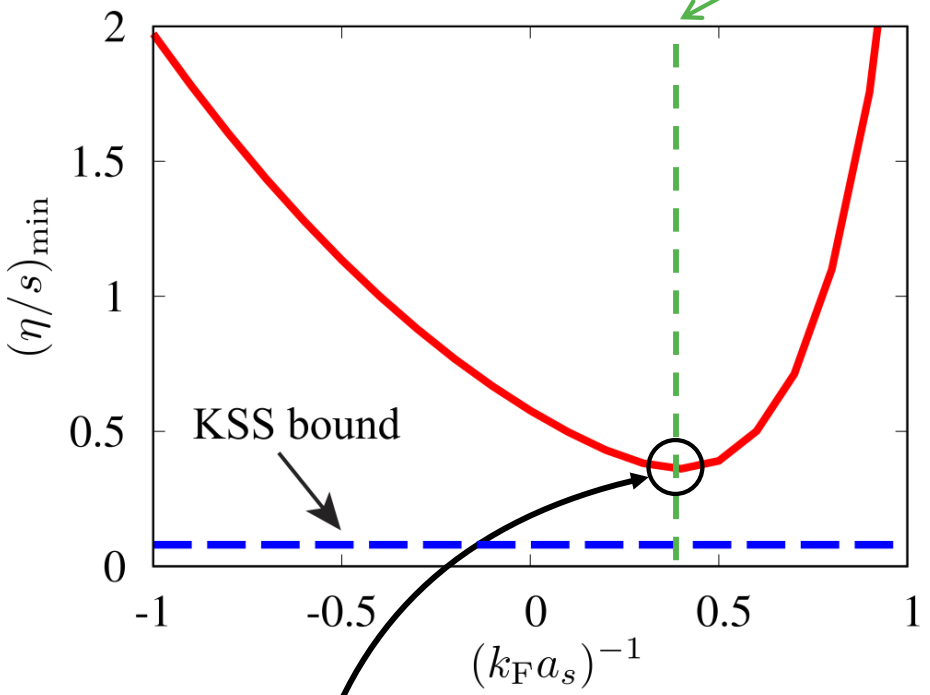
Kagamihara, Ohashi, JPSJ **89** (2020), 044005 (2020).

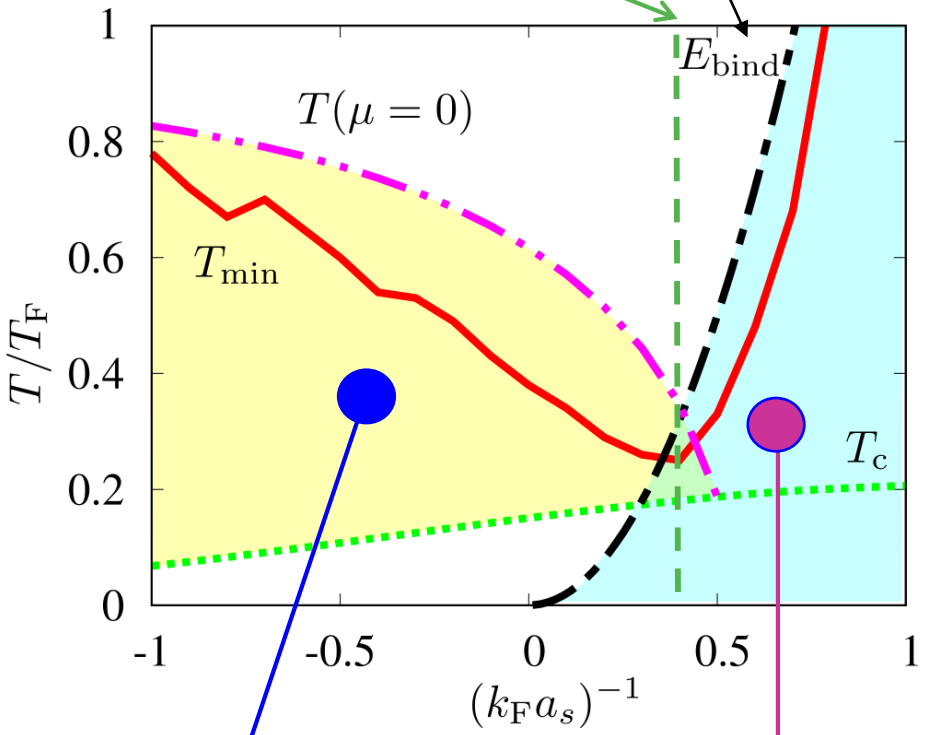
Assessment of KSS bound in cold Fermi gas physics

2-body binding energy

$$E_{bind} = \frac{1}{ma_s^2}$$

$(k_F a_s)^{-1} \simeq 0.4$



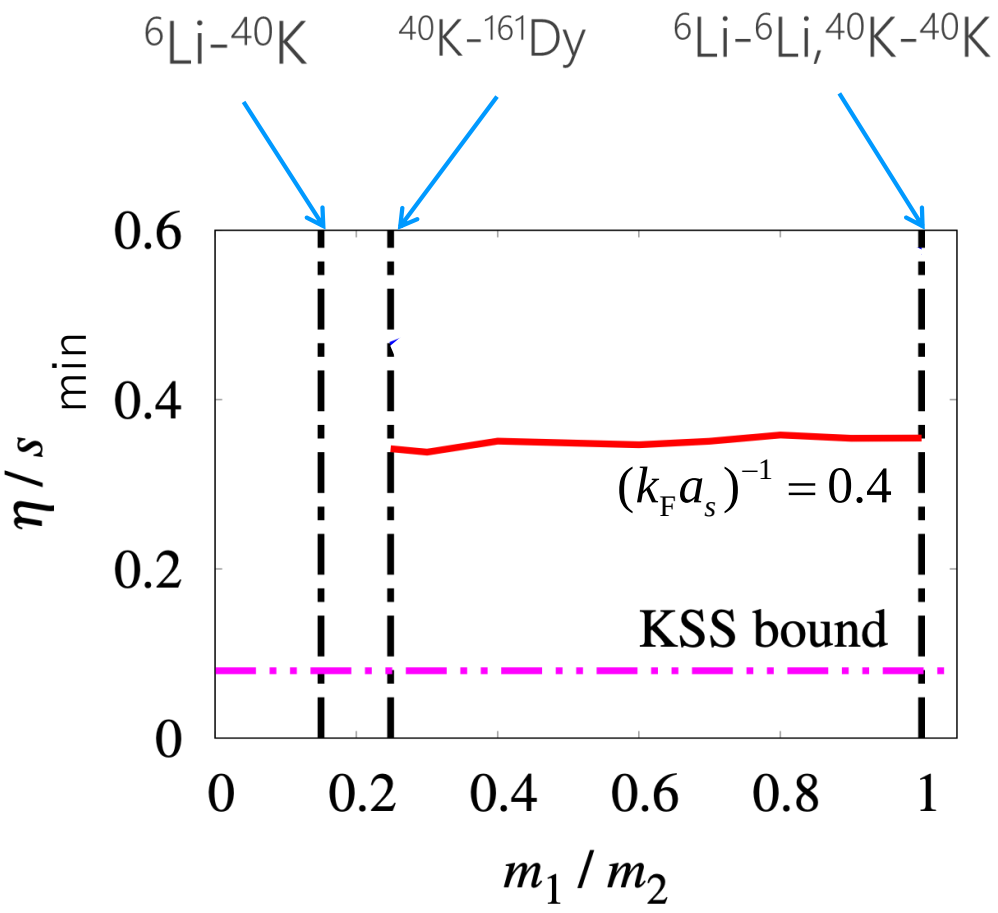


$$\left(\frac{\eta}{s}\right)_{\rm l.b.} \simeq 4.5 \times \frac{\hbar}{4\pi k_B}$$

Fermi degeneracy

Molecular gas

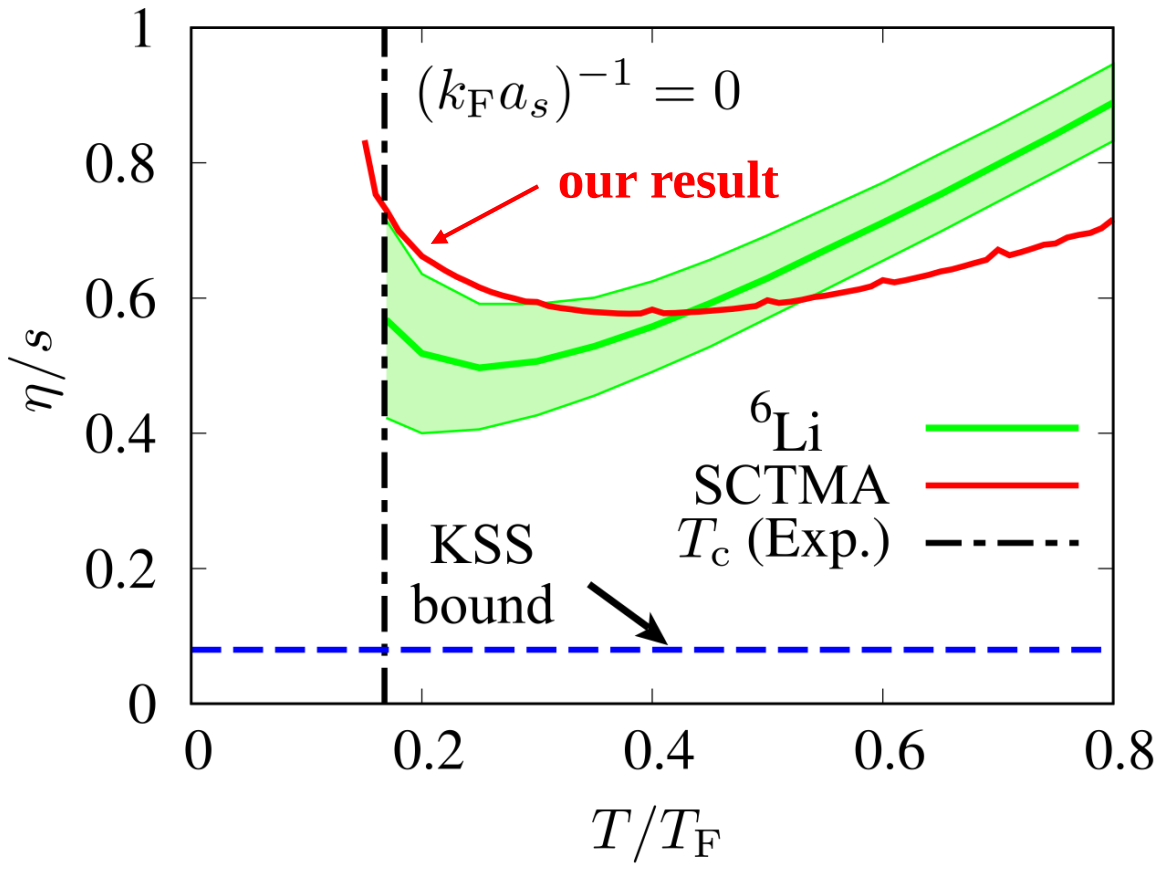
Universality of KSS bound

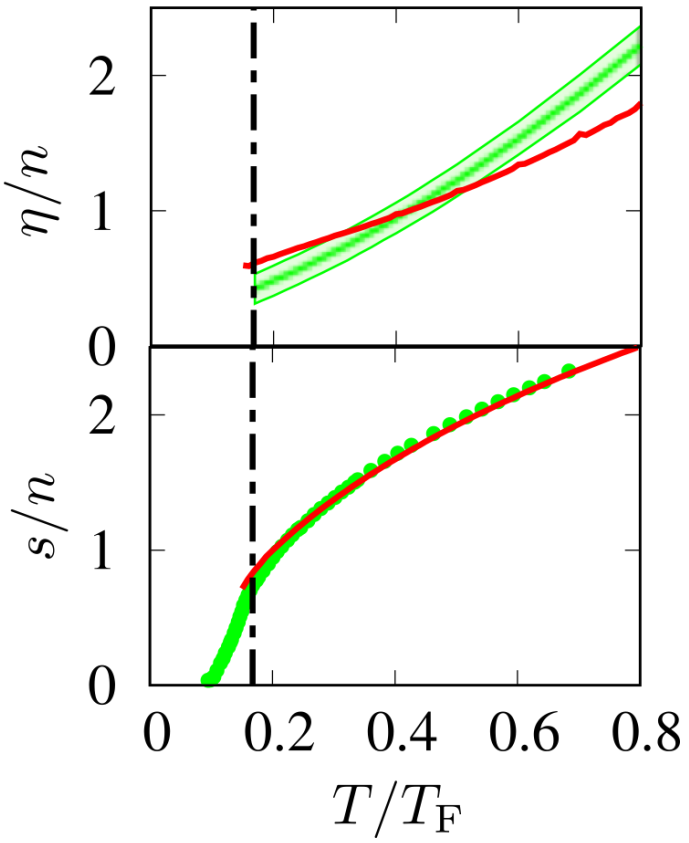


η/s always take the same minimum value at $(k_F a_s)^{-1}=0.4>0$.

$$\left(\frac{\eta}{s}\right)_{\text{l.b.}} \simeq 4.5 \times \frac{\hbar}{4\pi k_B}$$

Comparison with ^{6}Li Fermi gas experiment





^{6}Li: M. Bluhm et al. PRL **119** 065302 (2017)

Calculated η/s semi-quantitatively explains the recent experiment at the unitarity.

Summary

We have discussed the shear viscosity η in an ultracold Fermi gas. We showed that η is useful to see how the atom hierarchy changes to the molecule hierarchy. We also show that the KSS bound is obtained around the two hierarchy levels.