Strongly interacting one-dimensional fermions at finite temperature

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Outline

- Introduction
- Formalism
- Results
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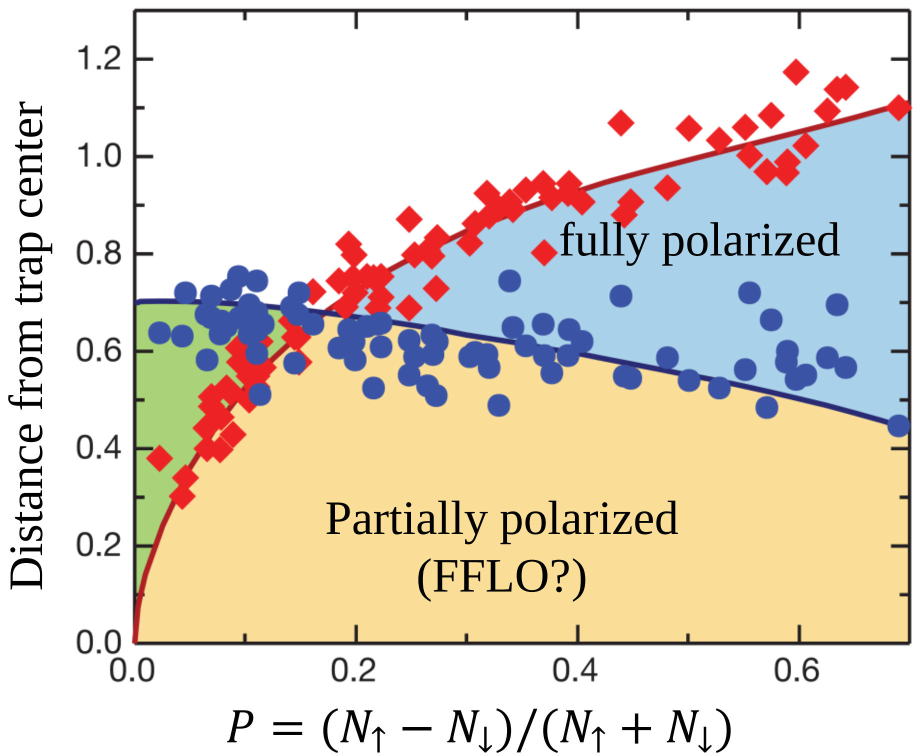
Gaudin-Yang model

One of the most fundamental model:
Two-component Fermi gas with two-body interaction in 1D

<u>Experiment for an imbalanced system</u>



Y. Liao, *et al*., Nature **467**, 567 (2010).

<u>Theoretical approaches</u>

Bethe ansatz,
Tomonaga-Luttinger liquid,
etc…

Path integral Monte Carlo method
(sign problem in an imbalanced system)

Complex Langevin method
(candidate for overcoming sign problem)

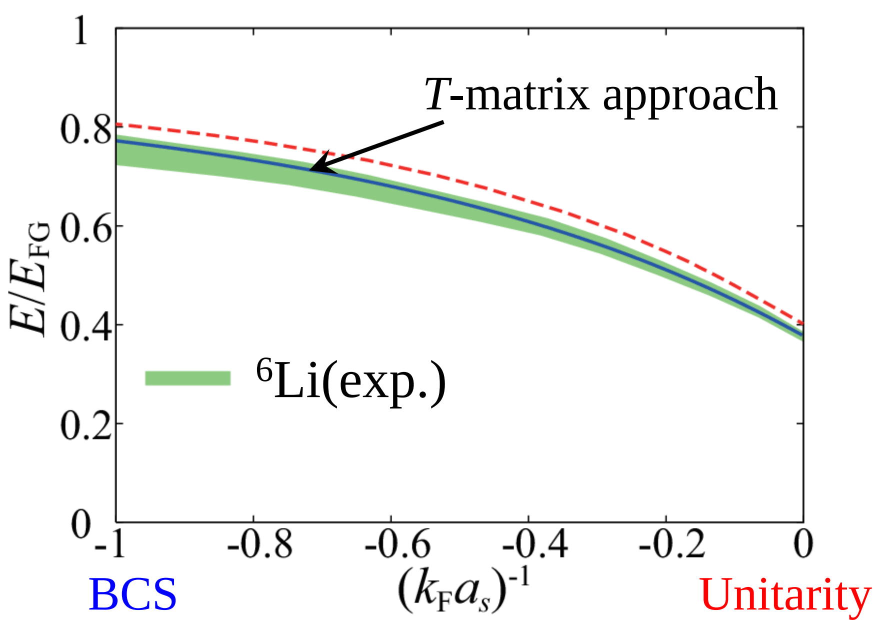
9/25 (tomorrow) See also the presentations by T. M. Doi and S. Tsutsui

Many-body *T*-matrix approach

<u>Review</u>: Y. Ohashi, <u>HT</u>, P. van Wyk, Prog. Part. Nucl. Phys. **111**, 103739 (2020).

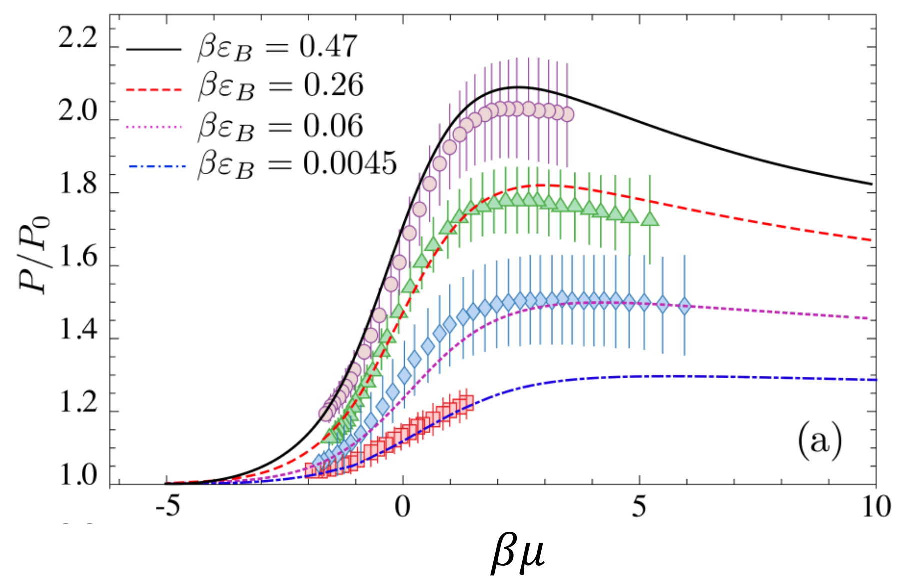
Does the many-body *T*-matrix approach work even in 1D?

▸ <u>Equation of state in a 3D Fermi gas</u>



M. Horikoshi, M. Koashi, <u>HT</u>, Y. Ohashi, and M. Kuwata-Gonokami, PRX, **7**, 041004 (2017).

▸ <u>Pressure in a 2D Fermi gas</u>



B. C. Mulkerin, K. Fenech, P. Dyke, C. J. Vale, X.-J. Liu, and H. Hu, PRA **92**, 063636 (2015).

Purpose of our study

- We theoretically investigate strongly interacting Fermi gases in one-dimension.
- We develop the many-body *T*-matrix approach and compare the results with other approaches (QMC, CL) for the case with the two-body interaction at finite temperature.

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Hamiltonian (Two-body interaction)

$$H = \sum_{p,\sigma} (\varepsilon_p - \mu_\sigma) c^\dagger_{p\sigma} c_{p\sigma} + g_2 \sum_{p,p\prime,q} c^\dagger_{p+q/2,\uparrow} c^\dagger_{-p+q/2,\downarrow} c_{-p\prime+q/2,\downarrow} c_{p\prime+q/2,\uparrow}$$

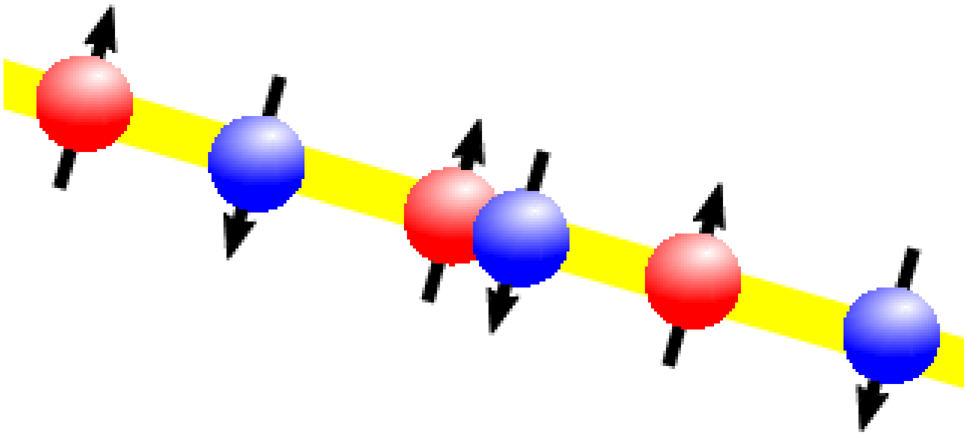
(Kinetic term) (attractive interaction)

$\varepsilon_p = \frac{p^2}{2m}$: kinetic energy

m : mass p : momentum

μ_σ : chemical potential

$\sigma = \uparrow, \downarrow$: pseudospin

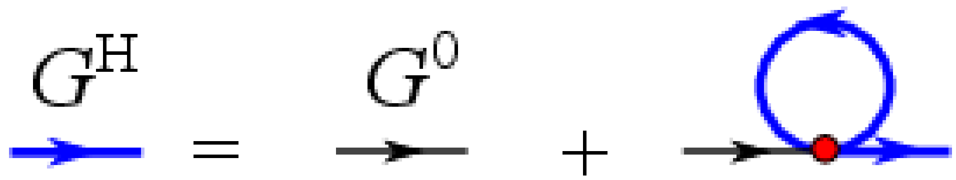


$c^{(\dagger)}_{p\sigma}$: annihilation (creation) operator of a fermion

$g_2 = -\frac{2}{ma}$: two-body coupling constant (a: 1D scattering length)

Self-consistent Hartree-Fock approximation (HF)

HT, S. Tsutsui, and T. M. Doi, Phys. Rev. Research **2**, 033441 (2020).



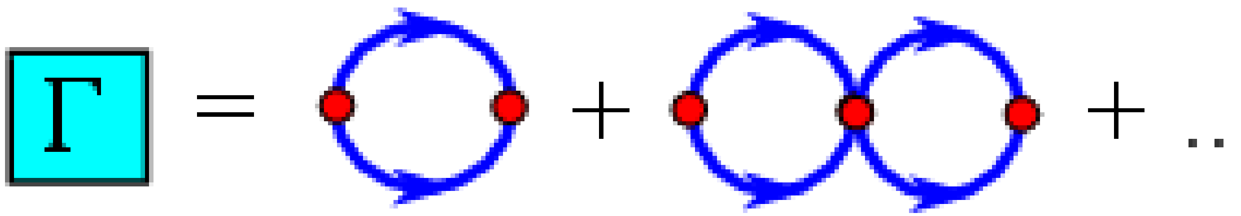
$$G_\sigma^{\mathrm{H}}(p, i\omega_n) = G_\sigma^{0}(p, i\omega_n) + G_\sigma^{0}(p, i\omega_n)\Sigma_\sigma^{\mathrm{H}} G_\sigma^{\mathrm{H}}(p, i\omega_n)$$

Bare Green's function: $G_\sigma^{0}(p, i\omega_n) = \dfrac{1}{i\omega_n - \varepsilon_p + \mu_\sigma}$

Hartree shift: $\Sigma_\sigma^{\mathrm{H}} = g N_{-\sigma}^{\mathrm{H}} = T \sum_{i\omega_n} \int \dfrac{dp}{2\pi} G_{-\sigma}^{\mathrm{H}}(p, i\omega_n)$

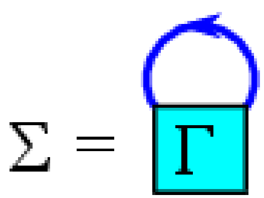
Many-body T-matrix approach

Many-body T-matrix:

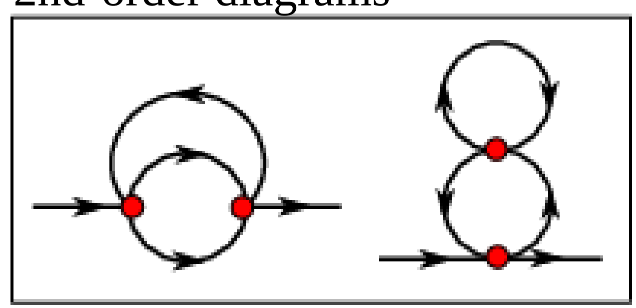


$$\Gamma(q, i\nu_\ell) = \frac{-g_2^2 \Pi(q, i\nu_\ell)}{1 + g_2 \Pi(q, i\nu_\ell)}$$

Self-energy:



2nd-order diagrams



$$\Sigma_\sigma(p, i\omega_n) = T \sum_{i\nu_\ell} \int \frac{dq}{2\pi} \Gamma(q, i\nu_\ell) G^{\mathrm{H}}_{-\sigma}(q - p, i\nu_\ell - i\omega_n)$$

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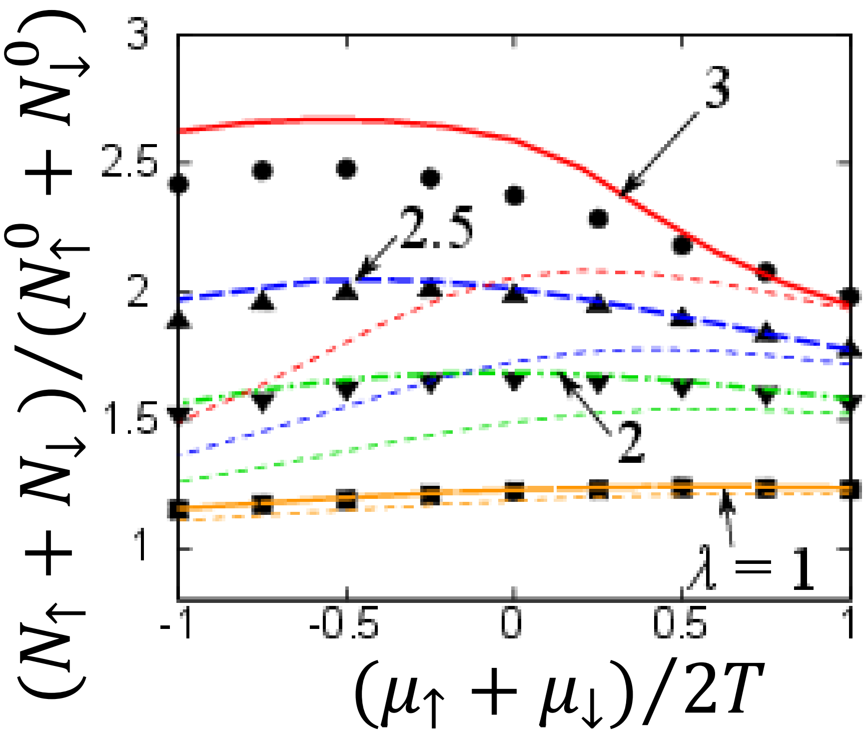
Number densities for unpolarized case

HT, S. Tsutsui, and T. M. Doi, Phys. Rev. Research **2**, 033441 (2020).

Comparison between TMA and QMC



We consider an equal mass case ($m_\uparrow = m_\downarrow$)

$$N_\sigma^0 = \int \frac{dp}{2\pi} \frac{1}{e^{\beta(p^2/2m - \mu_\sigma)} + 1}$$

: non-interacting counterpart

Dimensionless coupling parameter

$$\lambda = \frac{mg^2}{T} \equiv \frac{4E_\mathrm{b}}{T}$$

E_b: two-body binding energy

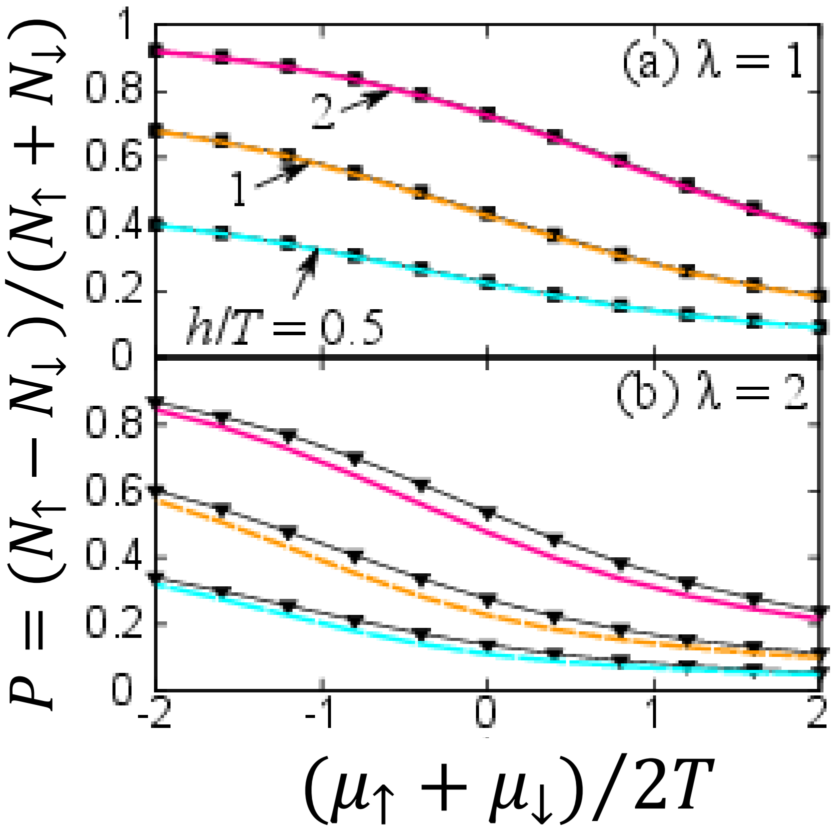
Dotted lines: self-consistent Hartree-Fock approximation

1D QMC (black symbols): M. Hoffmann, *et al.*, Phys. Rev. A **91**, 033618 (2015).

Spin polarization

HT, S. Tsutsui, and T. M. Doi, Phys. Rev. Research **2**, 033441 (2020).

Comparison between TMA and CLM



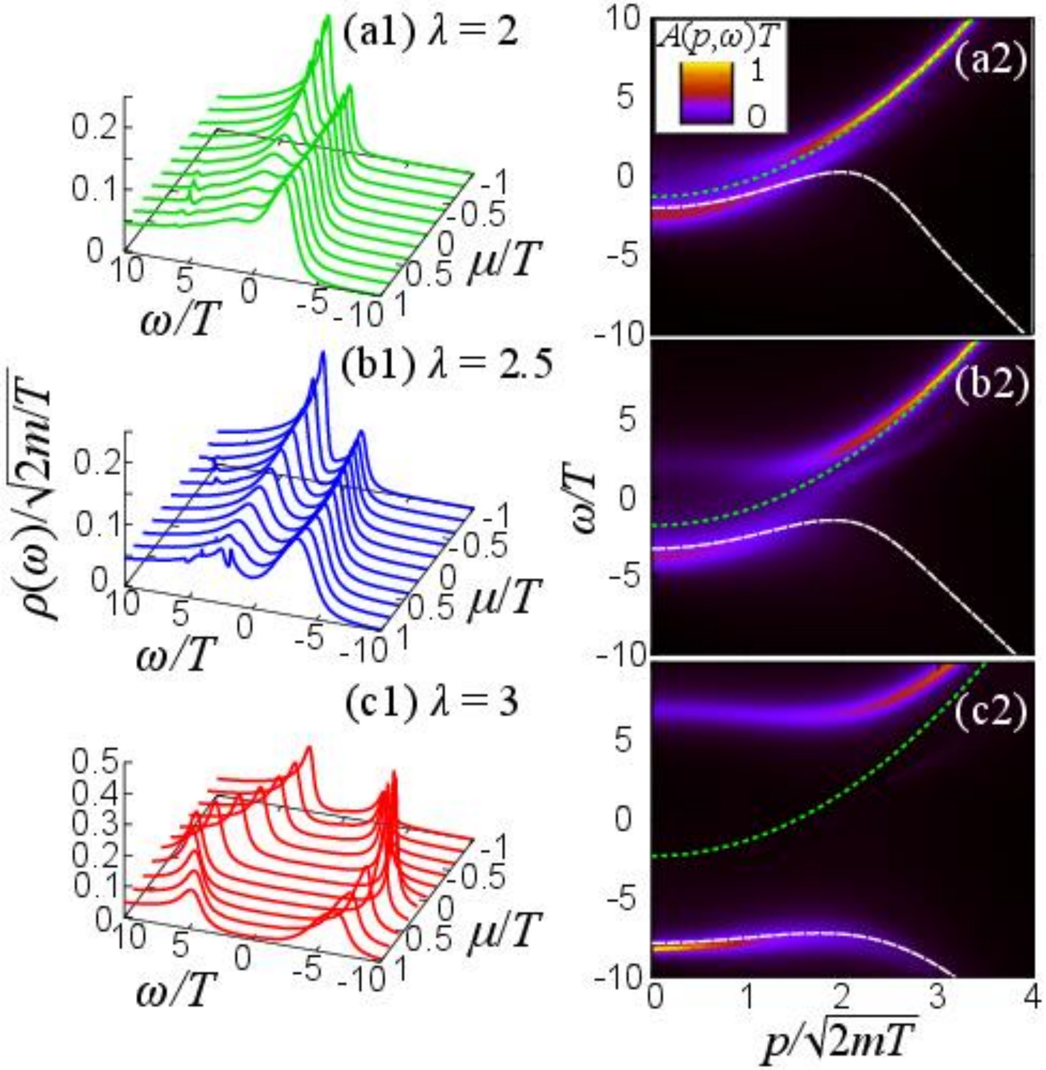
Our results of TMA show a good agreement with the previous results of complex Langevin method (CLM)

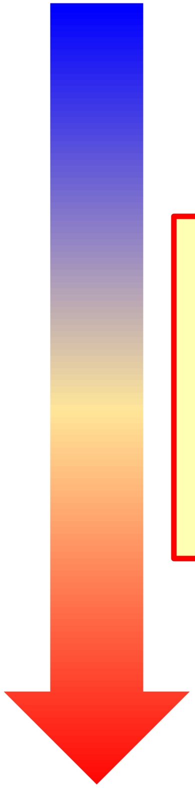
$$h = \frac{\mu_\uparrow - \mu_\downarrow}{2}$$:Fictitious magnetic field

1D complex Lagevin method (CLM, black symbols):
A. C. Loheac, *et al.*, PRD **98**, 054507 (2018).

Pseudogap effects in spectral functions

HT, S. Tsutsui, and T. M. Doi, Phys. Rev. Research **2**, 033441 (2020).

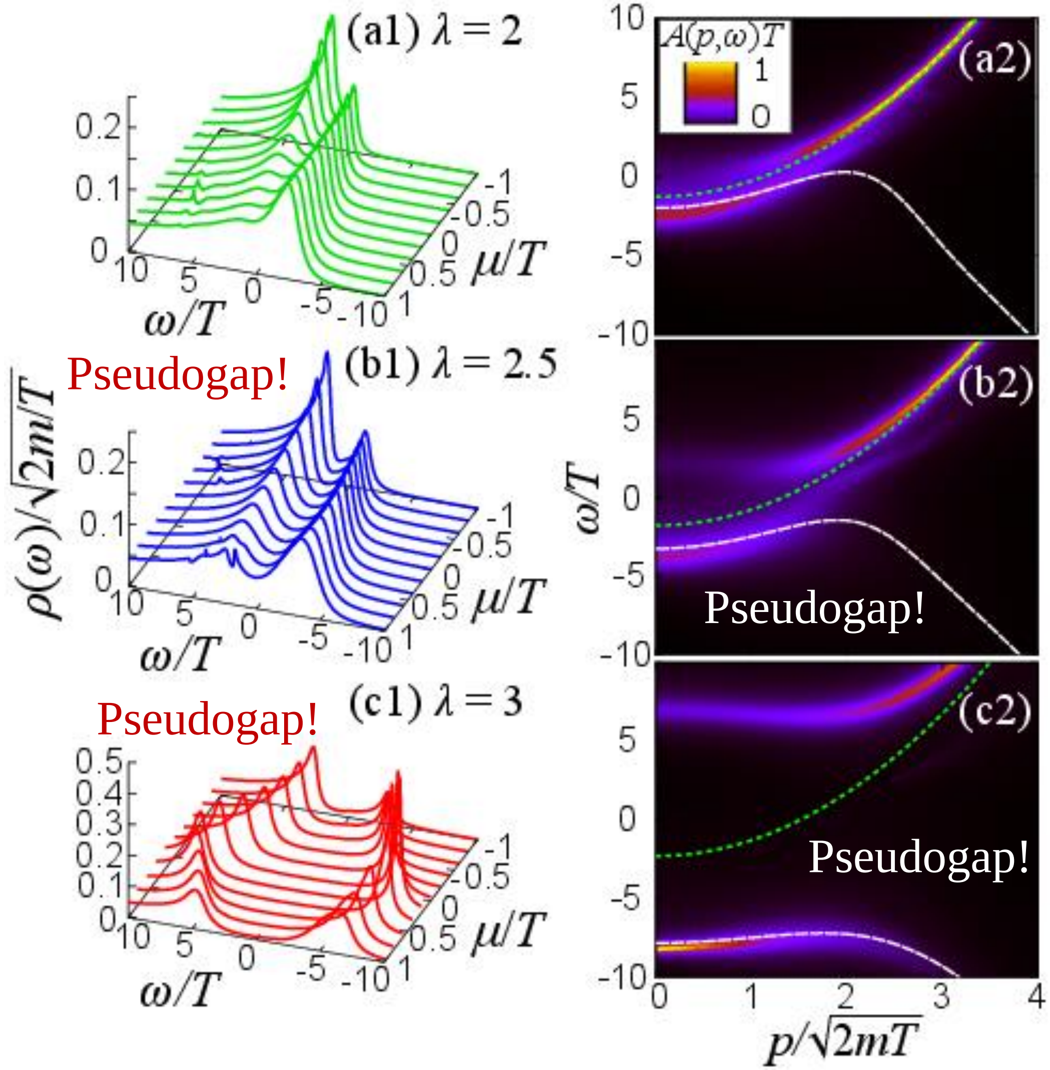




We found that the crossover from the quasiparticle state to the pseudogap state occurs even in 1D

Pseudogap effects in spectral functions

HT, S. Tsutsui, and T. M. Doi, Phys. Rev. Research **2**, 033441 (2020).



Weak coupling

Strong coupling

We found that the crossover from the quasiparticle state to the pseudogap state occurs even in 1D

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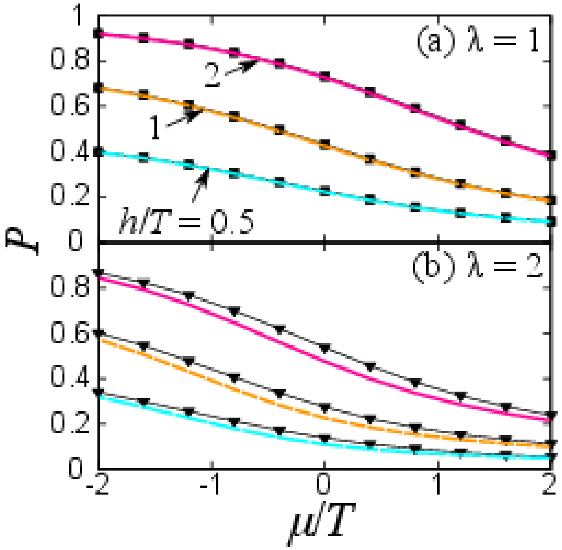
Summary

1. We investigate non-relativistic multi-component Fermi gases with attractive interactions in one-dimension.
2. We developed a many-body diagrammatic theory based on the in-medium two-body *T*-matrices. Our results well reproduce QMC and CL in the case of the two-body coupling.
3. The pseudogap effects are found.

HT, S. Tsutsui, and T. M. Doi, Phys. Rev. Research **2**, 033441 (2020).



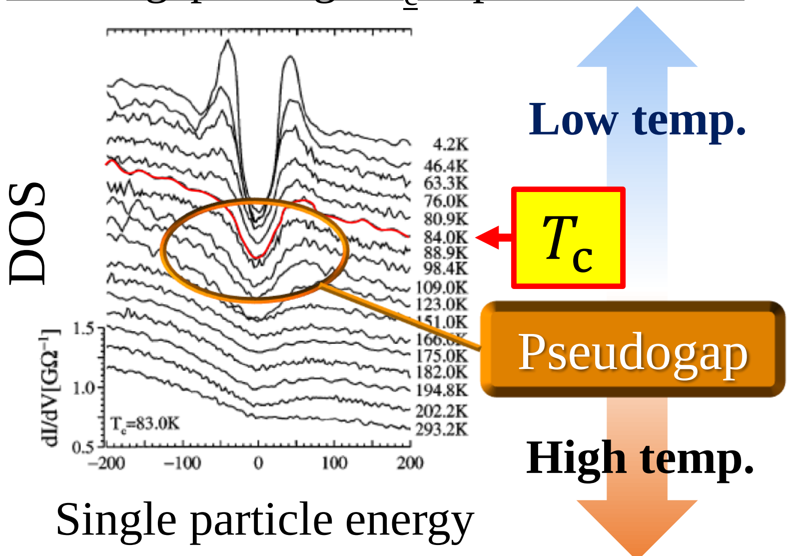
Future work:
higher dimensions,
mass-imbalance,
application to other systems
repulsive case, etc…

Appendix

Pseudogap effects associated with strong pairing fluctuations

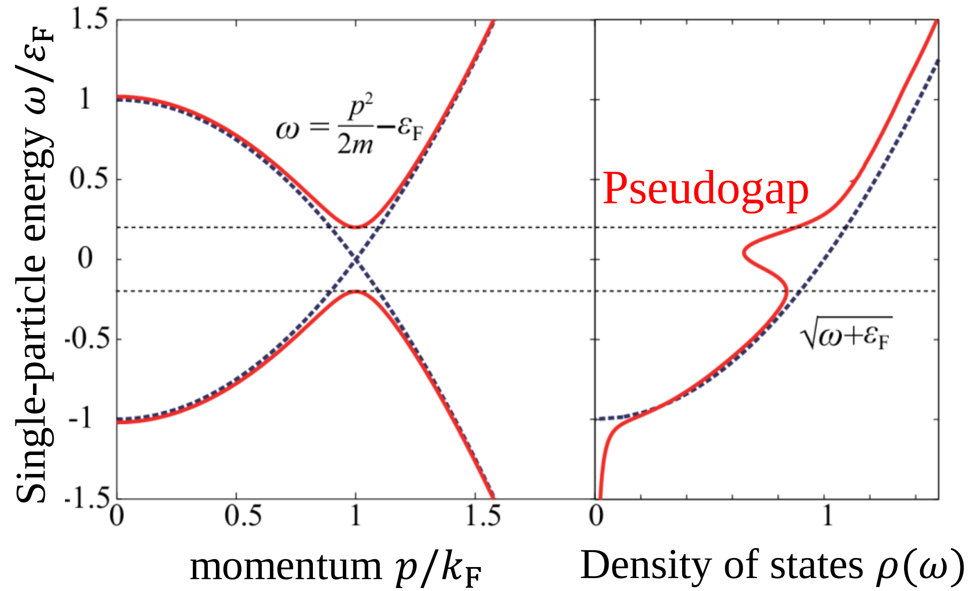
Review: Y. Ohashi, HT, P. van Wyk, Prog. Part. Nucl. Phys. **111**, 103739 (2020).

Pseudogap in high-T_c superconductors

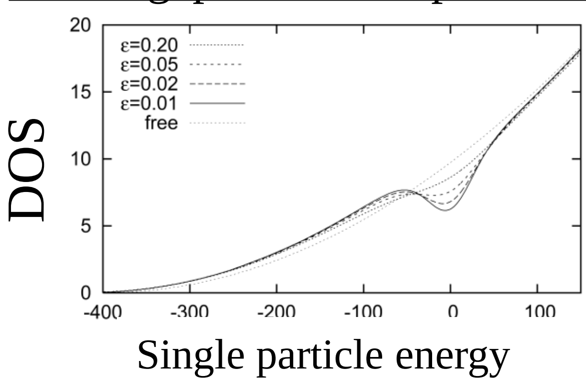


Ch. Renner, *et al.*, PRL **80**, 149 (1998).

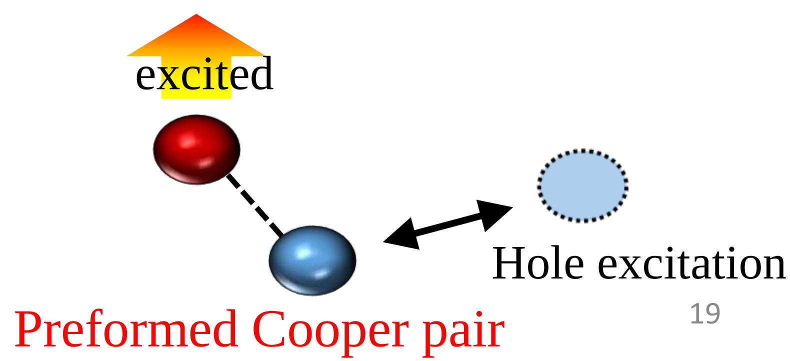
Single-particle dispersion and density of states



Pseudogap in color superconductors (NJL)



M. Kitazawa, *et al.*, PRD **70**, 056003 (2004).



Physical quantities in T-matrix approach

Number density

$$N_\sigma(\beta,\mu_\uparrow,\mu_\downarrow) = T\sum_{i\omega_n}\int\frac{d^d\boldsymbol{p}}{(2\pi)^d}G_\sigma(\boldsymbol{p},i\omega_n)e^{i\omega 0_+}$$

Single-particle spectral function

$$A_\sigma(\boldsymbol{p},\omega) = -\frac{1}{\pi}\mathrm{Im}G_\sigma(\boldsymbol{p},i\omega_n\to\omega+i\delta)$$

*For the analytic continuation we use the Pade approximation

Single-particle density of states (DOS)

$$\rho_\sigma(\omega) = \int\frac{d^d\boldsymbol{p}}{(2\pi)^d}A_\sigma(\boldsymbol{p},\omega)$$

1D Fermi Polaron energy

Comparison among TMA, CLM and TBA

Thermodynamic Bethe Ansatz (TBA)

$$U_{\mathrm{TBA}} = -\frac{2E_{F,\uparrow}}{\pi}\left[\frac{1}{k_{F,\uparrow}a} + \tan^{-1}\left(\frac{1}{k_{F,\uparrow}a}\right) - \left(\frac{1}{k_{F,\uparrow}a}\right)\left(\frac{\pi}{2} - \tan^{-1}\left(\frac{1}{k_{F,\uparrow}a}\right)\right)\right]$$

Exact solution at $T = 0, N_\downarrow/N_\uparrow \to 0$

J. B. Mcguire, J. Math. Phys. (N.Y.) 7, 123 (1966).

$E_{\mathrm{F},\uparrow} = \frac{k_{\mathrm{F},\uparrow}^2}{2m}$: Fermi energy of majority component

$k_{\mathrm{F},\uparrow} = \pi N_\uparrow$: Fermi momentum

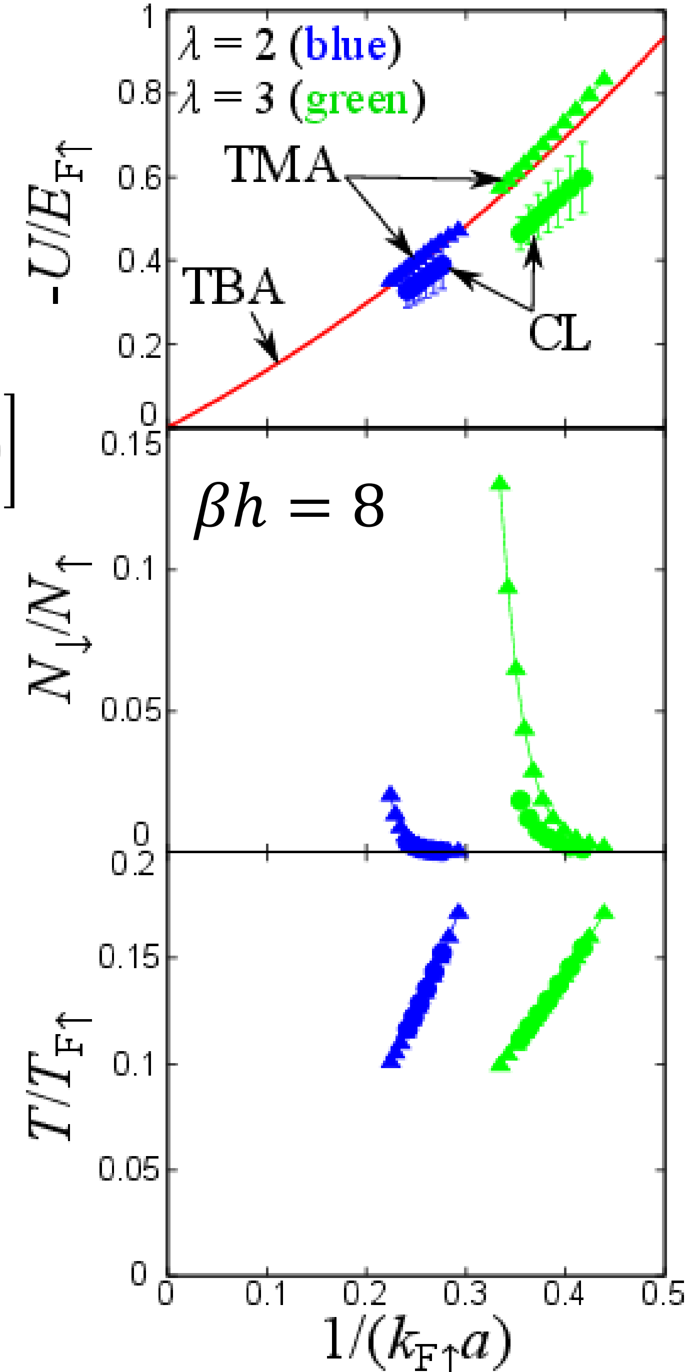
$a = -\frac{2}{mg}$: 1D scattering length

Three results almost agree with each other!

Slight deviations in the strong-coupling side.

☐ Lattice artifact?

☐ Finite density/temperature effects?



Limitation of *T*-matrix approach