Effective model approach to the NΩ interaction

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in collaboration with

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[1] T. S. , Y. Kamiya and T. Hyodo, Phys. Rev. C98 (2018) 015205.
[2] T. S. , Y. Kamiya and T. Hyodo, in preparation.

1. Introduction

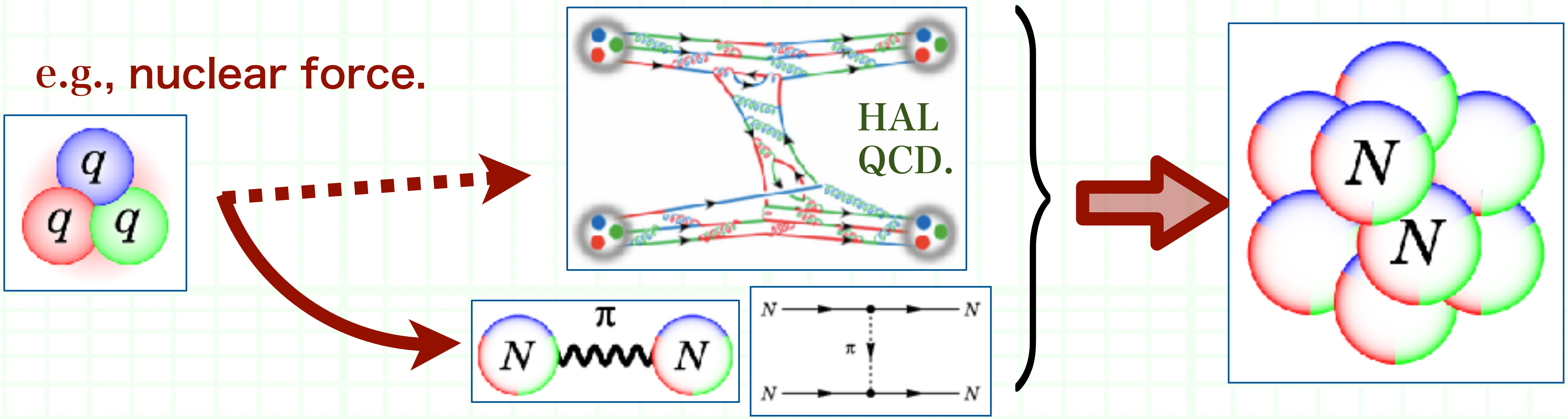
1. Introduction

++ Hadron-hadron interactions ++

■ We are interested in the hadron-hadron interactions, which are governed by QCD (quantum chromodynamics).

□ It belongs to low-energy phenomena, in which non-perturbative nature of QCD becomes significant.

e.g., nuclear force.



□ Traditional approaches to hadron-hadron interaction:

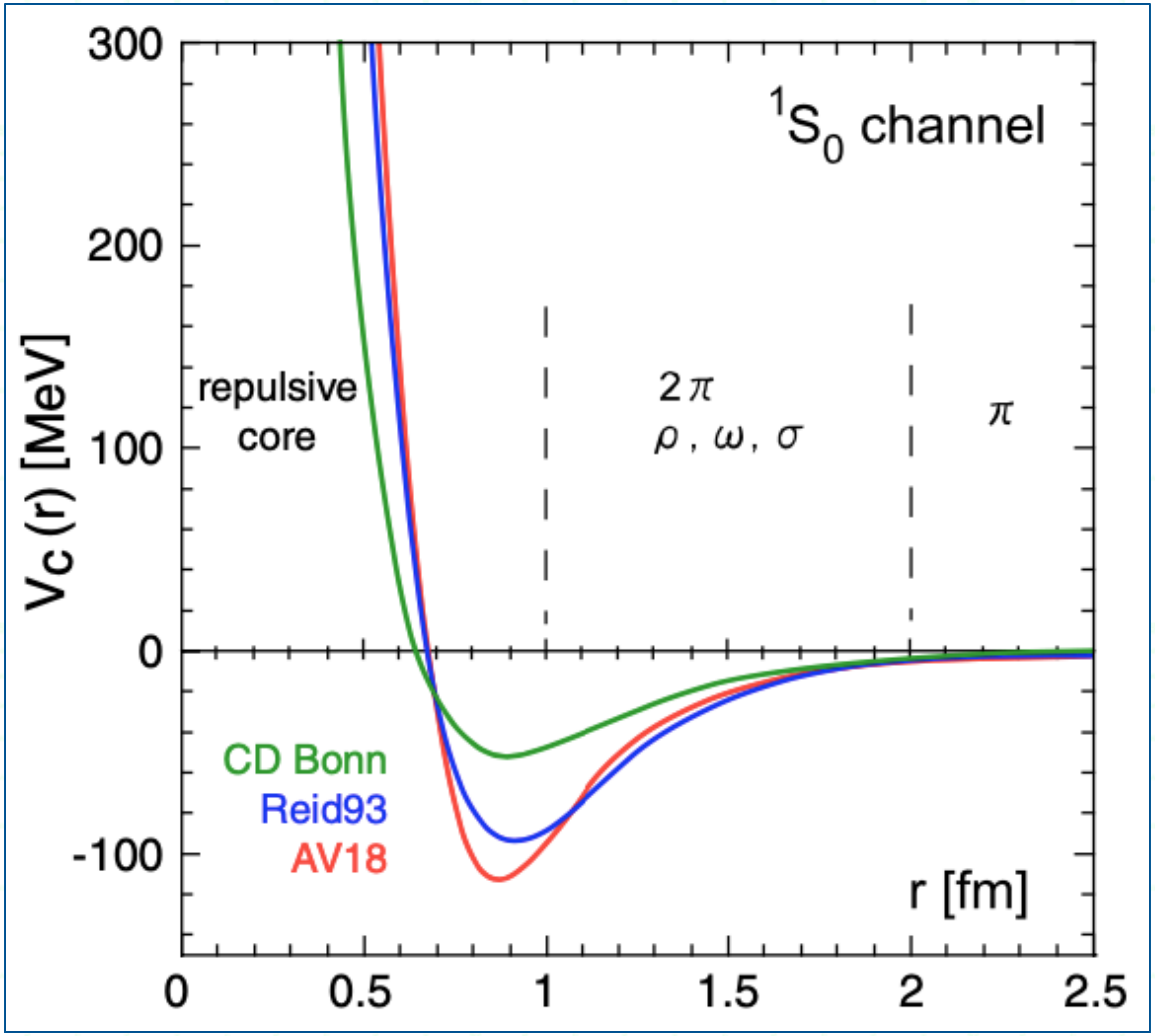
– Perform hadron-hadron scattering experiments.

– Construct phenomenological models.

1. Introduction

++ Nuclear force ++

■ For example, the nuclear force has been investigated as:



– They are constructed phenomenologically so as to reproduce the experimental NN data.

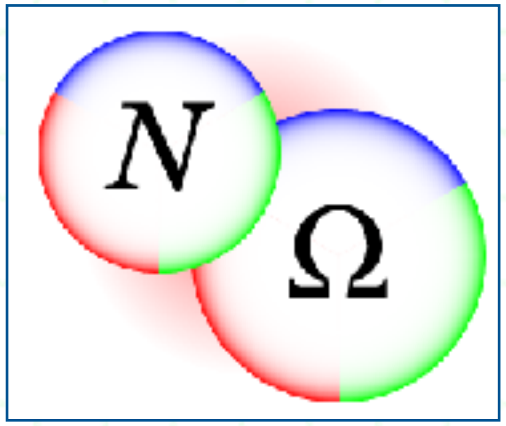
Ishii, Aoki, and Hatsuda, Phys. Rev. Lett. 99 (2007) 022001.

1. Introduction

++ Why NΩ interaction ? ++

■ Expected feature of the NΩ interaction.

□ Combination of N(uud / udd) [octet] + Ω(sss) [decuplet].
No same flavor. –> No repulsive core !?



□ Calculations in quark models.

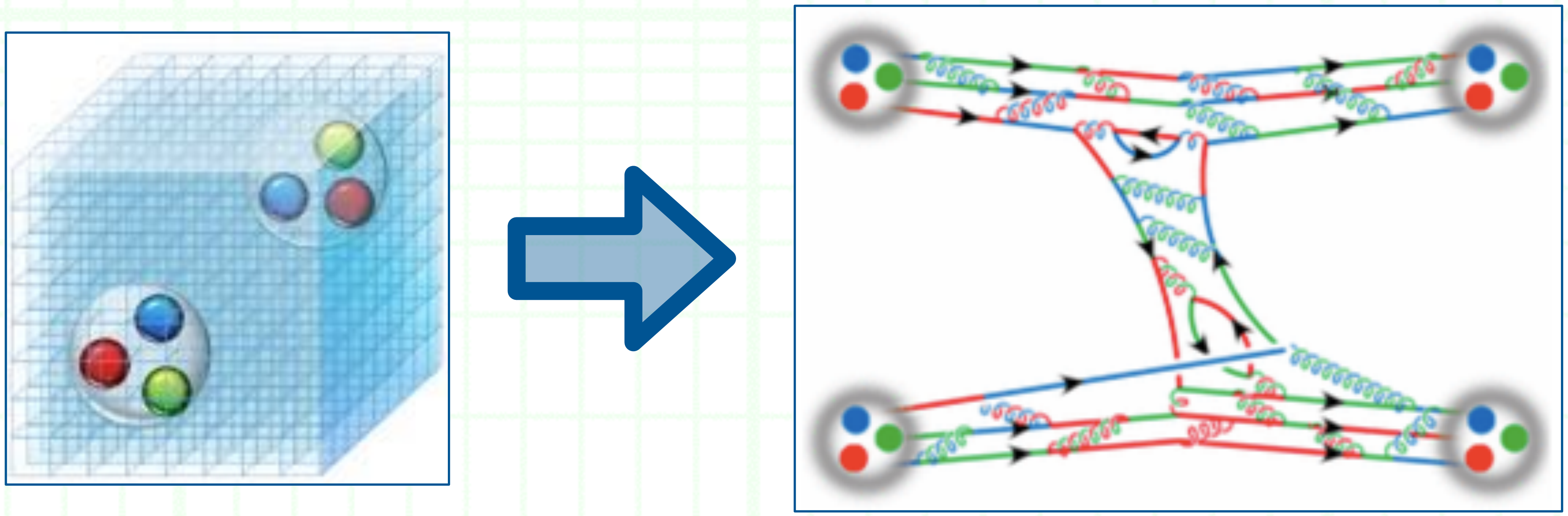
Goldman et al., Phys. Rev. Lett. 59 (1987) 627;
Oka, Phys. Rev. D38 (1988) 298;
Li and Shen, Eur. Phys. J. A8 (2000) 417;
Pang, Ping, Wang, Goldman and Zhao, Phys. Rev. C69 (2004) 065207;
Zhu, Huang, Ping and F. Wang, Phys. Rev. C92 (2015) 035210;
Huang, Ping and Wang, Phys. Rev. C92 (2015) 065202; ...

– Although the details are different, these calculations indicate the existence of the NΩ bound state.

1. Introduction

++ Recent breakthrough 1 ++

■ A way to calculate hadron-hadron interactions directly in lattice QCD simulation by the HAL QCD collaboration.



HAL QCD.

□ Measuring the correlation function of two-hadrons, they obtained hadron-hadron interactions from QCD.

$$\Psi_{\alpha\beta}(\boldsymbol{r},t) = \langle 0|n_\beta(\boldsymbol{y},t)p_\alpha(\boldsymbol{x},t)|B=2;W,\boldsymbol{P}=0\rangle \equiv \psi_{\alpha\beta}(\boldsymbol{r})e^{-iWt},$$

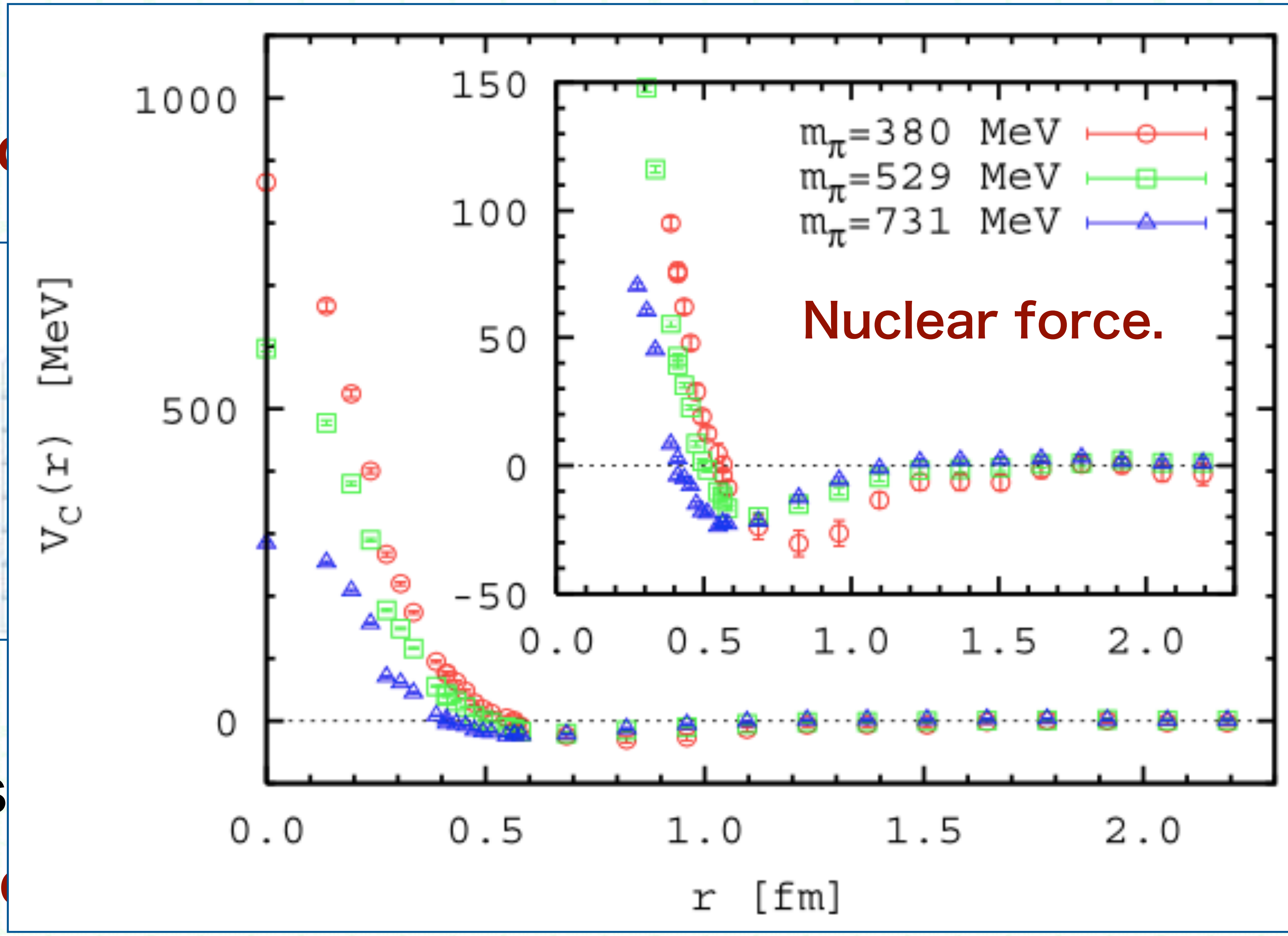
Aoki, Hatsuda, and Ishii, Prog. Theor. Phys. 123 (2010) 89.

1. Introduction

++ Recent breakthrough 1 ++

■ A way … directly in lattic… …oration.

HAL QCD.



□ Meas… …drons, they … …m QCD.

$$\Psi_{\alpha\beta}(\boldsymbol{r},t) = \langle 0|n_\beta(\boldsymbol{y},t)p_\alpha(\boldsymbol{x},t)|B=2;W,\boldsymbol{P}=0\rangle \equiv \psi_{\alpha\beta}(\boldsymbol{r})e^{-iWt},$$

Aoki, Hatsuda, and Ishii, Prog. Theor. Phys. 123 (2010) 89.

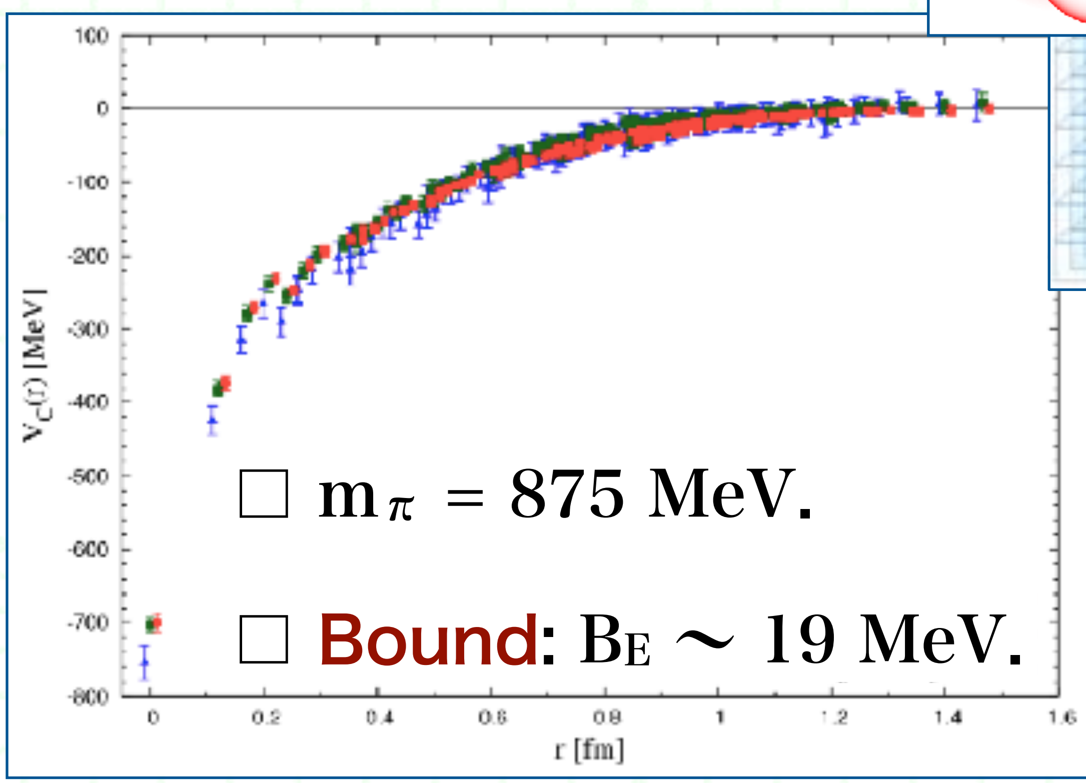
1. Introduction

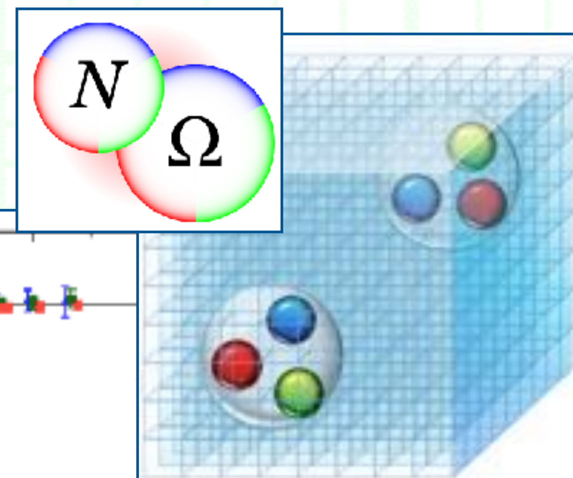
++ The NΩ system from lattice QCD ++

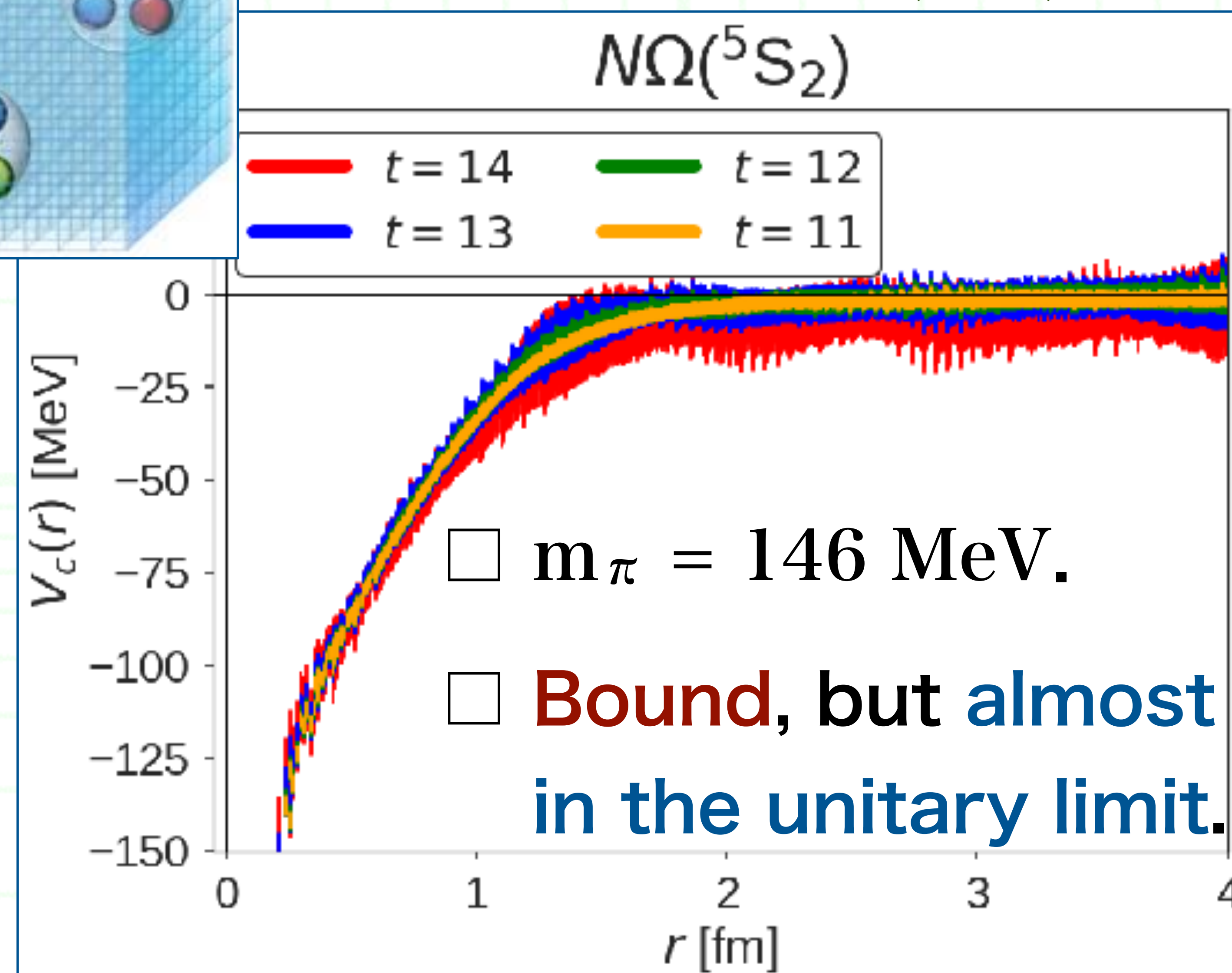
■ A way to calculate hadron-hadron interactions directly in lattice QCD simulation by the HAL QCD collaboration.

Etminan et al. [HAL QCD], Nucl. Phys. A928 (2014) 89.

Doi et al. [HAL QCD], EPJ Web of Conf. 175 (2018) 05009.







□ m_π = 875 MeV.

□ Bound: B_E ∼ 19 MeV.

□ m_π = 146 MeV.

□ Bound, but almost in the unitary limit.

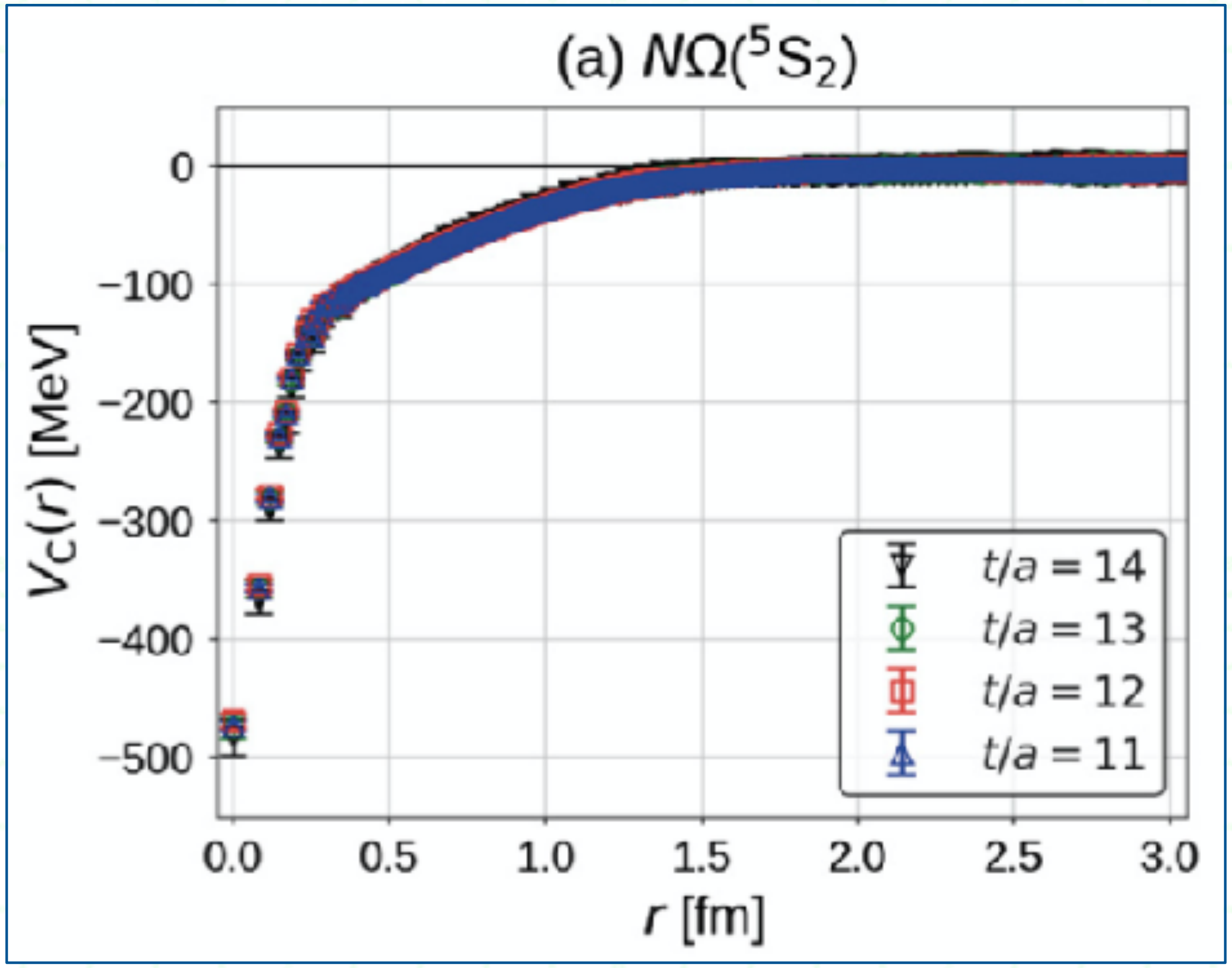
□ No repulsive core !

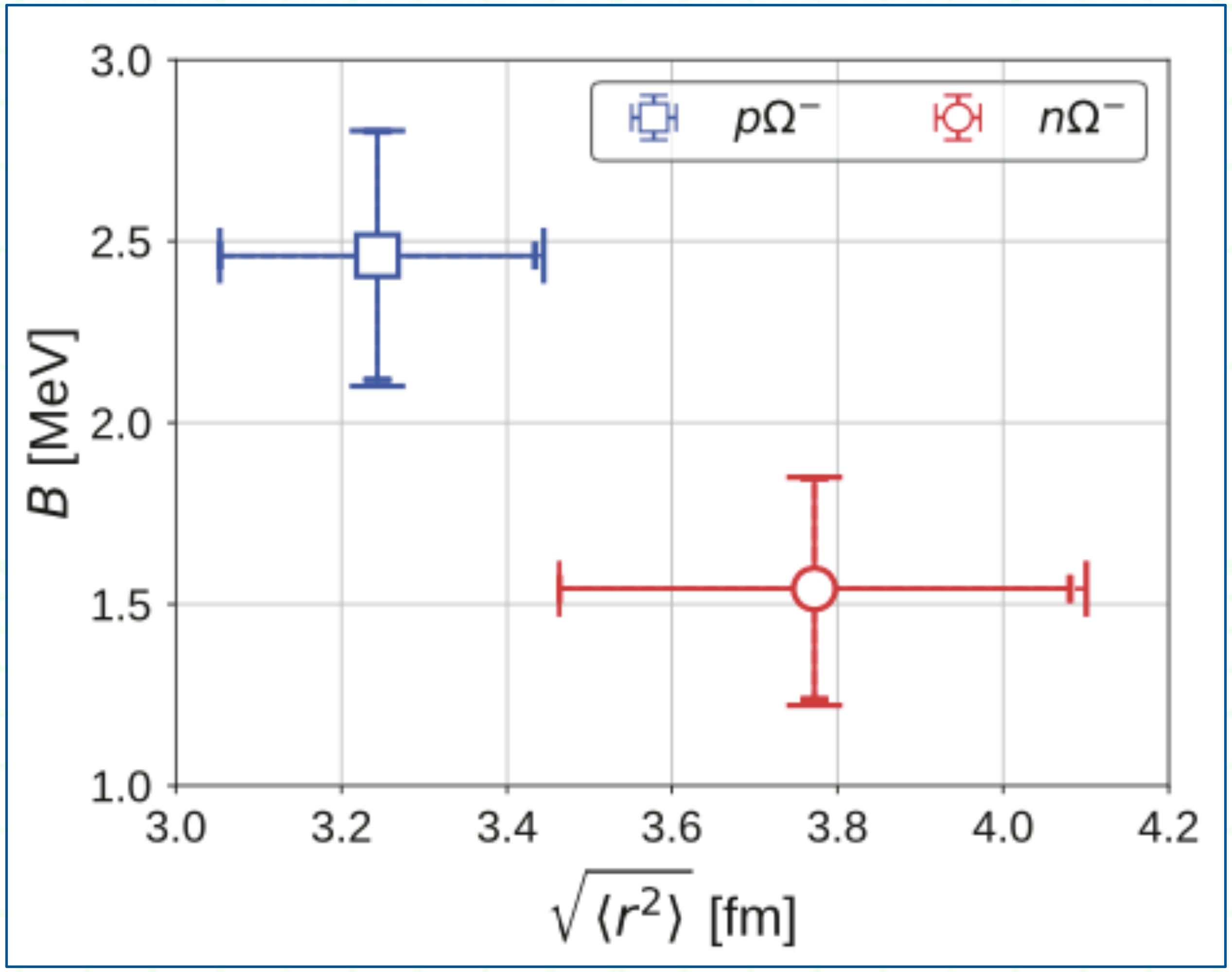
□ Implying NΩ is bound.

1. Introduction

++ The NΩ system from lattice QCD ++

- A way to calculate hadron-hadron interactions directly in lattice QCD simulation by the HAL QCD collaboration.
  - □ NΩ system. – m_Ω = 1712 MeV.



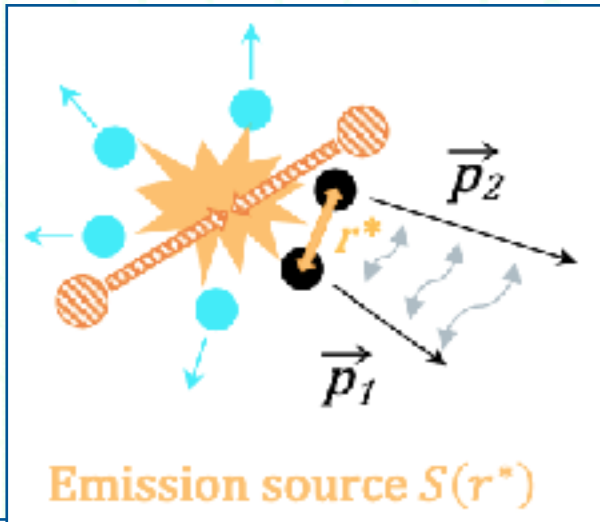


Iritani et al. [HAL QCD], Phys. Lett. B792 (2019) 284.

1. Introduction

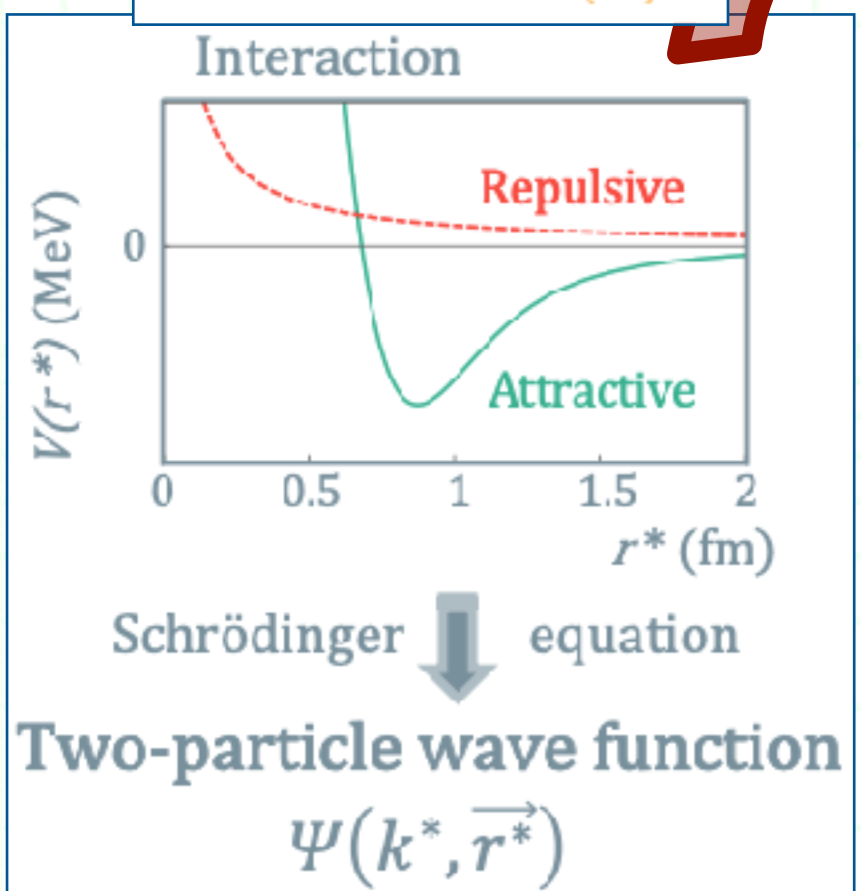
++ Recent breakthrough 2 ++

■ Hadron-hadron correlations in the pp collisions (7 - 13 TeV).

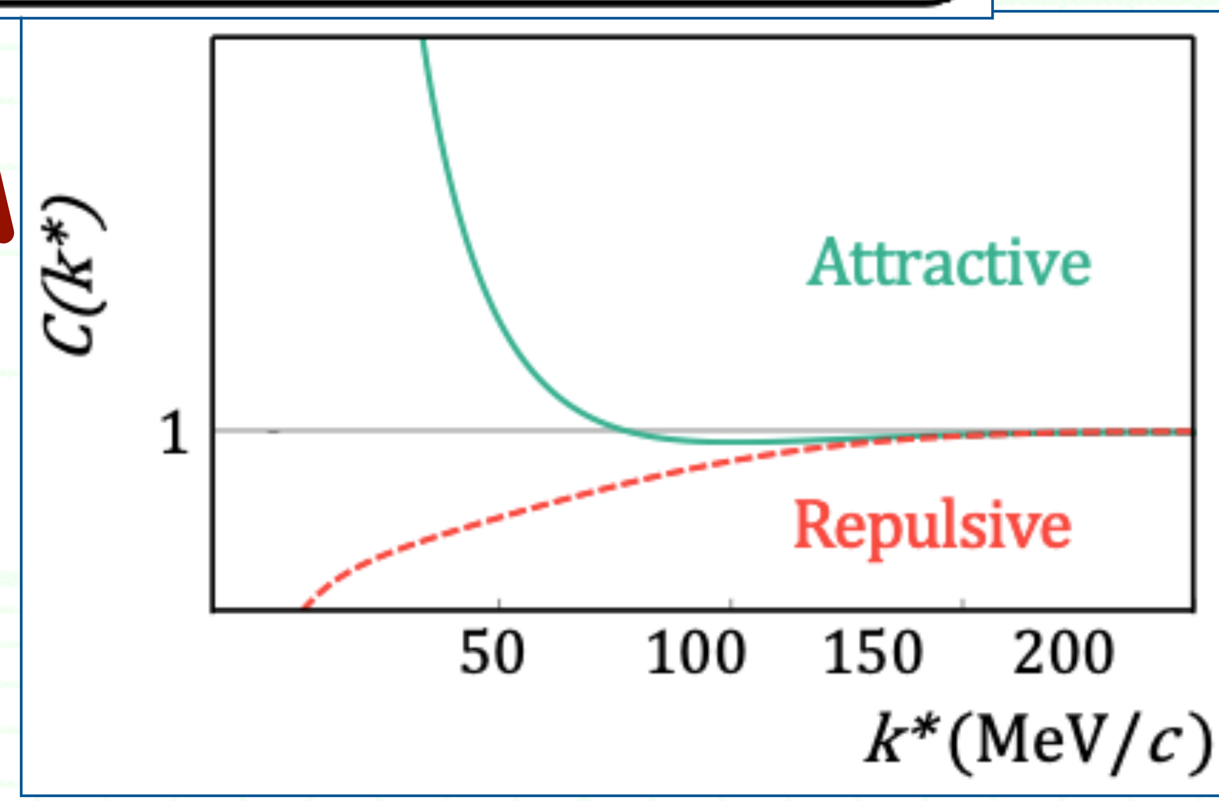


Correlation Function

$$C(k^*) = \int S(r^*) \left|\Psi(k^*, \vec{r^*})\right|^2 d^3r^* = \xi(k^*) \otimes \frac{N_{\mathrm{same}}(k^*)}{N_{\mathrm{mixed}}(k^*)}$$



ALICE, arXiv:2005.11495 (submitted to Nature).



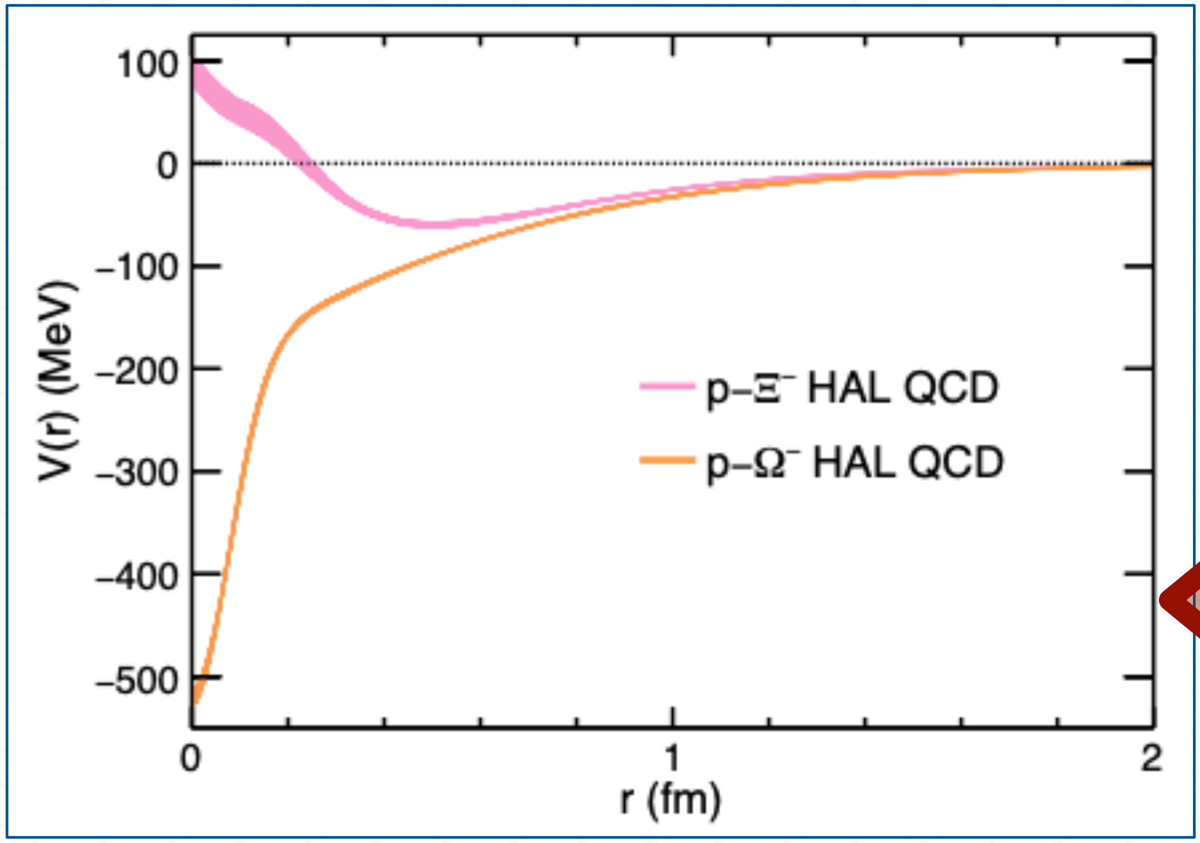
□ Correlation depends on the source size and interaction.

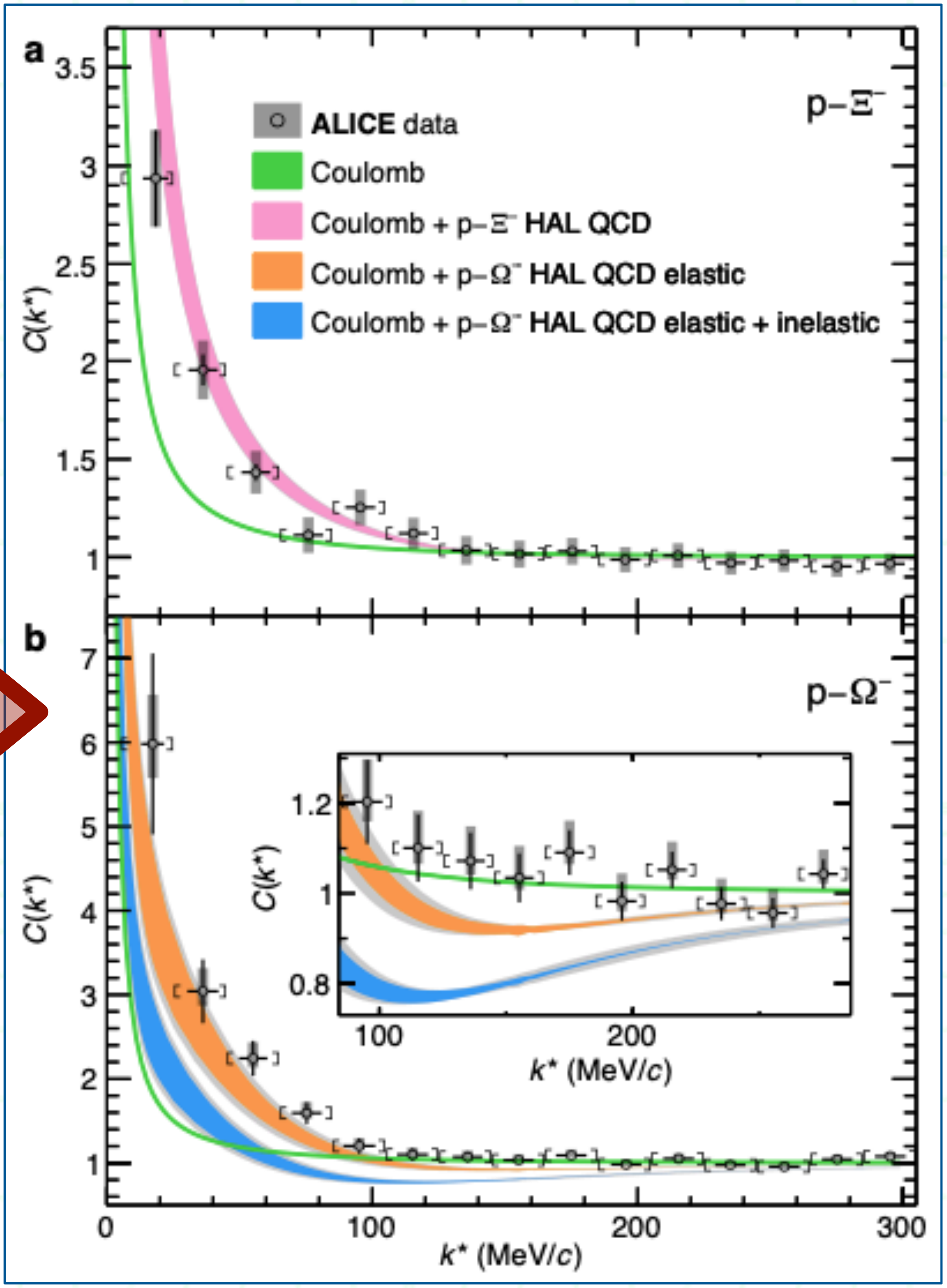
– The size can be fixed in another way.

1. Introduction

++ Recent breakthrough 2 ++

■ Hadron-hadron correlations in the pp collisions.





– Exp. data are consistent with the HAL QCD potentials !

ALICE, arXiv:2005.11495.

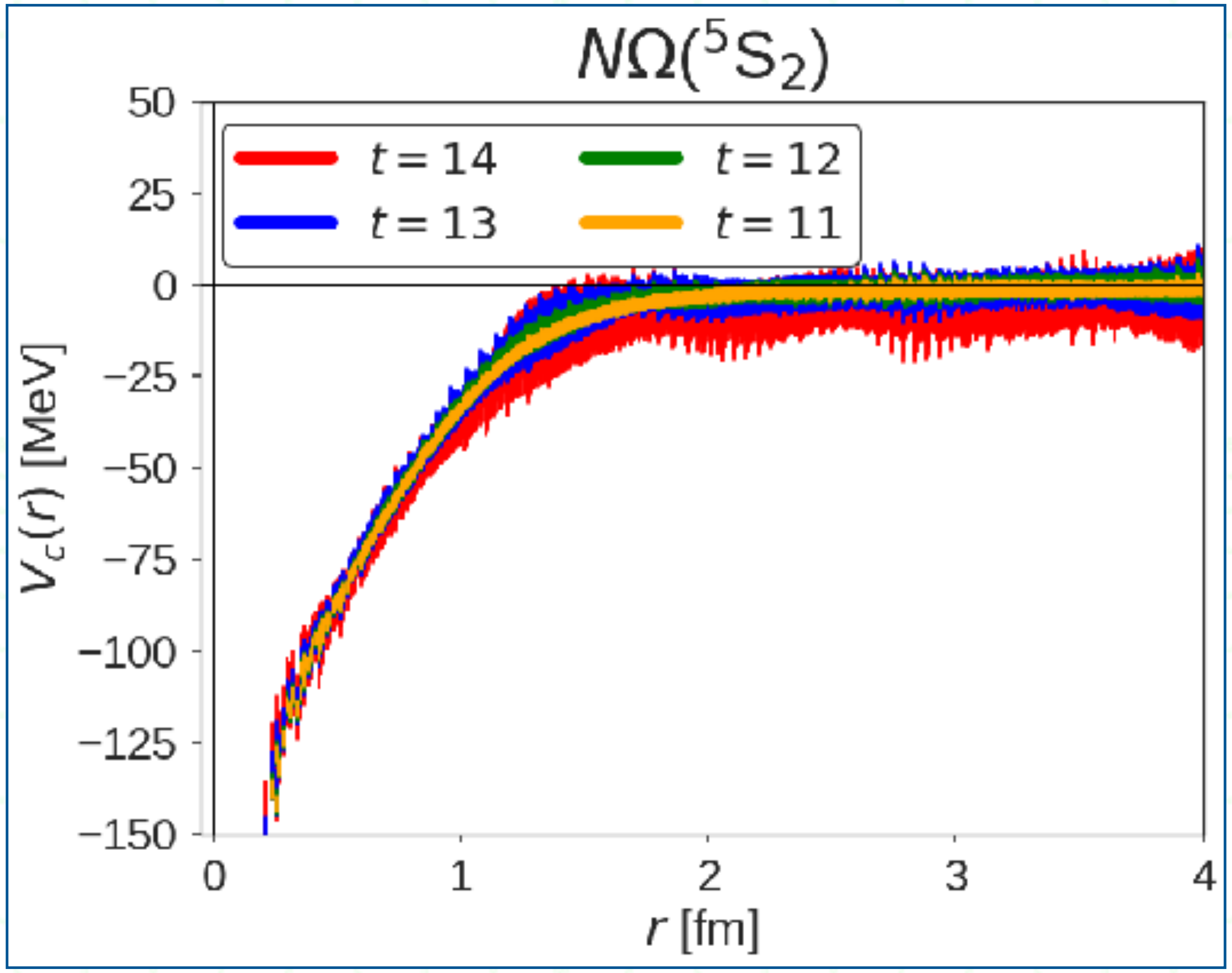
2. Meson exchange model for the NΩ interaction, version 2018

2. Model for NΩ, ver.2018

++ Motivation ++

- We want to understand the NΩ ($^5S_2$) interaction.
  - ☐ What is the origin of the attraction ?
  - – Physics behind it.
  - ☐ Extrapolate quark masses:
    m_π = 146 MeV to 138 MeV.
  - ☐ Discuss decay modes.
- We construct baryon-baryon interaction model with meson exchanges.

Doi et al. [HAL QCD],
EPJ Web of Conf. 175 (2018) 05009.



2. Model for NΩ, ver.2018

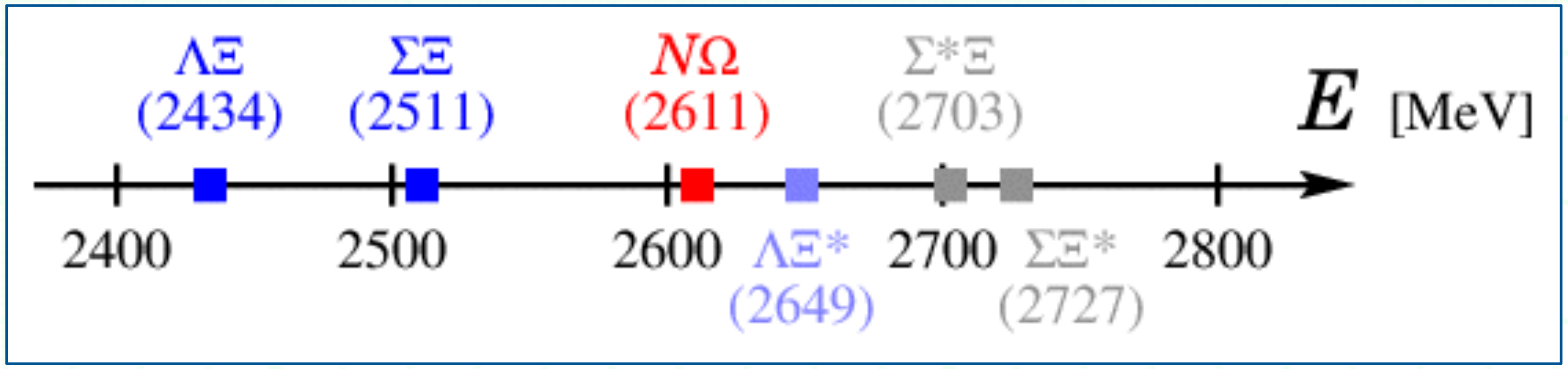
++ NΩ and coupled channels ++

■ Consider the NΩ ($^5S_2$, $J^P = 2^+$) coupled-channels.

□ Baryon-baryon systems in $S = -3$ & $I = 1/2$:

Channel	$J^P = 2^+$
1	$N\Omega\ (^5S_2)$
2	$\Lambda\Xi\ (^3D_2)$
3	$\Lambda\Xi\ (^1D_2)$
4	$\Sigma\Xi\ (^3D_2)$
5	$\Sigma\Xi\ (^1D_2)$
6	$\Lambda\Xi^*\ (^5S_2)$

($^{2S+1}L_J$)



□ Decay channels: ΛΞ, ΣΞ – Couple to the NΩ ($^5S_2$) state in D wave.

□ Closed channel: $\Lambda\Xi^* = \Lambda\Xi(1530)$ (the nearest).

2. Model for NΩ, ver.2018

++ Elastic NΩ interaction ++

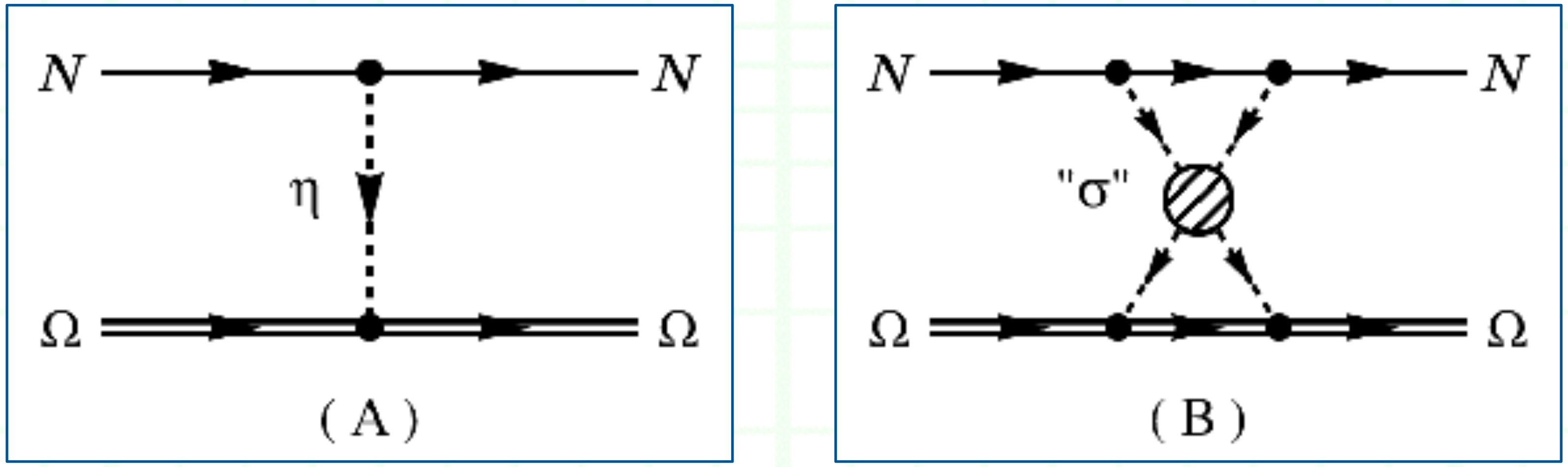
■ Possible mediating mesons in the elastic NΩ interaction:

☐ Pseudoscalar: the η meson.

☐ Scalar: the “σ” meson, which should be treated as as the correlated two pseudoscalar mesons (cf. nuclear force).

– We employ the dispersion relation approach.

☐ Vector: NO light vector mesons owing to the OZI rule.

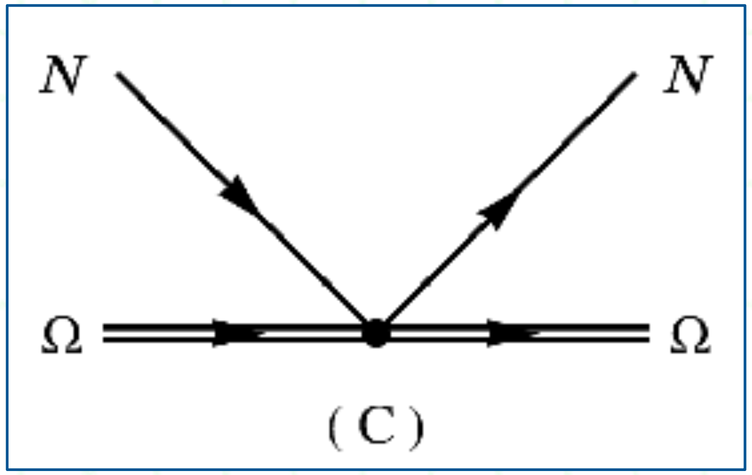


2. Model for NΩ, ver.2018

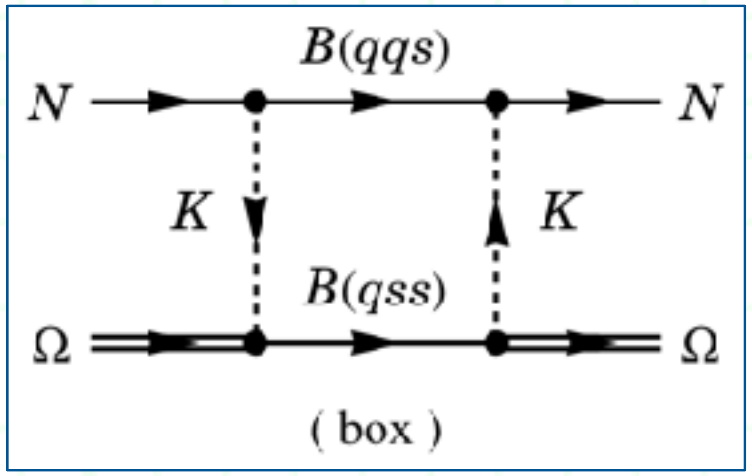
++ Further terms of NΩ interaction ++

- We may consider further contributions at short ranges:
  - ☐ Exchanges of heavier mesons.
  - ☐ Interactions at the quark-gluon level such as color magnetic interactions.
  - ☐ ...

–> We treat them as a contact term.

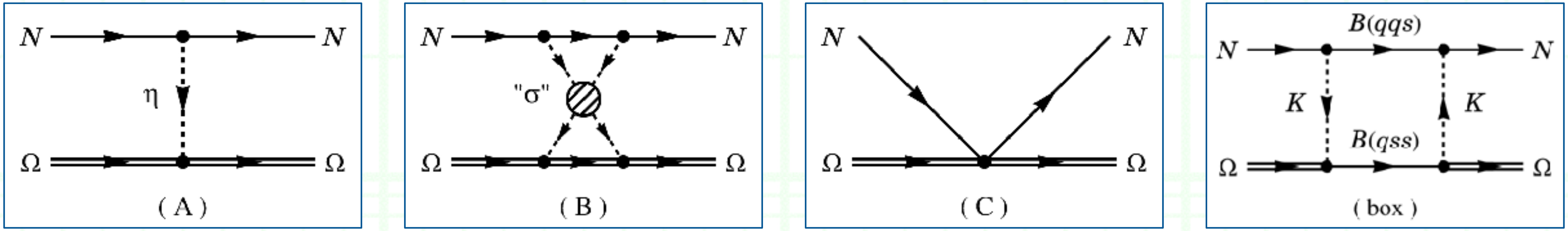


- Box diagrams with K meson exchange give the inelastic channel contributions.
  - ☐ ΛΞ, ΣΞ and ΛΞ*.



2. Model for NΩ, ver.2018

++ Summary of the NΩ interaction ++



■ Evaluate the NΩ ($^5S_2$) Int. with the above four diagrams.

$$V(E;\,p',\,p) = V_{\rm A}(p',\,p) + V_{\rm B}(p',\,p) + V_{\rm C}(p',\,p) + \sum_{j=2}^{6} V_{{\rm box}(j)}(E;\,p',\,p)$$

Channel	$J^P = 2^+$
1	$N\Omega\ (^5S_2)$
2	$\Lambda\Xi\ (^3D_2)$
3	$\Lambda\Xi\ (^1D_2)$
4	$\Sigma\Xi\ (^3D_2)$
5	$\Sigma\Xi\ (^1D_2)$
6	$\Lambda\Xi^*\ (^5S_2)$

□ Non-local interaction.

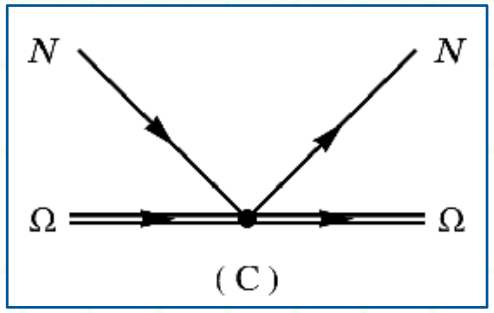
□ Energy dependence from the box terms.

□ Each vertex by effective Lagrangians.

– Free parameter only in the contact interaction.

(Cut-off is fixed as a typical hadron scale $\Lambda = 1$ GeV)

2. Model for NΩ, ver.2018



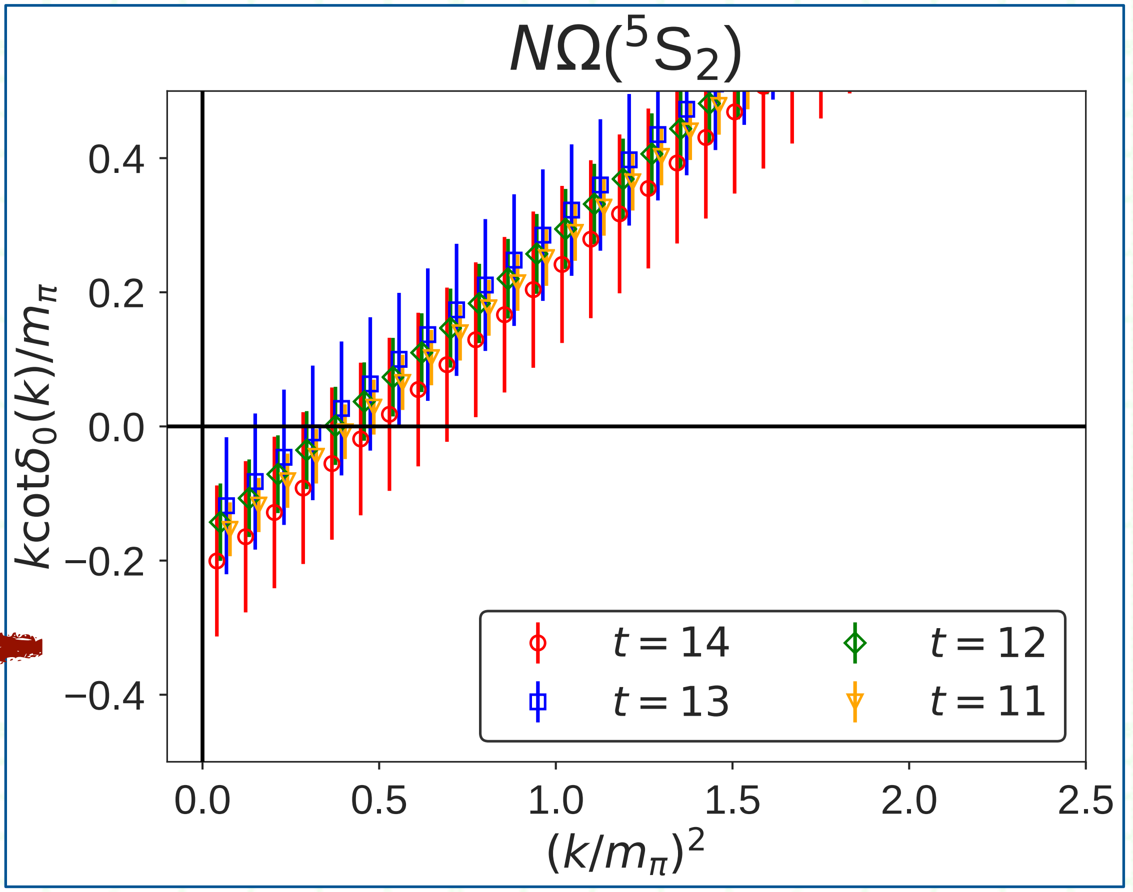
++ Model parameter ++

■ The contact coupling constant c is fixed by information from the HAL QCD analysis (up to 2018).

■ We reproduce the Scatt. length of the HAL QCD analysis on NΩ.

– Observable.

Scattering length
a = 7.4 ± 1.6 fm at t = 11.



□ HAL QCD provides the nearly physical quark masses on the lattice.

–> We fix $c = -22.1\ \mathrm{GeV}^{-2}$

to reproduce a = 7.4 fm with these lattice masses.

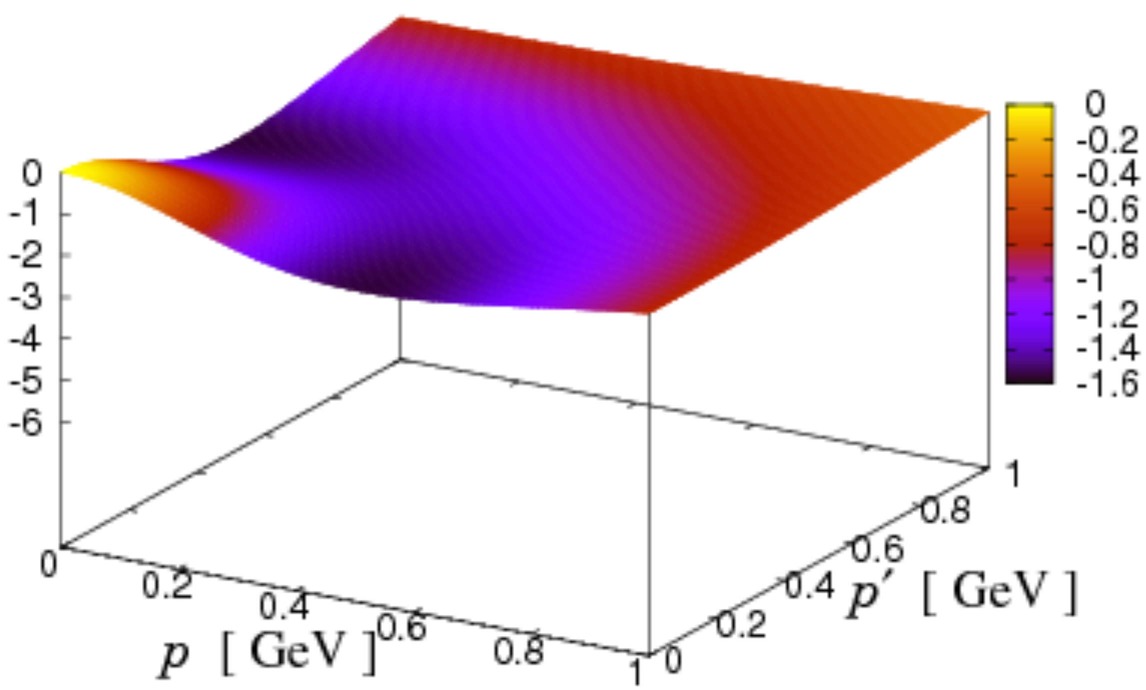
2. Model for NΩ, ver.2018

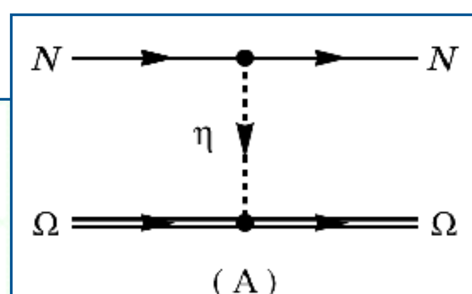
++ Elastic NΩ interaction ++

■ First, we show the NΩ ($^5S_2$) interaction in elastic channels:

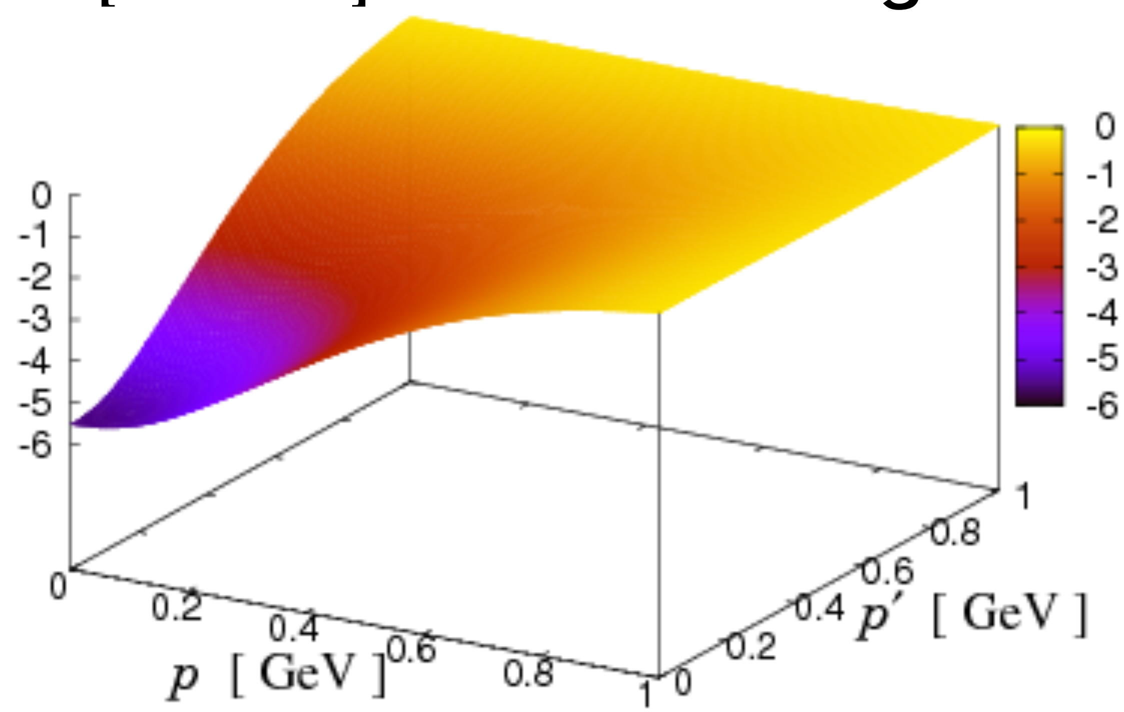
$$V(E;\,p',p) = \underline{V_{\rm A}(p',p) + V_{\rm B}(p',p) + V_{\rm C}(p',p)} + \sum_{j=2}^{6} V_{{\rm box}(j)}(E;\,p',p)$$

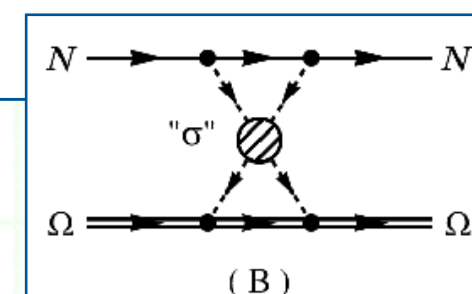
V_A [GeV^{-2}] – η exchange.





V_B [GeV^{-2}] – “σ” exchange.



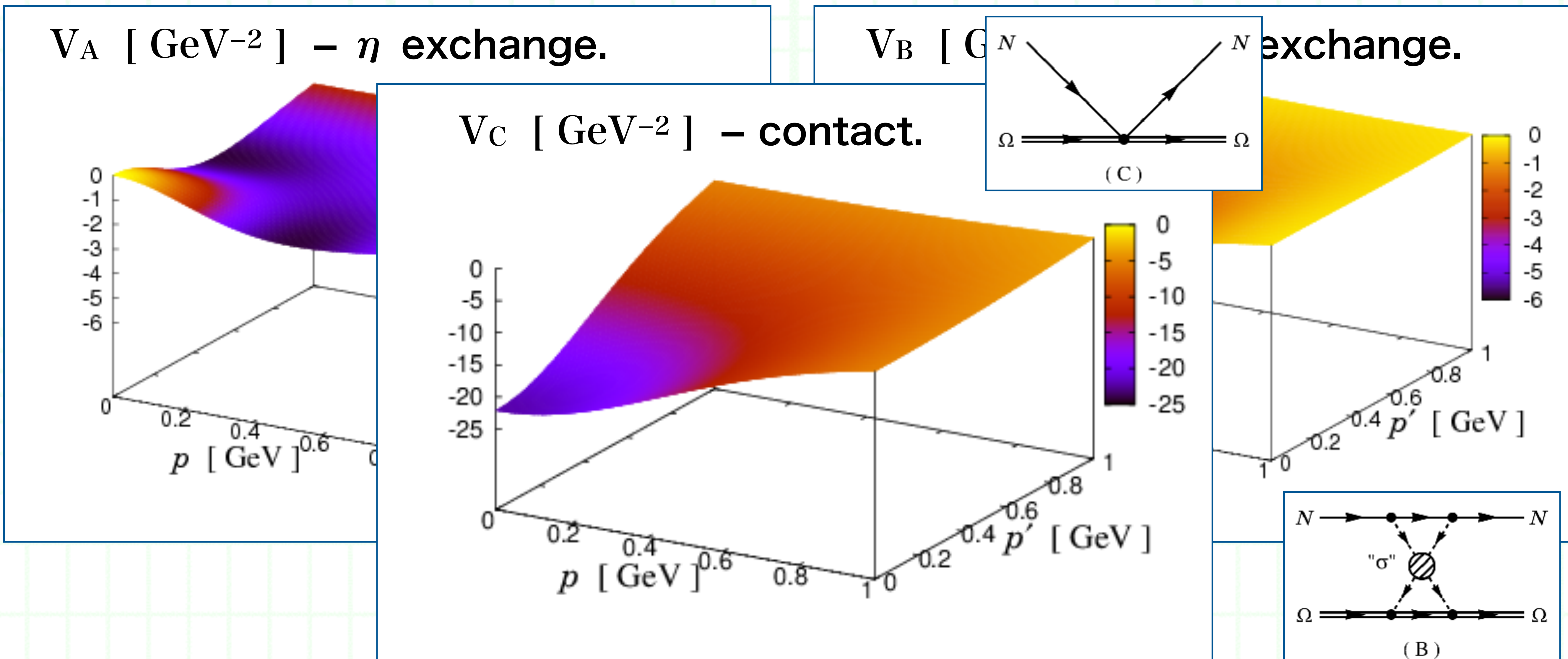


2. Model for NΩ, ver.2018

++ Elastic NΩ interaction ++

- First, we show the NΩ ($^5S_2$) interaction in elastic channels:

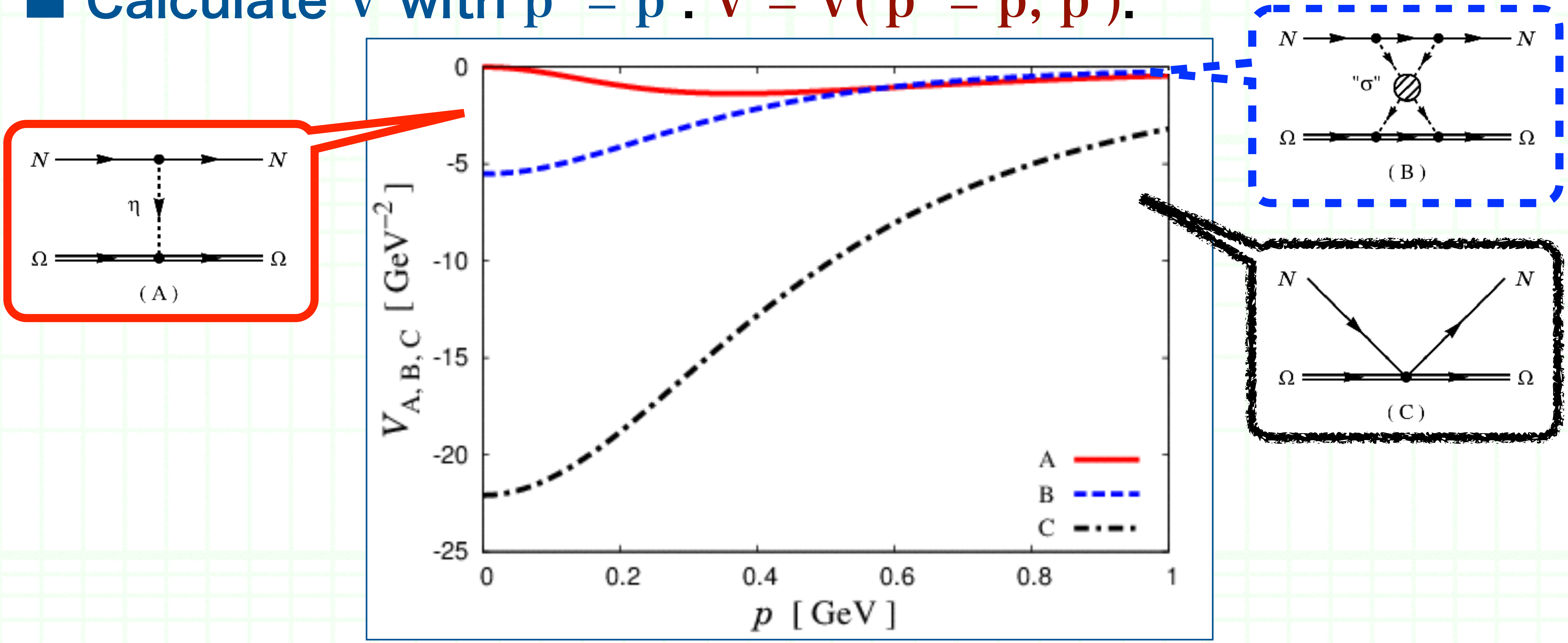
$$V(E;p',p) = V_A(p',p) + V_B(p',p) + V_C(p',p) + \sum_{j=2}^{6} V_{box(j)}(E;p',p)$$



2. Model for NΩ, ver.2018

++ Elastic NΩ interaction ++

■ Calculate V with p’ = p : V = V(p’ = p, p).



☐ The contact term is dominant.

☐ The η and “ σ ” exchanges are moderate.

– Small η NN and “ σ ” ΩΩ couplings.

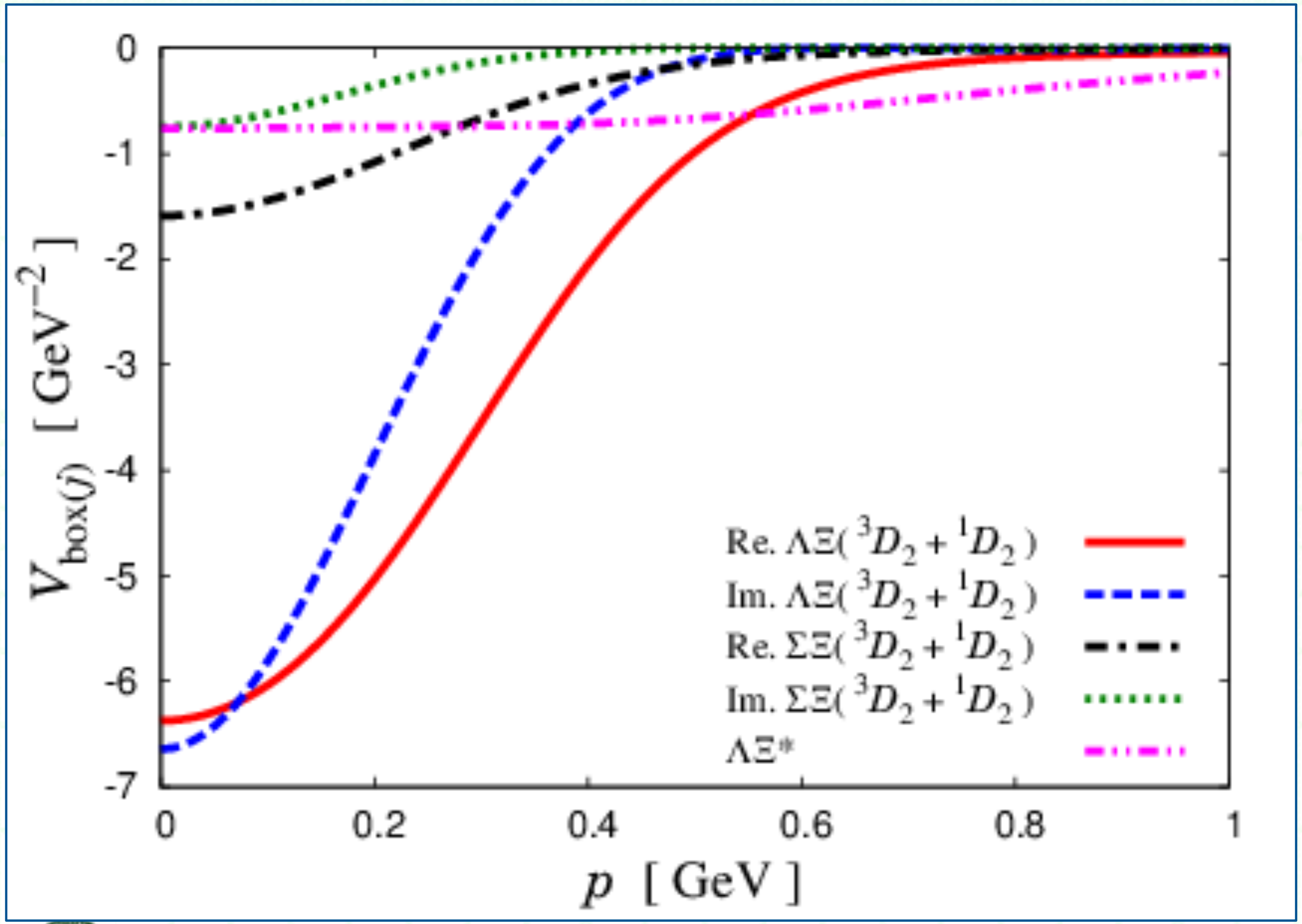
2. Model for NΩ, ver.2018

++ Inelastic NΩ interaction ++

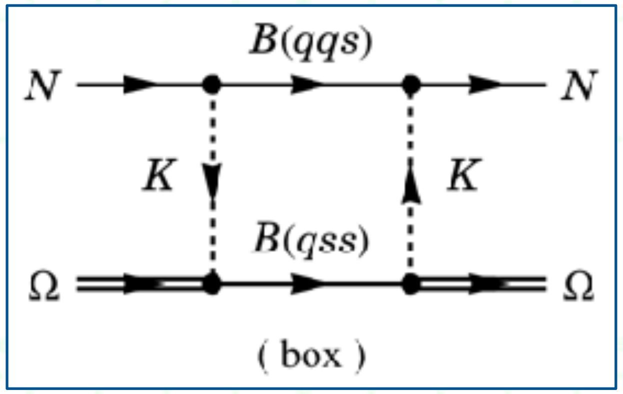
■ Next, we show the NΩ ($^5S_2$) Int. from inelastic channels:

$$V(E;p',p) = V_A(p',p) + V_B(p',p) + V_C(p',p) + \sum_{j=2}^{6} V_{box(j)}(E;p',p)$$

– Calculate V_{box} with p' = p and E = m_N + m_Ω:



V = V_{box}(m_N+m_Ω; p' = p, p).



□ ΛΞ is the largest among the box terms owing to the large $\bar{K}$NΛ coupling.

2. Model for NΩ, ver.2018

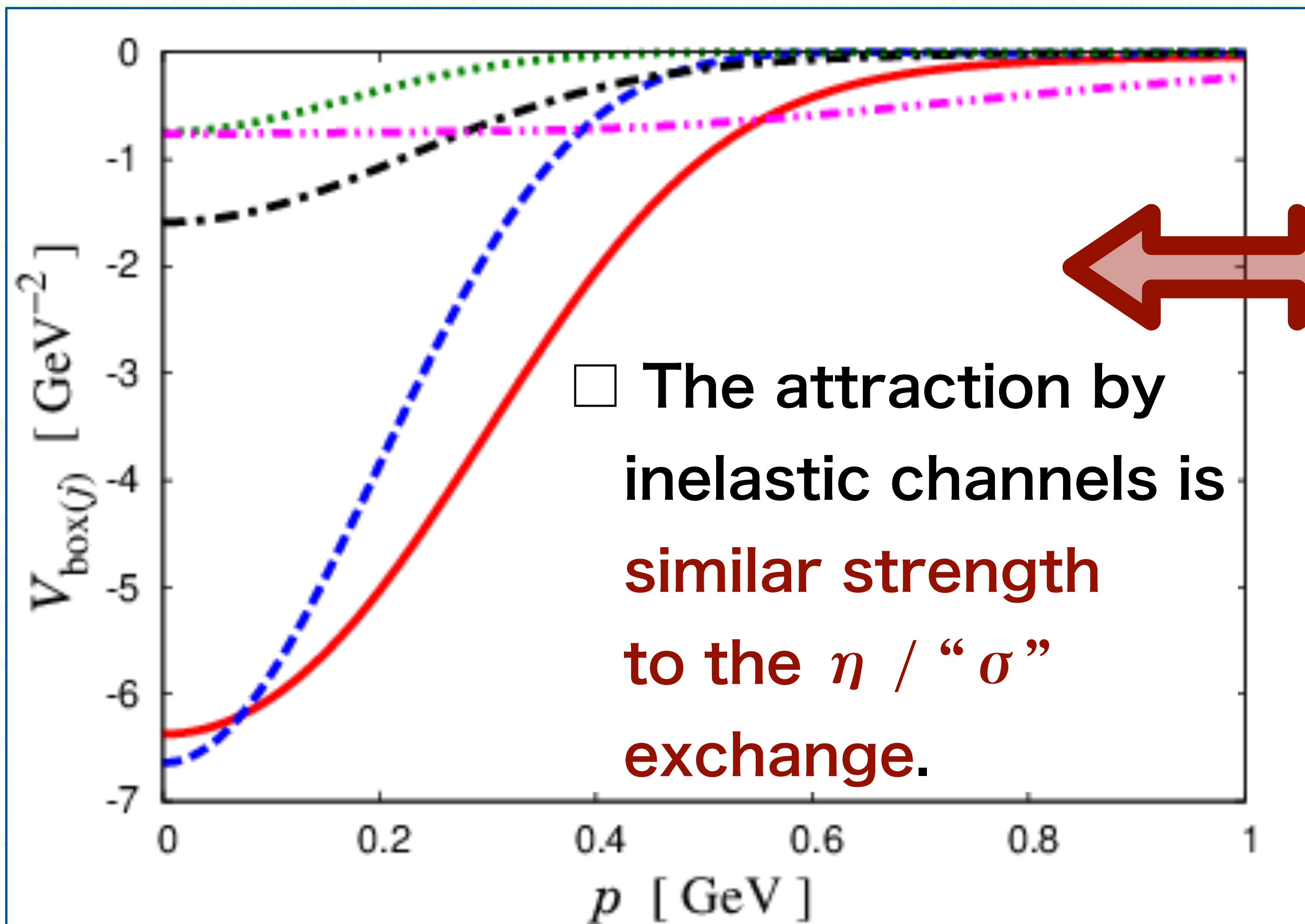
++ Inelastic NΩ interaction ++

■ Next, we show the NΩ ($^5S_2$) Int. from inelastic channels:

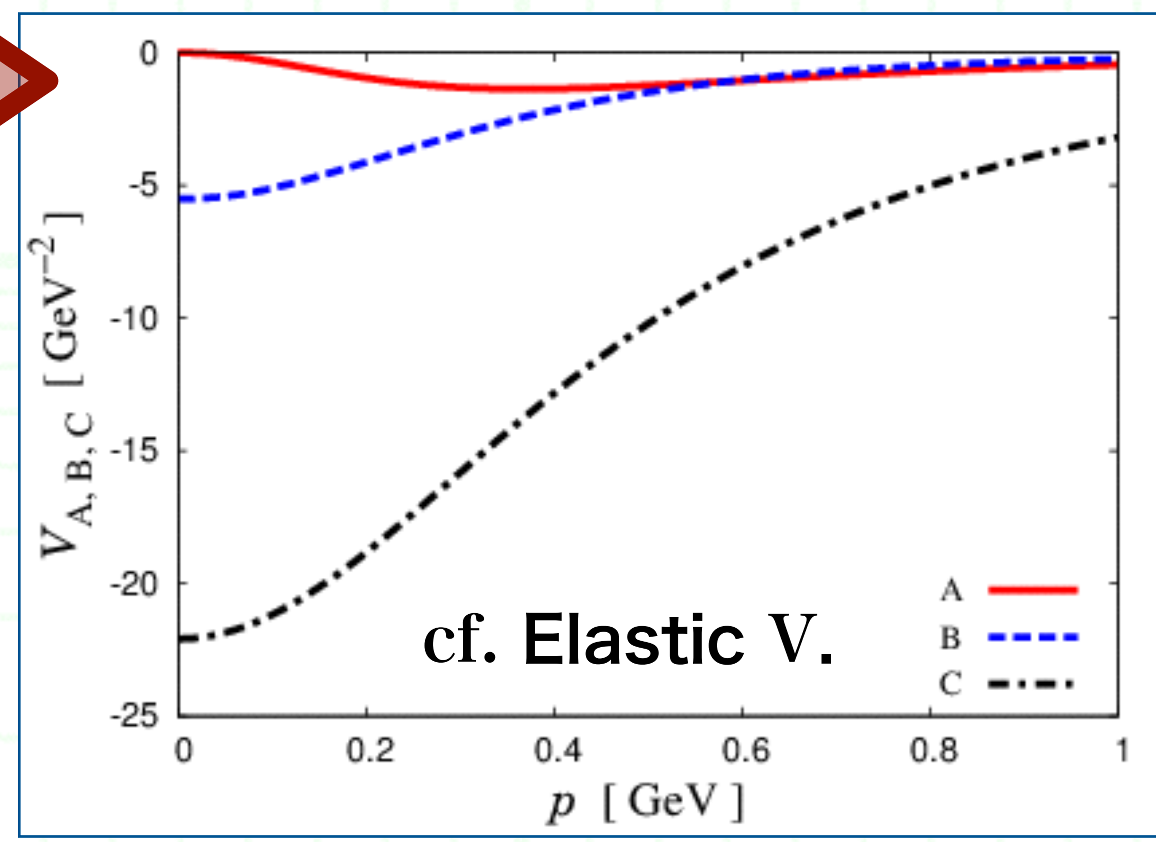
$$V(E;p',p) = V_A(p',p) + V_B(p',p) + V_C(p',p) + \sum_{j=2}^{6} V_{box(j)}(E;p',p)$$

– Calculate V_{box} with $p' = p$ and $E = m_N + m_\Omega$:

$V = V_{box}(\, m_N + m_\Omega;\, p' = p,\, p\,)$.



□ The attraction by inelastic channels is similar strength to the η / "σ" exchange.

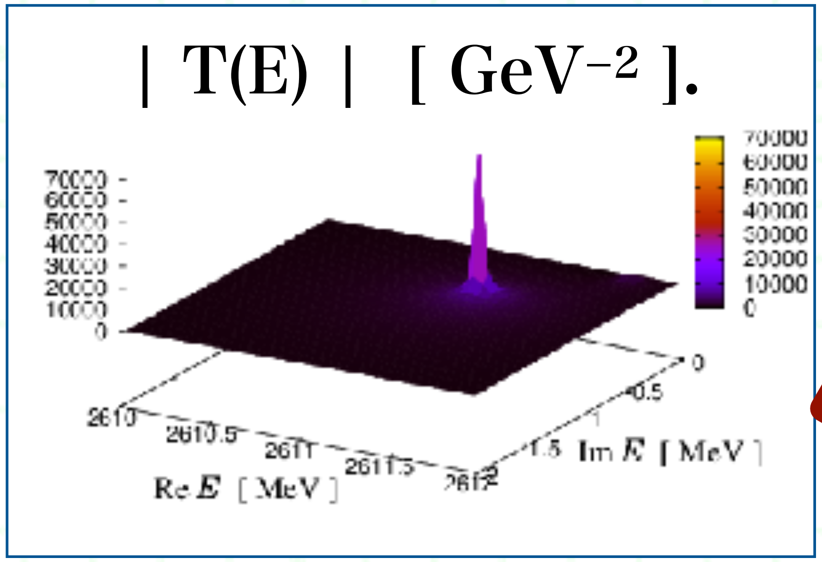


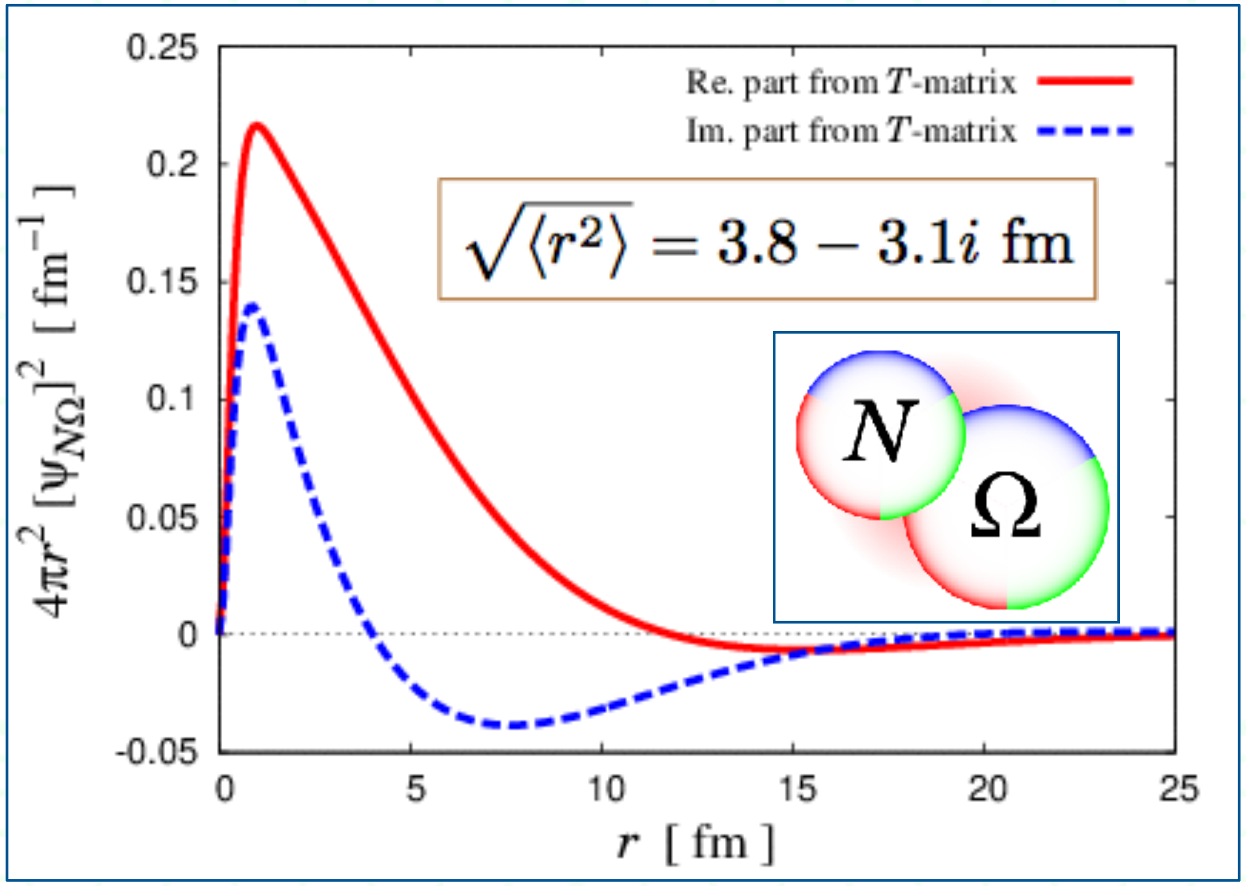
cf. Elastic V.

2. Model for NΩ, ver.2018

++ NΩ ($^5S_2$) quasi-bound state ++

■ The NΩ ($^5S_2$) Scatt. amplitude contains a resonance pole which corresponds to the NΩ ($^5S_2$) quasi-bound state !





■ E_{pole} = 2611.3 – 0.7 i MeV.

–> B_E = 0.1 MeV, Γ = 1.5 MeV.

■ We can extract the bound-state wave function $\psi_{N\Omega}$ from the residue of the resonance pole.

■ Coulomb assist for pΩ^-: $\Delta B_{\rm Coulomb} \sim 1$ MeV $\Delta\Gamma_{\rm Coulomb} \sim 1$ MeV

3. Meson exchange model for the NΩ interaction, version 202X

3. Model for NΩ, ver.202X

++ Update the NΩ interaction ++

■ We have constructed the NΩ ($^5S_2$) interaction in 2018, but it is not so satisfactory today. Mainly because:

1. Experimental data from ALICE as well as the final version of the HAL QCD NΩ potential are available.

2. Contact term was not interpreted.

3. Potential shape of HAL QCD vs. our model.

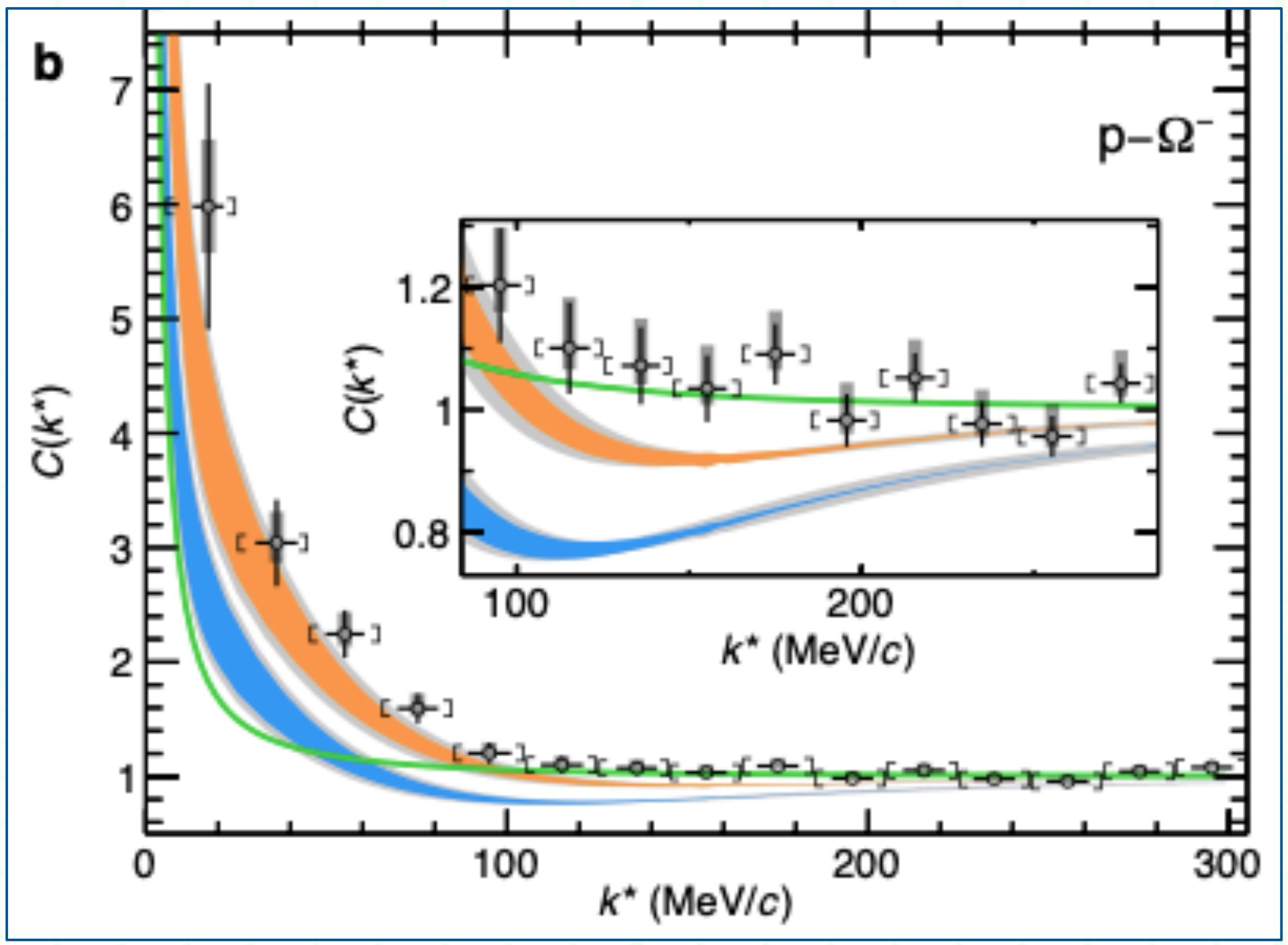
4. Only $^5S_2$.

–> Update is necessary !

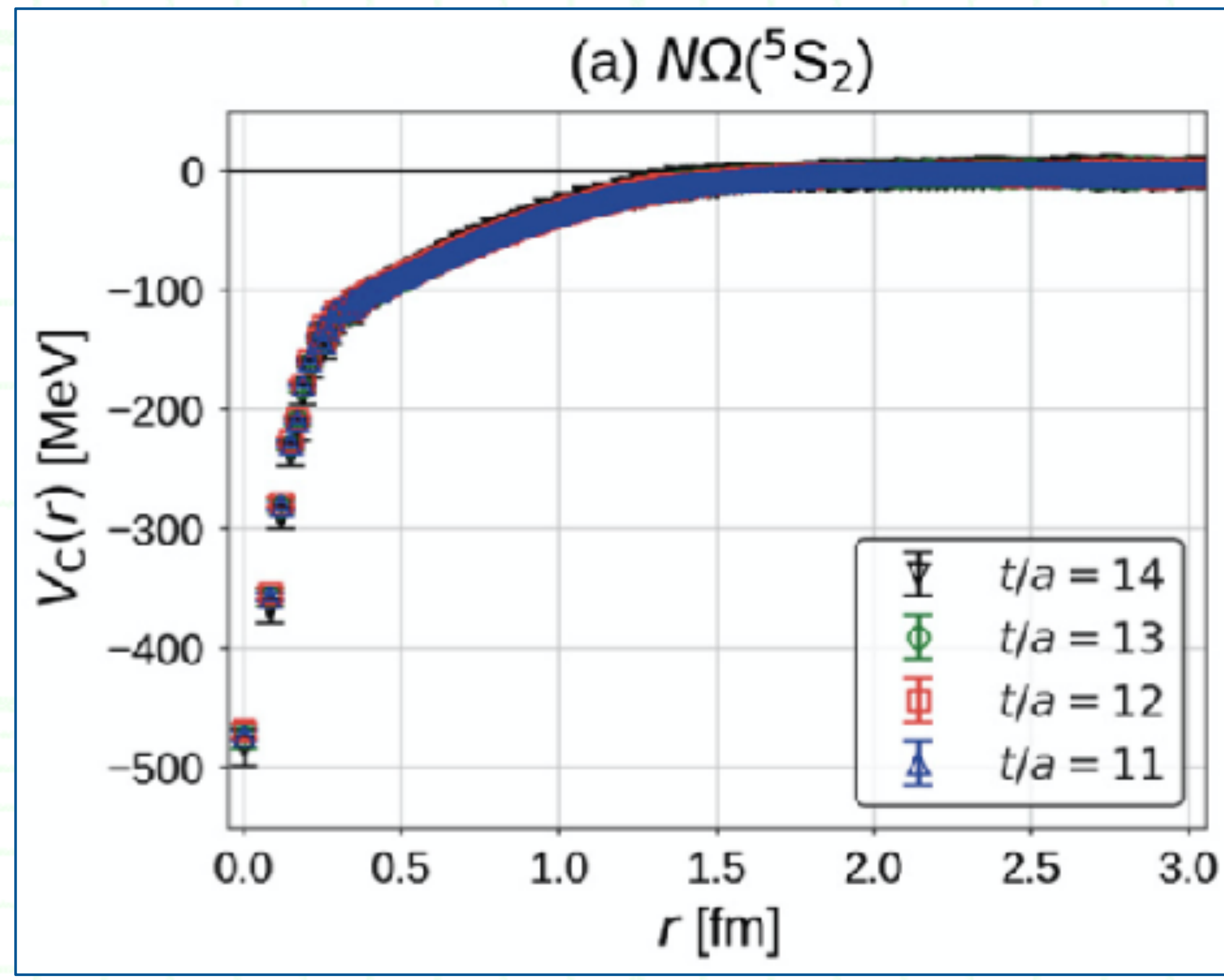
3. Model for NΩ, ver.202X

++ The latest data ++

- The latest data from ALICD and HAL QCD will help us to construct more reliable interaction.



ALICE, arXiv:2005.11495.

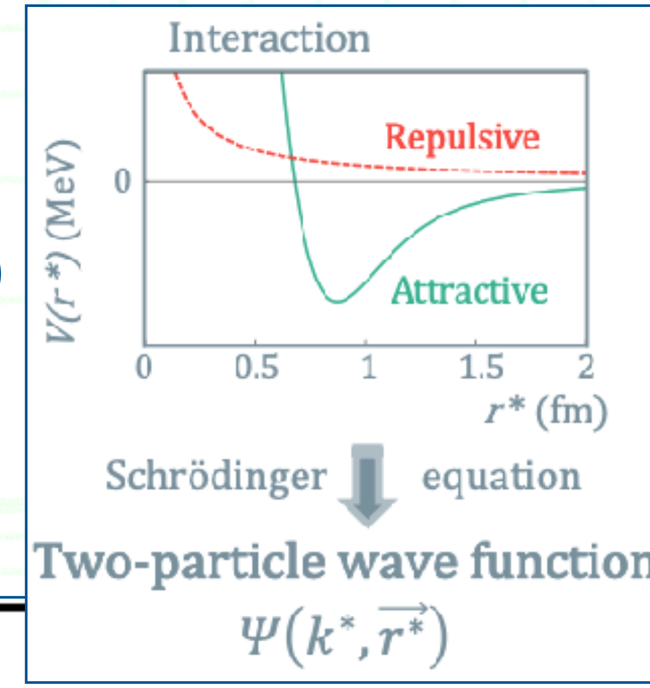


Iritani et al. [HAL QCD],
Phys. Lett. B792 (2019) 284.

3. Model for NΩ, ver.202X

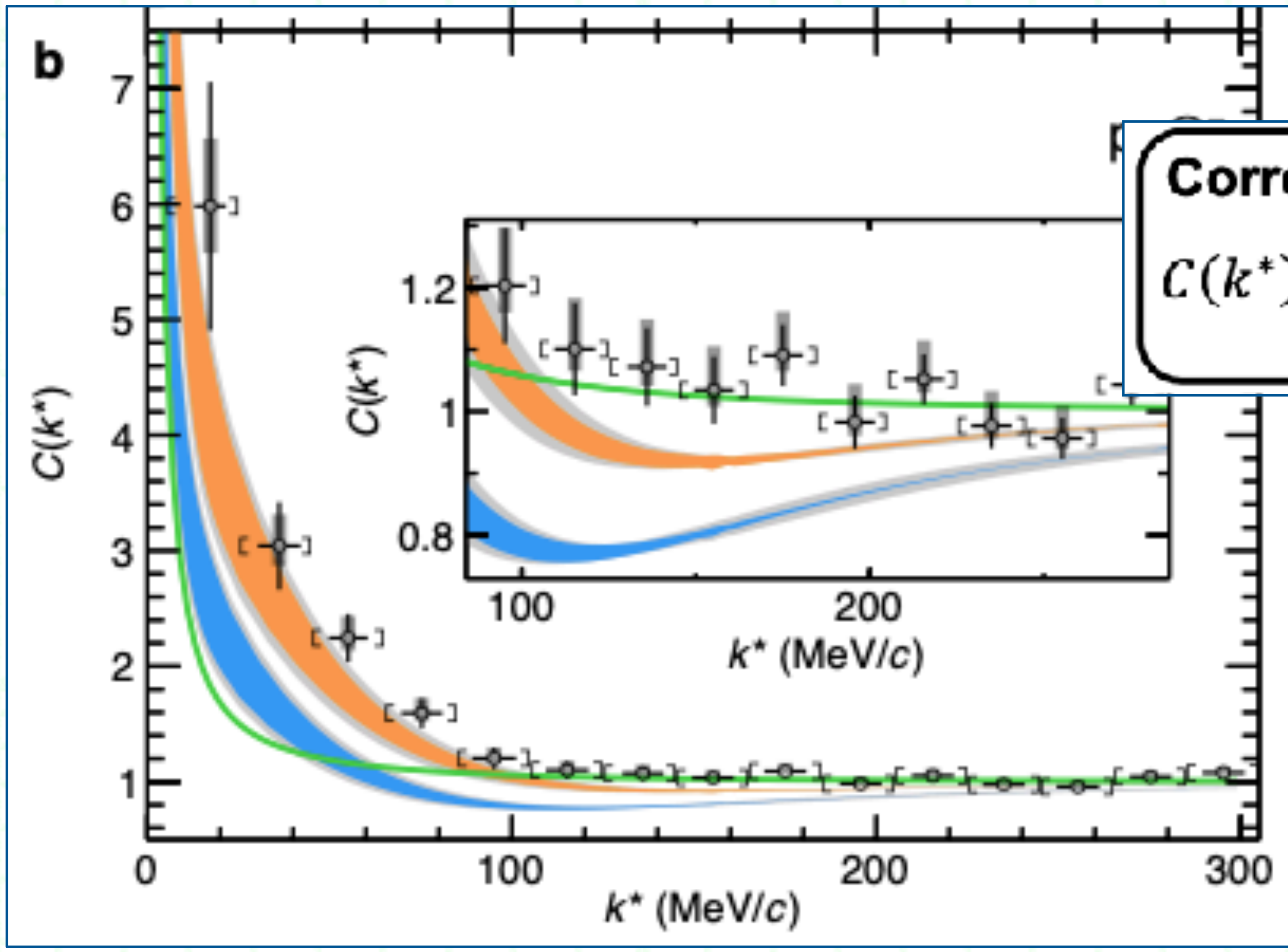
++ The latest data ++

■ The latest data from ALICD and HAL QCD to construct more reliable interaction.



Correlation Function

$$C(k^*) = \int S(r^*)\ \left|\Psi(k^*,\vec{r^*})\right|^2 d^3r^* = \xi(k^*) \otimes \frac{N_{\mathrm{same}}(k^*)}{N_{\mathrm{mixed}}(k^*)}$$



ALICE, arXiv:2005.11495.

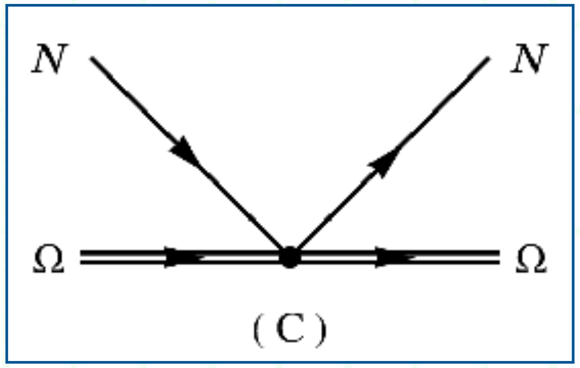
□ We can fix our model parameter (coupling constant) with the Exp. correlation function.

– In Exp., NΩ is a mixed state of various partial waves.

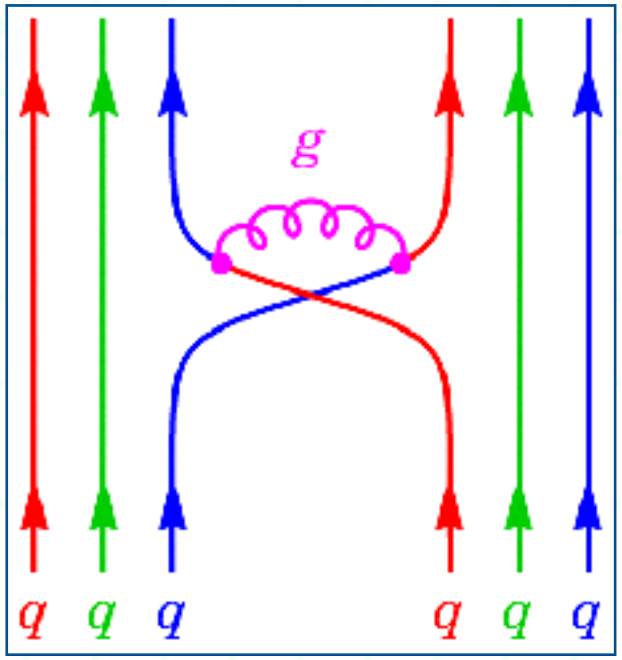
–> We need $^3S_1$ as well.

3. Model for NΩ, ver.202X

++ Short-range part ++



- We have introduced a phenomenological contact term which contributes at short range.



- ☐ What is the origin of the contact term ?
- ☐ Interaction at the quark-gluon level is expected to be dominant.

–> We can calculate the contact term by quark exchange.

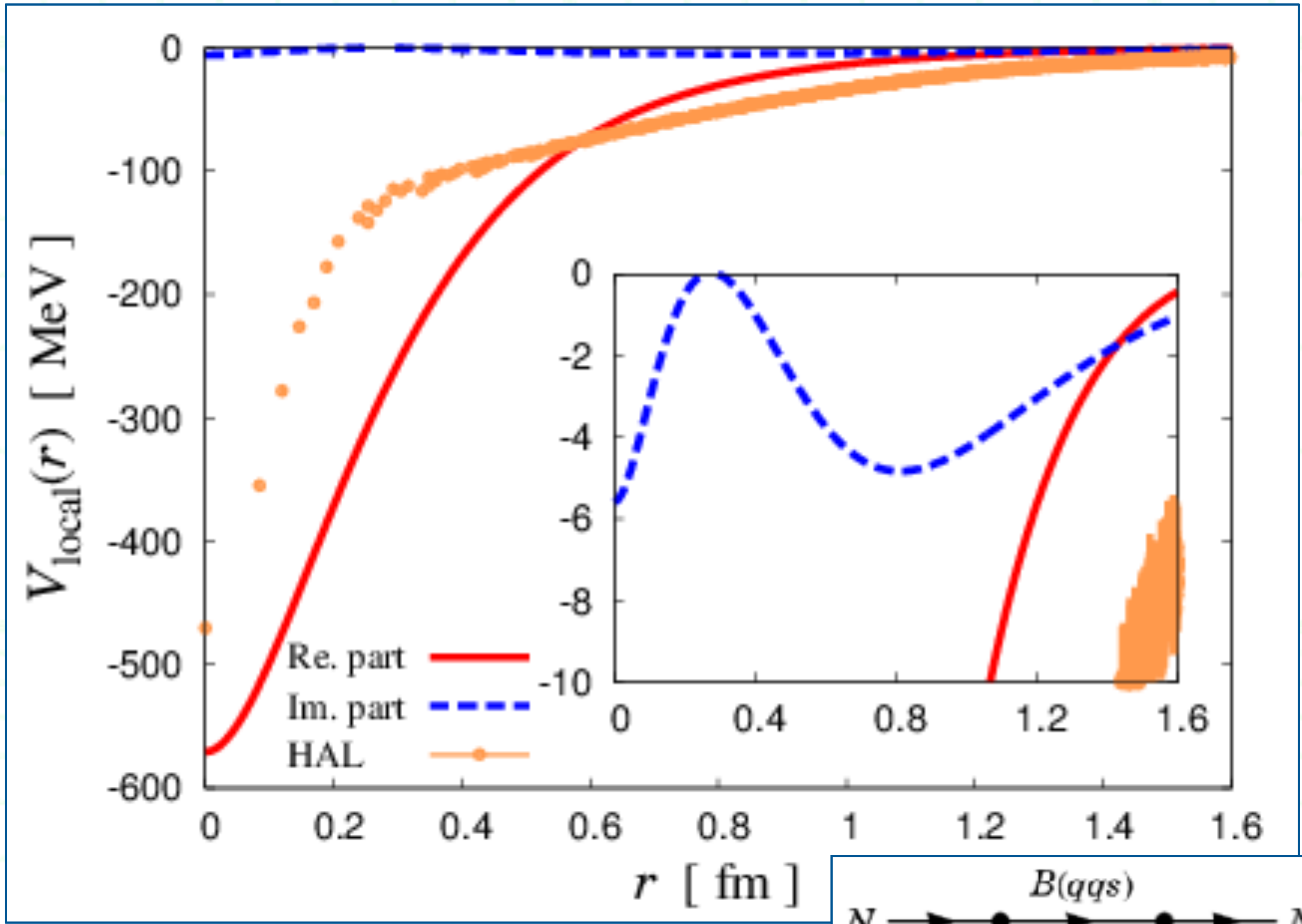
- In calculating this, the approach by Yonsei group will be helpful.

Park et al., Eur. Phys. J. A56 (2020) 93.

3. Model for NΩ, ver.202X

++ Comparison with the HAL QCD potential ++

■ We evaluate equivalent local NΩ ($^5S_2$) potential and compare it with the HAL QCD potential (m_π = 146 MeV).



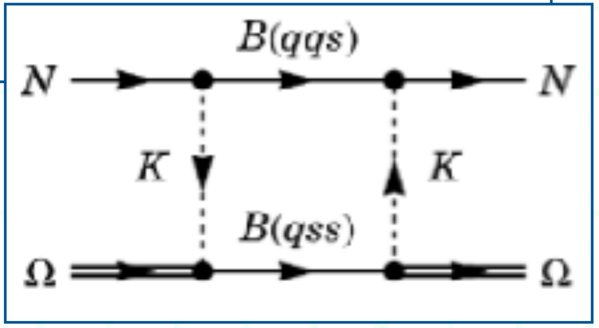
☐ Behavior is qualitatively consistent with the HAL QCD.

☐ However, the shape is different, in particular the long-range tail.

– Two-fold potential ?

Importance of K exchange ???

???

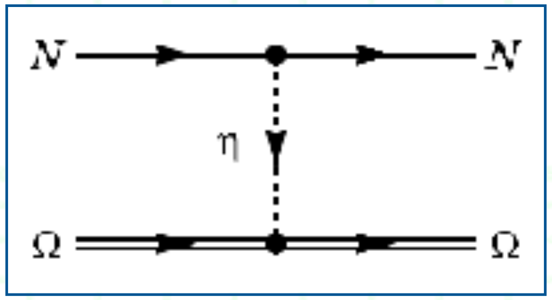


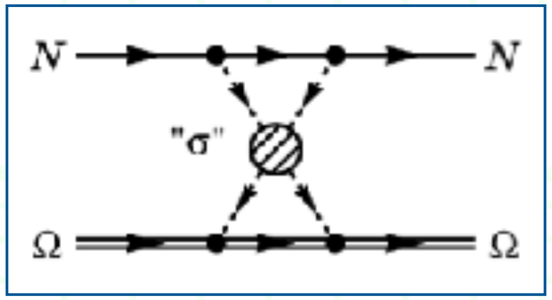
4. Summary

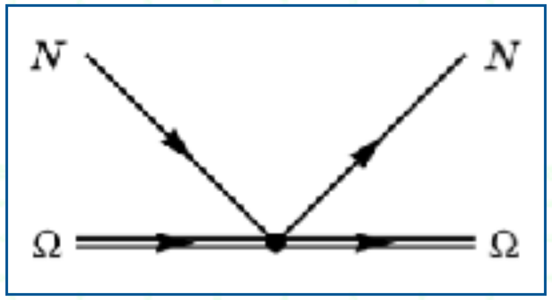
4. Summary

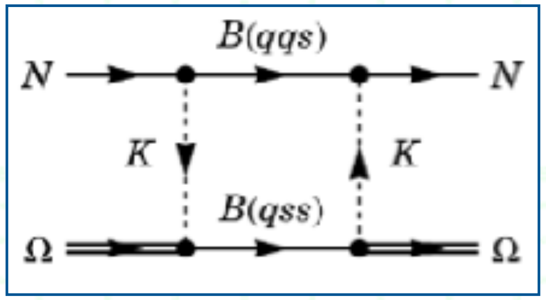
++ Summary ++

■ We constructed the NΩ ($^5S_2$) interaction according to:









□ The conventional exchanges of the η, "σ", and K (as boxes) mesons do not provide sufficient attraction.

□ Most of the attraction indicated in recent HAL QCD analysis is attributed to the short-range contact term.

■ Fitting parameter (contact coupling constant only) to scattering length in HAL QCD, we obtain the NΩ ($^5S_2$) quasi-bound state at $E_{pole} = 2611.3 - 0.7\,i$ MeV.

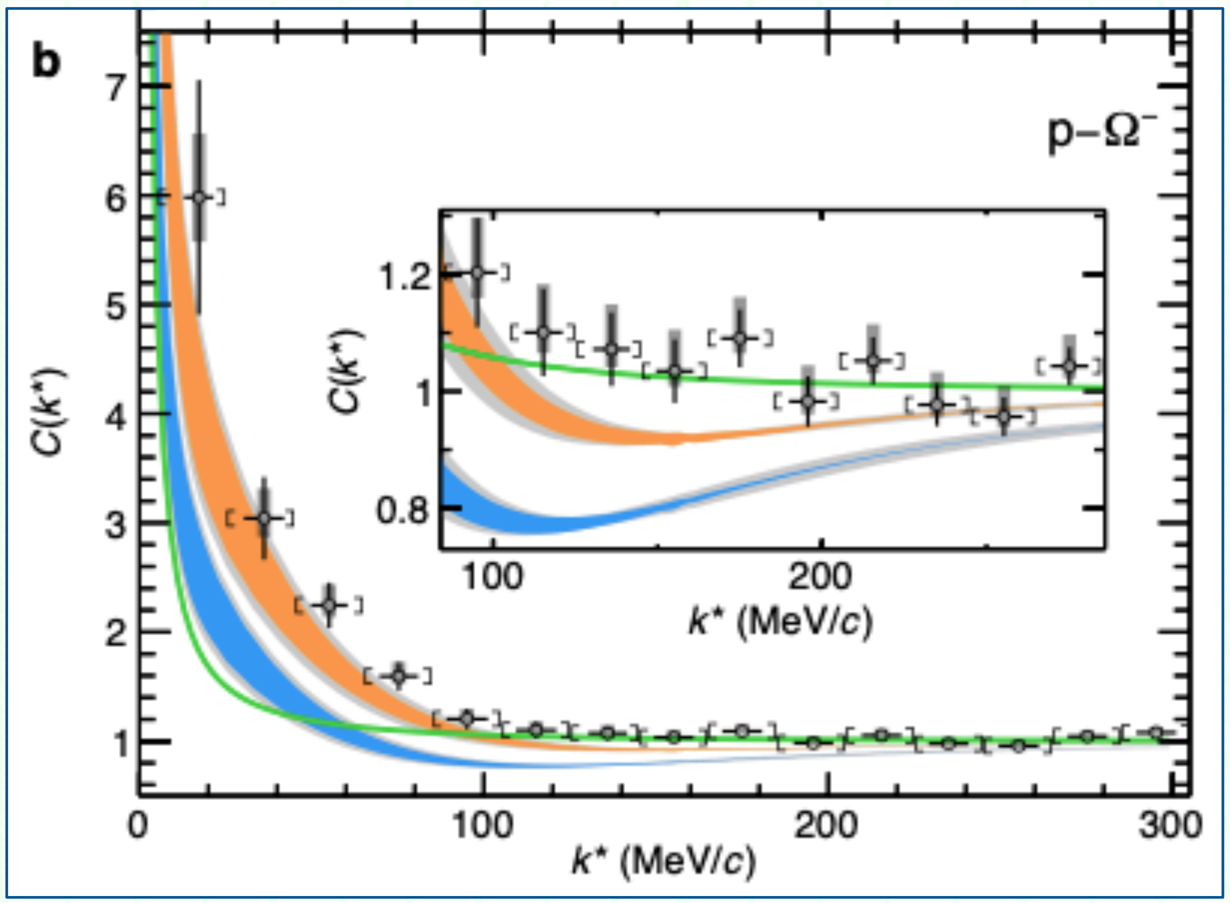
– $B_E = 0.1$ MeV, $\Gamma = 1.5$ MeV.

4. Summary

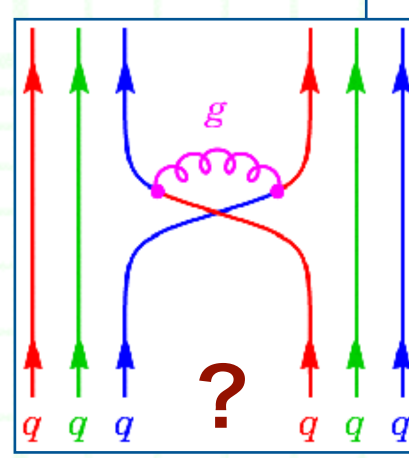
++ Outlook ++

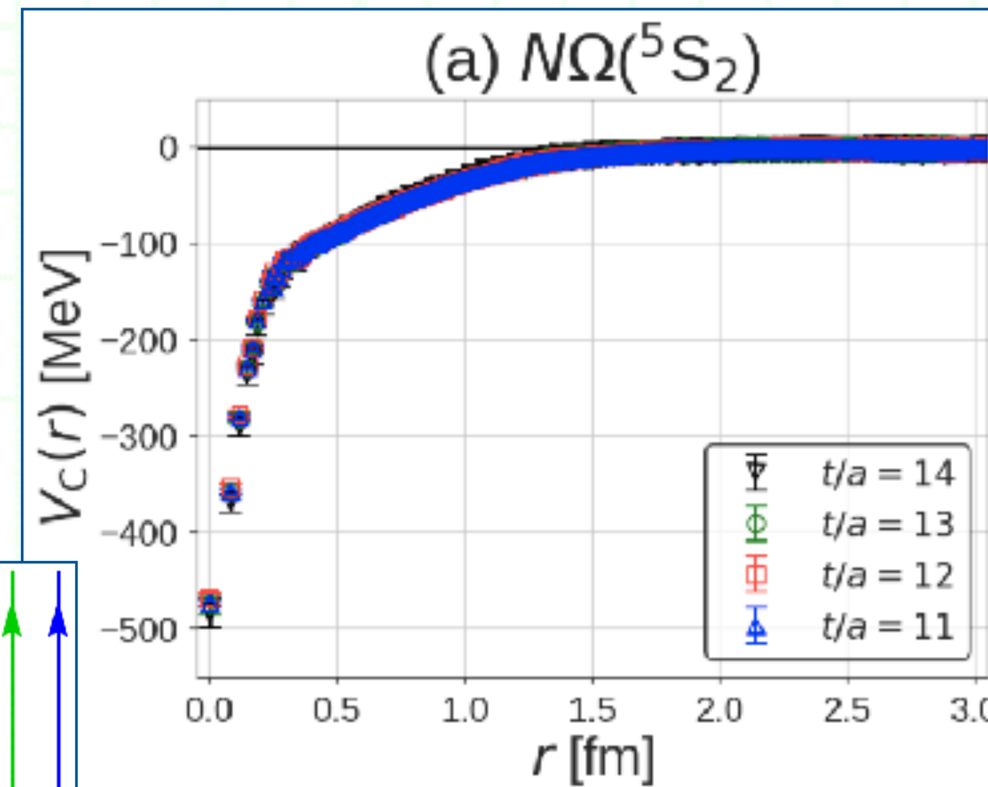
■ We have to update our NΩ interaction.

– In particular, Exp. data from ALICE as well as the final version of the HAL QCD NΩ potential are available.



ALICE, arXiv:2005.11495.





Iritani et al. [HAL QCD], Phys. Lett. B792 (2019) 284.

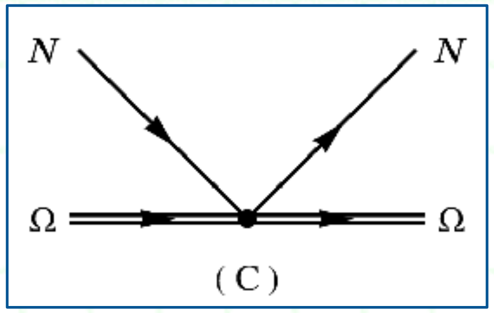
■ NΩ ($^5S_2$) quasi-bound state, exists or not ?

Thank you very much
for your kind attention !

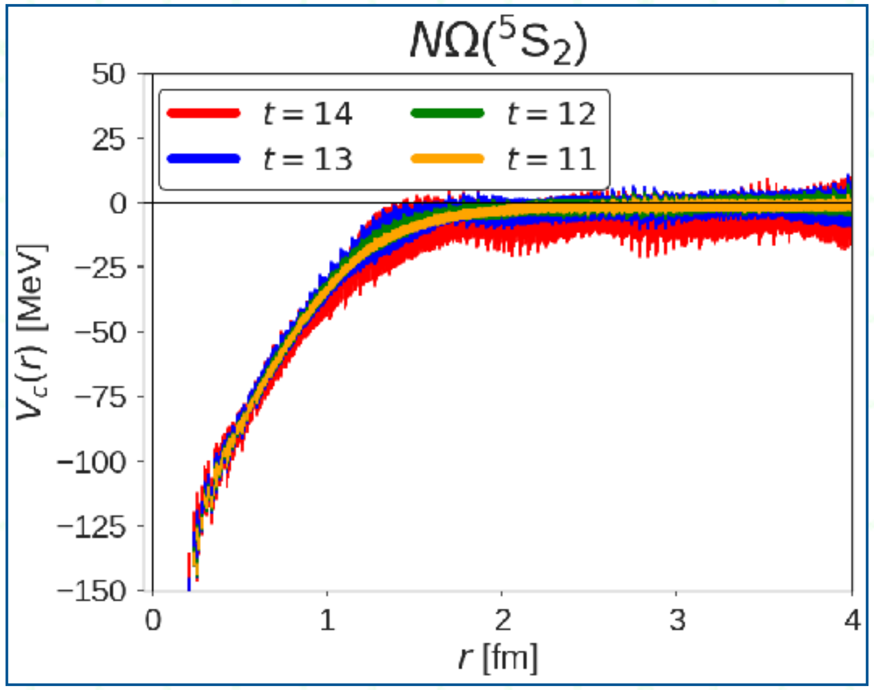
Appendix

Appendix

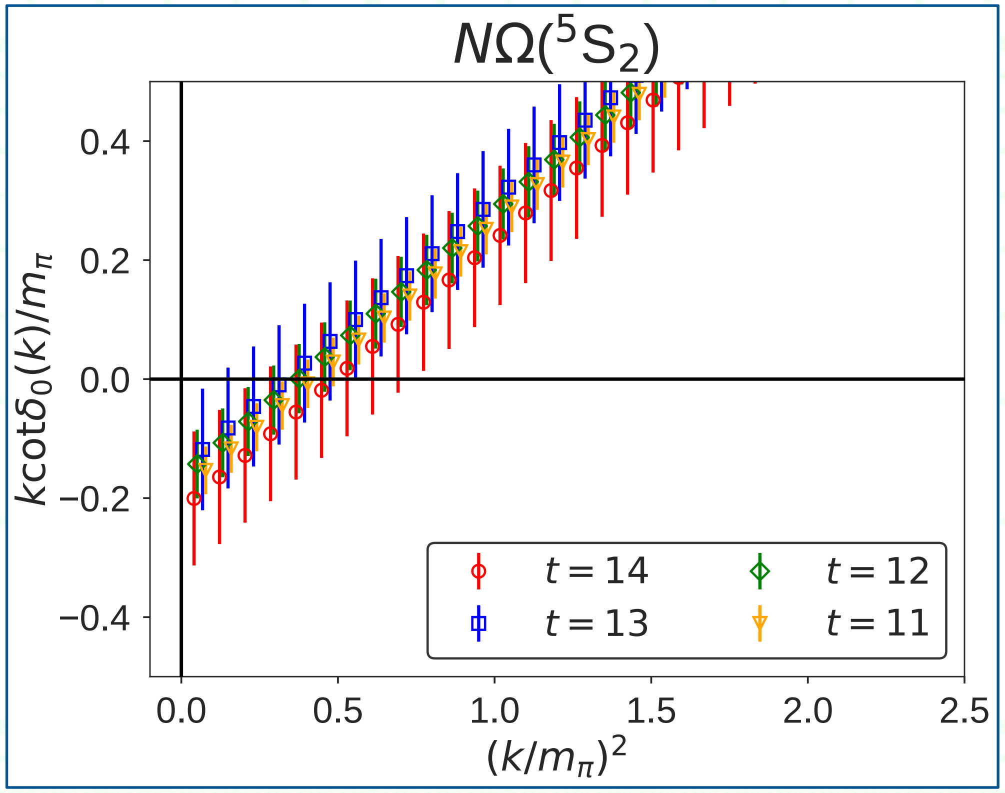
++ Model parameter ++



■ The contact coupling constant c is fixed by information from the HAL QCD analysis (up to 2018).



Doi et al. [HAL QCD], EPJ Web of Conf. 175 (2018) 05009.



– We could use potential V(r), but it is not observable in general (off-shell quantity). The potential may be non-local or change under the field redefinitions.

e.g., Epelbaum et al. Rev. Mod. Phys. 81 (2009) 1773.

–> It is safer to employ observable to fix coupling constant.

Appendix

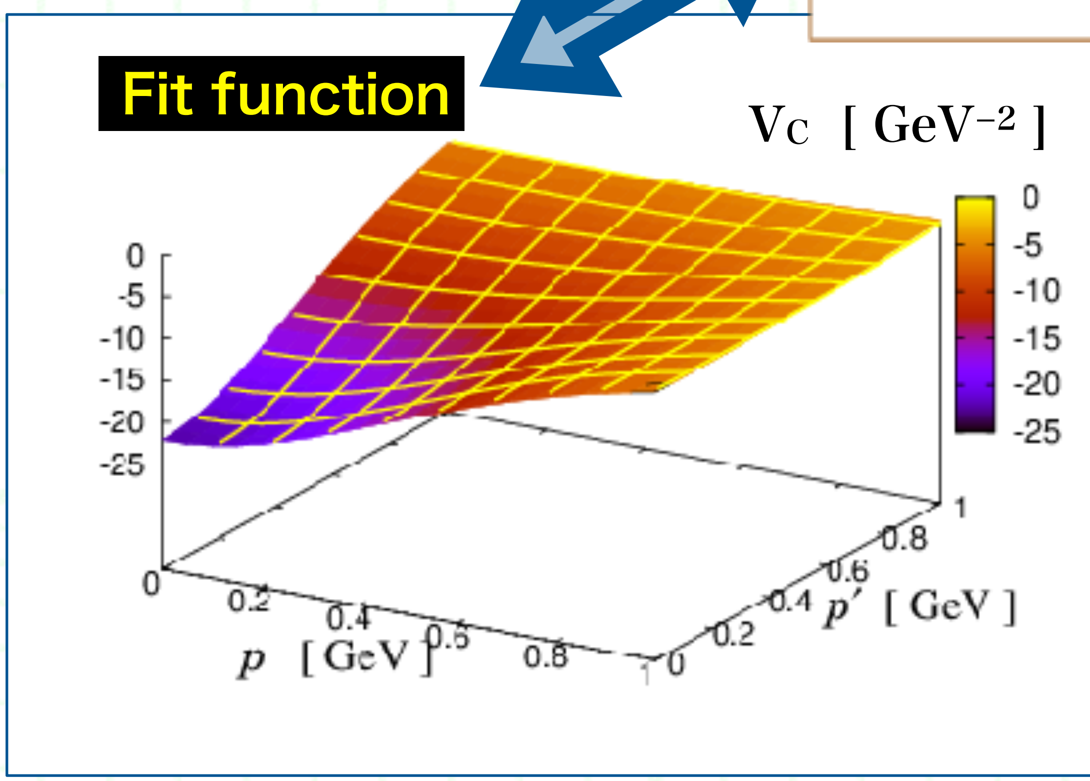
++ Equivalent local potential ++

■ Our NΩ ($^5S_2$) interaction is non-local.

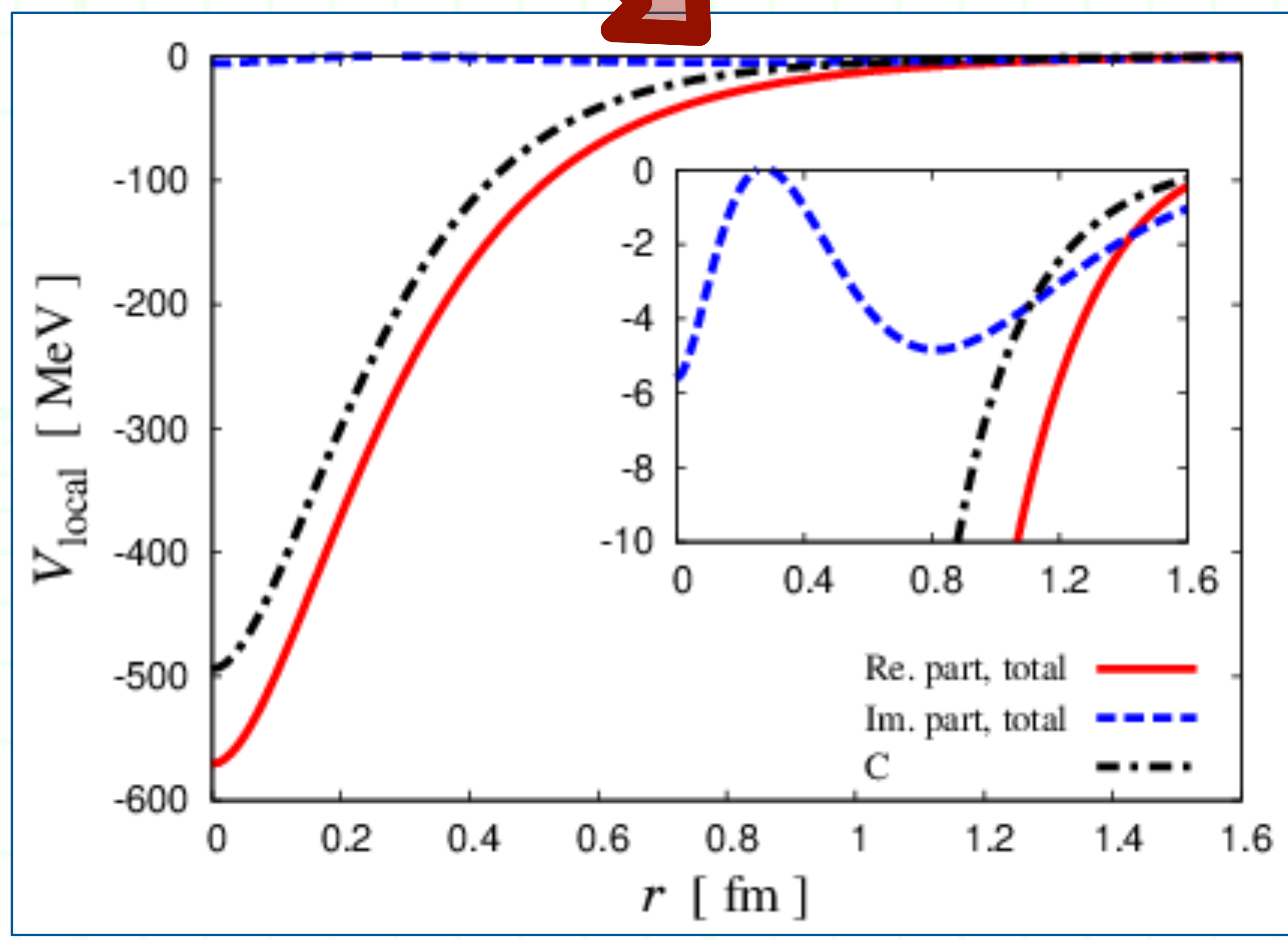
–> Construct a local potential as the sum of Yukawa form:

$$\tilde{V}_{\mathrm{local}}(q) = \sum_{n=1}^{9} \frac{C_n}{q^2 + m_n^2} \left(\frac{\Lambda^2}{\Lambda^2 + q^2} \right)^2$$

Fitted.



Coordinate space.



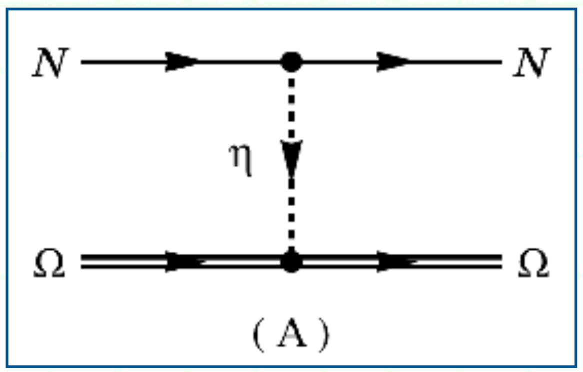
Parameter listed in
T.S., Y. Kamiya and T. Hyodo,
Phys. Rev. C98 (2018) 015205.

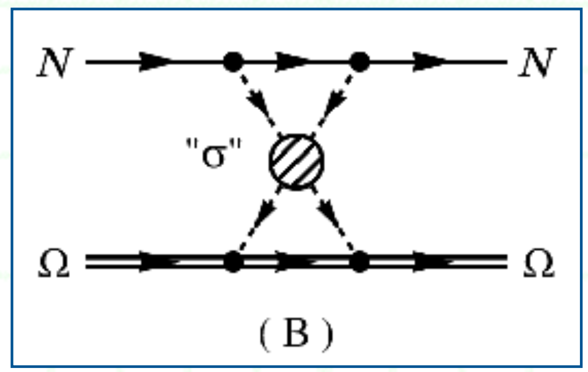
Appendix

++ Comparison with the HAL QCD potential ++

■ Mechanism to generate long-range tail of the potential ?

□ We have checked that the η and “σ” exchanges give only moderate attraction because of small ηNN and “σ” $\Omega\Omega$ couplings..



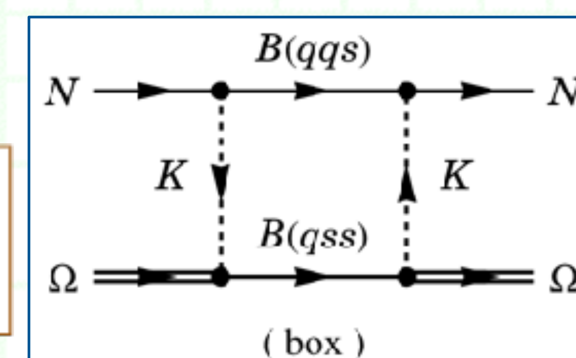


– Moderate attraction.

□ The K exchange to inelastic channels might help more.

– Exchanged K carries energy (neglected in ver.2018) and hence it becomes closer to its on mass shell.

$$\frac{1}{(p_K^\mu)^2 - m_K^2}, \qquad (p_K^\mu)^2 = E_K(\mathbf{p}_K)^2 - |\mathbf{p}_K|^2$$



Appendix

++ Comparison with the HAL QCD potential ++

- Nevertheless, we may not consider seriously the shape difference between the HAL QCD potential and ours.
  - □ HAL QCD potential is based on the approach with the Nambu-Bethe-Salpeter wave function:

$$(E_k - H_0)\varphi^W_{\alpha\beta}(\boldsymbol{x}) = \int U_{\alpha\beta;\gamma\delta}(\boldsymbol{x},\boldsymbol{y})\varphi^W_{\gamma\delta}(\boldsymbol{y})d^3y,$$

HAL QCD, Prog. Theor. Exp. Phys. 2012 01A105.

– We expect that the underlying theory is fine.

  - □ The potential is not observable in general (off-shell quantity), so it may change under the field redefinitions or may be non-local.

Appendix

++ Comparison with the HAL QCD potential ++

- Nevertheless, we may not consider seriously the shape difference between the HAL QCD potential and ours.
  - □ Non-locality might take place (???) to generate long-range interaction.

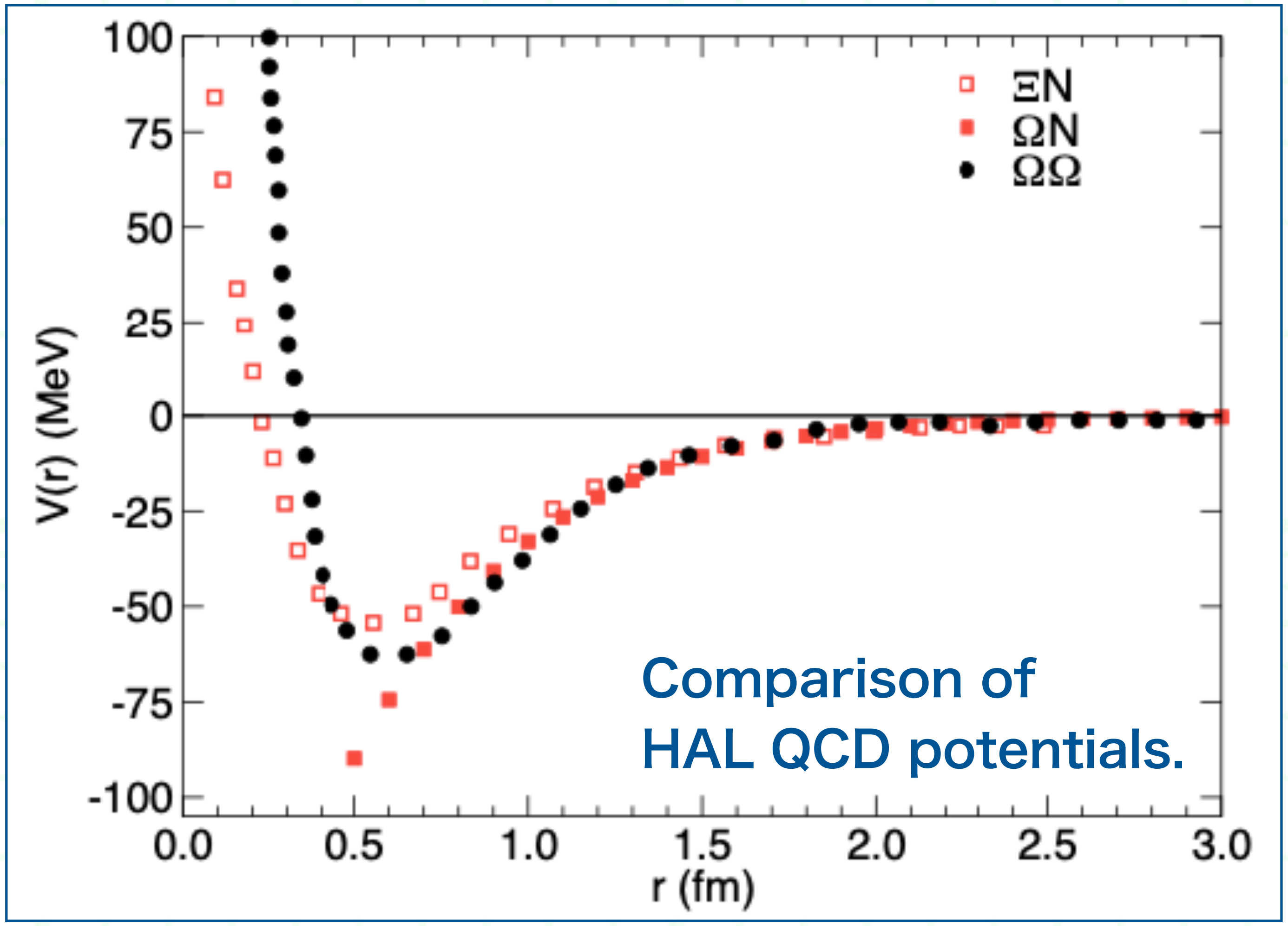
– ΞN, NΩ and ΩΩ give the same range of interactions.

ΞN: π exchange,

NΩ: NO π exchange,

ΩΩ: NO π exchange.

<– Why the same range ?



Haidenbauer et al., Eur. Phys. J. A55 (2019) 70.