# Electromagnetic response of halo nuclei and its cluster aspects

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Clusters & Hierarchies

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- Halo Nuclei and Universality
- Electric Dipole (E1) Response of Halo Nuclei -- introduction
- Coulomb breakup and E1 Response of 1n halo nuclei
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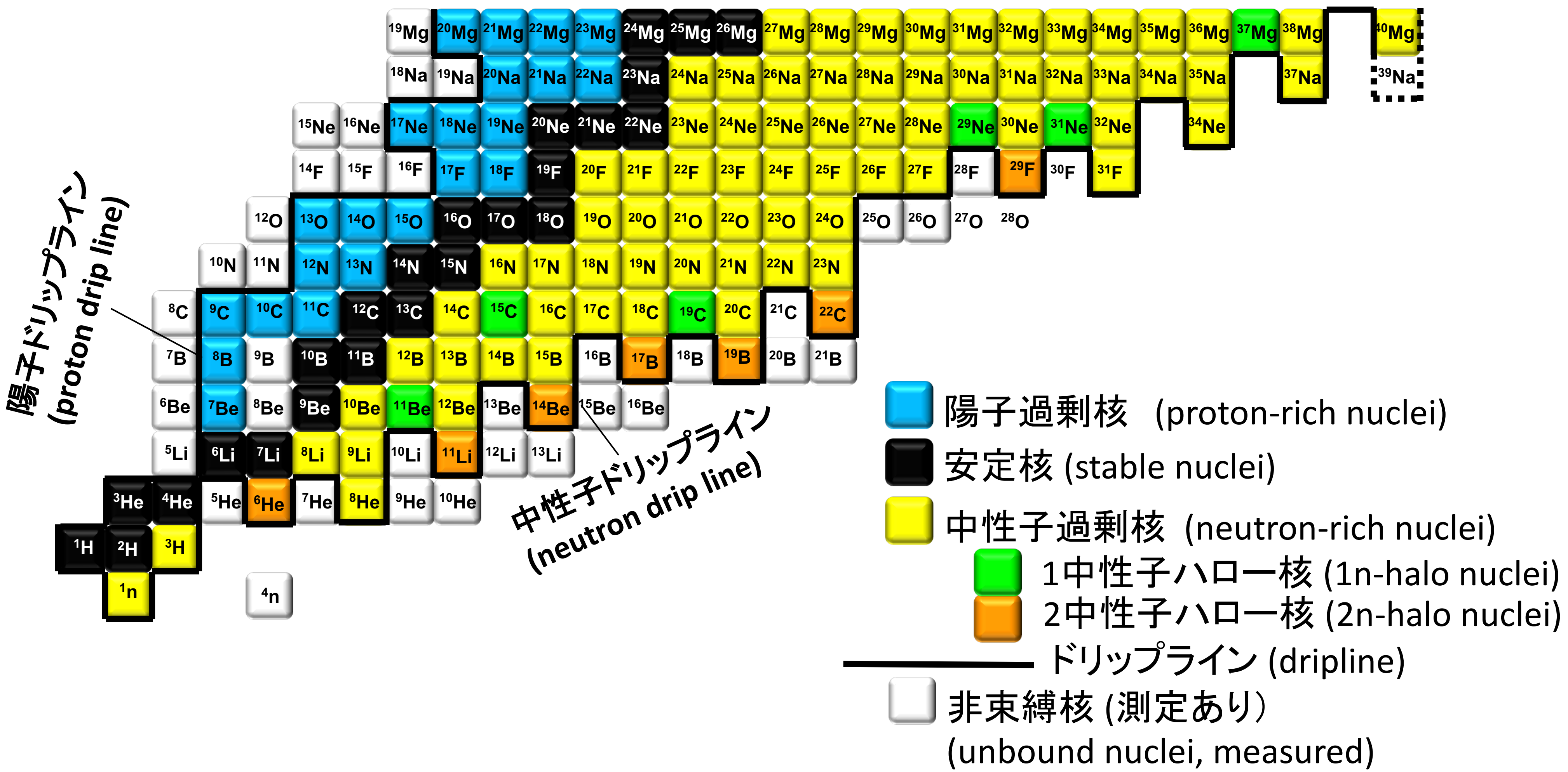
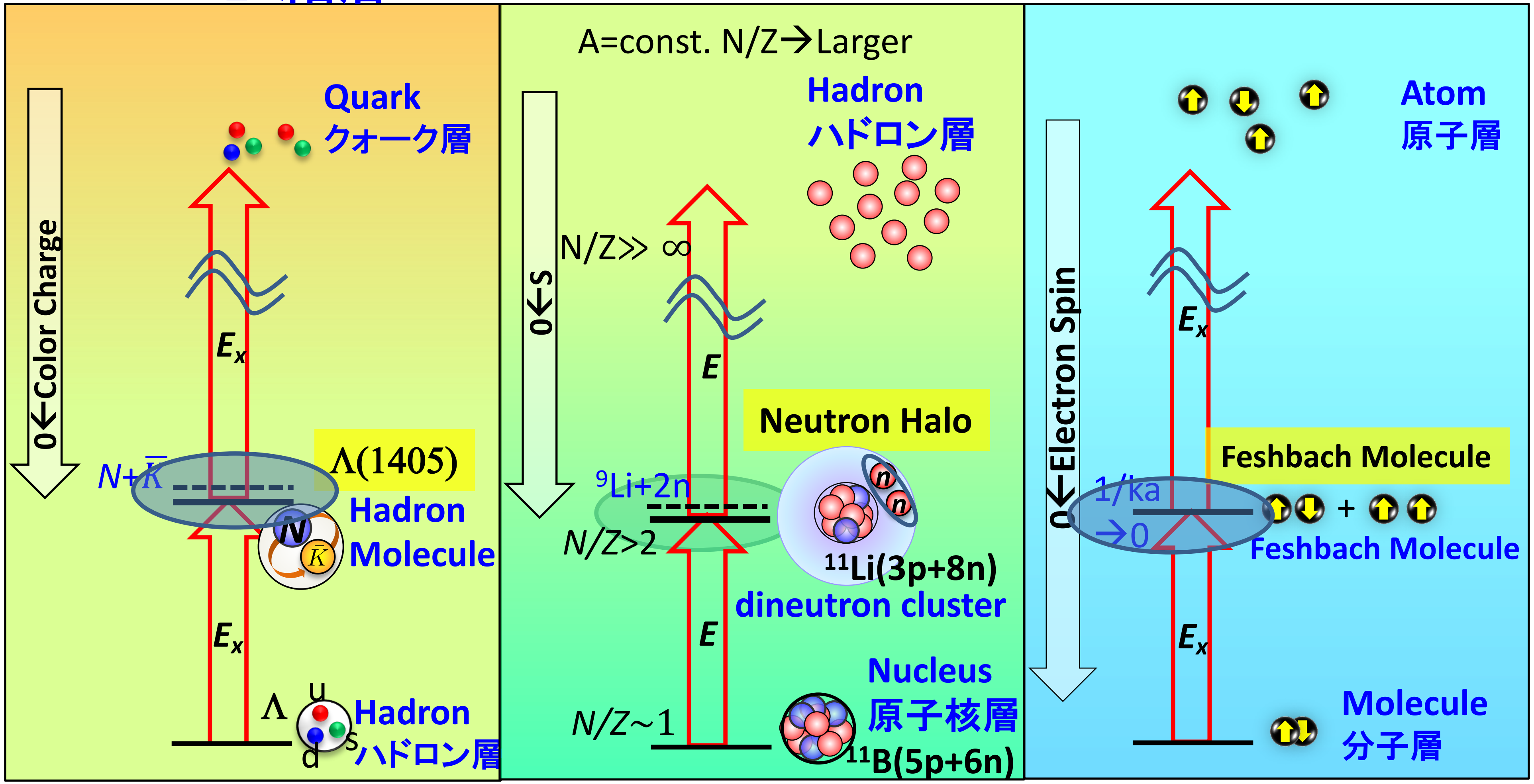
- Summary and Perspective

# Halo Nuclei and Universality

# Nuclear Landscape at the limit (with neutron-halo nuclei)

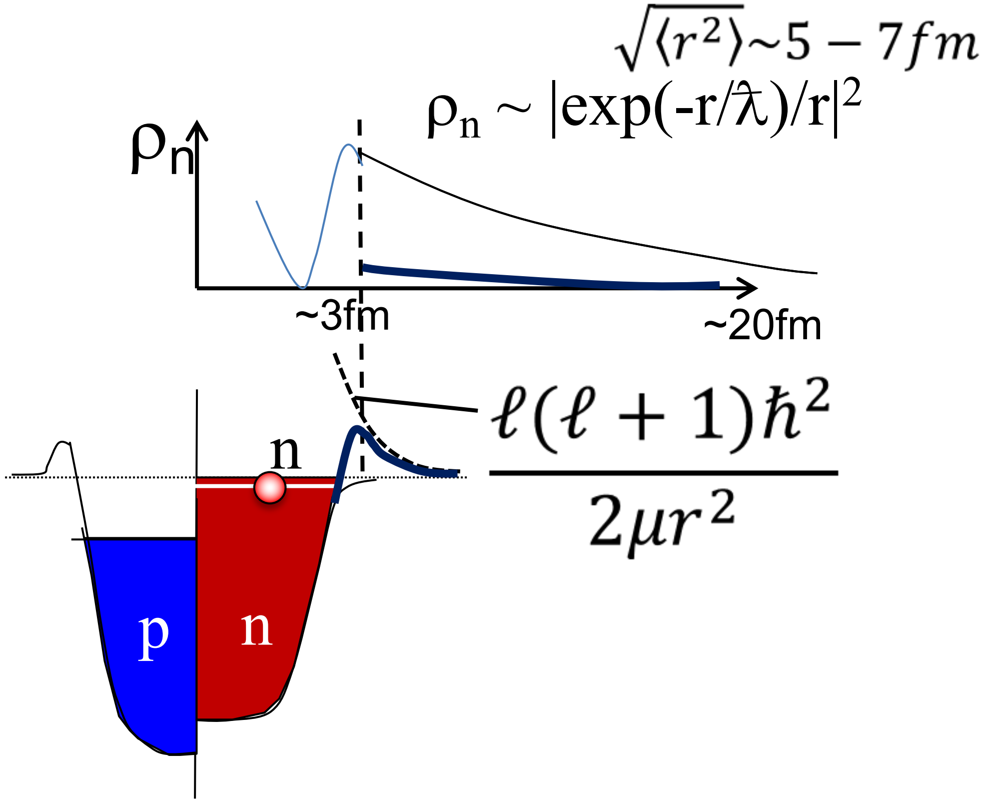


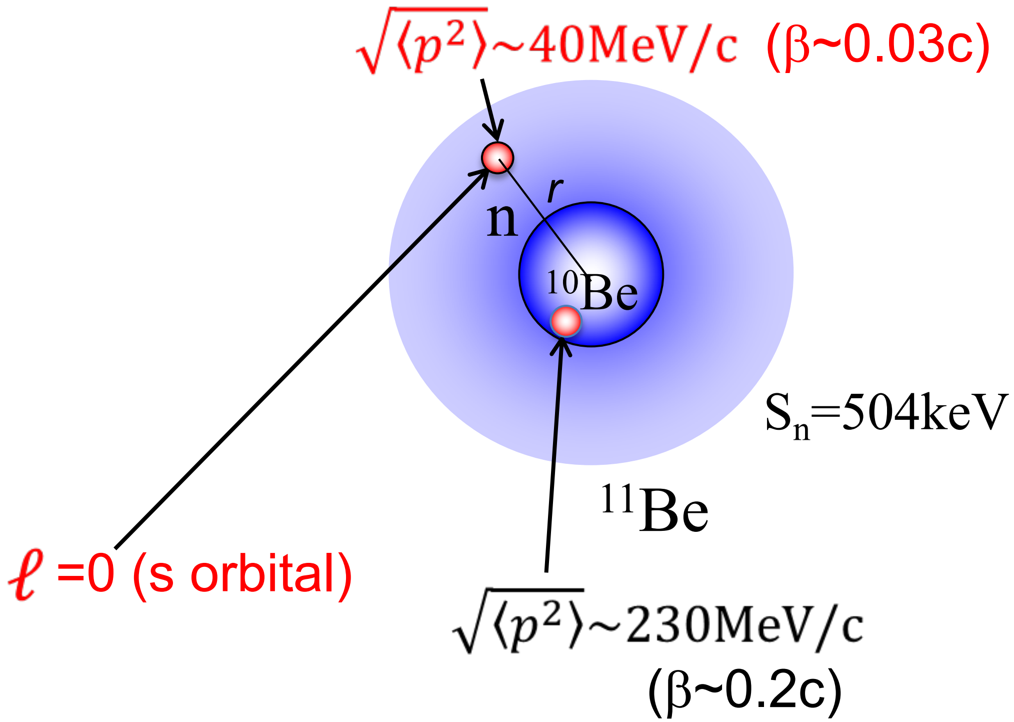
# Semi-Hierarchy: Clustering and Hierarchy of Matter
セミ階層




- ✓ Threshold: Clustering near Threshold → Semi−Hierarchy
- ✓ Degree of Freedom : Neutralization of Charge, Spin(S), Isospin(T)

# One Neutron Halo　1中性子ハロー




$\sqrt{\langle r^2 \rangle}$=5.77(16) fm

N.Fukuda, TN. PRC70, 054606 (2004).

- ✓ Smaller $S_n$:  $S_n$ < 1MeV << 8 MeV (standard value)
- ✓ Small Fermi momentum  (long wave length)
- ✓ Orbital Angular Momentum: l=0, 1 (s or p)
- ✓ Large radius of halo neutron:  5-7 fm > $R_{core}$(2~3 fm)

# Universality in 1n halo nuclei:

S-wave halo: $a_s \sim \lambda\!\!\!^{-}$ $\lambda\!\!\!^{-} = \lambda/2\pi$

**Atom/Molecule (a pair of Fermions)**

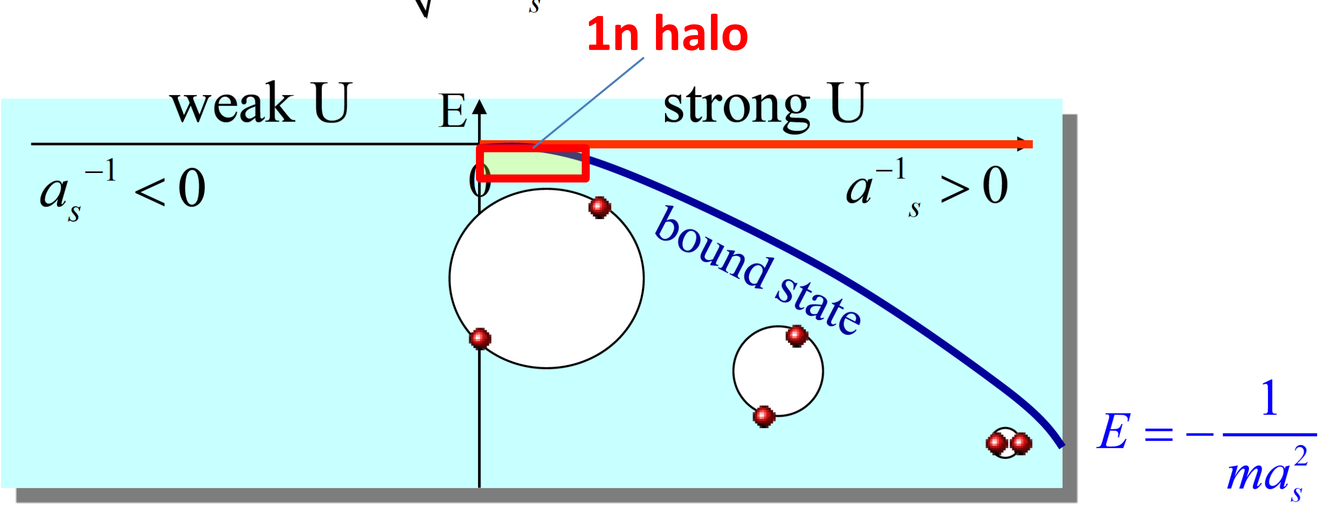
Slide: Y. Ohashi
School Mar.2019

**Bound state solution :** $a_s > 0$

$$E = -\frac{1}{m a_s^2}$$

$$\Psi(\mathbf{R} = \mathbf{r}_1 - \mathbf{r}_2) = \frac{1}{\sqrt{2\pi a_s}} \frac{e^{-R/a_s}}{R}$$

**Molecular size** $\sim a_s$




**S-wave 1n-halo nucleus**

$$E = -S_n = -\frac{\hbar}{2\mu \lambda\!\!\!^{-2}}$$

$$\psi(r) = C \frac{e^{-r/\lambda\!\!\!^{-}}}{r}$$

$$\lambda\!\!\!^{-} = a_s$$

$S_n$=0.502 MeV ($^{11}$Be) → $a_s$=6.77 fm
c.f. sqrt<r$^2$>=5.77(16)fm

S-wave halo nuclei known:
Halo ($S_n$<1 MeV): $^{11}$Be, $^{19}$C
Moderate Halo (1<$S_n$<3 MeV): d, $^{15}$C

c.f. $^{31}$Ne, $^{37}$Mg: p-wave halo

NOTE: $^{11}$Be is NOT a pure $^{10}$Be-n system

s-wave halo component    Non-halo components

$$^{11}\mathrm{Be(g.s)} = \alpha \left| ^{10}\mathrm{Be}(0^+) \otimes \nu(2s_{1/2}) \right\rangle + \beta \left| ^{10}\mathrm{Be}(2^+) \otimes \nu(1d_{5/2}) \right\rangle + \ldots$$

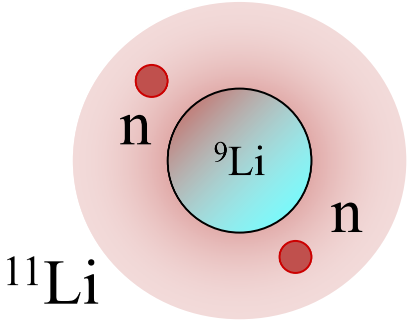
$\alpha^2 = 0.77$   $^{10}$Be(d,p)$^{11}$Be   B.Zwieglinski et al. NPA315,124(1979).

$\alpha^2 = 0.74$   $^{9}$Be($^{11}$Be, $^{10}$Be$\gamma$) X T.Aumann et al. PRL84,35(2000).

$\alpha^2 = 0.72(4)$   Coulomb Breakup   N.Fukuda, TN et. al., PRC2004

Spectroscopic Factor (分光学的因子)

# Two-neutron Halo




$S_{2n}$=0.37MeV

*Borromean Ring*

$^9$Li + n Barely Unbound

$$a_s = -22.4(4.8)\ fm$$

Yu. Aksyustina PLB666,430(2008).

n + n Barely Unbound

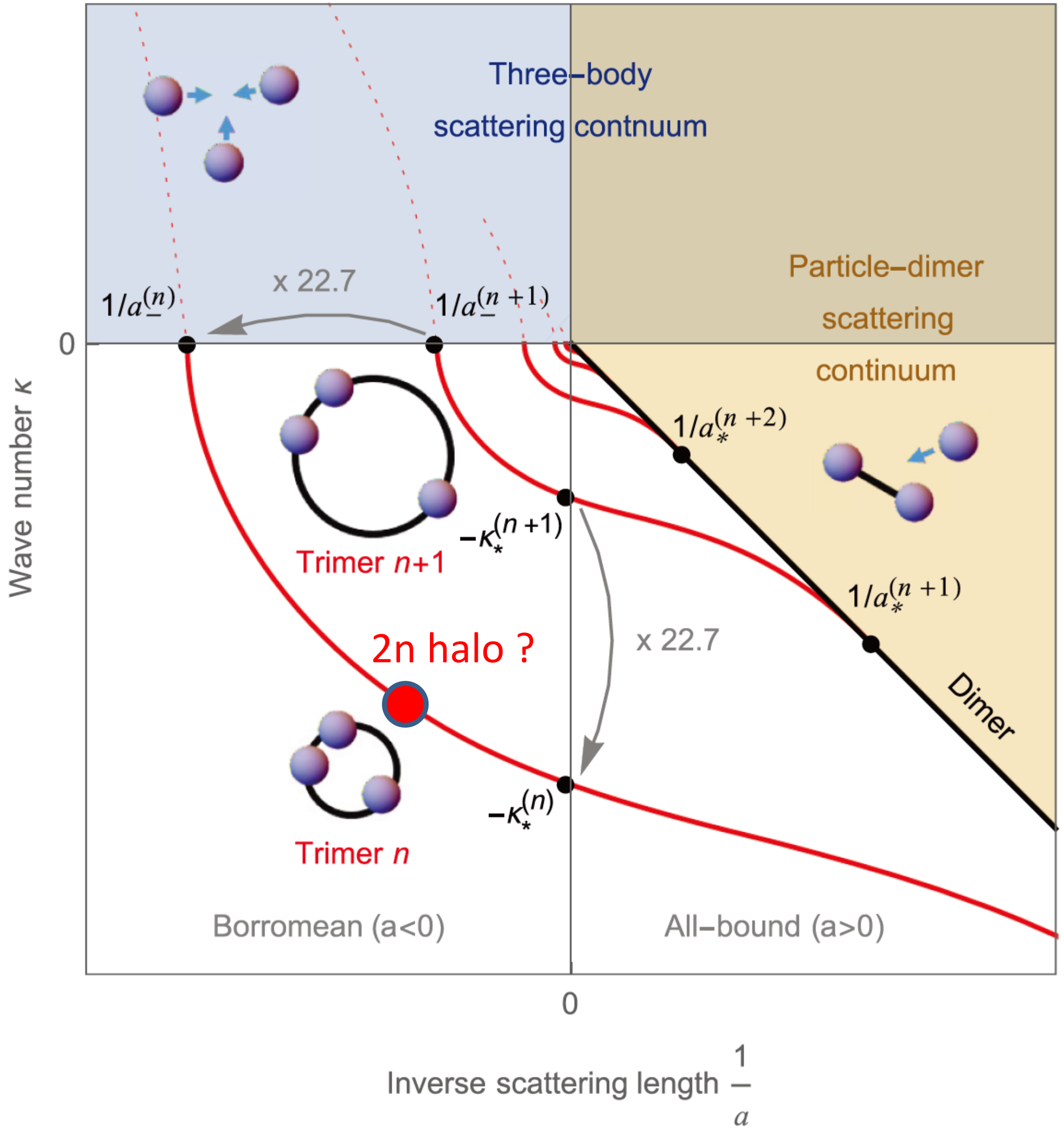
$$a_s = -18.9(4)\ fm$$

$^9$Li + n+n Bound

$S_{2n}$=0.369MeV

- ✓ Smaller $S_n$: $S_n$ < 1MeV << 8 MeV (standard value)
- ✓ Small Fermi momentum (long wave length)
- ✓ Orbital Angular Momentum: l=0, 1 (s or p)
- ✓ Large radius of halo neutron: 5-7 fm > $R_{core}$(2~3 fm)
- ✓ Any of the two-body constituents are UNBOUND

# Universality in 2n halo nuclei:




P.Naidon, S. Endo, Rep. Prog. Phys. 80, 056001

<u>2n Halo Nuclei</u>: Efimov States?

Likely Efimov Ground State,

But no Efimov resonances

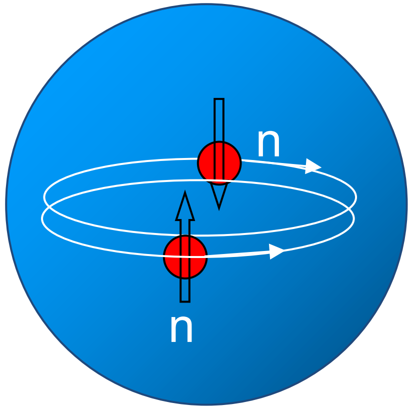
How about unbound resonance?

# *Dineutron Correlation?*

**What happens if "a pair of neutrons" in the external field?**

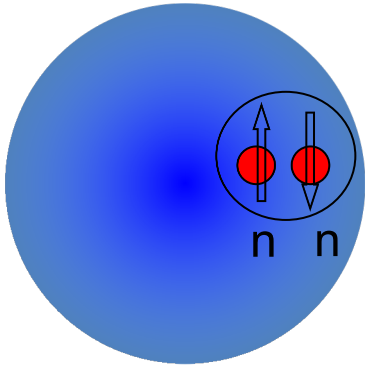
Stable Nuclei

BCS (long range)

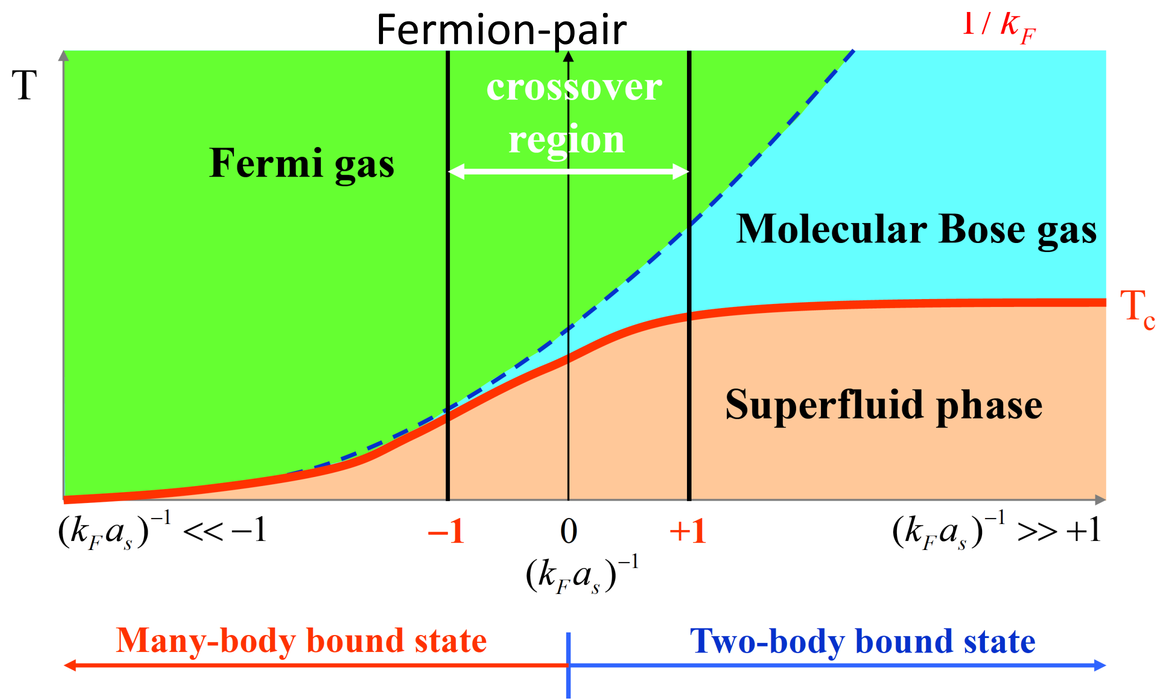



Halo (low density)

BEC/Crossover (short range)? (So far indirect evidence only)




A.B.Migdal 1973,
M. Matsuo 2010,
K. Hagino, H. Sagawa 2015
…

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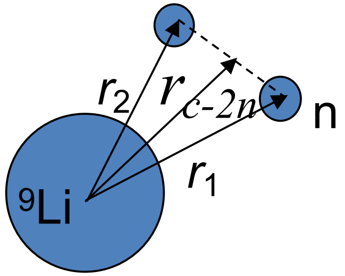
NOTE: $^{11}$Li is NOT a pure s-wave three-wave system




$$\Psi(^{11}Li_{g.s.}) = \Psi(^{9}Li_{g.s.}) \otimes [\alpha|(2s)^2\rangle + \beta|(1p)^2\rangle + \gamma|(1d)^2\rangle \ldots]$$

s-wave component

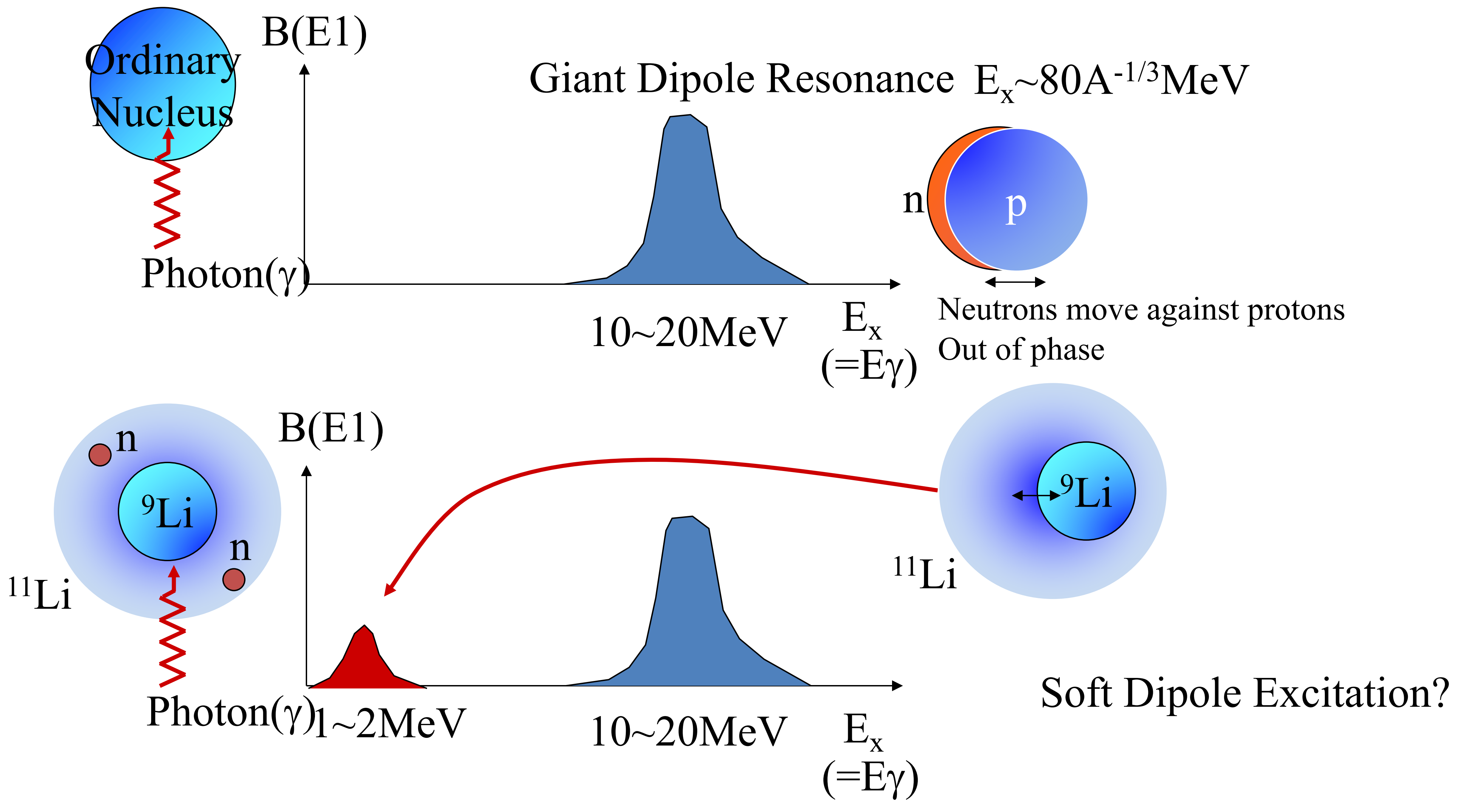
45(10)% ~50%

H. Simon

Phys.Rev.Lett.83,496(1999)

# Electric Dipole (E1) Response of Halo Nuclei

# When a nucleus absorbs a photon



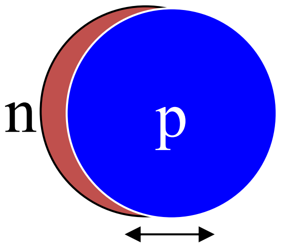
Rev.

# 原子核の電気双極子励起

## GDR( Giant Dipole Resonance)

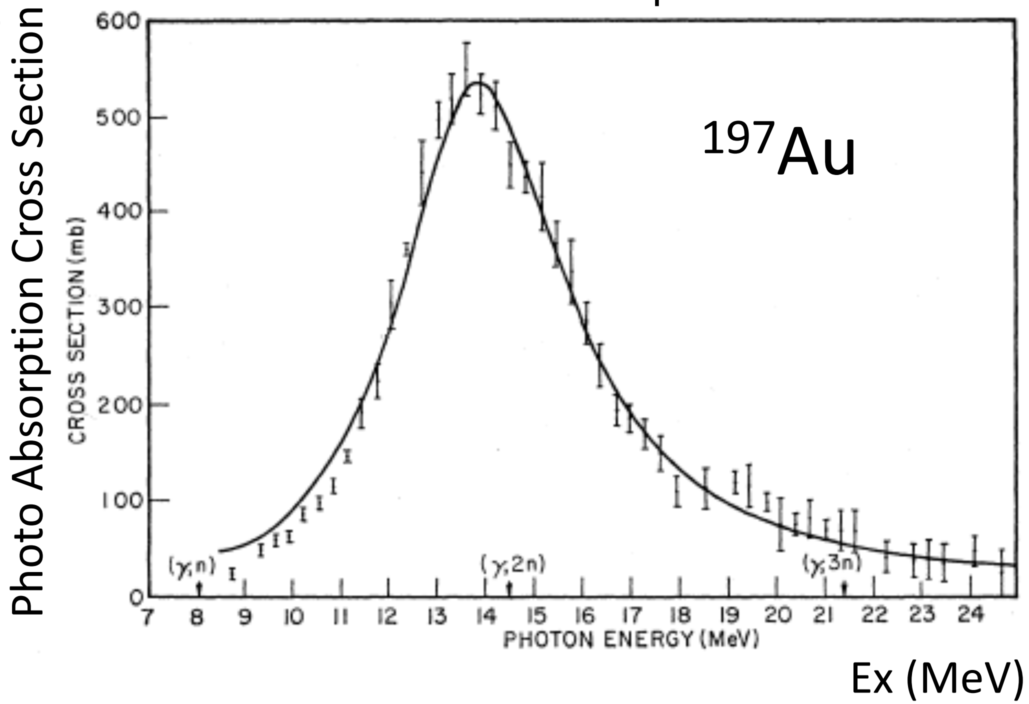
Dominant

$E_x \sim 80A^{-1/3}\mathrm{MeV}$




Giant
Dipole
Resonance
(GDR)

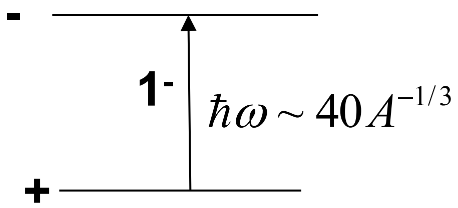
S.C.Fultz et al., PR127,1273(1962).
Bohr Mottelson II p. 475




## In Most Nuclei:

**E1 Excitation Strength is exhausted by GDR at high energies**

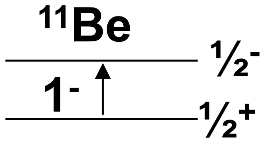
**(No E1 excitation at low energies!)**




E1: transition Between two major shells

**Strongest E1 excitation so far known**

T.Nakamura et al., PLB 394,11(1997).
S.S.Hanna et al., PRC3, 2198 (1971),
D.J.Millener et al., PRC28, 497(1983)




$$B(E1) = 0.099(10)e^2\mathrm{fm}^2 = 0.31(3)W.u.$$

Review

# Reduced Transition Probability $B(El)$

換算遷移確率

$$B(El) = \sum_{M_i, M_f, m} \left| \langle I_f M_f | O(Elm) | I_i M_i \rangle \right|^2 = \frac{1}{2I_i + 1} \left| \langle I_f \| O(El) \| I_i \rangle \right|^2$$

$$O(Elm) = Ze \sum_{k=1}^{Z} r_k^l Y_m^l(\Omega)$$

Wigner Eckart's Theorem

$$\langle I_f M_f | O(Elm) | I_i M_i \rangle = \frac{1}{\sqrt{2I_f + 1}} \langle I_f \| O(El) \| I_i \rangle (I_i M_i lm | I_f M_f)$$

Reduced Matrix element

Clebsch Gordan Coefficients (Geometry)

Unit: $e^2 fm^{2l}$

$|I_i M_i\rangle \xrightarrow[El]{} |I_f M_f\rangle$ Probability of γ-ray transition with $El$

## B(E1) E1遷移の行例要素

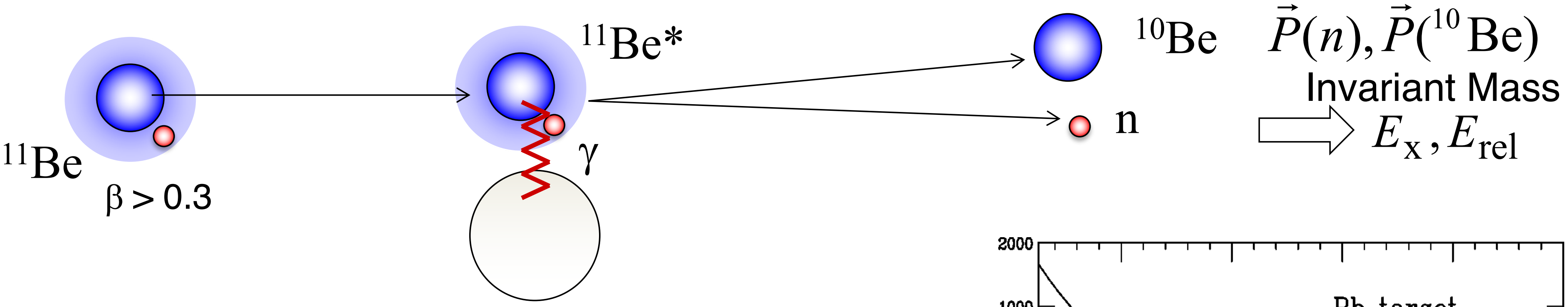
e.g.

For γ-ray decay

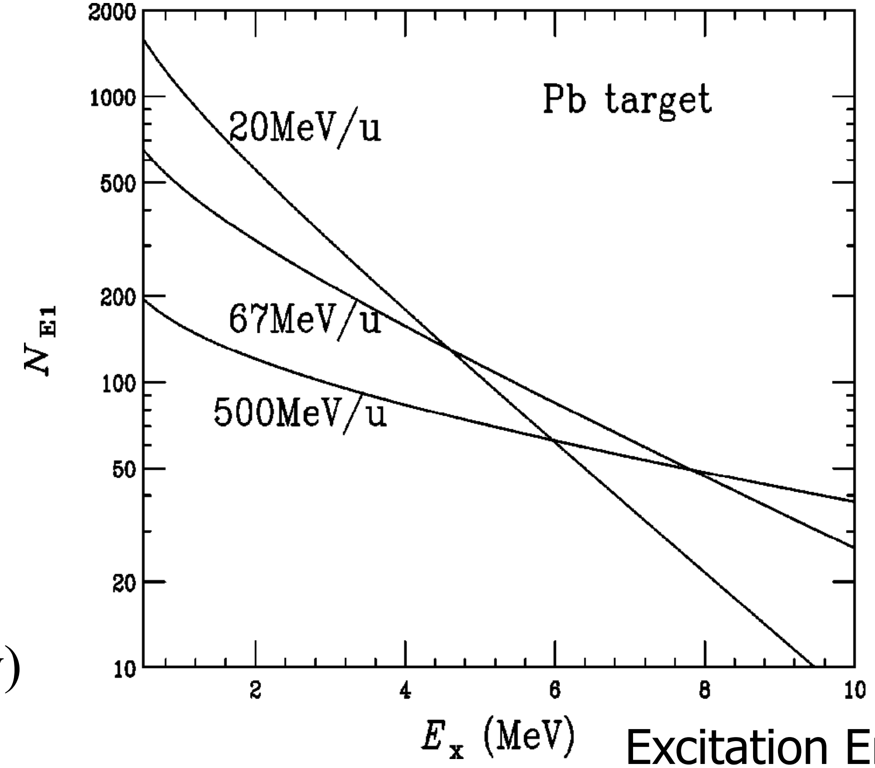
$$T = \frac{8\pi(l+1)}{l[(2l+1)!!]^2 \hbar} \left( \frac{E_\gamma}{\hbar c} \right)^{2l+1} B(El)$$

Transition Probability =1/(Life time)

# Coulomb Breakup(Dissociation) クーロン分解




Heavy Target (Pb)




Excitation Energy (=Photon Energy)

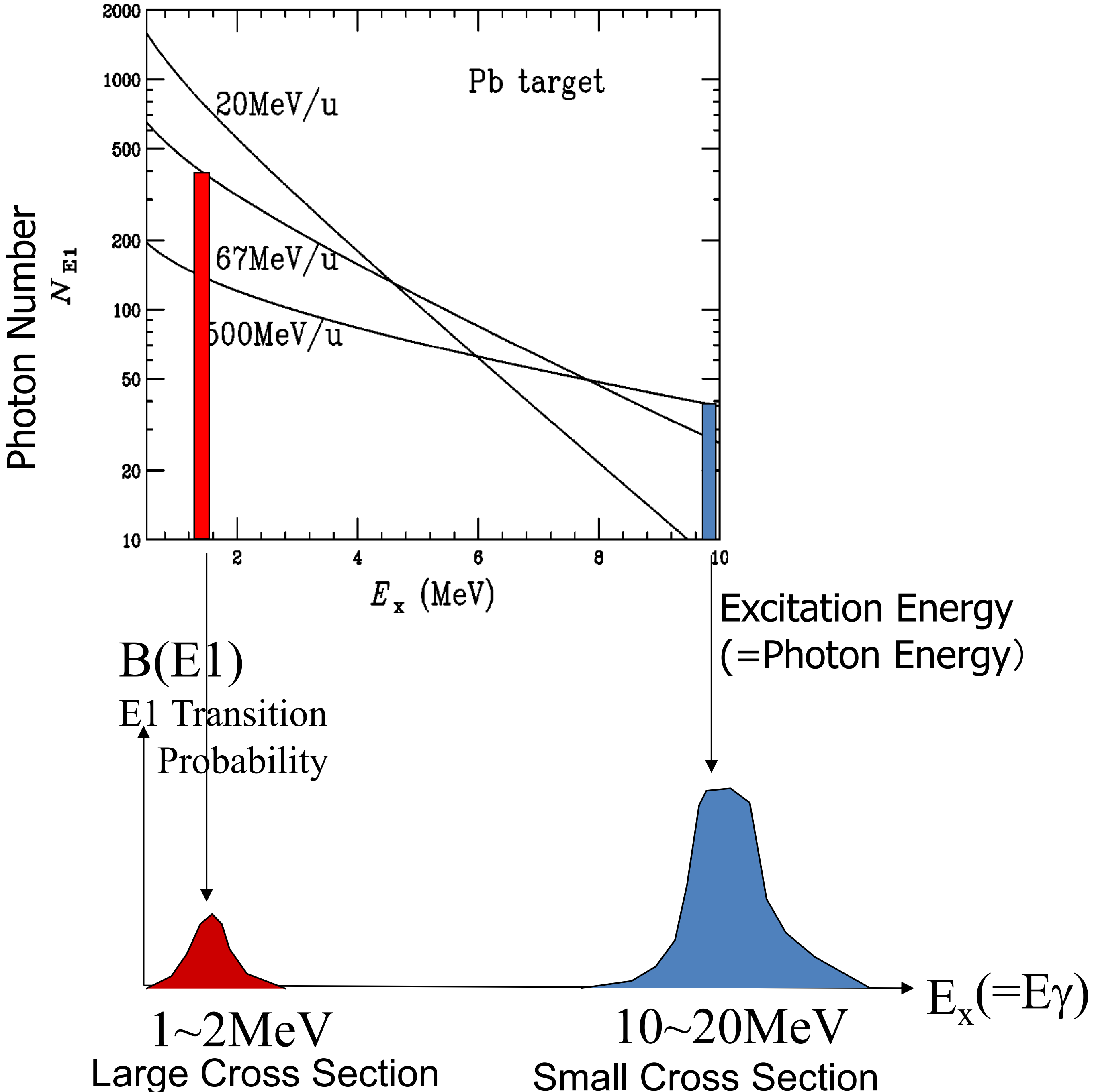
- Excitation by a Virtual Photon

$$\frac{d\sigma_{CD}}{dE_x} = \frac{16\pi^3}{9\hbar c} N_{E1}(E_x) \frac{dB(E1)}{dE_x}$$

Cross Section = (Photon Number) x (Transition Probability)

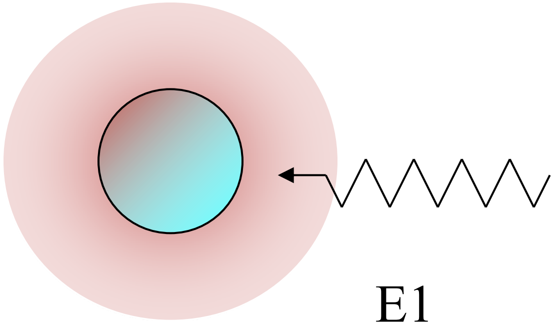
- Invariant Mass Spectroscopy

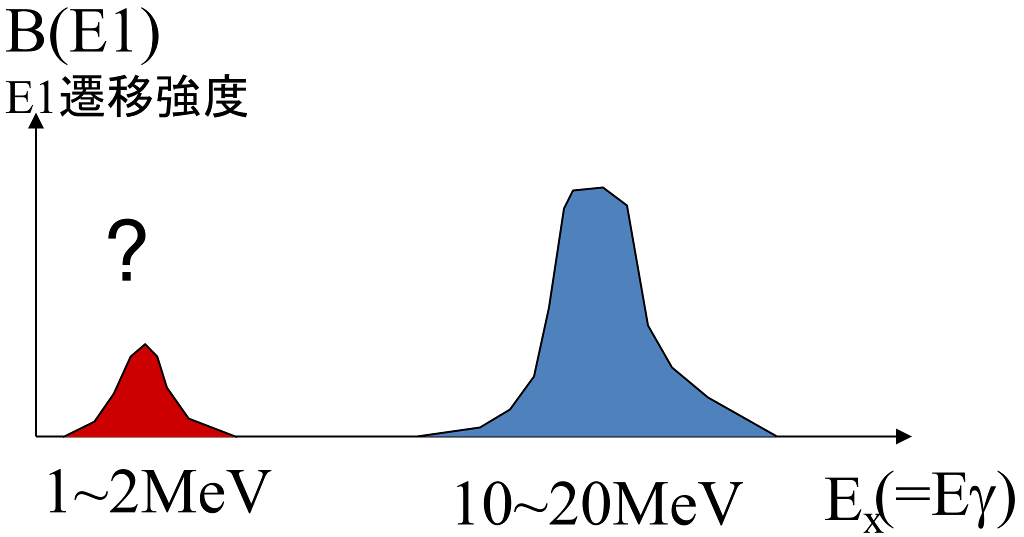
Coulomb Breakup
→Higher Sensitivity
at Low Excitation Energies
→Sensitive to Soft Mode



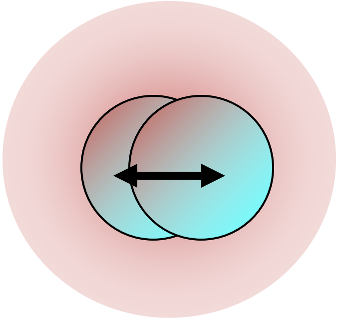
# Coulomb breakup and
# E1 Response of 1n-halo nuclei

# Mechanism of Soft E1



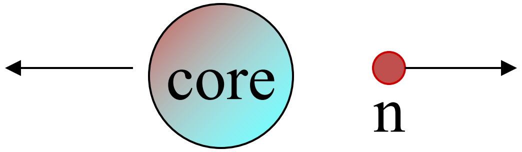




- Soft Dipole Resonance
（ソフトE１共鳴）



Slow Vibration
of core against halo
(Ikeda model)

- Direct Coulomb Breakup（直接分解）




$$\frac{dB(\mathrm{E1})}{dE_\mathrm{x}} \propto \left| \langle \exp(i\boldsymbol{q}\boldsymbol{r}) \right| \frac{Z}{A} \mathrm{r} \mathrm{Y}^1{}_\mathrm{m} \left| \Phi_\mathrm{gs} \rangle \right|^2$$

$$\propto \frac{\sqrt{S_\mathrm{n}}\,(E_\mathrm{x} - S_\mathrm{n})^{3/2}}{E_\mathrm{x}{}^4}$$

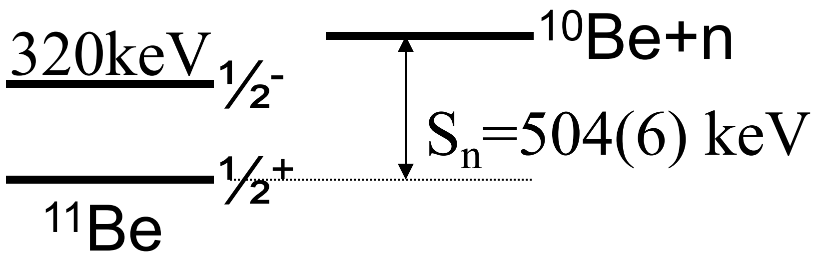
(plane wave approx.
& Yukawa w.f. for s-wave halo)

$$E_\mathrm{x}(\mathrm{Peak}) \propto \frac{8}{5} S_\mathrm{n} \quad B(\mathrm{E1}) \propto 1/\,S_\mathrm{n}$$

# $^{11}$Be

→ Suitable for studying basic mechanisms for halo phenomena

## Well-Known Structure

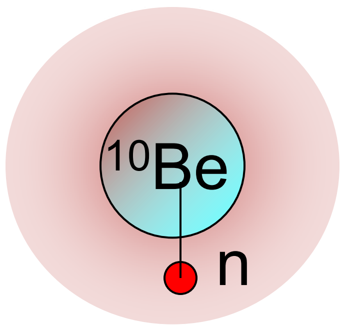



$$^{11}\mathrm{Be(g.s)} = \alpha \left| ^{10}\mathrm{Be}(0^+) \otimes \nu(2s_{1/2}) \right\rangle + \beta \left| ^{10}\mathrm{Be}(2^+) \otimes \nu(1d_{5/2}) \right\rangle$$

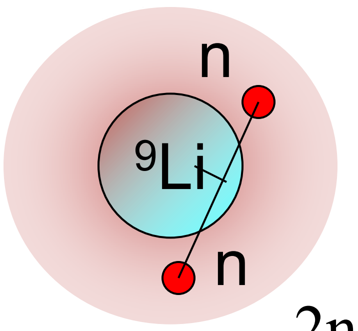
$\alpha^2$=0.77 $^{10}$Be(d,p)$^{11}$Be B.Zwieglinski et al. NPA315,124(1979).

$\alpha^2$=0.74 $^{9}$Be($^{11}$Be, $^{10}$Be$\gamma$) X T.Aumann et al. PRL84,35(2000).

## Simple One-neutron Halo Nucleus




$^{11}$Be

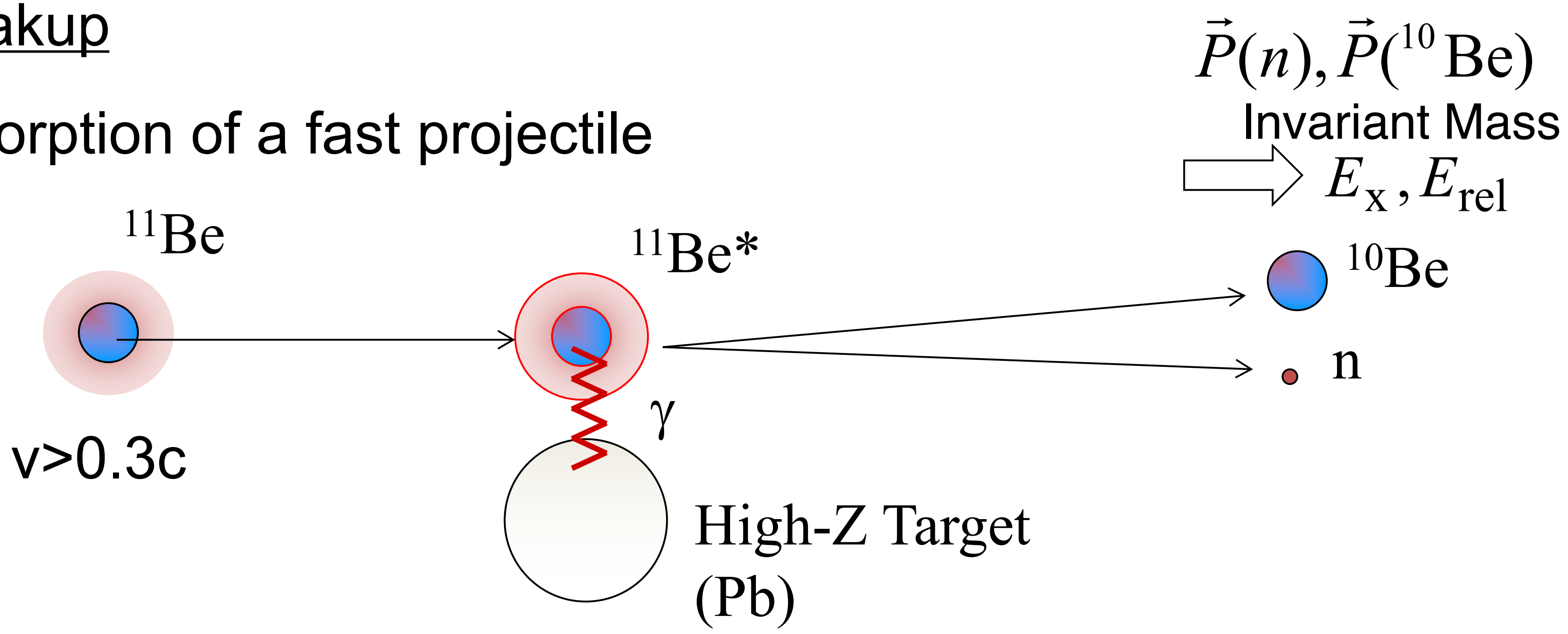



2n correlation?

$^{11}$Li

# Coulomb Breakup

→Photon absorption of a fast projectile

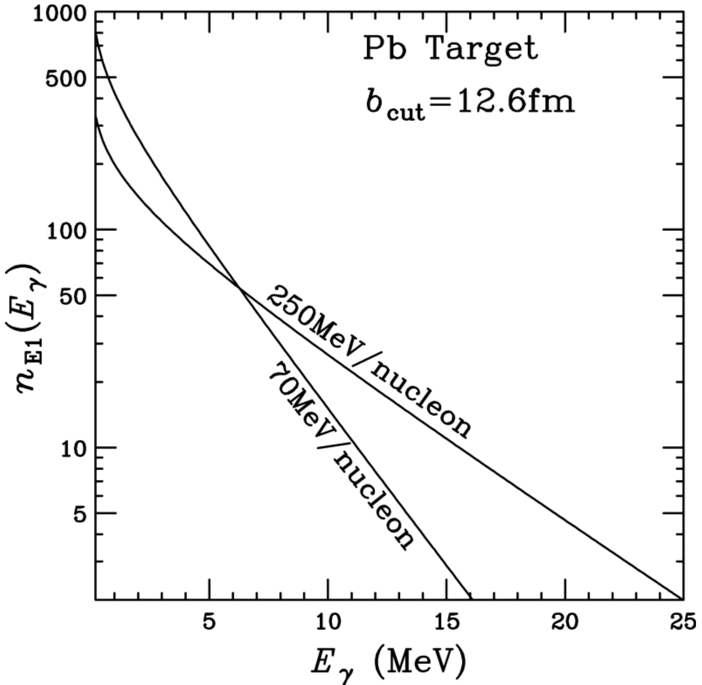



**Equivalent Photon Method**

$$\frac{d\sigma_{CB}}{dE_x} = \frac{16\pi^3}{9\hbar c} N_{E1}(E_x) \frac{dB(E1)}{dE_x}$$

Cross section = (Photon Number)x(Transition Probability)

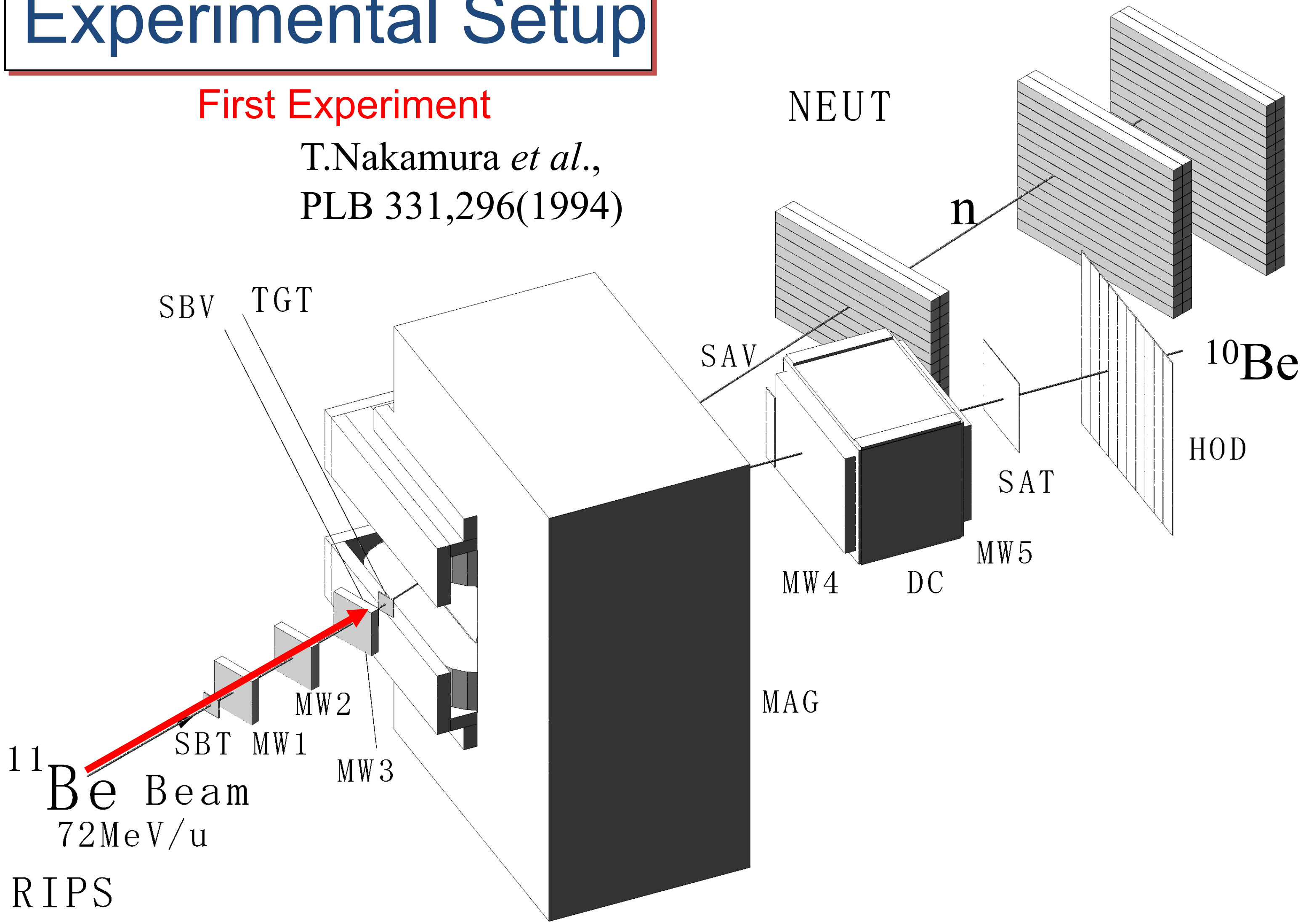
C.A. Bertulani, G. Baur, Phys. Rep. 163,299(1988).



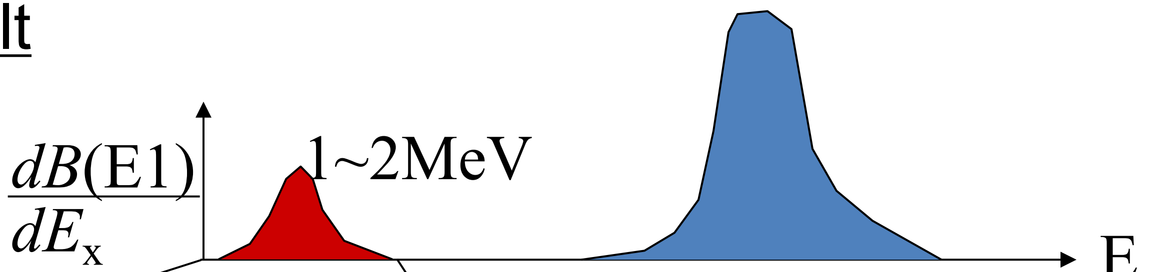
# Experimental Setup

First Experiment

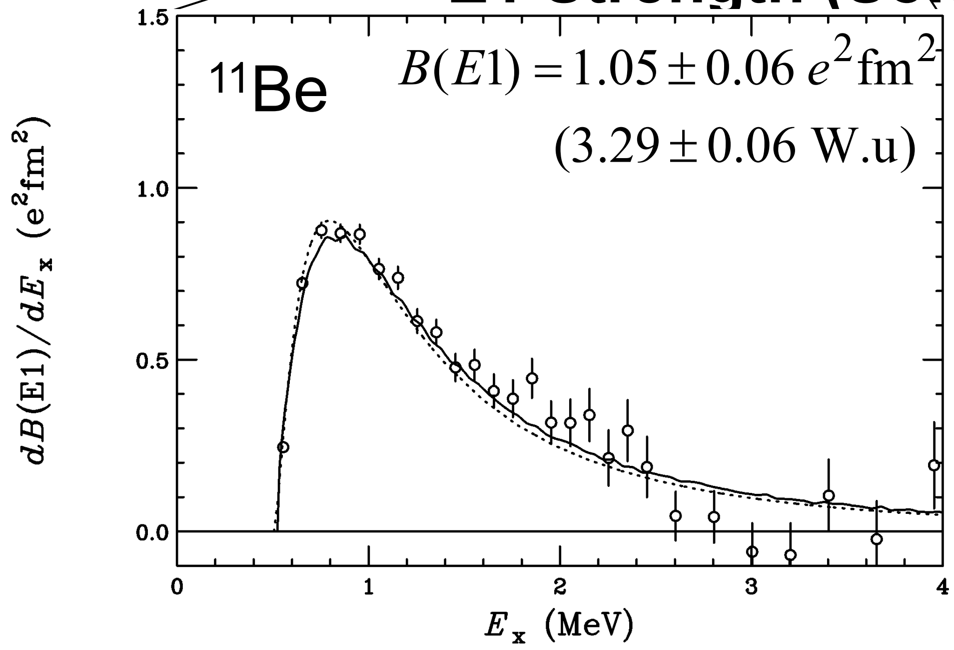
T.Nakamura *et al.*,
PLB 331,296(1994)



# E1 Response of $^{11}$Be: Result

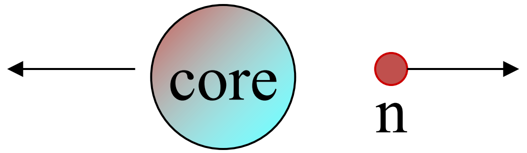



**Low-lying**
**E1 Strength (Soft E1 excitation)**



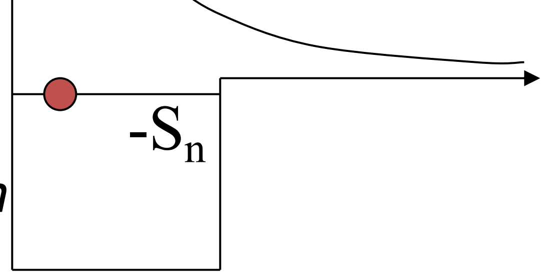

N.Fukuda, TN et al., PRC70, 054606 (2004)
TN *et al.*,PLB 331,296(1994)
Palit et al., PRC68, 034318(2003)

Direct Breakup Mechanism




$\rho \sim C^2S|\exp(-r/\lambda)/r|^2$

$C^2S = 0.72 \pm 0.04$



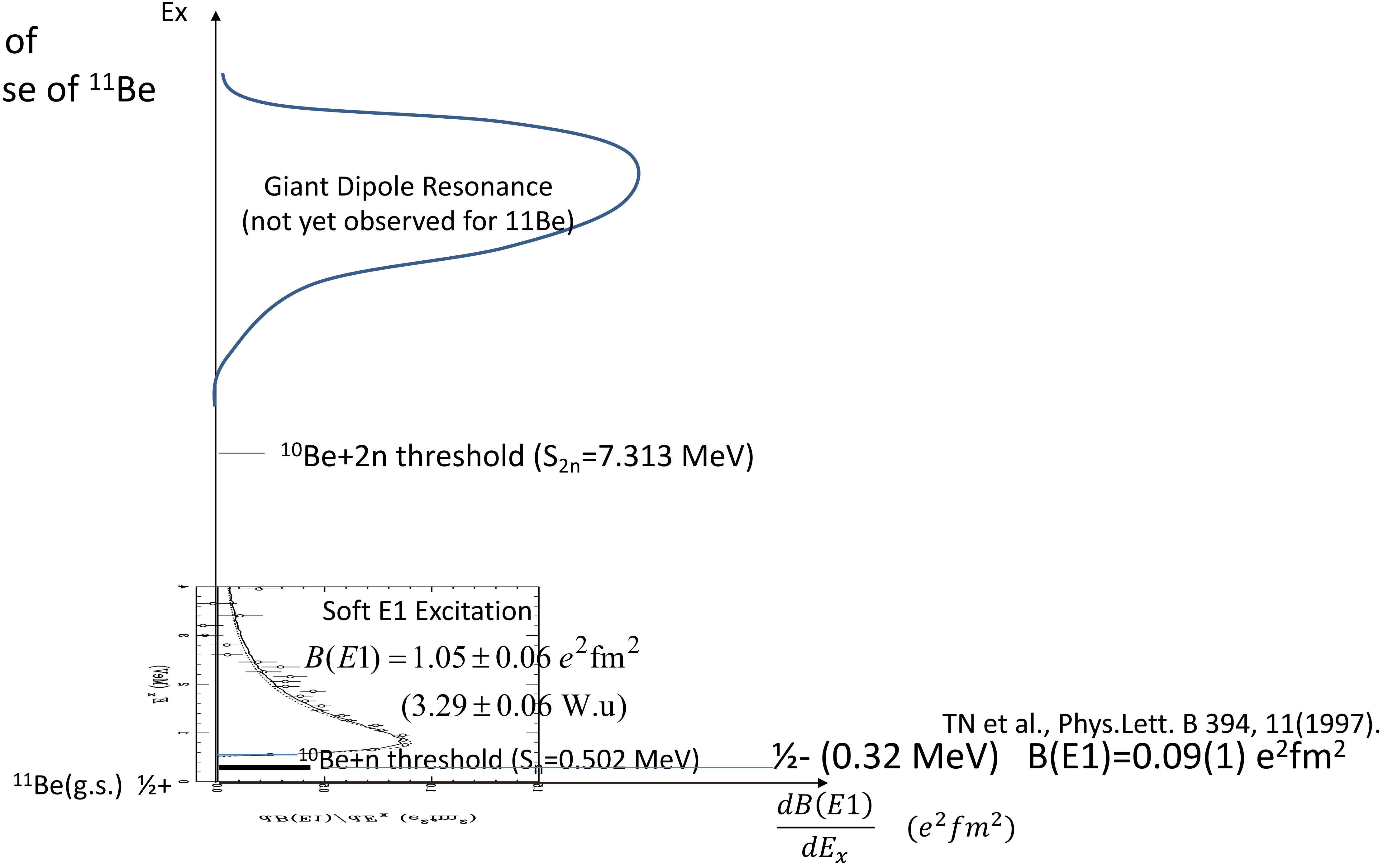

$$\frac{dB(E1)}{dE_x} \propto |\langle \exp(i\boldsymbol{qr})|\frac{Z}{A}rY^1{}_m|\Phi_{gs}\rangle|^2$$
$$\propto C^2S|\langle \exp(i\boldsymbol{qr})|\frac{Z}{A}rY^1{}_m|s_{1/2}\rangle|^2$$

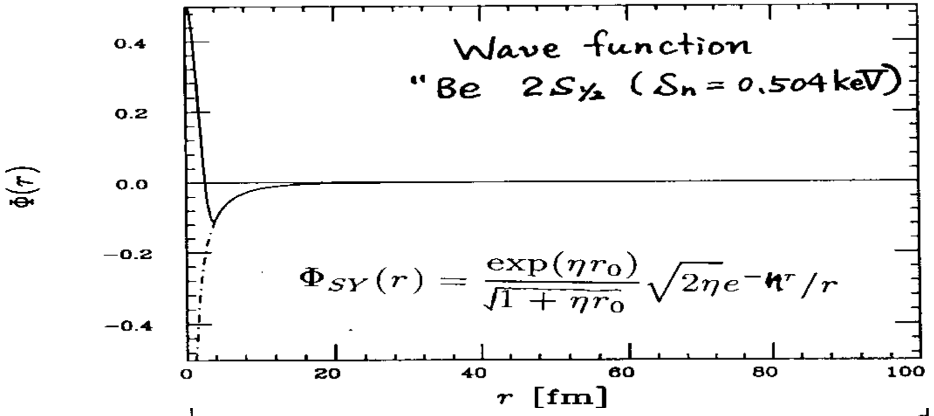
*Fourier Transform*

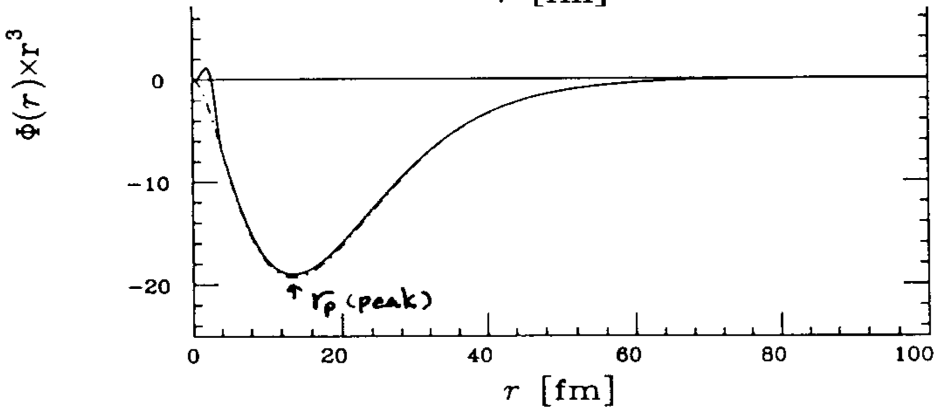
E1 Strength

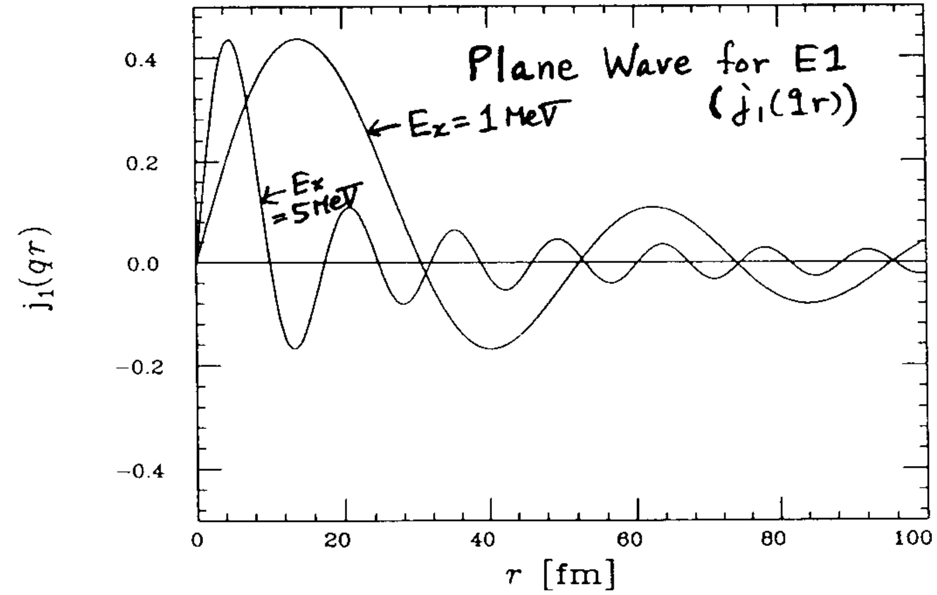
Halo State

# Summary of E1 response of $^{11}Be$






B(E1)
Sensitive strongly the Radial
Wave function of the valence neutron

$$B(E1) \propto \left| \left\langle e^{iqr} \left| rY_m^1 \right| \Phi(r) \right\rangle \right|^2$$

$$\left\langle e^{iqr} \left| rY_m^1 \right| \Phi(r) \right\rangle \propto \int e^{iqr} r^3 \Phi(r) dr$$

c.f. T.Otsuka et al., PRC49, R2289 (1994).

# B(E1) and Sum Rule

- Energy Weighted Sum Rule (TRK Sum Rule)

$$\int \sigma_\gamma(E_\gamma)dE_\gamma = \int \frac{16\pi^3}{9\hbar c} E_{\mathrm{x}} \frac{dB(E1)}{dE_{\mathrm{x}}} dE_{\mathrm{x}} = \frac{9}{4\pi}\frac{\hbar^2 e^2}{2m}\frac{NZ}{A}$$

38.1 $e^2$fm$^2$MeV for $^{11}$Be

- Cluster sum rule Y.Alhassid, M.Gai, and G.F.Bertsch PRL49,1482(1982)

Sum$=60\frac{NZ}{A} - 60\frac{N_c Z_c}{A_c} = 2.18\ e^2\mathrm{fm}^2\mathrm{MeV}$ For $^{11}$Be

Experiment ($E_x$<4 MeV)

$Sum = 1.52 \pm 0.22\ e^2\mathrm{fm}^2\mathrm{MeV}$= 4.0(5) % of TRK Sum =70(10) % of Cluster Sum

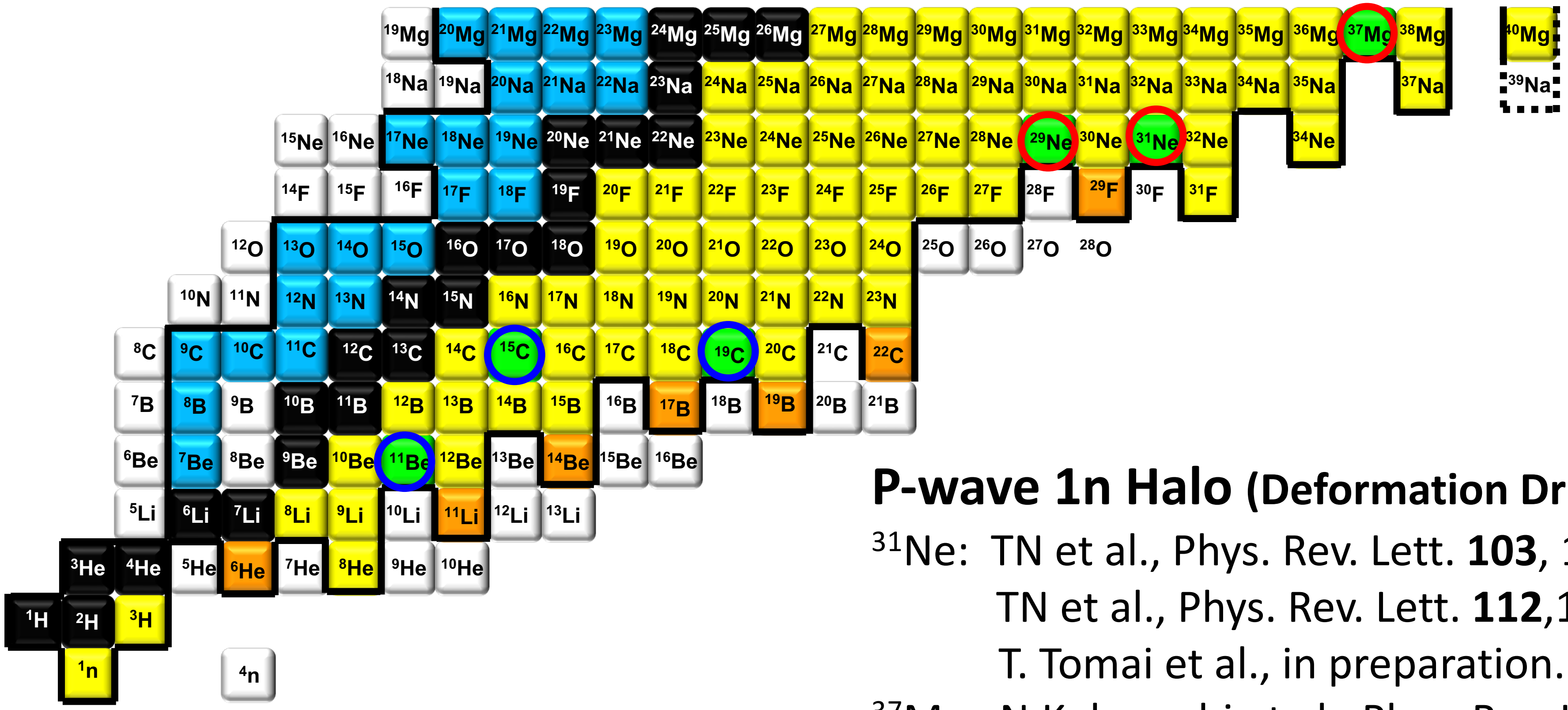
~ Spectroscopic Factor

- Non Energy Weighted Cluster Sum Rule H.Esbensen et al.,NPA542,310(1992)

$$B(E1) = \int_0^\infty \frac{dB(E1)}{dE_x} dE_x = \frac{3}{4\pi}\left(\frac{Ze}{A}\right)^2 \langle r^2 \rangle$$

Experiment: $B(E1) = 1.05 \pm 0.06\ e^2\mathrm{fm}^2 \Rightarrow \sqrt{\langle r^2 \rangle} = 5.77 \pm 0.16\mathrm{fm}$

# Coulomb Breakup of 1n Halo:




## P-wave 1n Halo (Deformation Driven Halo 変形ハロー)

$^{31}$Ne: TN et al., Phys. Rev. Lett. **103**, 1112 (1999).
TN et al., Phys. Rev. Lett. **112**,142501(2014).
T. Tomai et al., in preparation.

$^{37}$Mg: N.Kobayashi et al., Phys. Rev. Lett. **112**, 242501(2014).

$^{29}$Ne: N.Kobayashi et al., Phys. Rev. C**93**, 014613 (2016).
小林信之:日本物理学会若手奨励賞2020

## S-wave 1n Halo

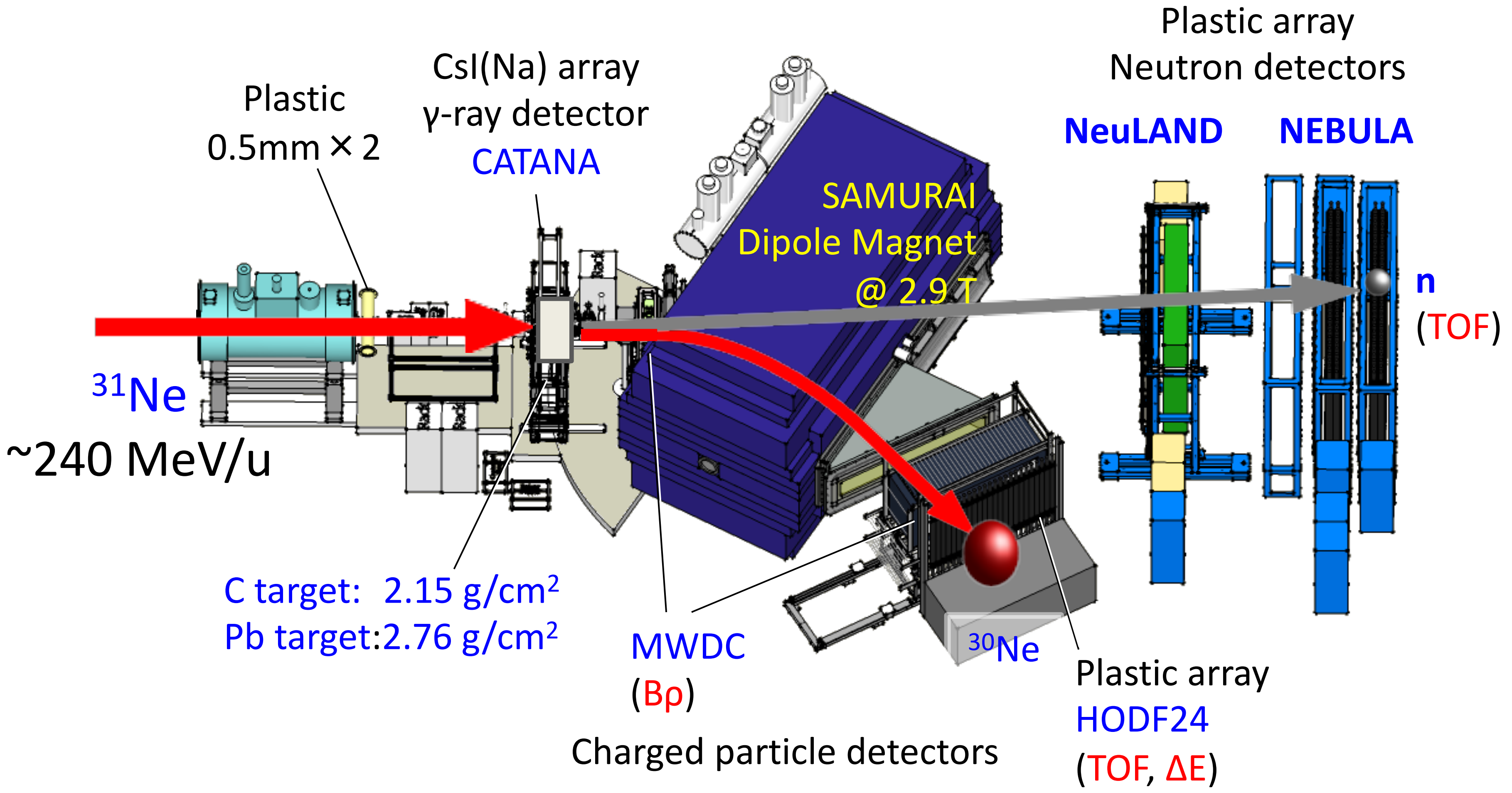
$^{19}$C: TN et al., Phys. Rev. Lett. **83**, 1112 (1999).

$^{15}$C: TN et al., Phys. Rev C **79**, 035805 (2009).

$^{11}$Be N.Fukuda et al., Phys. Rev. C**70**, 054606 (2004).
TN et al., Phys. Lett. B **331**, 296 (1994).

# Coulomb Breakup Measurement of $^{31}$Ne **T.Tomai et al.**

@SAMURAI@RIBF




Slide by T.Tomai

# Interim Summary (E1 Response of 1n Halo)

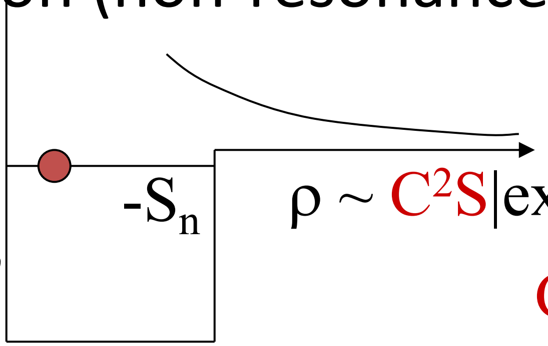
- E1 Response of 1n Halo Nuclei → Soft E1 Excitation (non-resonance origin)

$$\frac{dB(E1)}{dE_x} \propto |\langle \exp(i\boldsymbol{qr})|\frac{Z}{A} rY^1{}_m|\Phi_{gs}\rangle|^2$$

$$\propto C^2S|\langle \exp(i\boldsymbol{qr})|\frac{Z}{A} rY^1{}_m|s_{1/2}\rangle|^2$$

E1 Strength

⇔ *Fourier Transform*




$\rho \sim C^2S|\exp(-r/\lambda)/r|^2$

$C^2S = 0.72 \pm 0.0$

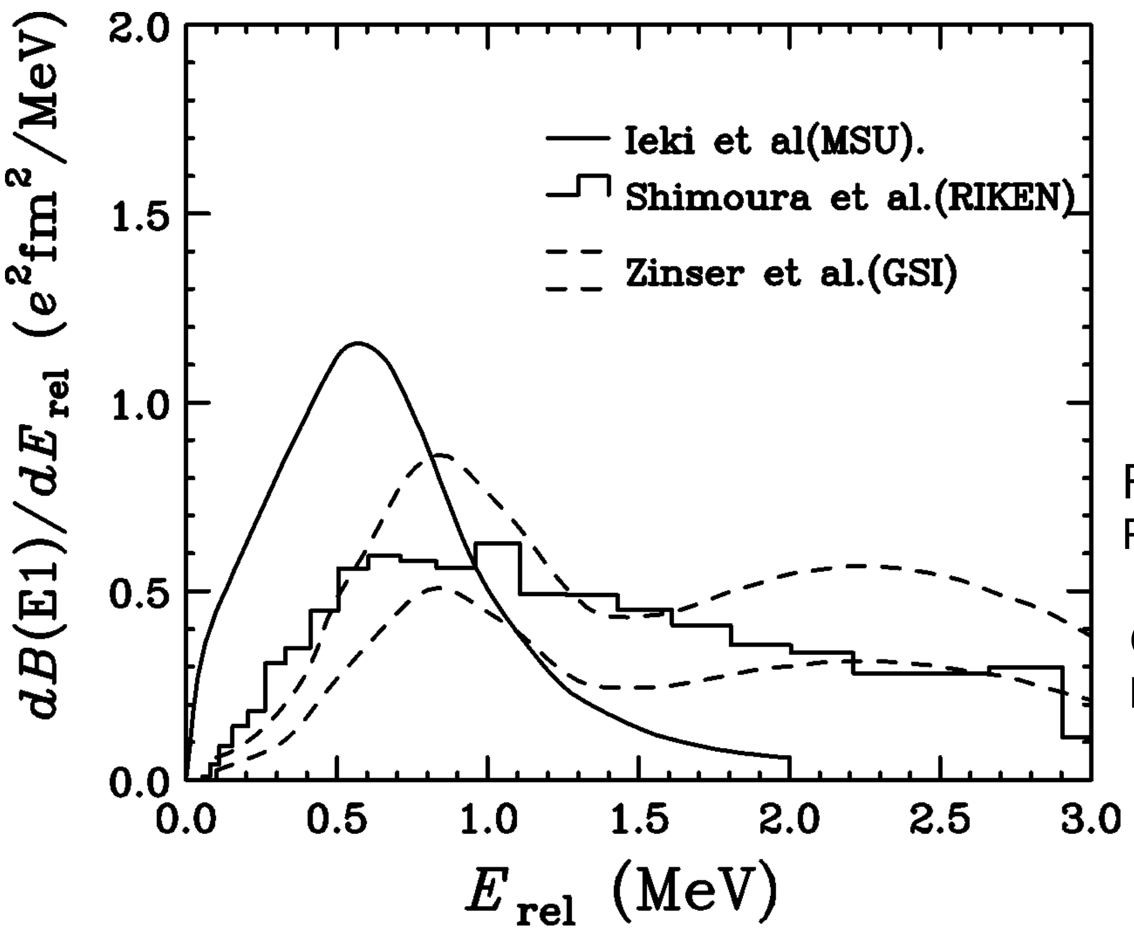
Halo State $^{11}$Be

- Spectroscopic tool for Halo state: $C^2S$, $S_n$, l
- Non-energy-weighted sum-rule → $\langle r^2 \rangle$
- Energy-weighted sum rule → Degree of Cluster (分離度）
- Deformation driven p-wave halo → Double-component halo?
- Universality: s-wave halo ←→ ultra-cold atom near unitary limit

# Coulomb breakup and
# E1 Response of 2n-halo nuclei

Focusing on $^{11}Li$ Result

# Coulomb Breakup of $^{11}$Li
# (Summary of Previous Results)



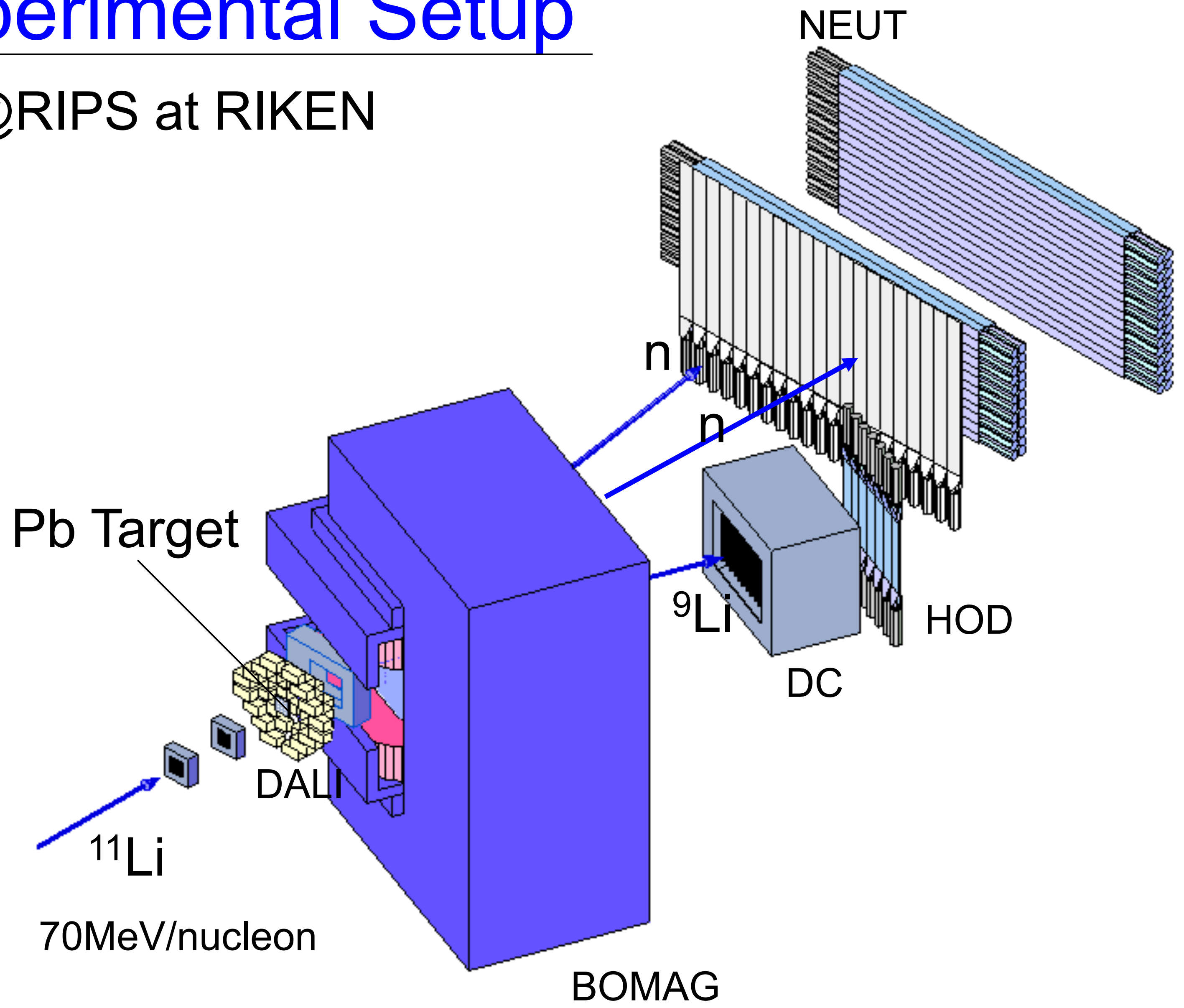

MSU@ 28MeV/nucleon
PRL 70 (1993) 730.
PRC 48(1993) 118.

RIKEN @ 43MeV/nucleon
PLB348 (1995) 29.

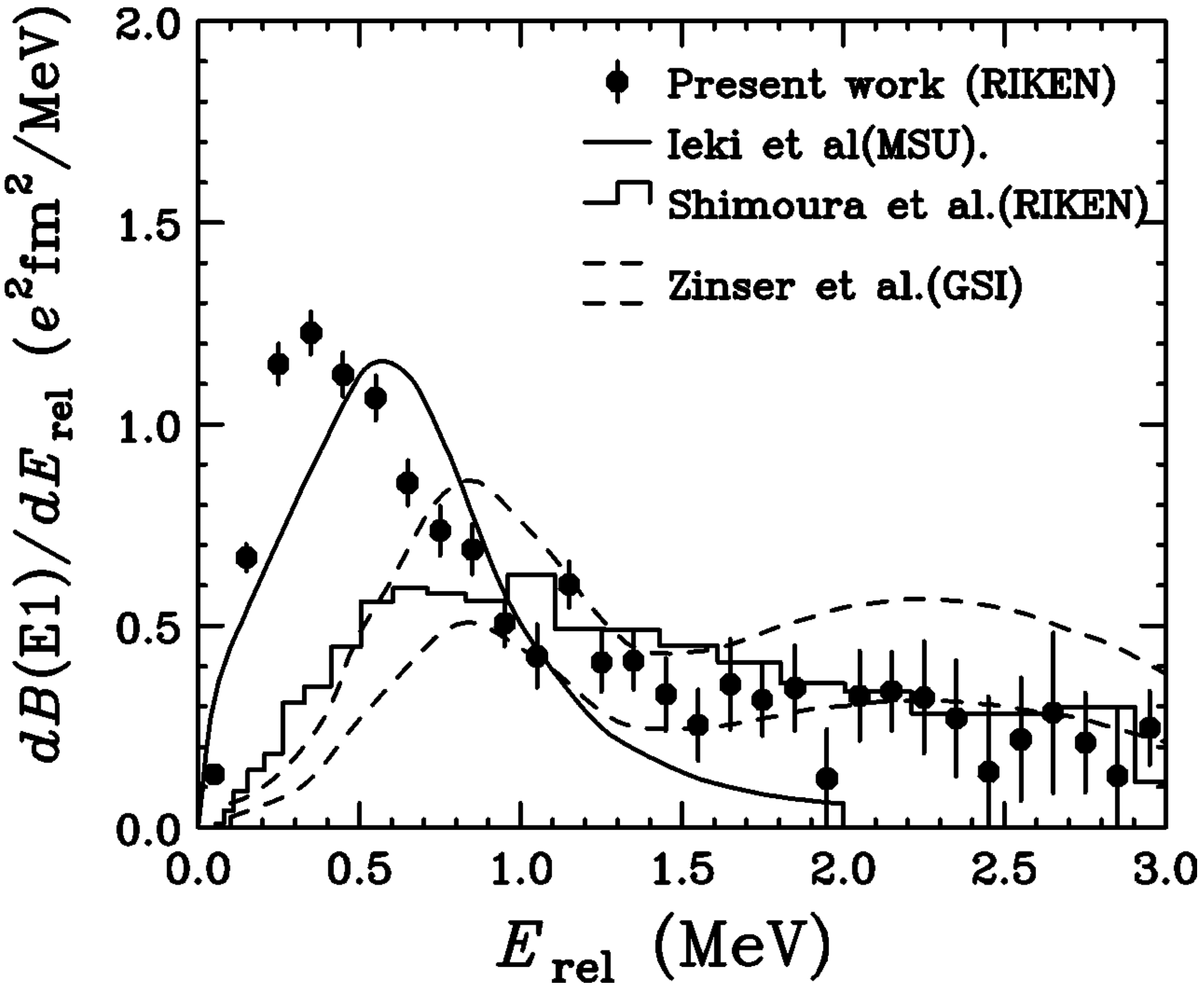
GSI @280MeV/nucleon
NPA 619 (1997) 151.

# Experimental Setup

@RIPS at RIKEN



# Result: E1 Response of $^{11}Li$

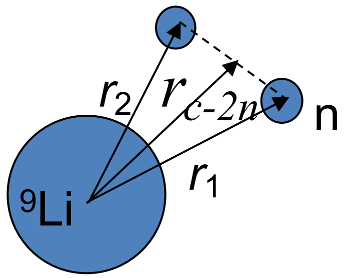


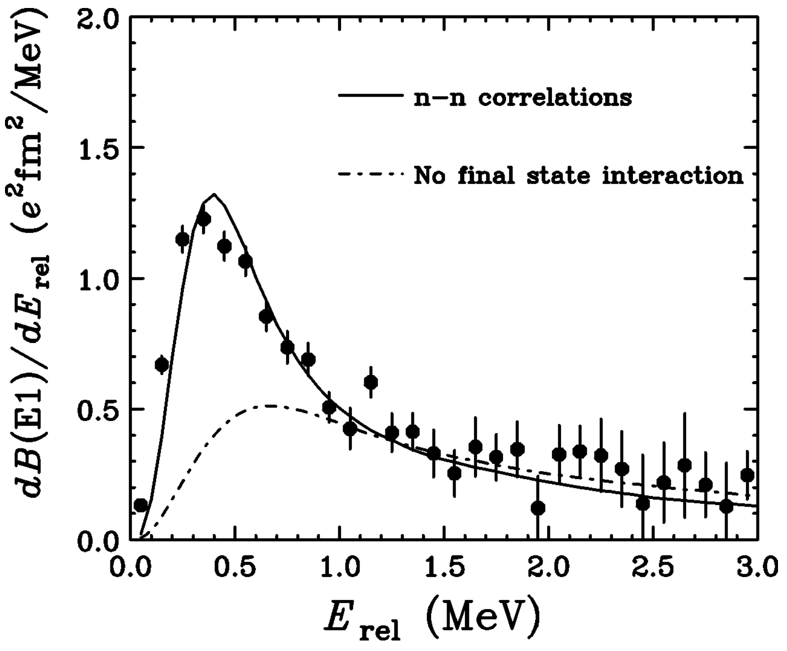

TN, AM Vinodkumar et al., Phys. Rev. Lett. **96**, 252502 (2006).

# Non-energy weighted E1 Cluster Sum Rule

$$B(E1) = \int_0^\infty \frac{dB(E1)}{dE_x} dE_x = \frac{3}{4\pi}\left(\frac{Ze}{A}\right)^2 \left\langle r_1^{\ 2} + r_2^{\ 2} + 2(\vec{r}_1 \cdot \vec{r}_2) \right\rangle$$

$$= \frac{3}{\pi}\left(\frac{Ze}{A}\right)^2 \left\langle r_{c-2n}^{\ \ 2} \right\rangle$$




$B(E1) = 1.42 \pm 0.18\ e^2 fm^2 (E_{\mathrm{rel}} \leq 3\mathrm{MeV})$

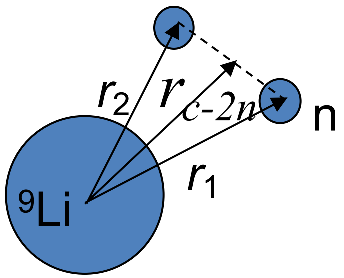
$\rightarrow 1.78(22)\ e^2 fm^2$ (Extrapolated value)

$\rightarrow \sqrt{\left\langle r_{c-2n} \right\rangle^2} = 5.01 \pm 0.32$ fm

~70% larger than non-correlated strength $\left(\vec{r}_1 \cdot \vec{r}_2 = 0\right)$

$\longrightarrow \left\langle \theta_{12} \right\rangle = 48^{+14}_{-18}$ deg

# Implication of the Narrow Opening Angle




Simple two-neutron shell model

$$\left|\Psi(^{11}\mathrm{Li})\right\rangle = Core \otimes \left[\alpha\left|(1s)^2\right\rangle + \beta\left|(0p)^2\right\rangle\right]$$

Melting of s(+ parity) and p(-parity) orbitals

$$\langle\cos\theta_{12}\rangle = \alpha^2\left\langle(1s)^2\middle|\cos\theta_{12}\middle|(1s)^2\right\rangle + \beta^2\left\langle(0p)^2\middle|\cos\theta_{12}\middle|(0p)^2\right\rangle + 2\alpha\beta\left\langle(0p)^2\middle|\cos\theta_{12}\middle|(1s)^2\right\rangle$$

$$= 2\alpha\beta\left\langle(0p)^2\middle|\cos\theta_{12}\middle|(1s)^2\right\rangle$$

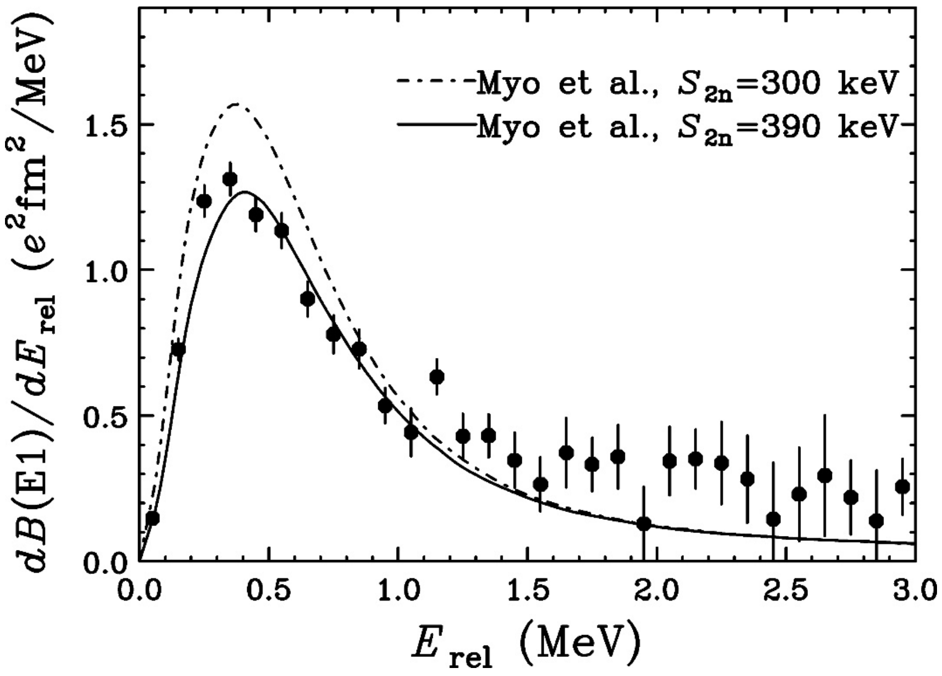
If only $(1s)^2$ or $(0p)^2$ → $\langle\cos\theta_{12}\rangle = 0, \quad \langle\theta_{12}\rangle = 90^\circ$

$$\langle\theta_{12}\rangle = 48^{+14}_{-18}\ \mathrm{deg} \implies$$

Mixture of different parity states is essential !

$$\alpha^2 = \beta^2 = 50\% \implies \langle\theta_{12}\rangle = 55\,\mathrm{deg}$$

# Comparison with theory



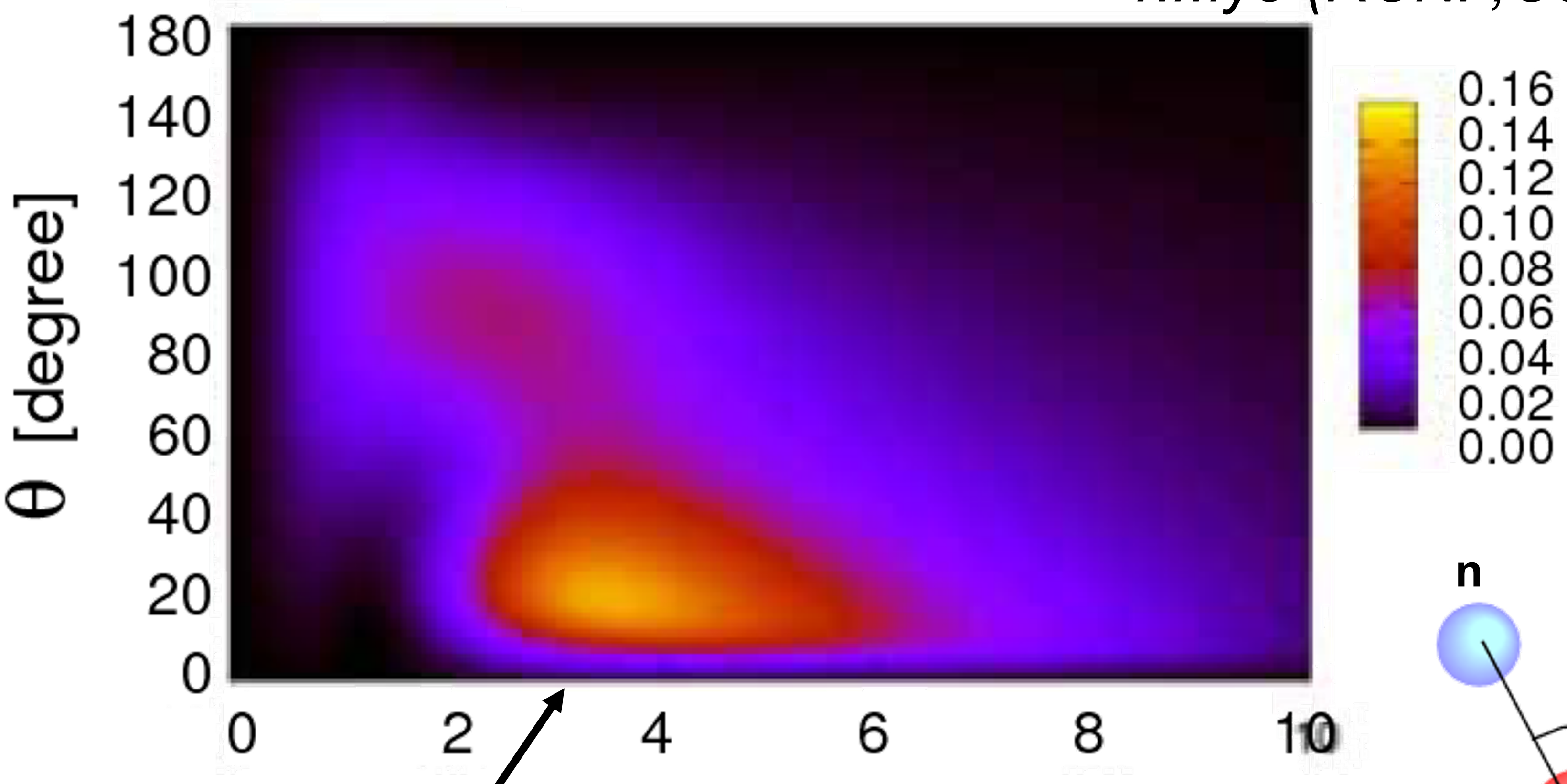

Myo et al.,PRC76,024305 (2007).

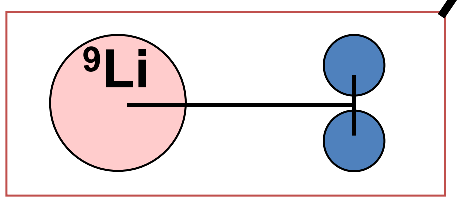
Core polarization
(Tensor correlation+
Pauli Principle)

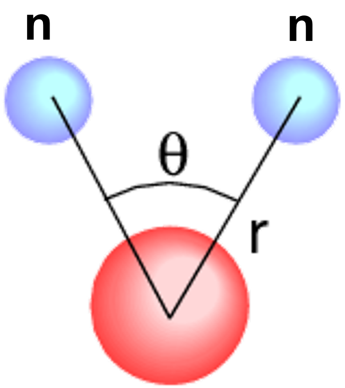
P($S^2$)~ 40%

$$\sqrt{\langle r_{c-2n} \rangle^2} = 5.69 \text{ fm}$$

# 2n correlation density in $^{11}$Li



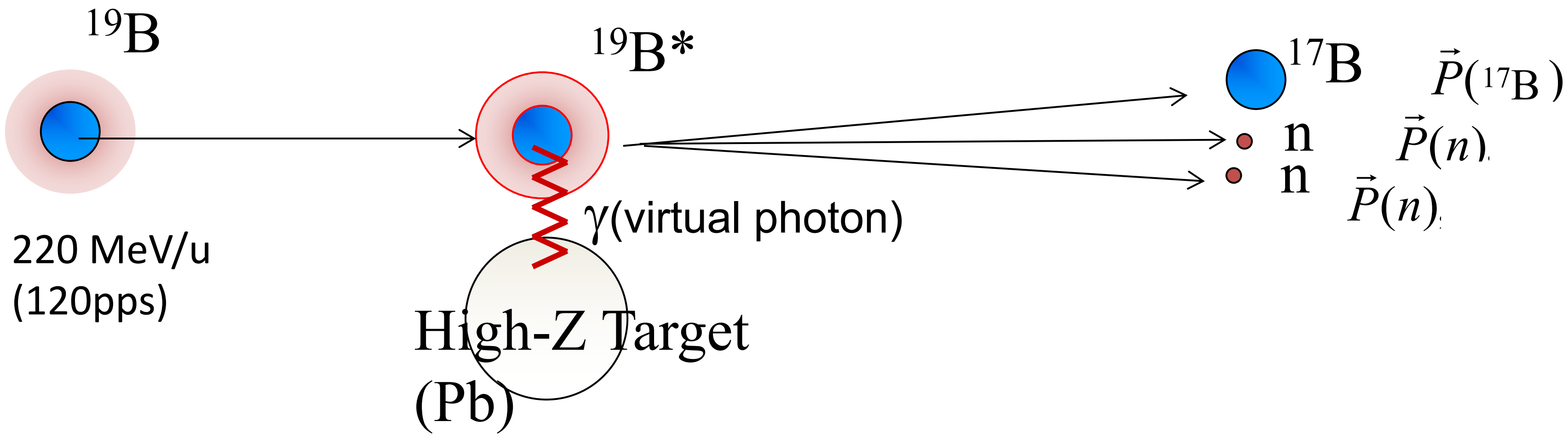



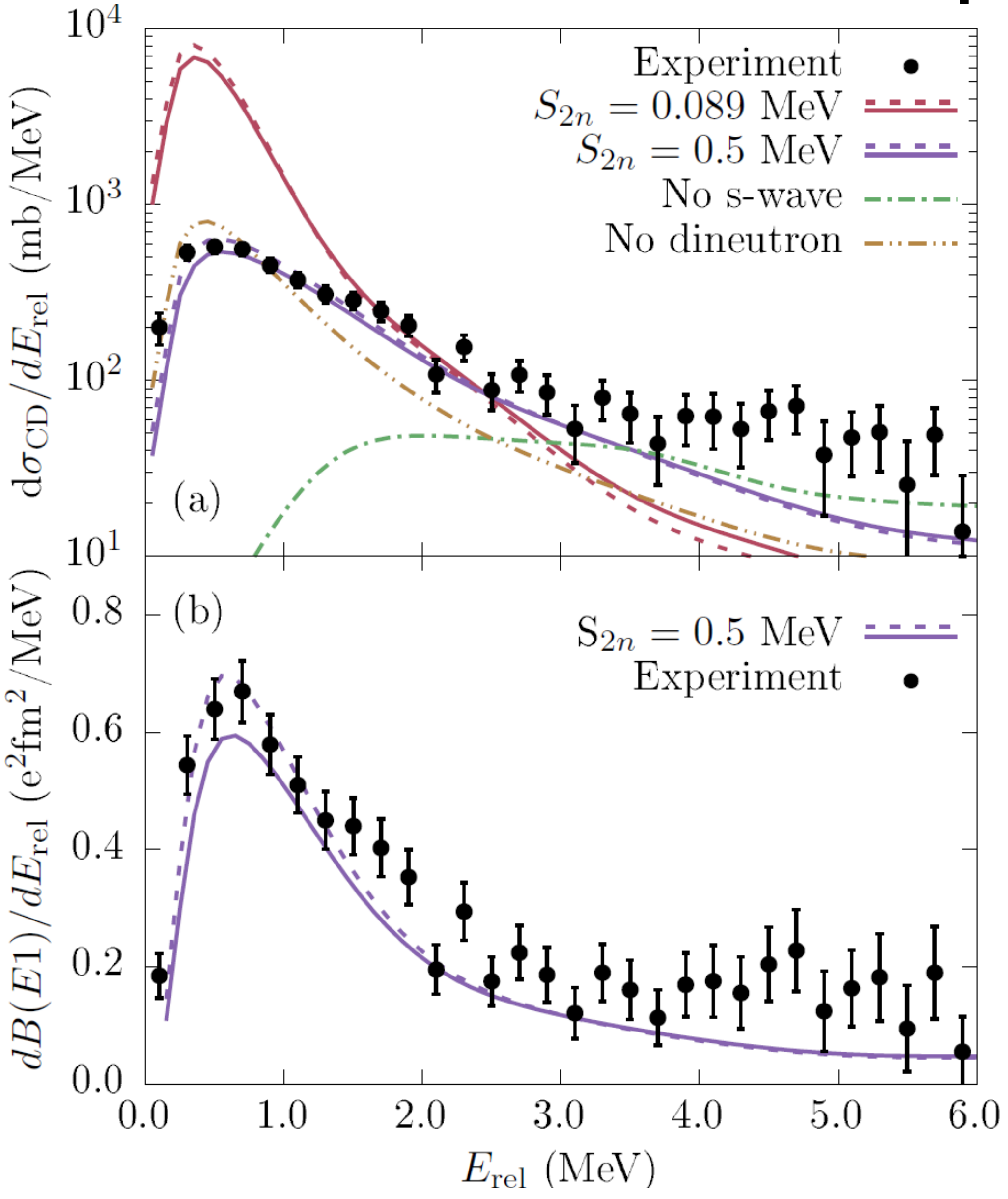

Cf. H.Esbensen and G.F.Bertsch, NPA542(1992)310

# Coulomb Breakup of $^{19}$B @ SAMURAI at RIBF

K. Cook, TN et al.



# E1 Response of $^{19}B$



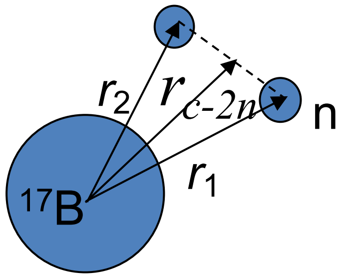

- B(E1) = 1.64 ± 0.06 (stat) ± 0.12 (sys) $e^2fm^2$ ($E_{rel}$ < 6 MeV). →Signature of a halo!

  Similar B(E1) to $^{11}Li$, $^{11}Be$.

  Core-2n distance (Sum rule)

  $$\sqrt{\langle r^2_{c-2n}\rangle} = 5.75 \pm 0.11(\mathrm{stat}) \pm 0.21(\mathrm{sys})\ \mathrm{fm}$$

- 3-body model calculations support $S_{2n}$ = 0.5 MeV, substantial s-wave component with a well-developed dineutron correlation.
- Consistent with large scattering length of $^{17}B$-n (a < - 50fm )

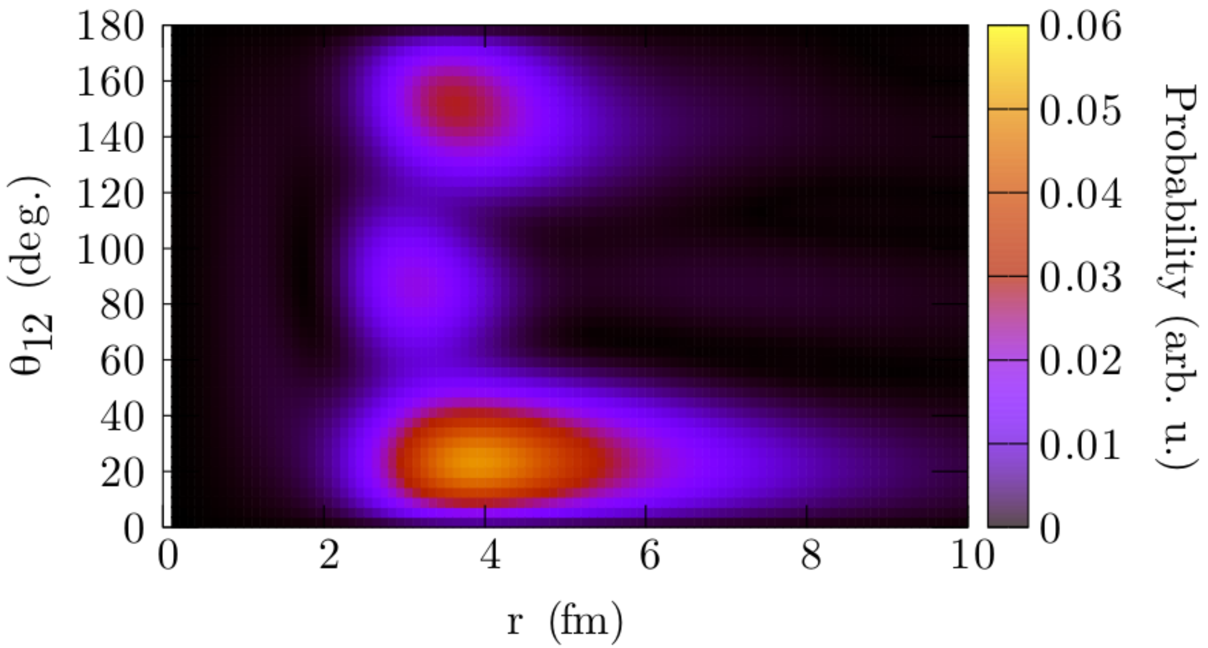



K.J. Cook, T. Nakamura, Y. Kondo, K. Hagino, K. Ogata *et al.* PRL **124**, 212503 (2020)

# Dineutron correlation in $^{19}$B

The 3-body model below (by K.Hagino) **reproduces dσ/dE_coul very well!**

Valence neutron density distribution for **$S_{2n}$ = 0.5 MeV, a = –50 fm.**




- Large amount of probability at around **$\theta_{12}$ ~ 25° → Dineutron correlation!**
- **The three peaked structure: due to the $d_{5/2}$ orbital** – seen also in calculations for $^{16}$C and $^{16}$Ne. Pure d-wave = three equal probability peaks. (Oishi 2010)
- **Asymmetry** : due to mixture of **negative parity configurations**
- Configurations: **negative parity states = 6%, s-wave = 35%, d-wave = 56%**

# Interim Summary (E1 Response of 2n halo)

- E1 Response of 2n Halo Nuclei→Soft E1 Excitation

  Coulomb breakup →Non-resonance origin Likely:

  c.f. Some work claiming soft E1 resonance

  (p,p') J.Tanaka et al., Phys. Lett. B 774, 268 (2017). **Ex=0.80(2) MeV, Γ=1.15(6) MeV**

  (d,d') R.Kanungo et al., Phys. Lev. Lett. **114**, 192502 (2015). **Ex=1.03(3) MeV, Γ=0.51(11) MeV**

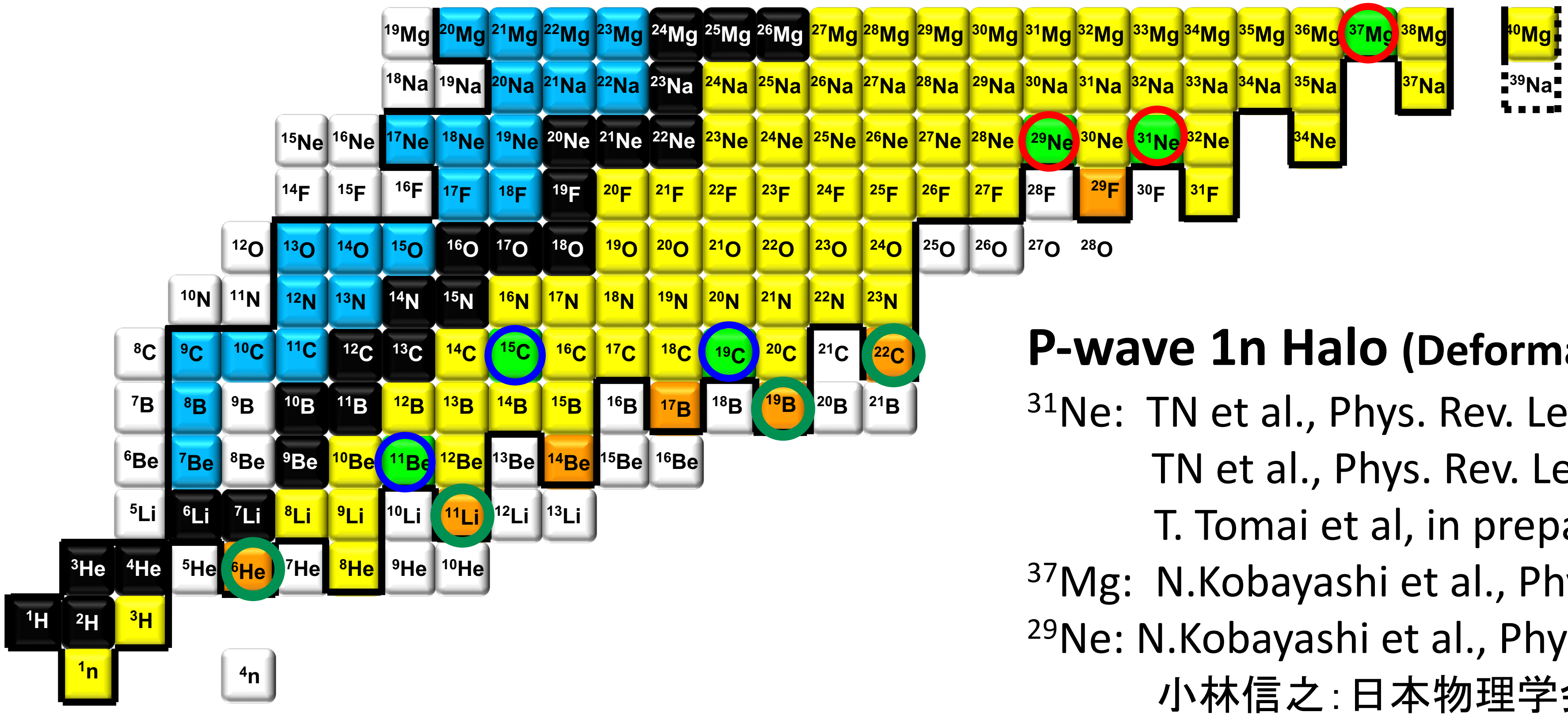
- Non-energy cluster sum rule → sqrt$\langle r_{c\text{-}2n}^2 \rangle$, $\langle \theta_{nn} \rangle$ →dineutron
- Spectroscopic tool for Halo state: mixture of different parities, dineutron
- Universality: s-wave halo←→Efimov ?

# Summary

- ✓ Halo Nuclei: Existing at the limit of stability (Boundary between Nuclear and nucleon(hadron) hierarchies
- ✓ Halo Nuclei: Some common features with ultra cold atoms at unitary limit
- ✓ Description by the scattering length/ Efimov
- ✓ Coulomb Breakup : Useful tool to probe E1 (Electric dipole) response of Halo Nuclei
- ✓ E1 Response of 1n-Halo Nuclei: Soft E1 Excitation: Non-resonant nature, Spectroscopic tool for C2S, Sn, and l
- ✓ E1 Response of 2n-halo nuclei: Soft E1 Excitation: Non-resonant nature (Some experiments showed resonance nature, though), Spectroscopic tool for nn correlation

# Coulomb Breakup of 1n and 2n Halo:

Review:
T. Aumann, TN, Phys. Scr T152, 014012 (2013).
TN, H. Sakurai, H. Watanabe, Prog. Part. Nucl. Phys. 97, 53 (2017).
TN, Y. Kondo, Clusters in Nuclei, Vol. 2. p67-119 (2012).




## P-wave 1n Halo (Deformation Driven Halo 変形ハロー)

$^{31}$Ne: TN et al., Phys. Rev. Lett. **103**, 1112 (1999).
TN et al., Phys. Rev. Lett. **112**,142501(2014).
T. Tomai et al, in preparation

$^{37}$Mg: N.Kobayashi et al., Phys. Rev. Lett. **112**, 242501(2014).

$^{29}$Ne: N.Kobayashi et al., Phys. Rev. C**93**, 014613 (2016).
小林信之：日本物理学会若手奨励賞2020

## S-wave 1n Halo

$^{19}$C: TN et al., Phys. Rev. Lett. **83**, 1112 (1999).

$^{15}$C: TN et al., Phys. Rev C **79**, 035805 (2009).

$^{11}$Be N.Fukuda et al., Phys. Rev. C**70**, 054606 (2004).
TN et al., Phys. Lett. B **331**, 296 (1994).

## 2n Halo

$^{11}$Li: TN et al., Phys. Rev. Lett. 96, 252502 (2006).

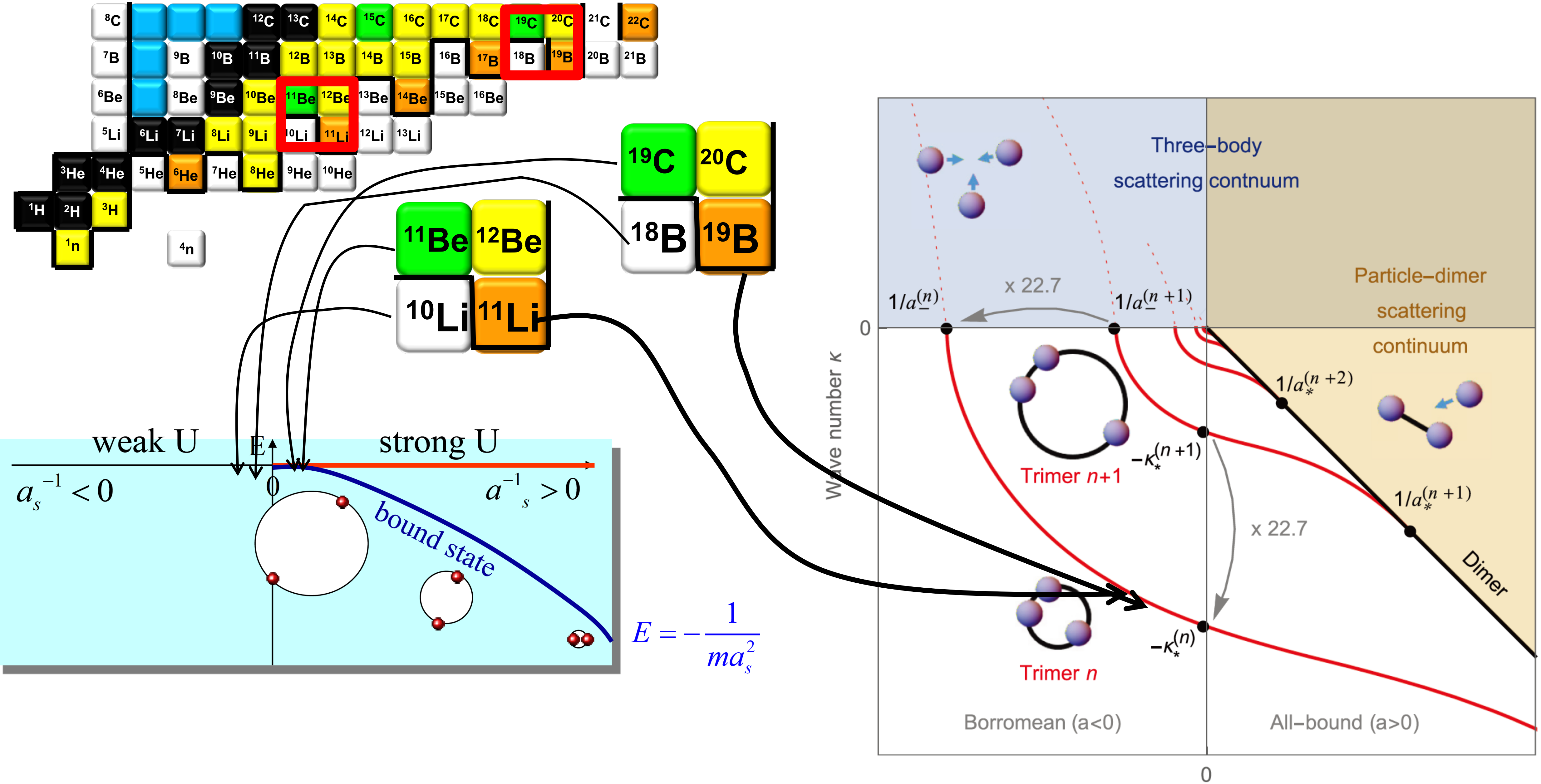
$^{19}$B: K.J. Cook et al., Phys. Rev. Lett. 124, 212503 (2020).

$^{6}$He: Y.Sun Submitted to PLB, A.T.Saito/C.Lehr, in preparaton

$^{22}$C: TN in preparation

# Perspective

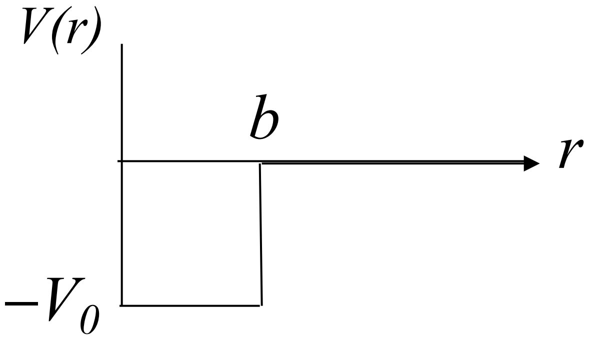
- Dineutron correlation: Direct evidence/ Mechanism to be explored in the near future
- 4n halo or giant halo?
- Halo is common for the whole nuclear chart?   Does halo exist in heavy nuclei?
  (37Mg is the heaviest halo nucleus known currently)
- How Halo is relevant for Shell evolution, and the location of the neutron drip line?
- How Halo can be understood as universal few-body quantum systems.



8C 12C 13C 14C 15C 16C 17C 18C 19C 20C 21C 22C
7B 9B 10B 11B 12B 13B 14B 15B 16B 17B 18B 19B 20B 21B
6Be 8Be 9Be 10Be 11Be 12Be 13Be 14Be 15Be 16Be
5Li 6Li 7Li 8Li 9Li 10Li 11Li 12Li 13Li
3He 4He 5He 6He 7He 8He 9He 10He
1H 2H 3H
1n 4n
19C 20C
18B 19B
11Be 12Be
10Li 11Li
weak U
strong U
E
$a_s^{-1} < 0$
0
$a_s^{-1} > 0$
bound state
$E = -\frac{1}{ma_s^2}$
Three–body
scattering contnuum
Particle–dimer
scattering
continuum
$1/a_-^{(n)}$
x 22.7
$1/a_-^{(n+1)}$
0
Wave number κ
$1/a_*^{(n+2)}$
$-\kappa_*^{(n+1)}$
Trimer n+1
x 22.7
$1/a_*^{(n+1)}$
Dimer
$-\kappa_*^{(n)}$
Trimer n
Borromean (a<0)
All–bound (a>0)
0

# memo  Scattering length in Nuclei

*V(r)*—Potential between *n* and *A*
(*A* can be $^9Li$ or *n* or whatever)

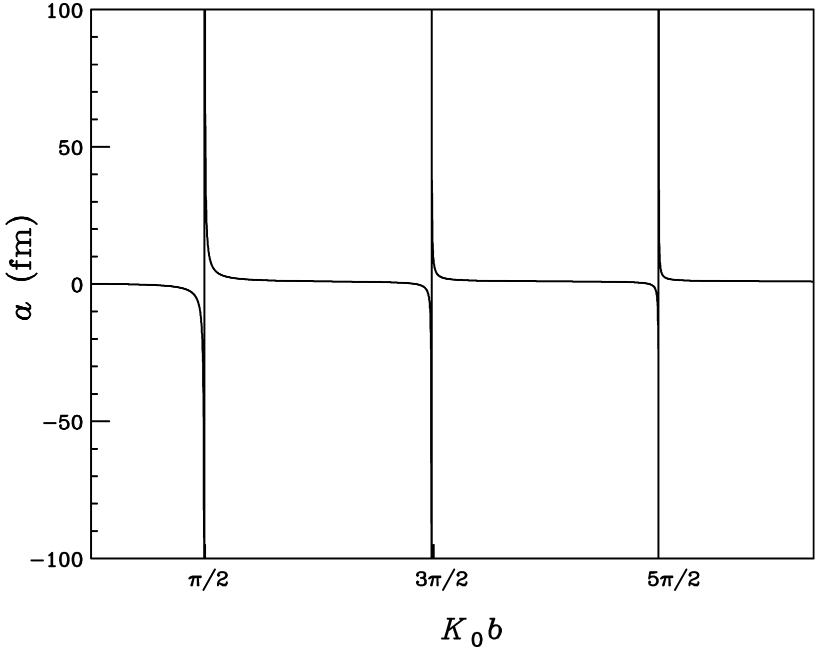



For a square well potential,

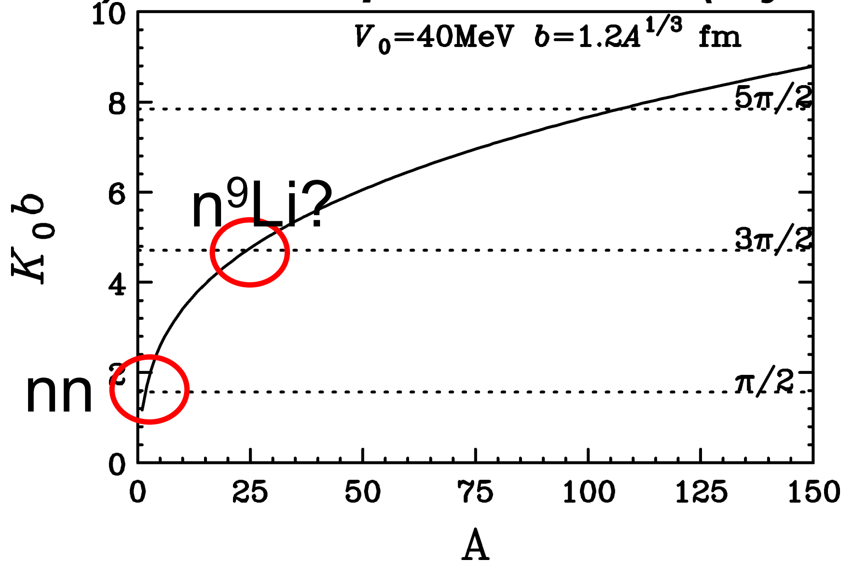
$$a = b\left[1 - \frac{\tan K_0 b}{K_0 b}\right], \quad K_0 = \frac{\sqrt{2\mu V_0}}{\hbar}$$

$$\lim_{k\to 0}\sigma = 4\pi a^2, \quad k\cot\delta = -\frac{1}{a} + \frac{1}{2}k^2 r_0$$

$a$ can be "negatively very large" only when
there is an unbound state under a specific potential condition




***V(n-9Li) is as deep as ~65MeV (Myo et al.)***




$$a(nn) = -18.5\text{fm},\ a(^9\text{Li}, n) = -10 \sim -25\text{fm}$$

# Backup

# Equivalent Photon Method

C.A.Bertulani and G.Baur,
Phys. Rep. 163, 299, (1988)

$$P(b) = \int N(E_x, b)\sigma_\gamma(E_x)\frac{dE_x}{E_x}$$

Probability of absorbing a photon at b = Virtual photon number x Photoabsorption cross section

光吸収断面積
(Photo-absorption Cross Section)
Probability of absorbing one photon

$$\sigma = \int_R^\infty P(b)2\pi b db = \int n(E_x)\sigma_\gamma(E_x)\frac{dE_x}{E_x}$$

Coulomb Breakup (Excitation) cross section

$$n(E_x) = \int_R^\infty N(E_x, b)2\pi b db$$

R: Minimum Impact parameter
最小衝突係数
(~R(P)+R(T))

Photo-absorption cross section vs B(Eλ) $\pi : E \text{ or } M, \quad \lambda : 1,2,\ldots$

$$\sigma_\gamma^{\pi\lambda} = \frac{(2\pi)^3(\lambda+1)}{\lambda[(2\lambda+1)!!]^2}\left(\frac{E_x}{\hbar c}\right)^{2\lambda-1}\frac{dB(\pi\lambda)}{dE_x}$$

$\frac{d\sigma}{dE_x db}, \quad \frac{d\sigma}{dE_x d\Omega}$ Can be described in a same manner.
Scattering angle is inversely proportional to b

Quiz: Explain the virtual photon spectrum (4 page back) qualitatively.

# Shape & Strength of B(E1) spectrum II

(from I.Hamamoto lecture)

For $\varepsilon_\ell < 0$ $\quad R_{\ell_b}(\varepsilon_b, r) \propto \alpha r h_{\ell b}(\alpha r)$ $\quad$ where $\alpha^2 = -\frac{2m}{\hbar^2}\varepsilon_b$

For $\varepsilon_\ell > 0$ $\quad$ (plane wave approximation)

$$R_{\ell_c}(\varepsilon_c, r) = \sqrt{\frac{2m}{\pi\hbar^2 k}}\, kr j_{\ell_c}(kr) \quad \text{where } k^2 = \frac{2m}{\hbar^2}\varepsilon_c$$

Note $\int_0^\infty dr R_{\ell_c}(E,r) R_{\ell_c}(E',r) = \delta(E - E')$

} assumed

| $\ell_b \to \ell_c$ | $\frac{dB(E1)}{dE} \propto \varepsilon_c^{\ell_c + 1/2}$ for very small $\varepsilon_c$ | $\frac{dB(E1)}{dE}$ is max. at |
|---|---|---|
| s → p | $\propto (\varepsilon_c)^{3/2}$ | $\varepsilon_c = \frac{3}{5}\varepsilon_b$ |
| p → s | $\propto (\varepsilon_c)^{1/2}$ | $\varepsilon_c \approx (0.18)\varepsilon_b$ |
| p → d | $\propto (\varepsilon_c)^{5/2}$ | $\varepsilon_c = \frac{5}{3}\varepsilon_b$ |
| d → p | $\propto (\varepsilon_c)^{3/2}$ | $\varepsilon_c = \frac{5}{3}\varepsilon_b$ |

# Efimov Effect

V.Efimov Phys.Lett.B33, 563(1970)
E.Braaten,H.-W.Hammer,Phys.Rep.428,259(2009)

3体系において

2体がぎりぎり束縛、またはぎりぎり非束縛→ $a \to \infty \quad (a >> r_0)$

Scattering length(散乱長)

2体力の
到達距離
（nn ~1.4fm)

$a = -17(1)\ \mathrm{fm}$
for nn

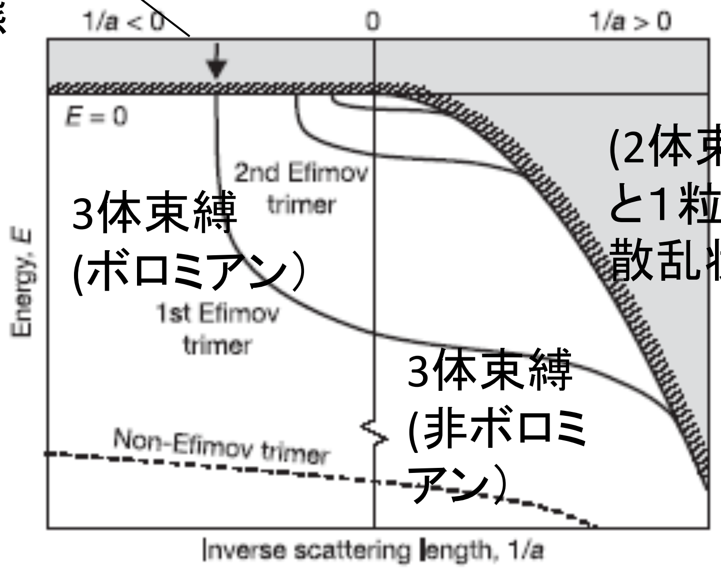
3体の弱束縛共鳴状態
が無限に現れる

$$\frac{a(n+1)}{a(n)} = \exp(\pi / s_0), \quad \frac{E(n+1)}{E(n)} = \exp(-2\pi / s_0)$$

$$N \approx \frac{1}{\pi} \ln\left(|a| / r_0\right)$$

Efimov
状態の数

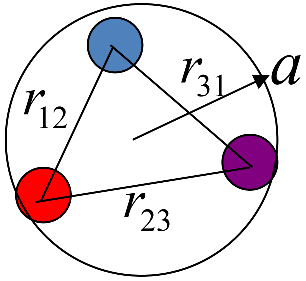
3体散乱
状態




Cs trimer(boson)
Nature Vol.440

Efimov 状態の物理的な描像

$a$ 程度の長距離力
（1/R²依存性
for R>$r_0$)が働く




$$R^2 = r_{12}{}^2 + r_{23}{}^2 + r_{31}{}^2$$

$$E_b = -\frac{\hbar^2}{ma^2}$$

Weakly bound
dimer

あたかも、相互作用の到達距離$r_0$よりも長い到達距離 $a$をもつように振舞う
相互作用によらない普遍的な現象（極低温3原子系でみつかっている）

前ページとaの符号が逆

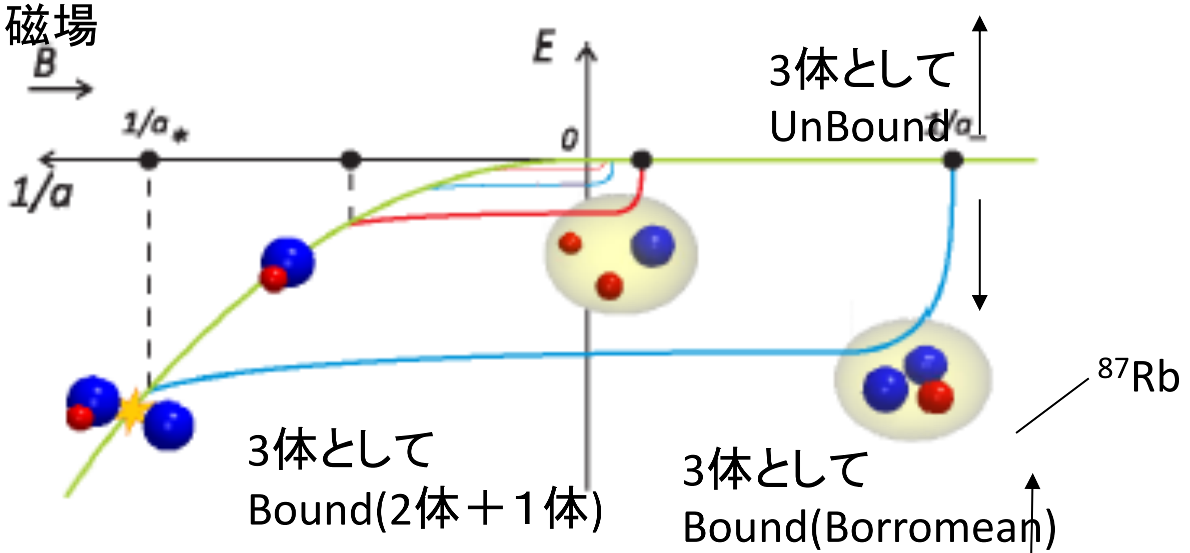


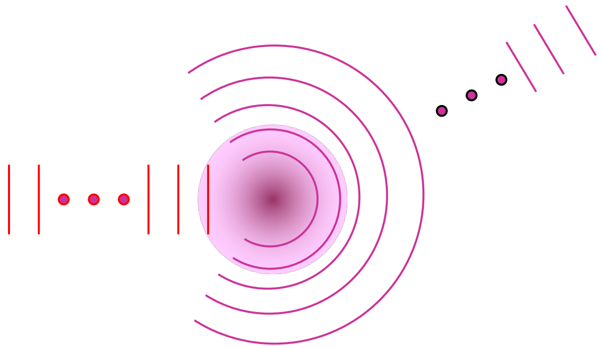

FIG. 1 (color online). Pictorial energy diagram of Efimov states for the double-species mixture of K and Rb, around an interspecies Feshbach resonance where the K-Rb scattering length diverges ($1/a = 0$). The Efimov resonances appear: (i) at the atom-dimer threshold for positive scattering lengths $a_*$; (ii) at the three-atoms threshold for negative scattering lengths $a_-$. Two distinct kinds of Efimov trimer are possible, KKRb (red or dark gray line) and KRbRb (blue or gray). The green or light gray line shows the dissociation threshold of the Efimov states.

K-K-RBあるいはK-Rb-RbでできたEfimov状態
C.Barontini et al. PRL103,043201 (2009)

原子核では
ボロミアン核
のほか3α,
Triton,$^{18}$C,$^{20}$C
などが候補
（tritonは
Efimov状態:
E.Braaten,
H.-W.Hammer,
Phys.Rep.428,259(2009)

# Scattering length

復習



散乱：漸近解

$$\Psi(\mathbf{r}) = e^{ikz} + \underline{f_k(\theta)}\frac{e^{ikr}}{r} \qquad \frac{d\sigma}{d\Omega} = |f_k(\theta)|^2$$

Scattering amplitude

低エネルギー極限（k→0)でどうなるか？

$$f_k(\theta) = -a \qquad \frac{d\sigma}{d\Omega} = a^2 \qquad \sigma(E) = 4\pi a^2$$

isotropic

弾性散乱断面積
＝半径　$a$の剛体球の散乱

低エネルギー散乱：Scattering length　$a$で特徴づけられる！

一般には　部分波展開

$$f_k(\theta) = \frac{1}{k}\sum_{\ell=0}^{\infty}(2\ell+1)e^{i\delta_\ell}\sin\delta_\ell P_\ell(\cos\theta) = \frac{1}{2ik}\sum_{\ell=0}^{\infty}(2\ell+1)(S_\ell - 1)P_\ell(\cos\theta)$$

$$\sigma(E) = \frac{4\pi}{k^2}\sum_{\ell=0}^{\infty}(2\ell+1)\sin^2\delta_\ell$$

低エネルギーで

$$k\cot\delta_0(k) = -\frac{1}{a} + \frac{1}{2}r_s k^2 \ldots$$