Deep learning in the analysis of hadron-hadron scattering

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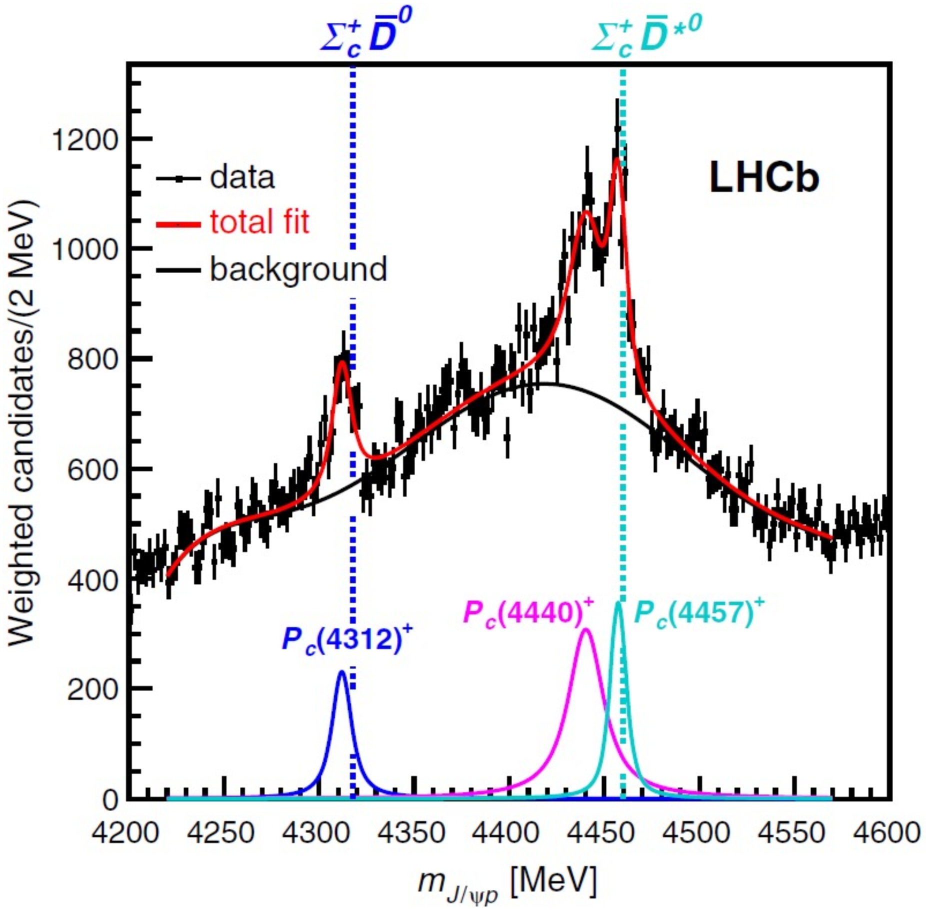
Motivation



10.1103/PhysRevLett.122.222001

Possible candidates

- Threshold cusp/ Triangle singularity
- Molecular state/ Virtual state
- Excited baryon or meson state/ Multiquark state

How to tell if a near-threshold enhancement is caused by a physical state?

- Phrase the question as a classification problem.
- **Deep learning approach** excels in solving a classification problem.

Use of data (simulated or real) to improve the performance of a model in accomplishing a specific task.

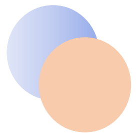
The case of deuteron

Deuteron exists.

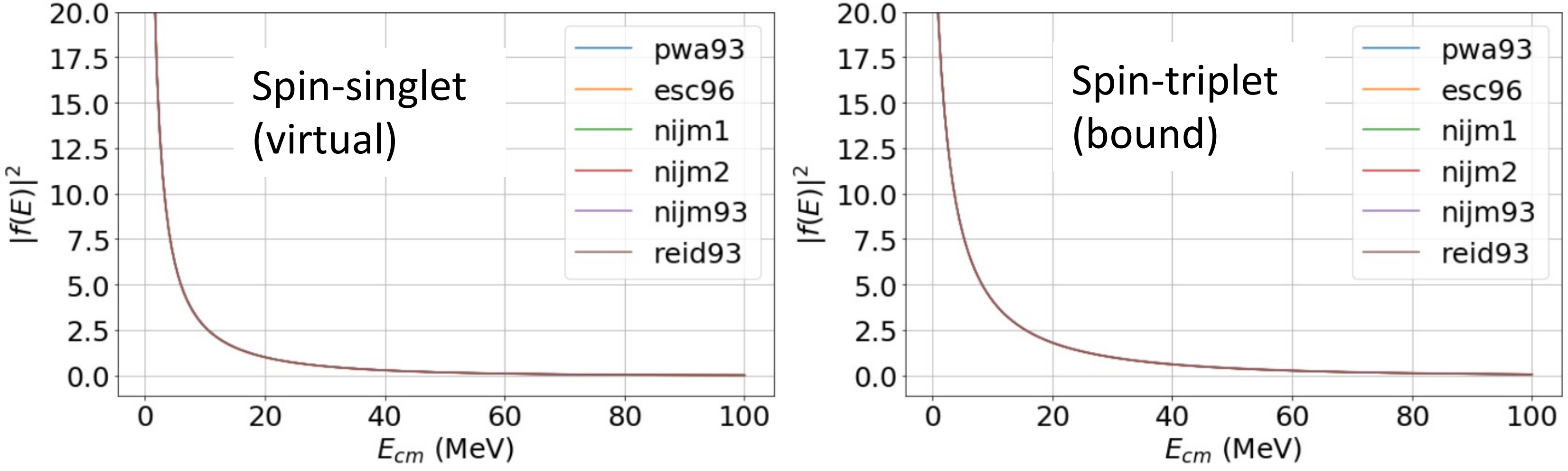
- We know the binding energy.
- We know the magnetic moment.

Given the scattering amplitudes, we can tell which threshold enhancement is caused by a bound state.

Deuteron



We can explain the cause of enhancement in the scattering data



http://nn-online.org/NN/?page=nnphs2

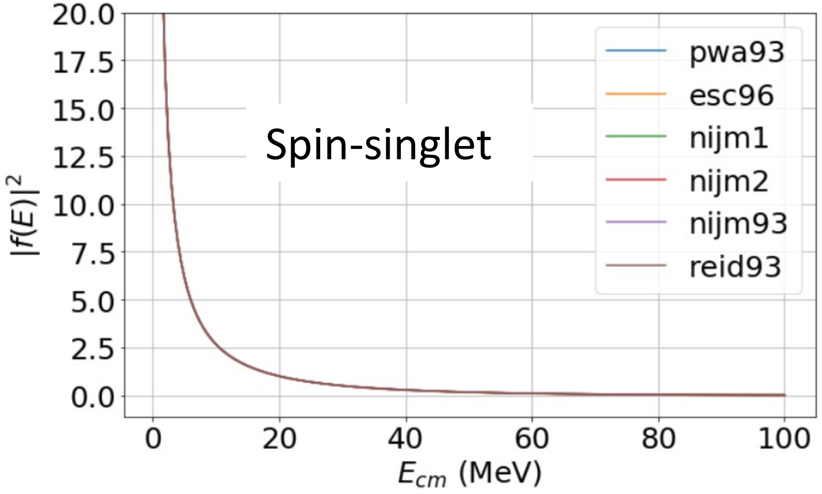
Trial problem

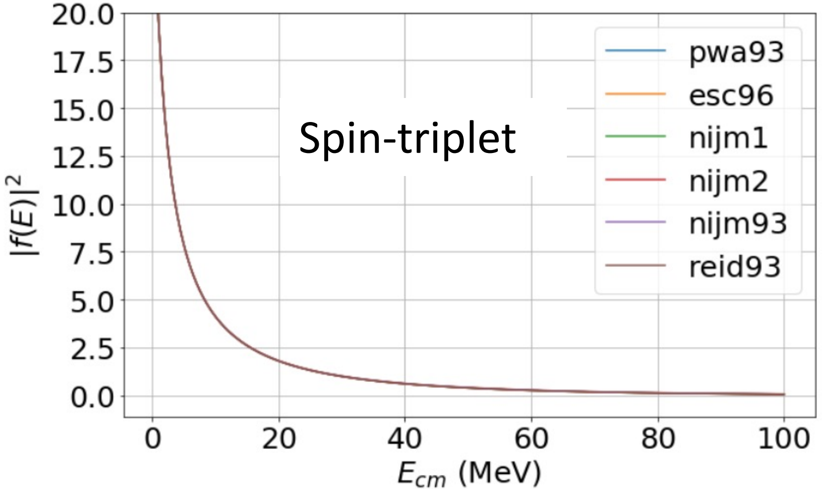
Assume that the deuteron is inaccessible.

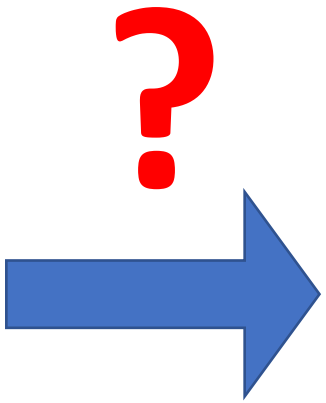
task: Identify which enhancement is caused by a two-hadron bound state?

[DLB Sombillo et al., 10.1103/PhysRevD.102.016024](https://doi.org/10.1103/PhysRevD.102.016024)

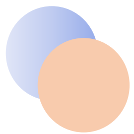
Scattering data







Deuteron

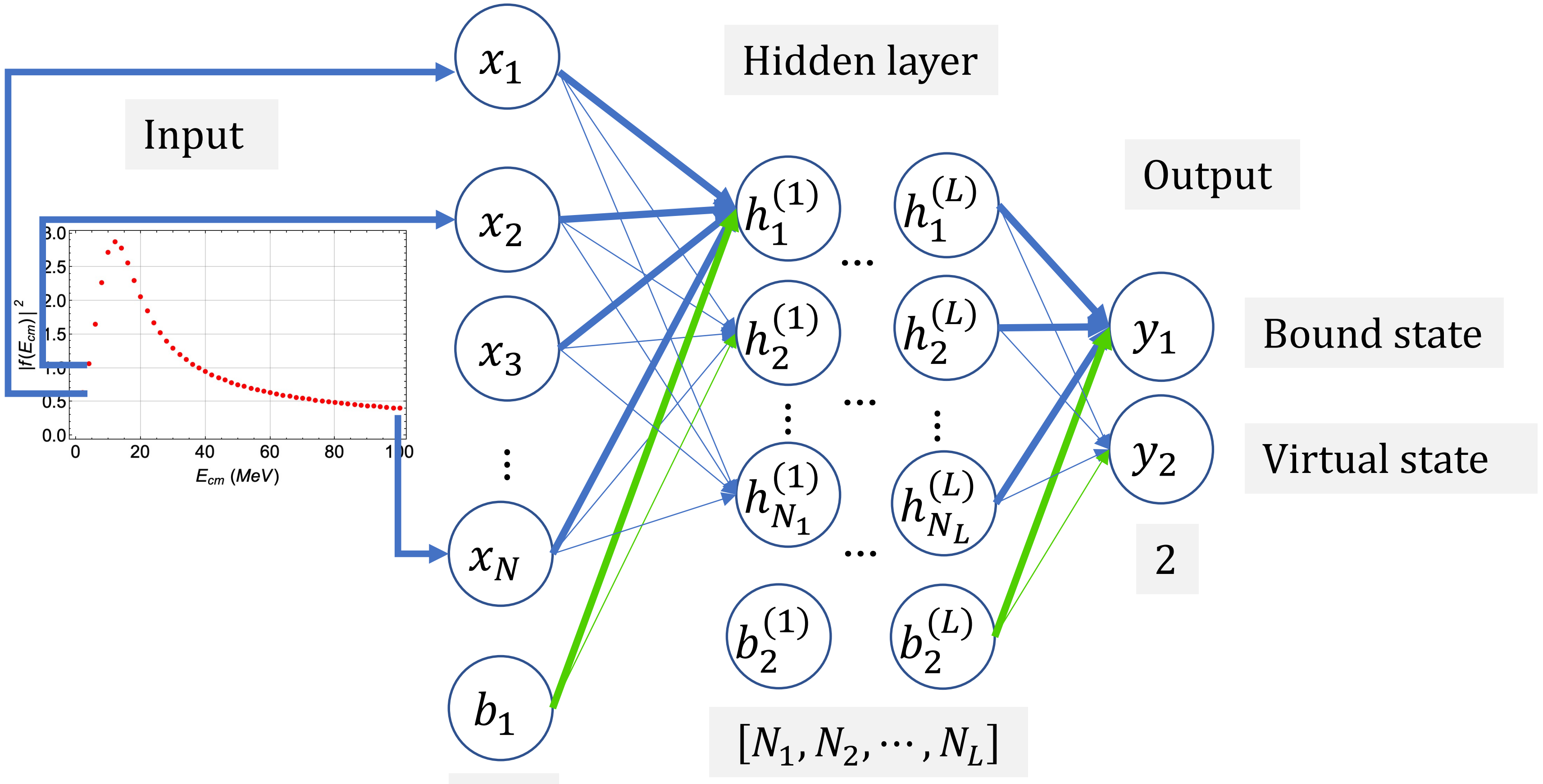


http://nn-online.org/NN/?page=nnphs2

Deep neural network models

model

- input and output nodes
- hidden layers
- nodes in each hidden layer
- optimizer to use



[250,100]
[250,100,50]
[250,250,250]

Optimizers: Adam or AMSGrad

Check which combination works best.

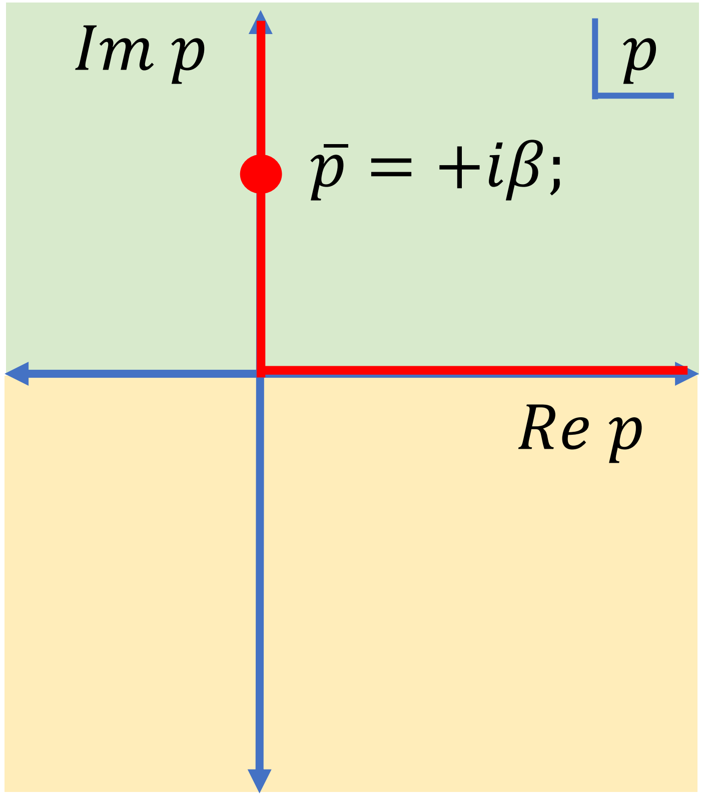
Chainer

https://dl.acm.org/doi/10.1145/3292500.3330756

Generation of simulated amplitudes

data
- S-matrix generic properties
  - Analyticity
  - Unitarity
- Label is based on pole position
- Introduce random background.

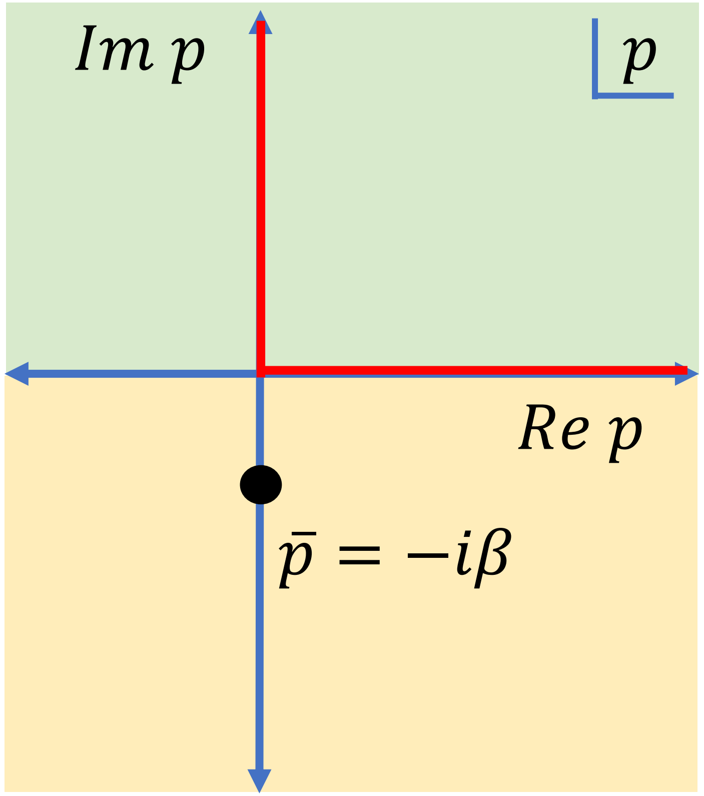
Bound state pole



Asymptotic solution is bounded:

$$\phi_{\ell,\bar{p}}(r) \sim e^{i(\bar{p}r - \ell\pi/2)} \xrightarrow[r\to\infty]{} 0$$

Virtual state pole



Asymptotic solution is unbounded:

$$\phi_{\ell,\bar{p}}(r) \sim e^{i(\bar{p}r - \ell\pi/2)} \xrightarrow[r\to\infty]{} \infty$$

Generation of simulated amplitudes

data

- S-matrix generic properties
  - Analyticity
  - Unitarity
- Label is based on pole position
- Introduce random background.

Bound/virtual:

$$S(p) = \exp\left[2i\eta \tan^{-1}\left(\frac{p}{\Lambda}\right)\right]\left(\frac{p+i\gamma_{far}}{p-i\gamma_{far}}\right)\left(\frac{p+i\gamma_{near}}{p-i\gamma_{near}}\right)$$

Branch cuts: $(-i\infty, -i\Lambda) \cup (i\Lambda, +i\infty)$

Pole background: $p = i\gamma_{far}$

Dataset generation:

$$|f(p)|^2 = \left|\frac{S(p)-1}{2ip}\right|^2$$

Output data:

- Bound $\gamma_{near} > 0$
- Virtual $\gamma_{near} \leq 0$

Background is needed to distinguish the cross-sections

$\eta \in (-4, -1)$	10 random values
$\Lambda \in (500\ MeV, 700\ MeV)$	20 random values
$\gamma_{near} \in (-0.9\Lambda, 200\ MeV)$	1000 random values
$\gamma_{far} \in (-2\Lambda, -\lvert\gamma_{near}\rvert)$	20 random values

4,000,000 input-output data

3,200,000 training data

- Used directly to modify weights and biases

800,000 testing data

- Will not modify network's parameters
- Check which architecture can generalize within the S-matrix model.

The training loop

improving the performance

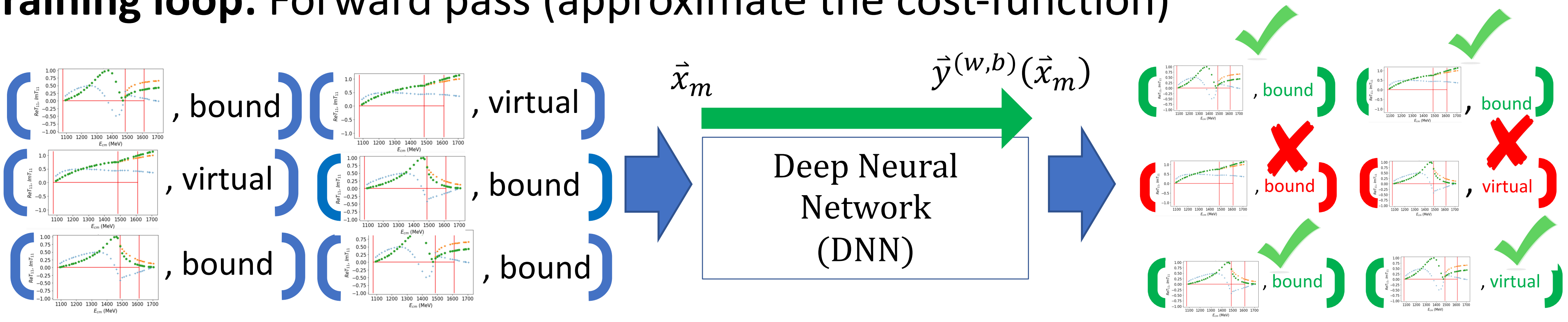
- Training loop
- Performance metric

3,200,000 training data

Correct prediction

Wrong prediction

Training loop: Forward pass (approximate the cost-function)



$$C(w,b) = \frac{1}{M}\sum_m \left\|\vec{y}^{(w,b)}(\vec{x}_m) - \vec{a}_m\right\|^2$$

The training loop

improving the performance

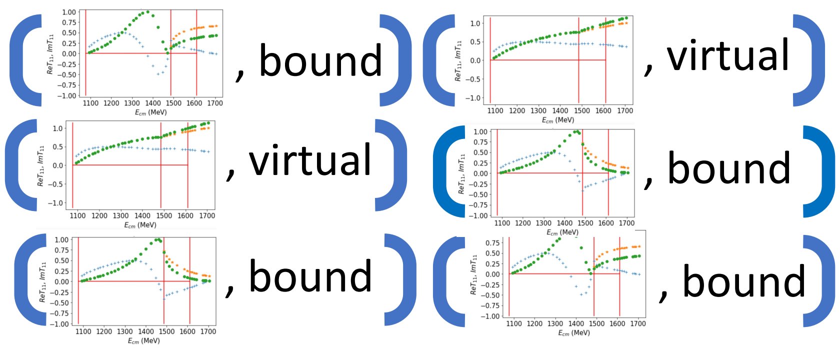
- Training loop
- Performance metric

3,200,000 training data

✓ Correct prediction

✗ Wrong prediction

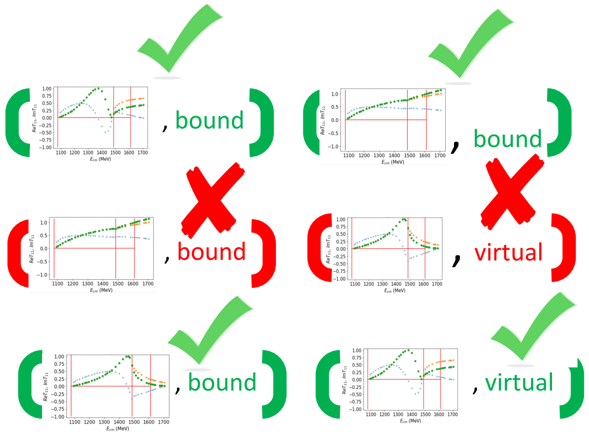
Training loop: Backpropagation (minimize cost)



$\vec{x}_m$

$\vec{y}^{(\boldsymbol{w},\boldsymbol{b})}(\vec{x}_m)$

Deep Neural Network (DNN)



$$C(\boldsymbol{w},\boldsymbol{b}) = \frac{1}{M}\sum_m \left\| \vec{y}^{(\boldsymbol{w},\boldsymbol{b})}(\vec{x}_m) - \vec{a}_m \right\|^2$$

Performance metric

improving the performance

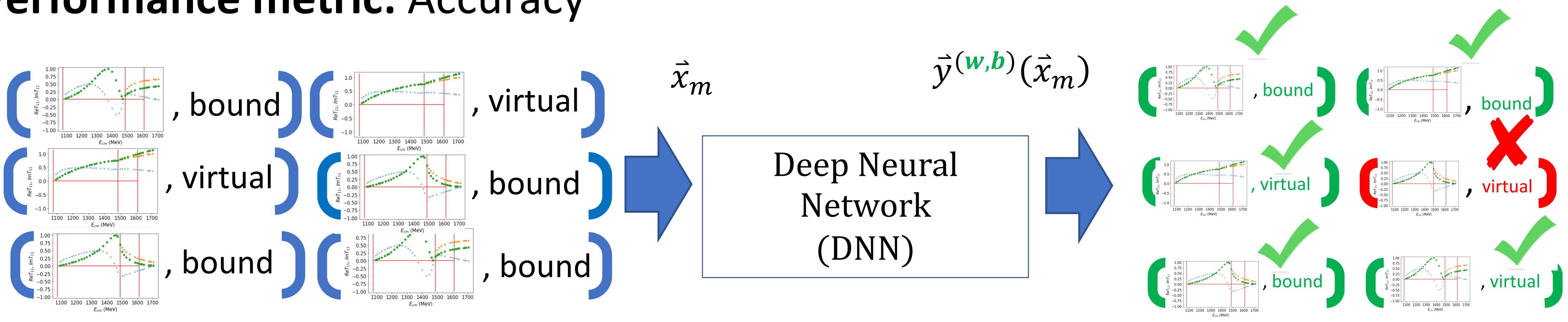
- Training loop
- Performance metric

3,200,000 training data

Correct prediction

Wrong prediction

Performance metric: Accuracy



$$Accuracy = \frac{Number\ of\ correct\ predictions}{Total\ number\ of\ items\ fed}$$

Performance metric

improving the performance
- Training loop
- Performance metric

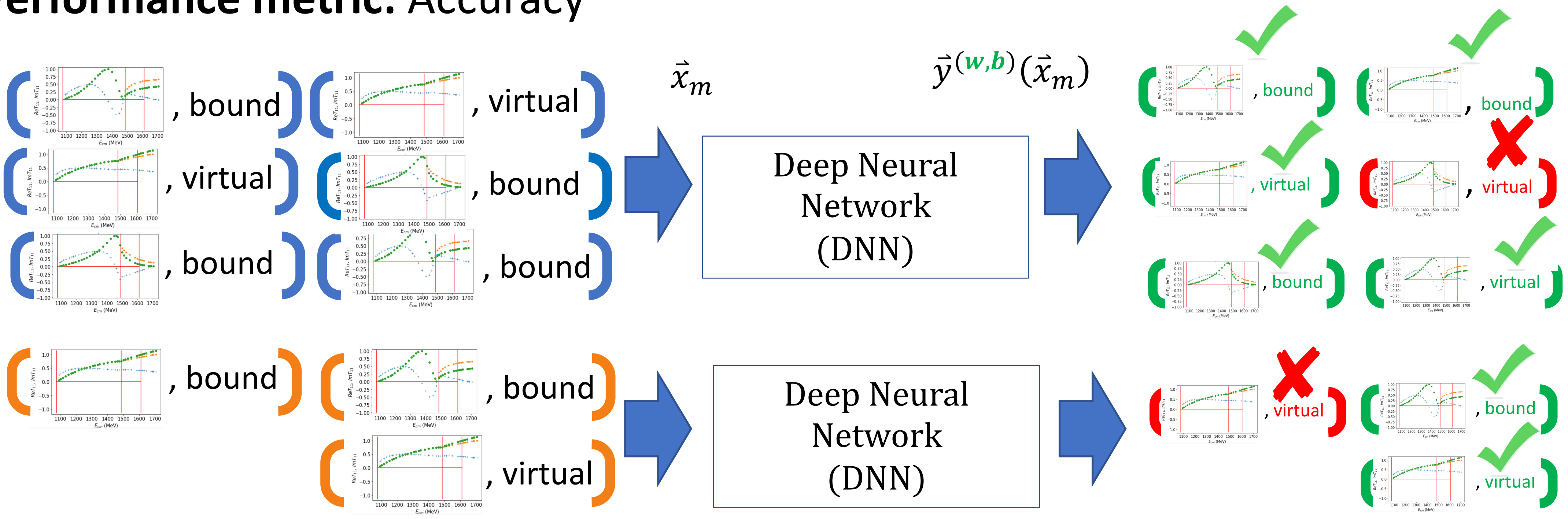
3,200,000 training data

800,000 testing data

✓ Correct prediction

✗ Wrong prediction

Performance metric: Accuracy

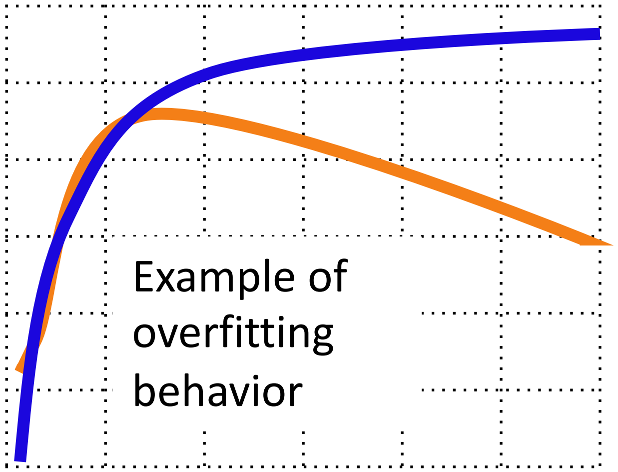


DNN models' performance

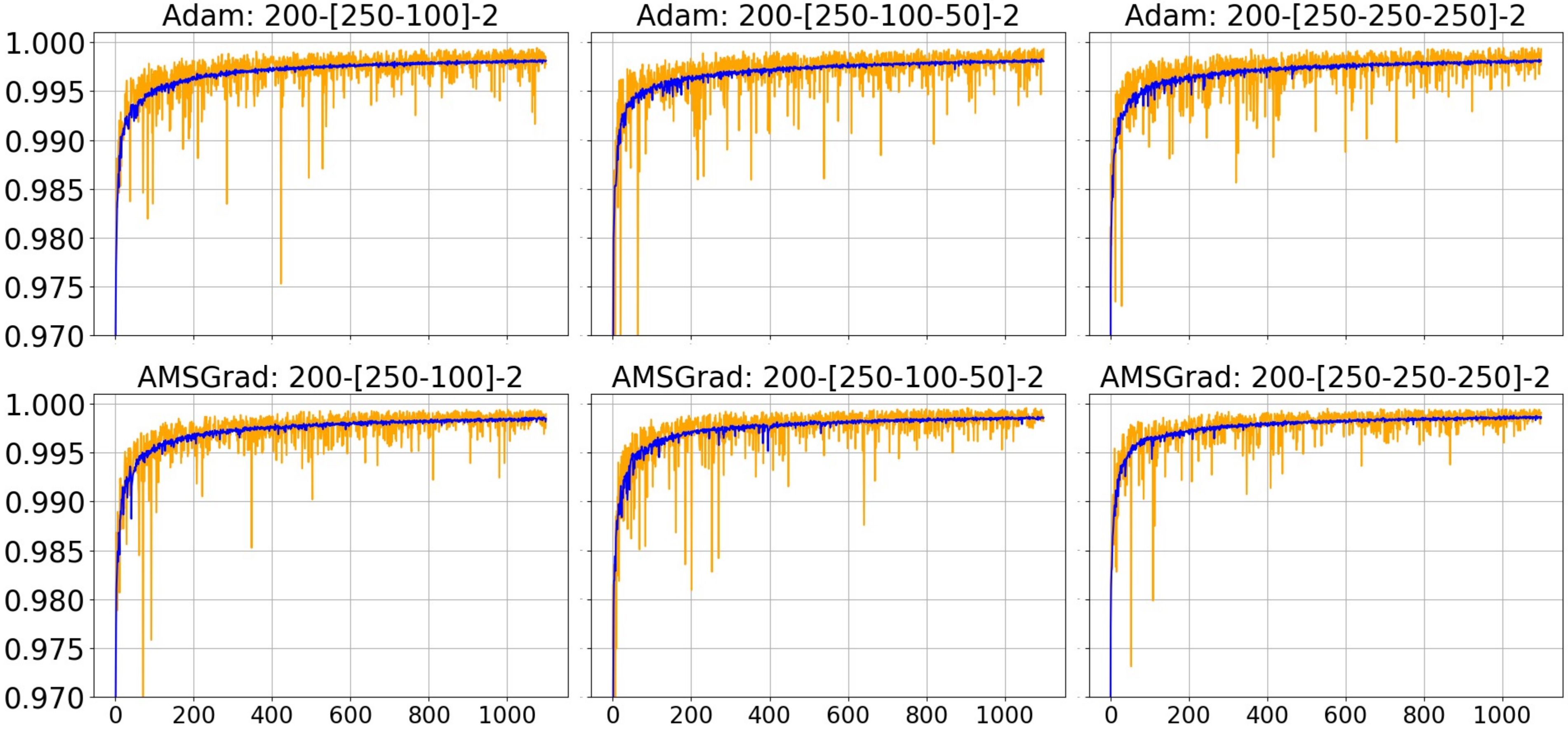
improving the performance

3,200,000 training data

800,000 testing data



Accuracy vs training epoch

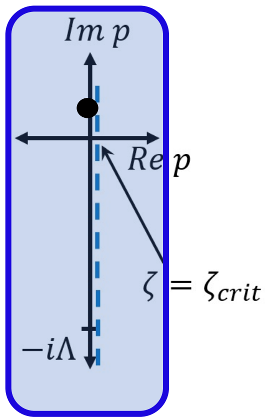


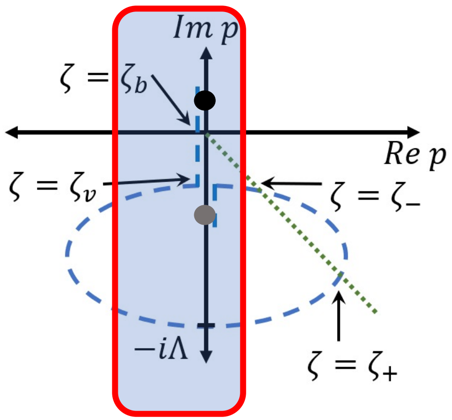
No overfitting observed (DNN is not just memorizing).
Generalization within the training S-matrix model is achieved.

Validation dataset (separable potential)

$$v(p,p') = \zeta\left(\frac{\Lambda^2}{p^2+\Lambda^2}\right)\left(\frac{\Lambda^2}{p'^2+\Lambda^2}\right)$$

Energy-independent coupling:





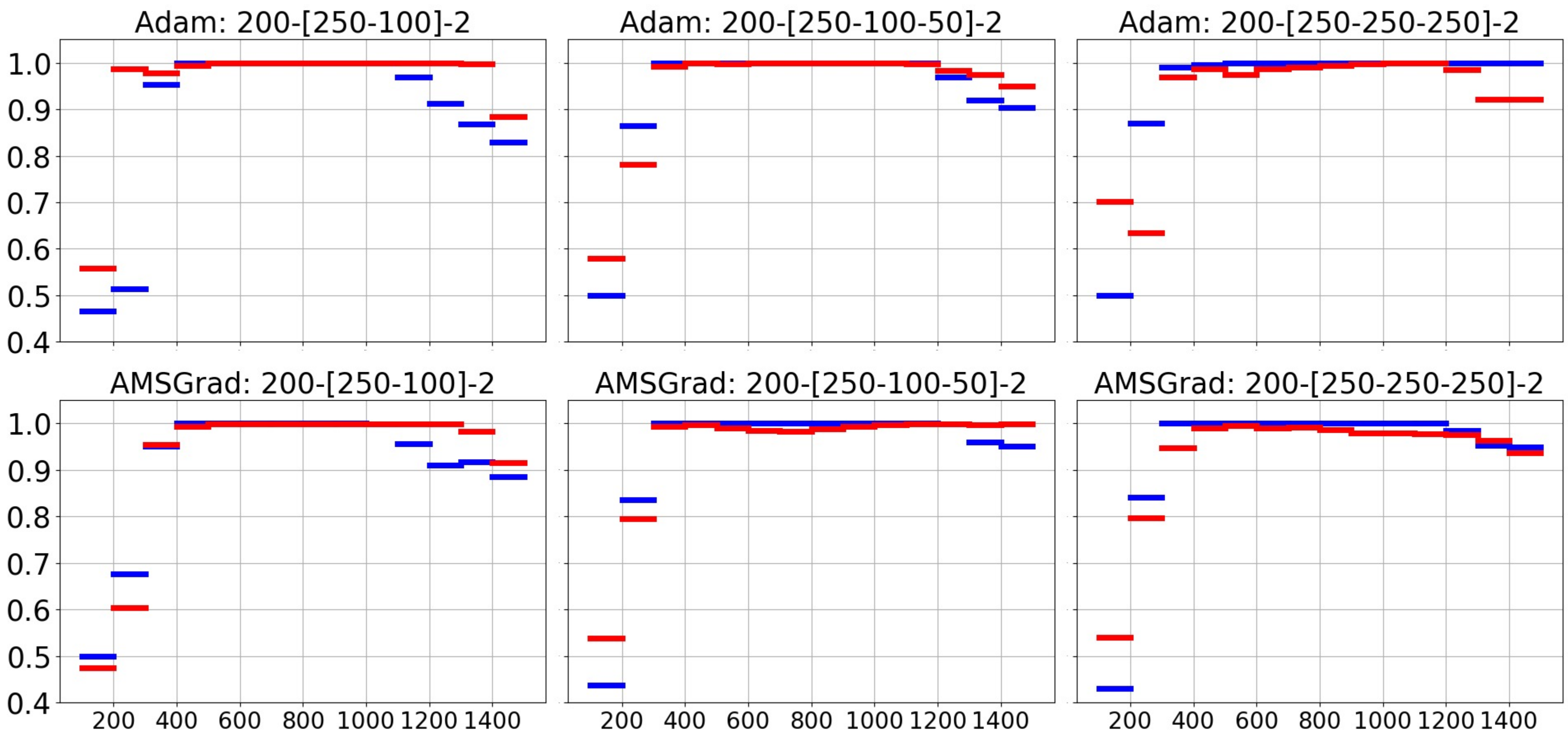
Energy-dependent coupling:
$\zeta \rightarrow (E - M_{sep})\zeta$

- Training dataset S-matrix background:
$$e^{2i\,\eta \tan^{-1}(p/\Lambda)}$$
- Validation dataset S-matrix background:
$$\left(\frac{p+i\Lambda}{p-i\Lambda}\right)^2$$

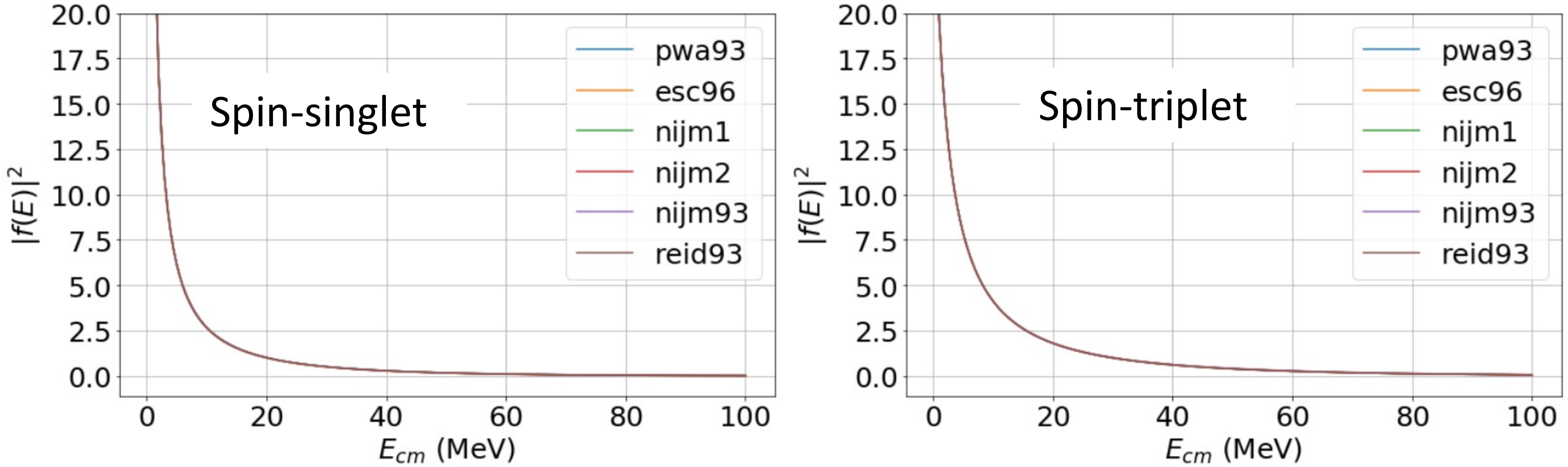
Validation performance

—Energy-independent coupling
—Energy-dependent coupling

Accuracy vs cutoff parameter Λ of separable potential



Single-channel bound-virtual classification



Deep Neural Network (DNN)

http://nn-online.org/NN/?page=nnphs2

	PWA93	ECS96	NijmI	NijmII	Nijm93	Reid93
N_{par}	39	14	41	47	15	50
χ^2/N	0.99	1.26	1.03	1.03	1.87	1.03

Deep Neural Network's predictions:

	PWA93	ECS96	NijmI	NijmII	Nijm93	Reid93
$^1S_0$	virtual	virtual	virtual	virtual	virtual	virtual
$^3S_1$	bound	bound	bound	bound	bound	bound

DLB Sombillo et al., 10.1103/PhysRevD.102.016024

https://github.com/sombillo/DNN-for-bound-virtual-classification

Conclusion and Outlook

- Demonstrate how deep learning can be applied in the study of near-threshold phenomena.
- Trained DNN can generalize beyond the training dataset.

- Extension to coupled-channel scattering.

  Extraction of S-matrix pole-configuration

  (manuscript in preparation)

Thank you for listening