新学術領域研究「量子クラスターで読み解く物質の階層構造」スクール 2021.3.23

Control of cluster formation in an ultracold Fermi gas through Fermi surface engineering

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Yoji Ohashi (Keio University)

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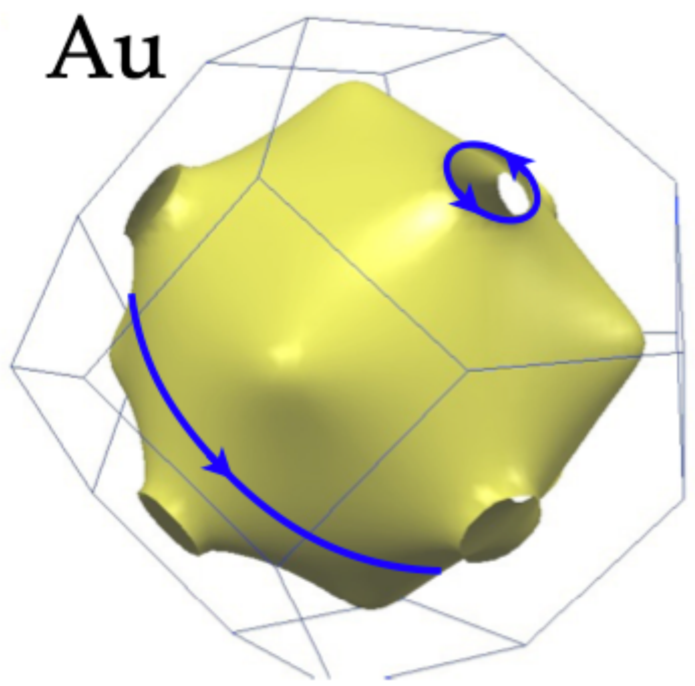
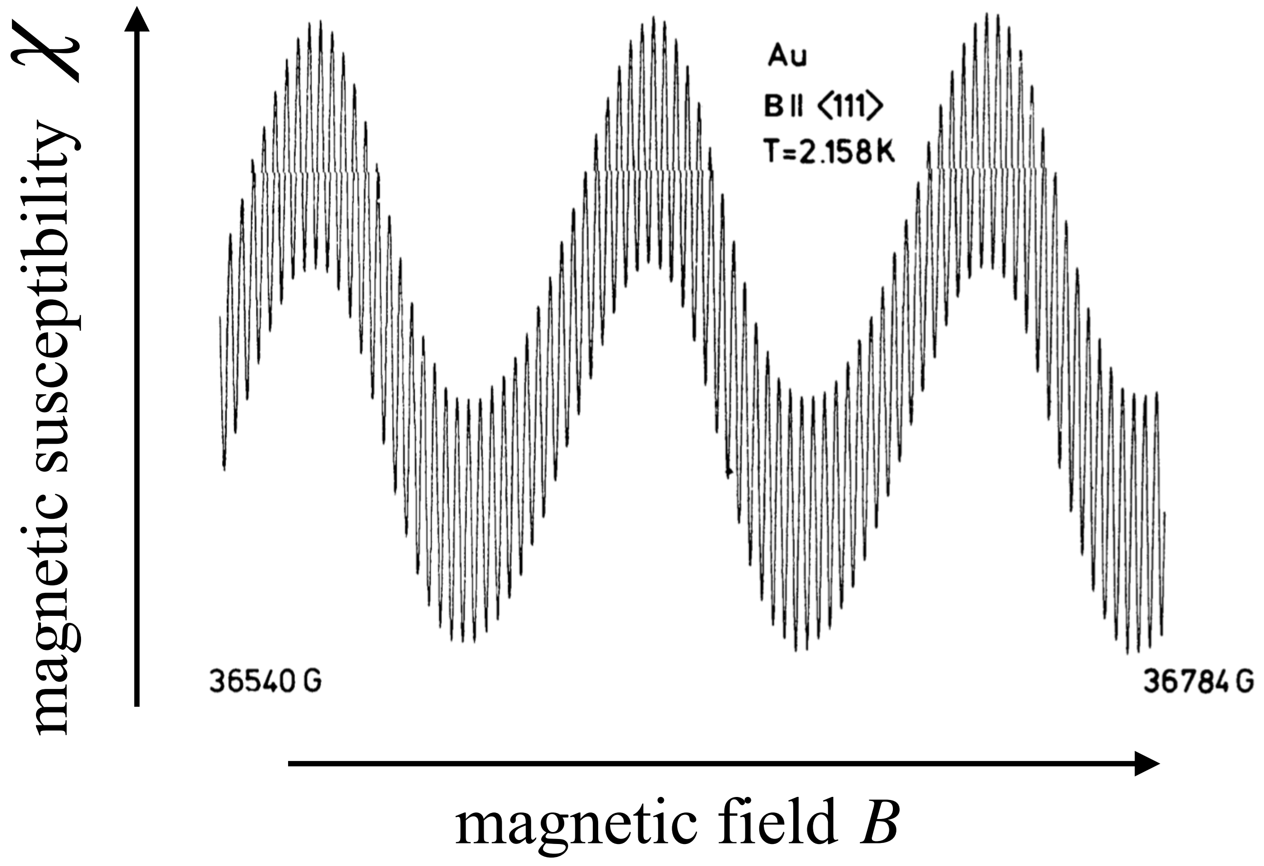
2. Application to realizing unconventional Fermi superfluids

3. Summary

Fermi surface “the face of metal”

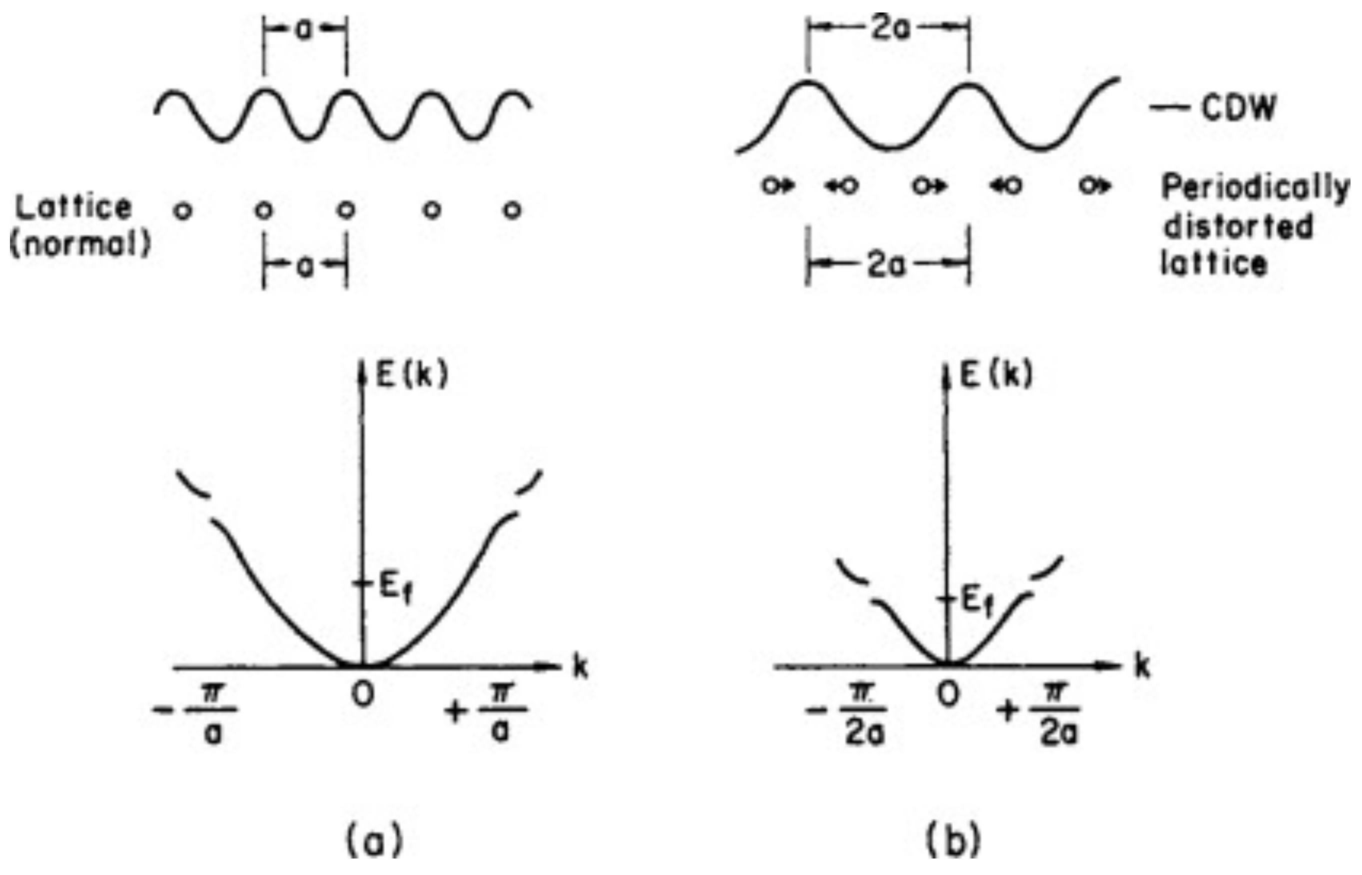
- quantum oscillation

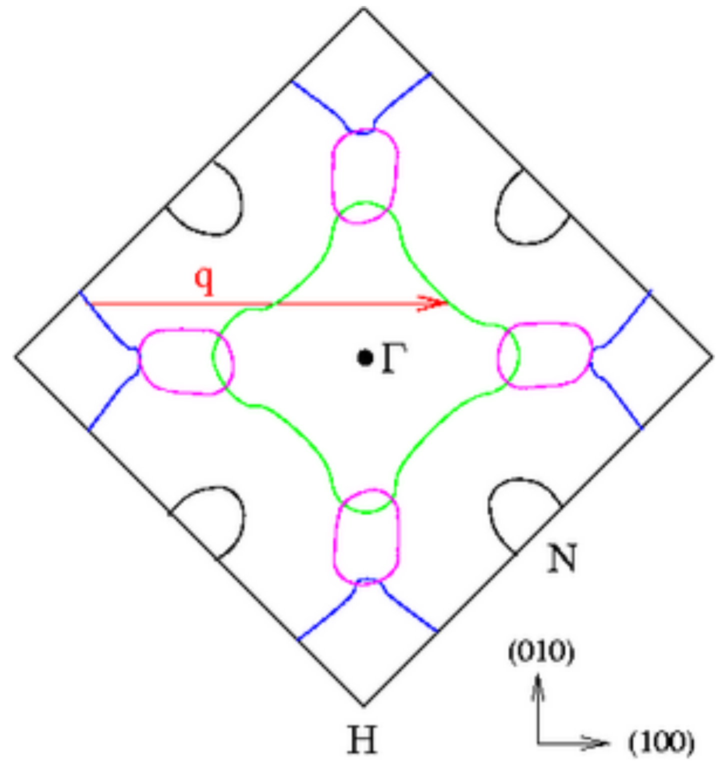




H. Ibach and H. Luth, “Solid-State Physics”

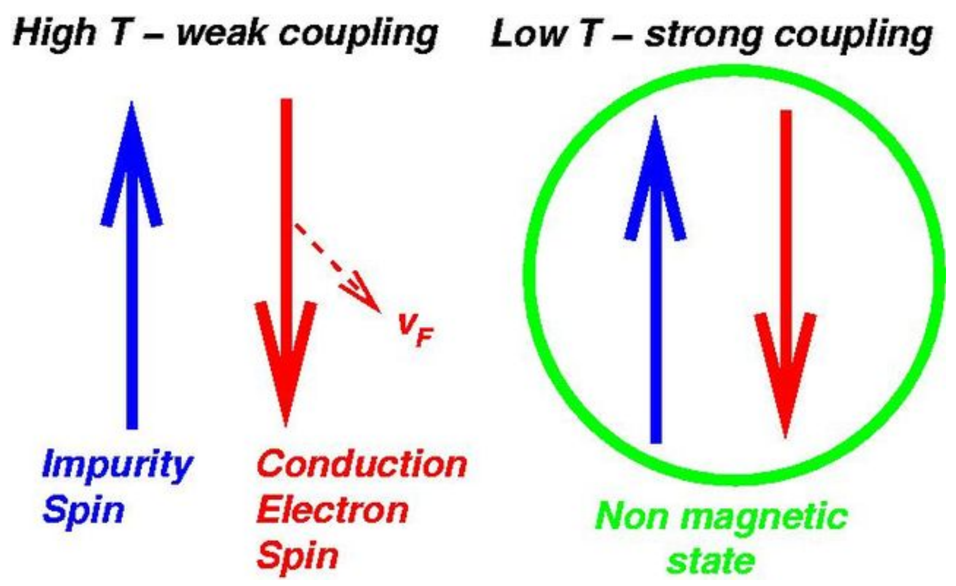
- charge- (spin-) density wave state

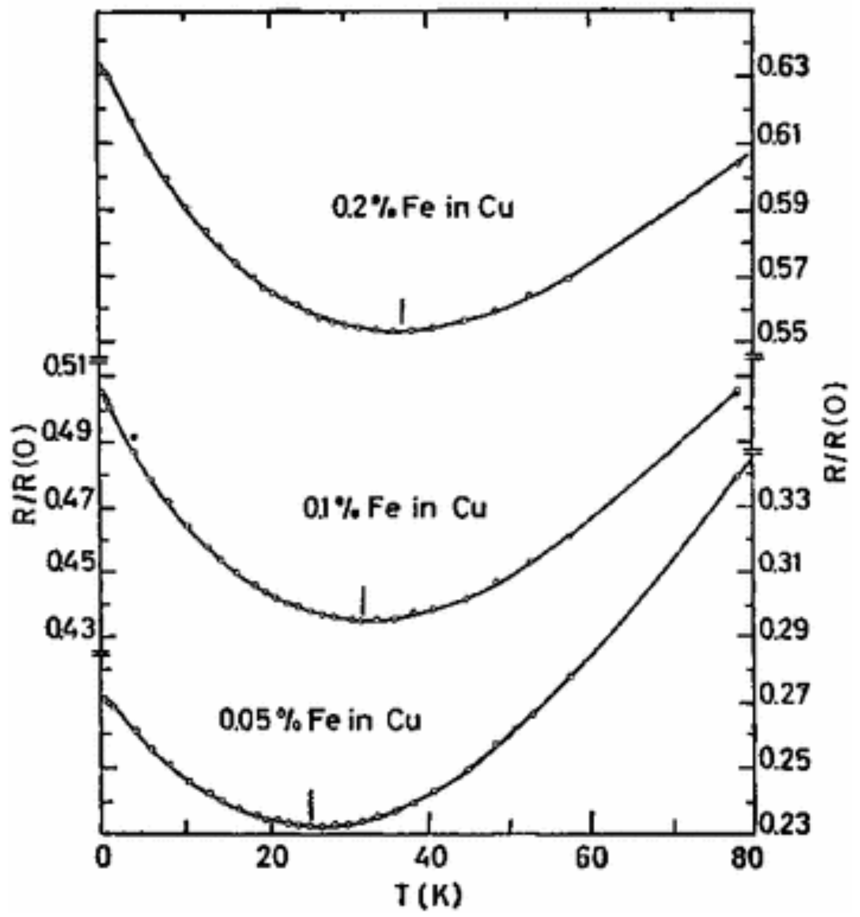




https://ja.wikipedia.org/wiki/スピン密度波

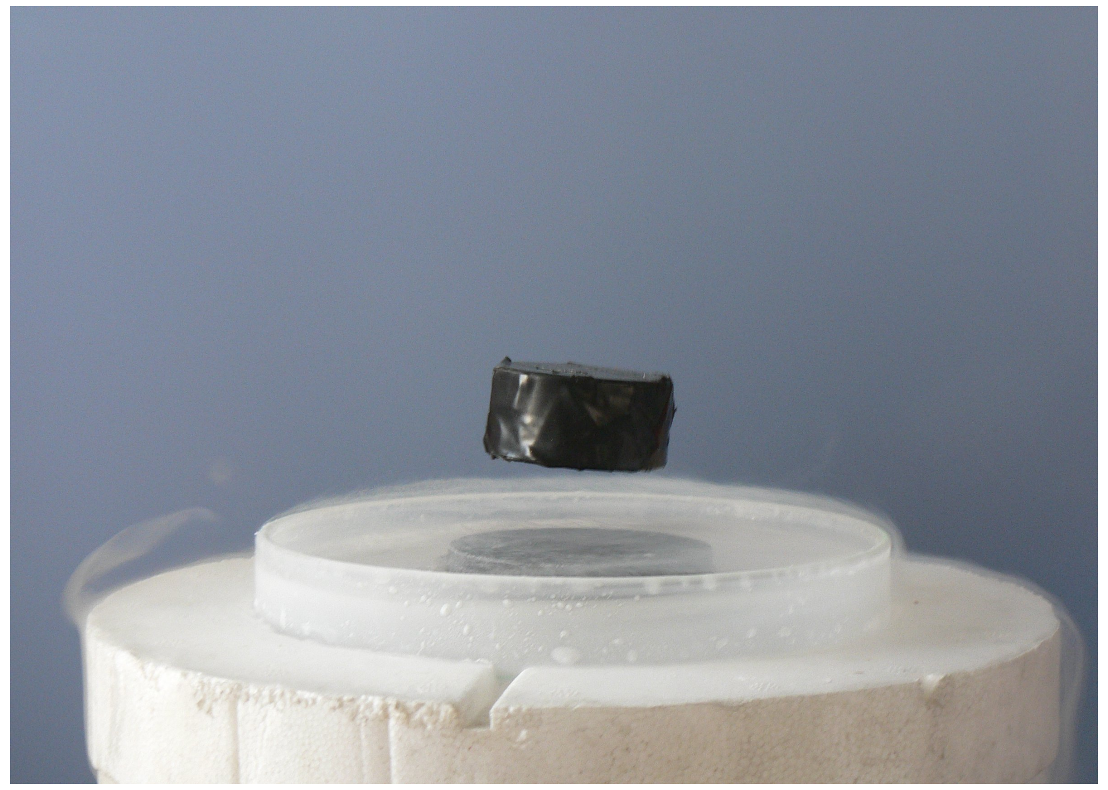
- Kondo effect

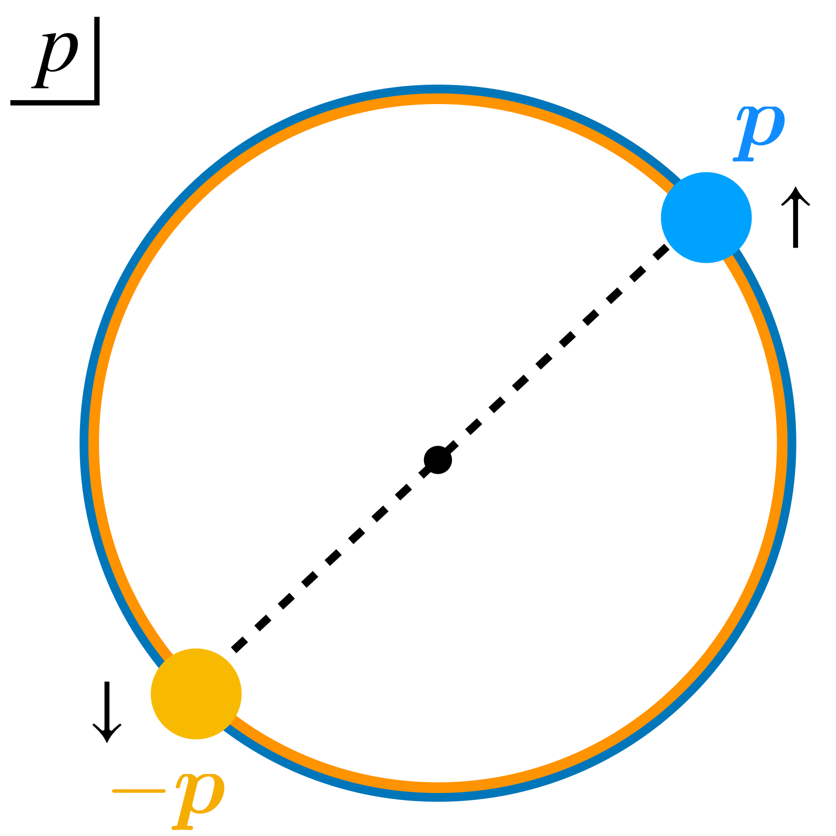




https://ja.wikipedia.org/wiki/近藤効果

- superconductivity

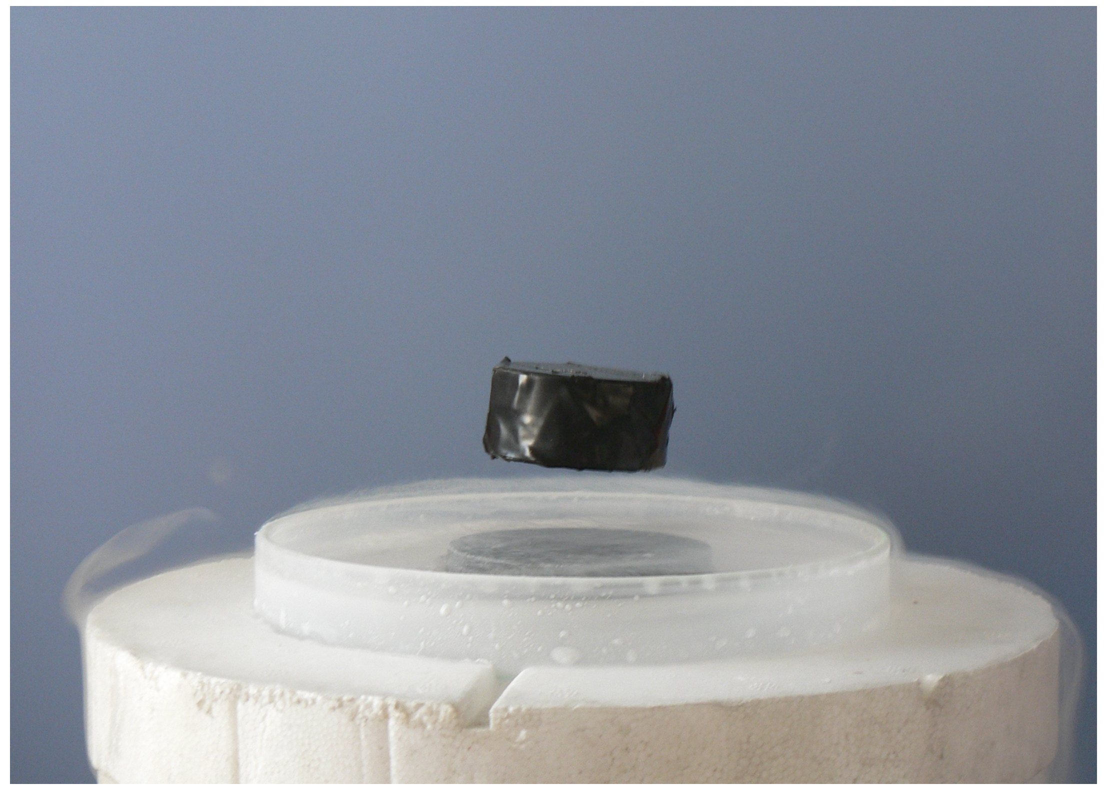


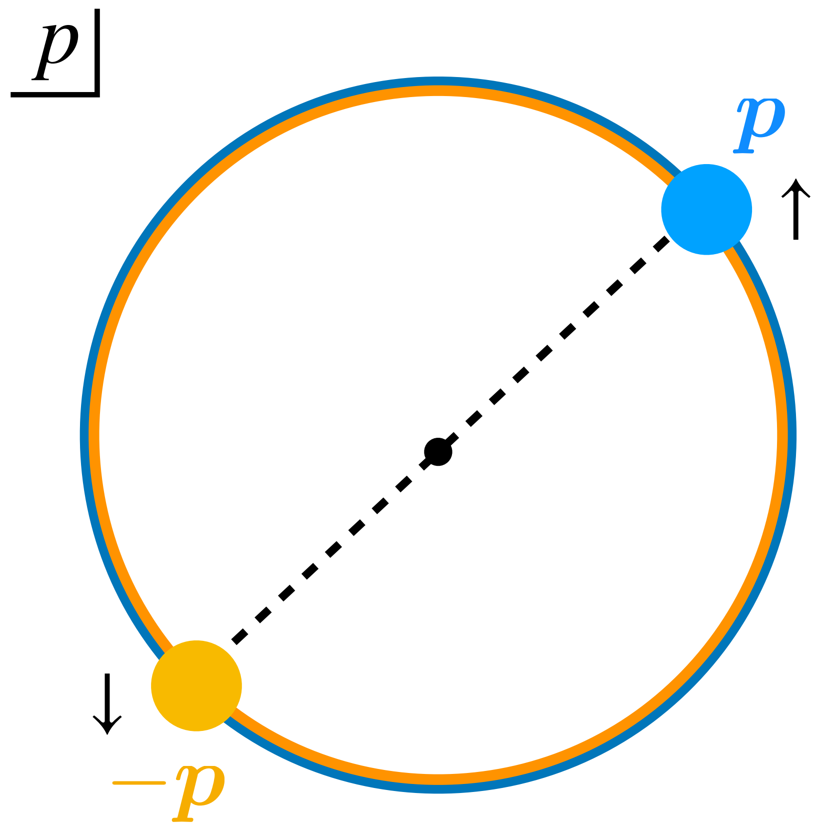


https://ja.wikipedia.org/wiki/超伝導

Fermi surface “the face of metal”

- superconductivity

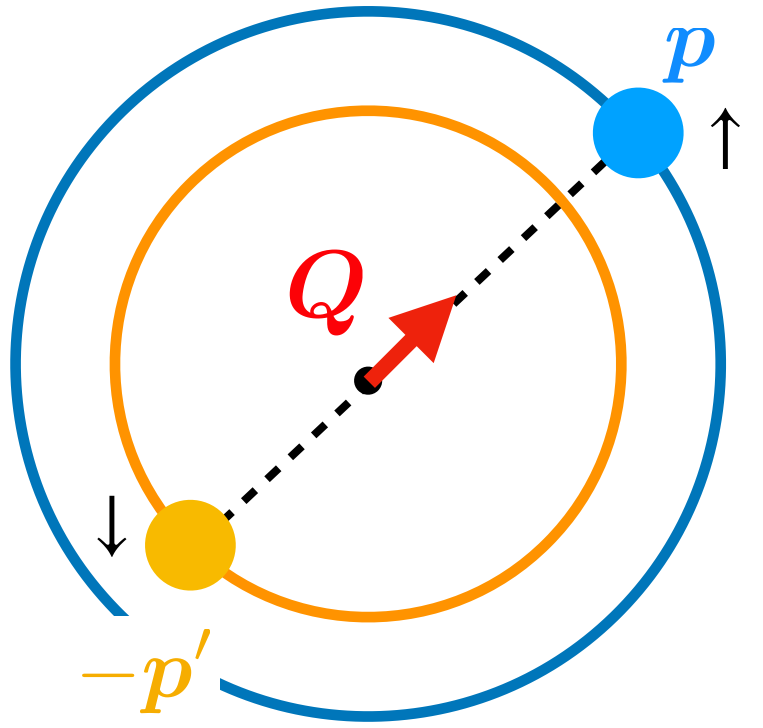




Cooper pair with zero center-of-mass momentum

BCS state Bardeen, Cooper, and Schrieffer (1957)

magnetic field ↓



Cooper pair with finite center-of-mass momentum

FF(LO) state Fulde and Ferrell (1964), Larkin and Ovchinnikov (1965)

Control of Fermi surfaces

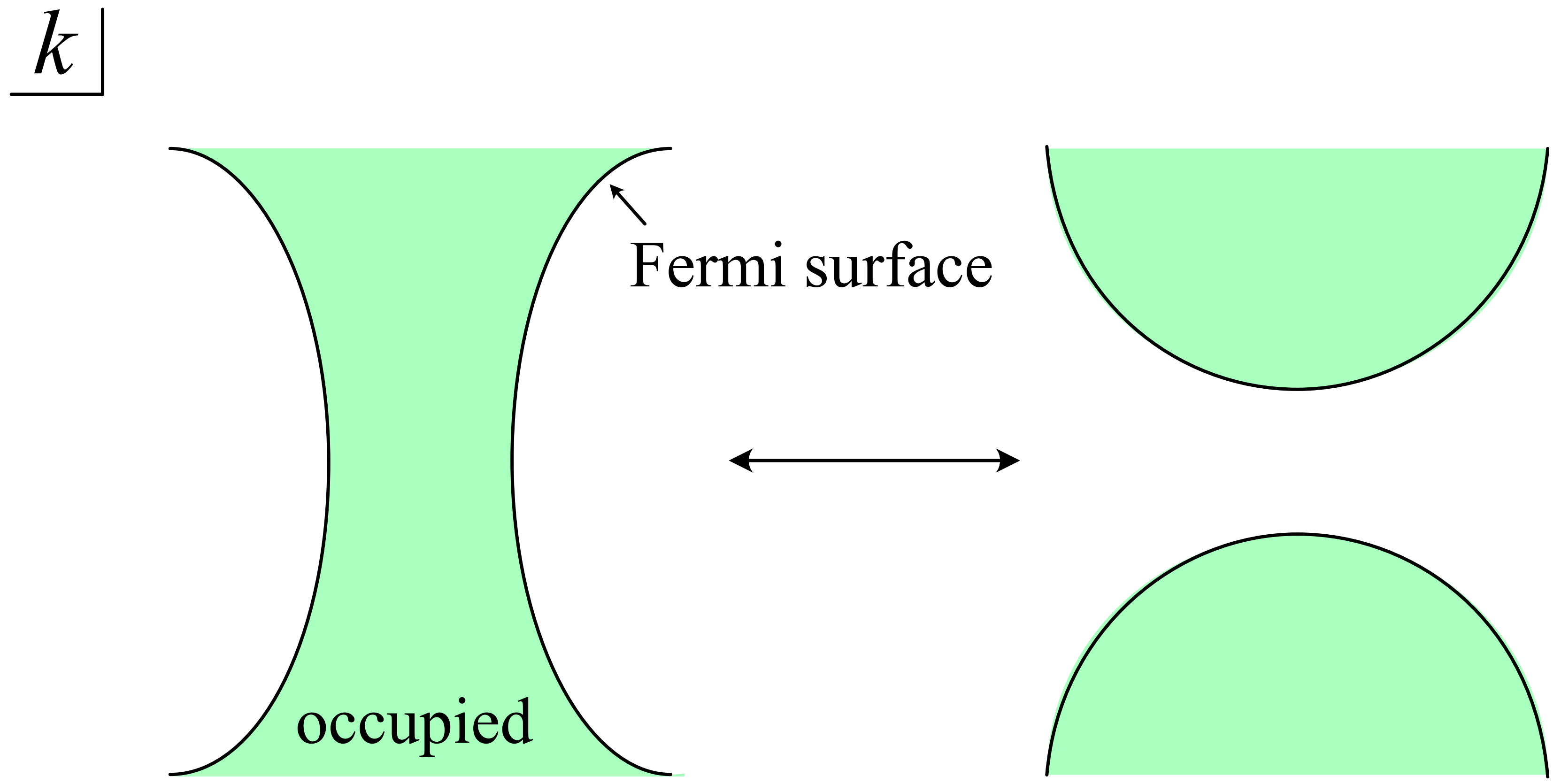
change a structure/topology of a Fermi surface

- pressure
  C. W. Chu, T. F. Smith, and W. E. Gardner, Phys. Rev. B **1**, 214 (1970)
  A. Rodriguez-Prieto, *et.al.*, Phys. Rev. B, **74**, 172104 (2006)
- strain
  L. R. Testardi and J. H. Condon, Phys. Rev. B **1**, 3928 (1970)
  J. M. V. Martins, *et.al.*, Phys. Rev. B **17**, 4633 (1978)
- doping
  N. P. Armitage, *et.al.*, Phys. Rev. Lett. **88**, 257001 (2002)
  A. Kaminski, *et.al.*, Phys. Rev. B **73**, 174511 (2006)

⋮

phase transitions trigged by changes in the topology of Fermi surfaces

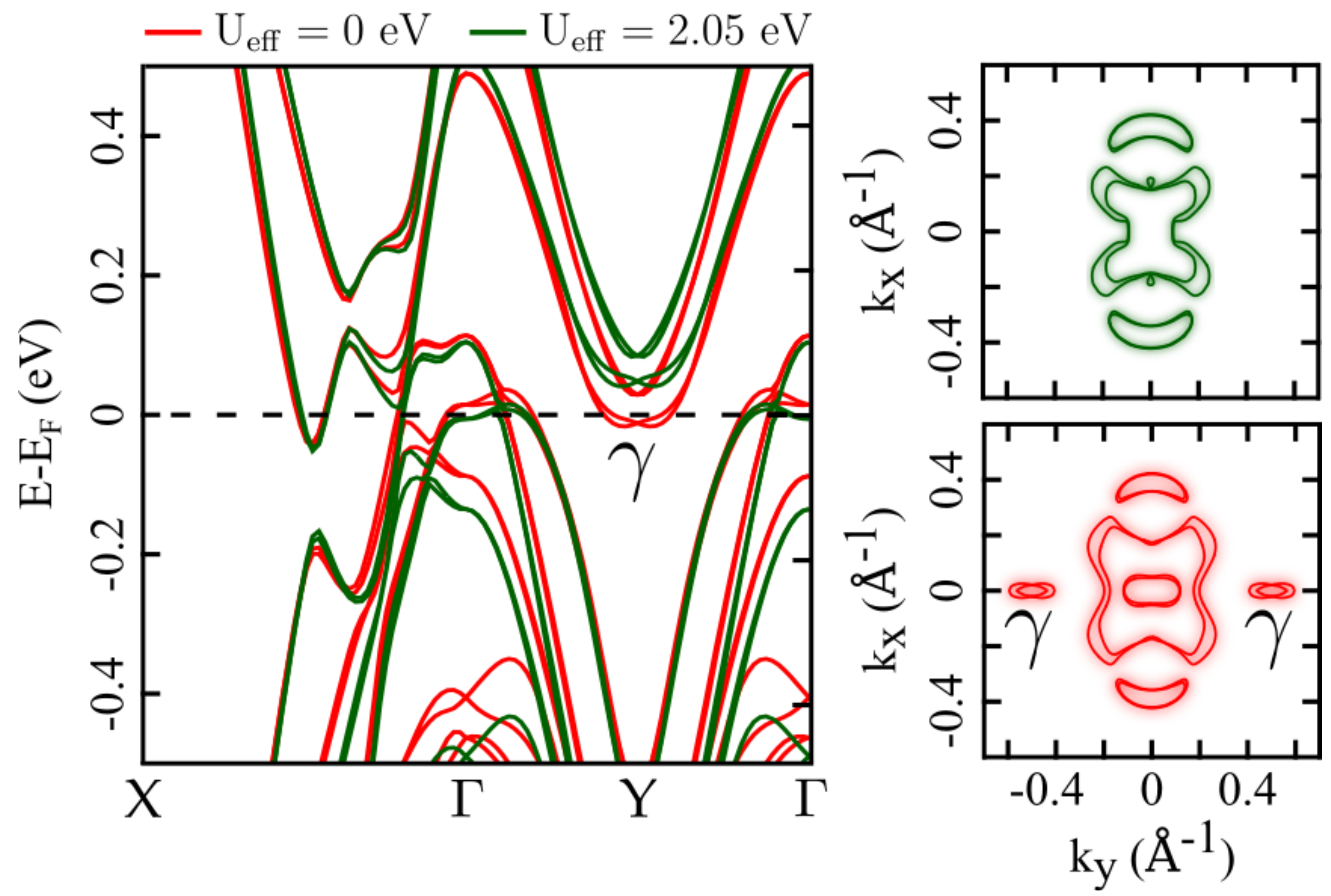
Lifshitz transition



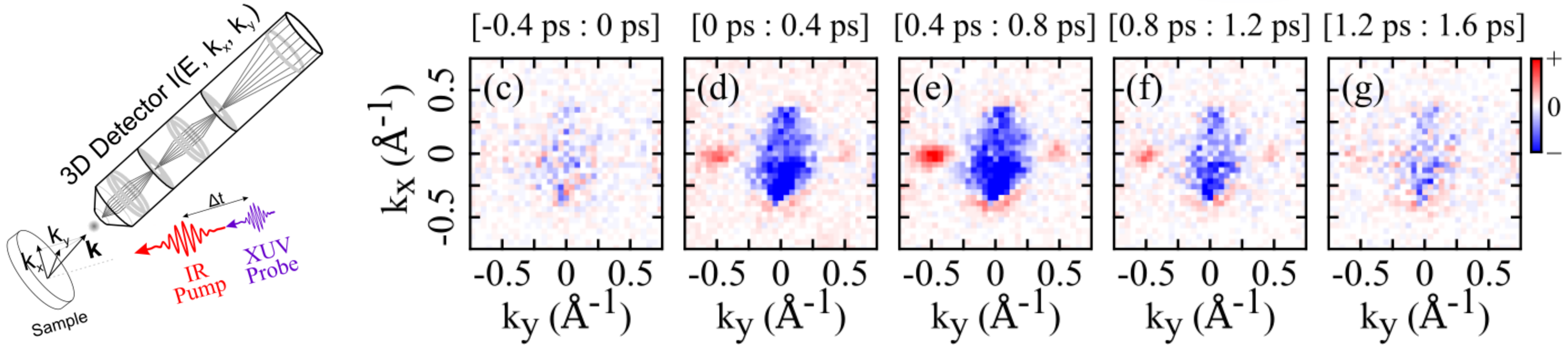
- Laser (non-equilibrium) S. Beaulieu, *et. al.*, arXiv:2003.04059

$T_d - MoTe_2$ Weyl semimetal

Theory：time-dependent self-consistent Hubbard U calculations

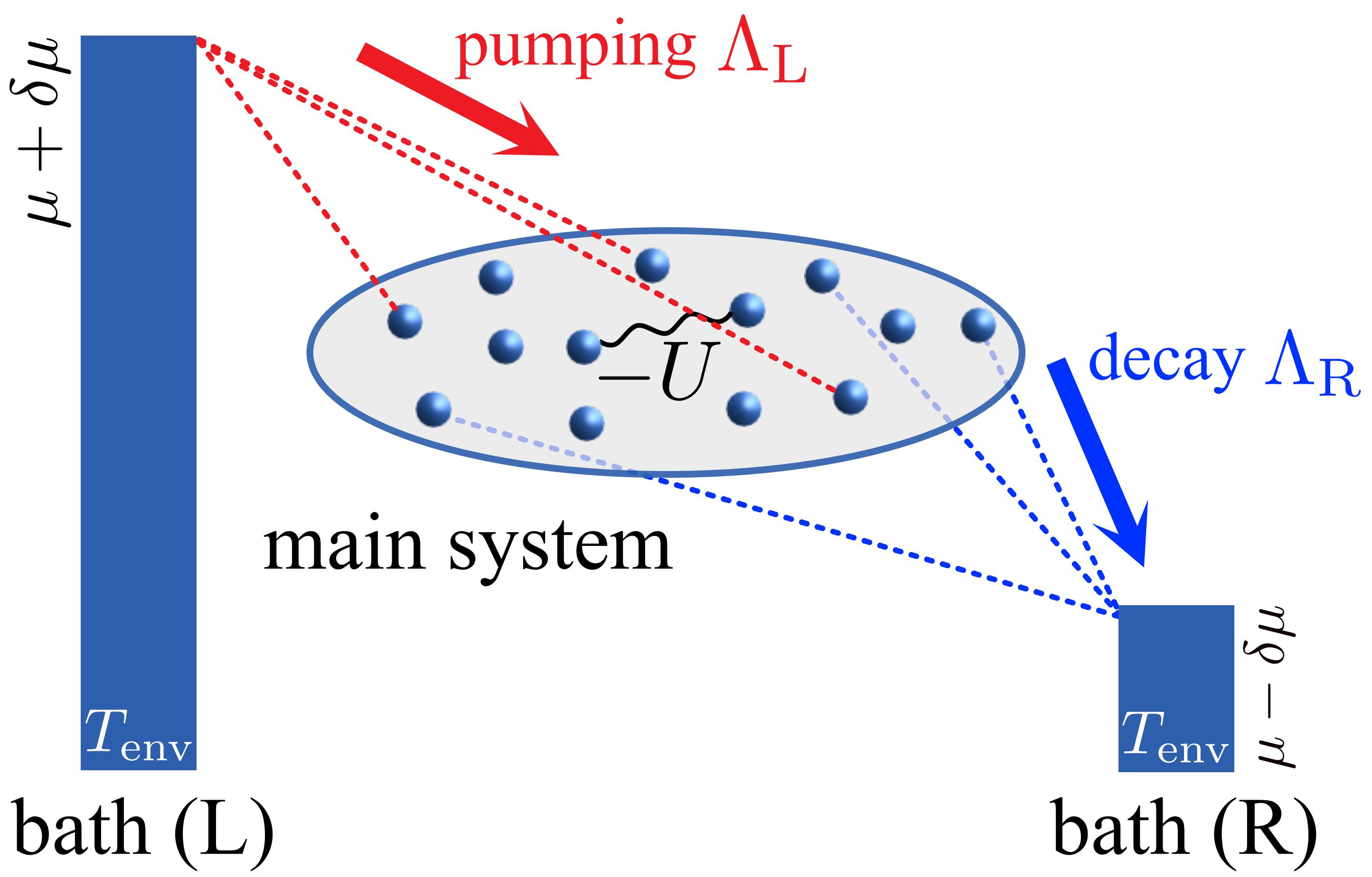


experiment：time-resolved multidimensional photoemission spectroscopy



Contents

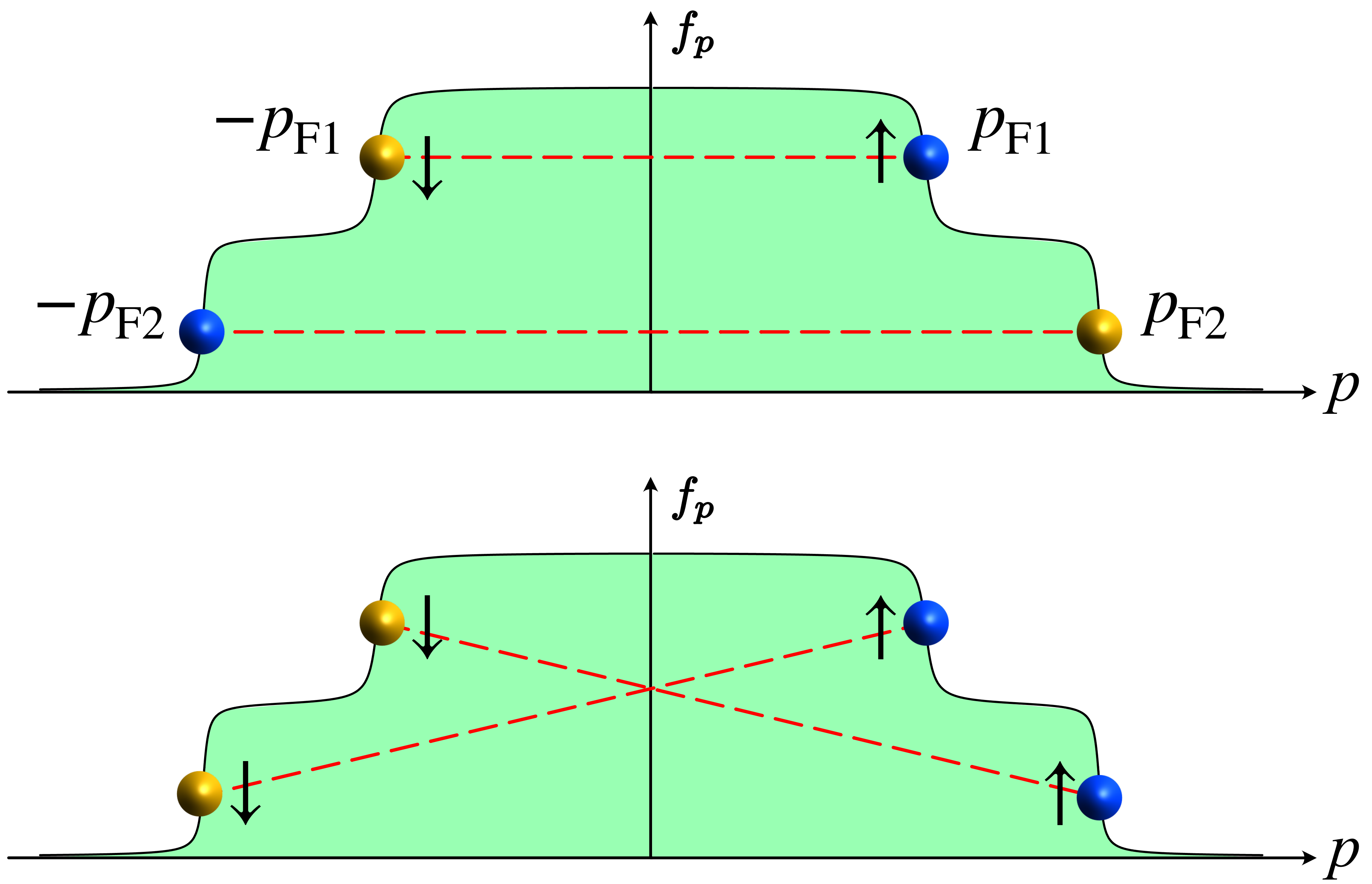
Driven-dissipative Fermi gas



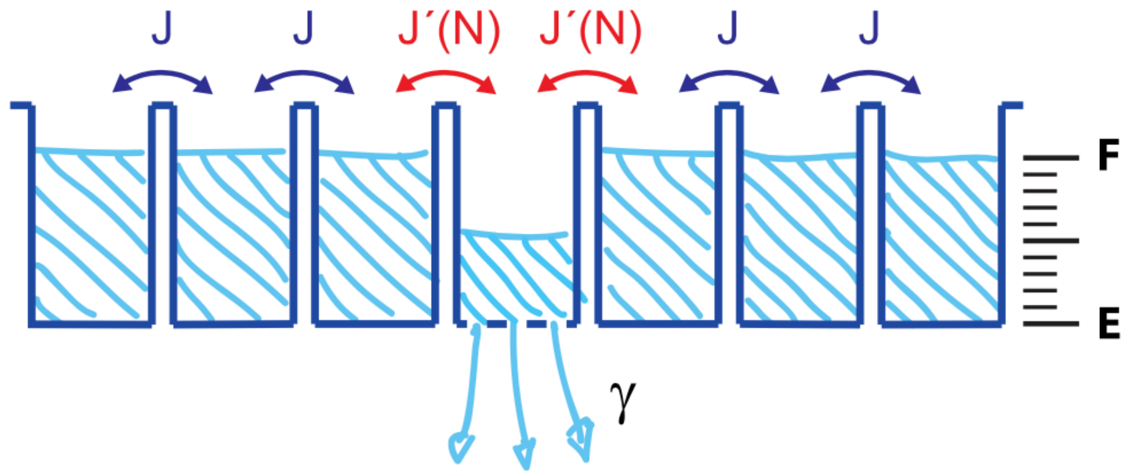
non-equilibrium steady state

pumping = decay

Cooper pairs associated with "effective Fermi surface"

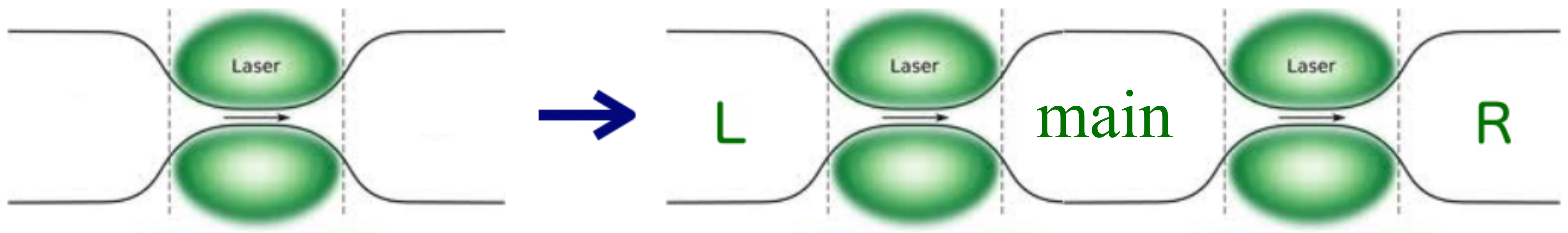


- driven-dissipative Josephson junction array



R. Labouvie, B. Santra, S. Heun, and H. Ott
Phys. Rev. Lett **116**, 235302 (2016)

- extension of the two-terminal configuration



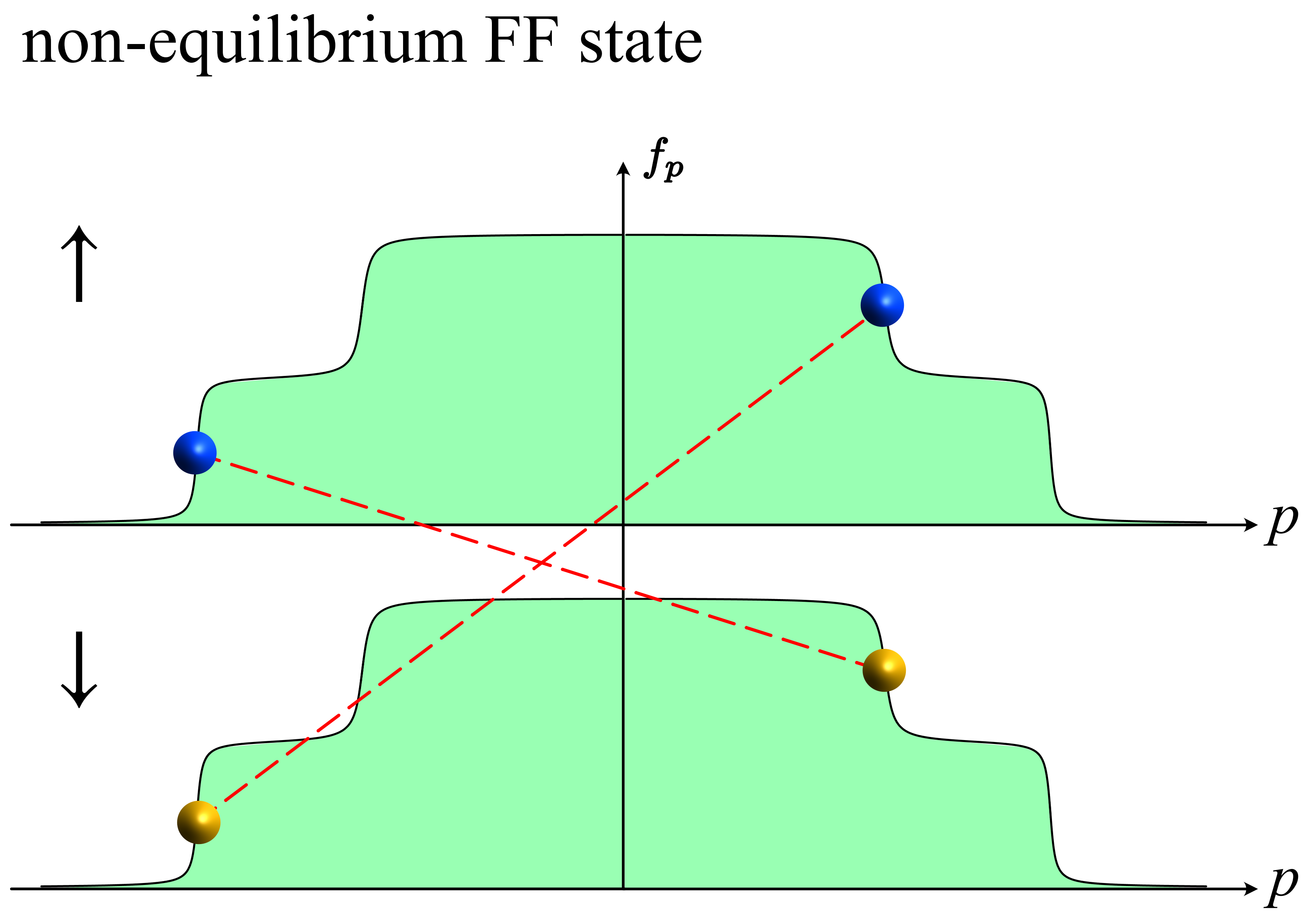
S. Krinner, D. Stadler, D. Husmann, J. P. Brantut, and T. Esslinger, Nature **517**, 64 (2015).

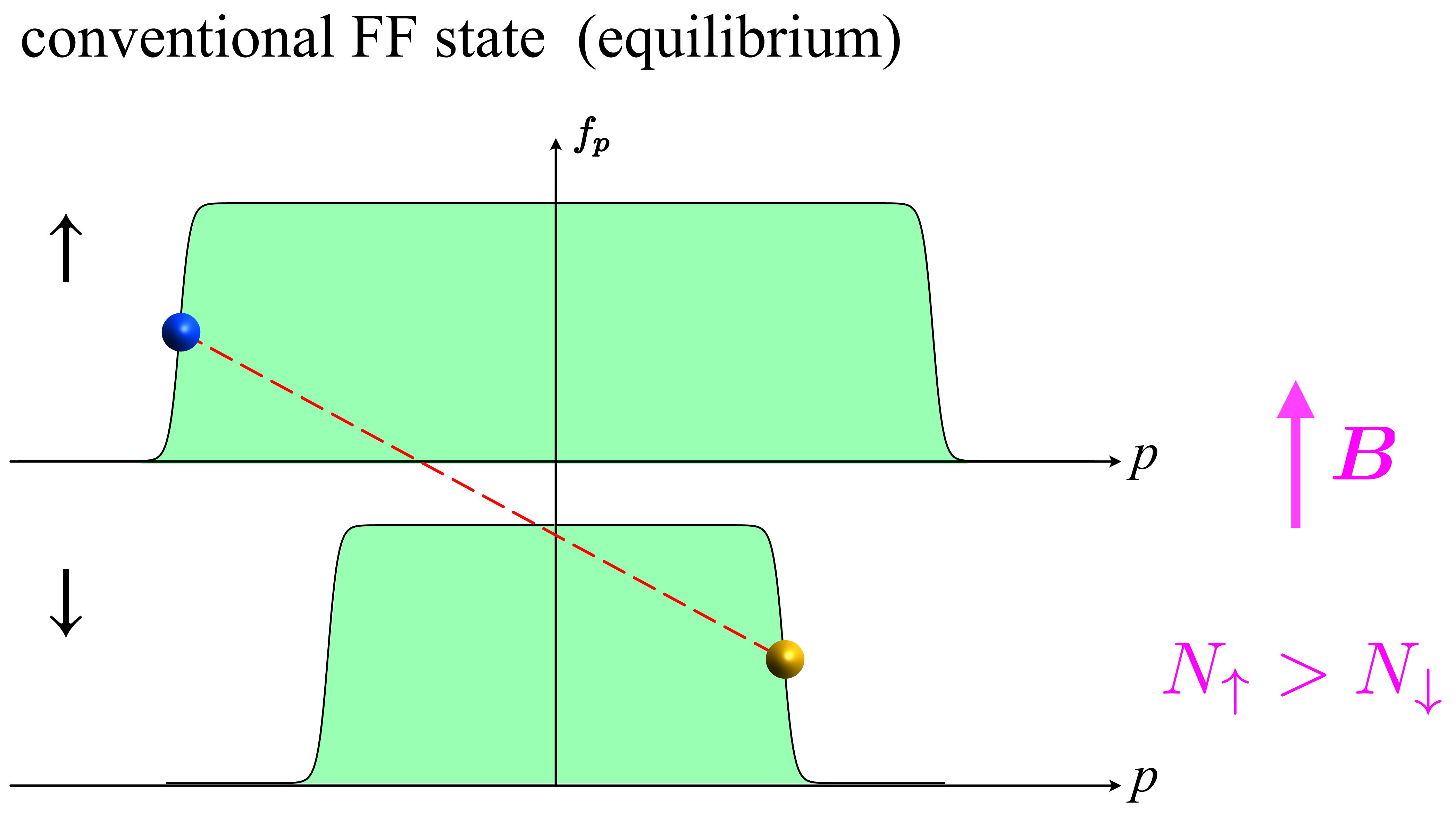
Cooper pairs with finite center-of-mass momentum

Fulde–Ferrell (FF) state

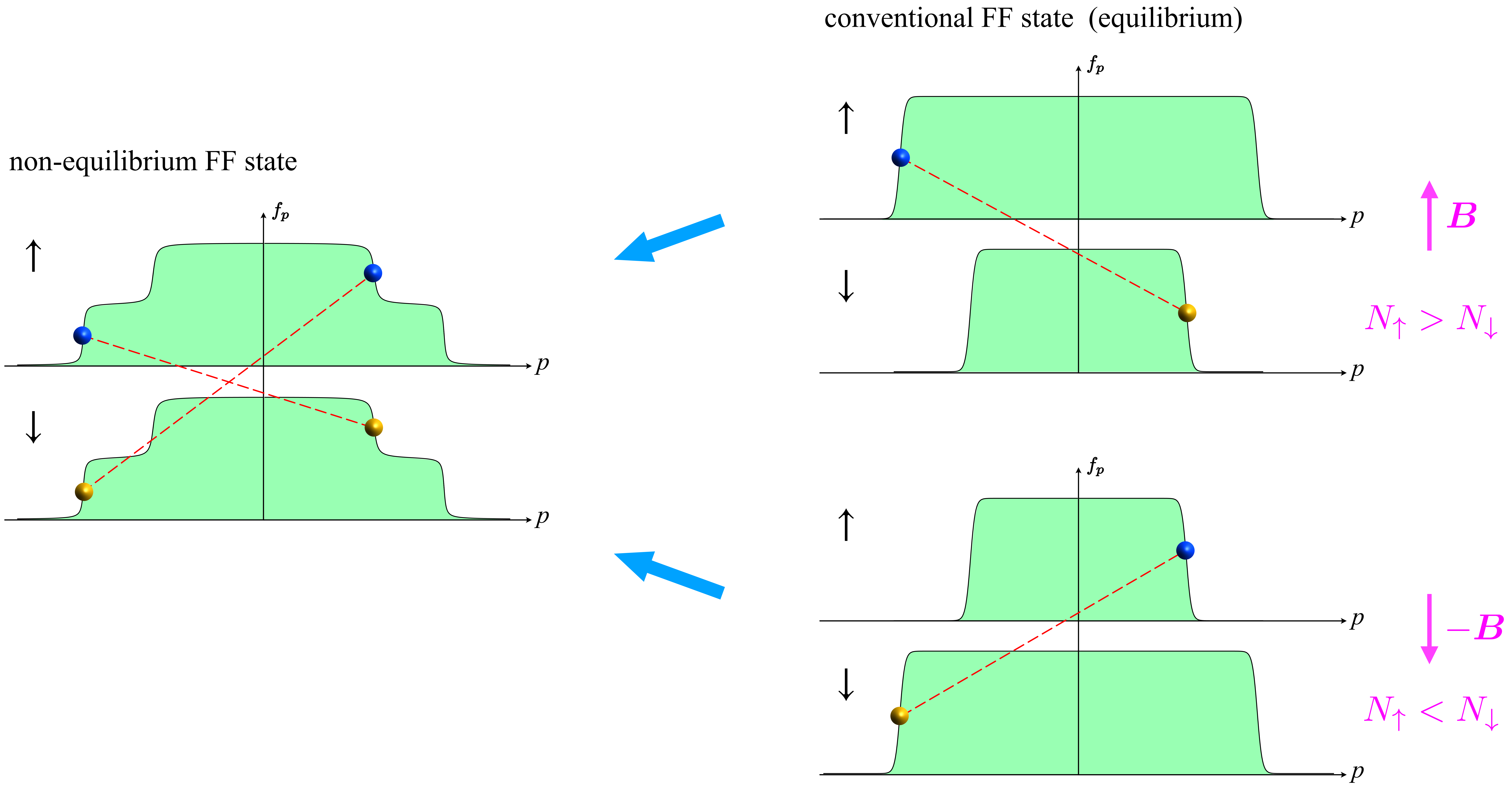
- metallic superconductivity under an external magnetic field
- spin-polarized Fermi gas
- color superconductivity in quantum chromodynamics

Driven-dissipative Fermi gas

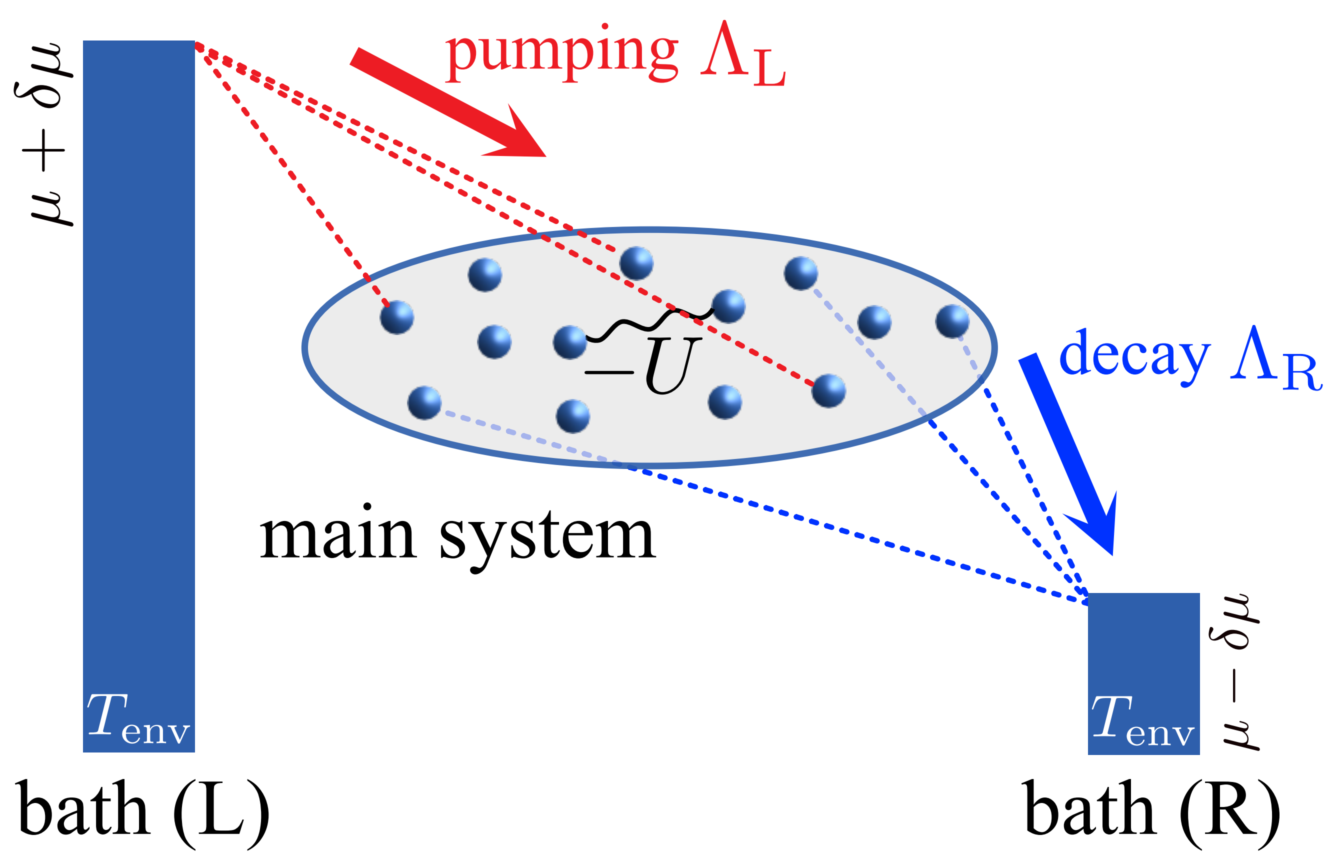




Driven-dissipative Fermi gas



Model Hamiltonian



- environment temperature T_{env}
- chemical potential bias $\delta\mu$
- dissipation strength $\gamma = \pi N_t \rho |\Lambda|^2$

$$\mathcal{H}_{tot} = \mathcal{H}_{sys} + \mathcal{H}_{env} + \mathcal{H}_{mix}$$

- main system $\mathcal{H}_{sys} = \sum_{\boldsymbol{p},\sigma=\uparrow,\downarrow} \varepsilon_{\boldsymbol{p}}\, a^\dagger_{\boldsymbol{p}\sigma} a_{\boldsymbol{p}\sigma} - U \sum_{\boldsymbol{p},\boldsymbol{p}',\boldsymbol{q}} a^\dagger_{\boldsymbol{p}+\boldsymbol{q}/2\uparrow} a^\dagger_{-\boldsymbol{p}+\boldsymbol{q}/2\downarrow} a_{-\boldsymbol{p}'+\boldsymbol{q}/2\downarrow} a_{\boldsymbol{p}'+\boldsymbol{q}/2\uparrow}$

attractive interaction

- reservoirs $\mathcal{H}_{env} = \sum_{\alpha=L,R} \sum_{\boldsymbol{p},\sigma} [\varepsilon_{\boldsymbol{p}} - \mu_\alpha]\, c^{\alpha\dagger}_{\boldsymbol{p}\sigma} c^{\alpha}_{\boldsymbol{p}\sigma}$ (free fermion bath)

- tunneling term $\mathcal{H}_{mix} = \sum_{\alpha=L,R} \sum_{j=1}^{N_t} \sum_{\boldsymbol{p},\boldsymbol{q},\sigma} \left[e^{i\mu_\alpha t} \Lambda c^{\alpha\dagger}_{\boldsymbol{q}\sigma} a_{\boldsymbol{p}\sigma} e^{-i\boldsymbol{q}\cdot\boldsymbol{R}^\alpha_j} e^{i\boldsymbol{p}\cdot\boldsymbol{r}_j} + \text{H.c.} \right]$

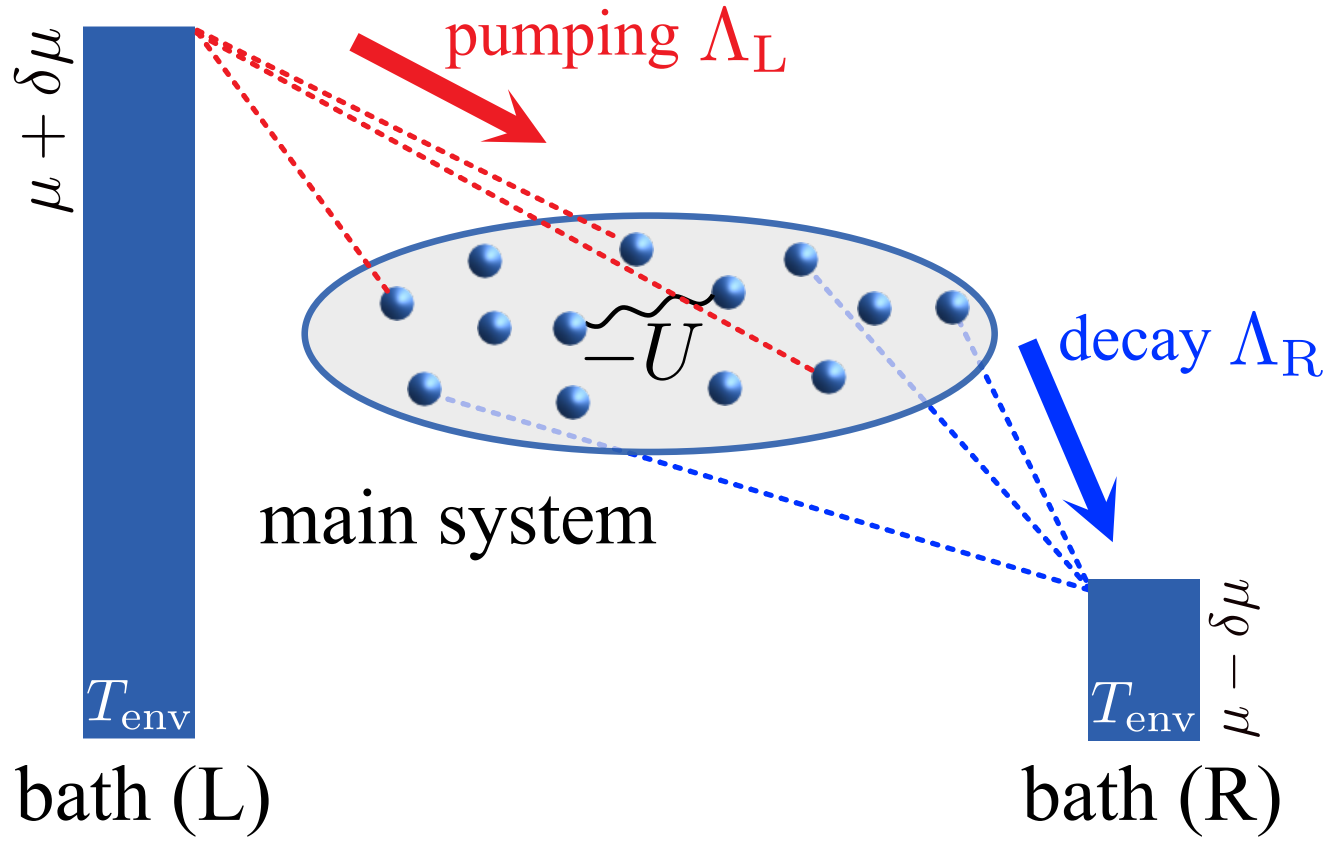
chemical potential bias — Spatially random tunneling

$\sigma = \uparrow, \downarrow$ pseudo-spin of atoms

$a_{\boldsymbol{p}\sigma}$ annihilation operator of a fermion in the main system

$c^{\alpha=L.R}_{\boldsymbol{p}\sigma}$ annihilation operator of a fermion in the reservoir

Model Hamiltonian



- environment temperature T_{env} $\left(T_{\mathrm{env}} = 0\right)$
- chemical potential bias $\delta\mu$
- dissipation strength $\gamma = \pi N_t \rho |\Lambda|^2$

$$\mathcal{H}_{\mathrm{tot}} = \mathcal{H}_{\mathrm{sys}} + \mathcal{H}_{\mathrm{env}} + \mathcal{H}_{\mathrm{mix}}$$

- main system $\mathcal{H}_{\mathrm{sys}} = \sum_{\boldsymbol{p},\sigma=\uparrow,\downarrow} \varepsilon_{\boldsymbol{p}}\, a^\dagger_{\boldsymbol{p}\sigma} a_{\boldsymbol{p}\sigma} - U \sum_{\boldsymbol{p},\boldsymbol{p}',\boldsymbol{q}} a^\dagger_{\boldsymbol{p}+\boldsymbol{q}/2\uparrow} a^\dagger_{-\boldsymbol{p}+\boldsymbol{q}/2\downarrow} a_{-\boldsymbol{p}'+\boldsymbol{q}/2\downarrow} a_{\boldsymbol{p}'+\boldsymbol{q}/2\uparrow}$

attractive interaction

- reservoirs $\mathcal{H}_{\mathrm{env}} = \sum_{\alpha=\mathrm{L,R}} \sum_{\boldsymbol{p},\sigma} [\varepsilon_{\boldsymbol{p}} - \mu_\alpha]\, c^{\alpha\dagger}_{\boldsymbol{p}\sigma} c^{\alpha}_{\boldsymbol{p}\sigma}$ (free fermion bath)

- tunneling term $\mathcal{H}_{\mathrm{mix}} = \sum_{\alpha=\mathrm{L,R}} \sum_{j=1}^{N_t} \sum_{\boldsymbol{p},\boldsymbol{q},\sigma} \left[e^{i\mu_\alpha t} \Lambda c^{\alpha\dagger}_{\boldsymbol{q}\sigma} a_{\boldsymbol{p}\sigma} e^{-i\boldsymbol{q}\cdot\boldsymbol{R}^\alpha_j} e^{i\boldsymbol{p}\cdot\boldsymbol{r}_j} + \mathrm{H.c.} \right]$

chemical potential bias

Spatially random tunneling

$\sigma = \uparrow, \downarrow$ pseudo-spin of atoms

$a_{\boldsymbol{p}\sigma}$ annihilation operator of a fermion in the main system

$c^{\alpha=\mathrm{L.R}}_{\boldsymbol{p}\sigma}$ annihilation operator of a fermion in the reservoir

Non-equilibrium superfluid phase (NESS: non-equilibrium steady state)

steady-state ansatz $\Delta(\boldsymbol{r}, t) = \Delta\, e^{i\boldsymbol{Q}\cdot\boldsymbol{r}} e^{-2i\mu t}$

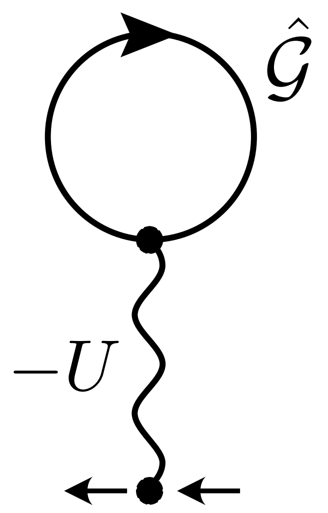
Fulde-Ferrell type order parameter

Nambu lesser Green's function

$$-i\mathcal{G}^{<}_{\boldsymbol{p}} = \begin{pmatrix} \langle a^\dagger_{\boldsymbol{p}\uparrow} a_{\boldsymbol{p}\uparrow}\rangle & \langle a_{-\boldsymbol{p}\downarrow} a_{\boldsymbol{p}\uparrow}\rangle \\ \langle a^\dagger_{\boldsymbol{p}\uparrow} a^\dagger_{-\boldsymbol{p}\downarrow}\rangle & \langle a_{-\boldsymbol{p}\downarrow} a^\dagger_{-\boldsymbol{p}\downarrow}\rangle \end{pmatrix}$$

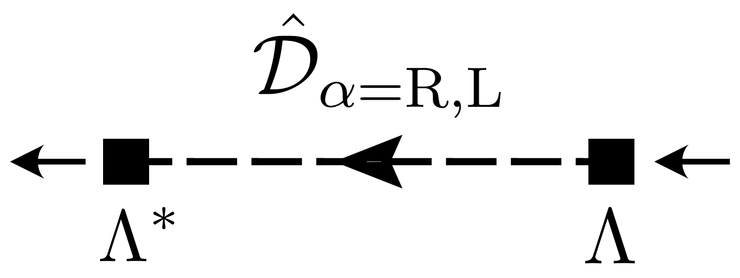
- diagonal component
  particle density
- off-diagonal component
  pair amplitude

- interaction effect



Hartree-Fock-Bogoliubov app.

- environment effect

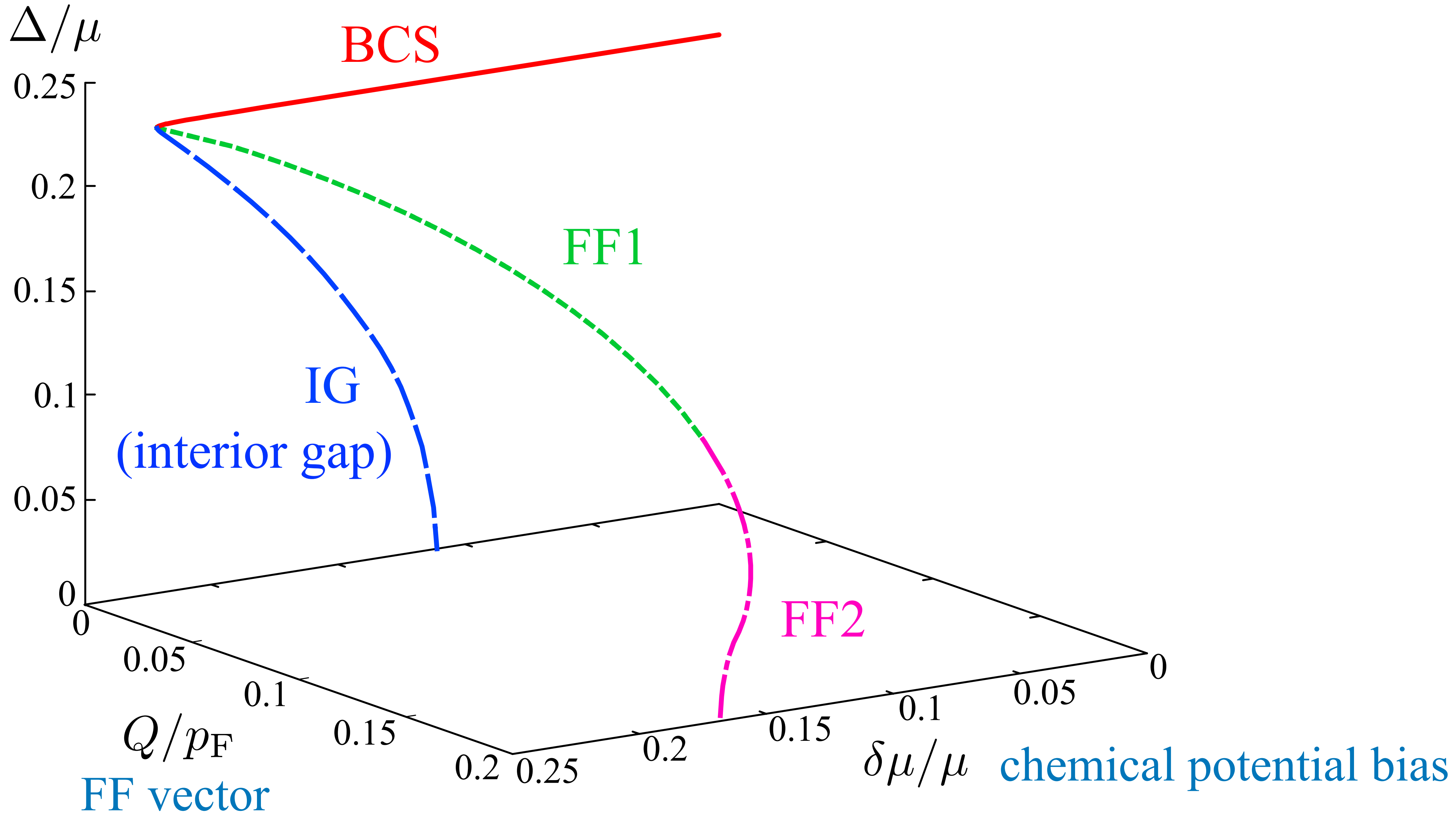


2nd Born app.

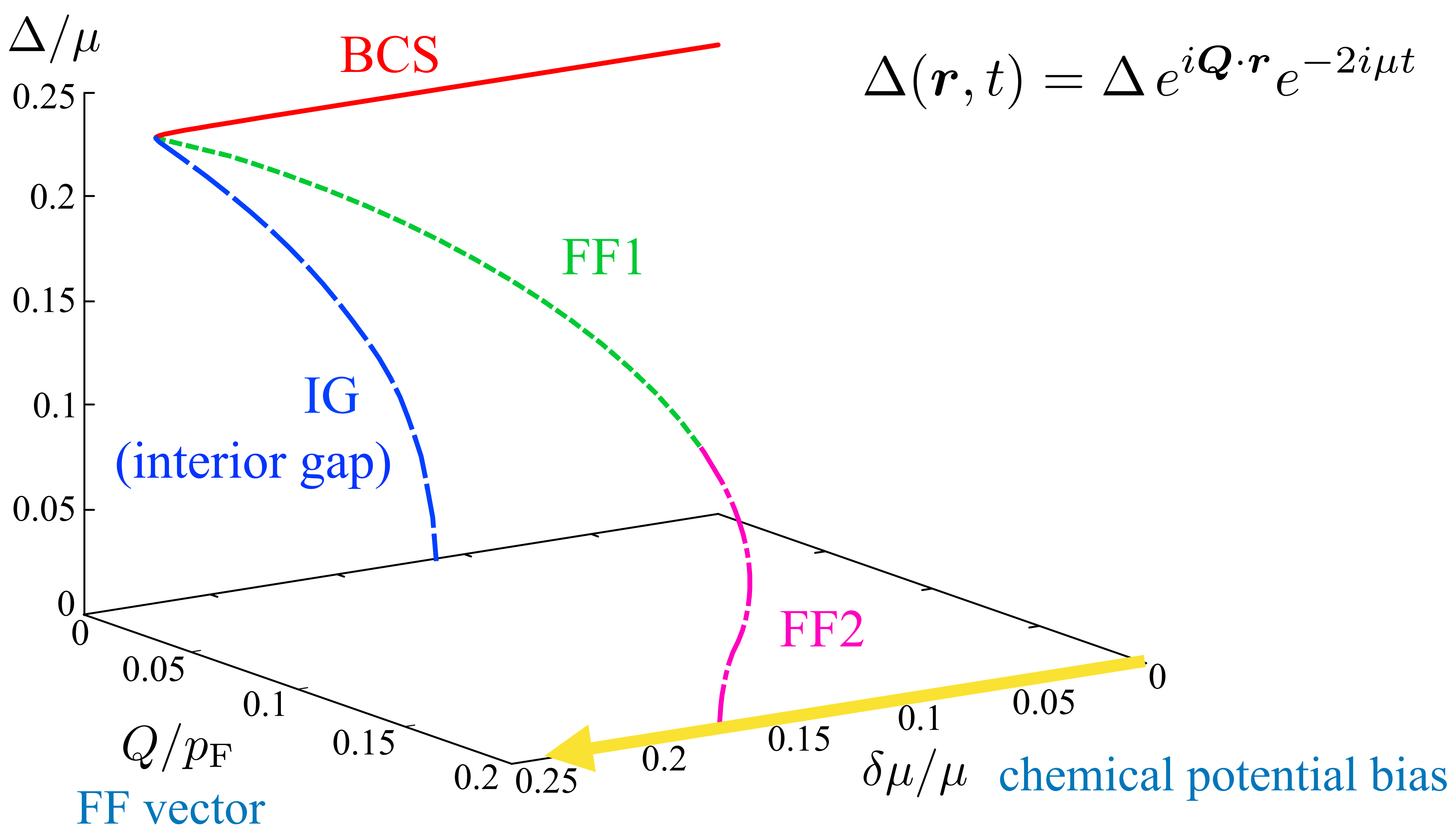
R. Hanai, P. B Littlewood, and Y. Ohashi, Phys. Rev. B **96**, 125206 (2017)

equations determining NESS solutions

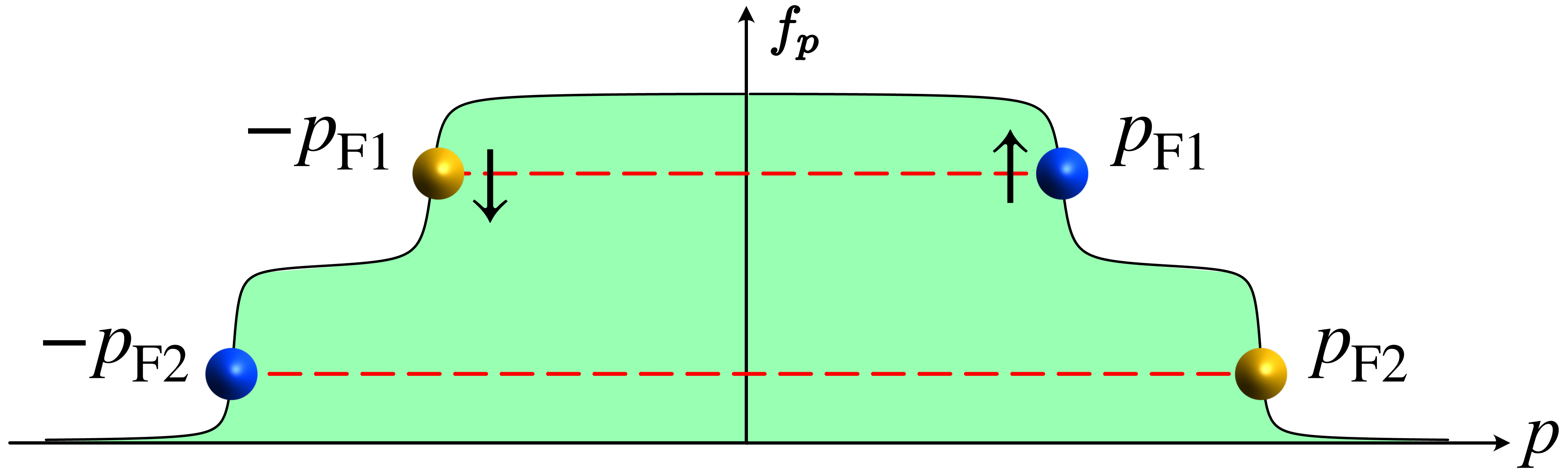
- NESS gap equation $\Delta = U \sum_{\boldsymbol{p}} \langle a_{-\boldsymbol{p}\downarrow} a_{\boldsymbol{p}\uparrow}\rangle$
- current condition $\boldsymbol{J} = \sum_{\sigma=\uparrow,\downarrow} \sum_{\boldsymbol{p}} [\boldsymbol{p} + \boldsymbol{Q}/2] \langle a^\dagger_{\boldsymbol{p}\sigma} a_{\boldsymbol{p}\sigma}\rangle = 0$



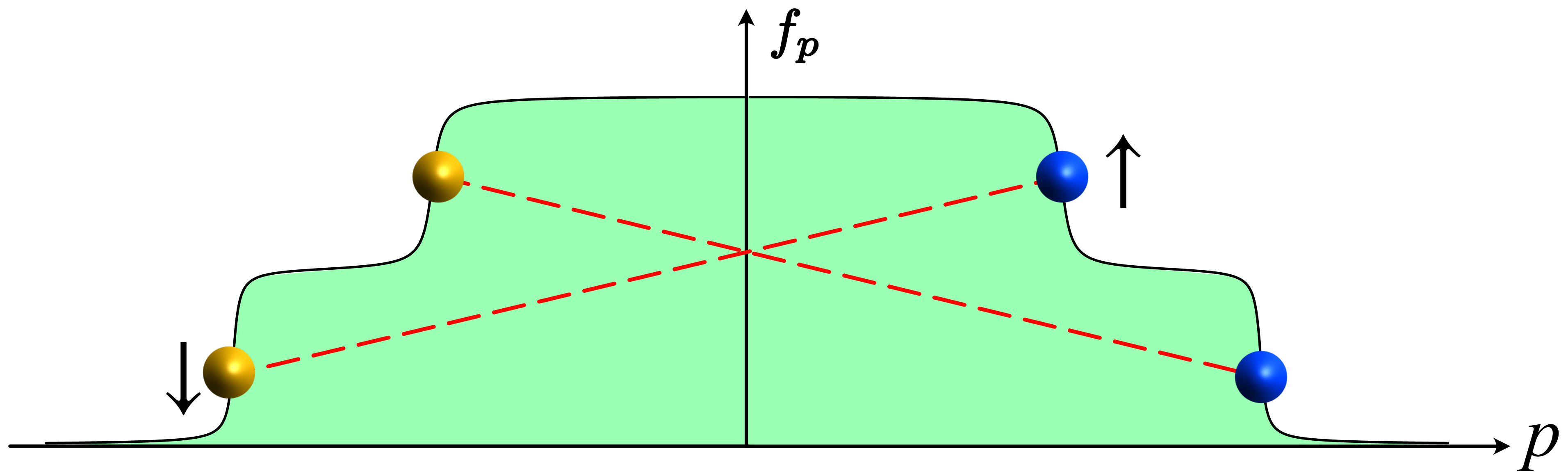
Non-equilibrium superfluid phase (NESS: non-equilibrium steady state)



- BCS, IG $(Q=0)$



- FF1, FF2 $(Q \neq 0)$

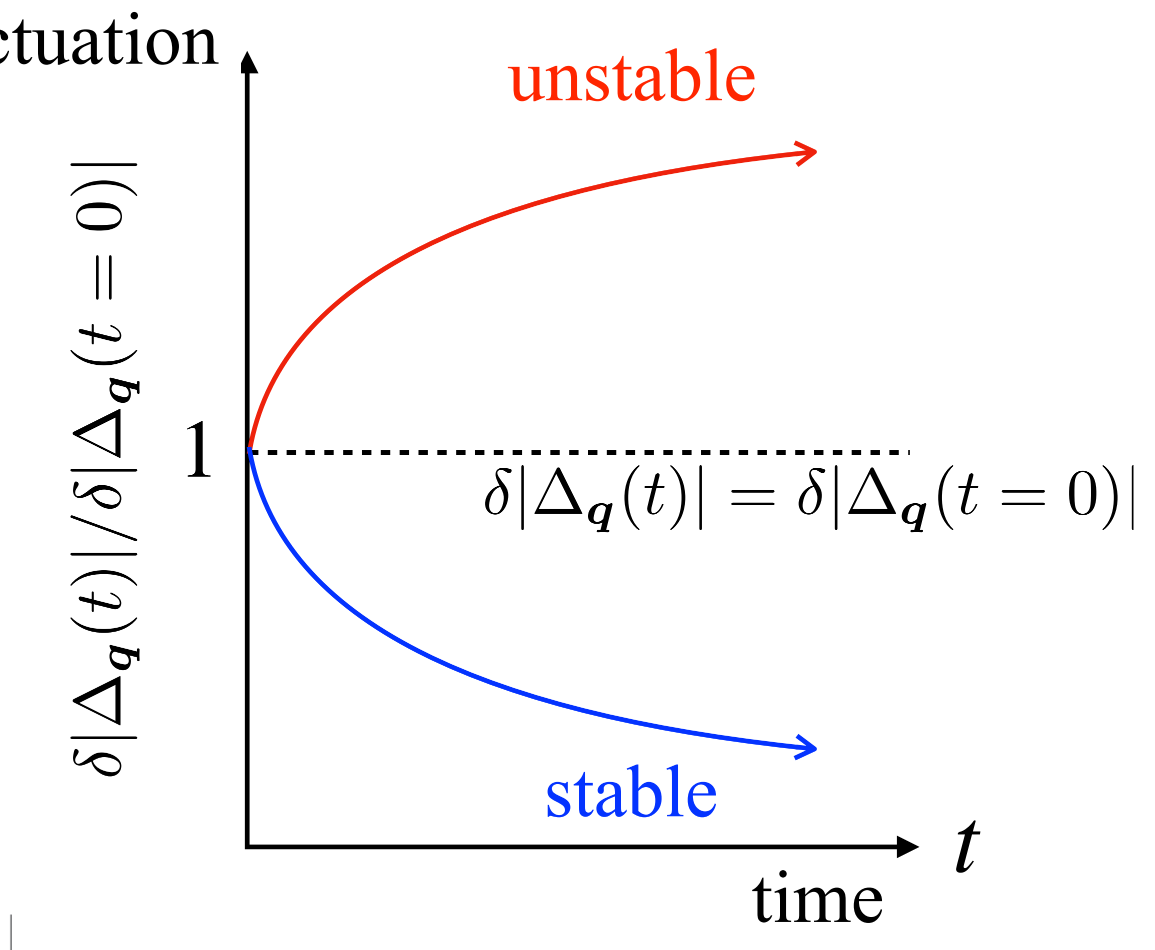


small → $\delta\mu$ → large

region	obtained solution
R1 $(0 < \delta\mu < 0.111\mu)$	BCS
R2 $(0.111\mu < \delta\mu < 0.135\mu)$	BCS, IG
R3 $(0.135\mu < \delta\mu < 0.152\mu)$	BCS, IG, FF1, FF2
R4 $(0.152\mu < \delta\mu < 0.183\mu)$	BCS, IG, FF1

Non-equilibrium superfluid phase (stability analysis)

fluctuations from the NESS $\delta|\Delta(\boldsymbol{r},t)| = |\Delta(\boldsymbol{r},t)| - \Delta_{\rm NESS}$



time-evolution of the superfluid order parameter Δ

$$\left[\mathcal{G}^{-1}\mathcal{G}^{<} - \mathcal{G}^{<}\mathcal{G}^{-1}\right](\boldsymbol{p},\boldsymbol{r},t) = \int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\left[\Sigma^{\rm R}\otimes\mathcal{G}^{<} - \mathcal{G}^{<}\otimes\Sigma^{\rm A} - \mathcal{G}^{\rm R}\otimes\Sigma^{<} + \Sigma^{<}\otimes\mathcal{G}^{\rm A}\right](\boldsymbol{p},\omega,\boldsymbol{r},t)$$

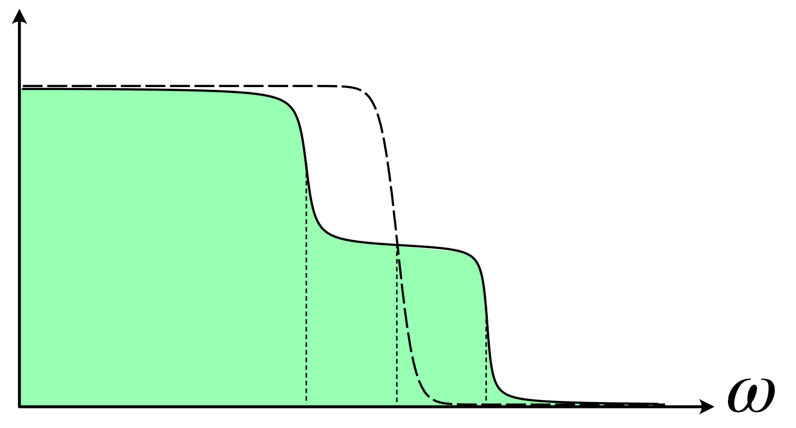
driving term — collision term

- ✤ self-energy $\Sigma =$ Hartree-Fock-Bogoliubov app. + 2nd Born app.
- ✤ gradient approximation $[A\otimes B](\boldsymbol{p},\omega,\boldsymbol{r},t) \simeq A(\boldsymbol{p},\omega,\boldsymbol{r},t)B(\boldsymbol{p},\omega,\boldsymbol{r},t)$

$$+\frac{i}{2}A(\boldsymbol{p},\omega,\boldsymbol{r},t)\left[\overleftarrow{\partial}_{\omega}\overrightarrow{\partial}_{t} - \overleftarrow{\partial}_{t}\overrightarrow{\partial}_{\omega} + \overleftarrow{\partial}_{\boldsymbol{r}}\cdot\overrightarrow{\partial}_{\boldsymbol{p}} - \overleftarrow{\partial}_{\boldsymbol{p}}\cdot\overrightarrow{\partial}_{\boldsymbol{r}}\right]B(\boldsymbol{p},\omega,\boldsymbol{r},t)$$

non-equilibrium distribution

$$F(\omega) = \frac{1}{2}\left[f(\omega+\delta\mu) + f(\omega-\delta\mu)\right]$$



$$i\frac{\partial}{\partial t}\mathcal{G}^{<}_{\boldsymbol{p}}(\boldsymbol{r},t) = \left[\,\xi_{\boldsymbol{p}} - \Delta(\boldsymbol{r},t)\tau_{+} - \Delta^{*}(\boldsymbol{r},t)\tau_{-}, \mathcal{G}^{<}_{\boldsymbol{p}}(\boldsymbol{r},t)\right] + \left[\frac{Q^2}{8m}\tau_3, \mathcal{G}^{<}_{\boldsymbol{p}}(\boldsymbol{r},t)\right] - \left[\frac{1}{8m}\tau_3, \nabla^2\mathcal{G}^{<}_{\boldsymbol{p}}(\boldsymbol{r},t)\right]$$

$$-\frac{i}{2}\left\{\boldsymbol{v}_{\boldsymbol{p}}, \mathcal{G}^{<}_{\boldsymbol{p}}(\boldsymbol{r},t)\right\} - \frac{i}{2}\left\{\frac{\boldsymbol{Q}}{2m}\tau_0, \nabla\mathcal{G}^{<}_{\boldsymbol{p}}(\boldsymbol{r},t)\right\} - 4i\gamma\mathcal{G}^{<}_{\boldsymbol{p}}(\boldsymbol{r},t) - 4\gamma\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}F(\omega)\mathcal{A}(\boldsymbol{p},\omega,\boldsymbol{r},t)$$

time-dependent spectra weight

environment effects

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Summary

- We theoretically propose an idea to process the structure of a Fermi surface (FS) with reservoirs so as to be suitable for the state which we want to realize.
- As an application of the FS reservoir-engineering, we have considered the driven-dissipative non-equilibrium Fermi gas.
- The “two effective FSs” processed by the FS reservoir-engineering are found to really work like two FSs and stabilize the exotic superfluid states, where the Cooper pair has a finite center-of-mass momentum.