Strong coupling theory for odd-frequency superfluidity

奇周波数超流動の強結合理論

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School for
"Clustering as a window on the hierarchical structure of quantum systems”
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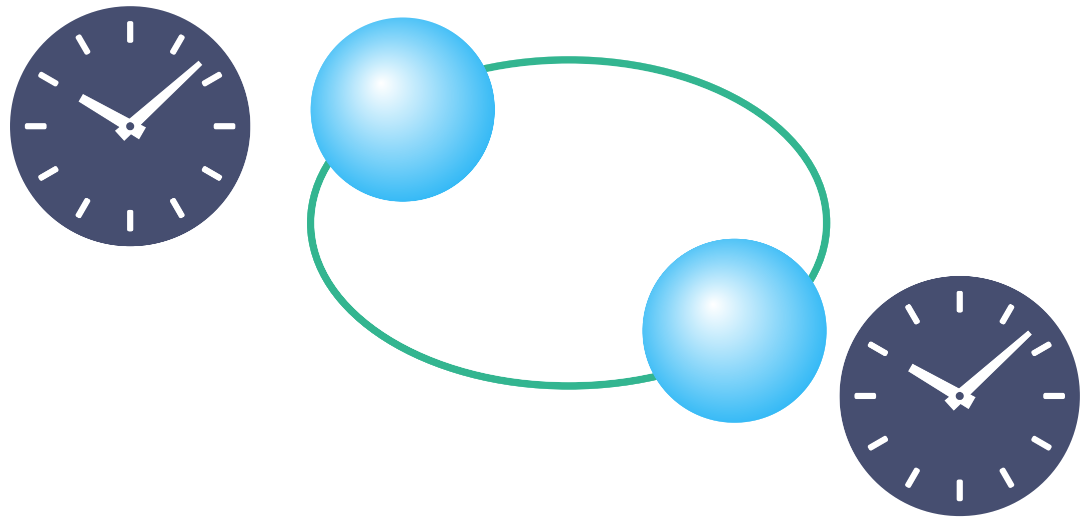
Contents

Odd frequency superfluidity (奇周波数超流動)

even frequency superfluidity



Ex. s-wave singlet, p-wave triplet

Superfluid order parameter

$$\Delta(t_2, t_1) = +\Delta(t_1, t_2)$$

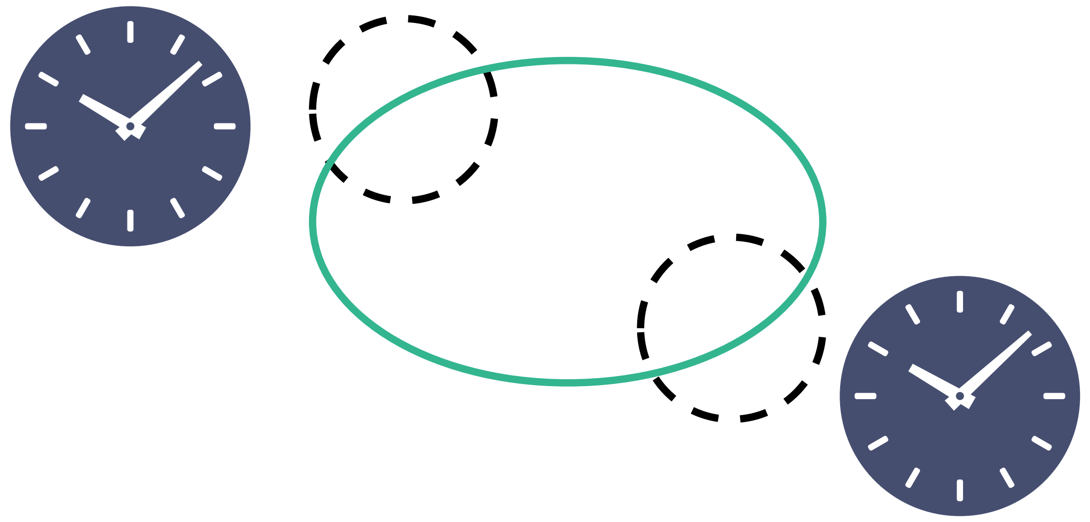
$$\Delta(-\omega) = +\Delta(\omega)$$

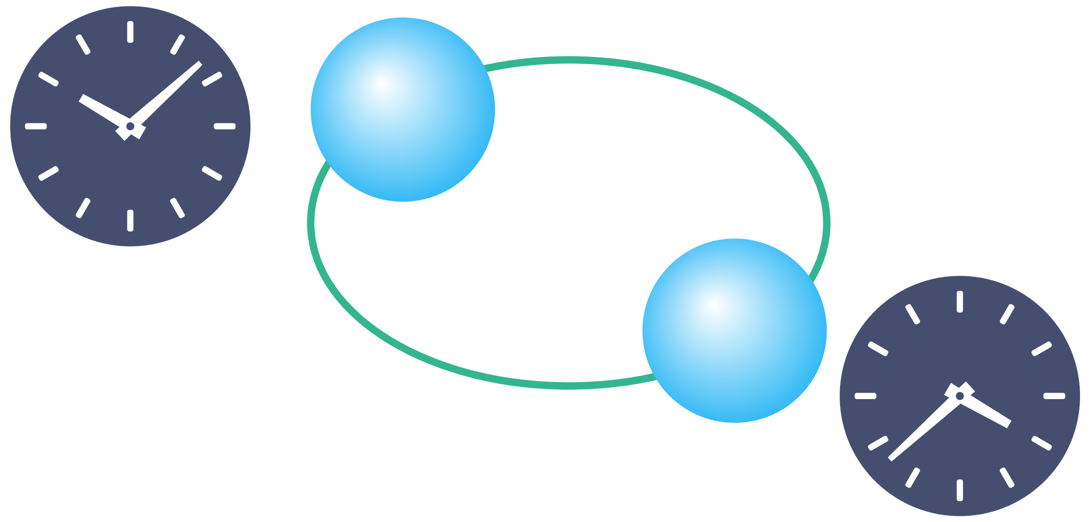
It is possible to form a pair with the particles at the same time.

odd-frequency superfluidity

$$\Delta(t_2, t_1) = -\Delta(t_1, t_2)$$

$$\Delta(-\omega) = -\Delta(\omega)$$



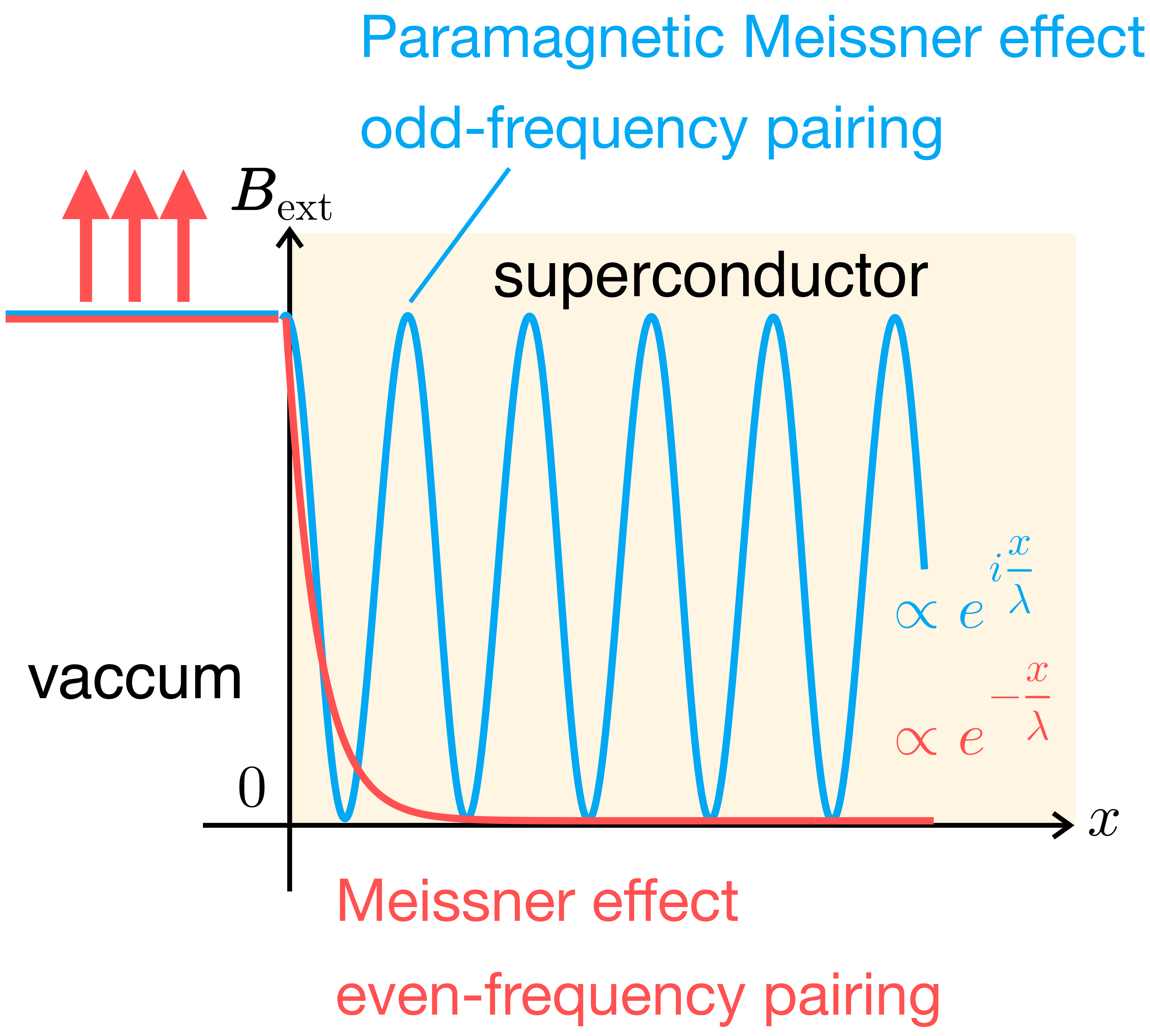


A pair with particles at the same time is prohibited.

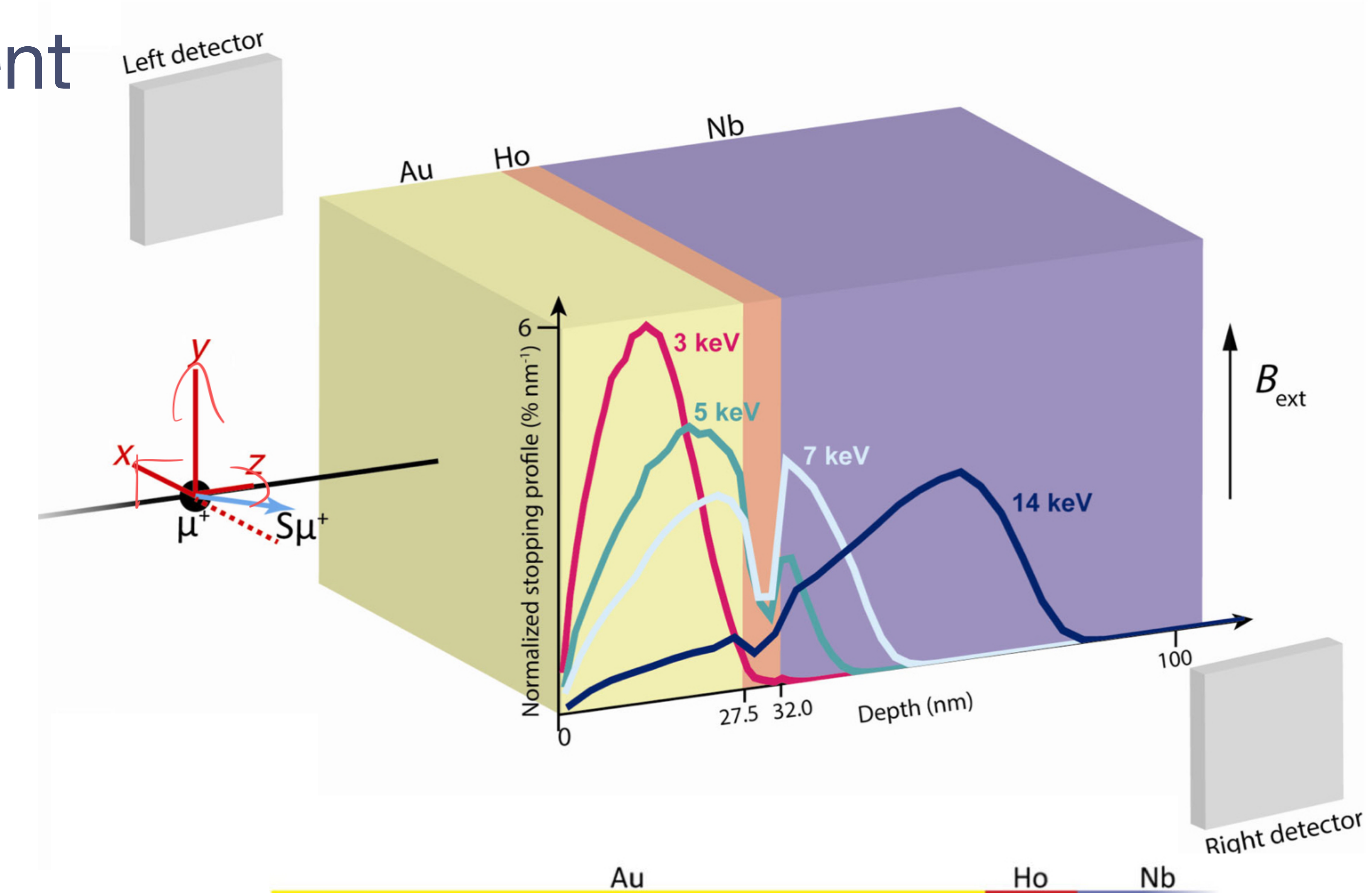
A pair only exists with particles at different times.

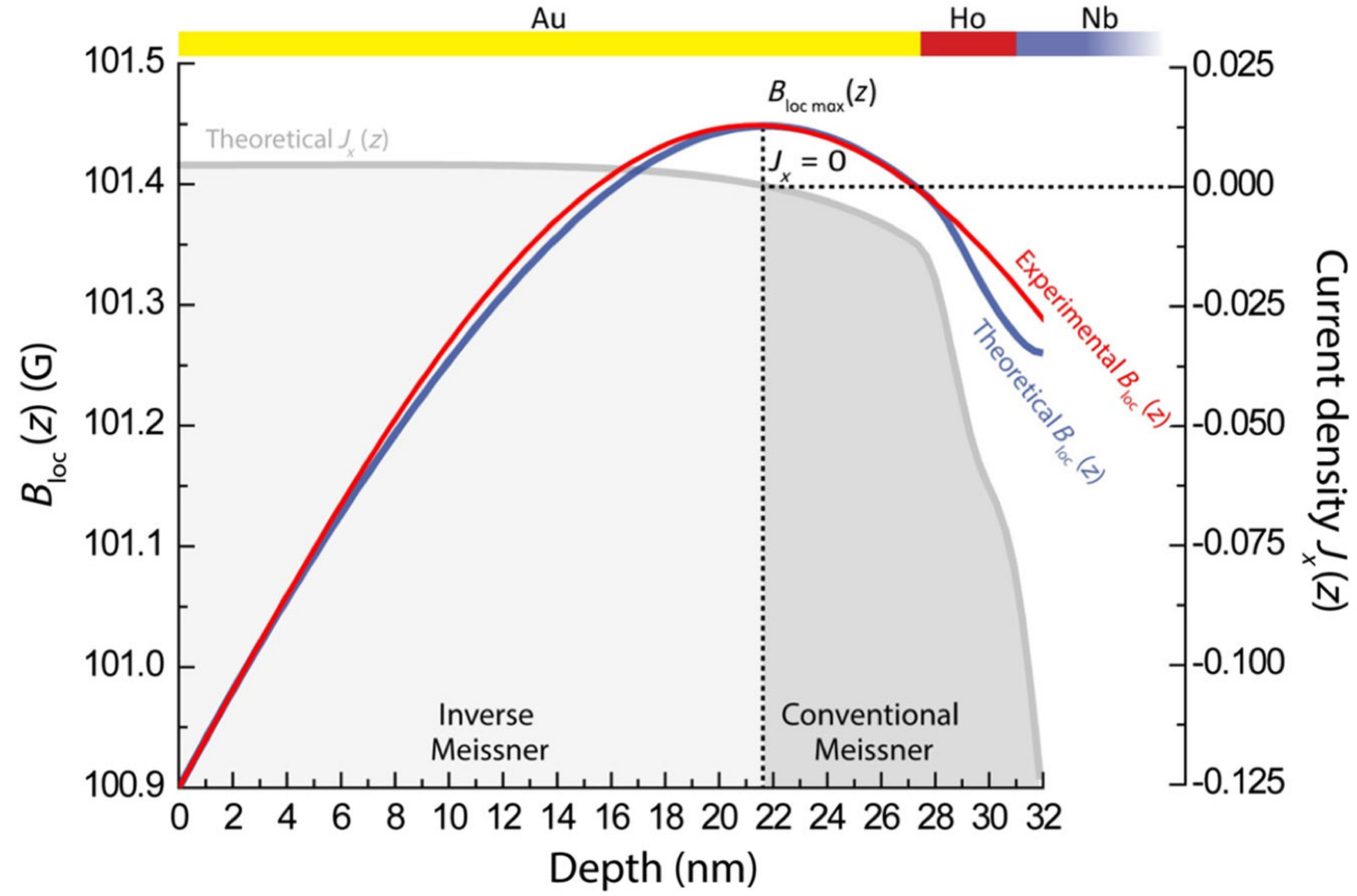
Experimental result (junction system)

Paramagnetic Meissner effect



μSR experiment



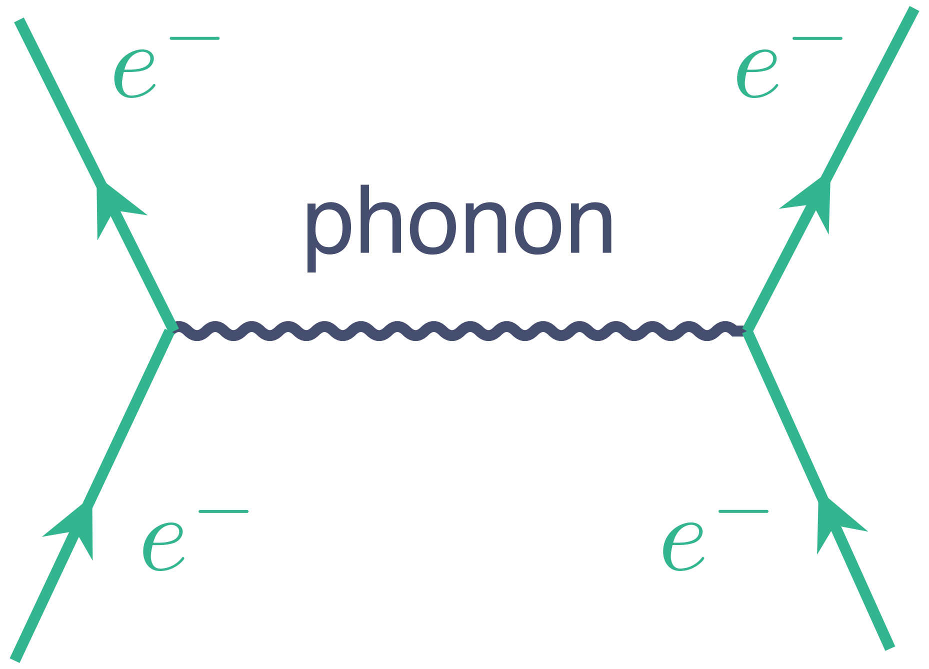


A. Di Bernardo et al Phys. Rev. X. **5** (2015) 041021

Theoretical proposal (bulk system)

Effect of retardation (Interaction propagates slowly) is important

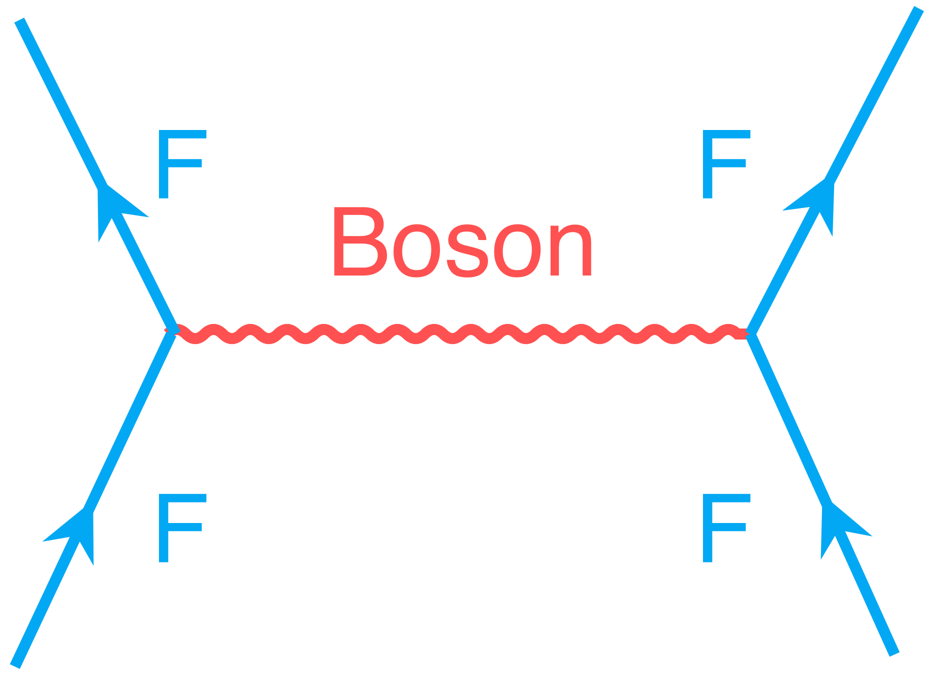
Electron-Phonon System



- Coulomb repulsive interaction between electrons at same time
- strong attractive interaction between electron by phonon

H. Kusunose, Y. Fuseya ,and K. Miyake
J. Phys. Soc. Jpn. 80 (2011) 044711

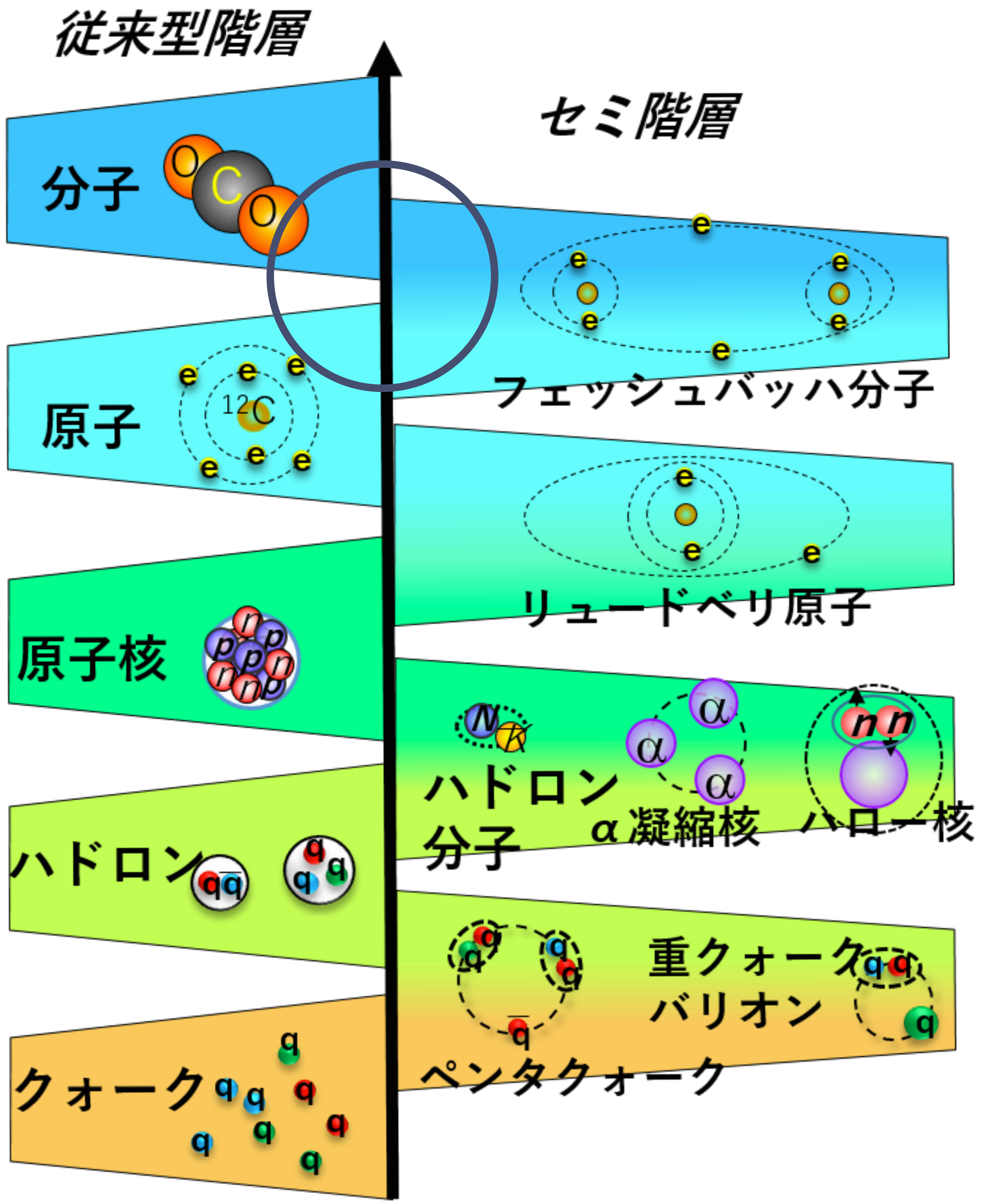
Mixture of bosonic and fermionic cold atoms



- strong attractive interaction between Fermion by boson
- s-wave odd-ω T_c > p-wave even-ω T_c

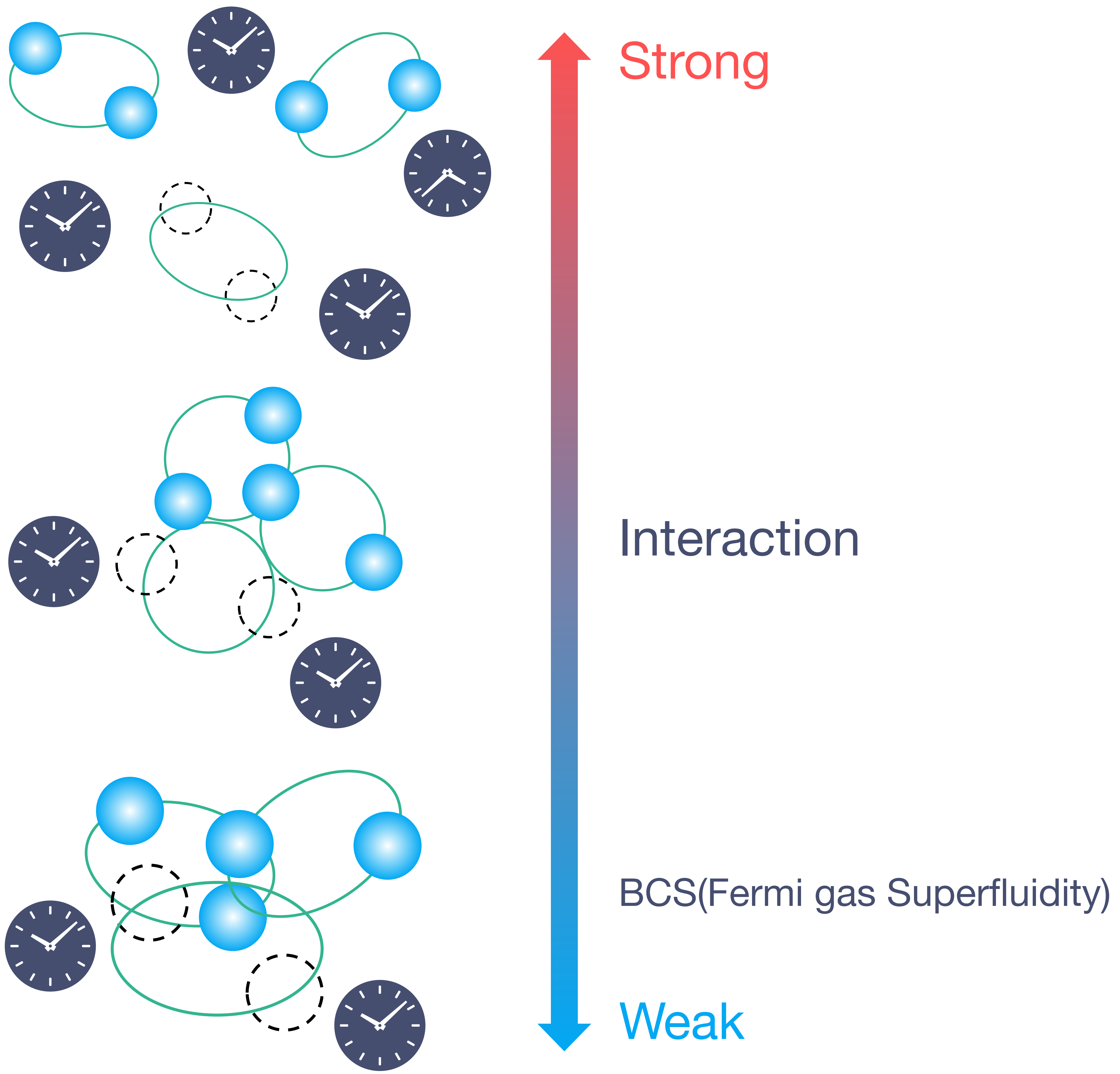
R. M. Kalas, A. V. Balatsky , and D. Mozyrsky
Phys. Rev. B. 78 (2008) 184513

Motivation



新学術領域「量子クラスターで読み解く物質の階層構造」
http://be.nucl.ap.titech.ac.jp/cluster/kenkyu.html

What is the molecule which only exists with atoms at different times ?



Research topic

We construct a strong-coupling theory for odd-frequency superfluidity.

We calculate

- superfluid order parameter $\Delta(\omega)$ at $T = 0$
- superfluid phase transition temperature T_c

Formalism

Action (1 component Fermi Gas) $S = S_0 + S_1$

$$S_0 = \sum_{\boldsymbol{p},\omega_n} \bar{\psi}_{\boldsymbol{p},\omega_n}(-i\omega_n + \xi_{\boldsymbol{p}})\psi_{\boldsymbol{p},\omega_n} \qquad \xi_p = \frac{p^2}{2m} - \mu$$

$$S_1 = \frac{1}{2\beta} \sum_{\boldsymbol{p_1},\omega_{n_1},\boldsymbol{p_2},\omega_{n_2},\boldsymbol{q},\nu_n} V_{\boldsymbol{p_1},\omega_{n_1},\boldsymbol{p_2},\omega_{n_2}} \bar{\psi}_{\boldsymbol{p_2}+\frac{\boldsymbol{q}}{2},\omega_{n_2}+\nu_n} \bar{\psi}_{-\boldsymbol{p_2}+\frac{\boldsymbol{q}}{2},-\omega_{n_2}} \psi_{-\boldsymbol{p_1}+\frac{\boldsymbol{q}}{2},-\omega_{n_1}} \psi_{\boldsymbol{p_1}+\frac{\boldsymbol{q}}{2},\omega_{n_1}+\nu_n}$$

Strongly retarded attractive interaction

(depending on frequency)

Partition function $Z = \int D\overline{\psi} D\psi e^{-S(\overline{\psi},\psi)}$

Separable interaction

Assumed frequency dependence: **separable** form

$$V_{\boldsymbol{p_1},\omega_{n_1},\boldsymbol{p_2},\omega_{n_2}} = -U\gamma_{\omega_{n_1}}\gamma_{\omega_{n_2}}$$

U : strength of the contact interaction

$$\gamma_{\omega_n} = \frac{\omega_n}{\sqrt{\omega_n^2 + \omega_0^2}}$$

Odd in ω_n → odd-frequency superfluidity

Renormalized interaction: a_s (cf. even frequency s-wave case)

$$\frac{1}{U} = -\frac{m}{4\pi a_s} + \sum_{p<p_{\max}} \frac{1}{2\epsilon_p}$$

a_s : s-wave scattering length

$-\infty$ $(k_{\mathrm{F}}a_s)^{-1}$ $+\infty$

Weak Coupling (BCS) ⟷ Strong Coupling (BEC)

Small U Large

$\Delta(\omega)$ at $T=0$: BCS-Leggett theory

$$Z=\int D\overline{\psi}D\psi e^{-S(\overline{\psi},\psi)} \;\longrightarrow\; Z=\int D\bar{\psi}D\psi\int D\bar{\Delta}D\Delta e^{-S(\bar{\psi},\psi,\bar{\Delta},\Delta)} \;\longrightarrow\; Z=\int D\bar{\Delta}D\Delta e^{-S_{\mathrm{eff}}(\Delta,\bar{\Delta})}$$

Hubbard-Stratonovich Trans.

Integrate out ψ

Saddle point approx. (= Mean Field approx.)

$\int D\bar{\Delta}D\Delta$ in Z is replaced by saddle point value: $Z_{\mathrm{MF}}\simeq e^{-S_{\mathrm{eff}}(\Delta,\bar{\Delta})}$

Free energy $\quad \Omega_{\mathrm{MF}}=-\frac{1}{\beta}\ln Z_{\mathrm{MF}}$

Gap equation $\quad \frac{\partial\Omega_{\mathrm{MF}}}{\partial\bar{\Delta}}=0$

Particle num. eq. $\quad N=-\frac{\partial\Omega_{\mathrm{MF}}}{\partial\mu}$

$\longrightarrow$ Δ_{MF}, μ

T_c : Saddle point approx.

$$Z = \int D\overline{\psi} D\psi e^{-S(\overline{\psi},\psi)} \rightarrow Z = \int D\bar{\psi} D\psi \int D\bar{\Delta} D\Delta e^{-S(\bar{\psi},\psi,\bar{\Delta},\Delta)} \rightarrow Z = \int D\bar{\Delta} D\Delta e^{-S_{\mathrm{eff}}(\Delta,\bar{\Delta})}$$

Hubbard-Stratonovich Trans.

Integrate out ψ

Saddle point approx. (= Mean Field approx.) at T_c

Gap equation $\frac{\partial \Omega_{\mathrm{MF}}}{\partial \bar{\Delta}} = 0 \quad \Delta_{\mathrm{MF}} = 0 \; (T \geq T_c)$

Particle num. eq. $N = N_0 = \sum_p f(\xi_p)$

Free particle

→ T_c, μ ✗

This approx. does not work when the interaction is strong at $T > 0$ because it ignores pairing fluctuation effects.

We improve this theory. → **Mean Field theory + fluctuation effect**

T_c : NSR theory

$$Z = \int D\overline{\psi} D\psi e^{-S(\overline{\psi},\psi)} \quad \rightarrow \quad Z = \int D\bar{\psi} D\psi \int D\bar{\Delta} D\Delta e^{-S(\bar{\psi},\psi,\bar{\Delta},\Delta)} \quad \rightarrow \quad Z = \int D\bar{\Delta} D\Delta e^{-S_{\text{eff}}(\Delta,\bar{\Delta})}$$

Hubbard-Stratonovich Trans.   Integrate out ψ

Gaussian fluctuation approx. around saddle point

$$\Delta_q = \Delta_{\text{MF}} + \underline{\eta_q}$$

Normal Fluid phase $\Delta_{\text{MF}} = 0 \ (T \geq T_c)$

$$Z \simeq \underline{Z_0} \times \underline{Z_{\text{NSR}}}$$

Free particle   Fluctuation

$$Z_{\text{NSR}} = \int D\bar{\eta} D\eta e^{-\frac{1}{2}\sum_q \bar{\eta}_q \Gamma_q^{-1} \eta_q}$$

Gap equation $$\frac{\partial \Omega_{\text{MF}}}{\partial \bar{\Delta}} = 0 \qquad \Delta_{\text{MF}} = 0 \ (T \geq T_c)$$

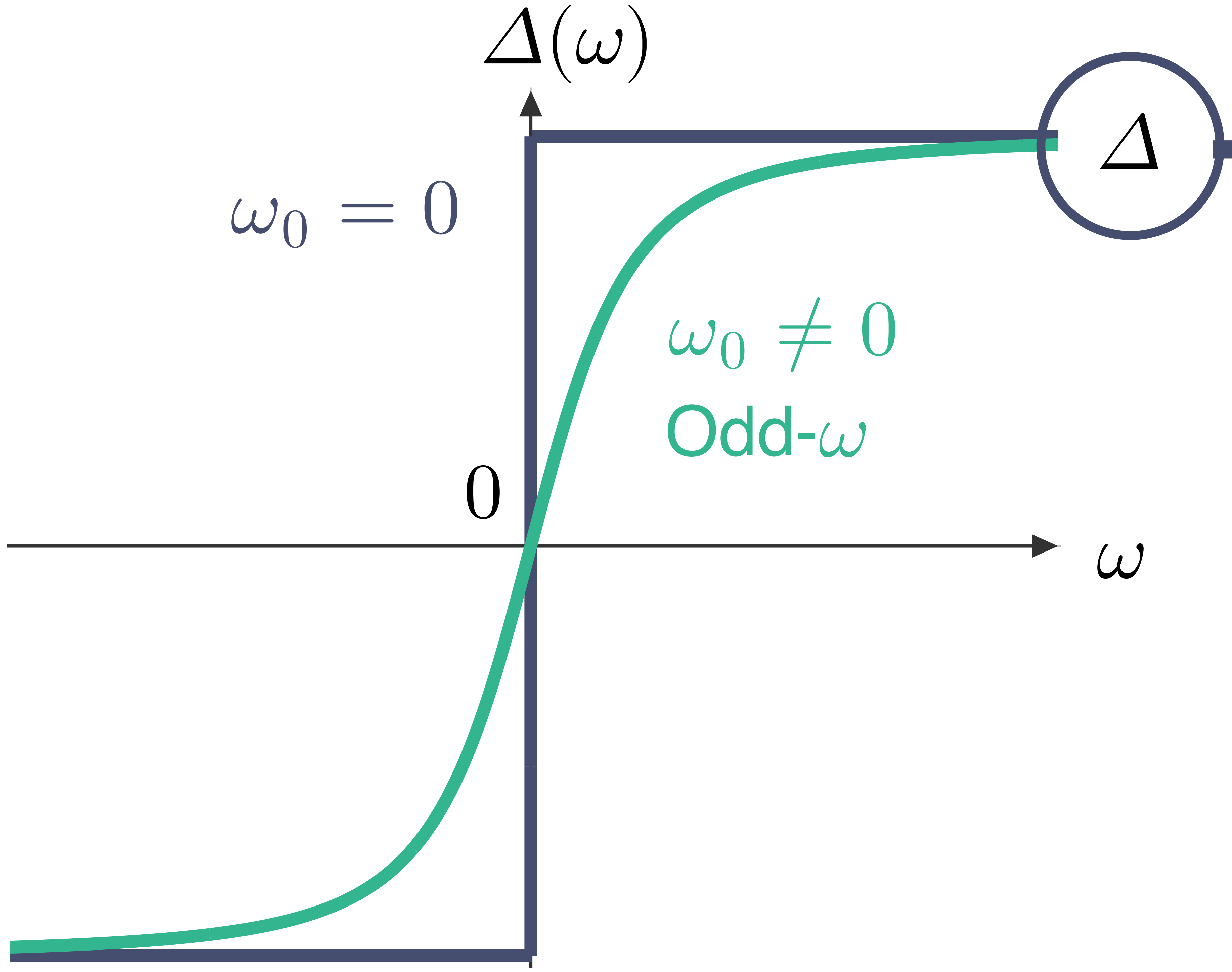
Particle num. eq. $$N = N_0 + \underline{N_{\text{NSR}}}$$ Fluctuation

$\rightarrow$ T_c, μ

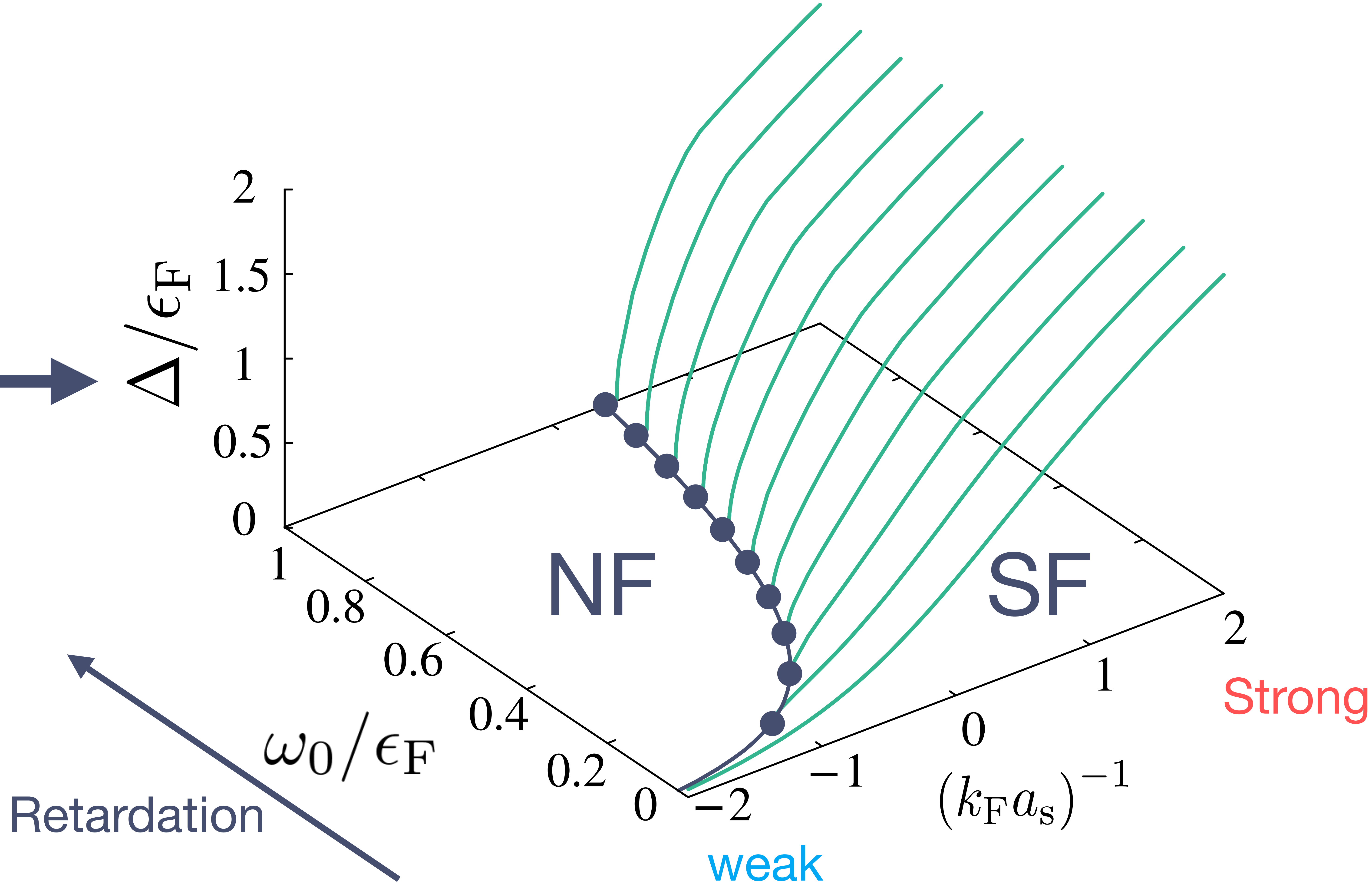
$\Delta(\omega)$ at $T = 0$

Frequency dependence of
Superfluid order parameter :

$$\Delta(\omega) = \frac{\Delta\omega}{\sqrt{\omega^2 + \omega_0^2}}$$

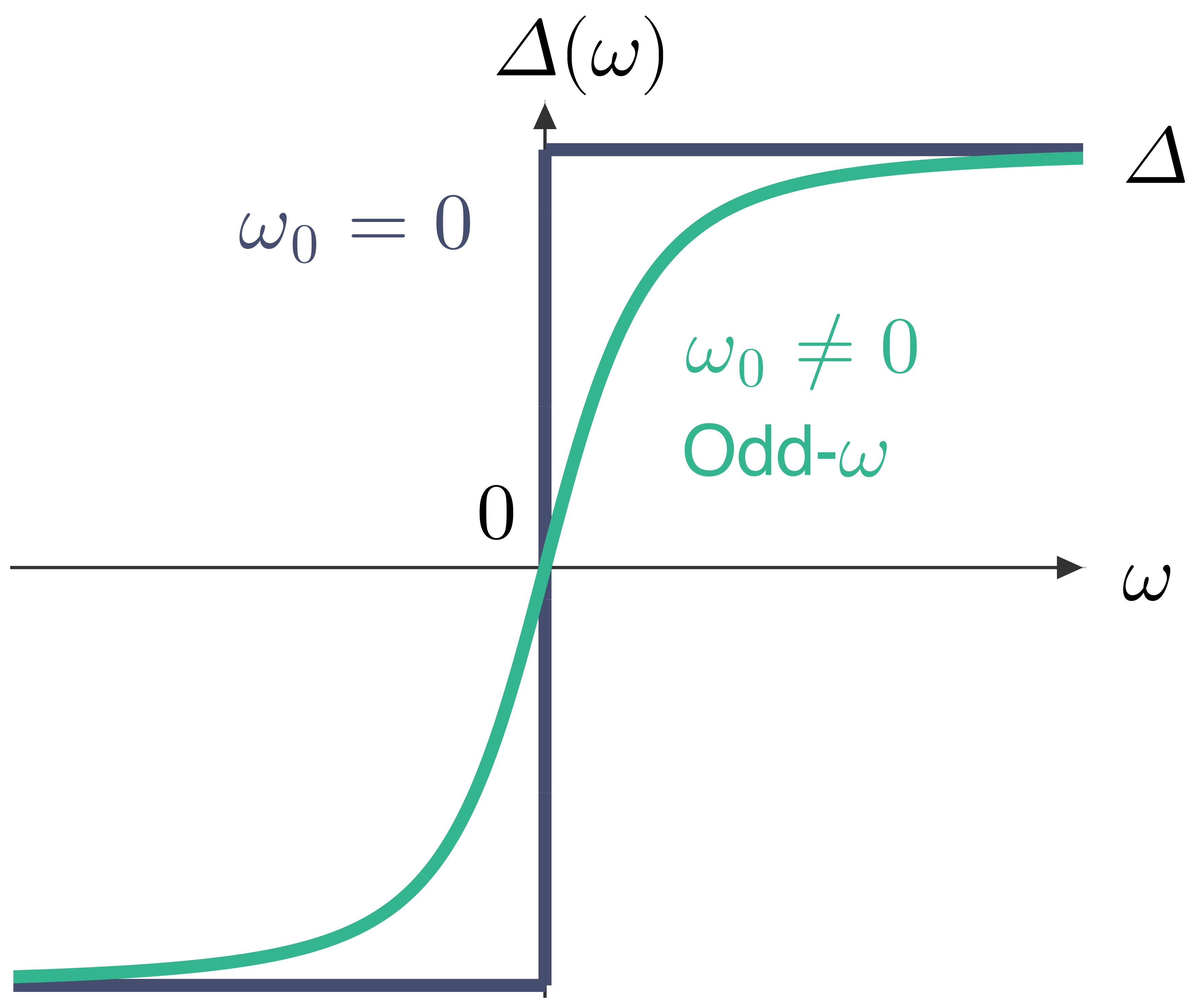


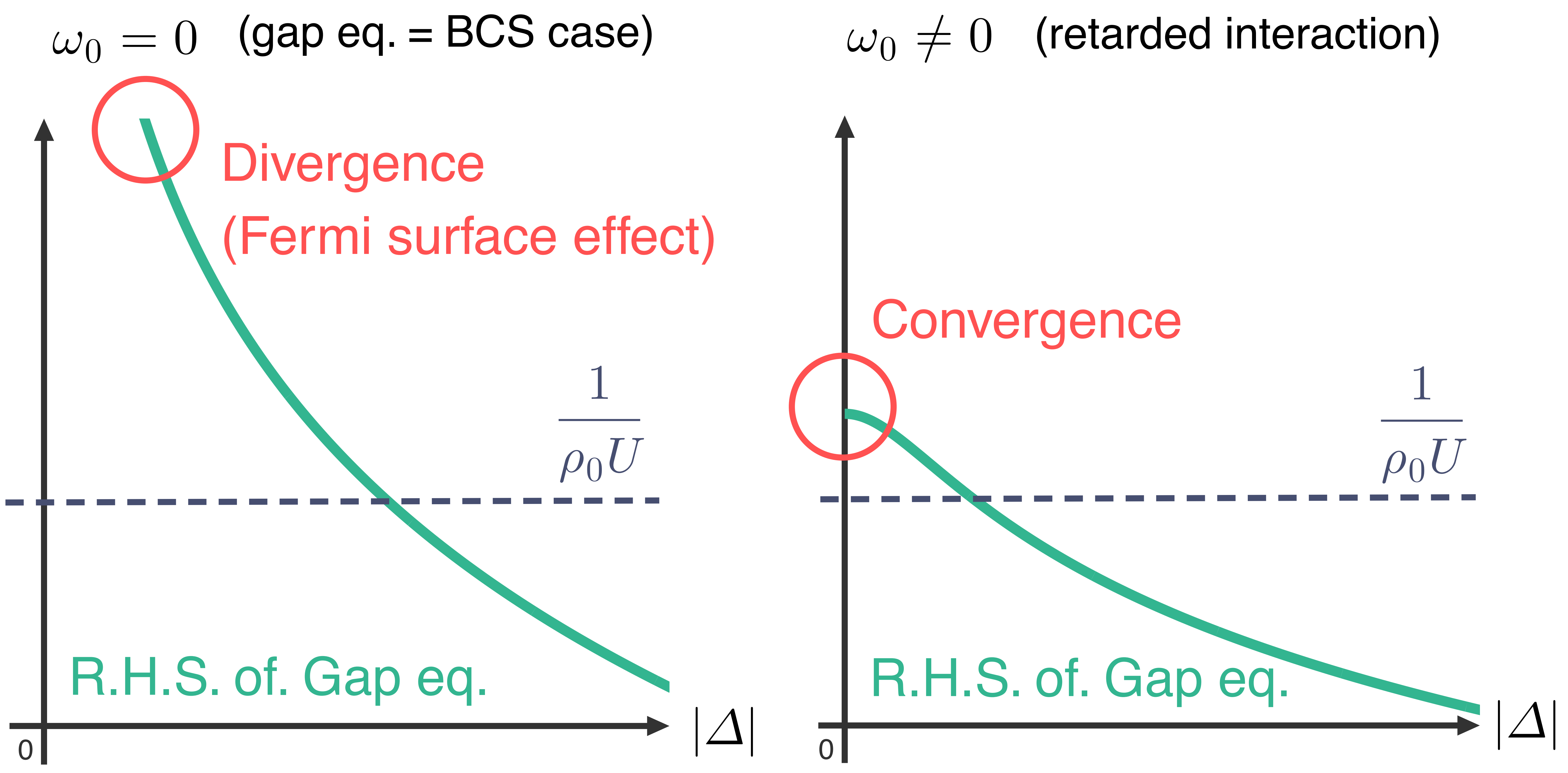
Odd-ω superfluid state prefers strong coupling regime at $T = 0$



Reason why $\Delta(\omega)$ disappears in BCS side

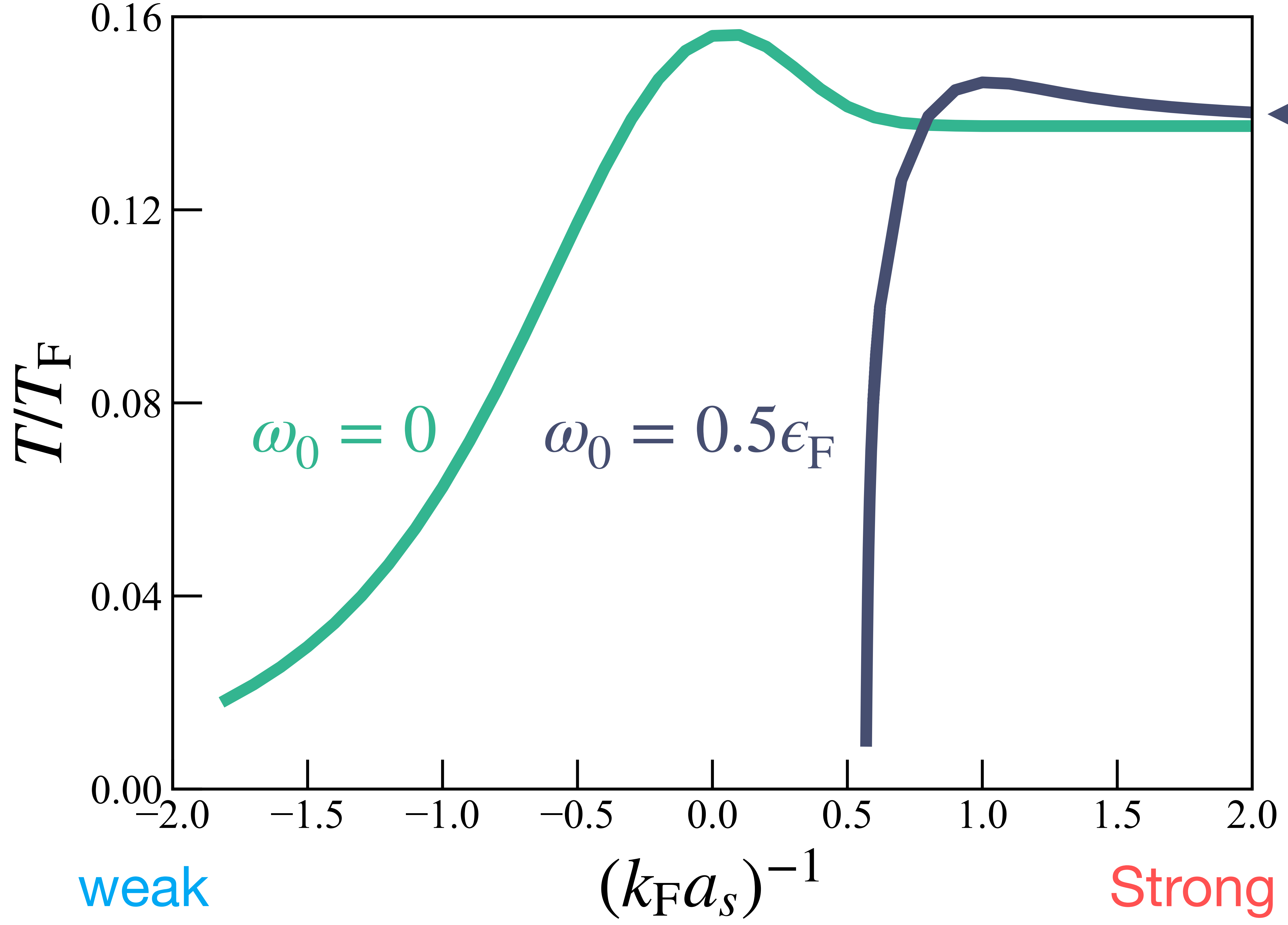
Retardation of interaction suppresses Fermi surface effect.





ρ_0 : DOS at Fermi energy U : strength of contact interaction

$\Delta(\omega)$ at $T = 0$ and T_c

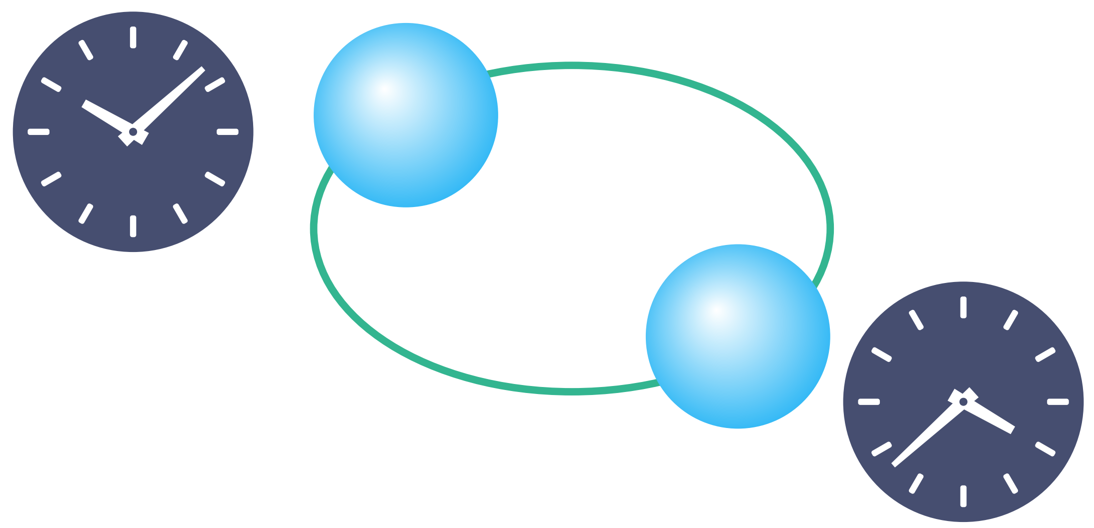


$T_c = 0.137 T_\mathrm{F}$

BEC transition temperature of $N/2$ molecules

$N = \frac{k_\mathrm{F}}{6\pi^3}$: particle num. of atoms

There are molecular bosons composed of atoms at **different times**.

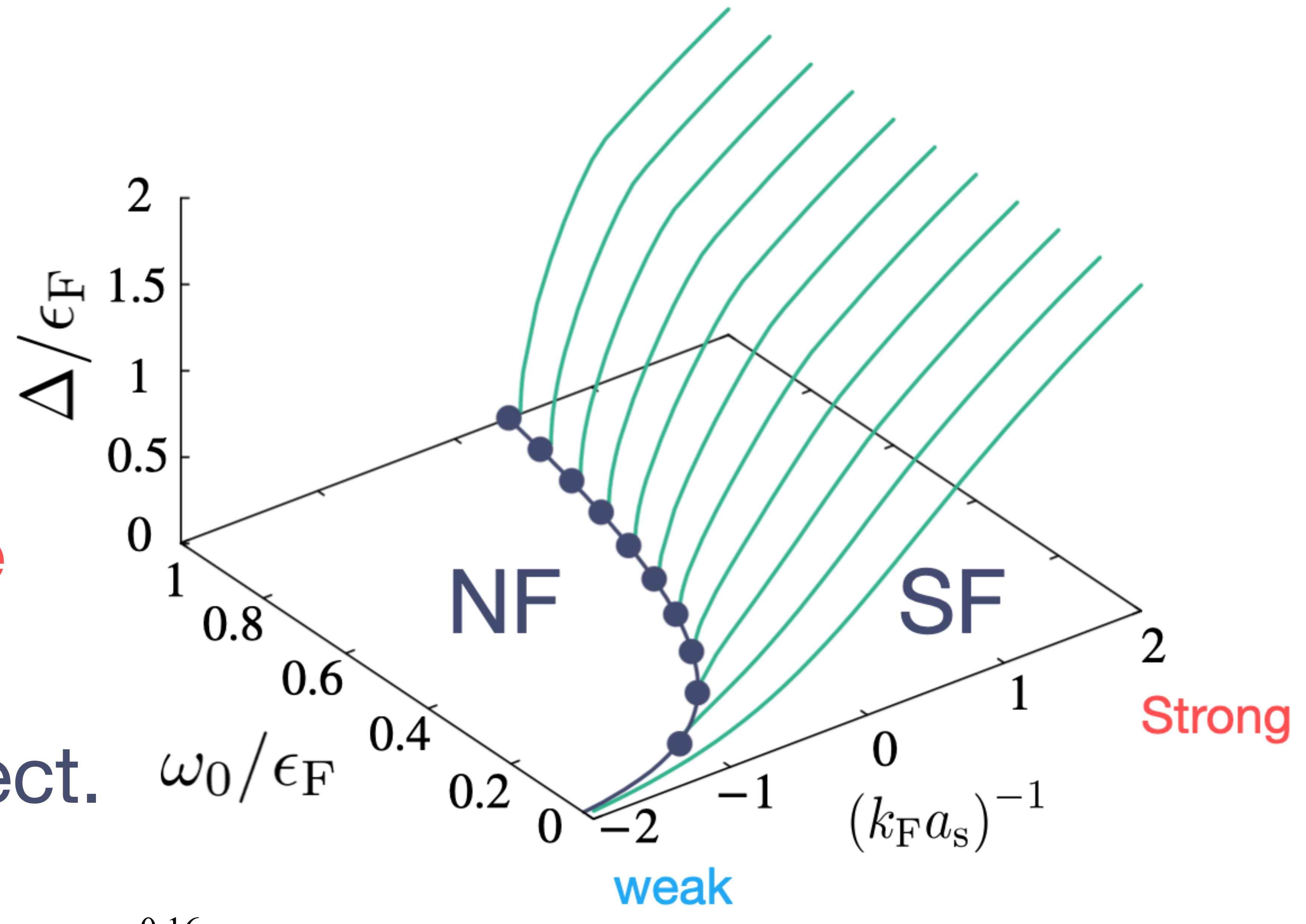


Effective molecular mass $= 2m$

m: atomic mass

Summary

We construct a **strong-coupling** theory for **odd-ω** superfluidity.

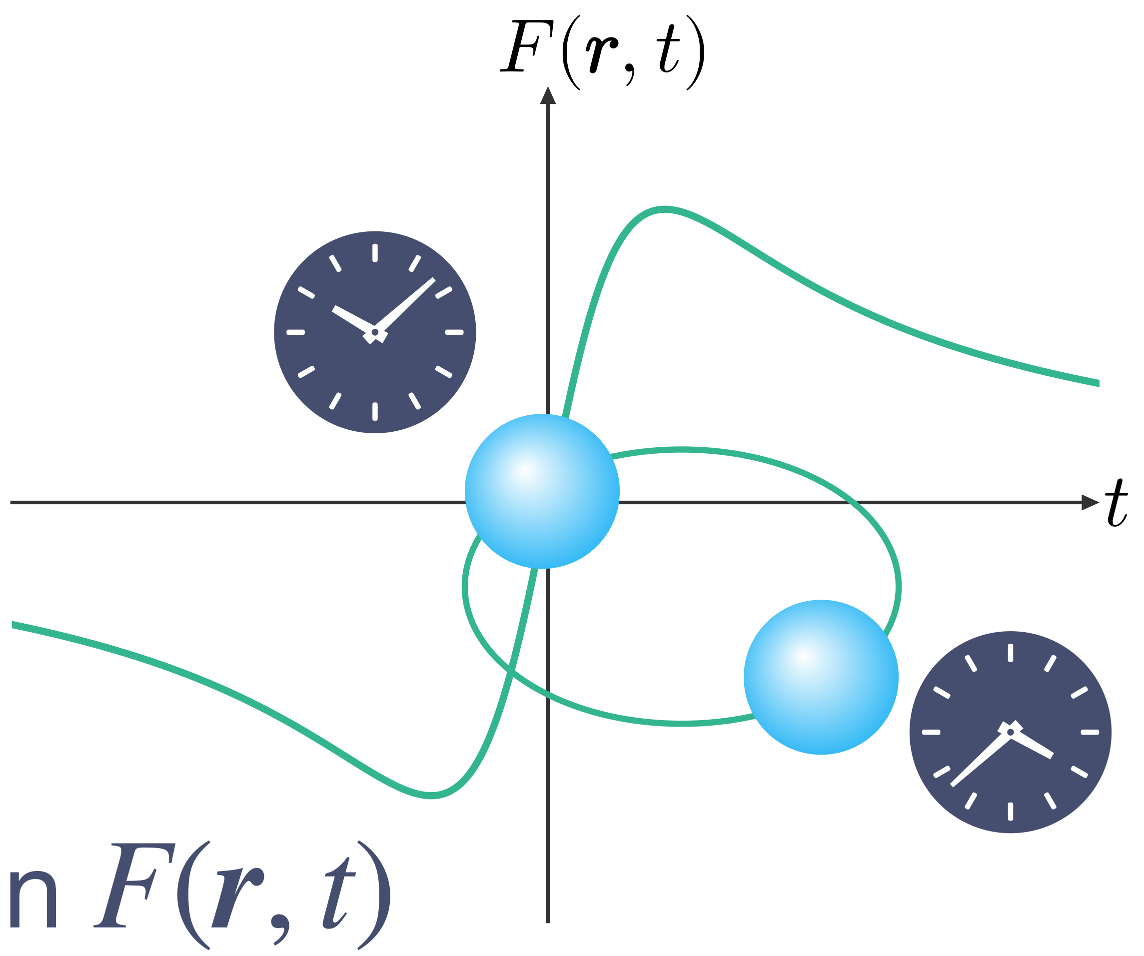


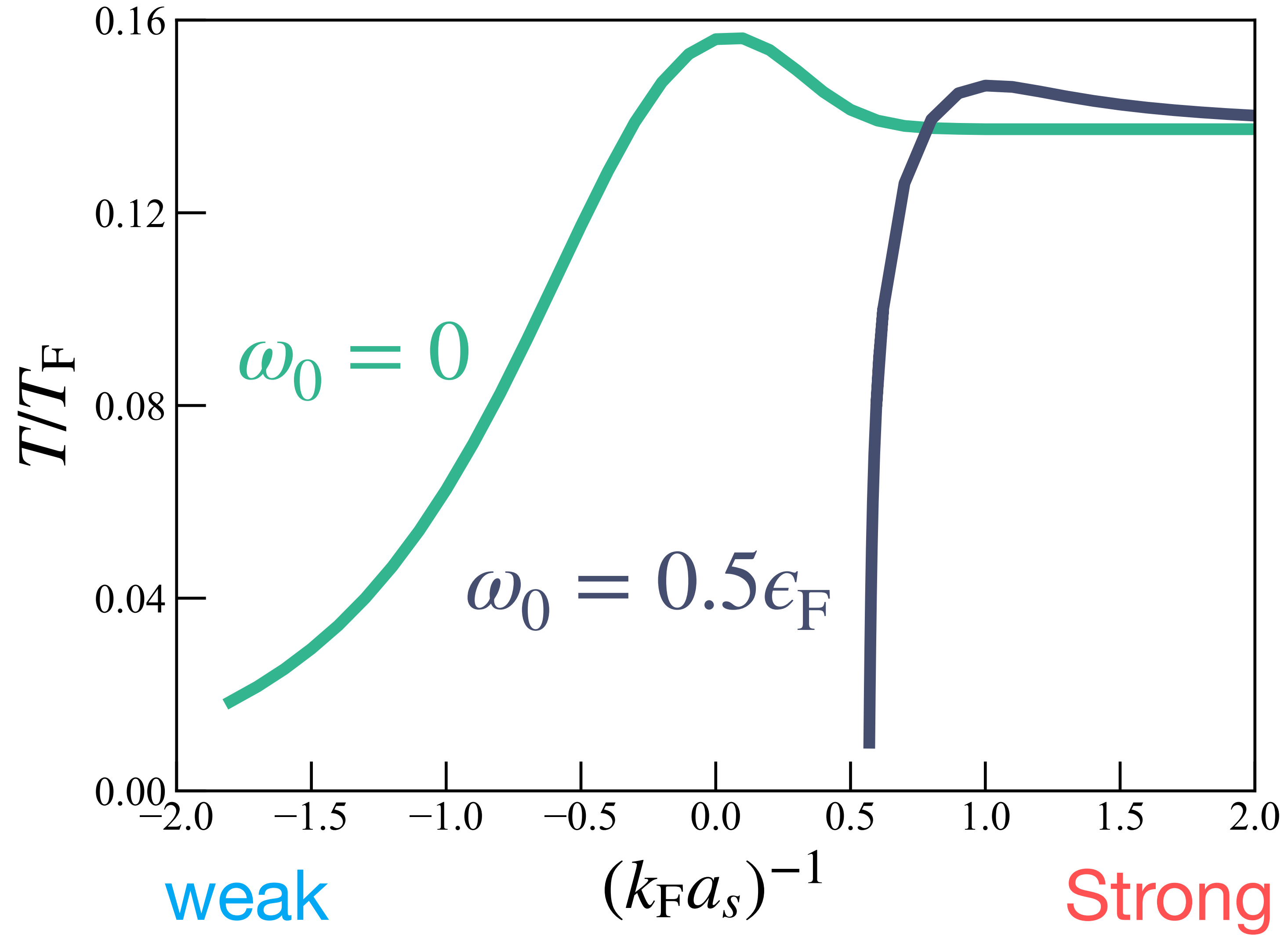
$\Delta(\omega)$ at $T=0$

- Odd-frequency superfluid state prefers **strong coupling regime**
- $\Delta(\omega)$ disappears at **weak coupling regime** because retardation of interaction suppresses Fermi surface effect.

T_c

- Strong coupling limit : $T_c \simeq 0.137 T_F$
- T_c disappears at **weak coupling regime**





Future work

Study on time structure of pair wave function $F(\boldsymbol{r}, t)$