

ハドロン分子状態の物理

Physics of hadronic molecules



Tetsuo Hyodo

Tokyo Metropolitan Univ.



2021, Mar. 22nd

Contents



Part I : Introduction

- What are “exotic hadrons”?
- Hadronic molecules and universality
- Unstable resonances



Part II : Structure of $\Lambda(1405)$ resonance

- Accurate $\bar{K}N$ scattering amplitude

Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011); NPA 881, 98 (2012)

- Compositeness of hadrons

Y. Kamiya, T. Hyodo, PRC93, 035203 (2016); PTEP2017, 023D02 (2017)

References



Review articles

- Compositeness

T. Hyodo, JPS journal Vol. 75 No. 8, 478 (2020)

- Exotic hadrons, etc.

T. Hyodo, M. Niiyama, arXiv: 2010.07592 [hep-ph], to appear in PPNP



Lecture notes (detailed calculation)

- English



Japanese



https://www.comp.tmu.ac.jp/hyodo/lecture_e.html#lec4

<https://www.comp.tmu.ac.jp/hyodo/2020Tokuron.html>

Introduction : What are “exotic hadrons”?

Observed hadrons

Particle Data Group (PDG) 2020 edition

<http://pdg.lbl.gov/>

p	$1/2^+$ ****	$\Delta(1232)$	$3/2^+$ ****	Σ^+	$1/2^+$ ****	Ξ^0	$1/2^+$ ****	Ξ_{cc}^{++}	***
n	$1/2^+$ ***	$\Delta(1600)$	$3/2^+$ ***	Σ^0	$1/2^+$ ***	Ξ^-	$1/2^+$ ***	Λ_b^0	$1/2^+$ ***
$N(1440)$	$1/2^+$ ***	$\Delta(1620)$	$1/2^-$ ***	Σ^-	$1/2^+$ ***	$\Xi(1530)$	$3/2^+$ ***	Λ_b^0	$1/2^+$ ***
$N(1520)$	$3/2^-$ ***	$\Delta(1700)$	$3/2^-$ ***	$\Sigma(1385)$	$3/2^+$ ***	$\Xi(1620)$	*	$\Lambda_b(5912)^0$	$1/2^-$ ***
$N(1535)$	$1/2^-$ ***	$\Delta(1750)$	$1/2^+$ *	$\Sigma(1580)$	$3/2^-$ *	$\Xi(1690)$	***	$\Lambda_b(5920)^0$	$3/2^-$ ***
$N(1650)$	$1/2^-$ ***	$\Delta(1900)$	$1/2^-$ ***	$\Sigma(1620)$	$1/2^-$ *	$\Xi(1820)$	$3/2^-$ ***	$\Lambda_b(6146)^0$	$3/2^+$ ***
$N(1675)$	$5/2^-$ ***	$\Delta(1905)$	$5/2^+$ ***	$\Sigma(1660)$	$1/2^+$ ***	$\Xi(1950)$	***	$\Lambda_b(6152)^0$	$5/2^+$ ***
$N(1680)$	$5/2^+$ ***	$\Delta(1910)$	$1/2^+$ ***	$\Sigma(1670)$	$3/2^-$ ***	$\Xi(2030)$	$\geq \frac{5}{2}$ ***	$\Lambda_b(6152)^0$	$5/2^+$ ***
$N(1700)$	$3/2^-$ ***	$\Delta(1920)$	$3/2^+$ ***	$\Sigma(1750)$	$1/2^-$ ***	$\Xi(2120)$	*	Σ_b	$1/2^+$ ***
$N(1710)$	$1/2^+$ ***	$\Delta(1930)$	$5/2^-$ ***	$\Sigma(1775)$	$5/2^+$ ***	$\Xi(2250)$	**	$\Sigma_b(6097)^+$	***
$N(1720)$	$3/2^+$ ***	$\Delta(1940)$	$3/2^-$ **	$\Sigma(1780)$	$3/2^+$ *	$\Xi(2370)$	**	$\Sigma_b(6097)^-$	***
$N(1860)$	$5/2^+$ **	$\Delta(1950)$	$7/2^+$ ***	$\Sigma(1880)$	$1/2^+$ **	$\Xi(2500)$	*	Ξ_b^0 , Ξ_b^-	$1/2^+$ ***
$N(1875)$	$3/2^-$ ***	$\Delta(2000)$	$5/2^+$ **	$\Sigma(1900)$	$1/2^-$ ***	Ξ_b^0	$1/2^+$ ***	$\Xi_b^0(5935)^0$	$1/2^+$ ***
$N(1880)$	$1/2^+$ ***	$\Delta(2150)$	$1/2^-$ *	$\Sigma(1910)$	$3/2^-$ ***	Ω^-	$3/2^+$ ***	$\Xi_b^0(5945)^0$	$3/2^+$ ***
$N(1895)$	$1/2^-$ ***	$\Delta(2200)$	$7/2^-$ ***	$\Sigma(1915)$	$5/2^+$ ***	$\Omega(2012)^-$?	$\Xi_b^0(5955)^0$	$3/2^+$ ***
$N(1900)$	$3/2^+$ ***	$\Delta(2300)$	$9/2^+$ **	$\Sigma(1940)$	$3/2^+$ *	$\Omega(2250)^-$	***	$\Xi_b(6227)^-$	***
$N(1990)$	$7/2^+$ **	$\Delta(2350)$	$5/2^-$ *	$\Sigma(2010)$	$3/2^-$ *	$\Omega(2380)^-$	**	Ω_b^-	$1/2^+$ ***
$N(2000)$	$5/2^+$ **	$\Delta(2390)$	$7/2^+$ ***	$\Sigma(2030)$	$7/2^+$ ***	$\Omega(2470)^-$	**	$\Xi_c(4312)^+$	*
$N(2040)$	$3/2^+$ *	$\Delta(2400)$	$9/2^-$ **	$\Sigma(2070)$	$5/2^+$ *	$\Xi_c(4312)^+$	*	$P_c(4312)^+$	*
$N(2060)$	$5/2^+$ ***	$\Delta(2420)$	$11/2^+$ ***	$\Sigma(2080)$	$3/2^+$ *	Λ_c^+	$1/2^+$ ***	$P_c(4380)^+$	*
$N(2100)$	$1/2^+$ ***	$\Delta(2750)$	$13/2^-$ ***	$\Sigma(2100)$	$7/2^-$ *	$\Lambda_c(2595)^0$	$1/2^-$ ***	$P_c(4440)^+$	*
$N(2120)$	$3/2^-$ ***	$\Delta(2950)$	$15/2^+$ **	$\Sigma(2160)$	$1/2^-$ *	$\Lambda_c(2625)^+$	$3/2^-$ ***	$P_c(4457)^+$	*
$N(2190)$	$7/2^+$ ***	$\Lambda(1670)$	$1/2^+$ ***	$\Sigma(2230)$	$3/2^+$ *	$\Lambda_c(2765)^+$	*	$\Xi_c(2860)^+$	$3/2^+$ ***
$N(2220)$	$9/2^+$ ***	$\Lambda(1670)$	$1/2^+$ ***	$\Sigma(2250)$	***	$\Lambda_c(2860)^+$	$3/2^+$ ***	$\Xi_c(2860)^+$	$3/2^+$ ***
$N(2250)$	$9/2^-$ ***	$\Lambda(1670)$	$1/2^-$ **	$\Sigma(2455)$	**	$\Lambda_c(2880)^+$	$5/2^+$ ***	$\Xi_c(2880)^+$	$5/2^+$ ***
$N(2300)$	$1/2^+$ **	$\Lambda(1405)$	$1/2^-$ ***	$\Sigma(2620)$	**	$\Lambda_c(2940)^+$	$3/2^-$ ***	$\Xi_c(2945)$	$1/2^+$ ***
$N(2570)$	$5/2^-$ **	$\Lambda(1520)$	$3/2^-$ ***	$\Sigma(2999)$	*	$\Sigma_c(2955)$	$1/2^+$ ***	$\Xi_c(2955)$	$1/2^+$ ***
$N(2600)$	$11/2^-$ ***	$\Lambda(1600)$	$1/2^+$ ***	$\Sigma(2999)$	*	$\Sigma_c(2955)$	$1/2^+$ ***	$\Xi_c(2955)$	$1/2^+$ ***
$N(2700)$	$13/2^+$ **	$\Lambda(1670)$	$1/2^-$ ***	$\Sigma(2999)$	*	$\Sigma_c(2955)$	$1/2^+$ ***	$\Xi_c(2955)$	$1/2^+$ ***
$\Lambda(1670)$	$1/2^-$ ***	$\Lambda(1690)$	$3/2^-$ ***	$\Sigma_c(2645)$	$1/2^+$ ***	$\Xi_c(2645)$	$3/2^-$ ***	$\Xi_c(2645)$	$3/2^-$ ***
$\Lambda(1710)$	$1/2^+$ *	$\Lambda(1800)$	$1/2^-$ ***	$\Sigma_c(2790)$	$1/2^-$ ***	$\Xi_c(2790)$	$1/2^-$ ***	$\Xi_c(2790)$	$1/2^-$ ***
$\Lambda(1810)$	$1/2^+$ ***	$\Lambda(1820)$	$5/2^+$ ***	$\Sigma_c(2815)$	$3/2^-$ ***	$\Xi_c(2815)$	$3/2^-$ ***	$\Xi_c(2815)$	$3/2^-$ ***
$\Lambda(1820)$	$5/2^+$ ***	$\Lambda(1830)$	$5/2^-$ ***	$\Sigma_c(2930)$	***	$\Xi_c(2930)$	***	$\Xi_c(2930)$	***
$\Lambda(1830)$	$5/2^-$ ***	$\Lambda(1890)$	$3/2^+$ ***	$\Sigma_c(2970)$	***	$\Xi_c(2970)$	***	$\Xi_c(2970)$	***
$\Lambda(1890)$	$3/2^+$ ***	$\Lambda(2000)$	$1/2^-$ *	$\Sigma_c(3055)$	***	$\Xi_c(3055)$	***	$\Xi_c(3055)$	***
$\Lambda(2000)$	$1/2^-$ *	$\Lambda(2050)$	$3/2^-$ *	$\Sigma_c(3080)$	***	$\Xi_c(3080)$	***	$\Xi_c(3080)$	***
$\Lambda(2050)$	$3/2^-$ *	$\Lambda(2070)$	$3/2^+$ *	$\Sigma_c(3123)$	*	$\Xi_c(3123)$	*	$\Xi_c(3123)$	*
$\Lambda(2070)$	$3/2^+$ *	$\Lambda(2080)$	$5/2^-$ **	$\Sigma_c(3123)$	$1/2^+$ ***	$\Xi_c(3123)$	$1/2^+$ ***	$\Xi_c(3123)$	$1/2^+$ ***
$\Lambda(2080)$	$5/2^-$ **	$\Lambda(2085)$	$7/2^+$ **	$\Sigma_c(3123)$	$5/2^+$ ***	$\Xi_c(3123)$	$5/2^+$ ***	$\Xi_c(3123)$	$5/2^+$ ***
$\Lambda(2085)$	$7/2^+$ **	$\Lambda(2100)$	$7/2^-$ ***	$\Sigma_c(3123)$	$7/2^+$ ***	$\Xi_c(3123)$	$7/2^+$ ***	$\Xi_c(3123)$	$7/2^+$ ***
$\Lambda(2100)$	$7/2^-$ ***	$\Lambda(2110)$	$5/2^+$ ***	$\Sigma_c(3123)$	$7/2^+$ ***	$\Xi_c(3123)$	$7/2^+$ ***	$\Xi_c(3123)$	$7/2^+$ ***
$\Lambda(2110)$	$5/2^+$ ***	$\Lambda(2120)$	$1/2^+$ ***	$\Sigma_c(3123)$	$1/2^+$ ***	$\Xi_c(3123)$	$1/2^+$ ***	$\Xi_c(3123)$	$1/2^+$ ***
$\Lambda(2120)$	$1/2^+$ ***	$\Lambda(2130)$	$1/2^-$ ***	$\Sigma_c(3123)$	*	$\Xi_c(3123)$	*	$\Xi_c(3123)$	*
$\Lambda(2130)$	$1/2^-$ ***	$\Lambda(2140)$	$1/2^+$ ***	$\Sigma_c(3123)$	*	$\Xi_c(3123)$	*	$\Xi_c(3123)$	*
$\Lambda(2140)$	$1/2^+$ ***	$\Lambda(2150)$	$1/2^-$ ***	$\Sigma_c(3123)$	*	$\Xi_c(3123)$	*	$\Xi_c(3123)$	*
$\Lambda(2150)$	$1/2^-$ ***	$\Lambda(2160)$	$1/2^+$ ***	$\Sigma_c(3123)$	*	$\Xi_c(3123)$	*	$\Xi_c(3123)$	*
$\Lambda(2160)$	$1/2^+$ ***	$\Lambda(2170)$	$1/2^-$ ***	$\Sigma_c(3123)$	*	$\Xi_c(3123)$	*	$\Xi_c(3123)$	*
$\Lambda(2170)$	$1/2^-$ ***	$\Lambda(2180)$	$1/2^+$ ***	$\Sigma_c(3123)$	*	$\Xi_c(3123)$	*	$\Xi_c(3123)$	*
$\Lambda(2180)$	$1/2^+$ ***	$\Lambda(2190)$	$1/2^-$ ***	$\Sigma_c(3123)$	*	$\Xi_c(3123)$	*	$\Xi_c(3123)$	*
$\Lambda(2190)$	$1/2^-$ ***	$\Lambda(2200)$	$1/2^+$ ***	$\Sigma_c(3123)$	*	$\Xi_c(3123)$	*	$\Xi_c(3123)$	*
$\Lambda(2200)$	$1/2^+$ ***	$\Lambda(2210)$	$1/2^-$ ***	$\Sigma_c(3123)$	*	$\Xi_c(3123)$	*	$\Xi_c(3123)$	*
$\Lambda(2210)$	$1/2^-$ ***	$\Lambda(2220)$	$1/2^+$ ***	$\Sigma_c(3123)$	*	$\Xi_c(3123)$	*	$\Xi_c(3123)$	*
$\Lambda(2220)$	$1/2^+$ ***	$\Lambda(2230)$	$1/2^-$ ***	$\Sigma_c(3123)$	*	$\Xi_c(3123)$	*	$\Xi_c(3123)$	*
$\Lambda(2230)$	$1/2^-$ ***	$\Lambda(2240)$	$1/2^+$ ***	$\Sigma_c(3123)$	*	$\Xi_c(3123)$	*	$\Xi_c(3123)$	*
$\Lambda(2240)$	$1/2^+$ ***	$\Lambda(2250)$	$1/2^-$ ***	$\Sigma_c(3123)$	*	$\Xi_c(3123)$	*	$\Xi_c(3123)$	*
$\Lambda(2250)$	$1/2^-$ ***	$\Lambda(2260)$	$1/2^+$ ***	$\Sigma_c(3123)$	*	$\Xi_c(3123)$	*	$\Xi_c(3123)$	*
$\Lambda(2260)$	$1/2^+$ ***	$\Lambda(2270)$	$1/2^-$ ***	$\Sigma_c(3123)$	*	$\Xi_c(3123)$	*	$\Xi_c(3123)$	*
$\Lambda(2270)$	$1/2^-$ ***	$\Lambda(2280)$	$1/2^+$ ***	$\Sigma_c(3123)$	*	$\Xi_c(3123)$	*	$\Xi_c(3123)$	*
$\Lambda(2280)$	$1/2^+$ ***	$\Lambda(2290)$	$1/2^-$ ***	$\Sigma_c(3123)$	*	$\Xi_c(3123)$	*	$\Xi_c(3123)$	*
$\Lambda(2290)$	$1/2^-$ ***	$\Lambda(2300)$	$1/2^+$ ***	$\Sigma_c(3123)$	*	$\Xi_c(3123)$	*	$\Xi_c(3123)$	*
$\Lambda(2300)$	$1/2^+$ ***	$\Lambda(2310)$	$1/2^-$ ***	$\Sigma_c(3123)$	*	$\Xi_c(3123)$	*	$\Xi_c(3123)$	*
$\Lambda(2310)$	$1/2^-$ ***	$\Lambda(2320)$	$1/2^+$ ***	$\Sigma_c(3123)$	*	$\Xi_c(3123)$	*	$\Xi_c(3123)$	*
$\Lambda(2320)$	$1/2^+$ ***	$\Lambda(2330)$	$1/2^-$ ***	$\Sigma_c(3123)$	*	$\Xi_c(3123)$	*	$\Xi_c(3123)$	*
$\Lambda(2330)$	$1/2^-$ ***	$\Lambda(2340)$	$1/2^+$ ***	$\Sigma_c(3123)$	*	$\Xi_c(3123)$	*	$\Xi_c(3123)$	*
$\Lambda(2340)$	$1/2^+$ ***	$\Lambda(2350)$	$1/2^-$ ***	$\Sigma_c(3123)$	*	$\Xi_c(3123)$	*	$\Xi_c(3123)$	*
$\Lambda(2350)$	$1/2^-$ ***	$\Lambda(2360)$	$1/2^+$ ***	$\Sigma_c(3123)$	*	$\Xi_c(3123)$	*	$\Xi_c(3123)$	*
$\Lambda(2360)$	$1/2^+$ ***	$\Lambda(2370)$	$1/2^-$ ***	$\Sigma_c(3123)$	*	$\Xi_c(3123)$	*	$\Xi_c(3123)$	*
$\Lambda(2370)$	$1/2^-$ ***	$\Lambda(2380)$	$1/2^+$ ***	$\Sigma_c(3123)$	*	$\Xi_c(3123)$	*	$\Xi_c(3123)$	*
$\Lambda(2380)$	$1/2^+$ ***	$\Lambda(2390)$	$1/2^-$ ***	$\Sigma_c(3123)$	*	$\Xi_c(3123)$	*	$\Xi_c(3123)$	*
$\Lambda(2390)$	$1/2^-$ ***	$\Lambda(2400)$	$1/2^+$ ***	$\Sigma_c(3123)$	*	$\Xi_c(3123)$	*	$\Xi_c(3123)$	*
$\Lambda(2400)$	$1/2^+$ ***	$\Lambda(2410)$	$1/2^-$ ***	$\Sigma_c(3123)$	*	$\Xi_c(3123)$	*	$\Xi_c(3123)$	*
$\Lambda(2410)$	$1/2^-$ ***	$\Lambda(2420)$	$1/2^+$ ***	$\Sigma_c(3123)$	*	$\Xi_c(3123)$	*	$\Xi_c(3123)$	*
$\Lambda(2420)$	$1/2^+$ ***	$\Lambda(2430)$	$1/2^-$ ***	$\Sigma_c(3123)$	*	$\Xi_c(3123)$	*	$\Xi_c(3123)$	*
$\Lambda(2430)$	$1/2^-$ ***	$\Lambda(2440)$	$1/2^+$ ***	$\Sigma_c(3123)$	*	$\Xi_c(3123)$	*	$\Xi_c(3123)$	*
$\Lambda(2440)$	$1/2^+$ ***	$\Lambda(2450)$	$1/2^-$ ***	$\Sigma_c(3123)$	*	$\Xi_c(3123)$	*	$\Xi_c(3123)$	*
$\Lambda(2450)$	$1/2^-$ ***	$\Lambda(2460)$	$1/2^+$ ***	$\Sigma_c(3123)$	*	$\Xi_c(3123)$	*	$\Xi_c(3123)$	*
$\Lambda(2460)$	$1/2^+$ ***	$\Lambda(2470)$	$1/2^-$ ***	$\Sigma_c(3123)$	*	$\Xi_c(3123)$	*	$\Xi_c(3123)$	*
$\Lambda(2470)$	$1/2^-$ ***	$\Lambda(2480)$	$1/2^+$ ***	$\Sigma_c(3123)$	*	$\Xi_c(3123)$	*	$\Xi_c(3123)$	*
$\Lambda(2480)$	$1/2^+$ ***	$\Lambda(2490)$	$1/2^-$ ***	$\Sigma_c(3123)$	*	$\Xi_c(3123)$	*	$\Xi_c(3123)$	*
$\Lambda(2490)$	$1/2^-$ ***	$\Lambda(2500)$	$1/2^+$ ***	$\Sigma_c(3123)$	*	$\Xi_c(3123)$	*	$\Xi_c(3123)$	*
$\Lambda(2500)$	$1/2^+$ ***	$\Lambda(2510)$	$1/2^-$ ***	$\Sigma_c(3123)$	*	$\Xi_c(3123)$	*	$\Xi_c(3123)$	*
$\Lambda(2510)$	$1/2^-$ ***	$\Lambda(2520)$	$1/2^+$ ***	$\Sigma_c(3123)$	*	$\Xi_c(3123)$	*	$\Xi_c(3123)$	*
$\Lambda(2520)$	$1/2^+$ ***	$\Lambda(2530)$	$1/2^-$ ***	$\Sigma_c(3123)$	*	$\Xi_c(3123)$	*	$\Xi_c(3123)$	*
$\Lambda(2530)$	$1/2^-$ ***	$\Lambda(2540)$	$1/2^+$ ***	$\Sigma_c(3123)$	*	$\Xi_c(3123)$	*	$\Xi_c(3123)$	*
$\Lambda(2540)$	$1/2^+$ ***	$\Lambda(2550)$	$1/2^-$ ***	$\Sigma_c(3123)$	*	$\Xi_c(3123)$	*	$\Xi_c(3123)$	*
$\Lambda(2550)$	$1/2^-$ ***	$\Lambda(2560)$	$1/2^+$ ***	$\Sigma_c(3123)$	*	$\Xi_c(3123)$	*	$\Xi_c(3123)$	*
$\Lambda(2560)$	$1/2^+$ ***	$\Lambda(2570)$	$1/2^-$ ***	$\Sigma_c(3123)$	*	$\Xi_c(3123)$	*	$\Xi_c(3123)$	*
$\Lambda(2570)$	<math								

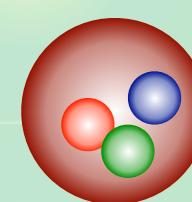
Color confinement

QCD Lagrangian : color $SU(3)$ symmetry

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4}G_{\mu\nu}^a G_a^{\mu\nu} + \bar{q}_\alpha(i\gamma^\mu D_\mu^{\alpha\beta} - m_q \delta^{\alpha\beta}) q_\beta$$

- **Quarks** q_α : color 3 
- **Antiquarks** \bar{q}_α : color $\bar{3}$ 
- **Gluons** A_μ^a : color 8

Color confinement: only color singlet states are observed

- **Mesons** $q\bar{q}$: $3 \otimes \bar{3} = \mathbf{1} \oplus \mathbf{8}$ 
- **Baryons** qqq : $3 \otimes 3 \otimes 3 = \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{10}$ 
- **Why are the $3, 8, 10, \dots$ states forbidden?**

Experimental fact, but not understood from QCD

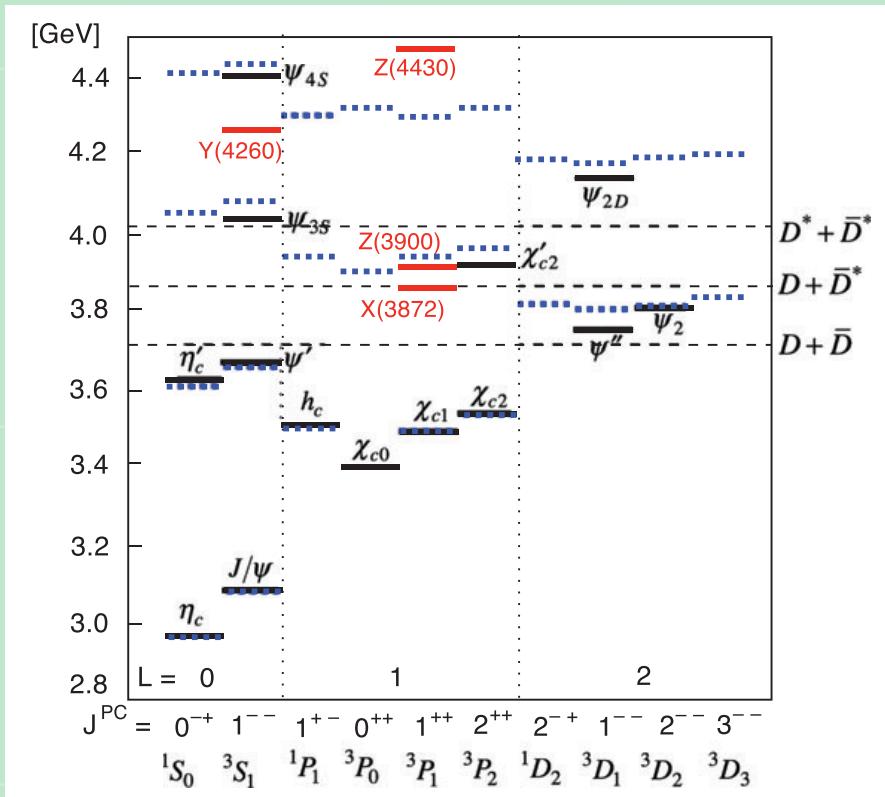
Exotic hadrons (naive)

Any other color singlet configurations?

- $qq\bar{q}\bar{q}$, $qqqq\bar{q}$, $q\bar{q}g$, gg , ...
- **Exotic hadrons : hadrons other than $q\bar{q}$, qqq**
- How can we **identify** them?

Standard criterion

- Constituent quark model
- Exotics : states which do not fit in the quark model



Charmonium spectrum

A. Hosaka *et al.*, PTEP 2016, 062C01 (2016)

Problem : quark model is not QCD!

Exotic hadrons from symmetries 1

Spatial symmetries in QCD

- **Parity** Z_2 : $\mathbf{r} \rightarrow -\mathbf{r}$
- **Rotation** $SO(3)$: $\mathbf{r} \rightarrow R\mathbf{r}$

—> Hadrons are classified by spin-parity J^P

Internal symmetries in QCD

- **Phase** $U(1)_V$: $q \rightarrow e^{i\theta} q, \quad \bar{q} \rightarrow e^{-i\theta} \bar{q}$
- > Hadrons are classified by baryon number B

$B = 0$ state : meson, $B = 1$ state : baryon

Definition by conserved quantum number

- Note : $n_q - n_{\bar{q}}$ is conserved
- > One cannot distinguish $q\bar{q}$ from $qq\bar{q}\bar{q}$ by B

Exotic hadrons from symmetries 2

Flavor symmetry

- **Isospin** $SU(2)$: $\begin{pmatrix} u \\ d \end{pmatrix} \rightarrow U \begin{pmatrix} u \\ d \end{pmatrix}$

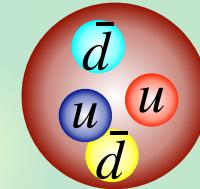
—> Hadrons are classified by isospin I

I of mesons made from only u, d quarks ($q, \bar{q} : I = 1/2$)

- $q\bar{q}$: $I = 0, 1$

No annihilation of $q\bar{q}$

- $qq\bar{q}\bar{q}$: $I = 0, 1, 2$



- $qqq\bar{q}\bar{q}\bar{q}$: $I = 0, 1, 2, 3$

—> $I \geq 2$ meson (charge $|Q| \geq 2$) cannot be $q\bar{q}$: exotic!

Definition by conserved quantum number

Extension to $SU(3)$: exoticness

T. Hyodo, D. Jido, A. Hosaka, PRL97, 192002 (2006); PRD75, 034002 (2007)

Exotic hadrons from symmetries 3

Exotic hadrons defined by conserved quantum numbers

Mesons $B = 0$

- $J^P = 0^-, I = 1$
- $J^P = 1^-, I = 1$
- $J^P = 0^-, I = 1/2, S = \pm 1$
- ...

Baryons $B = 1$

- $J^P = 1/2^+, I = 1/2$
- $J^P = 3/2^+, I = 3/2$
- $J^P = 1/2^-, I = 0, S = -1$
- ...

- $J^P = 0^-, I = 2$
- ...

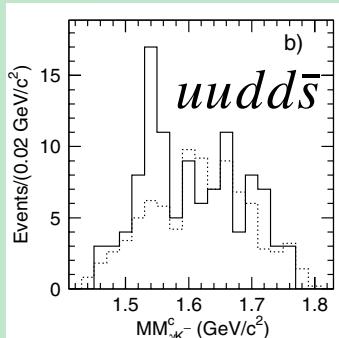
- $J^P = 1/2^+, I = 0, S = +1$
- ...

Quantum number exotics (require more than $\bar{q}q, qqq$)

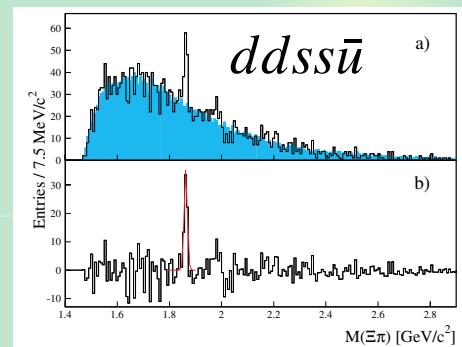
Exotic hadrons in experiments 1

Possible candidates of quantum number exotics

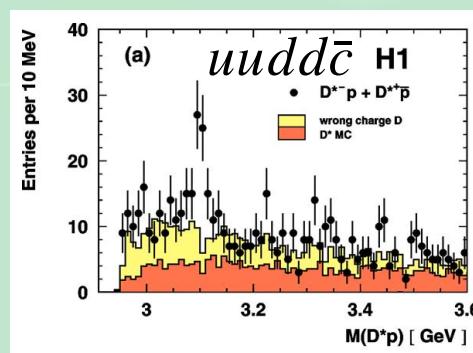
$\Theta^+(S = +1, B = 1)$ T. Nakano *et al.* (LEPS), PRL 91, 012002 (2003)



$\Xi^{--}(Q = -2, B = 1)$ C. Alt *et al.* (NA49), PRL 92, 042003 (2004)

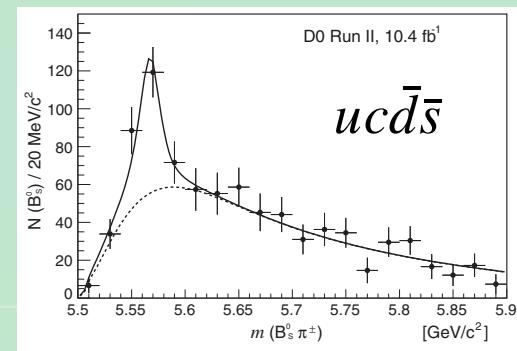


$\Theta_c(C = -1, B = 1)$



A. Aktas *et al.* (H1), PLB 588, 17 (2004)

$X(C = +1, I = 1)$



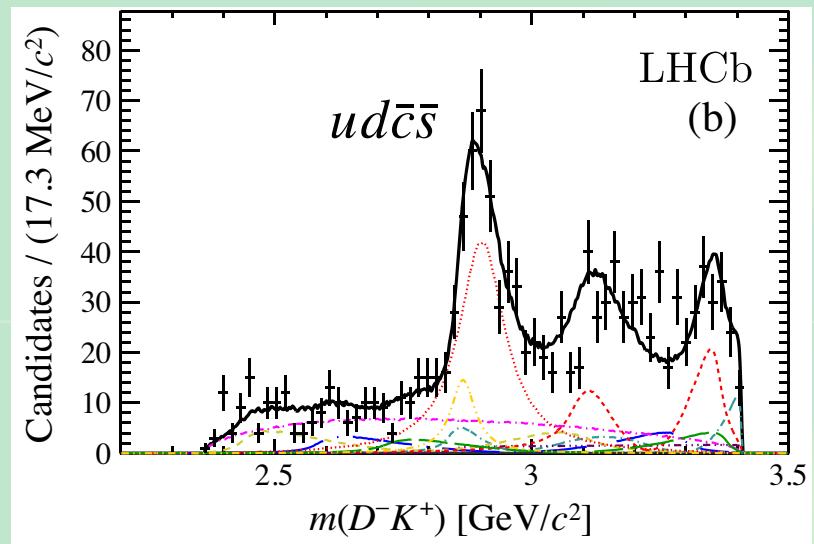
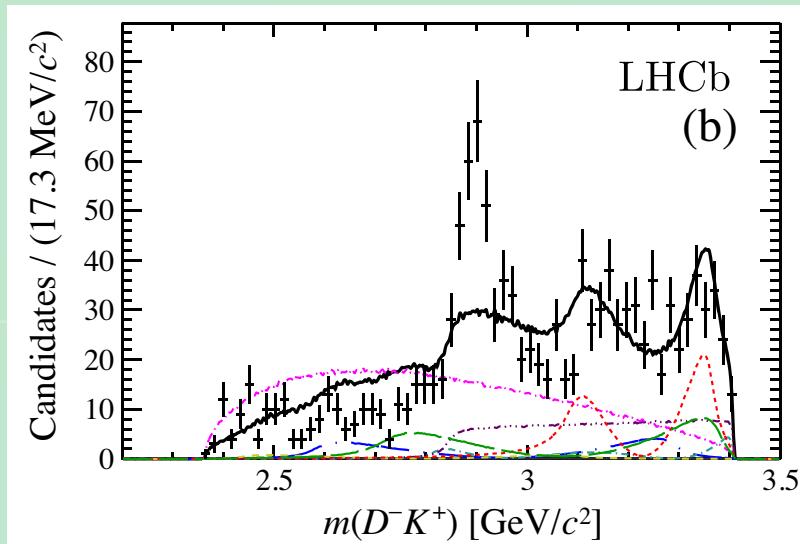
V.M. Abazov *et al.* (D0), PRL 117, 022003 (2016)

- $q\bar{q}$ annihilation is not possible : genuine exotic
- Excluded by higher statistics experiments

Exotic hadrons in experiments 2

New candidate $X(2900)$

$X(S = +1, C = -1)$ R. Aaij *et al.* (LHCb), PRD 102, 112003 (2020)



- Not included in PDG yet

No quantum number exotics are established

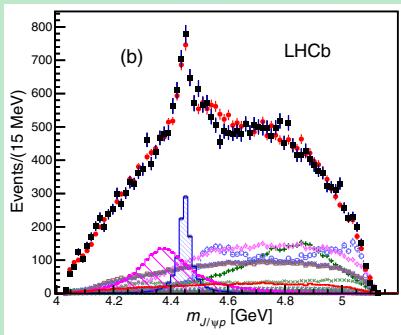
(except for J^{PC} exotics : $\pi_1(1400)$ and $\pi_1(1600)$ with $J^{PC} = 1^{-+}$)

Experimental fact, but not understood from QCD

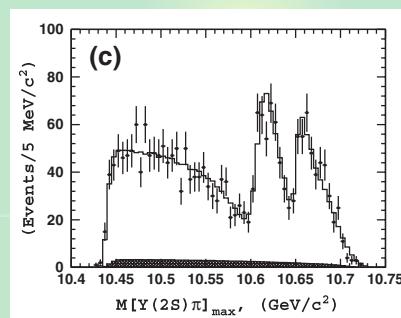
Quarkonium associated exotics

Pentaquarks P_c, P_{cs}, \dots , **tetraquarks** $X, Y, Z_c, Z_b, Z_{cs}, \dots$?

$P_c \sim \bar{c}cuud$ R. Aaij, et al., Phys. Rev. Lett. 115, 072001 (2015)



$Z_b \sim \bar{b}b\bar{u}d$ A. Bondar, et al., Phys. Rev. Lett. 108, 122001 (2012)



...

- $\bar{c}c, \bar{b}b$ can in principle be annihilated

- > Quantum number is **not** exotic

$$P_c \sim \bar{c}cuud \sim uud \sim N, \quad Z_b \sim \bar{b}b\bar{u}d \sim \bar{u}d \sim b_1(J^{PC} = 1^{+-})$$

- OZI rule : existence of heavy quark pair is almost certain

- > Clues to understand the quantum number exotics

Summary of part I



Hadrons are classified by conserved quantum numbers. Non-observation of quantum number exotics is highly nontrivial.

Contents



Part I : Introduction

- What are “exotic hadrons”?

- Hadronic molecules and universality

- Unstable resonances



Part II : Structure of $\Lambda(1405)$ resonance

- Accurate $\bar{K}N$ scattering amplitude

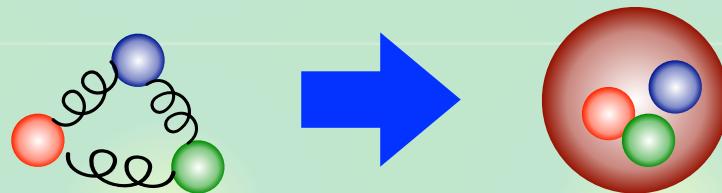
Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011); NPA 881, 98 (2012)

- Compositeness of hadrons

Y. Kamiya, T. Hyodo, PRC93, 035203 (2016); PTEP2017, 023D02 (2017)

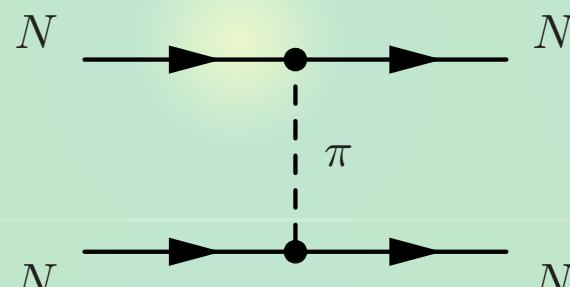
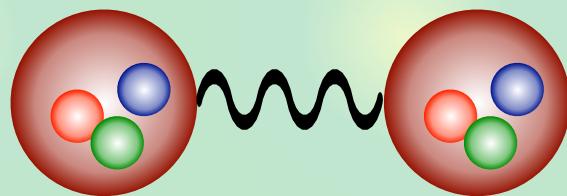
Hadronic interactions

Strong interaction : hadrons \leftarrow quarks



- c.f. EM interaction : atoms \leftarrow electrons and nucleus

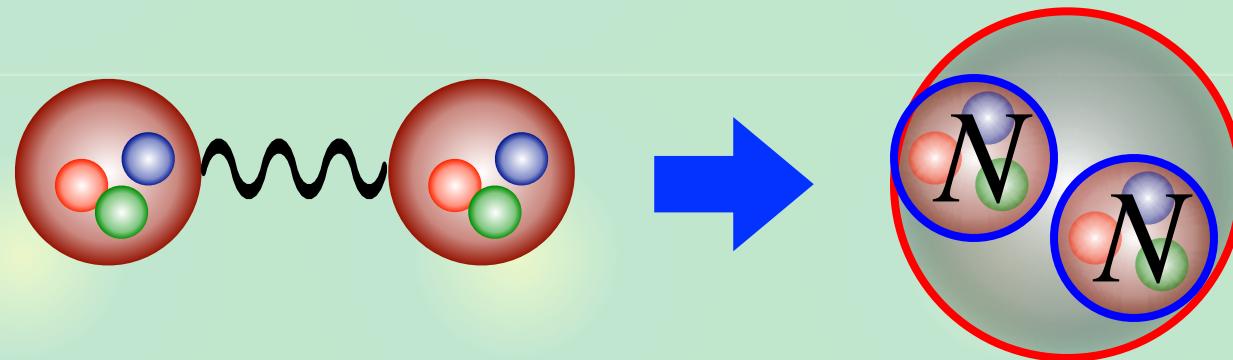
Hadron-hadron interaction (e.g. nuclear force)



- stems eventually from QCD
- Hadrons are (color) charge neutral : van der Waals force?
- Meson ($q\bar{q}$) exchange force, ...

Hadronic molecules (naive)

If the hadronic interaction is sufficiently attractive...



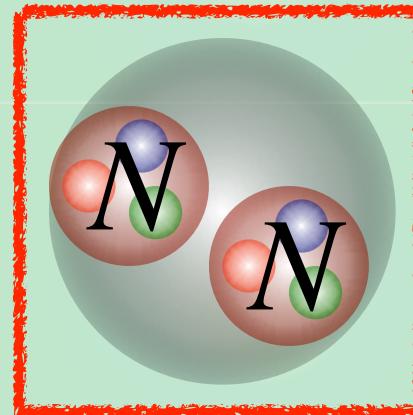
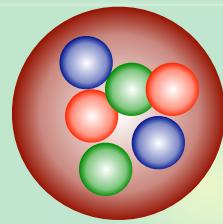
- (quasi-)bound state can be formed : **hadronic molecules**
- Constituent hadrons keep their identity
- Clustering of quarks

Known examples : deuteron d , nuclei

- Binding energy of **nuclei** : $\mathcal{O}(1)$ MeV
 - Binding energy of **quarks** : ∞ or $\mathcal{O}(100)$ MeV
- > Hadronic molecules : **shallow bound states**

Compositeness

Can we distinguish hadronic molecules from others?



- Same conserved quantum numbers $J^P = 1^+, I = 0, B = 2$
- Physical state = superposition of possible configurations

$$|d\rangle \stackrel{?}{=} C_{6q} |qqqqqq\rangle + C_{NN} |NN\rangle + \dots$$

- Orthogonal basis?

Expansion in terms of hadrons $|d\rangle = \sqrt{Z} |d_0\rangle + \boxed{\sqrt{X}} |NN\rangle + \dots$

- Hadrons are asymptotic states of QCD
- > compositeness X (part II)

Two-body universal physics

Shallow s-wave bound states : low-energy universality

E. Braaten, H.-W. Hammer, Phys. Rept. 428, 259 (2006);

P. Naidon, S. Endo, Rept. Prog. Phys. 80, 056001 (2017)

- Scattering length $|a| \gg$ interaction range R_{typ}
- Size of (quasi-)bound state $R \sim a$: loosely bound
- Relation with eigenenergy E

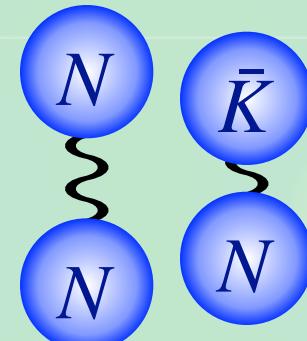
$$a(E) \sim \frac{i}{k} = \frac{i}{\sqrt{2\mu E}} \quad (|E| \ll 1)$$

Examples: d , $\Lambda(1405)$, ${}^4\text{He}$ dimer

	NN [fm]	$\bar{K}N$ [fm]	${}^4\text{He}$ [a_0]
$a(E)$	4.3	$1.2 - 0.8i$	178
a_{emp}	5.1	$1.4 - 0.9i$	189
R_{typ}	1.4	0.4	10

vdW

strong



${}^4\text{He}$

Wave function

Why is a weakly bound s-wave state spatially large?

- Radial Schrödinger equation

$$-\frac{\hbar^2}{2\mu} \frac{d^2}{dr^2} u_0(r) + V(r)u_0(r) = Eu_0(r), \quad \psi_0(r) = Y_0^0 \frac{u_0(r)}{r}$$

- At large distance ($V(r) = 0$) with zero energy ($E = 0$)

$$u_0(r) = C(r - a), \quad (r > R_{\text{typ}})$$

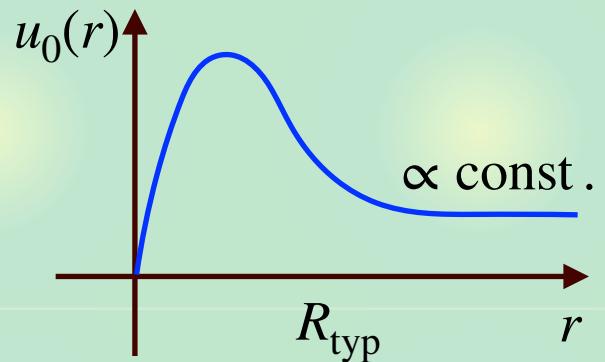
- Scattering length a : intercept of $u_0(r)$

- Bound state with $B = 0 \Rightarrow |a| = \infty$

Wave function is not normalizable

$$\int d^3r |\psi_0(r)|^2 = \int_0^\infty dr |u_0(r)|^2 = \infty$$

$B = 0$ state is not a bound state (zero-energy resonance)



Consequences

Mean squared radius

$$\langle r^2 \rangle = \int d^3r \ r^2 |\psi_0(r)|^2 = \int_0^\infty dr \ r^2 |u_0(r)|^2 = \infty$$

—> Size of $B = 0$ state is **infinitely large** ($a = R$)

Compositeness X (weight of the composite component)

T. Hyodo, Phys. Rev. C 90, 055208 (2014)

$$\Psi = \begin{pmatrix} \tilde{\psi}_0(p) \\ c \\ \vdots \end{pmatrix} \quad X = \int \frac{d^3p}{(2\pi)^3} |\tilde{\psi}_0(p)|^2 = \int d^3r |\psi_0(r)|^2 \quad \text{infinite}$$

$$Z = |c|^2 + \dots \quad \text{finite}$$

$$X + Z = 1 \quad \leftarrow |\Psi|^2 = 1$$

—> $B = 0$ state is **completely composite** ($X = 1$, $Z = 0$)

Weakly bound state ($B \neq 0$, except for fine tuning)

- Deviation from $a = R$: **weak-binding relation**

Summary of part I

- ➊ Hadrons are classified by conserved quantum numbers. Non-observation of quantum number exotics is **highly nontrivial**.
- ➋ Hadronic molecules are shallow (quasi-)bound states of hadrons. Different hierarchies can be related by low-energy **universality**.

Contents



Part I : Introduction

- What are “exotic hadrons”?
- Hadronic molecules and universality
- Unstable resonances



Part II : Structure of $\Lambda(1405)$ resonance

- Accurate $\bar{K}N$ scattering amplitude

Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011); NPA 881, 98 (2012)

- Compositeness of hadrons

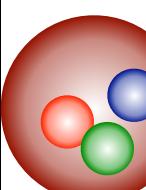
Y. Kamiya, T. Hyodo, PRC93, 035203 (2016); PTEP2017, 023D02 (2017)

Unstable states via strong interaction

Stable/unstable hadrons

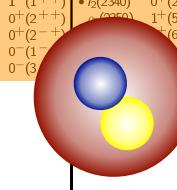
<http://pdg.lbl.gov/>

p	$1/2^+$ ****	$\Delta(1232)$	$3/2^+$ ****	Σ^+	$1/2^+$ ****	Ξ^0	$1/2^+$ ****	Ξ_{cc}^{++}	***
n	$1/2^+$ ***	$\Delta(1600)$	$3/2^+$ ***	Σ^0	$1/2^+$ ***	Ξ^-	$1/2^+$ ***	Λ_b^0	$1/2^+$ ***
$N(1440)$	$1/2^+$ ***	$\Delta(1620)$	$1/2^-$ ***	Σ^-	$1/2^+$ ***	$\Xi(1530)$	$3/2^+$ ***	Ξ_b^0	$1/2^+$ ***
$N(1520)$	$3/2^-$ ***	$\Delta(1700)$	$3/2^-$ ***	$\Sigma(1885)$	$3/2^+$ ***	$\Xi(1620)$	*	$\Lambda_b(5912)^0$	$1/2^-$ ***
$N(1535)$	$1/2^-$ ***	$\Delta(1750)$	$1/2^+$ *	$\Sigma(1850)$	$3/2^-$ *	$\Xi(1690)$	***	$\Lambda_b(5920)^0$	$3/2^-$ ***
$N(1650)$	$1/2^-$ ***	$\Delta(1900)$	$1/2^-$ ***	$\Sigma(1620)$	$1/2^+$ *	$\Xi(1820)$	$3/2^-$ ***	$\Lambda_b(6146)^0$	$3/2^+$ ***
$N(1675)$	$5/2^-$ ***	$\Delta(1905)$	$5/2^+$ ***	$\Sigma(1660)$	$1/2^+$ ***	$\Xi(1950)$	***	$\Lambda_b(6152)^0$	$5/2^+$ ***
$N(1680)$	$5/2^+$ ***	$\Delta(1910)$	$1/2^+$ ***	$\Sigma(1670)$	$3/2^-$ ***	$\Xi(2030)$	$\geq \frac{5}{2}?$ ***	Σ_b^-	$1/2^+$ ***
$N(1700)$	$3/2^-$ ***	$\Delta(1920)$	$3/2^+$ ***	$\Sigma(1750)$	$1/2^-$ ***	$\Xi(2120)$	*	Σ_b^+	$3/2^+$ ***
$N(1710)$	$1/2^+$ ***	$\Delta(1930)$	$5/2^-$ ***	$\Sigma(1775)$	$5/2^+$ ***	$\Xi(2250)$	**	$\Sigma_b(6097)^+$	***
$N(1720)$	$3/2^+$ ***	$\Delta(1940)$	$3/2^-$ **	$\Sigma(1780)$	$3/2^+$ *	$\Xi(2370)$	**	$\Xi_b(6097)^-$	***
$N(1860)$	$5/2^+$ **	$\Delta(1950)$	$7/2^+$ ***	$\Sigma(1880)$	$1/2^+$ **	$\Xi(2500)$	*	Ξ_b^0 , Ξ_b^-	$1/2^+$ ***
$N(1875)$	$3/2^-$ ***	$\Delta(2000)$	$5/2^+$ **	$\Sigma(1900)$	$1/2^-$ ***	$\Xi(2600)$	$\Xi_b(5935)^-$	$1/2^+$ ***	
$N(1880)$	$1/2^+$ ***	$\Delta(2150)$	$1/2^-$ *	$\Sigma(1910)$	$3/2^-$ ***	Ω^-	$3/2^+$ ***	$\Xi_b(5945)^0$	$3/2^+$ ***
$N(1895)$	$1/2^-$ ***	$\Delta(2200)$	$7/2^-$ ***	$\Sigma(1915)$	$5/2^+$ ***	$\Omega(2012)^?$	***	$\Xi_b(5955)^-$	$3/2^+$ ***
$N(1900)$	$3/2^+$ ***	$\Delta(2300)$	$9/2^+$ **	$\Sigma(1940)$	$3/2^+$ *	$\Omega(2250)^-$	***	$\Xi_b(6227)^-$	***
$N(1990)$	$7/2^+$ **	$\Delta(2350)$	$5/2^-$ *	$\Sigma(2010)$	$3/2^-$ *	$\Omega(2380)^-$	**	Ω_b^-	$1/2^+$ ***
$N(2000)$	$5/2^+$ **	$\Delta(2390)$	$7/2^+$ ***	$\Sigma(2030)$	$7/2^+$ ***	$\Omega(2470)^-$	**	$P_c(4312)^+$	*
$N(2040)$	$3/2^+$ *	$\Delta(2400)$	$9/2^-$ **	$\Sigma(2070)$	$5/2^+$ *	Λ_c^+	$1/2^+$ ***	$P_c(4380)^+$	*
$N(2060)$	$5/2^+$ ***	$\Delta(2420)$	$11/2^+$ ***	$\Sigma(2080)$	$3/2^+$ *	$\Lambda_c(2595)^+$	$1/2^-$ ***	$P_c(4440)^+$	*
$N(2100)$	$1/2^+$ ***	$\Delta(2750)$	$13/2^-$ **	$\Sigma(2100)$	$7/2^-$ *	$\Lambda_c(2625)^+$	$3/2^-$ ***	$P_c(4457)^+$	*
$N(2120)$	$3/2^-$ ***	$\Delta(2950)$	$15/2^+$ **	$\Sigma(2230)$	$3/2^+$ *	$\Lambda_c(2765)^+$	*		
$N(2190)$	$7/2^+$ ***	Λ	$1/2^+$ ***	$\Sigma(2250)$	***	$\Lambda_c(2860)^-$	$3/2^+$ ***		
$N(2220)$	$9/2^+$ ***	Λ	$1/2^+$ ***	$\Sigma(2455)$	***	$\Lambda_c(2880)^+$	$5/2^+$ ***		
$N(2250)$	$9/2^-$ ***	Λ	$1/2^-$ **	$\Sigma(2620)$	**	$\Lambda_c(2940)^+$	$3/2^-$ ***		
$N(2300)$	$1/2^+$ **	$\Lambda(1405)$	$1/2^-$ ***	$\Sigma(3000)$	*	$\Sigma_c(2455)$	$1/2^+$ ***		
$N(2570)$	$5/2^-$ **	$\Lambda(1520)$	$3/2^-$ ***	$\Sigma(2520)$	$3/2^-$ ***	$\Sigma_c(2520)$	$3/2^-$ ***		
$N(2600)$	$11/2^-$ ***	$\Lambda(1600)$	$1/2^+$ ***	$\Sigma(3170)$	*	$\Sigma_c(2800)$	***		
$N(2700)$	$13/2^+$ **	$\Lambda(1670)$	$1/2^-$ ***			Ξ_c^+	$1/2^+$ ***		
		$\Lambda(1690)$	$3/2^-$ ***			Ξ_c^0	$1/2^+$ ***		
		$\Lambda(1710)$	$1/2^+$ *			Ξ_c^-	$1/2^+$ ***		
		$\Lambda(1800)$	$1/2^-$ ***			Ξ_c^+	$1/2^+$ ***		
		$\Lambda(1810)$	$1/2^+$ ***			Ξ_c^0	$1/2^+$ ***		
		$\Lambda(1820)$	$5/2^+$ ***			$\Xi_c(2645)^-$	$3/2^+$ ***		
		$\Lambda(1830)$	$5/2^-$ ***			$\Xi_c(2790)$	$1/2^-$ ***		
		$\Lambda(1890)$	$3/2^+$ ***			$\Xi_c(2815)$	$3/2^-$ ***		
		$\Lambda(2000)$	$1/2^-$ *			$\Xi_c(2930)$	**		
		$\Lambda(2050)$	$3/2^-$ *			$\Xi_c(2970)$	***		
		$\Lambda(2070)$	$3/2^+$ *			$\Xi_c(3055)$	***		
		$\Lambda(2080)$	$5/2^-$ *			$\Xi_c(3080)$	***		
		$\Lambda(2085)$	$7/2^+$ **			$\Xi_c(3123)$	*		
		$\Lambda(2100)$	$7/2^-$ ***			Ω_c^0	$1/2^+$ ***		
		$\Lambda(2110)$	$5/2^+$ ***			Ω_c^0	$3/2^+$ ***		
		$\Lambda(2325)$	$3/2^-$ *			Ω_c^0	$5/2^+$ ***		
		$\Lambda(2350)$	$9/2^+$ ***			Ω_c^0	$7/2^+$ ***		
		$\Lambda(2585)$	**						



162 baryons

LIGHT UNFLAVORED ($S = C = B = 0$)		STRANGE ($S = \pm 1, C = B = 0$)		CHARMED, STRANGE ($C = S = \pm 1$)		$c\bar{c}$ continued $F_c(f_c)$	
$\bullet \pi^\pm$	$1^- (0^-)$	$\bullet \pi(1670)$	$1^- (2^-)$	K^\pm	$1/2(0^-)$	$\bullet D_S^\pm$	$0(0^-)$
$\bullet \eta^0$	$1^- (0^+)$	$\bullet \eta(1680)$	$0^- (1^-)$	K^0	$1/2(0^-)$	$\bullet D_{s1}^0(2317)^{\pm}$	$0(0^+)$
$\bullet f_0(500)$	$0^+(0^+)$	$\bullet f_0(1690)$	$1^+(3^-)$	K_S^0	$1/2(0^-)$	$\bullet D_{s1}(2460)^{\pm}$	$0(0^+)$
$\bullet \eta(770)$	$1^+(1^-)$	$\bullet \eta(1700)$	$1^-(2^+)$	K_L^0	$1/2(0^-)$	$\bullet D_{s1}(2536)^{\pm}$	$0(0^+)$
$\bullet \omega(782)$	$0^-(1^-)$	$\bullet \omega(1710)$	$0^+(0^+)$	K_0^0	$1/2(2^+)$	$\bullet D_{s2}(2573)^{\pm}$	$0(0^+)$
$\bullet \rho'(958)$	$0^+(0^+)$	$\bullet \rho(1720)$	$0^+(0^-)$	$K(700)$	$1/2(0^+)$	$\bullet D_{s1}(2700)^{\pm}$	$0(1^-)$
$\bullet f_0(980)$	$0^+(0^+)$	$\bullet \pi(1800)$	$1^- (0^-)$	$K(1400)$	$1/2(1^+)$	$\bullet D_{s1}(2860)^{\pm}$	$0(1^-)$
$\bullet \chi_b(980)$	$1^- (0^+)$	$\bullet f_0(1810)$	$0^+(2^+)$	$K(1410)$	$1/2(2^-)$	$\bullet D_{s3}(2860)^{\pm}$	$0(3^-)$
$\bullet \omega(1020)$	$0^-(1^-)$	$\bullet X(1835)$	$?^2 (0^+)$	$K_0^0(1430)$	$1/2(0^+)$	$\bullet D_J(3040)^{\pm}$	$0(0^?)$
$\bullet h_b(1170)$	$0^-(1^-)$	$\bullet \phi(1850)$	$0^-(3^-)$	$K_0^0(1430)$	$1/2(2^+)$		
$\bullet b_1(1235)$	$1^+(1^+)$	$\bullet \nu(1870)$	$0^+(2^-)$	$K(1460)$	$1/2(2^-)$		
$\bullet f_2(1270)$	$0^+(2^+)$	$\bullet \pi(1900)$	$1^+(1^-)$	$K(1530)$	$1/2(2^-)$		
$\bullet f_1(1285)$	$0^+(1^+)$	$\bullet f_2(1910)$	$0^+(2^+)$	$K(1650)$	$1/2(1^+)$	$\bullet B_c^+$	$1/2(1^-)$
$\bullet \omega(1295)$	$0^+(0^+)$	$\bullet \alpha(1950)$	$1^- (0^+)$	$K(1690)$	$1/2(1^+)$	$\bullet B_c^0$	$1/2(0^-)$
$\bullet \pi(1300)$	$1^- (0^+)$	$\bullet f_2(1950)$	$0^+(2^-)$	$K(1700)$	$1/2(2^-)$	$\bullet B_c^0/B_c^0/B_c^0$	$ADMIXTURE$
$\bullet \omega(1320)$	$1^-(2^+)$	$\bullet \alpha(1970)$	$1^- (4^-)$	$K_0^0(1780)$	$1/2(3^-)$	$\bullet B_c^0/B_c^0/B_c^0/b$	$baryon$
$\bullet f_0(1370)$	$0^+(0^+)$	$\bullet \nu(1990)$	$1^+(3^-)$	$K_0^0(1820)$	$1/2(2^+)$	$\bullet Z_d(4240)$	CKM
$\bullet \pi(1400)$	$1^- (1^-)$	$\bullet \pi(2005)$	$1^-(2^-)$	$K(1830)$	$1/2(0^+)$	$\bullet Z_d(4250)^{\pm}$	$1^-(2^+)$
$\bullet h_b(1415)$	$0^-(1^-)$	$\bullet \nu(2020)$	$0^+(0^+)$	$K(1950)$	$1/2(0^+)$	$\bullet B_c(4260)^{\pm}$	$0^+(1^+)$
$\bullet \alpha(1420)$	$1^-(1^+)$	$\bullet f_0(2050)$	$0^+(4^+)$	$K(1980)$	$1/2(2^+)$	$\bullet B_c(4274)^{\pm}$	$0^+(0^+)$
$\bullet f_1(1420)$	$0^+(1^+)$	$\bullet \pi(2100)$	$1^-(2^-)$	$K_0^0(2250)$	$1/2(2^-)$	$\bullet Z_d(4300)$	$0^+(0^-)$
$\bullet \omega(1420)$	$0^-(1^-)$	$\bullet \phi(2100)$	$0^+(0^+)$	$K_0^0(2250)$	$1/2(2^-)$	$\bullet Z_d(4415)$	$0^+(1^-)$
$\bullet f_2(1430)$	$0^+(2^+)$	$\bullet f_2(2150)$	$0^+(2^-)$	$K_0^0(2320)$	$1/2(3^+)$	$\bullet B_c(4590)^{\pm}$	$1/2(0^+)$
$\bullet \alpha(1450)$	$1^-(0^+)$	$\bullet \nu(2150)$	$1^+(1^-)$	$K_0^0(2380)$	$1/2(5^-)$	$\bullet Z_d(4590)^{\pm}$	$1/2(0^2)$
$\bullet \gamma(1475)$	$0^+(1^+)$	$\bullet \phi(2170)$	$0^-(1^-)$	$K_0^0(2500)$	$1/2(4^+)$	$\bullet Z_d(4600)$	$1/2(0^2)$
$\bullet f_0(1500)$	$0^+(0^+)$	$\bullet \phi(2200)$	$0^+(2^+)$	$K(2500)$	$1/2(2^+)$	$\bullet Z_d(4640)$	$1/2(0^2)$
$\bullet f_1(1510)$	$0^+(1^+)$			$K(2510)$	$1^+(4^+)$	$\bullet Z_d(4680)$	$1/2(0^2)$
$\bullet f_2'(1525)$	$0^+(2^+)$	$\eta(2225)$	$0^+(0^-)$	$K(2550)$	$1/2(3^-)$	$\bullet B_c(4690)^{\pm}$	$1/2(0^+)$
$\bullet f_2(1565)$	$0^+(2^+)$	$\pi(2250)$	$1^+(3^-)$	$K_0^0(2550)$	$1/2(3^-)$	$\bullet B_c(4730)^{\pm}$	$0(0^-)$
$\bullet \pi(1570)$	$1^+(1^-)$	$\bullet f_2(2300)$	$0^+(2^+)$	$K_0^0(2595)$	$1/2(0^-)$	$\bullet D(2007)^{\pm}$	$1/2(0^-)$
$\bullet h_b(1595)$	$0^+(1^-)$	$\bullet f_2(2300)$	$0^+(4^+)$	$K_0^0(2630)$	$1/2(0^+)$	$\bullet D(2101)^{\pm}$	$1/2(0^-)$
$\bullet \pi_1(1600)$	$1^-(1^-)$	$\bullet f_2(2330)$	$0^+(0^+)$	$D_0^*(2300)$	$1/2(0^+)$	$\bullet D(2020)^{\pm}$	$1/2(0^-)$
$\bullet \alpha_1(1640)$	$1^-(1^+)$	$\bullet f_2(2340)$	$0^+(2^+)$	$D_0^*(2300)$	$1/2(0^+)$	$\bullet D(2120)^{\pm}$	$1/2(0^-)$
$\bullet f_2(1640)$	$1^-(1^+)$	$\bullet f_2(2350)$	$0^+(1^+)$	$D_0^*(2420)$	$1/2(0^+)$	$\bullet D(2240)^{\pm}$	$1/2(0^-)$
$\bullet \nu_2(1645)$	$0^+(2^+)$	$\bullet f_2(2350)$	$0^+(6^+)$	$D_0^*(2420)$	$1/2(0^+)$	$\bullet D(2340)^{\pm}$	$1/2(0^-)$
$\bullet \omega_2(1650)$	$0^-(1^-)$			$D_0^*(2430)$	$1/2(0^+)$	$\bullet D(2460)^{\pm}$	$1/2(0^-)$
$\bullet \alpha_3(1670)$	$0^-(3^-)$			$D_0^*(2550)^0$	$1/2(0^+)$	$\bullet D_2^*(2460)^{\pm}$	$0(0^-)$
				$D_0^*(2550)^0$	$1/2(0^+)$	$\bullet D_2^*(2550)^{\pm}$	$0(0^-)$
				$D_0^*(2640)^0$	$1/2(0^+)$	$\bullet D_2^*(2740)^{\pm}$	$1/2(0^+)$
				$D_0^*(2740)^0$	$1/2(0^+)$	$\bullet D_2^*(2740)^{\pm}$	$1/2(0^+)$
				$D_0^*(2780)^0$	$1/2(0^+)$	$\bullet D_2^*(2780)^{\pm}$	$1/2(0^+)$
				$D(3000)^0$	$1/2(0^+)$	$\bullet D_2^*(3000)^{\pm}$	$1/2(0^+)$

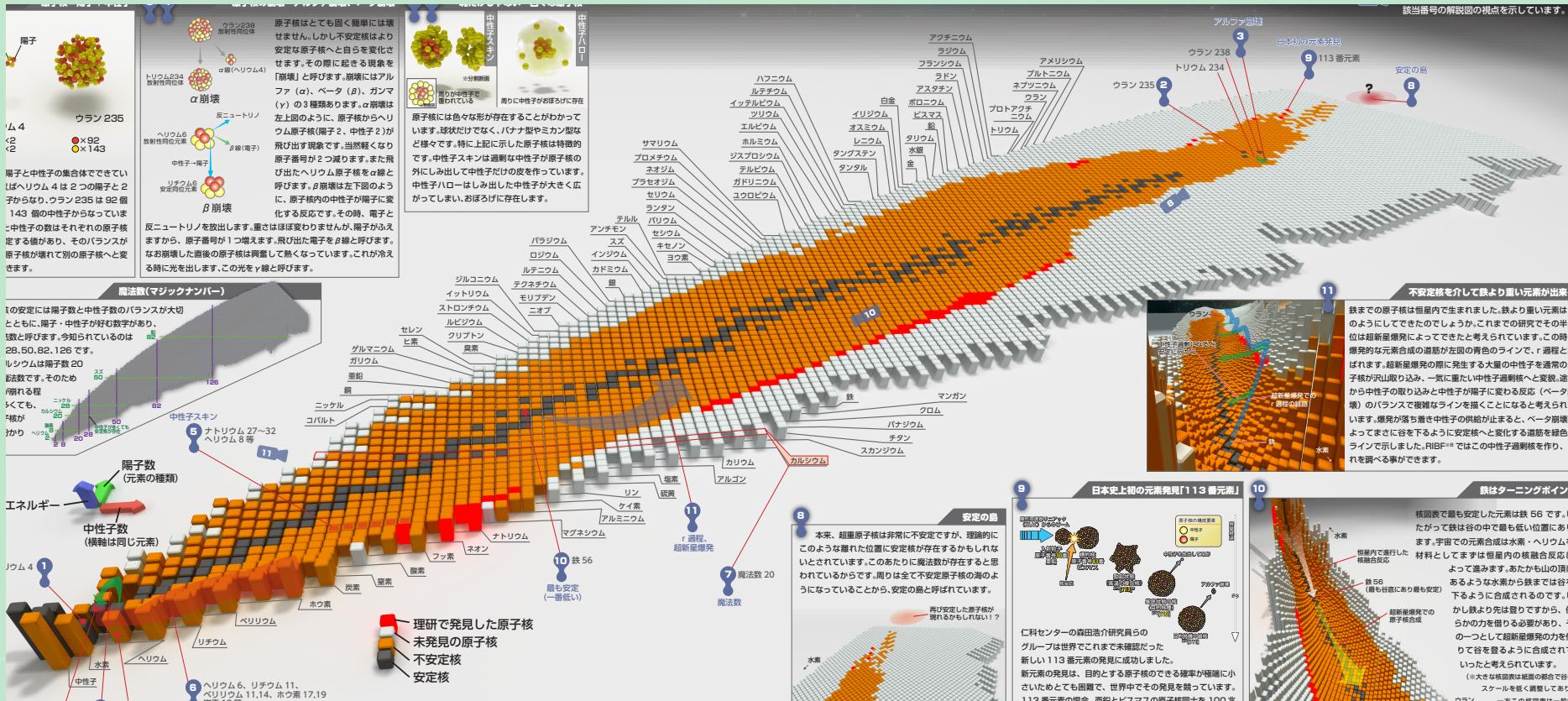


209 mesons

$\bullet n_c(1S)$	$0^+(0^-)$
-------------------	------------

Relation to unstable nuclei

Stable nuclei (~300), unstable nuclei (~2000)



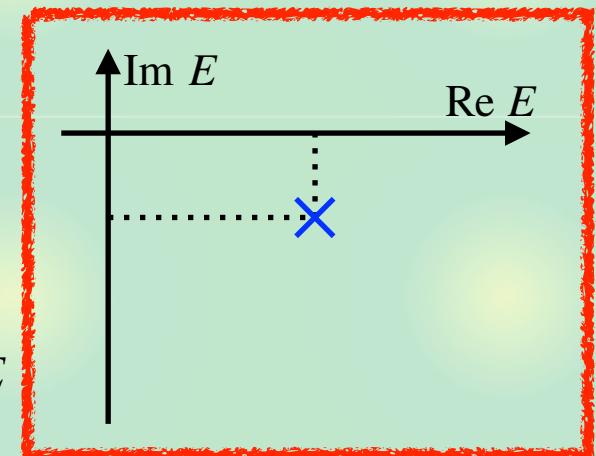
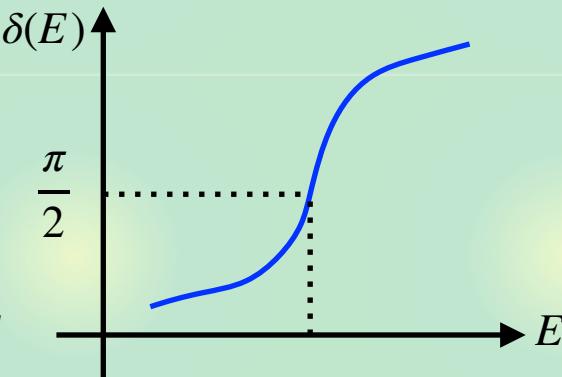
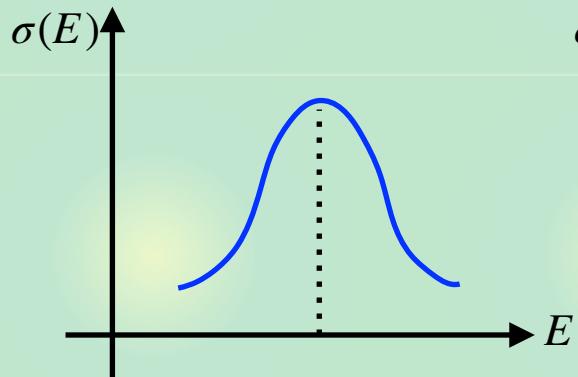
<https://www.nishina.riken.jp/enjoy/kakuzu/index.html>

Structure of unstable nuclei

- Clustering, halo nuclei, Efimov effect, ...

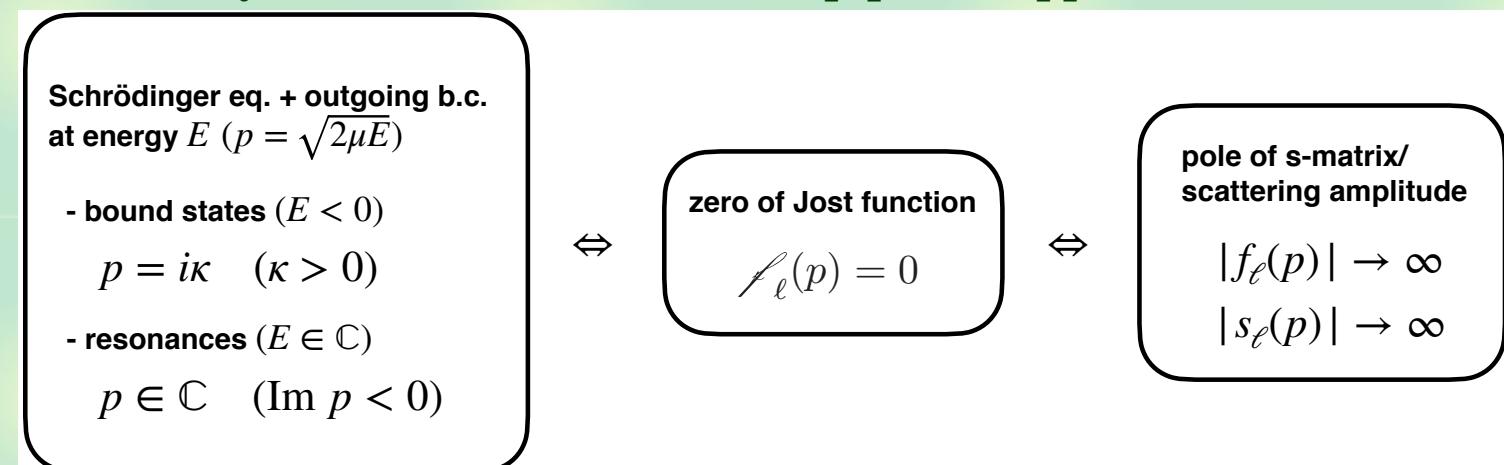
Pole of resonances

Signals of a resonance



Well-defined characterization : pole of scattering amplitude

T. Hyodo, M. Niiyama, arXiv: 2010.07592 [hep-ph], to appear in PPNP



Theoretical analysis to pin down the pole position

Nature of resonances

Resonance as an “eigenstate” of Hamiltonian

- Complex energy

G. Gamow, Z. Phys. 51, 204 (1928)

Zur Quantentheorie des Atomkernes.

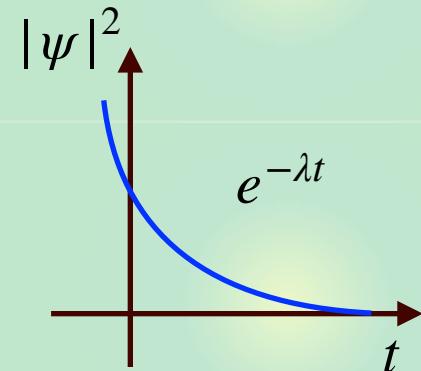
Von G. Gamow, z. Zt. in Göttingen.

Mit 5 Abbildungen. (Eingegangen am 2. August 1928.)

Um diese Schwierigkeit zu überwinden, müssen wir annehmen, daß die Schwingungen gedämpft sind, und E komplex setzen:

$$E = E_0 + i \frac{\hbar \lambda}{4\pi},$$

wo E_0 die gewöhnliche Energie ist und λ das Dämpfungsdekkrement (Zerfallskonstante). Dann sehen wir aber aus den Relationen (2a) und (2b),



- Time dependence : probability decreasing

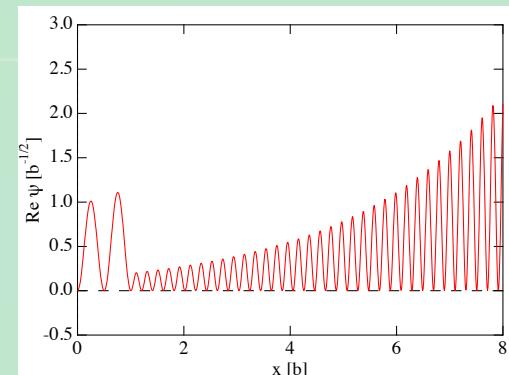
$$\psi = \psi(q) \cdot e^{+\frac{2\pi i E}{\hbar} t} \propto e^{+2\pi i E_0 t / \hbar} e^{-(\lambda/2)t}, \quad |\psi|^2 \propto e^{-\lambda t}$$

- Spatial distribution : divergence at large r

$$\Psi(r) \sim e^{ikr}, \quad |\Psi(r)|^2 \sim e^{2k_I r}, \quad k = k_R - ik_I$$

- complex expectation value (norm, $\langle r^2 \rangle$)

- interpretation problem...



Summary of part I

- ➊ Hadrons are classified by conserved quantum numbers. Non-observation of quantum number exotics is **highly nontrivial**.
- ➋ Hadronic molecules are shallow (quasi-)bound states of hadrons. Different hierarchies can be related by low-energy **universality**.
- ➌ Most of hadrons are **unstable** against the strong decay. Internal structure of hadrons should be discussed with unstable nature.

Contents



Part I : Introduction

- What are “exotic hadrons”?
- Hadronic molecules and universality
- Unstable resonances



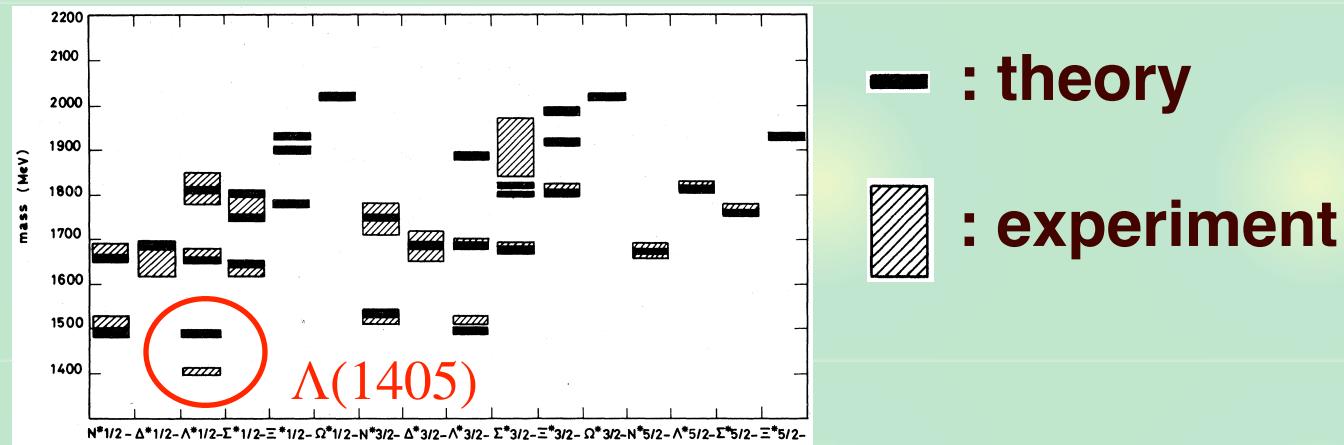
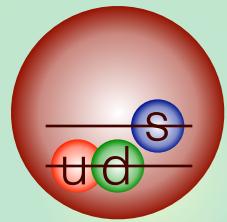
Part II : Structure of $\Lambda(1405)$ resonance

- $\bar{K}N$ scattering and $\Lambda(1405)$ poles
[Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 \(2011\); NPA 881, 98 \(2012\)](#)
- Compositeness of $\Lambda(1405)$
[Y. Kamiya, T. Hyodo, PRC93, 035203 \(2016\); PTEP2017, 023D02 \(2017\)](#)

$\Lambda(1405)$ and $\bar{K}N$ scattering

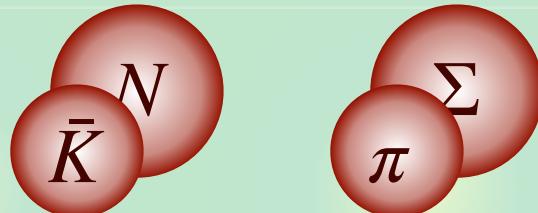
$\Lambda(1405)$ does not fit in standard picture \rightarrow exotic candidate

N. Isgur and G. Karl, Phys. Rev. D18, 4187 (1978)



Resonance in coupled-channel scattering

- Coupling to MB states



Detailed analysis of $\bar{K}N-\pi\Sigma$ scattering is necessary

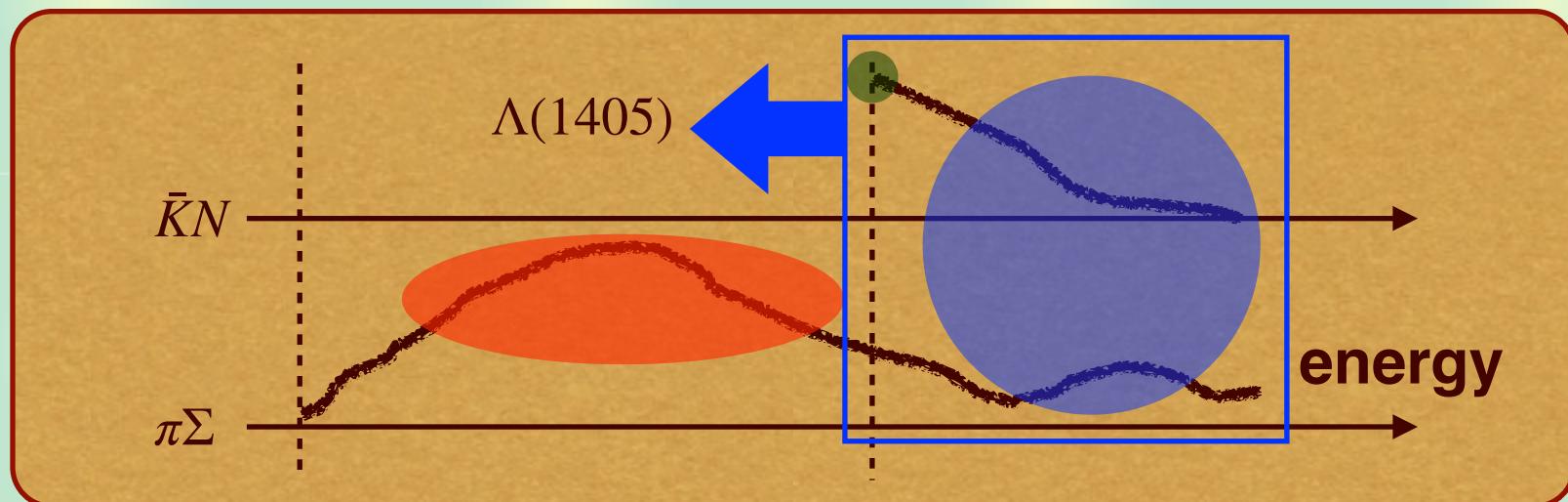
Strategy for $\bar{K}N$ interaction

Above the $\bar{K}N$ threshold : direct constraints

- $K^- p$ total cross sections (old data)
- $\bar{K}N$ threshold branching ratios (old data)
- $K^- p$ scattering length (new data : SIDDHARTA)

Below the $\bar{K}N$ threshold: indirect constraints

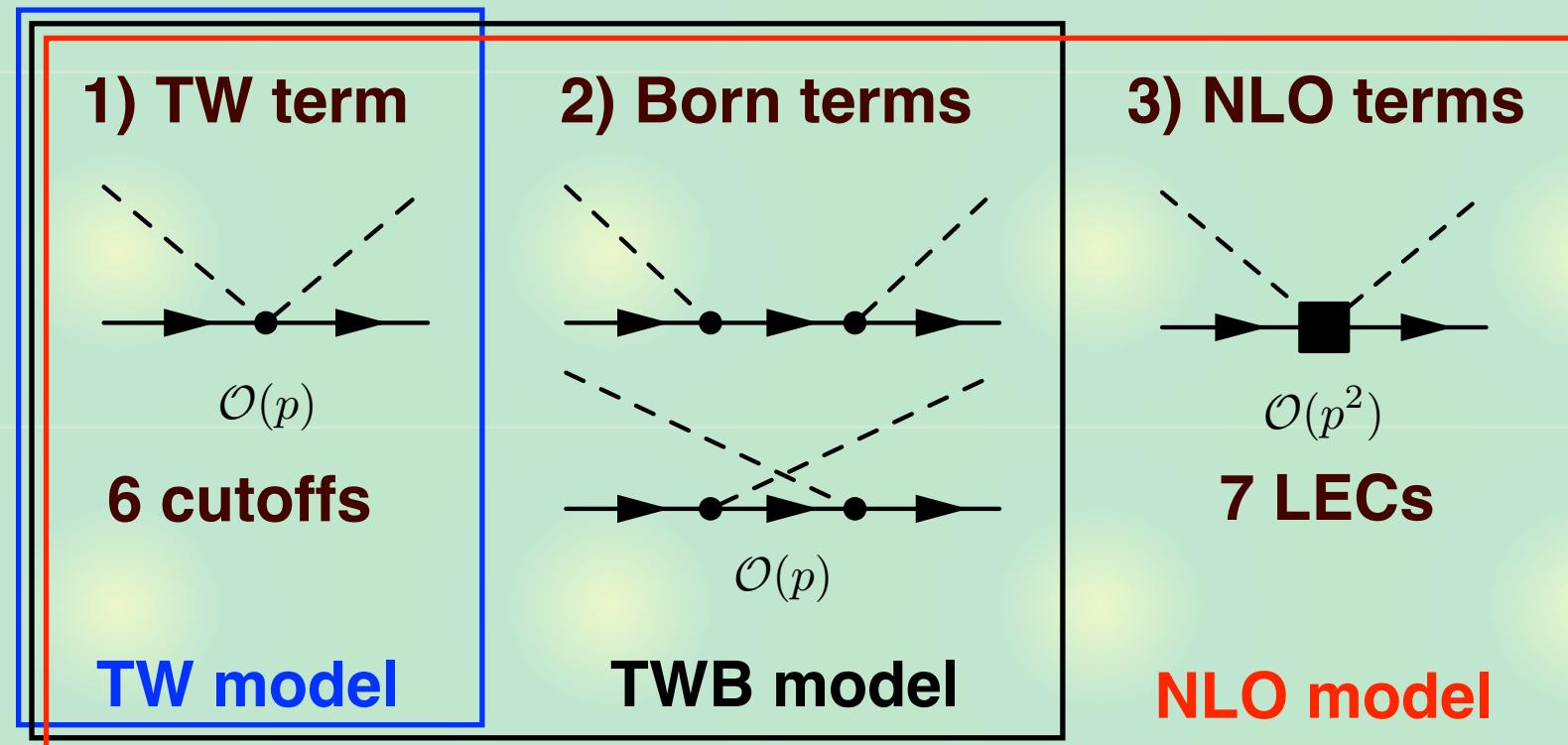
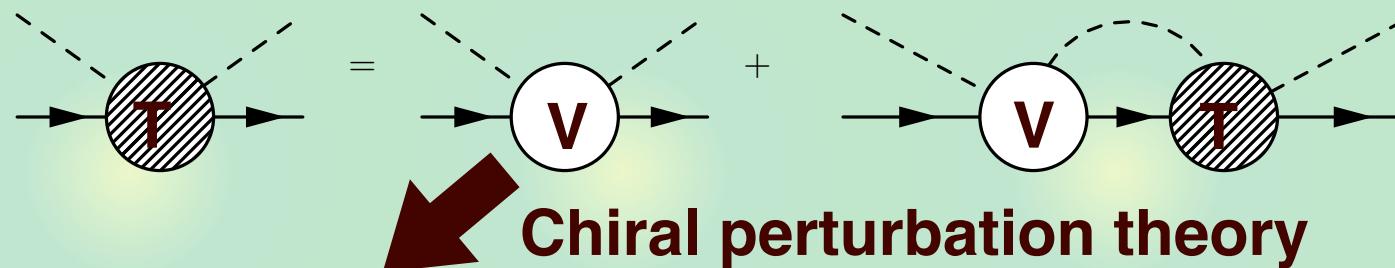
- $\pi\Sigma$ mass spectra (new data : LEPS, CLAS, HADES, ...)



Construction of the realistic amplitude

Chiral SU(3) coupled-channels ($\bar{K}N, \pi\Sigma, \pi\Lambda, \eta\Lambda, \eta\Sigma, K\Xi$) approach

Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011); NPA 881 98 (2012)

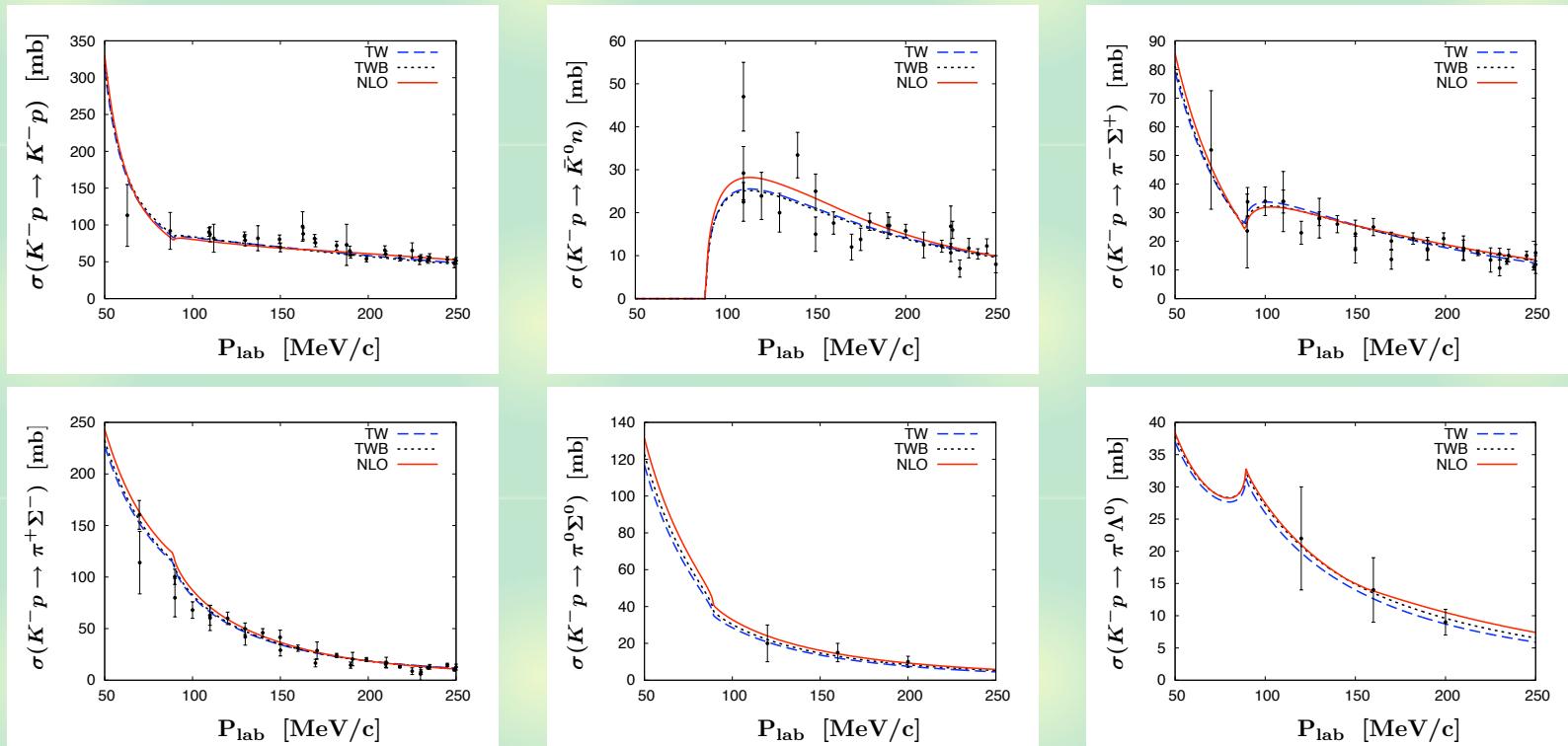


Best-fit results

 K at rest

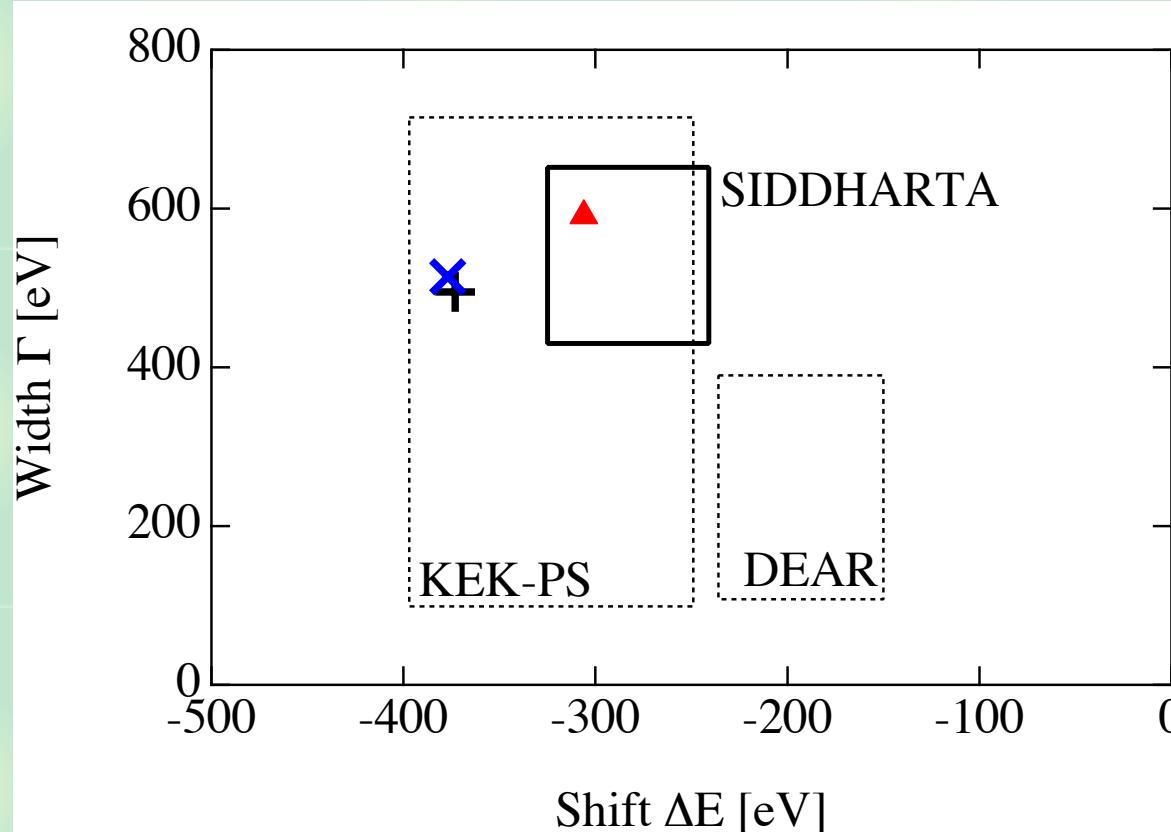
	TW	TWB	NLO	Experiment	
ΔE [eV]	373	377	306	$283 \pm 36 \pm 6$	[10]
Γ [eV]	495	514	591	$541 \pm 89 \pm 22$	[10]
γ	2.36	2.36	2.37	2.36 ± 0.04	[11]
R_n	0.20	0.19	0.19	0.189 ± 0.015	[11]
R_c	0.66	0.66	0.66	0.664 ± 0.011	[11]
$\chi^2/\text{d.o.f}$	1.12	1.15	0.96		

} SIDDHARTA
} Branching ratios

 $K^- p$ cross sectionsAccurate description of all existing data ($\chi^2/\text{d.o.f} \sim 1$)

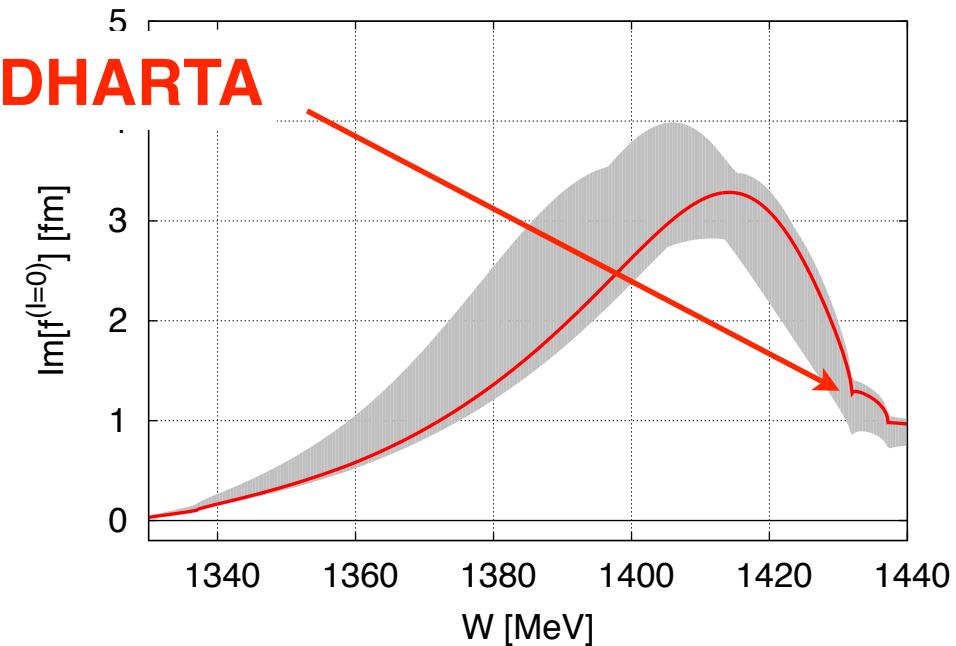
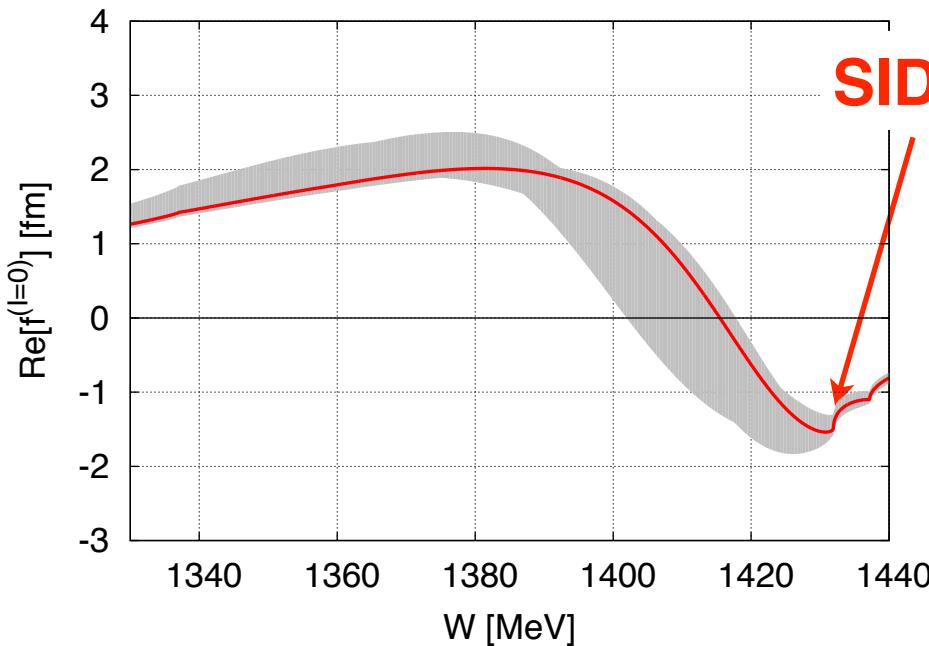
Comparison with SIDDHARTA

	TW	TWB	NLO
$\chi^2/\text{d.o.f.}$	1.12	1.15	0.957



TW and TWB are reasonable, while best-fit requires NLO

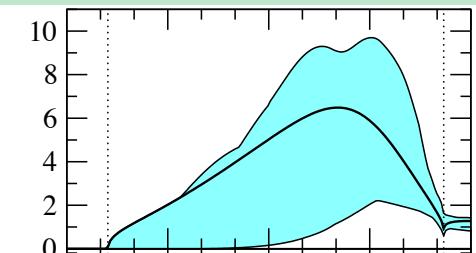
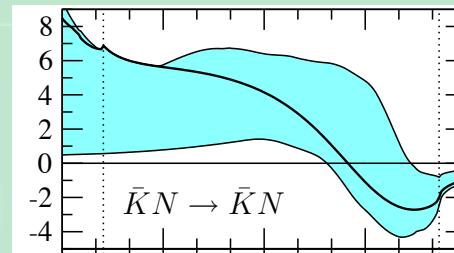
Subthreshold extrapolation

Uncertainty of $\bar{K}N \rightarrow \bar{K}N(I = 0)$ amplitude below threshold

Y. Kamiya, K. Miyahara, S. Ohnishi, Y. Ikeda, T. Hyodo, E. Oset, W. Weise,
NPA 954, 41 (2016)

- c.f. without SIDDHARTA

R. Nissler, Doctoral Thesis (2007)



SIDDHARTA is essential for subthreshold extrapolation

Extrapolation to complex energy: two poles

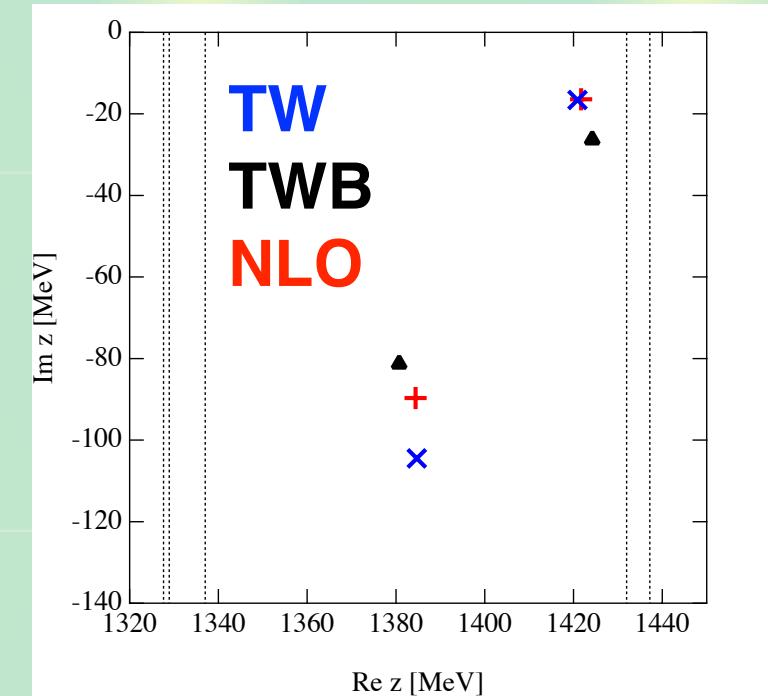
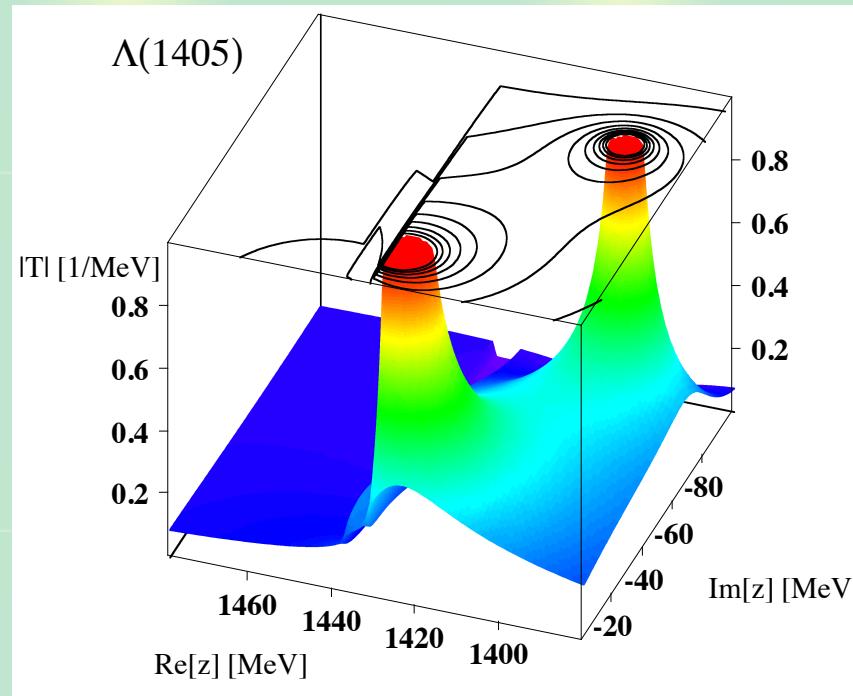
Two poles : superposition of two eigenstates

J.A. Oller, U.G. Meißner, PLB 500, 263 (2001);

D. Jido, J.A. Oller, E. Oset, A. Ramos, U.G. Meißner, NPA 723, 205 (2003);

U.G. Meißner, Symmetry 12, 981 (2020); M. Mai, arXiv: 2010.00056 [nucl-th];

T. Hyodo, M. Niiyama, arXiv: 2010.07592 [hep-ph]



T. Hyodo, D. Jido, Prog. Part. Nucl. Phys. 67, 55 (2012)

NLO analysis confirms the two-pole structure

PDG has changed

2020 update of PDG

Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011); NPA 881, 98 (2012); ▲

Z.H. Guo, J.A. Oller, PRC87, 035202 (2013); ×

M. Mai, U.G. Meißner, EPJA51, 30 (2015) ■ ○

- Particle Listing section:

Citation: P.A. Zyla *et al.* (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020)

$\Lambda(1405) \frac{1}{2}^-$

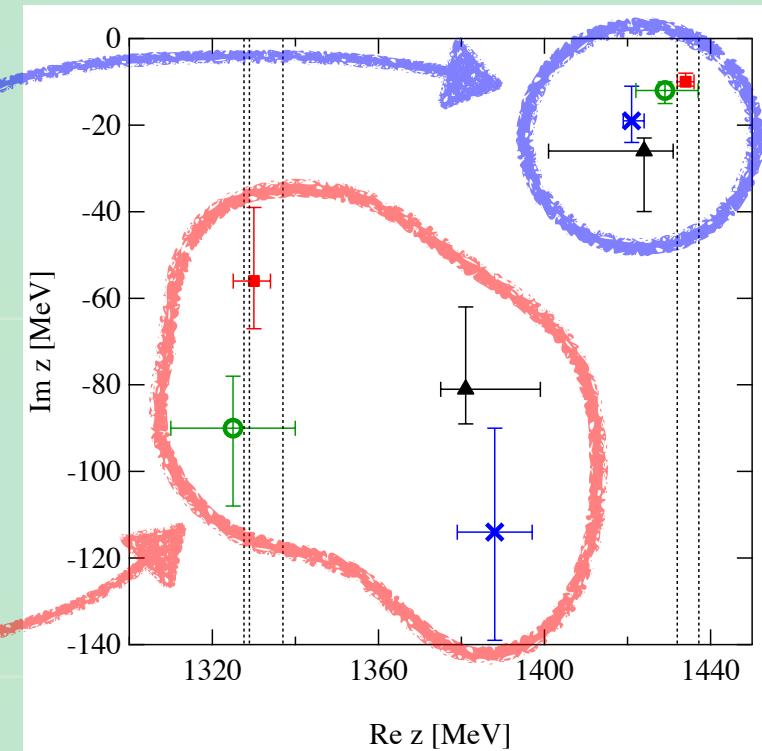
$I(J^P) = 0(\frac{1}{2}^-)$ Status: * * * *

Citation: P.A. Zyla *et al.* (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020)

$\Lambda(1380) \frac{1}{2}^-$

$J^P = \frac{1}{2}^-$ Status: * *

new!



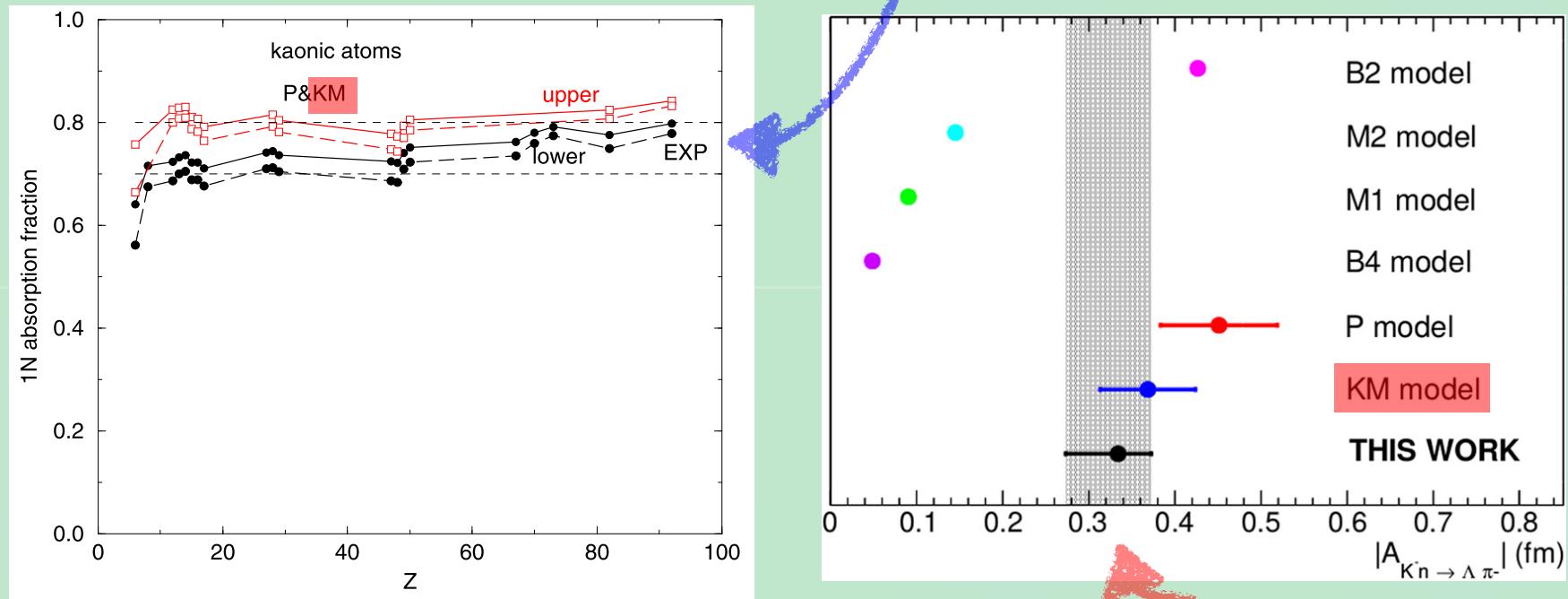
T. Hyodo, M. Niiyama, arXiv: 2010.07592 [hep-ph], to appear in PPNP

- “ $\Lambda(1405)$ ” is no longer at 1405 MeV but ~ 1420 MeV.
- Lower pole : two-star resonance $\Lambda(1380)$

Further check of amplitude

Single-nucleon absorption on kaonic atoms

E. Friedman, A. Gal, NPA959, 66 (2017)

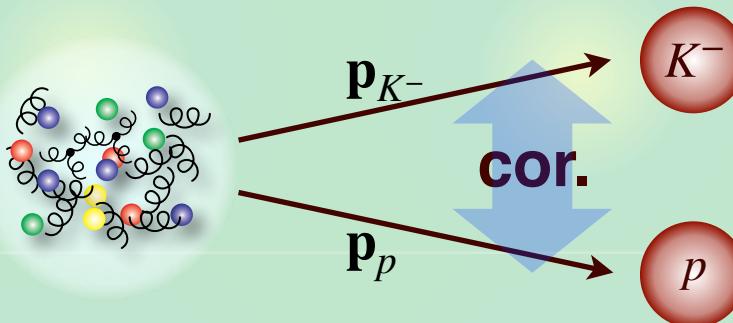
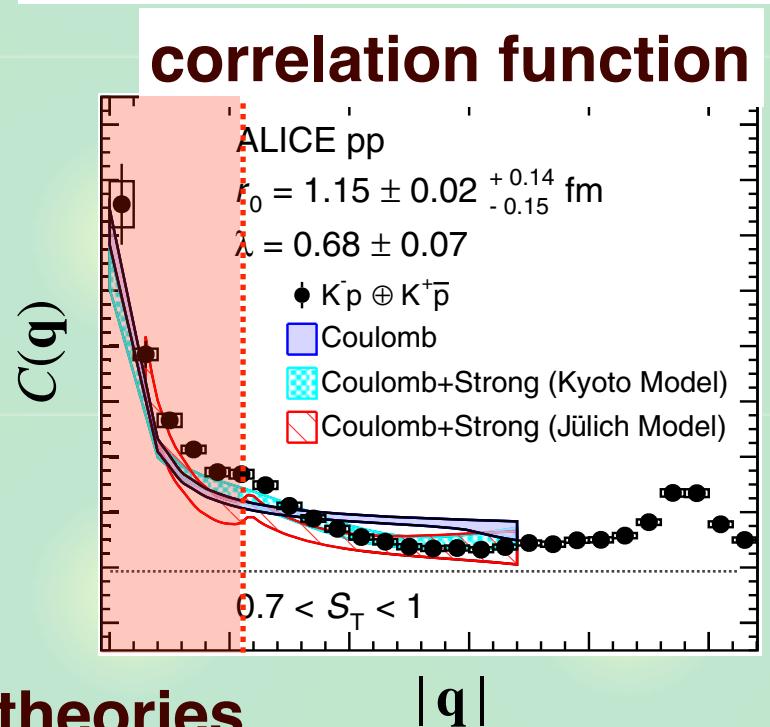
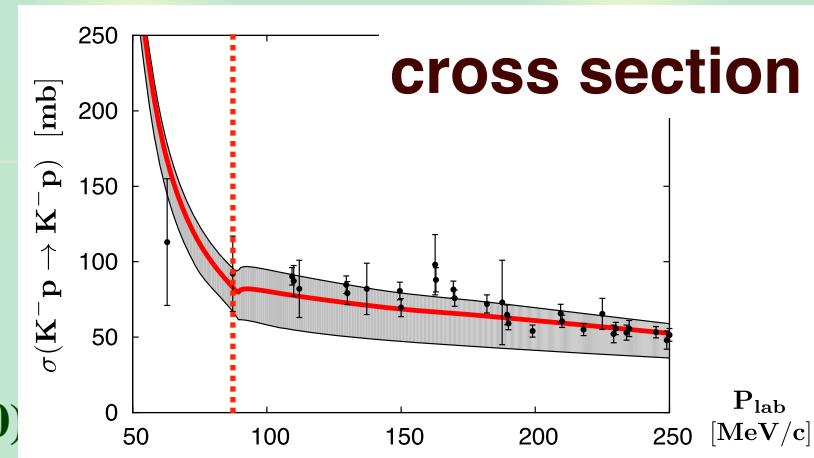
 $|f_{K^-n \rightarrow \pi^- \Lambda}|$ from K^- absorption on ${}^4\text{He}$ at DAΦNEK. Piscicchia, *et al.*, PLB782, 339 (2018)Our amplitude (**KM model**) is compatible with these analyses

New data : K^-p correlation function K^-p total cross sectionsY. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011)

- Old bubble chamber data

 K^-p correlation functionS. Acharya *et al.* (ALICE), PRL 124, 092301 (2020)

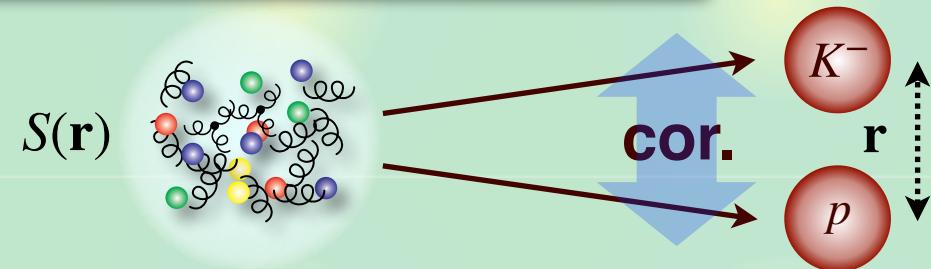
$$C(\mathbf{q}) = \frac{N_{K^-p}(\mathbf{p}_{K^-}, \mathbf{p}_p)}{N_{K^-}(\mathbf{p}_{K^-})N_p(\mathbf{p}_p)}$$

- Excellent precision ($\bar{K}^0 n$ cusp)- Low-energy data below $\bar{K}^0 n$ -> Important constraint on $\Lambda(1405)$ theories

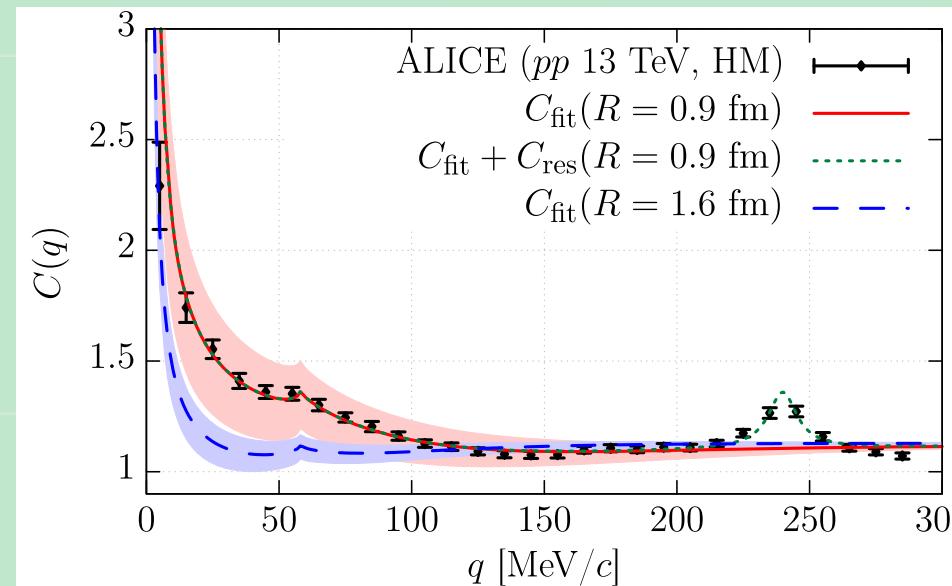
Prediction from chiral SU(3) dynamics

Theoretical calculation of $C(q)$

$$C(\mathbf{q}) \simeq \int d^3\mathbf{r} S(\mathbf{r}) |\Psi_{\mathbf{q}}^{(-)}(\mathbf{r})|^2$$



- Wave function $\Psi_{\mathbf{q}}^{(-)}(\mathbf{r})$: coupled-channel $\bar{K}N$ - $\pi\Sigma$ - $\pi\Lambda$ potential
- Source function $S(\mathbf{r})$: estimated by K^+p data

Y. Kamiya, T. Hyodo, K. Morita, A. Ohnishi, W. Weise. PRL124, 132501 (2020)

Correlation function is well reproduced

Contents



Part I : Introduction

- What are “exotic hadrons”?
- Hadronic molecules and universality
- Unstable resonances



Part II : Structure of $\Lambda(1405)$ resonance

- $\bar{K}N$ scattering and $\Lambda(1405)$ poles

Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011); NPA 881, 98 (2012)

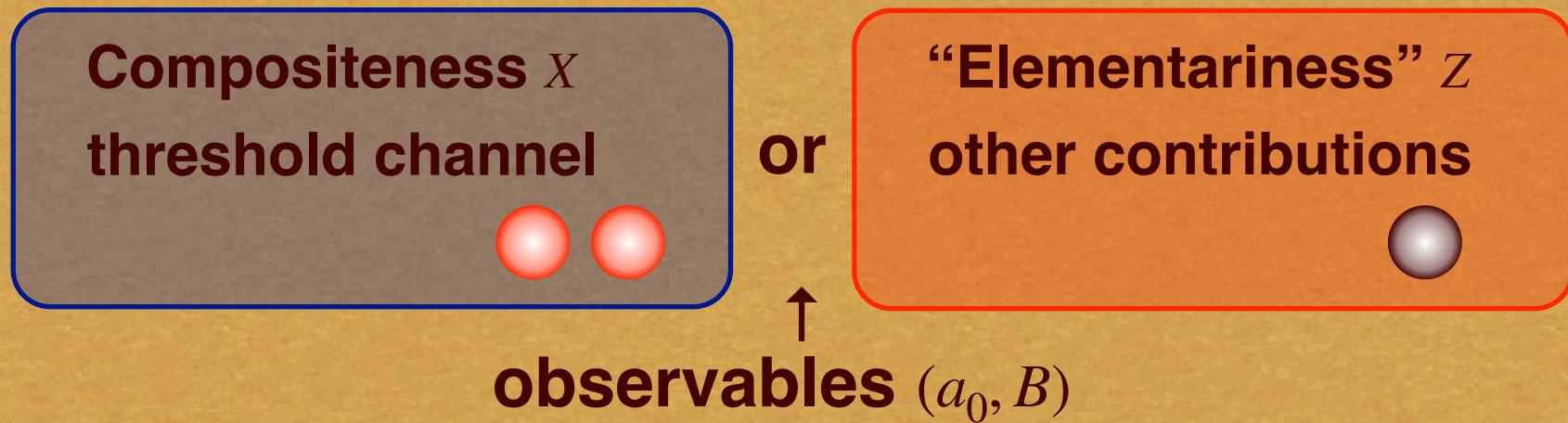
- Compositeness of $\Lambda(1405)$

Y. Kamiya, T. Hyodo, PRC93, 035203 (2016); PTEP2017, 023D02 (2017)

Compositeness of hadrons

- Structure of a given resonance (pole)?
- Weak binding relation for stable bound states

S. Weinberg, Phys. Rev. 137, B672 (1965)



- Effective field theory —> description of low-energy scattering amplitude, generalization to **unstable** resonances

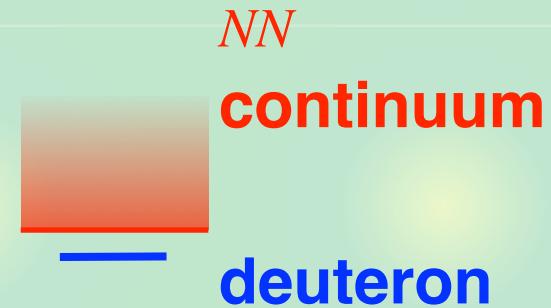
Weak-binding relation for stable states

Compositeness X of s-wave weakly bound state ($R \gg R_{\text{typ}}$)

S. Weinberg, Phys. Rev. 137, B672 (1965);

T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013)

$$|d\rangle = \sqrt{X} |NN\rangle + \sqrt{1-X} |\text{others}\rangle$$



$$a_0 = R \left\{ \frac{2X}{1+X} + \mathcal{O}\left(\frac{R_{\text{typ}}}{R}\right) \right\}, \quad R = \frac{1}{\sqrt{2\mu B}}$$

↓

↑ scattering length

↑ radius of state

- Deuteron is NN composite : $a_0 \sim R \Rightarrow X \sim 1$
- Internal structure from **observable** (a_0, B)

Problem: applicable only for stable states

Effective field theory

Low-energy scattering with near-threshold bound state

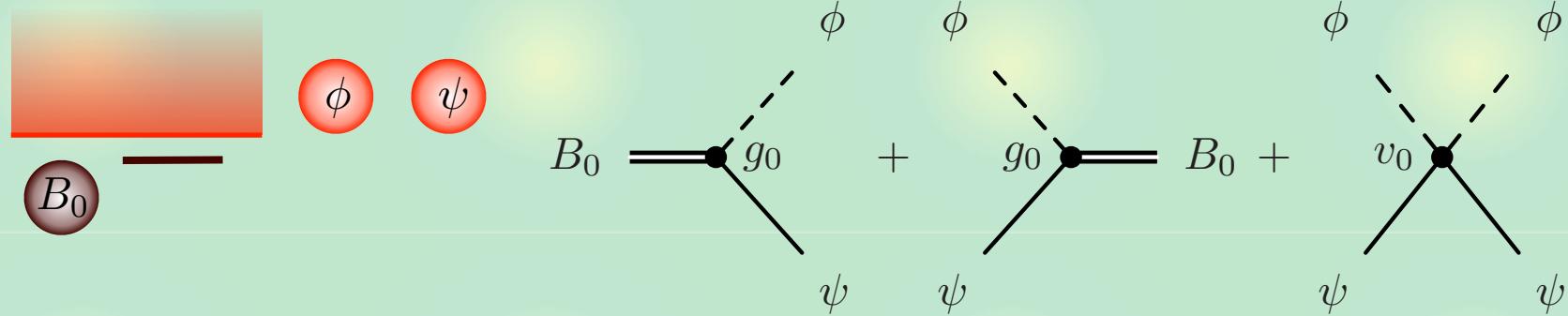
- **Nonrelativistic EFT with contact interaction**

D.B. Kaplan, Nucl. Phys. B494, 471 (1997)

E. Braaten, M. Kusunoki, D. Zhang, Annals Phys. 323, 1770 (2008)

$$H_{\text{free}} = \int d\mathbf{r} \left[\frac{1}{2M} \nabla \psi^\dagger \cdot \nabla \psi + \frac{1}{2m} \nabla \phi^\dagger \cdot \nabla \phi + \frac{1}{2M_0} \nabla B_0^\dagger \cdot \nabla B_0 + \omega_0 B_0^\dagger B_0 \right]$$

$$H_{\text{int}} = \int d\mathbf{r} \left[g_0 (B_0^\dagger \phi \psi + \psi^\dagger \phi^\dagger B_0) + v_0 \psi^\dagger \phi^\dagger \phi \psi \right]$$



- **Cutoff** : $\Lambda \sim 1/R_{\text{typ}}$ (**interaction range of microscopic theory**)
- **At low momentum** $p \ll \Lambda$, **interaction \sim contact**

Compositeness and “elementariness”

Eigenstates

$$H_{\text{free}} |B_0\rangle = \omega_0 |B_0\rangle, \quad H_{\text{free}} |\mathbf{p}\rangle = \frac{\mathbf{p}^2}{2\mu} |\mathbf{p}\rangle$$

free (discrete + continuum)

$$(H_{\text{free}} + H_{\text{int}}) |B\rangle = -B |B\rangle$$

full (bound state)

- Normalization of $|B\rangle$ + completeness relation

$$\langle B | B \rangle = 1, \quad 1 = |B_0\rangle\langle B_0| + \int \frac{d\mathbf{p}}{(2\pi)^3} |\mathbf{p}\rangle\langle\mathbf{p}|$$

- Projections onto free eigenstates

$$1 = Z + X, \quad Z \equiv |\langle B_0 | B \rangle|^2, \quad X \equiv \int \frac{d\mathbf{p}}{(2\pi)^3} |\langle \mathbf{p} | B \rangle|^2$$

“elementarity”



compositeness

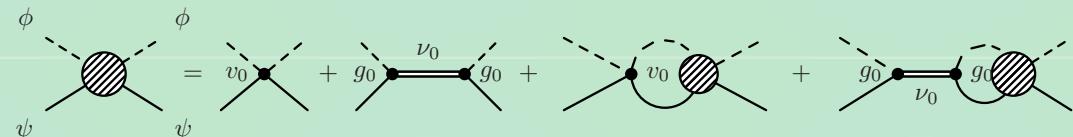


$Z, X : \text{real and nonnegative} \rightarrow \text{interpreted as probability}$

Weak binding relation

$\psi\phi$ scattering amplitude (exact result)

$$f(E) = -\frac{\mu}{2\pi} \frac{1}{[v(E)]^{-1} - G(E)}$$



$$v(E) = v_0 + \frac{g_0^2}{E - \omega_0}, \quad G(E) = \frac{1}{2\pi^2} \int_0^\Lambda dp \frac{p^2}{E - p^2/(2\mu) + i0^+}$$

Compositeness $X \leftarrow v(E), G(E)$

$$X = \frac{G'(-B)}{G'(-B) - [1/v(-B)]'}$$

$1/R = \sqrt{2\mu B}$ expansion of scattering length a_0

$$a_0 = -f(E=0) = R \underbrace{\left\{ \frac{2X}{1+X} + \overline{\mathcal{O}\left(\frac{R_{\text{typ}}}{R}\right)} \right\}}_{\text{renormalization independent}} \text{renormalization dependent}$$

If $R \gg R_{\text{typ}}$, correction terms neglected: $X \leftarrow (a_0, B)$

Inclusion of decay channel

Introduce decay channel

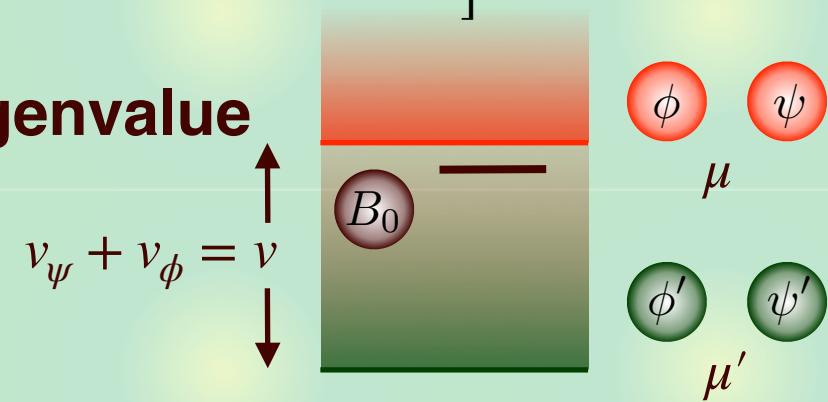
$$H'_{\text{free}} = \int d\mathbf{r} \left[\frac{1}{2M'} \nabla \psi'^{\dagger} \cdot \nabla \psi' - \nu_{\psi} \psi'^{\dagger} \psi' + \frac{1}{2m'} \nabla \phi'^{\dagger} \cdot \nabla \phi' - \nu_{\phi} \phi'^{\dagger} \phi' \right]$$

$$H'_{\text{int}} = \int d\mathbf{r} \left[g'_0 \left(B_0^{\dagger} \phi' \psi' + \psi'^{\dagger} \phi'^{\dagger} B_0 \right) + v'_0 \psi'^{\dagger} \phi'^{\dagger} \phi' \psi' + v'_0 (\psi'^{\dagger} \phi'^{\dagger} \phi' \psi' + \psi'^{\dagger} \phi'^{\dagger} \phi' \psi') \right]$$

Quasi-bound state : complex eigenvalue

$$H = H_{\text{free}} + H'_{\text{free}} + H_{\text{int}} + H'_{\text{int}}$$

$$H|h\rangle = E_h |h\rangle, \quad E_h \in \mathbb{C}$$



Generalized relation : correction from threshold difference

$$a_0 = R \left\{ \frac{2X}{1+X} + \mathcal{O}\left(\left|\frac{R_{\text{typ}}}{R}\right|\right) + \underline{\mathcal{O}\left(\left|\frac{\ell}{R}\right|^3\right)} \right\}, \quad R = \frac{1}{\sqrt{-2\mu E_h}}, \quad \ell \equiv \frac{1}{\sqrt{2\mu\nu}}$$

Y. Kamiya, T. Hyodo, PRC93, 035203 (2016); PTEP2017, 023D02 (2017)

If $|R| \gg (R_{\text{typ}}, \ell)$, correction terms neglected: $X \leftarrow (a_0, E_h)$

Complex compositeness

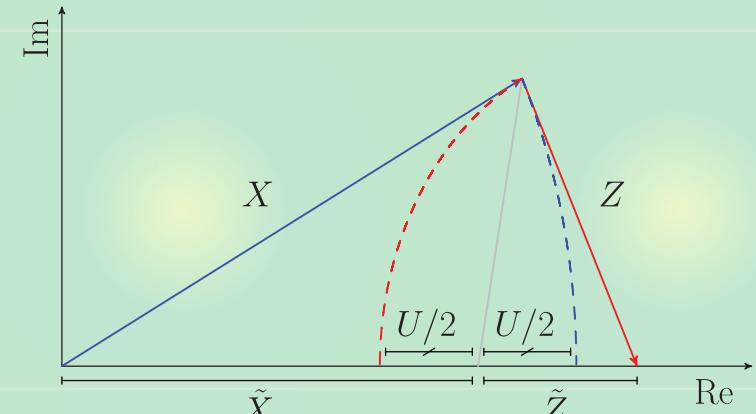
Unstable states \rightarrow complex Z and X

$$Z + X = 1, \quad Z, X \in \mathbb{C}$$

- Probabilistic interpretation?

New definition

$$\tilde{Z} = \frac{1 - |X| + |Z|}{2}, \quad \tilde{X} = \frac{1 - |Z| + |X|}{2}$$



- Interpreted as **probabilities** $\tilde{Z} + \tilde{X} = 1, \quad \tilde{Z}, \tilde{X} \in [0, 1]$
- reduces to Z and X in the bound state limit

$U/2$: uncertainty of interpretation

$$U = |Z| + |X| - 1$$

c.f. T. Berggren, Phys. Lett. 33B, 547 (1970)

- Sensible interpretation only for small $U/2$ case

Evaluation of compositeness

Generalized weak-binding relation

$$a_0 = R \left\{ \frac{2X}{1+X} + \mathcal{O}\left(\left|\frac{R_{\text{typ}}}{R}\right|\right) + \mathcal{O}\left(\left|\frac{\ell}{R}\right|^3\right) \right\}, \quad R = \frac{1}{\sqrt{-2\mu E_h}}, \quad \ell \equiv \frac{1}{\sqrt{2\mu\nu}}$$

(a_0, E_h) determinations by several groups

- Neglecting correction terms:

	E_h [MeV]	a_0 [fm]	$X_{\bar{K}N}$	$\tilde{X}_{\bar{K}N}$	$U/2$
Set 1 [35]	$-10 - i26$	$1.39 - i0.85$	$1.2 + i0.1$	1.0	0.3
Set 2 [36]	$-4 - i8$	$1.81 - i0.92$	$0.6 + i0.1$	0.6	0.0
Set 3 [37]	$-13 - i20$	$1.30 - i0.85$	$0.9 - i0.2$	0.9	0.1
Set 4 [38]	$2 - i10$	$1.21 - i1.47$	$0.6 + i0.0$	0.6	0.0
Set 5 [38]	$-3 - i12$	$1.52 - i1.85$	$1.0 + i0.5$	0.8	0.3

- In all cases, $X \sim 1$ with small $U/2$ (complex nature)

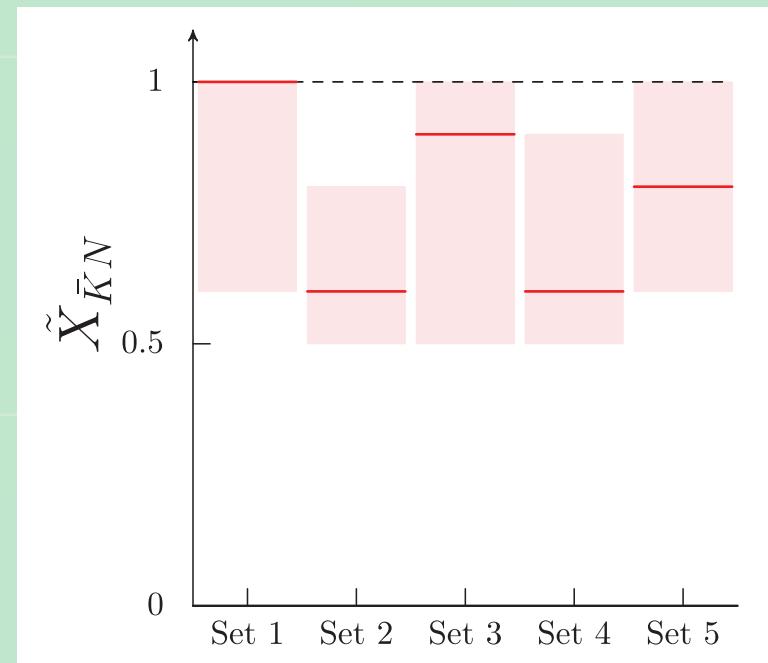
$\Lambda(1405)$: $\bar{K}N$ composite dominance \leftarrow observables

Uncertainty estimation

Estimation of correction terms: $|R| \sim 2$ fm

$$a_0 = R \left\{ \frac{2X}{1+X} + \mathcal{O}\left(\left|\frac{R_{\text{typ}}}{R}\right|\right) + \mathcal{O}\left(\left|\frac{\ell}{R}\right|^3\right) \right\}, \quad R = \frac{1}{\sqrt{-2\mu E_h}}, \quad \ell \equiv \frac{1}{\sqrt{2\mu\nu}}$$

- ρ meson exchange picture: $R_{\text{typ}} \sim 0.25$ fm
- Energy difference from $\pi\Sigma$: $\ell \sim 1.08$ fm



$\bar{K}N$ composite dominance holds even with correction terms.

Summary of part II



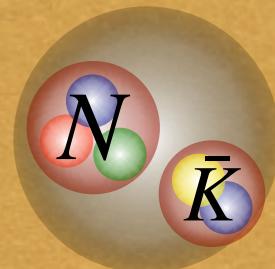
Pole structure of the $\Lambda(1405)$ region is now well constrained by the experimental data.

“ $\Lambda(1405)$ ” $\rightarrow \Lambda(1405)$ and $\Lambda(1380)$

Y. Ikeda, T. Hyodo, W. Weise, PLB 706, 63 (2011); NPA 881, 98 (2012)



Compositeness of hadrons can be studied by observables through the weak-binding relation. Generalized weak-binding relation shows that (higher-energy) $\Lambda(1405)$ is dominated by $\bar{K}N$ molecular component.



Y. Kamiya, T. Hyodo, PRC93, 035203 (2016); PTEP2017, 023D02 (2017)