Chiral Effective Theory of Diquark Clusters and Exotic Hadrons

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第６回クラスター階層領域研究会

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Recent works

M. Harada, Y.R. Liu, M.O., K. Suzuki, "*Chiral effective theory of diquarks and $U_A(1)$ anomaly*", Phys. Rev. D101, 054038 (2020)

Chiral effective Lagrangian for the scalar (0^+) and psudoscalar (0^-) diquarks

Y. Kim, E. Hiyama, M.O., K. Suzuki, "*Spectrum of singly heavy baryons from a chiral effective theory of diquarks*", Phys. Rev. D102, 014004 (2020)

Diquark-heavy-quark model of singly heavy baryons with scalar diquark

Y. Kawakami, M. Harada, M.O., K. Suzuki, "*Suppression of decay widths in singly heavy baryons induced by the $U_A(1)$ anomaly*", Phys. Rev. D102, 114004 (2020)

Goldberger-Treiman relation and decays of the P-wave excited state of Λ_c baryon

Y. Kim, Y.R. Liu, M.O., K. Suzuki, "*Heavy baryon spectrum with chiral multiplets of scalar and vector diquarks*", *ArXiv*. 2105.09087 (2021)

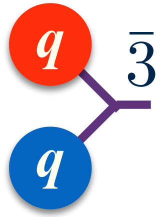
Chiral effective theory of axialvector (1^+) and vector (1^-) diquarks and its application to the spectrum of singly heavy baryons

Diquark

- **Diquark: the simplest *colorful cluster* in hadrons**

  "bound" qq state

  color $3 \otimes 3 = \bar{3} \oplus 6$ **spin :** $\frac{1}{2} \otimes \frac{1}{2} = 0 \oplus 1$

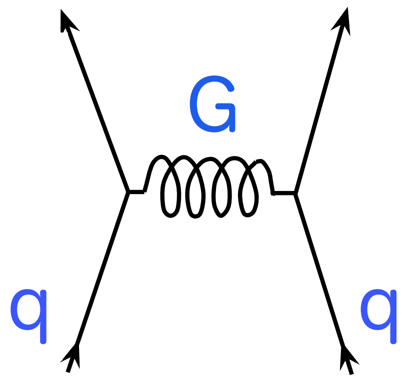


- **S-wave color 3bar diquarks: S(0^+) and A(1^+)**

- **Spin dependent force from magnetic gluon exchange predicts strong attraction in S(0^+).**

  Color-Magnetic Interaction

$$\Delta_{\rm CM} \equiv \langle - \sum_{i<j} (\vec{\lambda}_i \cdot \vec{\lambda}_j)(\vec{\sigma}_i \cdot \vec{\sigma}_j) \rangle$$

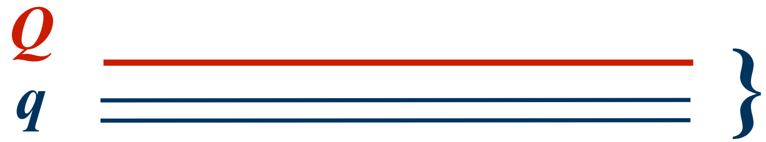


S(0^+) color 3bar $\Delta_{\rm CM} = -8$

A(1^+) color 3bar $\Delta_{\rm CM} = +8/3$

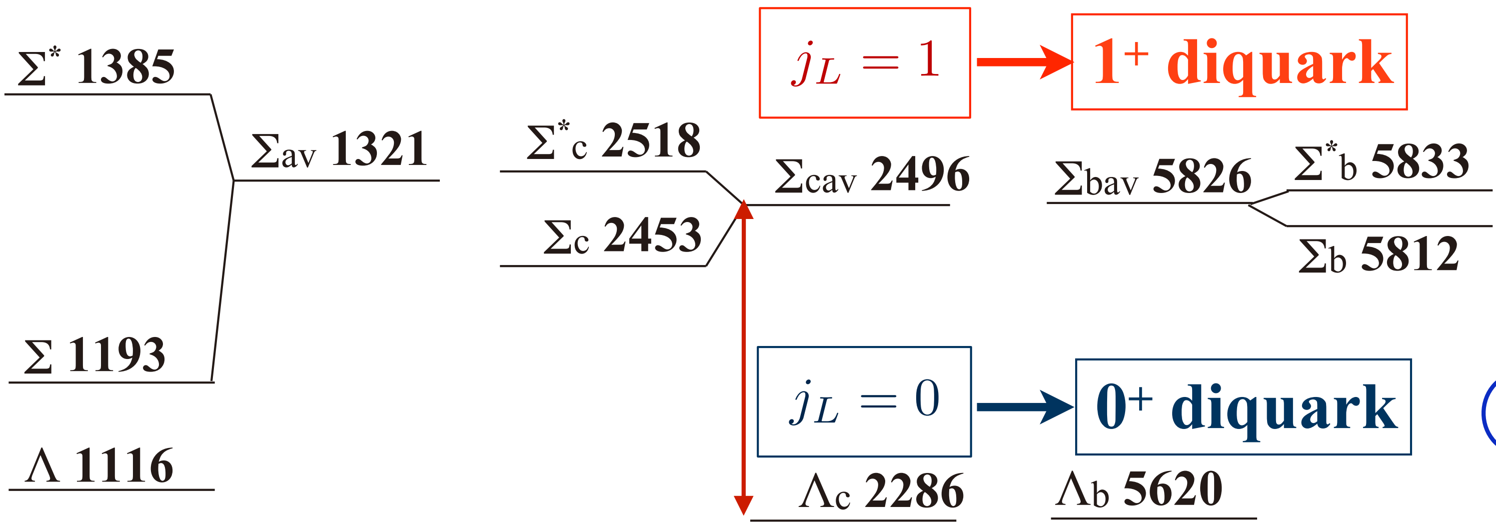
Diquark in Heavy Baryons

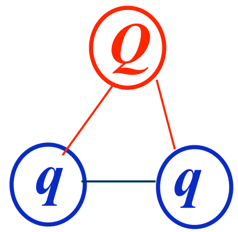
HQ spin symmetry $[S_Q, H] = O\left(\frac{1}{m_Q}\right)$



$\vec{J} = \vec{S}_Q + \vec{j}_L \qquad \vec{j}_L = \vec{S}_q + \vec{L}_q$

$J = j_L \pm \frac{1}{2}$ **states are degenerate in the HQ limit.**





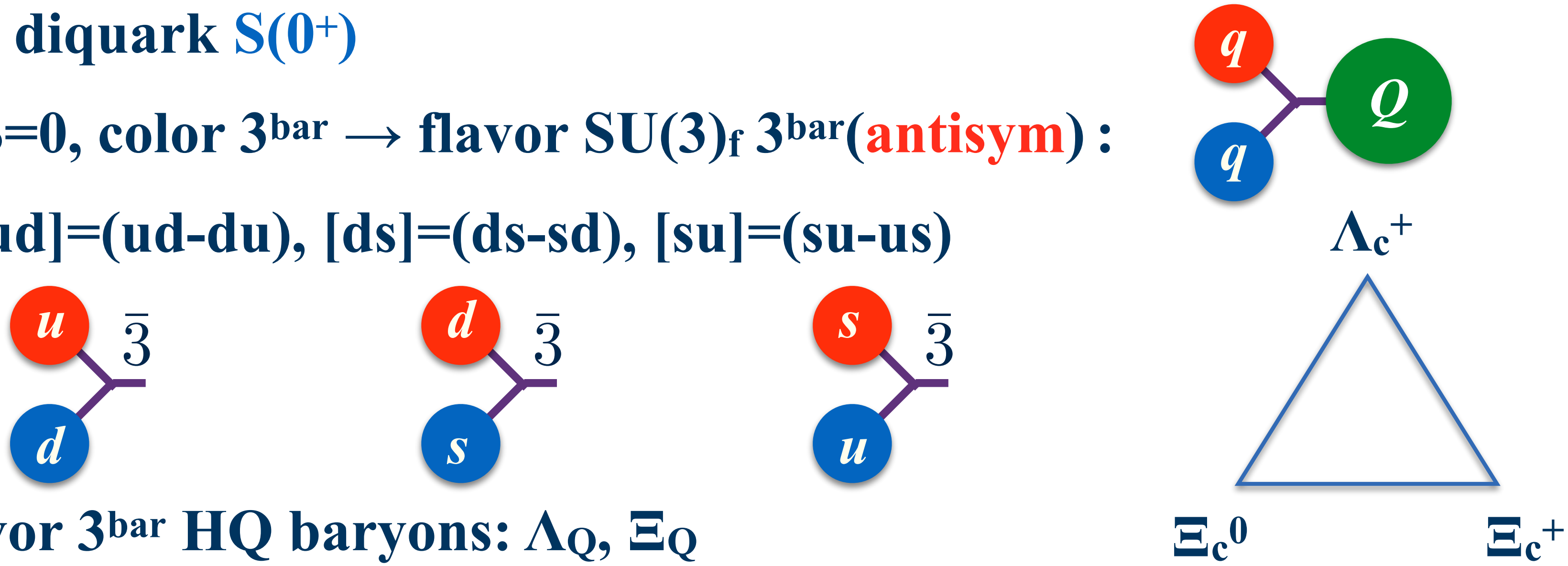
Spectroscopy of Light Diquarks

Diquark in Heavy Baryons

- Scalar diquark S(0^+)

  L=0, S=0, color $\bar{3}$ → flavor $SU(3)_f$ $\bar{3}$(antisym) :

  [ud]=(ud-du), [ds]=(ds-sd), [su]=(su-us)



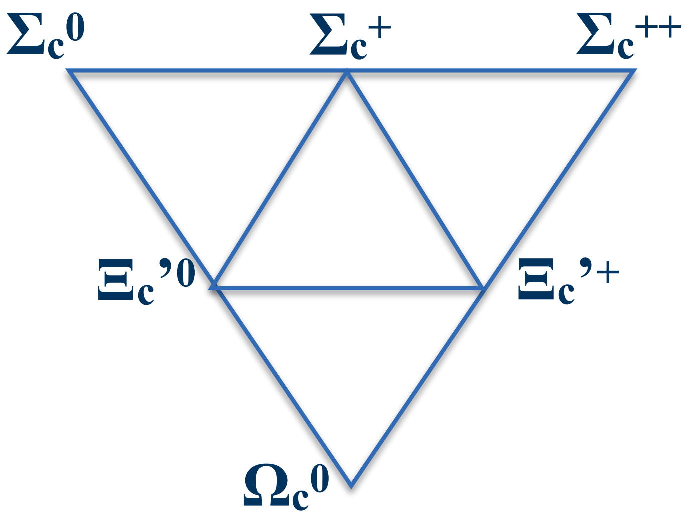
  ⇒ flavor $\bar{3}$ HQ baryons: Λ_Q, Ξ_Q

- Axial vector diquark A(1^+)

  L=0, S=1, color $\bar{3}$ → $SU(3)_f$ 6 (sym)

  uu, {ud}, dd, {us}, {ds}, ss

  ⇒ flavor 6 HQ baryons: Σ_Q, Ξ_Q', Ω_Q



Chiral Effective Theory

- **Chiral symmetry SU(3)$_R$ x SU(3)$_L$**

$q^a_{\alpha i}$ a (color), α (Dirac), i (flavor)

$$q^a_{iR} = P_R\, q^a_i, \quad q^a_{iL} = P_L\, q^a_i \qquad P_{R,L} \equiv \frac{1 \pm \gamma_5}{2}$$

$$q_R \to U_R\, q_R = (U_R)_{ij} q_{jR}, \quad U_R \in SU(3)_R$$

$$q_L \to U_L\, q_L = (U_L)_{ij} q_{jL}, \quad U_L \in SU(3)_L$$

- **Scalar chiral diquarks (color 3^{bar})**

$d^a_{iR} \equiv \epsilon_{ijk} (q^T_{jR}\, C\, q_{kR})^{\bar{3}}$ Right scalar diquark, chiral $(\bar{3}$,1), color $\bar{3}$

$d^a_{iL} \equiv \epsilon_{ijk} (q^T_{jL}\, C\, q_{kL})^{\bar{3}}$ Left scalar diquark, chiral (1, $\bar{3}$), color $\bar{3}$

- **Parity eigenstates: 0$^+$, 0$^-$ diquarks**

$$S^a_i = d^a_{iR} - d^a_{iL} = \epsilon_{ijk} (q^T_j\, C\gamma_5\, q_k)^{\bar{3}}$$

$$P^a_i = d^a_{iR} + d^a_{iL} = \epsilon_{ijk} (q^T_j\, C\, q_k)^{\bar{3}}$$

$$(\bar{3}, 1) + (1, \bar{3})$$

Chiral Effective Theory

M. Harada, Y.R. Liu, M.O., K. Suzuki, PR D101, 054038 (2020)

$$\mathcal{L} = \mathcal{D}_\mu d_{R,i} (\mathcal{D}^\mu d_{R,i})^\dagger + \mathcal{D}_\mu d_{L,i} (\mathcal{D}^\mu d_{L,i})^\dagger$$

$$-m_0^2 (d_{R,i} d_{R,i}^\dagger + d_{L,i} d_{L,i}^\dagger)$$

chiral invariant mass term

$$-\frac{m_1^2}{f} (d_{R,i} \Sigma_{ij}^\dagger d_{L,j}^\dagger + d_{L,i} \Sigma_{ij} d_{R,j}^\dagger)$$

$U_A(1)$ anomaly

CSB mass terms

$$-\frac{m_2^2}{2f^2} \epsilon_{ijk} \epsilon_{\ell mn} (d_{R,k} \Sigma_{\ell i} \Sigma_{mj} d_{L,n}^\dagger + d_{L,k} \Sigma_{\ell i}^\dagger \Sigma_{mj}^\dagger d_{R,n}^\dagger)$$

$$+\frac{1}{4} \mathrm{Tr} \left[\partial^\mu \Sigma^\dagger \partial_\mu \Sigma \right] + V(\Sigma).$$

$$\Sigma_{ij} \equiv \sigma_{ij} + i\pi_{ij}$$

- For the SSB vacuum $\langle \Sigma \rangle = f$, the mass term of the right and left diquarks are given by

$$M^2 = \begin{pmatrix} m_0^2 & m_1^2 + m_2^2 \\ m_1^2 + m_2^2 & m_0^2 \end{pmatrix} \quad \text{for } \langle \Sigma_{ij} \rangle = f \delta_{ij}$$

Chiral Effective Theory

- **The mass eigenstates are given by**

$$M^2 = \begin{pmatrix} m_0^2 & m_1^2 + m_2^2 \\ m_1^2 + m_2^2 & m_0^2 \end{pmatrix}$$

Scalar diquark

$$S_i^a = \frac{1}{\sqrt{2}}(d^a_{R,i} - d^a_{L,i})$$

$$\longrightarrow M(0^+) = \sqrt{m_0^2 - m_1^2 - m_2^2},$$

Pseudo-scalar diquark

$$P_i^a = \frac{1}{\sqrt{2}}(d^a_{R,i} + d^a_{L,i})$$

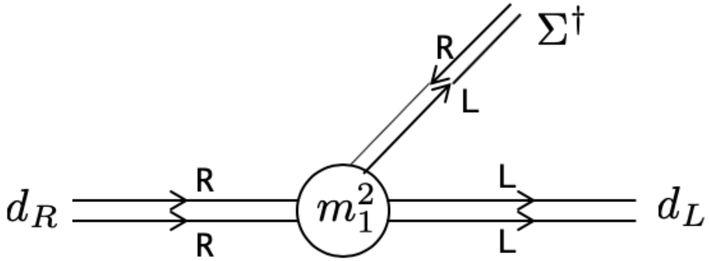
$$\longrightarrow M(0^-) = \sqrt{m_0^2 + m_1^2 + m_2^2},$$

$U_A(1)$ anomaly

- **$U_A(1)$ anomaly in the diquark effective theory**

$$-\frac{m_1^2}{f}(\underline{d_{R,i}\Sigma^\dagger_{ij}d^\dagger_{L,j}} + d_{L,i}\Sigma_{ij}d^\dagger_{R,j})$$

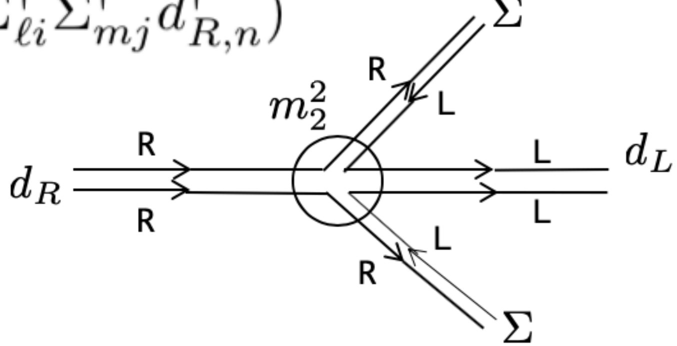
3 left quarks and 3 right antiquarks
flavor antisymmetric
induces anomalous singlet current



$$\partial_\mu J_A^{\mu 0} = \frac{3m_1^2}{2}(S\lambda_0 P^\dagger - P\lambda_0 S^\dagger)$$

- **non-anomalous term**

$$-\frac{m_2^2}{2f^2}\epsilon_{ijk}\epsilon_{\ell mn}(d_{R,k}\Sigma_{\ell i}\Sigma_{mj}d^\dagger_{L,n} + d_{L,k}\Sigma^\dagger_{\ell i}\Sigma^\dagger_{mj}d^\dagger_{R,n})$$



Chiral/Flavor symmetry breaking

- **Masses of diquarks**

$$A \equiv \frac{f_s}{f_\pi}\left(1+\frac{m_s}{g_s f_s}\right) \sim \frac{5}{3}$$

***i*=3 (ud)**

$$M_3(0^+) = \sqrt{m_0^2 - A m_1^2 - m_2^2}, \qquad M_3(0^-) = \sqrt{m_0^2 + A m_1^2 + m_2^2}.$$

***i*=1,2 (ds), (us)**

$$M_1(0^+) = M_2(0^+) = \sqrt{m_0^2 - m_1^2 - A m_2^2}, \qquad M_1(0^-) = M_2(0^-) = \sqrt{m_0^2 + m_1^2 + A m_2^2},$$

- **Inverse mass hierarchy**

$$[M_{1,2}(0^+)]^2 - [M_3(0^+)]^2 = [M_3(0^-)]^2 - [M_{1,2}(0^-)] = (A-1)(m_1^2 - m_2^2). > 0$$

(ds), (us) (ud) (ud) (ds), (us)

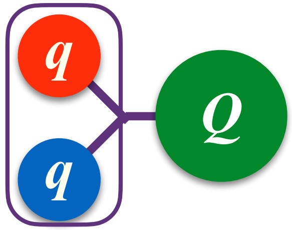
$U_A(1)$ anomaly mass

$$M_1(0^-) < M_3(0^-)$$

(ds), (us) (ud)

M. Harada, Y.R. Liu, M.O., K. Suzuki, PR D101, 054038 (2020)

Diquark-Heavy-Quark model



- Heavy baryon system as a bound state of Q + d
- Input parameters

  From the masses of the ground state baryons

  * $M(\Lambda_c, 1/2^+) = 2286.46$ MeV,
  * $M(\Xi_c, 1/2^+) = \frac{1}{2}(M(\Xi_c^+) + M(\Xi_c^0)) = 2469.42$ MeV.

  0^+ diquark (ud) mass from lattice QCD * $M_{(ud)}(0^+) = 725$ MeV

  0^- diquark (ud) mass from lattice * $M_{(ds)}(0^-) = 1265$ MeV

- A linear + Coulomb potential between Q and diquark

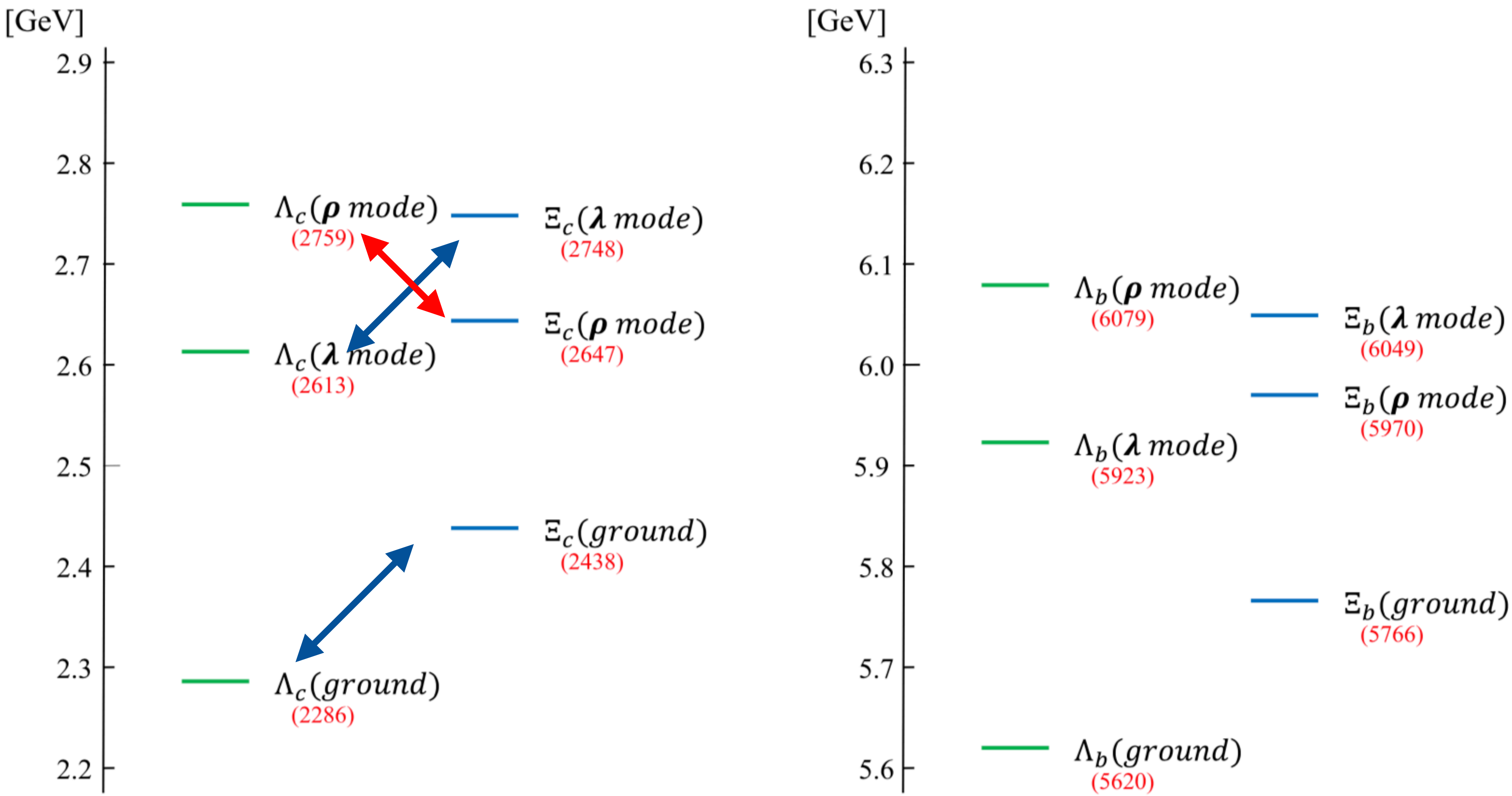
$$V(r) = -\frac{\alpha}{r} + \lambda r + C$$

α	$\lambda(\mathrm{GeV}^2)$	$C_c(\mathrm{GeV})$	$C_b(\mathrm{GeV})$	$M_c(\mathrm{GeV})$	$M_b(\mathrm{GeV})$
$(2/3)\times 90/\mu$	0.165	−0.58418362	−0.58829590	1.750	5.112

T. Yoshida, E. Hiyama, A. Hosaka, M. Oka, K. Sadato, PR D 92, 114029 (2015)

Inverse mass hierarchy

Y. Kim, E. Hiyama, M. Oka, K. Suzuki, *PRD 102, 014004 (2020)*



Axialvector/Vector Diquarks

Y. Kim, Y.R.Liu, M.O., K. Suzuki, ArXiv. 2105.09087

- **The $1^+/1^-$ diquarks in (3,3) representation**

$$d_{ij}^{\mu a} \equiv \epsilon_{abc}(q_{iL}^{bT}\, C\gamma^\mu\, q_{jR}^c) = \epsilon_{abc}(q_{jR}^{bT}\, C\gamma^\mu\, q_{iL}^c) \quad \text{chiral (3,3) vector diquark}$$

$$d_{V[ij]}^{\mu a} = d_{ij}^{\mu a} - d_{ji}^{\mu a} = \epsilon_{abc}(q_i^{bT}\, C\gamma^\mu\gamma^5\, q_j^c) \quad \text{Vector } 1^- \text{ diquark, flavor } \bar{3}$$

$$d_{A\{ij\}}^{\mu a} = d_{ij}^{\mu a} + d_{ji}^{\mu a} = \epsilon_{abc}(q_i^{bT}\, C\gamma^\mu\, q_j^c) \quad \text{Axial-vector } 1^+ \text{ diquark, flavor } 6$$

$$d^\mu \longrightarrow U_L d^\mu U_R^T, \quad (3,3) \qquad d^{\mu\dagger} \longrightarrow U_R^{T\dagger} d^\mu U_L^\dagger \quad (\bar{3},\bar{3})$$

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}[F^{\mu\nu}F_{\mu\nu}^\dagger] + m_0^2\mathrm{Tr}[d^\mu d_\mu^\dagger] + \frac{m_1^2}{f_\pi^2}\mathrm{Tr}[\Sigma^\dagger d^\mu \Sigma^T d_\mu^{\dagger T}] + \frac{2m_2^2}{f_\pi^2}\mathrm{Tr}[\Sigma^\dagger \Sigma d^{\mu T} d_\mu^{\dagger T}]$$

$$F^{\mu\nu} = D^\mu d^\nu - D^\nu d^\mu$$

- **All the terms are chiral and $U_A(1)$ invariant.**

Axialvector/Vector Diquarks

- **Using the masses of the 6-irrep single charm baryons, we determine the diquark masses.**

Y. Kim, Y.R. Liu, MO, K. Suzuki, *ArXiv. 2105.09087*

$$[M_{qq}(1^+)]^2 = m_{V0}^2 + m_{V1}^2 + 2m_{V2}^2,$$
$$[M_{qs}(1^+)]^2 = m_{V0}^2 + m_{V1}^2 + 2m_{V2}^2 + \epsilon(m_{V1}^2 + 2m_{V2}^2)$$
$$[M_{ss}(1^+)]^2 = m_{V0}^2 + m_{V1}^2 + 2m_{V2}^2 + 2\epsilon(m_{V1}^2 + 2m_{V2}^2)$$
$$[M_{qq}(1^-)]^2 = m_{V0}^2 - m_{V1}^2 + 2m_{V2}^2,$$
$$[M_{qs}(1^-)]^2 = m_{V0}^2 - m_{V1}^2 + 2m_{V2}^2 + \epsilon(-m_{V1}^2 + 2m_{V2}^2)$$
$$\epsilon = A - 1 = \frac{f_s}{f_\pi}\left(1 + \frac{m_s}{g_s f_s}\right) - 1 \sim \frac{2}{3}$$

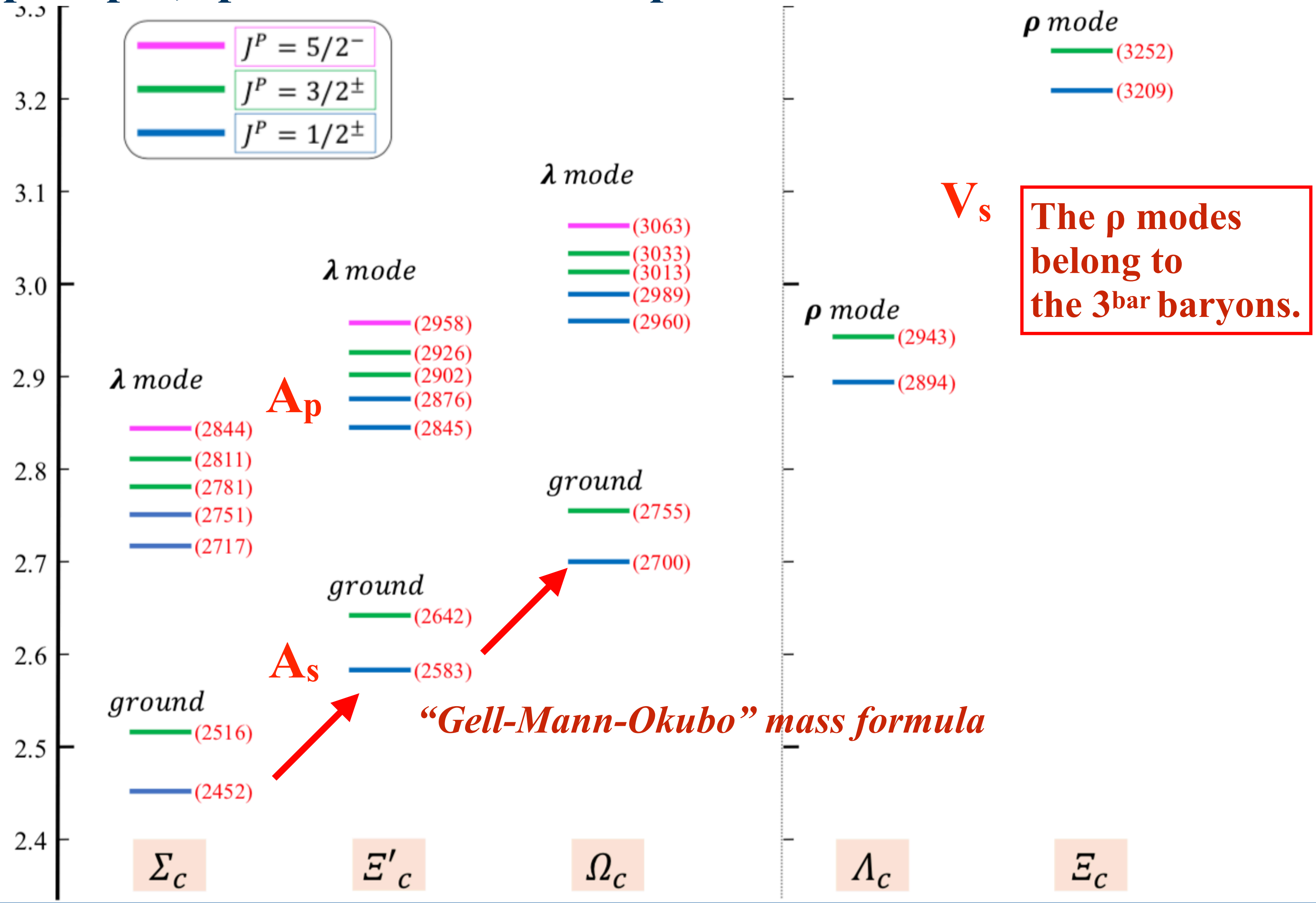
$M_{qq}(1^+)$ (MeV)	973.41
$M_{qs}(1^+)$ (MeV)	1115.98
$M_{ss}(1^+)$ (MeV)	1242.29
$M_{qq}(1^-)$ (MeV)	1446.72
$M_{qs}(1^-)$ (MeV)	1776.10
m_0^2 (MeV^2)	$(707.60)^2$
m_1^2 (MeV^2)	$-(756.79)^2$
m_2^2 (MeV^2)	$(713.99)^2$

- **The diquark masses satisfy the generalized "Gell-Mann-Okubo" mass formula approximately.**

$$M_{ss}^2(1^+) - M_{qs}^2(1^+) = M_{qs}^2(1^+) - M_{qq}^2(1^+)$$

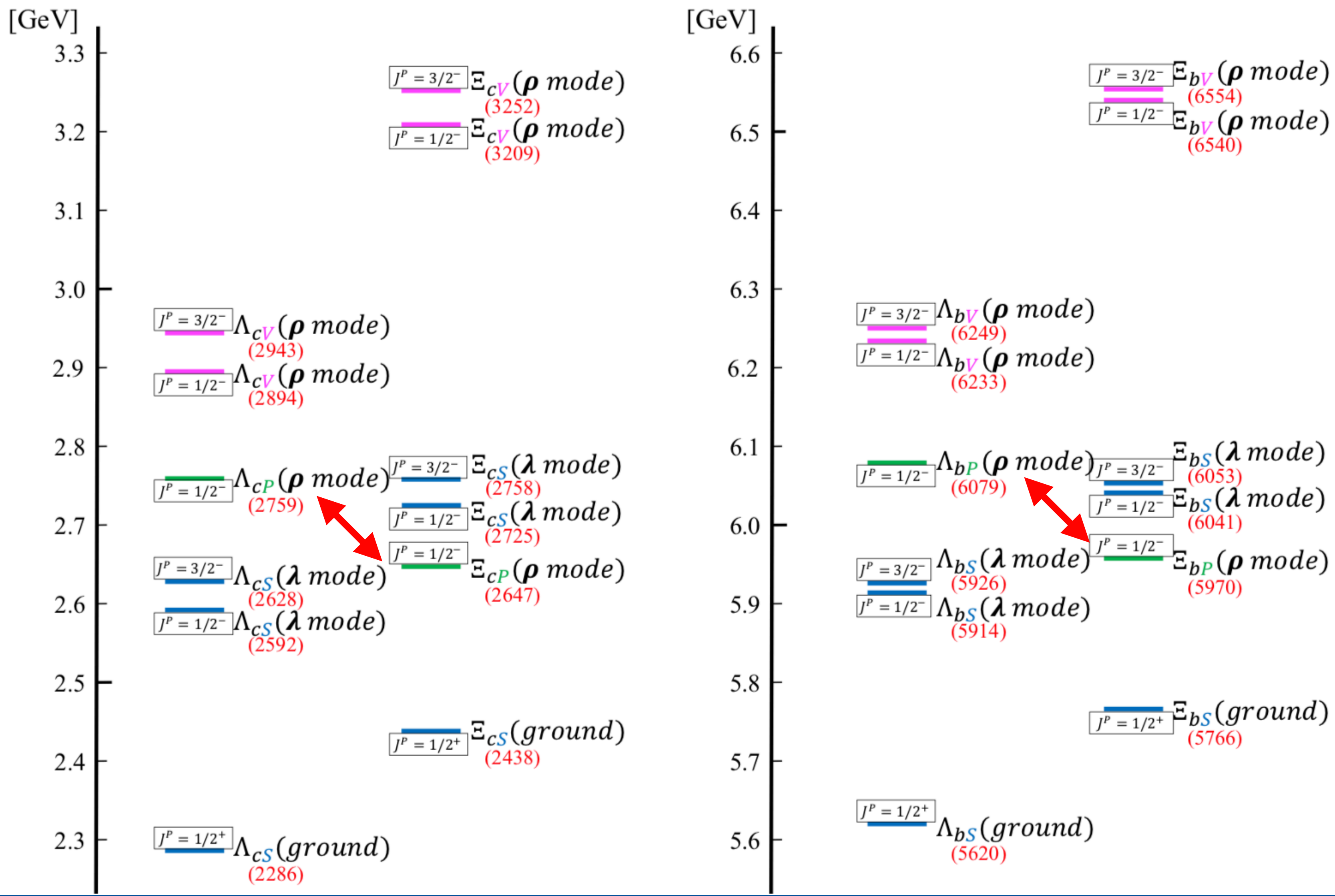
Spectrum of Σ_c, Ξ_c', and Ω_c

- **The spin-spin, spin-orbit and tensor potentials are introduced.**



Full Spectrum of Λ_Q and Ξ_Q

- **The spectrum of Λ_Q and Ξ_Q with S-P and A-V diquarks**



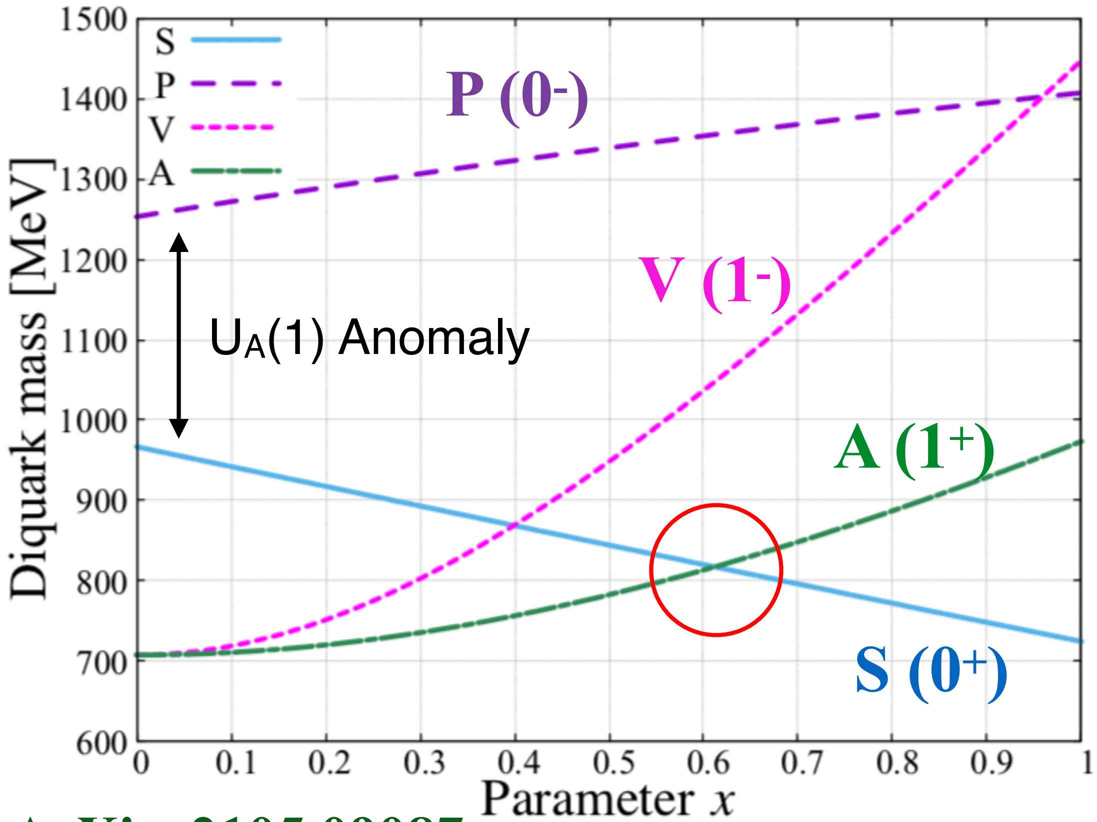
Chiral symmetry vs Diquarks

- **Masses of the 0+ and 1+ diquarks under chiral restoration**

$$M(0^+) = \sqrt{m_{S0}^2 - (x+\epsilon)m_{S1}^2 - x^2 m_{S2}^2},$$
$$M(1^+) = \sqrt{m_{V0}^2 + x^2(m_{V1}^2 + 2m_{V2}^2)}.$$

$$m_{S0}^2 = (1031\ \text{MeV})^2$$
$$m_{S1}^2 = (606\ \text{MeV})^2$$
$$m_{S2}^2 = -(274\ \text{MeV})^2$$

$$m_{V0}^2 = (708\ \text{MeV})^2$$
$$m_{V1}^2 = -(760\ \text{MeV})^2$$
$$m_{V2}^2 = (714\ \text{MeV})^2$$



Y. Kim, Y.R.Liu, M.O., K. Suzuki, ArXiv. 2105.09087

Goldberger-Treiman relation

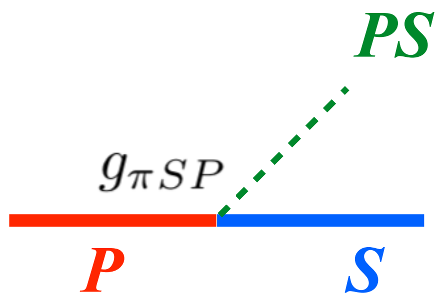
- π_p **(pseudo-scalar nonet) diquark couplings**

$$\mathcal{L}_{\pi SP} = \frac{i(m_1^2+m_2^2)}{f}\pi_p(S\lambda_p P^\dagger - P\lambda_p S^\dagger) \qquad \textit{octet + singlet}$$

$$-\frac{3im_2^2}{f}\pi_0(S\lambda_0 P^\dagger - P\lambda_0 S^\dagger) \qquad \textit{singlet}$$

- **SU(3) "Goldberger-Treiman" relation** ***for octet mesons***

$$g_{\pi SP} \equiv \frac{m_1^2+m_2^2}{f} = \frac{M^2(0^-)-M^2(0^+)}{2f}$$



- **For chiral partner of heavy baryons**

$$g = \frac{\Delta M}{2f} = \frac{M(1/2^-)-M(1/2^+)}{2f}$$

***B*(1/2⁻)** ***B*(1/2⁺)**

$$\Lambda_c^* \to \Lambda_c + \eta_8$$
$$\Xi_c^* \to \Lambda_c + \bar{K}$$

Goldberger-Treiman relation

- **Chiral effective theory for flavor 3^bar Baryons**

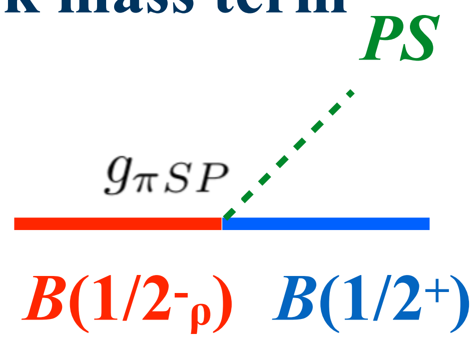
  Y. Kawakami, M. Harada, *PR D97 (2018) 114024, PR D99 (2019) 094016*

$$\mathcal{L} = \bar{S}_{R,i}(iv^\mu\partial_\mu)S_{R,i} + \bar{S}_{L,i}(iv^\mu\partial_\mu)S_{L,i} - M_{B0}\left(\bar{S}_{R,i}S_{R,i} + \bar{S}_{L,i}S_{L,i}\right)$$
$$-\frac{M_{B1}}{f}\left(\bar{S}_{R,i}\Sigma^T_{ij}S_{L,j} + \bar{S}_{L,i}\Sigma^{T\dagger}_{ij}S_{R,j}\right)$$
$$-\frac{M_{B2}}{2f^2}\epsilon_{ijk}\epsilon_{lmn}\left(\bar{S}_{L,k}\Sigma^T_{li}\Sigma^T_{mj}S_{R,n} + \bar{S}_{R,k}\Sigma^{T\dagger}_{li}\Sigma^{T\dagger}_{mj}S_{L,n}\right)$$

$U_A(1)$ **anomaly term**

- **SU(3) symmetry breaking thru the quark mass term**

$$\langle\Sigma\rangle \to \langle\tilde{\Sigma}\rangle = \langle\Sigma\rangle + \mathrm{diag}\{1,1,A\}$$
$$M_q^\pm = M_{B0} \mp (M_{B1} + AM_{B2})$$
$$M_s^\pm = M_{B0} \mp (AM_{B1} + M_{B2})$$
$$g = \frac{M_{B1}+M_{B2}}{f} = \frac{\Delta M_q + \Delta M_s}{2f(A+1)} < \frac{\Delta M_s}{2f}$$



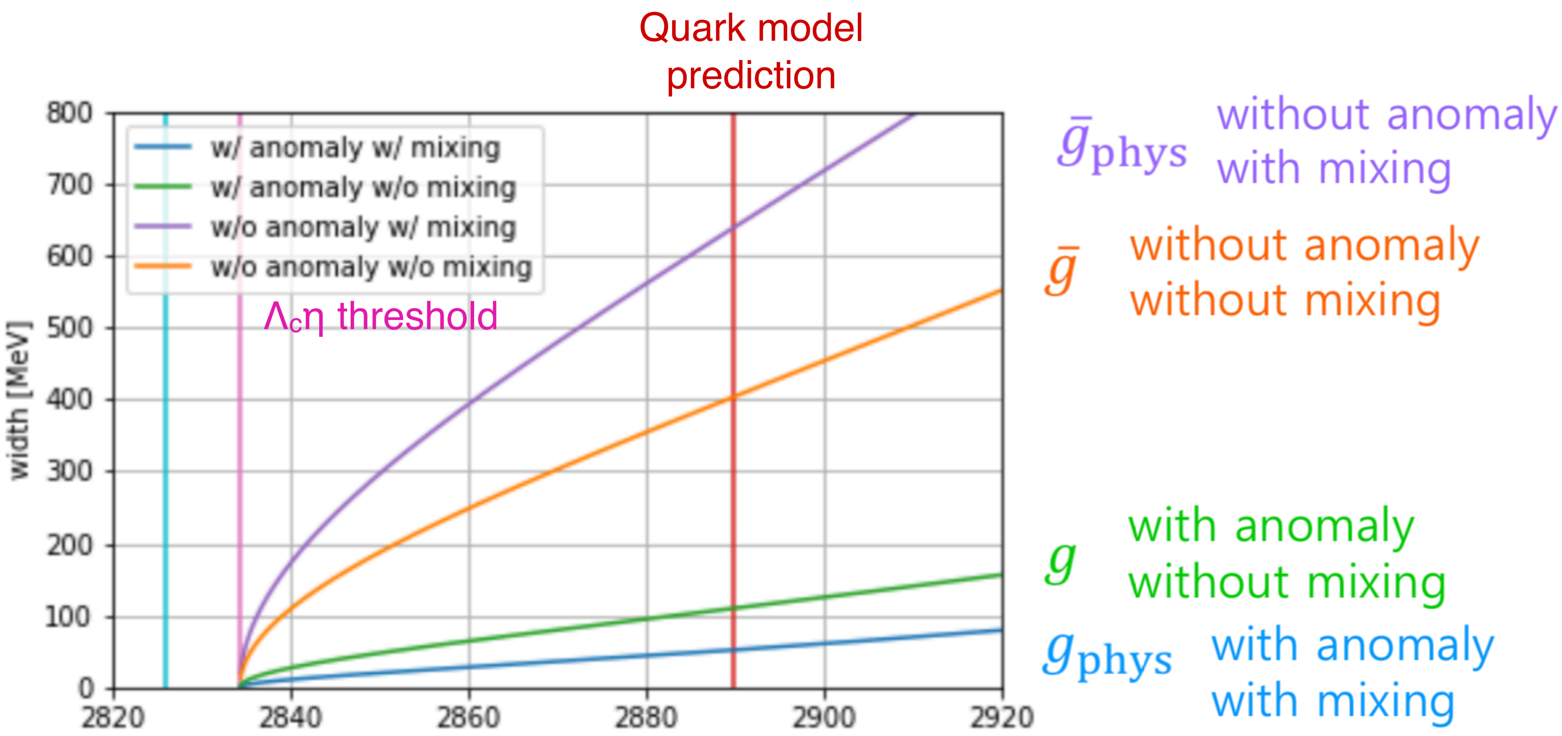
- $\Lambda_c^* \to \Lambda_c + \eta$ **decay with *η*, *η*' mixing**

  $(1/2^-_\rho)$ $(1/2^+)$

  Y. Kawakami, M. Harada, M.O., K. Suzuki, *PR D102, 114004 (2020)*

Decays of Chiral Partner

- **Suppression of decay coupling constant due to the $U_A(1)$ anomaly**



Unknown mass of the chiral partner $\Lambda_c(1/2^-)$

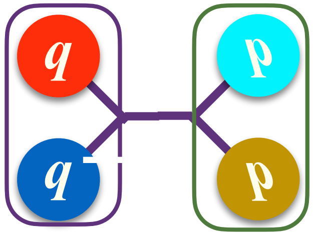
Y. Kawakami, M. Harada, M.O., K. Suzuki, ***PR D102, 114004 (2020)***

Summary

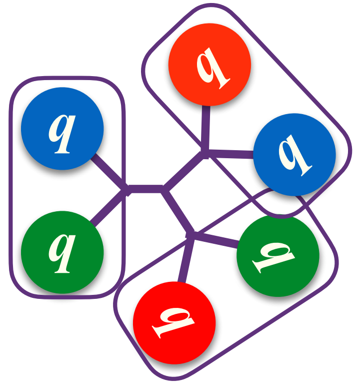
- We have constructed a chiral SU(3)×SU(3) effective theory of diquarks. The S/P diquarks are paired in $(\bar{3},1)+(1,\bar{3})$, while the A/V diquarks are in (3,3) representations. Effects of SCSB and $U_A(1)$ anomaly to the diquark masses are studied.
- We predict an inverse mass hierarchy for the pseudoscalar (0^-) diquarks.
- The parameters of the effective theory are fixed by the use of lattice QCD data and experimental data of heavy baryons.
- Heavy baryon spectra are analyzed by a diquark-heavy-quark potential model. The spectra of Λ_Q and Ξ_Q look very different, while the spectra of Σ_Q, Ξ'_Q, Ω_Q are similar with an energy shift.
- We predict mass inversion of $S(0^+)$-$A(1^+)$ diquarks under chiral symmetry restoration.
- We have found that the $U_A(1)$ anomaly (and the η-η' mixing) strongly suppress the decay of the $1/2^-$ Λ_Q to $\Lambda_Q(1/2^+) + \eta$.

Perspectives

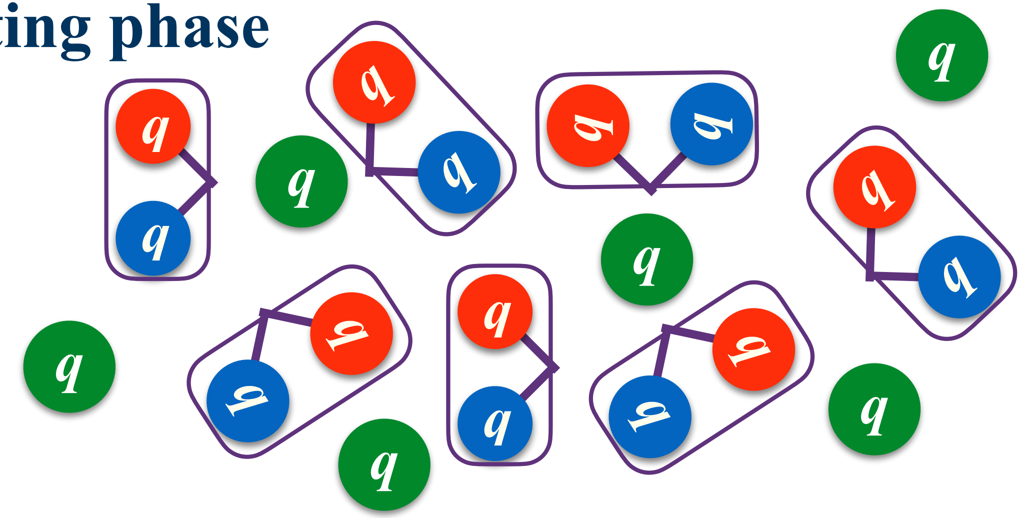
- Use of Diquarks D_q



$D_q D_q^{bar}$ = qq $q^{bar}q^{bar}$ = Tetraquark
D_q Q = qq Q = HQ Baryon
$D_q D_q Q^{bar}$ = qq qq Q^{bar} = Pentaquark
$D_q D_q D_q$ = qq qq qq = Hexaquark (Dibaryon)



- Diquarks may BE condensate in dense hadronic matter.
  => color-superconducting phase



Perspectives

- **Double Heavy Tetraquarks QQ$\overline{ud}$ (J^π=1$^+$, I=0)**

Qi Meng et al., Physics Letters B 814 (2021) 136095