האוניברסיטה העברית בירושלים
THE HEBREW UNIVERSITY OF JERUSALEM

Baryonic EFT for Light Hypernuclei

Nir Barnea

CLUSHIQ2022, Sendai
October 31 - July 3, 2022

Collaboration

Jerusalem, Israel

A. Gal, B. Bazak, M. Bagnarol

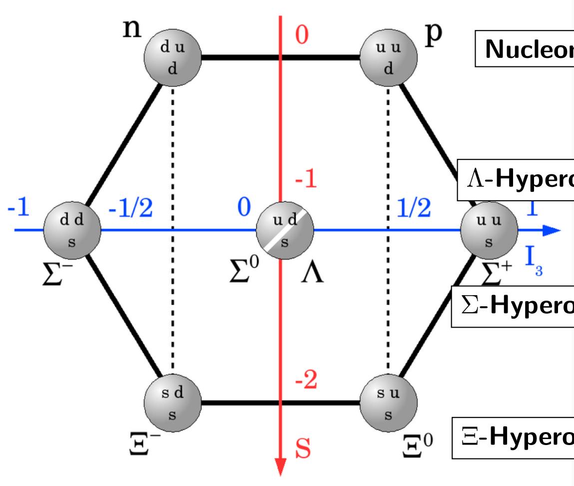
CEA, Saclay, France

L. Contessi

Rez/Prague, Czech Republic

M. Schäfer, J. Mareš

Introduction - the baryon octet

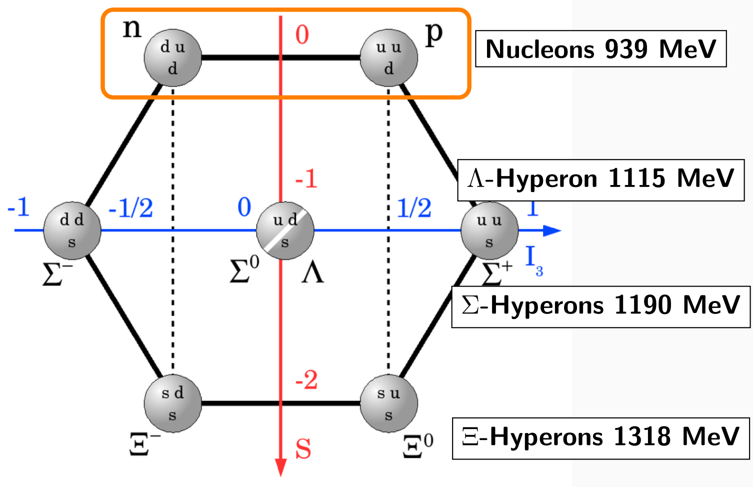


Nucleons 939 MeV

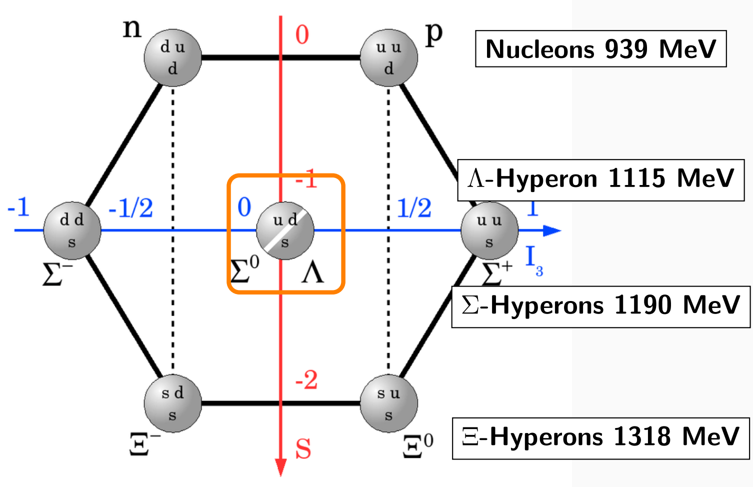
Λ-Hyperon 1115 MeV

Σ-Hyperons 1190 MeV

Ξ-Hyperons 1318 MeV



n
d u
d
0
u u
d
p
-1
-1
d d
s
-1/2
0
u d
s
1/2
u u
s
1
I_3
Σ^-
Σ^0
Λ
Σ^+
-2
s d
s
s u
s
Ξ^-
S
Ξ^0
Nucleons 939 MeV
Λ-Hyperon 1115 MeV
Σ-Hyperons 1190 MeV
Ξ-Hyperons 1318 MeV


n
d u
d
p
u u
d
0
-1
-1/2
-1
0
1/2
1
I
I3
d d
s
u d
s
u u
s
Σ−
Σ0
Λ
Σ+
s d
s
s u
s
-2
S
Ξ−
Ξ0
Nucleons 939 MeV
Λ-Hyperon 1115 MeV
Σ-Hyperons 1190 MeV
Ξ-Hyperons 1318 MeV

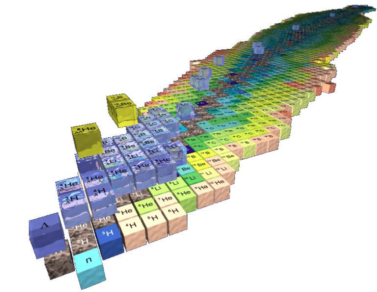
האוניברסיטה העברית בירושלים
THE HEBREW UNIVERSITY OF JERUSALEM

Nuclei & Hypernuclei

≈3300 nuclear isotopes

≈40 single Lambda hypernuclei

3 double Lambda hypernuclei

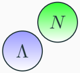


QCD → EFT

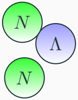
The program:

- Use obsereved hyperons properties
- Precise few-body methods
- Effective description of nature

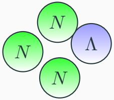
What do we have?

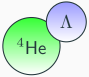


Not bound
scarce scattering data

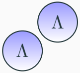


$^{3}_{\Lambda}$**H,** $B_\Lambda \approx 0.1$ **MeV**

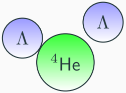




$^{4}_{\Lambda}$**H**^{0,1}**,**$^{4}_{\Lambda}$**He**^{0,1} $B_\Lambda \approx 3$ **MeV**
$^{5}_{\Lambda}$**He,** $B_\Lambda \approx 3$ **MeV**



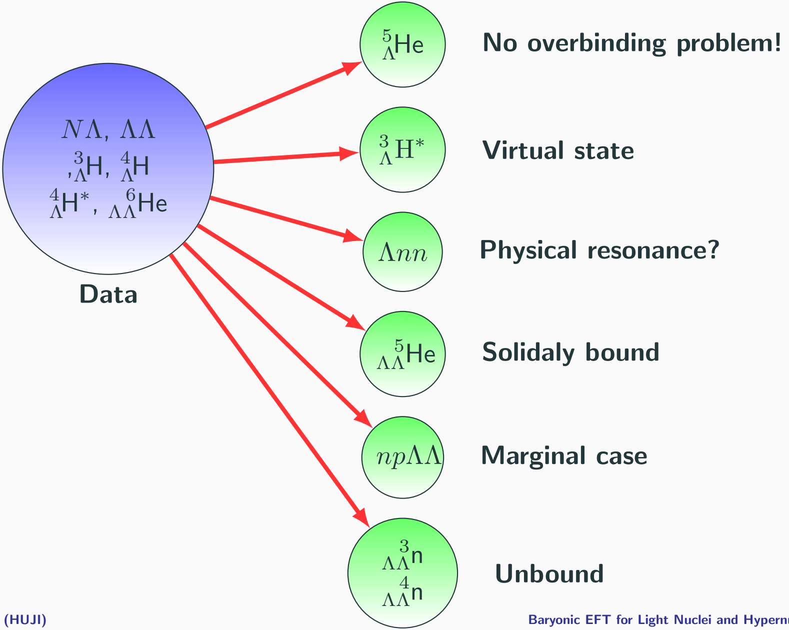
Not bound
no scattering data



$^{\;\;6}_{\Lambda\Lambda}$**He,** $B_\Lambda \approx 3$ **MeV**

Hypernuclear BEFT/$\not\pi$EFT in a nut shell

האוניברסיטה העברית בירושלים
THE HEBREW UNIVERSITY OF JERUSALEM



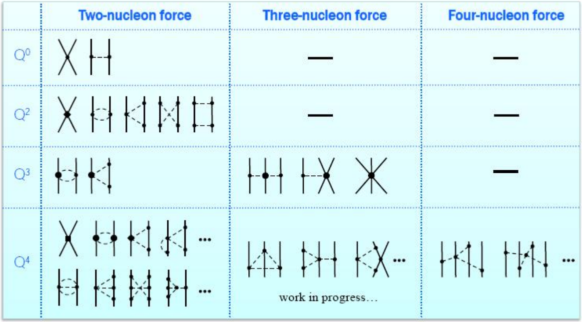
Baryonic EFT

	2-body	3-body	4-body	5-body
LO	C_1	D_1	–	–
NLO	C_2	–	E_2	–
N^{2}LO	C_3	D_3	?	?

The Nuclear Interaction - χEFT

Weinberg, van Kolck, Epelbaum, Machleidt, Meissner, ...

$$\boxed{\mathcal{L}_{QCD}(q,G) \longrightarrow \mathcal{L}_{\chi EFT}(B,\pi,K)}$$

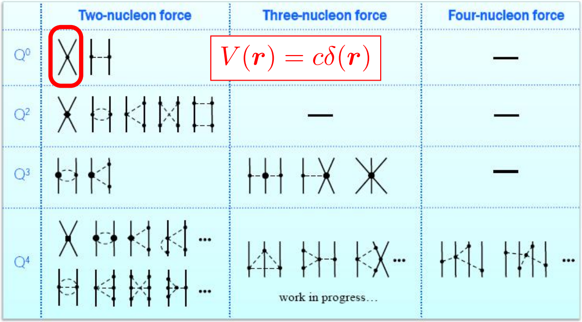


$$V_{LO} = \underbrace{V_\pi(\boldsymbol{r})}_{1-\text{pion exchange}} + \underbrace{(c_S + c_T \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)\delta(\boldsymbol{r})}_{\delta \text{ interactions}}$$

The Nuclear Interaction - χEFT

Weinberg, van Kolck, Epelbaum, Machleidt, Meissner, ...

$$\mathcal{L}_{QCD}(q,G) \longrightarrow \mathcal{L}_{\chi EFT}(B,\pi,K)$$

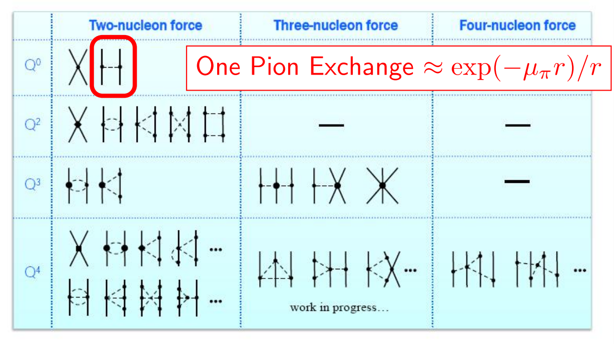


$$V_{LO} = \underbrace{V_\pi(\boldsymbol{r})}_{1-\text{pion exchange}} + \underbrace{(c_S + c_T \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)\delta(\boldsymbol{r})}_{\delta \text{ interactions}}$$

The Nuclear Interaction - χEFT

Weinberg, van Kolck, Epelbaum, Machleidt, Meissner, ...

$$\boxed{\mathcal{L}_{QCD}(q,G) \longrightarrow \mathcal{L}_{\chi EFT}(B,\pi,K)}$$



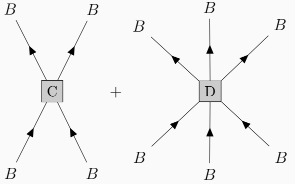
One Pion Exchange $\approx \exp(-\mu_\pi r)/r$

$$V_{LO} = \underbrace{V_\pi(\boldsymbol{r})}_{1-\text{pion exchange}} + \underbrace{(c_S + c_T \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)\delta(\boldsymbol{r})}_{\delta \text{ interactions}}$$

Baryonic EFT aka $\not\pi$EFT

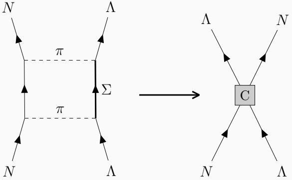
- $B = n, p, \Lambda$ are the only DOF.

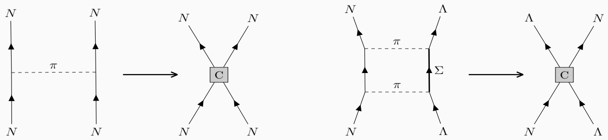
$$\boxed{\mathcal{L}_{QCD}(q,G) \longrightarrow \mathcal{L}_{\chi EFT}(B,\pi,K) \longrightarrow \mathcal{L}(B)}$$



- $\mathcal{L}$ is expanded in powers of Q/M_h.
- Include contact terms and derivatives.
- Not too many parameters

$$\boxed{\begin{aligned}\mathcal{L} =& N^\dagger\left(i\partial_0 + \frac{\nabla^2}{2m}\right)N + \Lambda^\dagger\left(i\partial_0 + \frac{\nabla^2}{2m}\right)\Lambda \\ &+ \mathcal{L}_{2B} + \mathcal{L}_{3B} + \ldots\end{aligned}}$$





Accuracy for light nuclei

Nuclei The pion mass is our breaking scale M_h

$$\left(\frac{Q}{M_h}\right) = \frac{\sqrt{2B_N M_N}}{m_\pi} \approx 0.5 - 0.8$$

Seems to work better in practice as $\Delta B(^4\text{He}) \approx 10\%$

Hypernuclei No OPE therefore breaking scale is $2m_\pi$

$$\left(\frac{Q}{M_h}\right) = \frac{\sqrt{2B_\Lambda M_\Lambda}}{2m_\pi} \approx 0.3$$

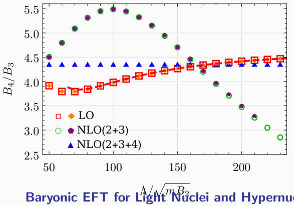
At LO accuracy goes as $(Q/M_h)^2$

1. **Universality**
   - @LO the only 2-body inputs are scattering lengths
   - BEFT is well suited for studying universality

2. **The Wigner Bound** Phillips, Beane and Cohen (1997-1998)
   - The effective range is bounded by the cutoff $r_{\text{eff}} \leq W/\lambda$
   - All orders but LO are perturbation (Kaplan, van Kolck, ...).

3. **The Thomas collapse** Bedaque, Hammer, and van Kolck (1999)
   - With LO 2-body interaction $B_3 \propto \hbar\lambda^2/m.$
   - A 3-body counter term must be introduced at LO.

4. **NLO - 4-body force** Bazak et al. (2019)
   - At NLO the 4-body system is unstable.
   - **Conclusion:** the 4-body force must be promoted to NLO.



The Nuclear Interaction - BEFT/$\not{\pi}$EFT

Kaplan, van Kolck, Bedaque, Hammer, Bazak, ...

	2-body	3-body	4-body	5-body
LO	C_1	D_1	–	–
NLO	C_2	–	E_2	–
N^2LO	C_3	D_3	?	?

Bazak, Kirscher, König, Pavón Valderrama, Barnea, and van Kolck, PRL **122**, 143001 (2019)

Hammer, König and van Kolck, Rev. Mod. Phys. **92**, 025004 (2020)

	2-body		3-body	
Strangeness = 0	C_{IS}	C_{01}, C_{10}	D_{IS}	$D_{\frac{1}{2}\frac{1}{2}}$
Strangeness = -1	C_{IS}	$C_{\frac{1}{2}0}$, $C_{\frac{1}{2}1}$	D_{IS}	$D_{0\frac{1}{2}}$, $D_{0\frac{1}{2}}$, $D_{1\frac{3}{2}}$
Strangeness = -2	C_{IS}	C_{00}	D_{IS}	$D_{\frac{1}{2}\frac{1}{2}}$

2-body & 3-body diagrams:

Contact terms - minimal amount of parameters

LECs - constrainted by exp. data

Strange = 0 - #LECs = 3

Strange = -1 -#LECs = 5

Strange = -2 - #LECs = 2

L. Contessi, M. Schafer, N. Barnea, A. Gal, J. Mareš, PLB 797 (2019) 134893

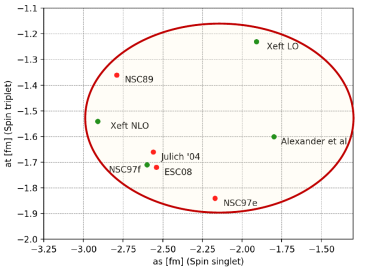
YN scattering data

- Cross-section data for $p_{lab} \geq 100$ MeV/c
- 12 d.p. for $\lambda + p \to \Lambda + p$
- 22 d.p. for $\Sigma + N \to \Lambda + N, \Sigma + N$

- Spin dependence not resolved

- **Alexander et al.** PR173, 1452 (1968)
  $$a^0_{\Lambda N} = -1.8 \text{ fm}$$
  $$a^1_{\Lambda N} = -1.6 \text{ fm}$$

- **Sechi-Zorn et al.** PR175, 1735 (1968)
  $$0 > a^0_{\Lambda N} > -9.0 \text{ fm}$$
  $$-0.8 > a^1_{\Lambda N} > -3.2 \text{ fm}$$



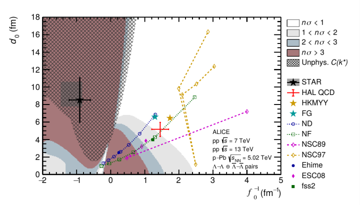
A. Gal et al.,

Rev. Mod. Phys. 88 (2016) 035004

$|lambdaN$ and $\Lambda\Lambda$ scattering lengths

ΛN scattering length RMP 88, 035004 (2016)

Exp./Model	$a^0_{\Lambda N}$ [fm]	$a^1_{\Lambda N}$ [fm]
Alexander	-1.80	-1.60
NSC89	-2.79	-1.36
NSC97e	-2.17	-1.84
NSC97f	-2.60	-1.71
ESC08c	-2.54	-1.72
Julich 04	-2.56	-1.66
χEFT(LO)	-1.91	-1.23
χEFT(NLO)	-2.91	-1.54



ALICE Collaboration, PLB 797, 134822 (2019)

$\Lambda\Lambda$ scattering length

Exp./Model	$a^0_{\Lambda\Lambda}$ [fm]	
^{12}C$(K^-,K^+)\Lambda\Lambda X$	$-1.2(6)$	PRC 85, 015204 (2012)
HALQCD	$-0.81 \pm 0.23^{0.0}_{-0.1}$	NPA 998, 121737 (2020)
χEFT(LO;600)	-1.52	PLB 653, 29 (2007)
χEFT(NLO;600)	-0.66	NPA 954, 273 (2016)
Femtoscopy	$-0.79^{+0.29}_{-1.13}$	PRC 91, 024916 (2016)
		PRL 114, 022301 (2015)

What do we have?

- LO and NLO $\not\pi$EFT fitted to low-energy experimental constraints
- The Schrödinger equation

What do we want to know?

- **Bound states**
- **Resonances**
- **Scattering**

How do we get there?

- Gaussian basis functions
- Few-body bound states $\Rightarrow$ SVM (Suzuki and Varga)
- Scattering $\Rightarrow$ Busch formula
- Complex rotation, analytic continuation $\Rightarrow$ Resonances

- **Nuclear scattering**

  Elastic s-wave scattering @NLO for $A \leq 5$

- **Λ hypernuclei ($^{A}_{\Lambda}Z$)**

  s-shell hypernuclei - overbinding of $^{5}_{\Lambda}$He

  Hypernuclear resonances

- **$\Lambda\Lambda$ hypernuclei ($^{A}_{\Lambda\Lambda}Z$)**

  Onset of binding, A=4 or 5?

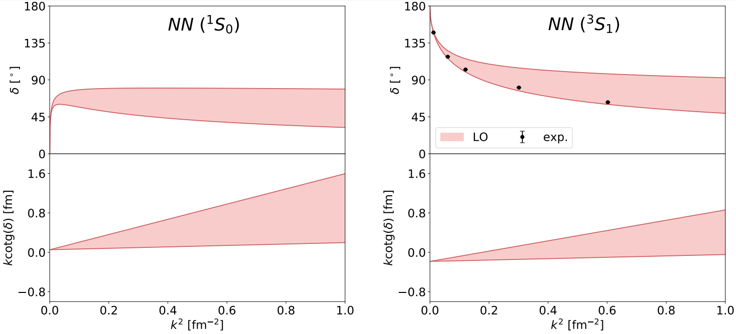
- **Charge symmetry breaking**

  The Dalitz von Hippel paramaters from SU(3) symmetry.

- **Few nucleons in a box**

  EFT matching of LQCD calcs.

The nuclear sector



Leading order (LO):

(exp. constraints)

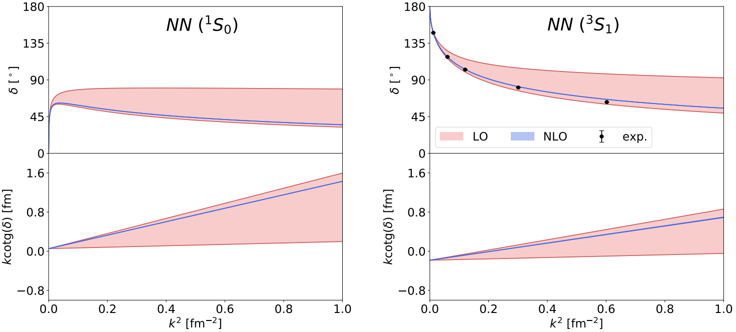
$a_0^{nn} = \; -18.95(40)$ fm

$a_1^{np} = \; 5.419(7)$ fm

$B(^3\text{H}) = 8.482$ MeV

Effective range expansion:

$$k\text{cotg}(\delta) = -\frac{1}{a} + \frac{1}{2} r k^2 + \dots$$



Leading order (LO):
(exp. constraints)

$a_0^{nn} = \; -18.95(40)$ fm

$a_1^{np} = \; 5.419(7)$ fm

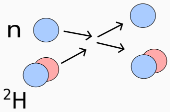
$B(^3\mathrm{H}) = 8.482$ MeV

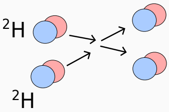
Next-to-leading order (NLO):
(exp. constraints)

$r_0^{nn} = \; 2.75(11)$ fm

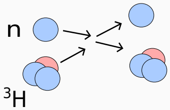
$r_1^{np} = \; 1.753(8)$ fm

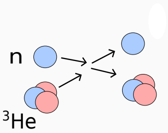
$B(^4\mathrm{He}) = 28.296$ MeV





Few-body s-wave scattering





Universal fermionic relations (STM, Petrov, Deltuva,...)

Atom-Dimer scattering

$$\frac{a_{ad}}{a_{aa}} = 1.1791 + 0.553\frac{r_{aa}}{a_{aa}} \quad ; \quad \frac{r_{ad}}{a_{aa}} = -0.038 + 1.04\frac{r_{aa}}{a_{aa}}$$

Dimer-Dimer scattering

$$\frac{a_{dd}}{a_{aa}} = 0.5986 + 0.105\frac{r_{aa}}{a_{aa}} \quad ; \quad \frac{r_{dd}}{a_{aa}} = 0.133 + 0.51\frac{r_{aa}}{a_{aa}}$$

These results are reproduced for spin saturated system:

- Neutron-Deuteron $S = \frac{3}{2}$ scattering
- Deuteron-Deuteron $S = 2$ scattering.

$n + d\ (S = \frac{1}{2}, T = \frac{1}{2})$ scattering

1. **Near-threshold $^3\mathrm{H}^*$ virtual state**

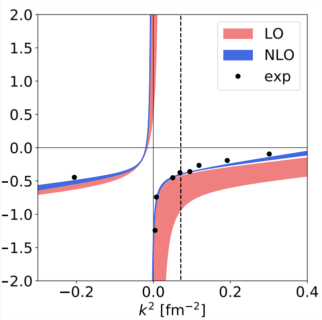
   $\Rightarrow$ pole of S-matrix

2. **Near-threshold zero in S-matrix**

$$\frac{1}{k\mathrm{cotg}(\delta) - ik} = 0$$

$$\lim_{k \to k_0} k\mathrm{cotg}(\delta) = \pm\infty$$

$\Rightarrow$ modified ERE

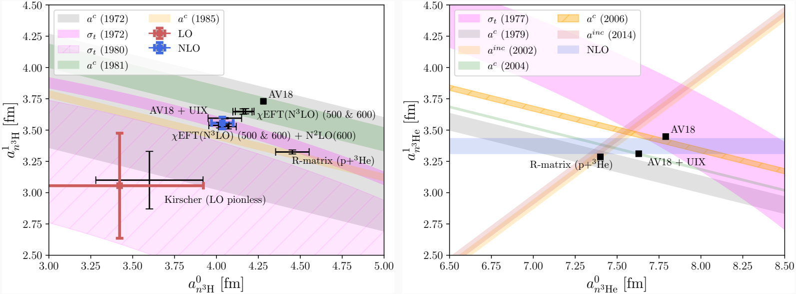


Oers and Seagrave, PLB 24, 11 (1967)

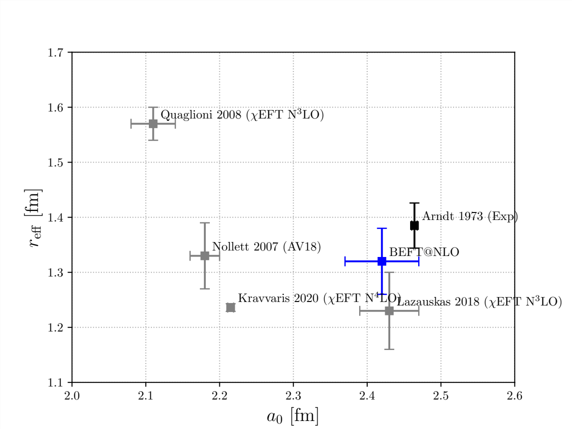
$$a^{1/2}_{n^2\mathrm{H}} = 0.29 \text{ fm}$$
$$r^{1/2}_{n^2\mathrm{H}} = 1.70 \text{ fm}$$

$$k\mathrm{cotg}(\delta) = A + B\ k^2 + \frac{C}{(1 + D\ k^2)} \quad ; \quad a = -\frac{1}{A + C} \quad \text{and} \quad r = 2B$$

Experiment & Theory : $n + {}^3\mathrm{H}$ and $n + {}^3\mathrm{He}$ scattering lengths



Phys. Rev. C 42 (1990) 438; Phys. Rev. C 102 (2020) 034007; Few-Body Syst. 34 (2004) 105; Phys. Lett B 721 (2013) 355; Phys. Rev. C 68(R) (2003) 021002

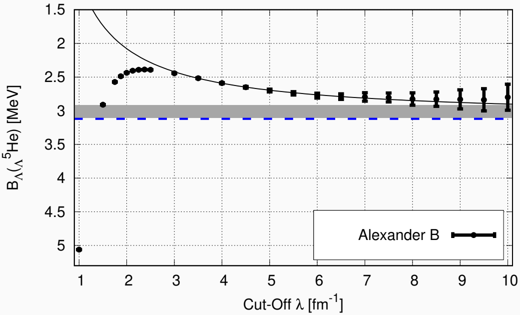


Exp	a_0 [fm]	r_{eff} [fm]
Arndt 1973	2.4641 ± 0.0037	1.385 ± 0.041
Haun 2020	$2.4746^{\pm 0.0017\ \text{[stat]}}_{\pm 0.0011\ \text{[syst]}}$	-

Light Hypernuclei @LO

The ${}^{5}_{\Lambda}$He binding energy

$B_\Lambda({}^{5}_{\Lambda}\mathrm{He})$ vs. cut-off λ in LO BEFT

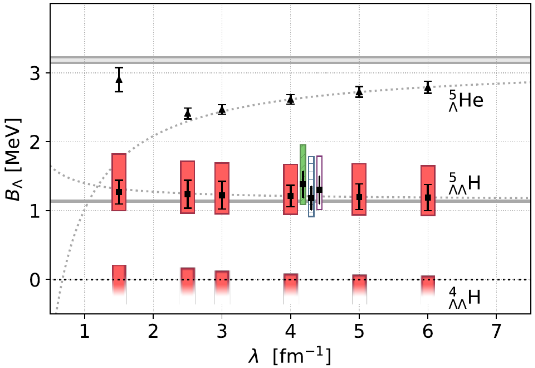


L.Contessi N.Barnea A.Gal, PRL 121 (2018) 102502

With Alexander & χEFT(NLO) scattering lengths a_s, a_t
$B_\Lambda({}^{5}_{\Lambda}\mathrm{He})$ is reproduced within theoretical error

Cut-off dependence

$$\frac{B_\Lambda(\lambda)}{B_\Lambda(\infty)} = 1 + \frac{\alpha}{\lambda} + \frac{\beta}{\lambda^2} + \dots$$



Contessi-Schafer-Barnea-Gal-Mareš, PLB 797 (2019) 134893.

Double-Λ systems:

- The neutral systems $\Lambda\Lambda n$, $\Lambda\Lambda nn$ are far from threshold
- ${}_{\Lambda\Lambda}^{4}$H on verge of binding. Better data is needed for clarification.
- In our theory ${}_{\Lambda\Lambda}^{5}$H is comfortably bound

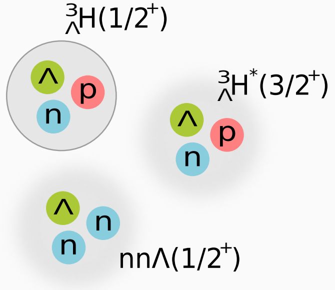
האוניברסיטה העברית בירושלים
THE HEBREW UNIVERSITY OF JERUSALEM

$^3_\Lambda\mathrm{H}^*(3/2^+)$

- no experimental evidence
- JLab C12-19-002 proposal

$\Lambda nn(1/2^+)$

- experiment (HypHI)
- JLab E12-17-003 experiment
- structure of neutron-rich Λ-hypernuclei



Calculating resonance states is a non-trivial task

We have used two techniques:

- Complex scaling method (CSM)
- Inverse analysis continuation in coupling constraint (IACCC)

Λnn and ${}^{3}_{\Lambda}\mathrm{H}^{*}$ - Theory

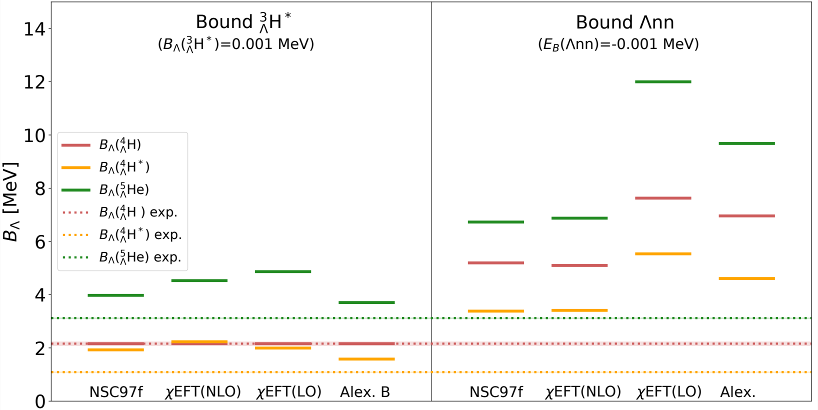
- **R. H. Dalitz, B. W. Downs** (PR110, 958, 1958; PR111, 967, 1958; PR114, 593, 1959)
  first calculation, variational approach, unbound $\Lambda\mathrm{nn}$
- **H. Garcilazo** (J. Phys. G: Nucl. Phys. 13, 63, 1987)
  Faddeev equations, separable potentials, unbound $\Lambda\mathrm{nn}$
- ...
- **V. B. Belyaev et al.** (NPA803, 210, 2008)
  first resonance calculation, 3-body Jost function, phenomenological potential
  $\Lambda\mathrm{nn}$ pole just above/below the threshold, large widths
- **E. Hiyama et al.** (PRC89, 061302(R), 2014)
  YN model equivalent to NSC97f; changing ${}^{3}V^{T}_{N\Lambda-N\Sigma}$, ${}^{0}V_{NN}$ to bind $\Lambda\mathrm{nn}$
  nonexistence of bound $\Lambda\mathrm{nn}$ (${}^{3}_{\Lambda}\mathrm{H}$, ${}^{3}_{\Lambda}\mathrm{H}^{*}$, ${}^{4}_{\Lambda}\mathrm{H}$, ${}^{3}\mathrm{H}$)
- **A. Gal, H. Garcilazo** (PLB736, 93, 2014)
  Faddeev equations, separable potentials
  nonexistence of bound $\Lambda\mathrm{nn}$ ($\sigma_{\Lambda p}$, ${}^{3}_{\Lambda}\mathrm{H}$, and ${}^{4}_{\Lambda}\mathrm{H}$ exc. energy)
- **I. R. Afnan, B. F. Gibson** (PRC92, 054608, 2015)
  Faddeev equations, $\Lambda\mathrm{nn}$ resonance calculations, separable potentials
  subthreshold (non-physical) $\Lambda\mathrm{nn}$ resonance

$\Lambda\mathrm{nn}$ and ${}^{3}_{\Lambda}\mathrm{H}^*$ - Exp.

- **HypHI Collaboration** (PRC88, 041001(R), 2013)
  suggestion of bound $\Lambda\mathrm{nn}$, ${}^{6}\mathrm{Li} + {}^{12}\mathrm{C}$ @ 2A GeV

- **JLab E12-17-003 Experiment** (PTEP92 2022, 013D01, 2022)
  ^{3}H$(e, e'K^{+})\Lambda nn$
  No significant structures observed

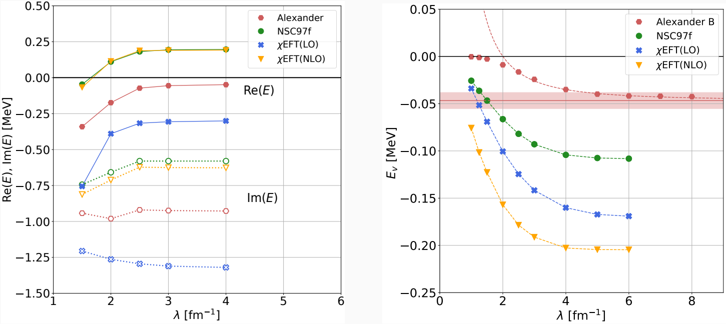
Implications of just bound Λnn and ${}^3_\Lambda H^*$ ($\lambda = 6\ \text{fm}^{-1}$)



- $B_\Lambda({}^3_\Lambda H)$ is used to fix three-body force in $I, S = 0, 1/2$ channel and remains unaffected

$\Lambda\mathrm{nn}$ system and ${}^{3}_{\Lambda}\mathrm{H}^{*}(J^{\pi} = 3/2^{+})$ excited state

$\Lambda\mathrm{nn}(J^{\pi} = 1/2^{+})$ ${}^{3}_{\Lambda}\mathrm{H}^{*}(J^{\pi} = 3/2^{+})$



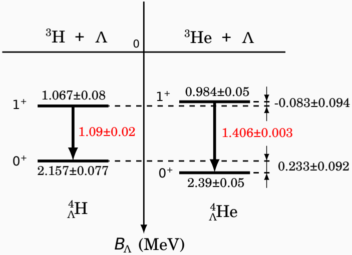
$\Lambda\mathrm{nn}$ predicted as a near-threshold resonance

→ large width $1.16 \leq \Gamma \leq 2.00$ MeV

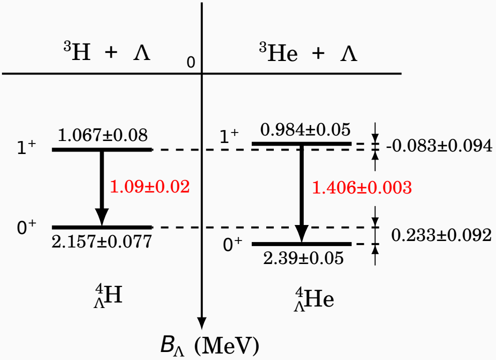
${}^{3}_{\Lambda}\mathrm{H}^{*}$ obtained as a near-threshold virtual state

→ enhanced s-wave $\Lambda + {}^{2}\mathrm{H}$ phaseshifts in $J^{\pi} = 3/2^{+}$ channel

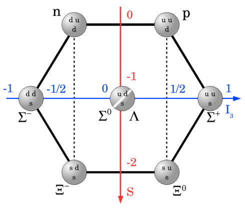
Charge symmetry breaking



$A = 4$ hypernuclear level scheme



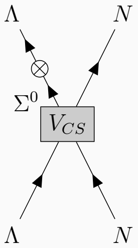
- **Charge symmetry:** invarince under $n \leftrightarrow p$, e.g. $^3\mathrm{H} \leftrightarrow {}^3\mathrm{He}$
- **Nuclei:** for ^{3}He - ^{3}H, ΔE_{CSB} without Coulomb is about 70 keV
- For ^{3}He - ^{3}H: $\Delta E_{CSB}/\Delta E \approx 0.01$
- **Hypernuclei:** CSB in $^4_\Lambda$He-$^4_\Lambda$H: $\Delta E_{CSB}/\Delta E \approx 0.22$



Dalitz, von Hippel Phys. Lett. 10, 153 (1964)
$\Lambda - \Sigma^0$ mixing in SU$(3)_f$ (following Coleman & Glashow)

$$\mathcal{A}^{(0)}_{I=1} = -\frac{\langle \Sigma^0 | \delta M | \Lambda \rangle}{M_{\Sigma^0} - M_\Lambda} = -0.0148(6)$$

CSB OPE contribution by $g_{\Lambda\Lambda\pi} = 2\mathcal{A}^{(0)}_{I=1} g_{\Lambda\Sigma\pi}$



- **A. Gal** Phys. Lett. B 744, 352, (2015)
  Generalization of DvH

$$\langle \Lambda N | V_{\rm CSB} | \Lambda N \rangle = -\frac{2}{\sqrt{3}} \mathcal{A}^{(0)}_{I=1} \langle \Sigma N | V_{\rm CS} | \Lambda N \rangle \, \tau_z .$$

$\Delta B_\Lambda(0^+) \approx 240$ keV $\quad \Delta B_\Lambda(1^+) \approx 35$ keV

- **Gazda, Gal** PRL 116, 122501 (2016)
  generalized DvH; LO χEFT YN interaction; NSCM

$\Delta B_\Lambda(0^+) \approx 180 \pm 130$ keV $\quad \Delta B_\Lambda(1^+) \approx -200 \pm 30$ keV

Observations:

- π, K terms negligible
- CSB is short range physics
- 2 d.p. and 2 parameters C_s^{CSB}, C_t^{CSB}
- $S = 0, 1$ have opposite signs
- Spin singlet dominance

$$|C_s^{CSB}| \gg |C_t^{CSB}|$$

Question:

Can BEFT explain these last 2 observations?

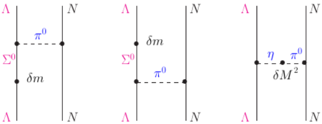


Fig. 1 CSB contributions involving pion exchange, according to Dalitz and von Hippel [1], due to $\Lambda - \Sigma^0$ mixing (left two diagrams) and $\pi^0 - \eta$ mixing (right diagram).

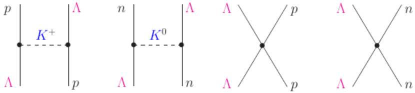


Fig. 2 CSB contributions from $K^{\pm}/K^0$ exchange (left) and from contact terms (right).

Λ	NLO13		NLO19	
	C_s^{CSB}[MeV^{-2}]	C_t^{CSB}[MeV^{-2}]	C_s^{CSB}[MeV^{-2}]	C_t^{CSB}[MeV^{-2}]
500	4.691×10^{-3}	-9.294×10^{-4}	5.590×10^{-3}	-9.505×10^{-4}
550	6.724×10^{-3}	-8.625×10^{-4}	6.863×10^{-3}	-1.260×10^{-3}
600	9.960×10^{-3}	-9.870×10^{-4}	9.217×10^{-3}	-1.305×10^{-3}
650	1.500×10^{-2}	-1.142×10^{-3}	1.240×10^{-2}	-1.395×10^{-3}

Haidenbauer et al., Few-body sys. (2019)

Observations:

- π, K terms negligible
- CSB is short range physics
- 2 d.p. and 2 parameters C_s^{CSB}, C_t^{CSB}
- $S = 0, 1$ have opposite signs
- Spin singlet dominance

$$|C_s^{CSB}| \gg |C_t^{CSB}|$$

Question:

Can BEFT explain these last 2 observations?

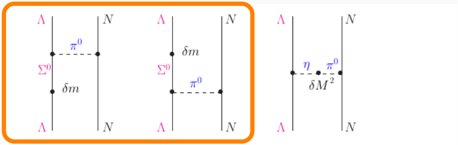


Fig. 1 CSB contributions involving pion exchange, according to Dalitz and von Hippel [1], due to $\Lambda - \Sigma^0$ mixing (left two diagrams) and $\pi^0 - \eta$ mixing (right diagram).

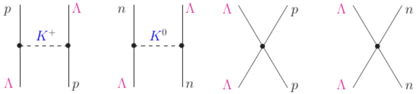


Fig. 2 CSB contributions from $K^{\pm}/K^0$ exchange (left) and from contact terms (right).

Λ	NLO13		NLO19	
	C_s^{CSB}[MeV^{-2}]	C_t^{CSB}[MeV^{-2}]	C_s^{CSB}[MeV^{-2}]	C_t^{CSB}[MeV^{-2}]
500	4.691×10^{-3}	-9.294×10^{-4}	5.590×10^{-3}	-9.505×10^{-4}
550	6.724×10^{-3}	-8.625×10^{-4}	6.863×10^{-3}	-1.260×10^{-3}
600	9.960×10^{-3}	-9.870×10^{-4}	9.217×10^{-3}	-1.305×10^{-3}
650	1.500×10^{-2}	-1.142×10^{-3}	1.240×10^{-2}	-1.395×10^{-3}

Haidenbauer et al., Few-body sys. (2019)

Observations:

- π, K terms negligible
- CSB is short range physics
- 2 d.p. and 2 parameters C_s^{CSB}, C_t^{CSB}
- $S = 0, 1$ have opposite signs
- Spin singlet dominance

$$|C_s^{CSB}| \gg |C_t^{CSB}|$$

Question:

Can BEFT explain these last 2 observations?

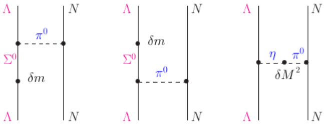


Fig. 1 CSB contributions involving pion exchange, according to Dalitz and von Hippel [1], due to $\Lambda - \Sigma^0$ mixing (left two diagrams) and $\pi^0 - \eta$ mixing (right diagram).

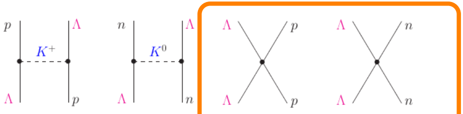


Fig. 2 CSB contributions from $K^{\pm}/K^0$ exchange (left) and from contact terms (right).

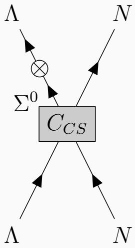
Λ	NLO13		NLO19	
	C_s^{CSB}[MeV^{-2}]	C_t^{CSB}[MeV^{-2}]	C_s^{CSB}[MeV^{-2}]	C_t^{CSB}[MeV^{-2}]
500	4.691×10^{-3}	-9.294×10^{-4}	5.590×10^{-3}	-9.505×10^{-4}
550	6.724×10^{-3}	-8.625×10^{-4}	6.863×10^{-3}	-1.260×10^{-3}
600	9.960×10^{-3}	-9.870×10^{-4}	9.217×10^{-3}	-1.305×10^{-3}
650	1.500×10^{-2}	-1.142×10^{-3}	1.240×10^{-2}	-1.395×10^{-3}

Haidenbauer et al., Few-body sys. (2019)

The DvH mechanism in BEFT

Dalitz, von Hippel for BEFT

$$\langle \Lambda N | C_{\mathrm{CSB}} | \Lambda N \rangle = -\frac{2}{\sqrt{3}} \, \mathcal{A}^{(0)}_{I=1} \, \langle \Sigma N | C_{\mathrm{CS}} | \Lambda N \rangle \, \tau_z .$$



Assuming $\mathrm{SU(3)_f}$ symmetry we can relate $C^S_{\Lambda N,\Sigma N}$ to the NN and ΛN LECs:

$$C^0_{\Lambda N,\Sigma N} = -3(C^0_{NN} - C^0_{\Lambda N}),$$
$$C^1_{\Lambda N,\Sigma N} = \quad (C^1_{NN} - C^1_{\Lambda N}).$$

Dover, Feshbach, Ann. Phys. (NY) 198, 321 (1990)

The resulting CSB LECs are opposite in sign and

$$|C_s^{CSB}| \gg |C_t^{CSB}|$$

Having $\mathcal{A}^{(0)}_{I=1}$ we have **no free parameters**.

Now we can go in the other direction and **predict** $\mathcal{A}^{(0)}_{I=1}$ from the hypernuclear spectrum.

The DvH mechanism in BEFT

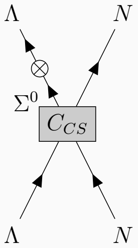
האוניברסיטה העברית בירושלים
THE HEBREW UNIVERSITY OF JERUSALEM

Dalitz, von Hippel for BEFT

$$\langle \Lambda N | C_{\rm CSB} | \Lambda N \rangle = -\frac{2}{\sqrt{3}} \, \mathcal{A}^{(0)}_{I=1} \, \langle \Sigma N | C_{\rm CS} | \Lambda N \rangle \, \tau_z \, .$$

Assuming $\mathrm{SU(3)_f}$ symmetry we can relate $C^S_{\Lambda N, \Sigma N}$ to the NN and ΛN LECs:

$$C^0_{\Lambda N, \Sigma N} = -3(C^0_{NN} - C^0_{\Lambda N}),$$
$$C^1_{\Lambda N, \Sigma N} = \quad (C^1_{NN} - C^1_{\Lambda N}).$$



`Dover, Feshbach, Ann. Phys. (NY) 198, 321 (1990)`

The resulting CSB LECs are opposite in sign and

$$|C_s^{CSB}| \gg |C_t^{CSB}|$$

Having $\mathcal{A}^{(0)}_{I=1}$ we have **no free parameters**.

Now we can go in the other direction and **predict** $\mathcal{A}^{(0)}_{I=1}$ from the hypernuclear spectrum.

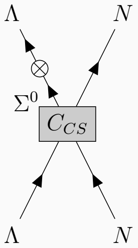
Dalitz, von Hippel for BEFT

$$\langle \Lambda N | C_{\rm CSB} | \Lambda N \rangle = -\frac{2}{\sqrt{3}} \mathcal{A}^{(0)}_{I=1} \langle \Sigma N | C_{\rm CS} | \Lambda N \rangle \, \tau_z .$$

Assuming SU(3)$_{\rm f}$ symmetry we can relate $C^S_{\Lambda N,\Sigma N}$ to the NN and ΛN LECs:

$$C^0_{\Lambda N,\Sigma N} = -3(C^0_{NN} - C^0_{\Lambda N}),$$
$$C^1_{\Lambda N,\Sigma N} = \quad (C^1_{NN} - C^1_{\Lambda N}).$$

Dover, Feshbach, Ann. Phys. (NY) 198, 321 (1990)



The resulting CSB LECs are opposite in sign and

$$|C_s^{CSB}| \gg |C_t^{CSB}|$$

Having $\mathcal{A}^{(0)}_{I=1}$ we have **no free parameters**.

Now we can go in the other direction and **predict** $\mathcal{A}^{(0)}_{I=1}$ from the hypernuclear spectrum.

The DvH mechanism in BEFT

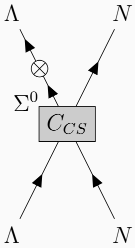
Dalitz, von Hippel for BEFT

$$\langle \Lambda N | C_{\text{CSB}} | \Lambda N \rangle = -\frac{2}{\sqrt{3}} \mathcal{A}^{(0)}_{I=1} \langle \Sigma N | C_{\text{CS}} | \Lambda N \rangle \, \tau_z .$$

Assuming SU(3)$_{\text{f}}$ symmetry we can relate $C^S_{\Lambda N,\Sigma N}$ to the NN and ΛN LECs:

$$C^0_{\Lambda N,\Sigma N} = -3(C^0_{NN} - C^0_{\Lambda N}),$$
$$C^1_{\Lambda N,\Sigma N} = \quad (C^1_{NN} - C^1_{\Lambda N}).$$

Dover, Feshbach, Ann. Phys. (NY) 198, 321 (1990)



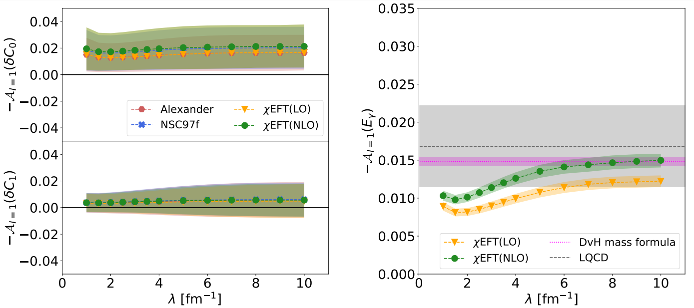
The resulting CSB LECs are opposite in sign and

$$|C_s^{CSB}| \gg |C_t^{CSB}|$$

Having $\mathcal{A}^{(0)}_{I=1}$ we have **no free parameters**.

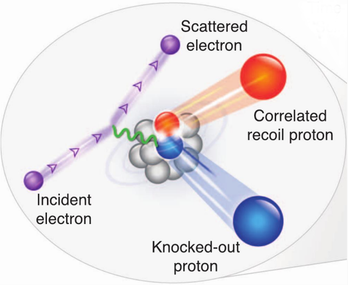
Now we can go in the other direction and **predict** $\mathcal{A}^{(0)}_{I=1}$ from the hypernuclear spectrum.

Extracting the DvH parameter



Method/Input	$-\mathcal{A}_{I=1}$
SU(3)$_f$ [DvH64]	0.0148 ± 0.0006
LQCD [LQCD20]	0.0168 ± 0.0054
$\not\pi$EFT(LO)/χEFT(LO) [Polinder06]	0.0139 ± 0.0013
$\not\pi$EFT(LO)/χEFT(NLO) [Haidenbauer13]	0.0168 ± 0.0014

Short Range Correlations - The Generalized Contact Formalism



The short range wave function - Universality

We start with the 2-body Schrodinger ...

$$\left[-\frac{\hbar^2}{m}\nabla^2 + V(\boldsymbol{r})\right]\psi = E\psi$$

Vanishing distance, $r \longrightarrow 0$

- The energy becomes negligible $E \ll \hbar^2/mr^2$
- The w.f. ψ assumes an asymptotic **energy independent** form φ

$$\left[-\frac{\hbar^2}{m}\nabla^2 + V(r)\right]\varphi(\boldsymbol{r}) = 0 \qquad r\varphi(r) = 0|_{r=0}$$

- φ is a universal function (in the **weak** sense - V dependent)

Factorization, Short range observable - The Contact

Tan, Braaten & Platter,...

The 2-body system

$$\psi(\boldsymbol{r}) \xrightarrow[r \to 0]{} \varphi(\boldsymbol{r})$$

The N-body system

$$\Psi(\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_N) \xrightarrow[r_{12} \to 0]{} \varphi(\boldsymbol{r}_{12}) A(\boldsymbol{R}_{12}, \boldsymbol{r}_3, \ldots, \boldsymbol{r}_N)$$

Factorization, Short range observable - The Contact

Tan, Braaten & Platter,...

The 2-body system

$$\psi(\boldsymbol{r}) \xrightarrow[r \to 0]{} \varphi(\boldsymbol{r})$$

$$O_{12} \approx \delta(\boldsymbol{r}_{12}) \Longrightarrow \langle\psi|O_{12}|\psi\rangle \approx C_2\langle\varphi|O_{12}|\varphi\rangle$$

The N-body system

$$\Psi(\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_N) \xrightarrow[r_{12} \to 0]{} \varphi(\boldsymbol{r}_{12})A(\boldsymbol{R}_{12}, \boldsymbol{r}_3, \ldots, \boldsymbol{r}_N)$$

Factorization, Short range observable - The Contact

Tan, Braaten & Platter,...

The 2-body system

$$\psi(\boldsymbol{r}) \xrightarrow[r \to 0]{} \varphi(\boldsymbol{r})$$

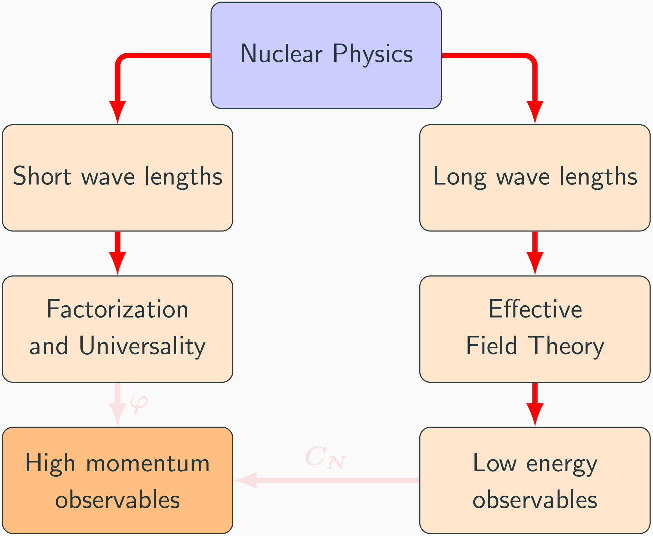
$$O_{12} \approx \delta(\boldsymbol{r}_{12}) \Longrightarrow \langle\psi|O_{12}|\psi\rangle \approx C_2\langle\varphi|O_{12}|\varphi\rangle$$

The N-body system

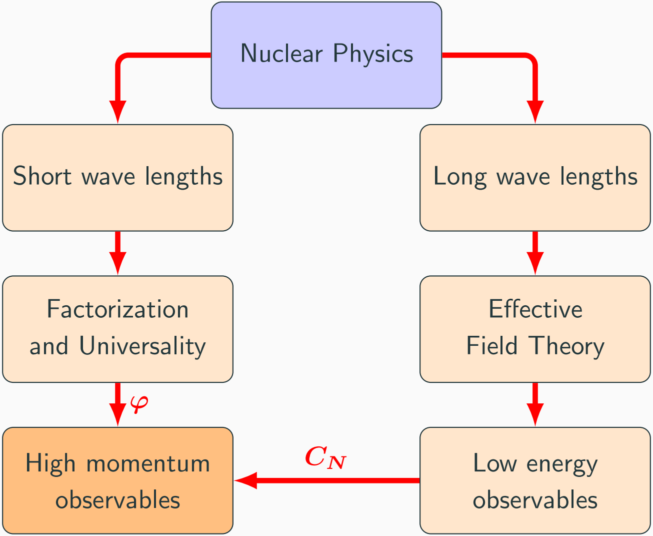
$$\Psi(\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_N) \xrightarrow[r_{12} \to 0]{} \varphi(\boldsymbol{r}_{12}) A(\boldsymbol{R}_{12}, \boldsymbol{r}_3, \ldots, \boldsymbol{r}_N)$$

$$\langle\Psi|\sum O_{ij}|\Psi\rangle \approx \underbrace{\tfrac{N(N-1)}{2}\langle A|A\rangle}_{C_N}\langle\varphi|O_{12}|\varphi\rangle$$

Short and Long



Short and Long

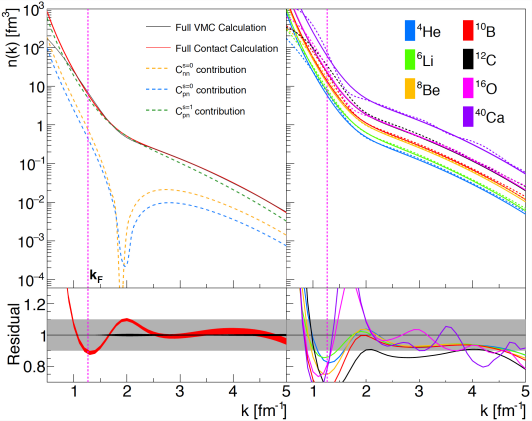


האוניברסיטה העברית בירושלים
THE HEBREW UNIVERSITY OF JERUSALEM

The **asymptotic** 1-body momentum distribution

$$n_n(\boldsymbol{k}) \longrightarrow C_{np}^{s=0}|\tilde{\varphi}_{np}^{s=0}(\boldsymbol{k})|^2 + C_{np}^{s=1}|\tilde{\varphi}_{np}^{s=1}(\boldsymbol{k})|^2 + 2C_{nn}^{s=0}|\tilde{\varphi}_{nn}^{s=0}(\boldsymbol{k})|^2$$

Comparing with the VMC data:



Surprisingly, the agreement holds for $k_F \leq k \leq 6\ \text{fm}^{-1}$

1-body neutron and proton momentum distributions

$$n_n(\boldsymbol{k}), \;\; n_p(\boldsymbol{k})$$

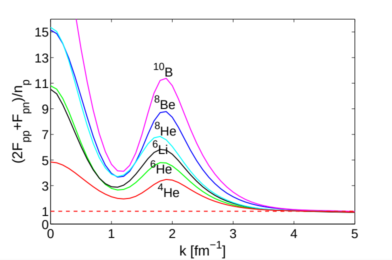
2-body nn, np, pp momentum distributions

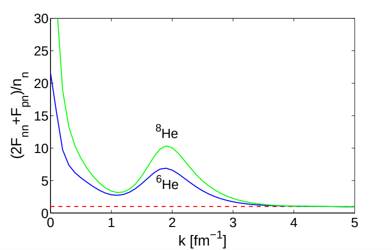
$$F_{nn}(\boldsymbol{k}), \;\; F_{pn}(\boldsymbol{k}), \;\; F_{pp}(\boldsymbol{k})$$

SRC relations

$$n_p(\boldsymbol{k}) \xrightarrow[k\to\infty]{} 2F_{pp}(\boldsymbol{k}) + F_{pn}(\boldsymbol{k})$$

$$n_n(\boldsymbol{k}) \xrightarrow[k\to\infty]{} 2F_{nn}(\boldsymbol{k}) + F_{pn}(\boldsymbol{k})$$



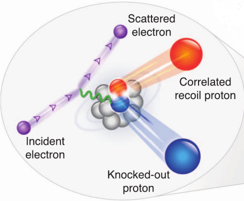


VMC calculations of light nuclei

R. B. Wiringa, *et al.*, PRC **89**, 024305 (2014)

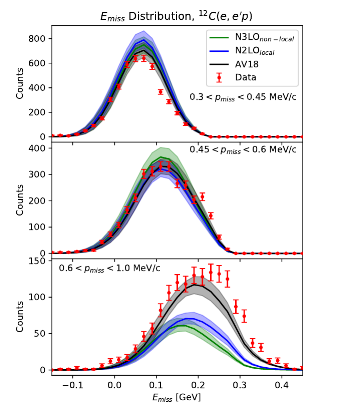
- Series of 1-body, 2-body momentum distributions
- The data is available for $2 \leq A \leq 10$ and $A = 12, 16, 40$
- The calculations were done with the VMC method
- Potential - AV18+UX

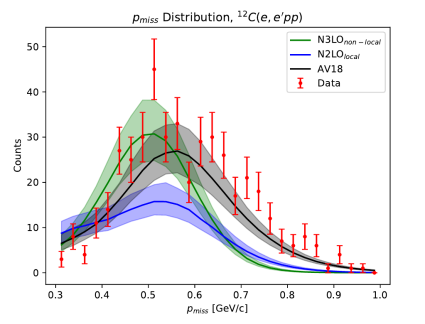
The momentum relations holds for $4\ \mathrm{fm}^{-1} \leq k \leq 5\ \mathrm{fm}^{-1}$



Experiments at $1.4 < x_B \leq 2$

A. Schmidt *et al.* (CLAS Collaboration), Nature (2020)





Contacts taken from ab-initio calculations

σ_{CM} taken from previous experiments.

E^*_{A-2} is modified in the range $(0,30)$MeV.

summary

STRONG@HUJI

האוניברסיטה העברית בירושלים
THE HEBREW UNIVERSITY OF JERUSALEM

Thank you !