Ab initio electroweak reactions with nuclei

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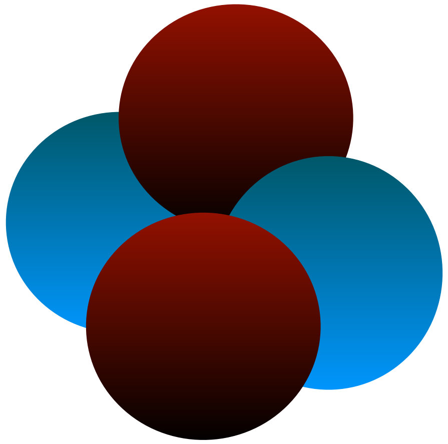
International Symposium on
"Clustering as a window on the hierarchical structure of quantum systems"
Sendai, Japan

Clusters & Hierarchies

Ab initio nuclear theory



- Start from neutrons and protons as building blocks (centre of mass coordinates, spins, isospins)

- Solve the non-relativistic quantum mechanical problem of A-interacting nucleons

$$H|\psi_i\rangle = E_i|\psi_i\rangle$$

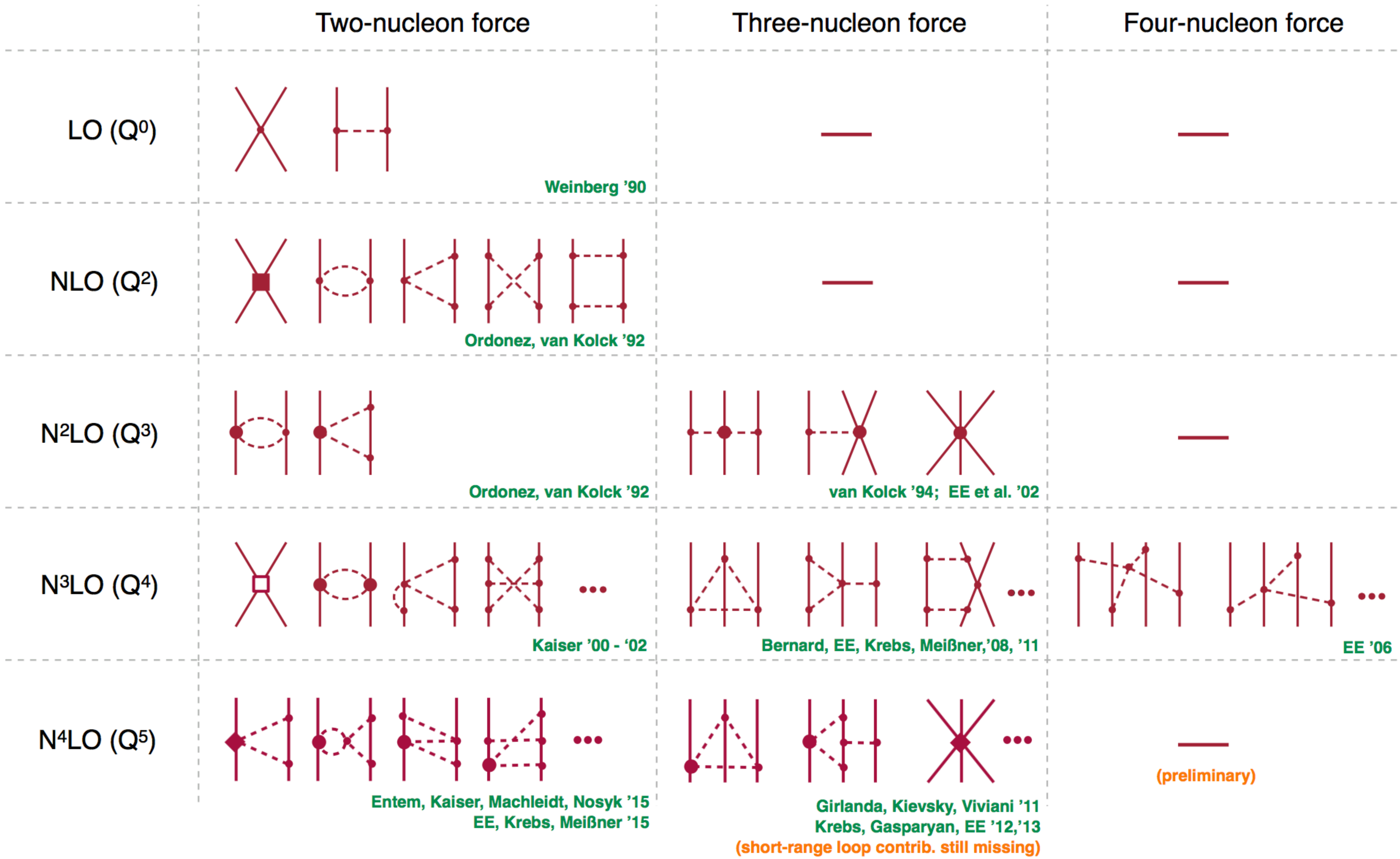
$$H = T + V_{NN}(\Lambda) + V_{3N}(\Lambda) + \ldots$$

using phenomenological potentials or interactions from chiral effective field theory (χEFT)

- Find numerical solutions with no approximations or controllable approximations

Hierarchical Structure of χEFT

from E. Epelbaum (2018), see yesterday's talk



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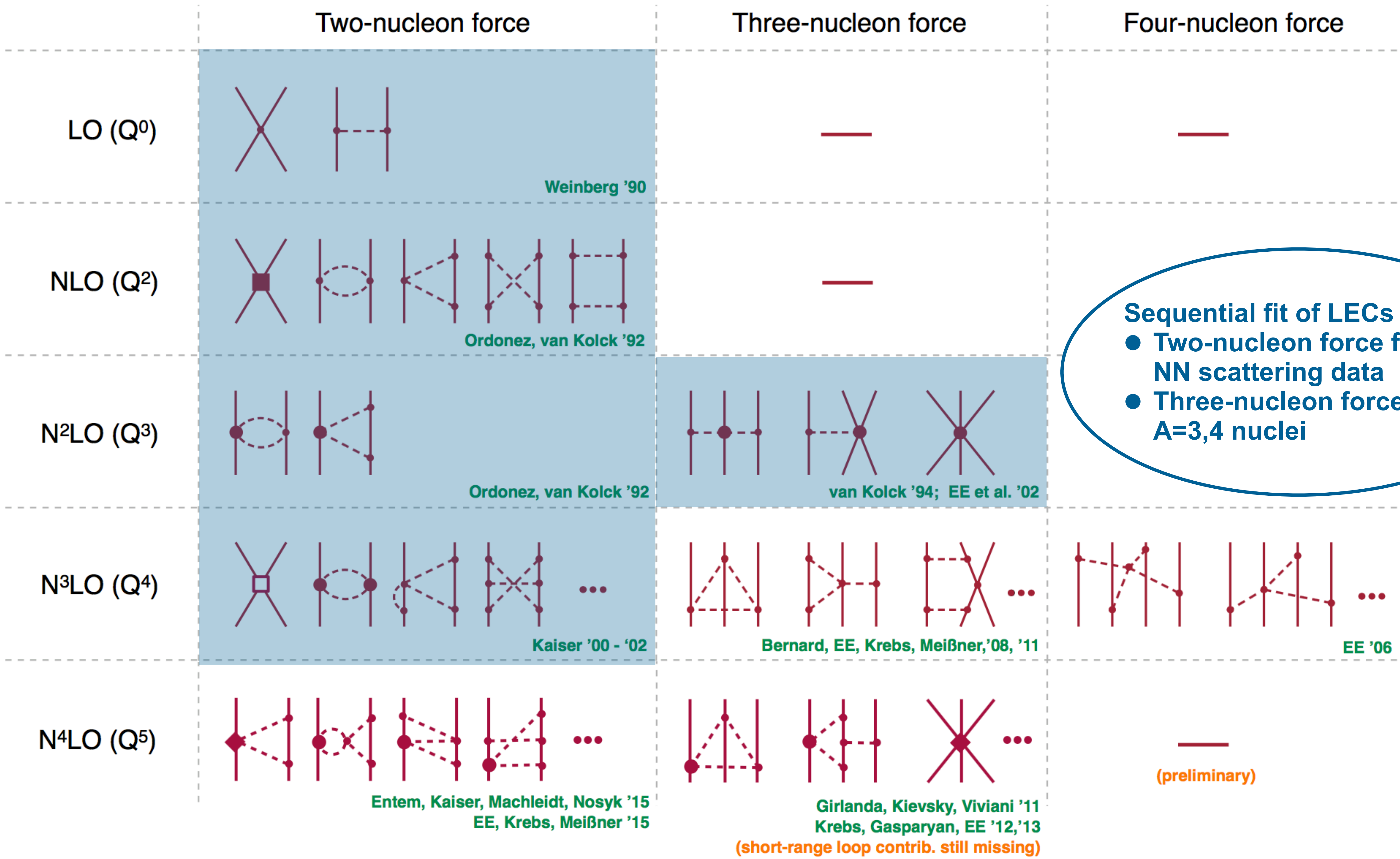
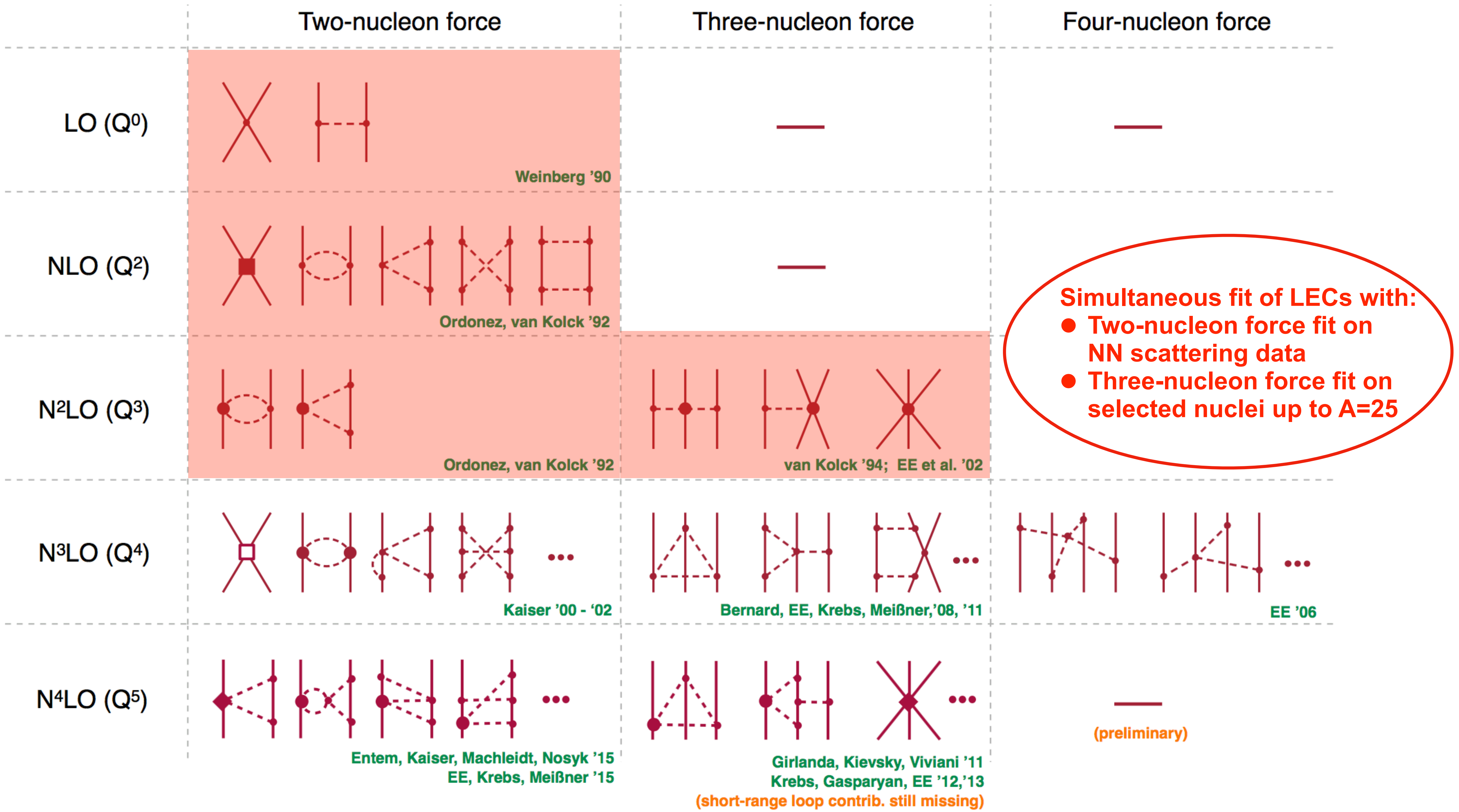


Sequential fit of LECs with:
- **Two-nucleon force fit on NN scattering data**
- **Three-nucleon force fit on A=3,4 nuclei**

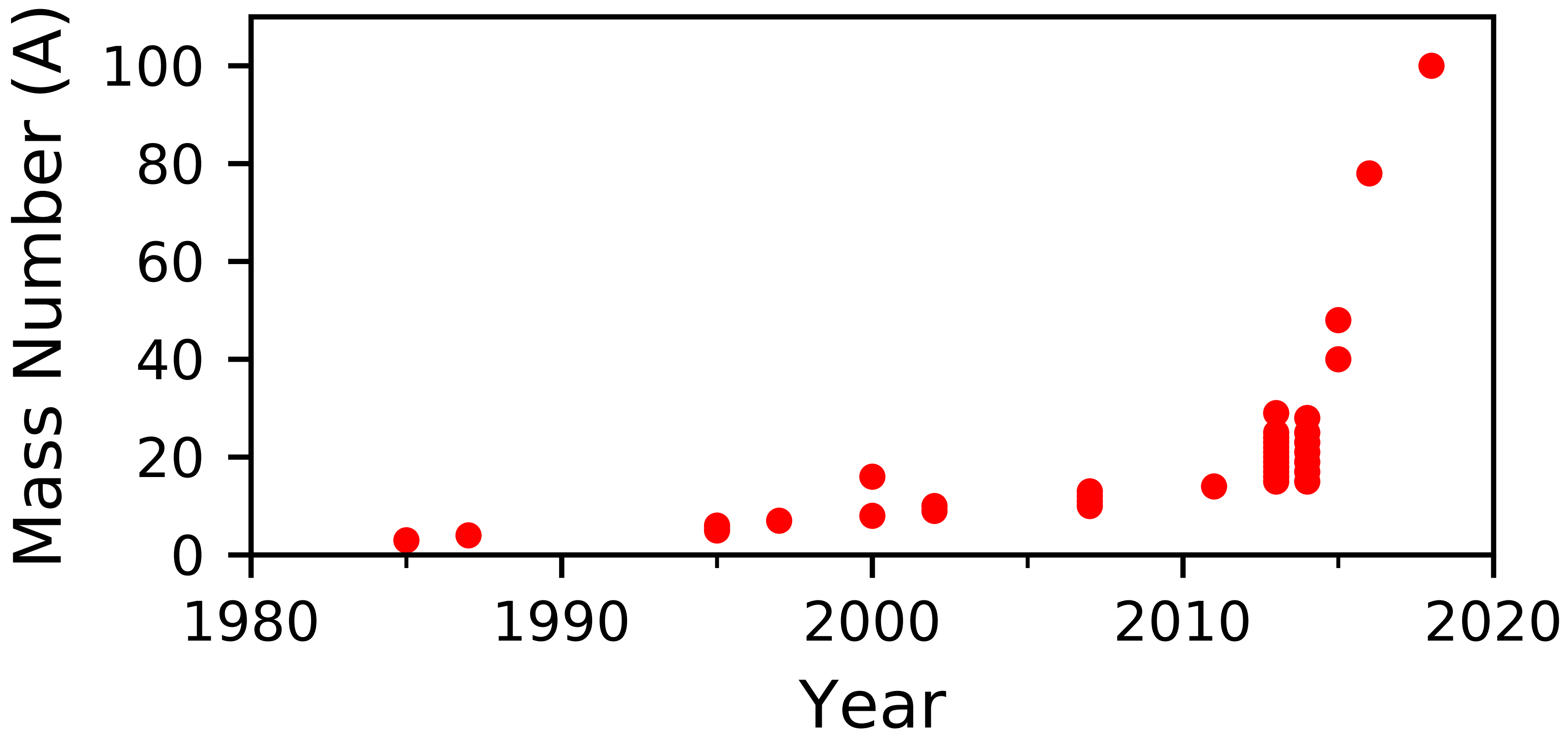
Hierarchical Structure of χEFT

from E. Epelbaum (2018), see yesterday's talk



Progress of ab initio theory

Ab initio calculations starting from
NN+3N interactions



J.Simonis, SB, G.Hagen, Eur. Phys. J. A **55**, 241 (2019).

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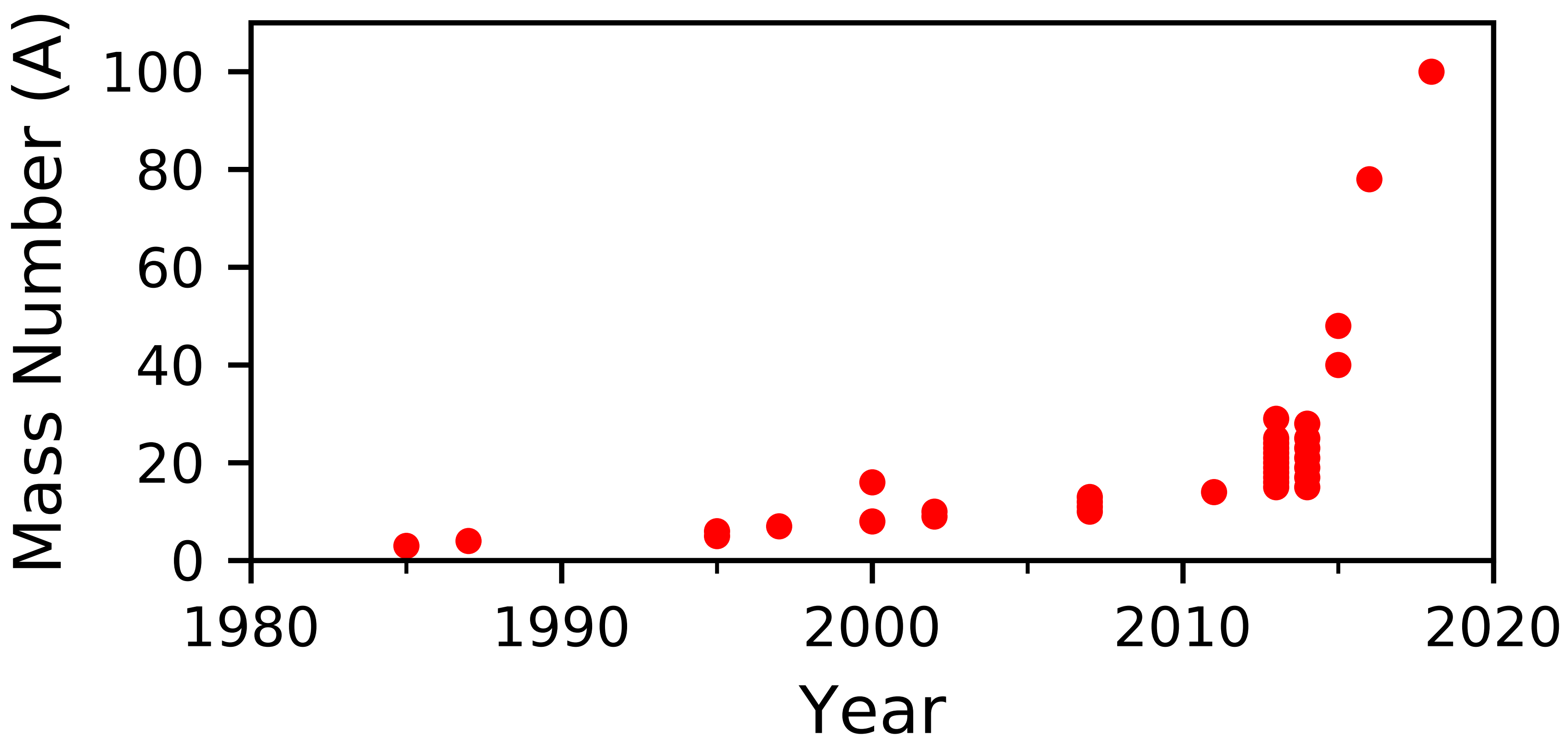
Nature Phys. 18, 1196 (2022)



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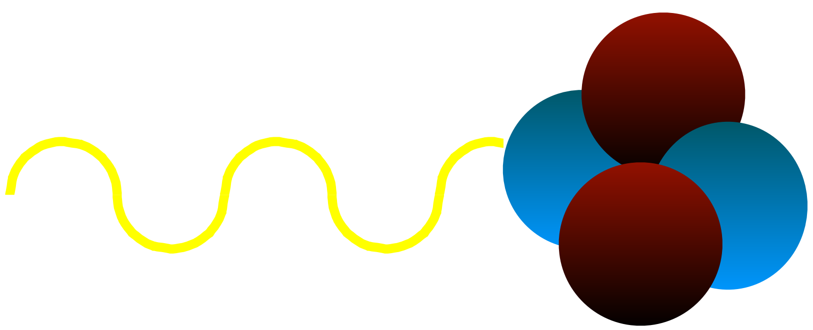
Electroweak reactions



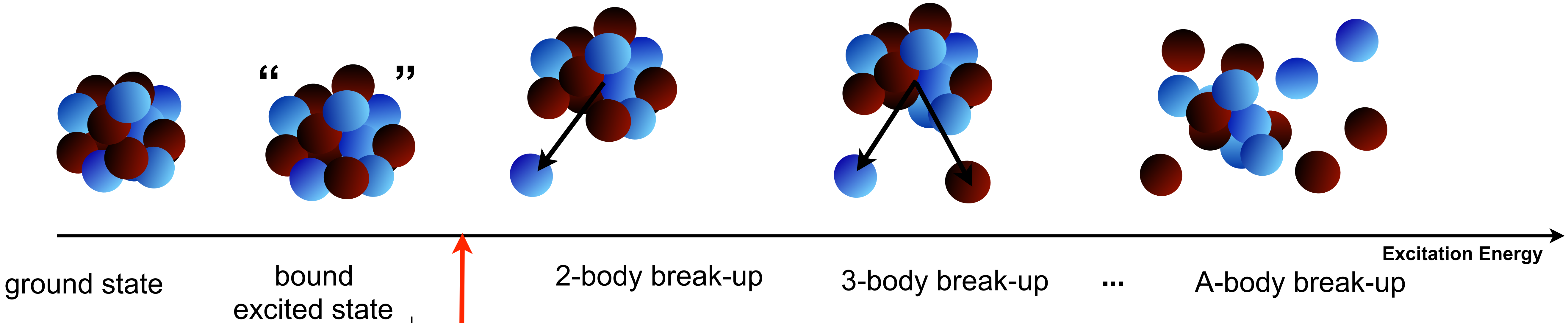
Cross Section $\sigma_{ew} \sim R(\omega) = \sum_f \left| \left\langle \psi_f \left| \Theta \right| \psi_0 \right\rangle \right|^2 \delta(E_f - E_0 - \omega)$

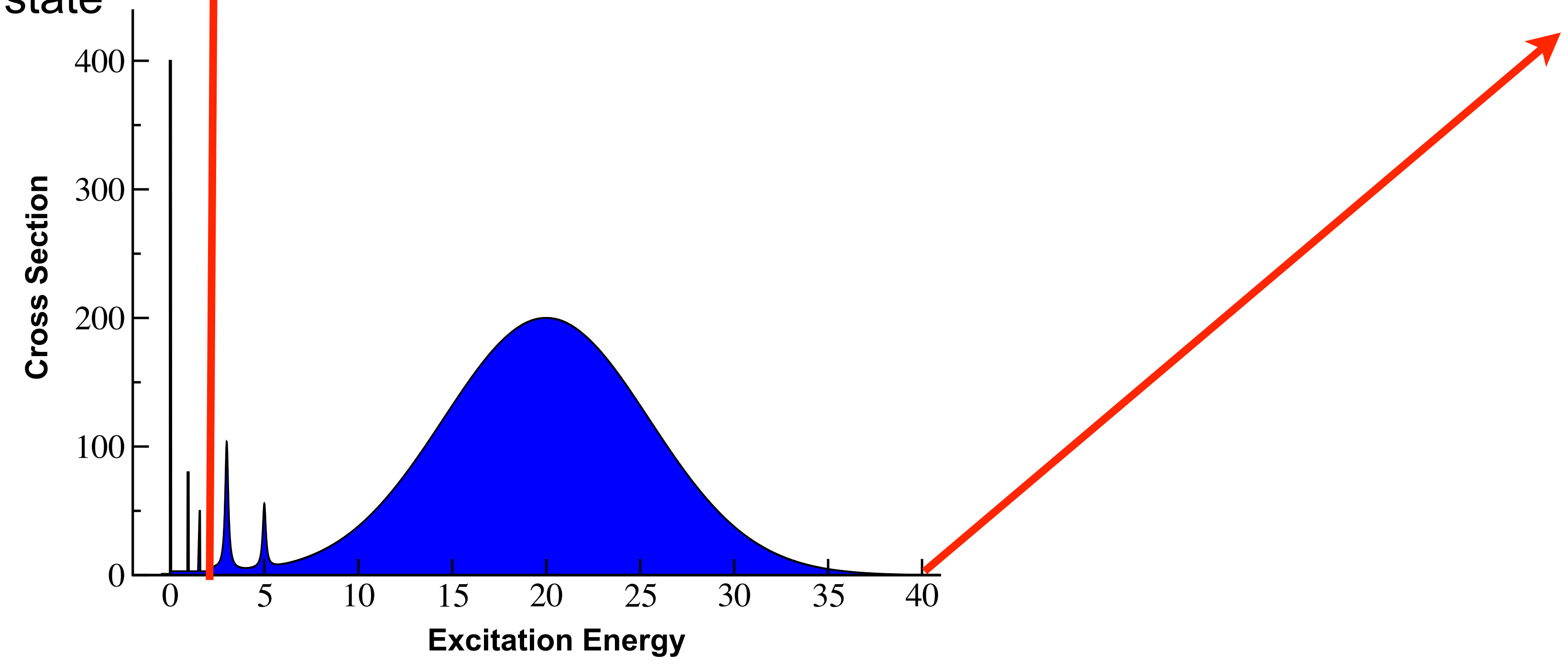
Electroweak operator

The continuum problem

$$R(\omega) = \sum_f \left| \left\langle \psi_f \left| \Theta \right| \psi_0 \right\rangle \right|^2 \delta(E_f - E_0 - \omega)$$

Depending on E_f, many channels may be involved





Lorentz Integral Transform

$$R(\omega) = \sum\!\!\!\!\!\!\!\int_f \left| \left\langle \psi_f \right| \Theta \left| \psi_0 \right\rangle \right|^2 \delta(E_f - E_0 - \omega)$$

Exact knowledge limited in
energy and mass number

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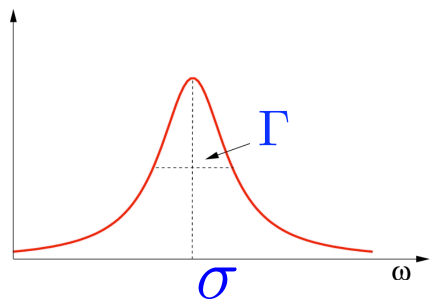
$$L(\sigma,\Gamma) = \frac{\Gamma}{\pi} \int d\omega \frac{R(\omega)}{(\omega - \sigma)^2 + \Gamma^2} = \langle \tilde{\psi} | \tilde{\psi} \rangle$$

Efros, *et al.*, JPG.Nucl.Part.Phys.**34** (2007) R459



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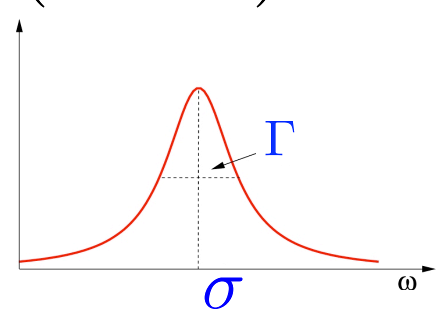

$$(H - E_0 - \sigma + i\Gamma) \mid \tilde{\psi} \rangle = \Theta \mid \psi_0 \rangle$$

Bound-state-like equation

Lorentz Integral Transform

$$R(\omega) = \sum_f \left| \left\langle \psi_f \right| \Theta \left| \psi_0 \right\rangle \right|^2 \delta(E_f - E_0 - \omega)$$

Exact knowledge limited in energy and mass number

inversion

$$L(\sigma, \Gamma) = \frac{\Gamma}{\pi} \int d\omega \frac{R(\omega)}{(\omega - \sigma)^2 + \Gamma^2} = \langle \tilde{\psi} | \tilde{\psi} \rangle$$

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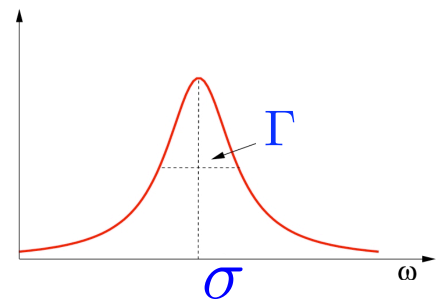

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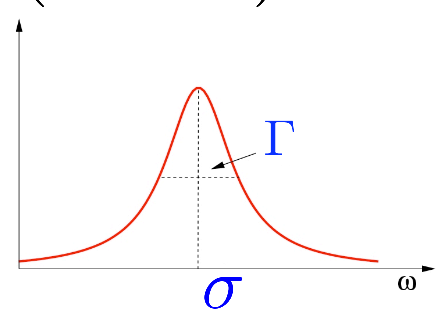

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Bound-state-like equation

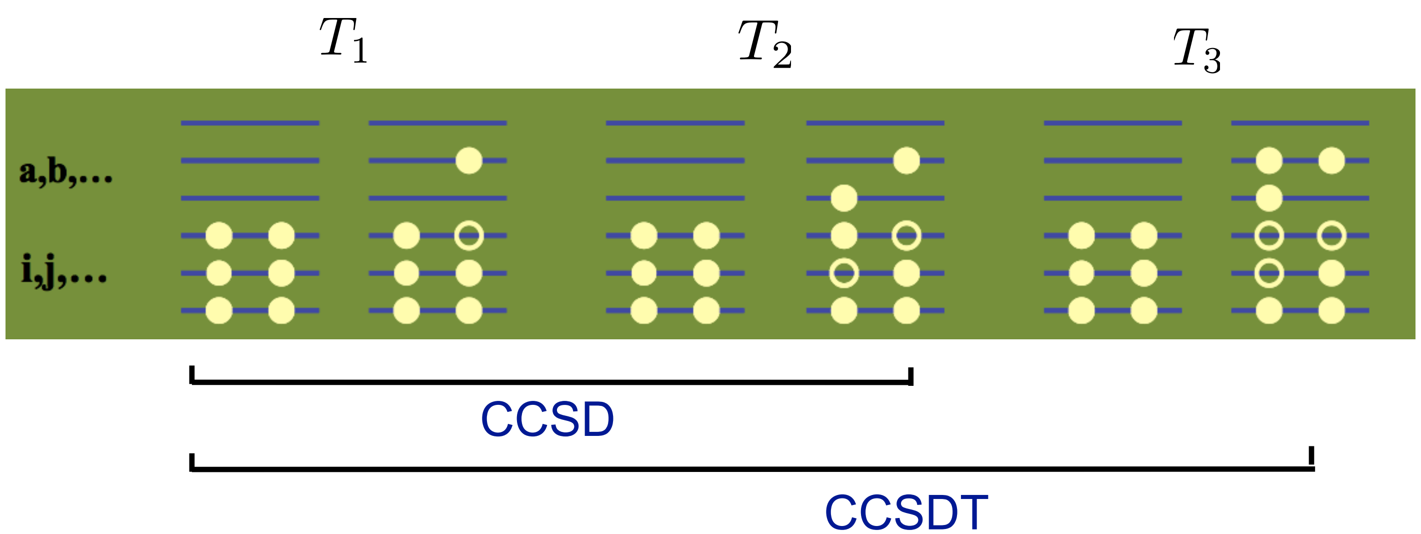
Solved with:

- Hyper-spherical Harmonics
- No core shell model
- Coupled Cluster theory

Coupled-cluster theory

$$|\psi_0(\vec{r}_1, \vec{r}_2, ..., \vec{r}_A)\rangle = e^T |\phi_0(\vec{r}_1, \vec{r}_2, ..., \vec{r}_A)\rangle \qquad T = \sum T_{(A)}$$

cluster expansion



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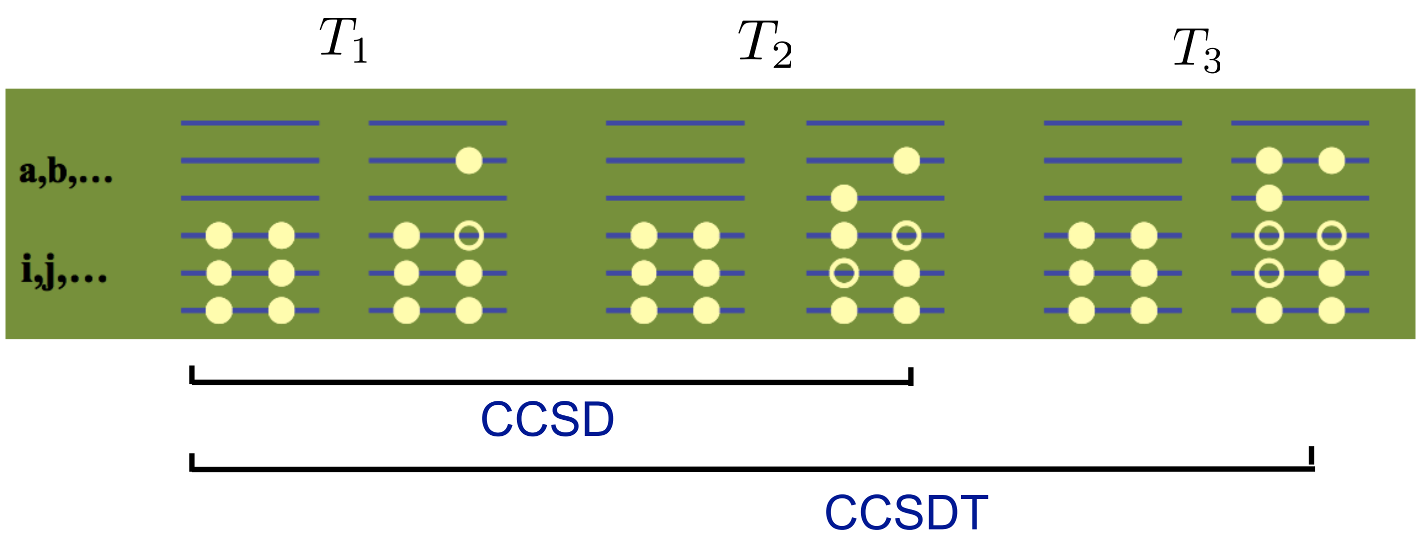

SB *et al.*, Phys. Rev. Lett. **111**, 122502 (2013)

$$(\bar{H} - E_0 - \sigma + i\Gamma)|\tilde{\Psi}_R\rangle = \bar{\Theta}|\Phi_0\rangle$$

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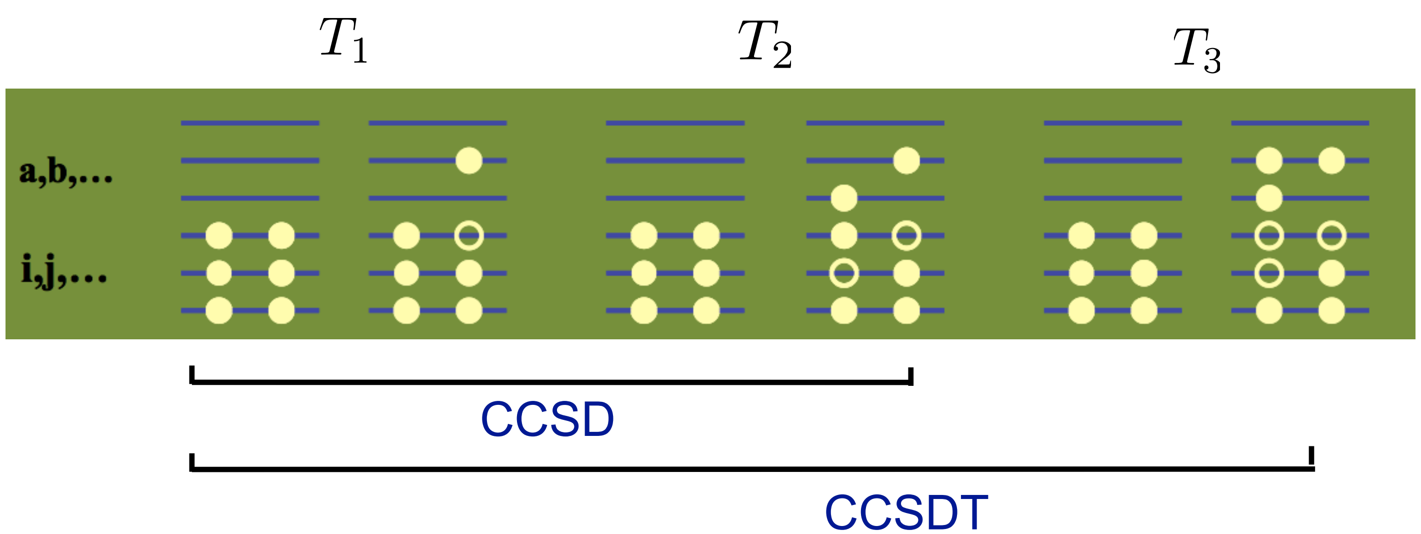

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$$(\bar{H} - E_0 - \sigma + i\Gamma)|\tilde{\Psi}_R\rangle = \bar{\Theta}|\Phi_0\rangle$$

$$\begin{cases} \bar{H} = e^{-T}He^{T} \\ \bar{\Theta} = e^{-T}\Theta e^{T} \\ |\tilde{\Psi}_R\rangle = \hat{R}|\Phi_0\rangle \end{cases}$$

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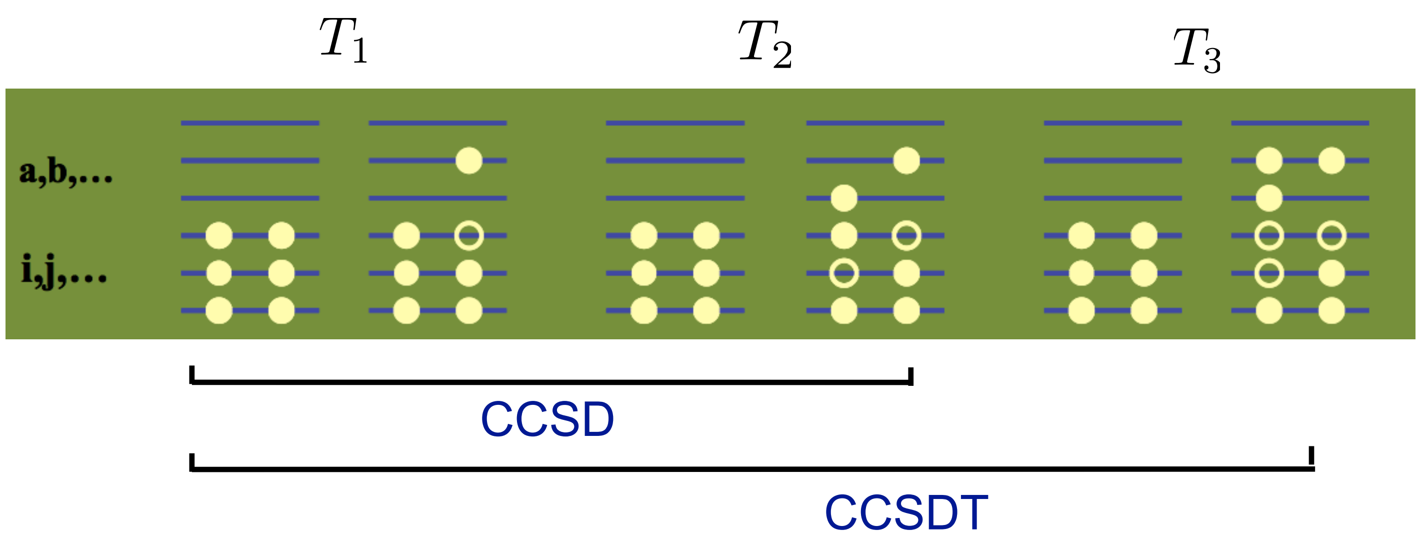

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$$(\bar{H}-E_0-\sigma+i\Gamma)|\tilde{\Psi}_R\rangle = \bar{\Theta}|\Phi_0\rangle \quad\rightarrow\quad \begin{cases} \bar{H}=e^{-T}He^{T} \\ \bar{\Theta}=e^{-T}\Theta e^{T} \\ |\tilde{\Psi}_R\rangle=\hat{R}|\Phi_0\rangle \end{cases}$$

$$\mathcal{R}(z)=r_0(z)+\sum_{ai} r_i^a(z)a_a^\dagger a_i+\frac{1}{4}\sum_{abij} r_{ij}^{ab}(z)a_a^\dagger a_b^\dagger a_j a_i+...$$

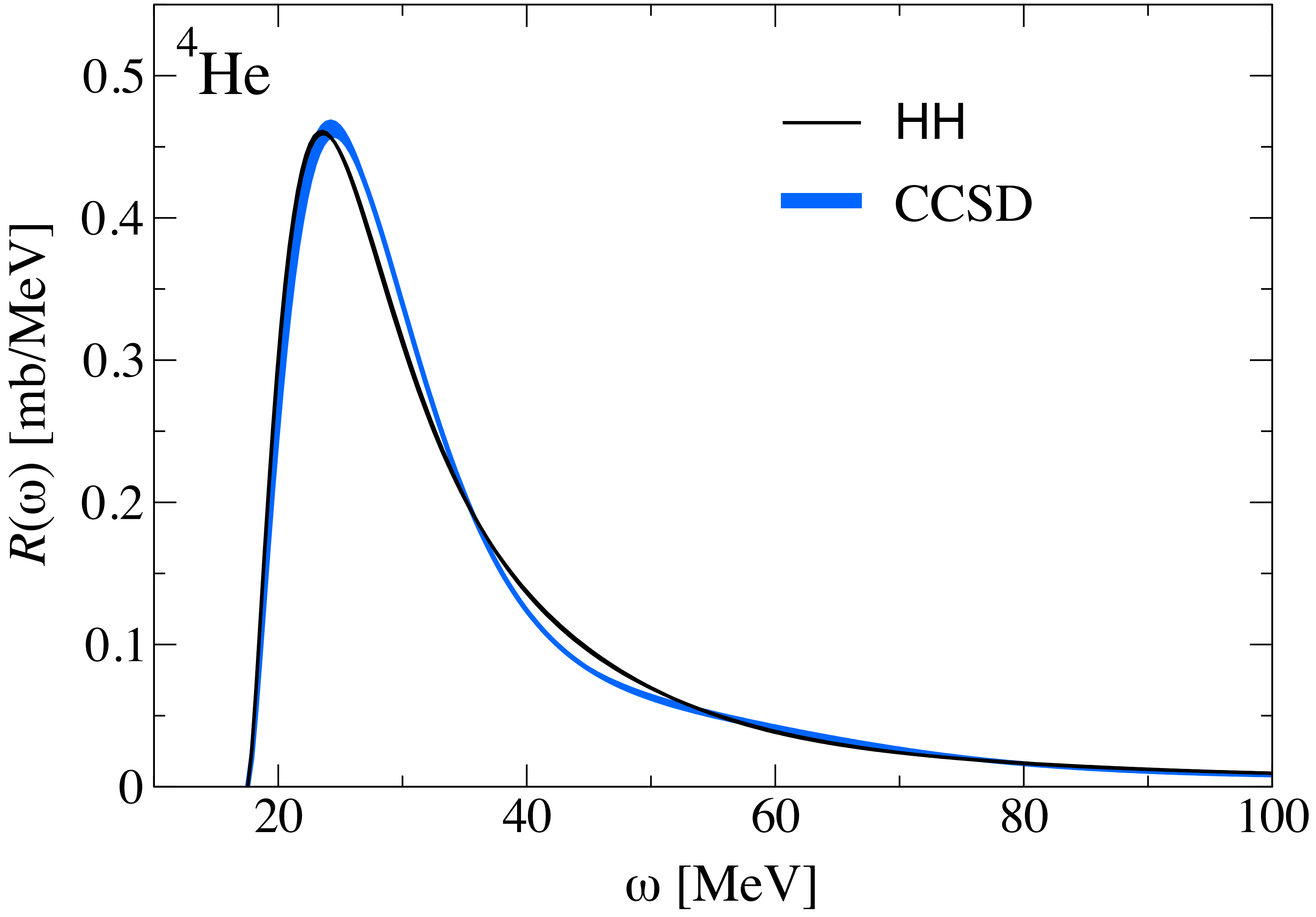
Results with implementation at CCSD level + some study of triples contributions

Validation in ^{4}He

Dipole response function

Comparison of CCSD with exact hyperspherical harmonics with NN forces at N^3LO

SB *et al.*, Phys. Rev. Lett. **111**, 122502 (2013)

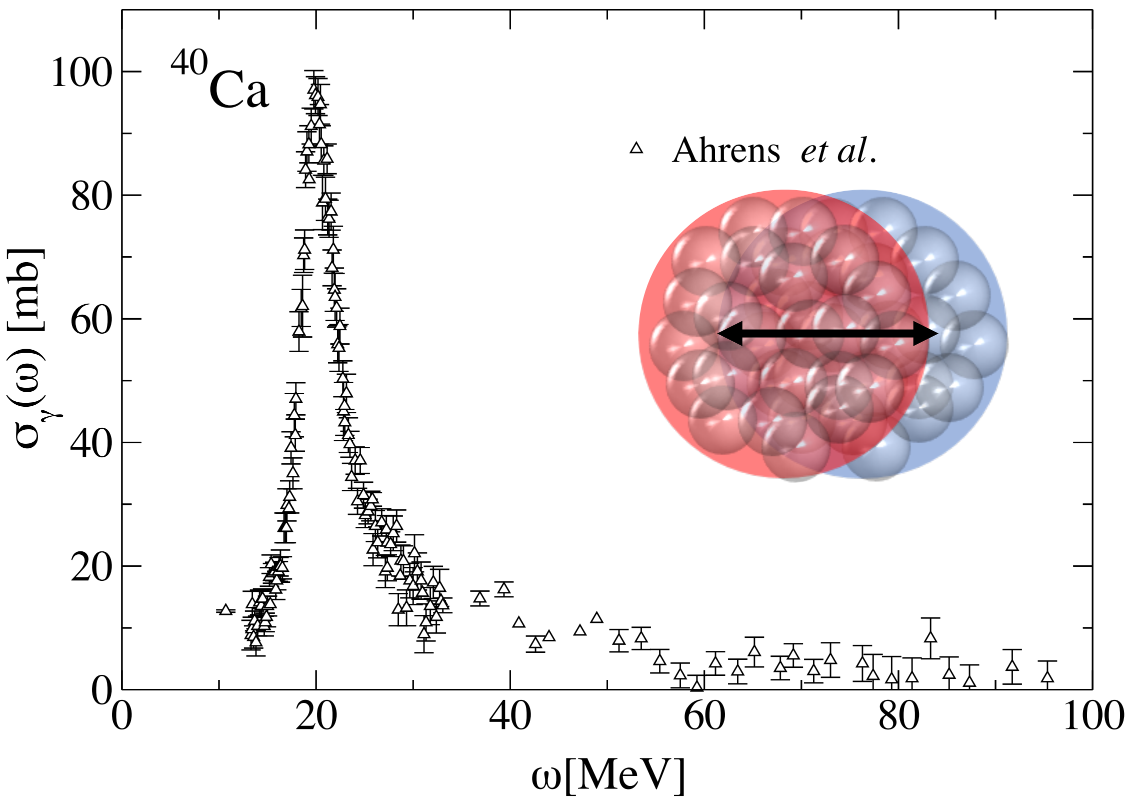


Emergence of structures in nuclei

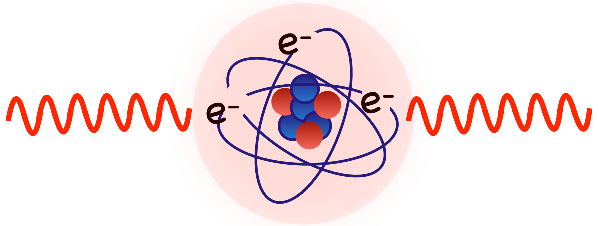
Stable Nuclei

We have data on ~180 stable nuclei

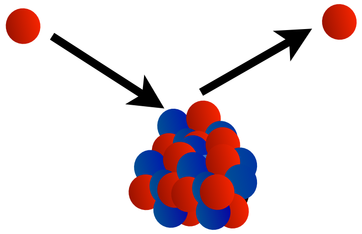
Giant dipole resonances



From photoabsorption experiments



(p,p') experiments



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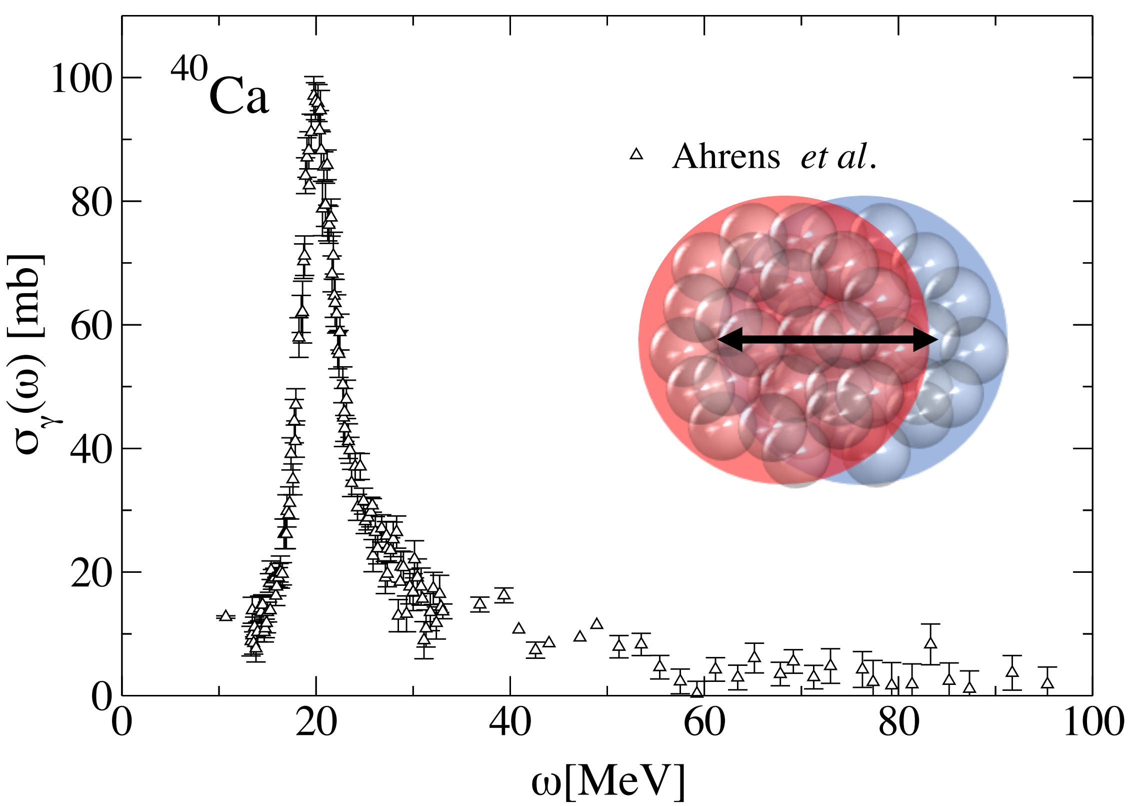

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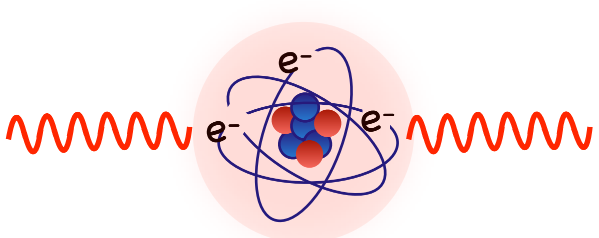

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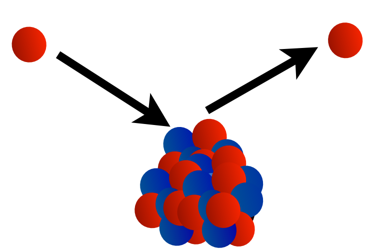
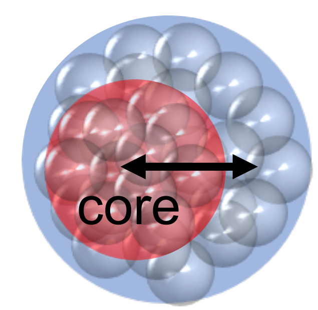
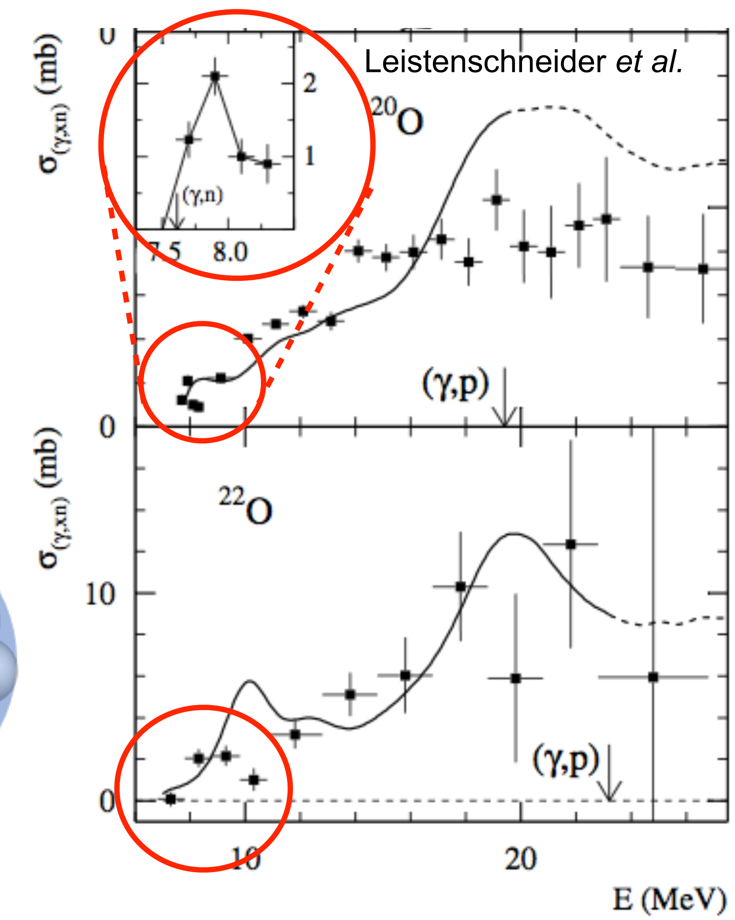


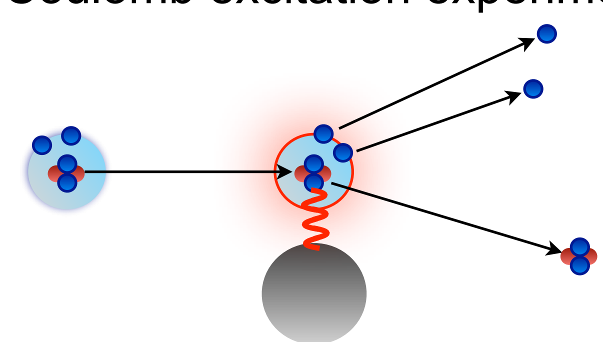
Unstable Nuclei

Fewer data, pigmy dipole resonances





From Coulomb excitation experiments



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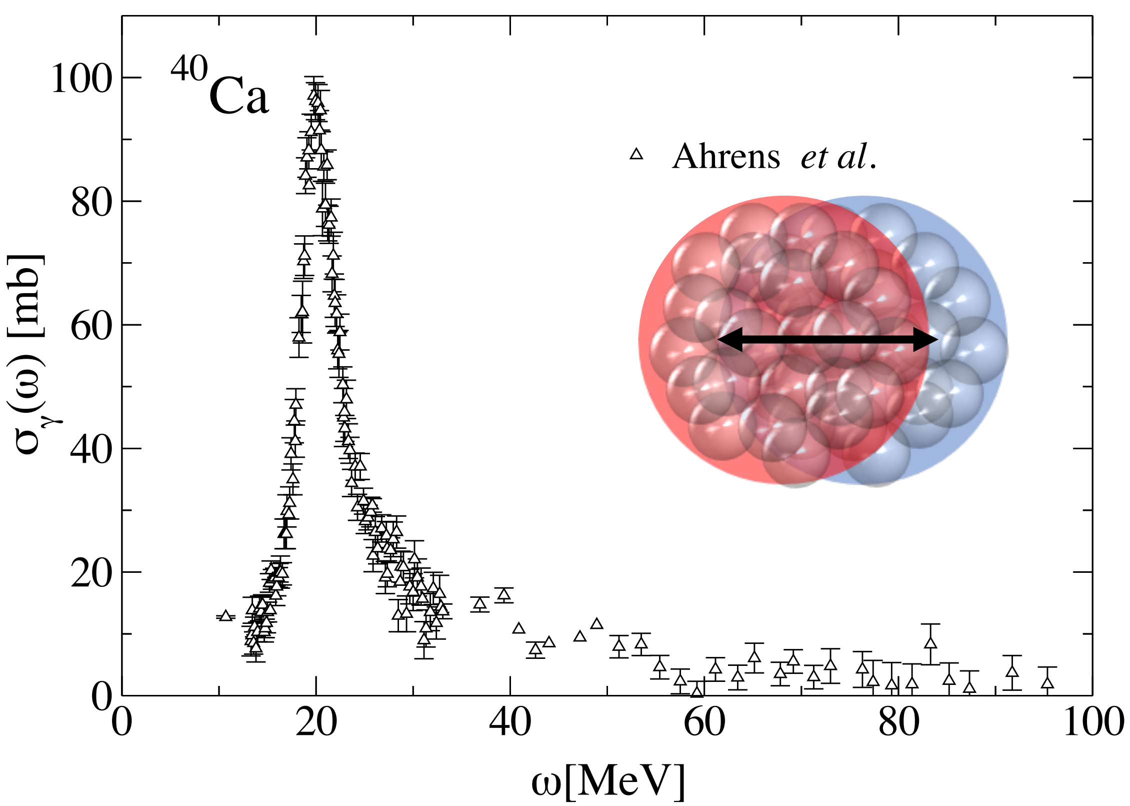

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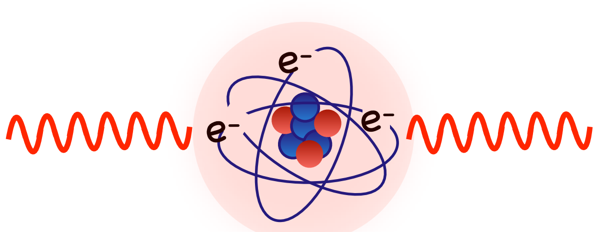
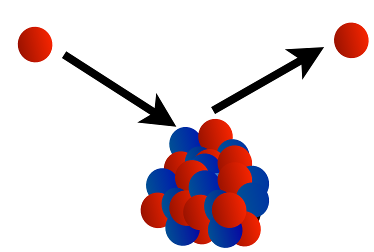


(p,p') experiments



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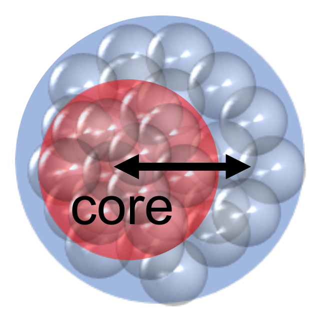

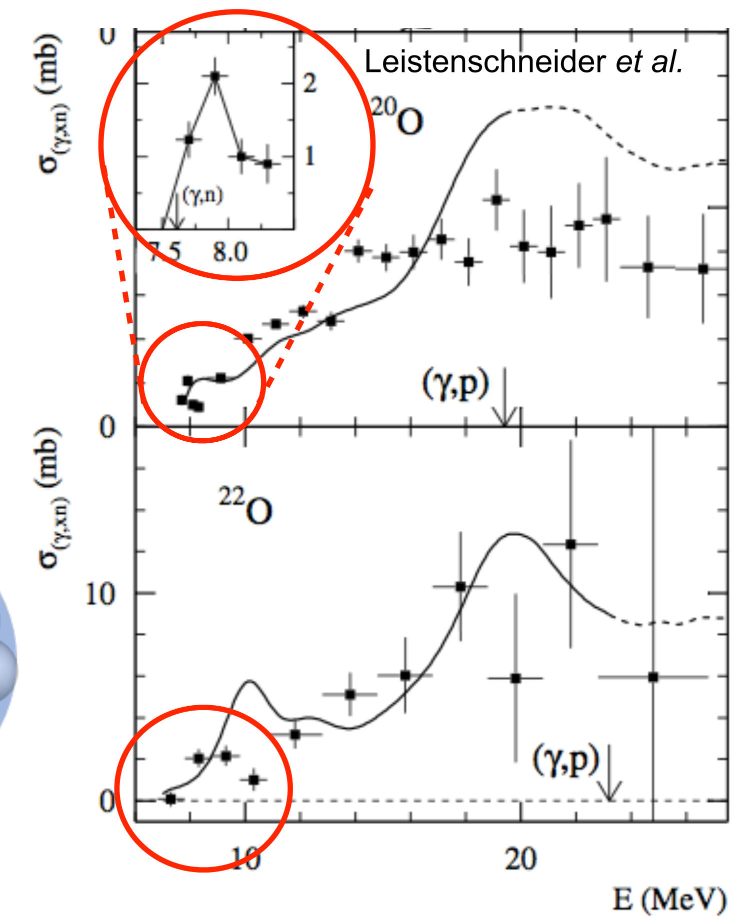

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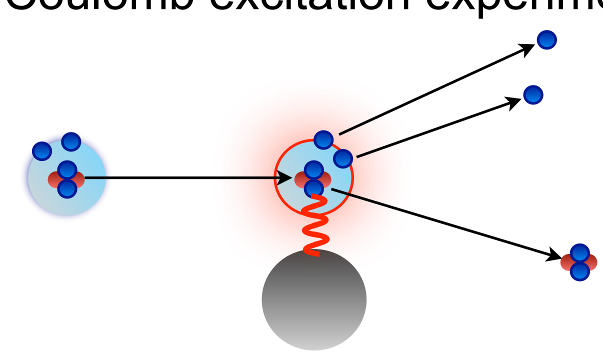
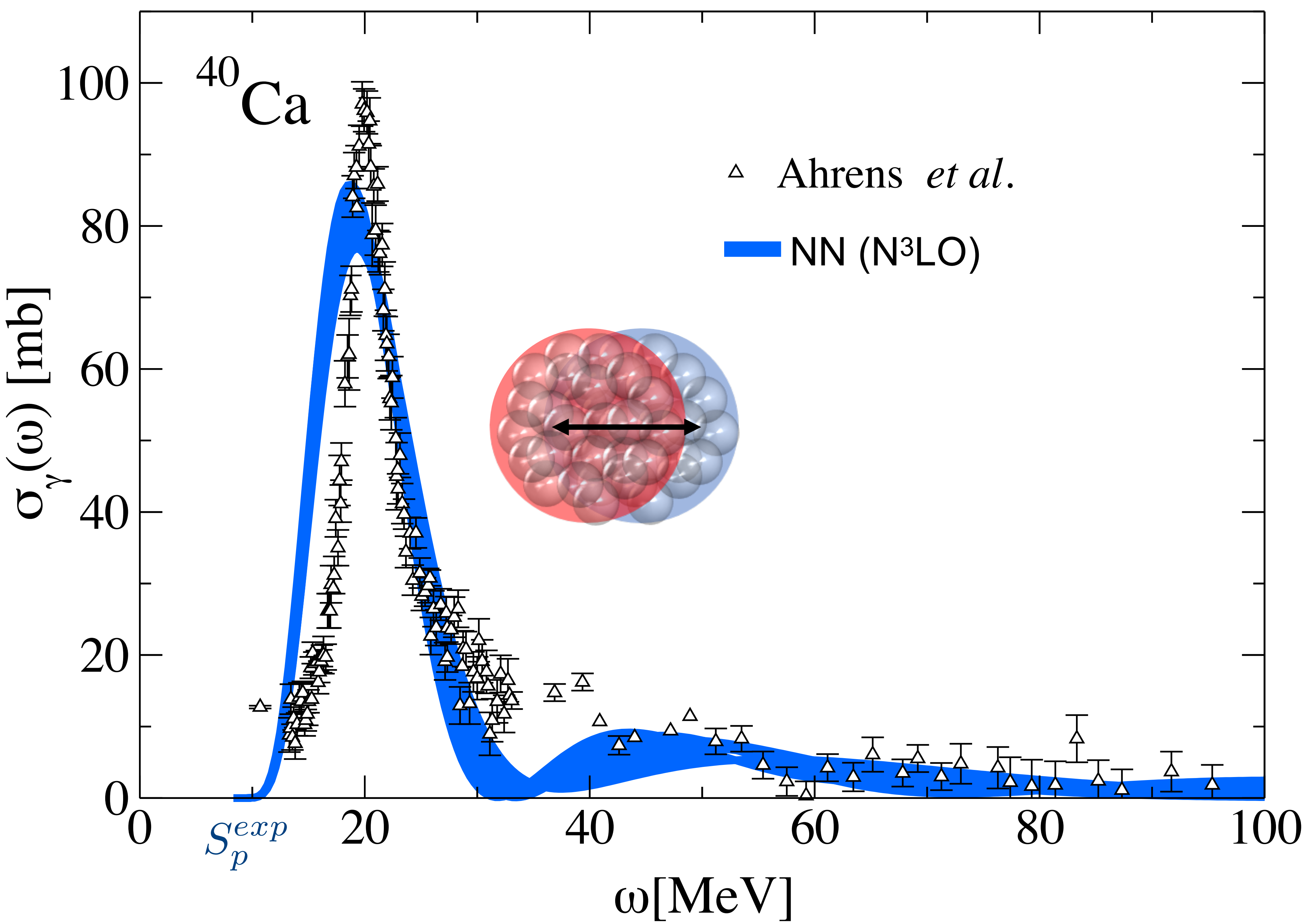


Do we see the emergence of collective motions from first principle calculations?

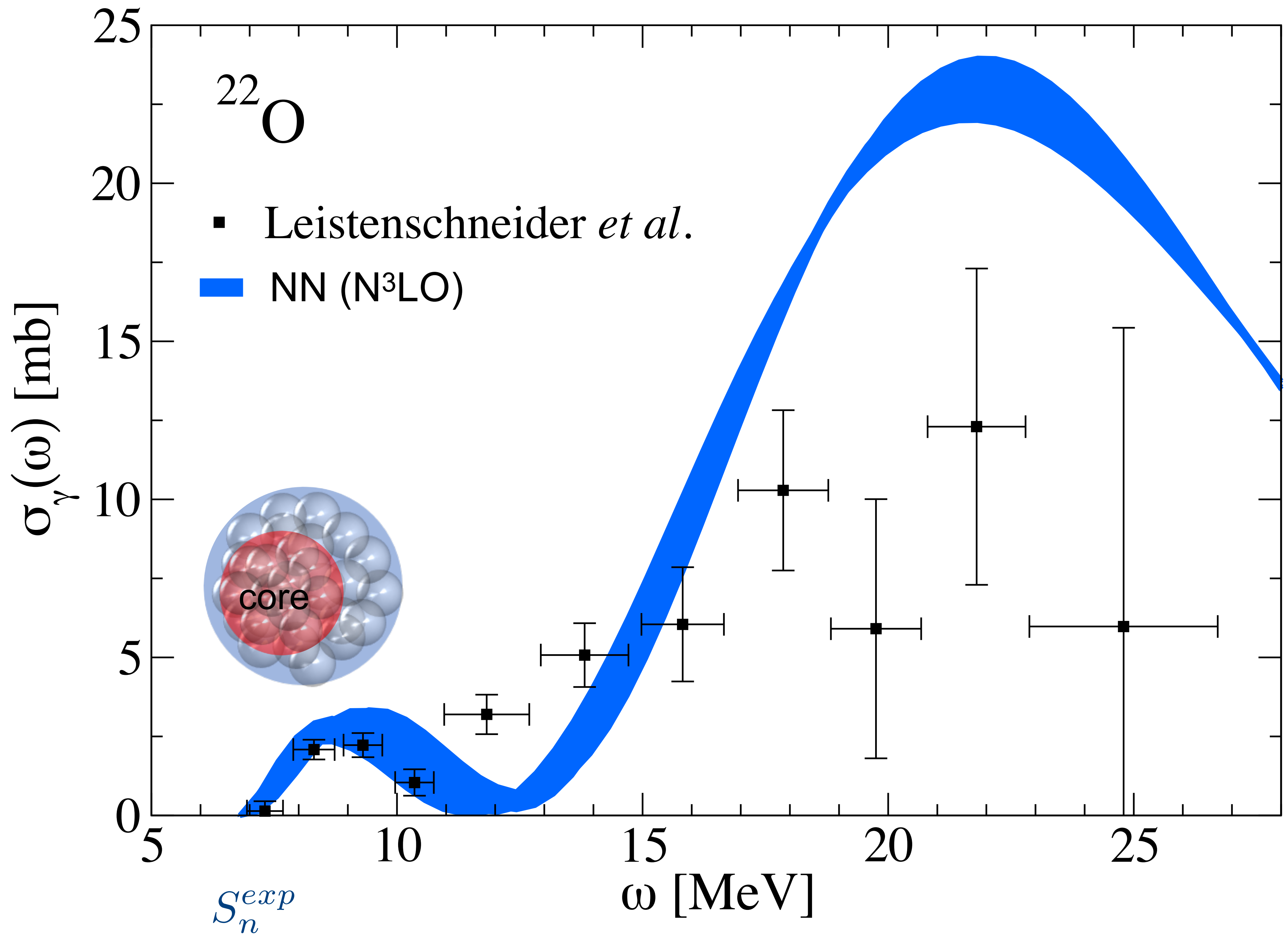
Giant dipole resonances

SB et al., PRC **90**, 064619 (2014)



Pygmy dipole resonance

SB et al., PRC **90**, 064619 (2014)



Connections to astrophysics

Nuclear Equation of State

$$E(\rho,\delta) = E(\rho,0) + S(\rho)\delta^2 + \mathcal{O}(\delta^4) \qquad \rho = \rho_n + \rho_p, \quad \delta = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$$

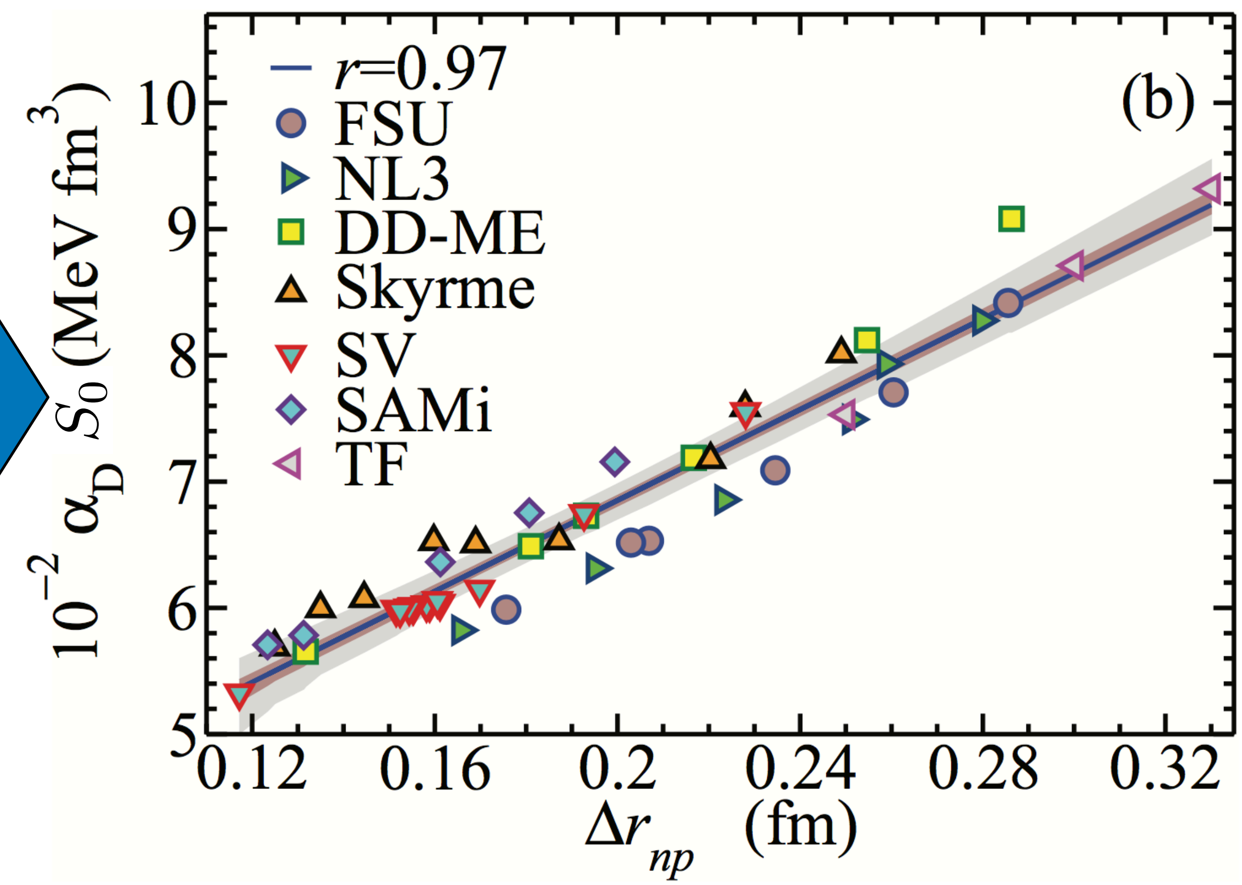
$$S(\rho) = S_0 + \frac{L}{3\rho_0}(\rho - \rho_0) + \frac{K_{sym}}{18\rho_0^2}(\rho-\rho_0)^2 + \dots$$

Symmetry energy at saturation density

Slope parameter, related to pressure of pure neutron matter at saturation density

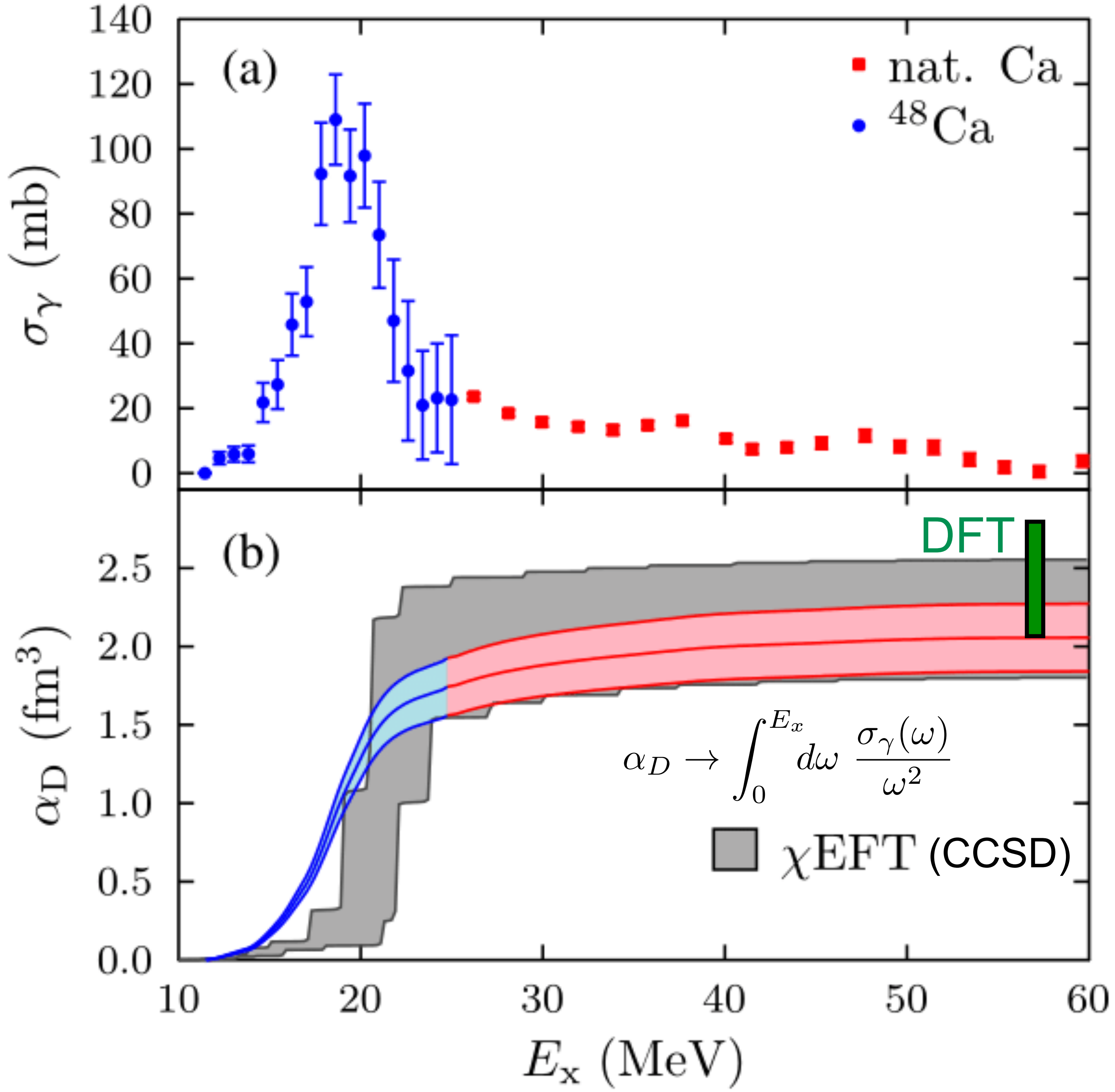
Laboratory measurements on finite nuclei are crucial

Roca-Maza et al.,Phys. Rev. C **88**, 024316 (2013)

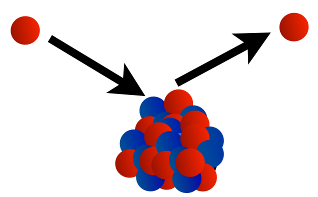


The ^{48}Ca nucleus

Birkhan, Miorelli, SB *et al.*, Phys. Rev. Lett. **118,** 252501 (2017)



RCPN (p,p') experiment



New constraint on the EOS

$$28.5 \leq S_0 \leq 33.3 \quad \text{MeV}$$
$$43.8 \leq L \leq 48.6 \quad \text{MeV}$$

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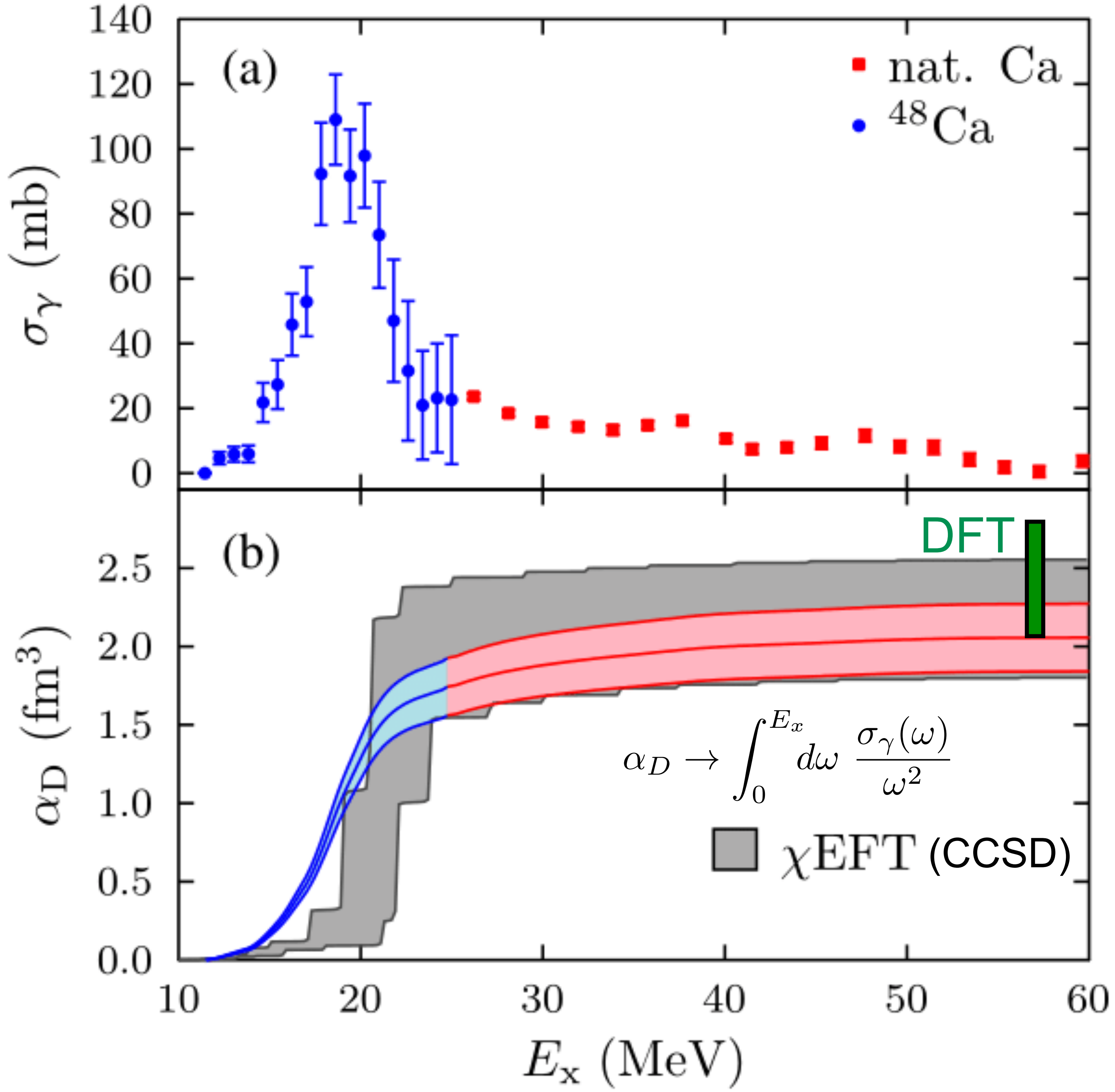

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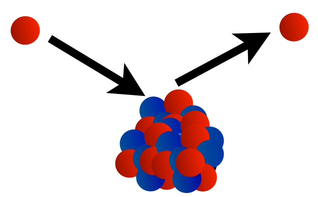

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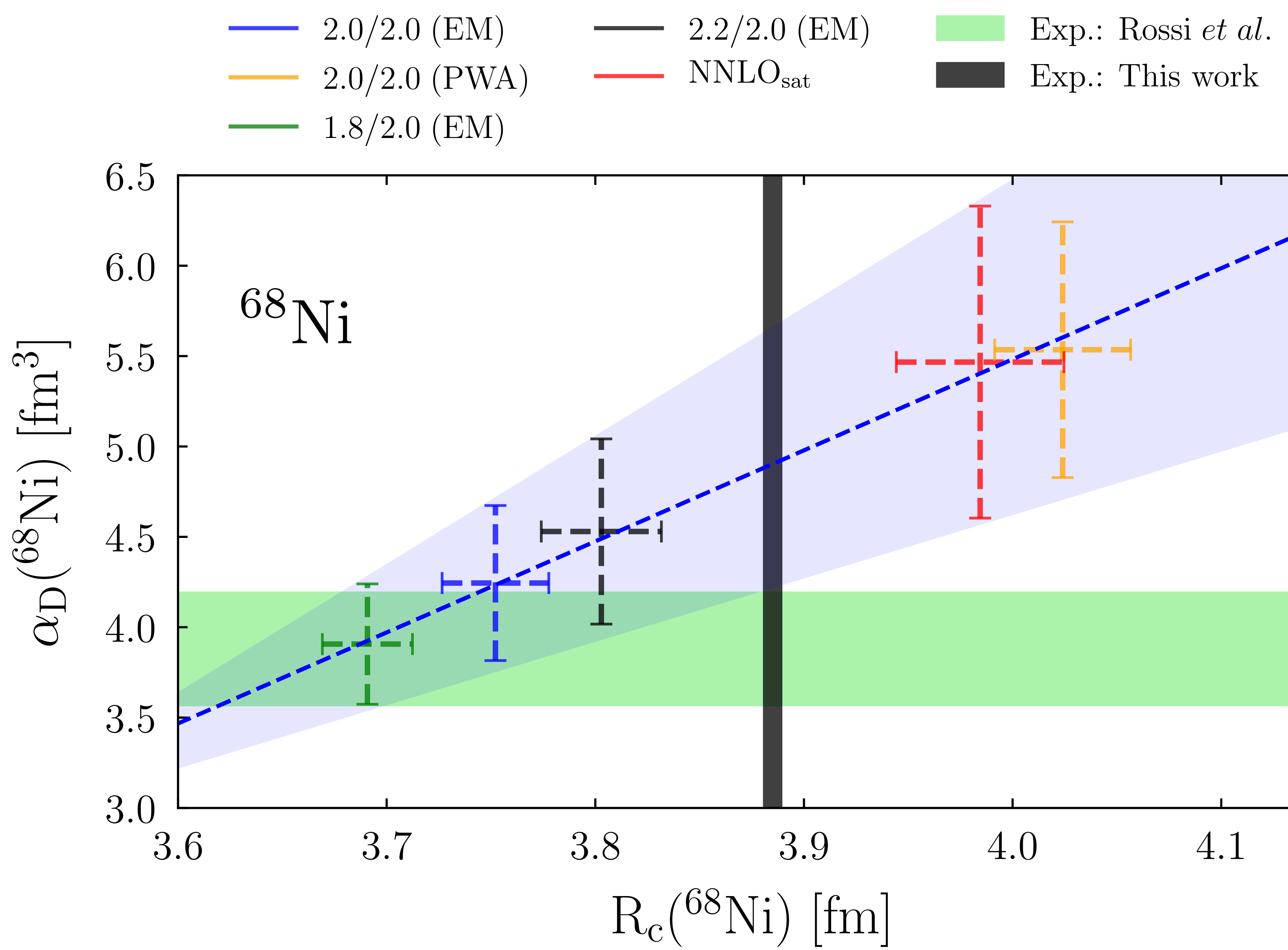
Improved correlations: CCSD-T1

Miorelli, SB et al., PRC **98**, 014324 (2018)

Simonis, SB, Hagen, EPJA **55**, 241 (2019)

The ^{68}Ni nucleus

S.Kaufmann, J. Simonis, SB *et al.*, PRL **104** (2020) 132505



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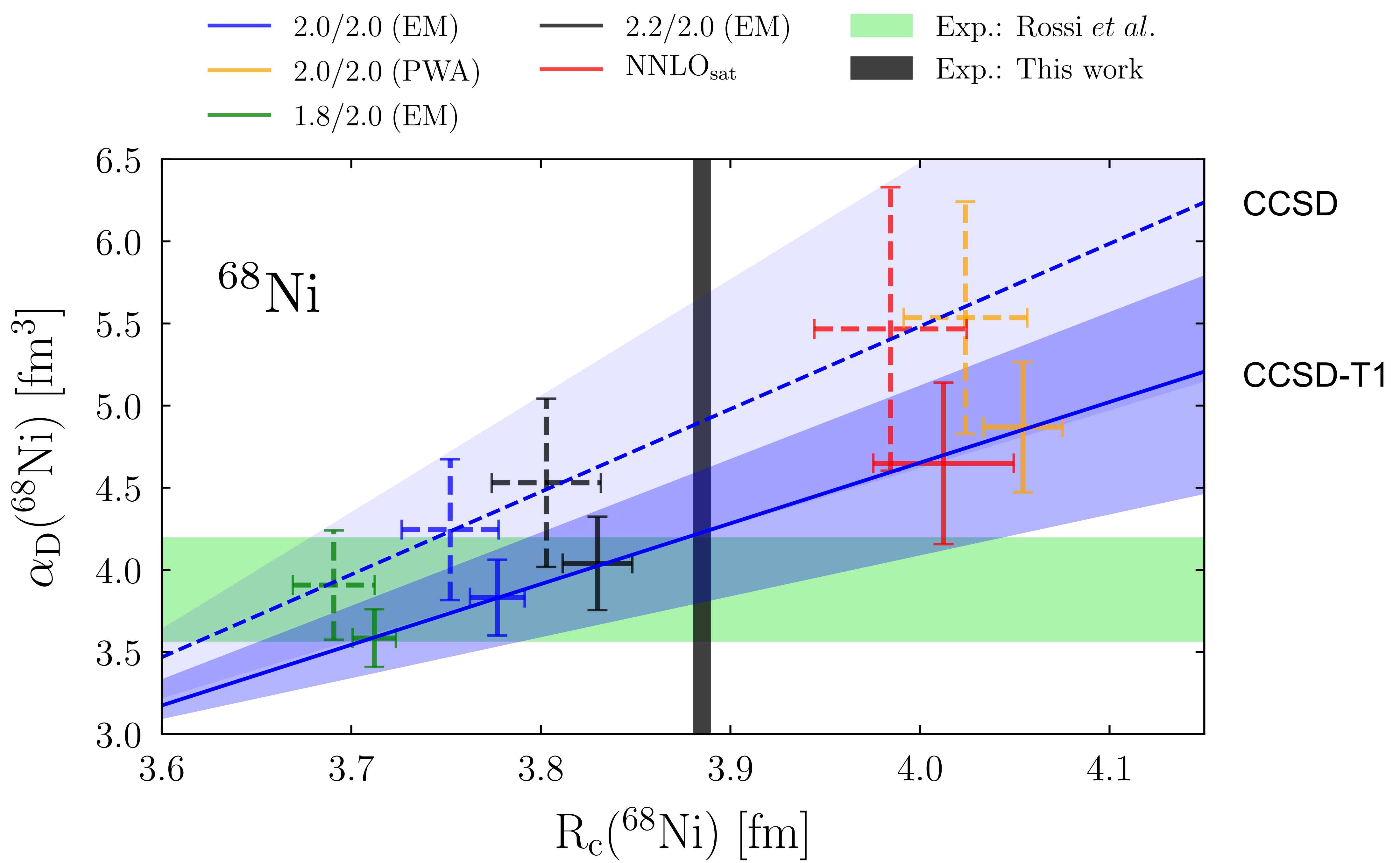
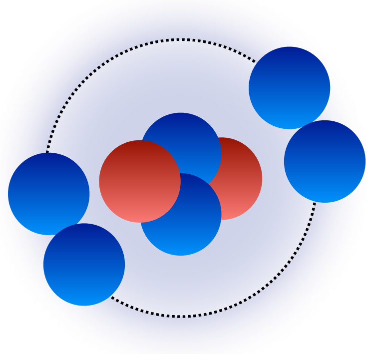
S.Kaufmann, J. Simonis, SB *et al.*, PRL **104** (2020) 132505

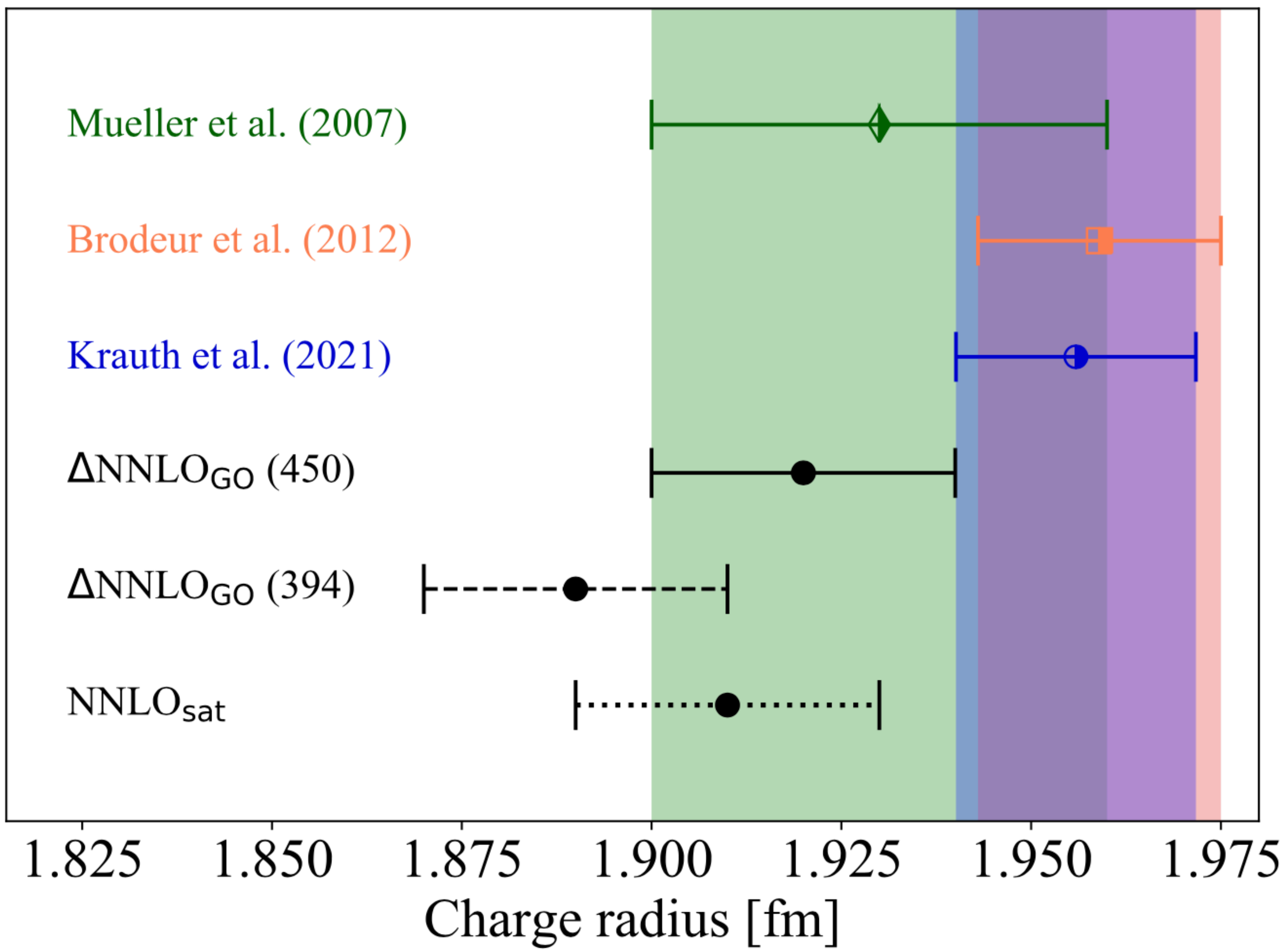


The Halo ^8He nucleus

^8He



F. Bonaiti, SB, G.Hagen, PRC 105, 034313 (2022)



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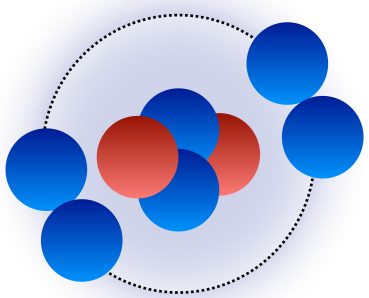
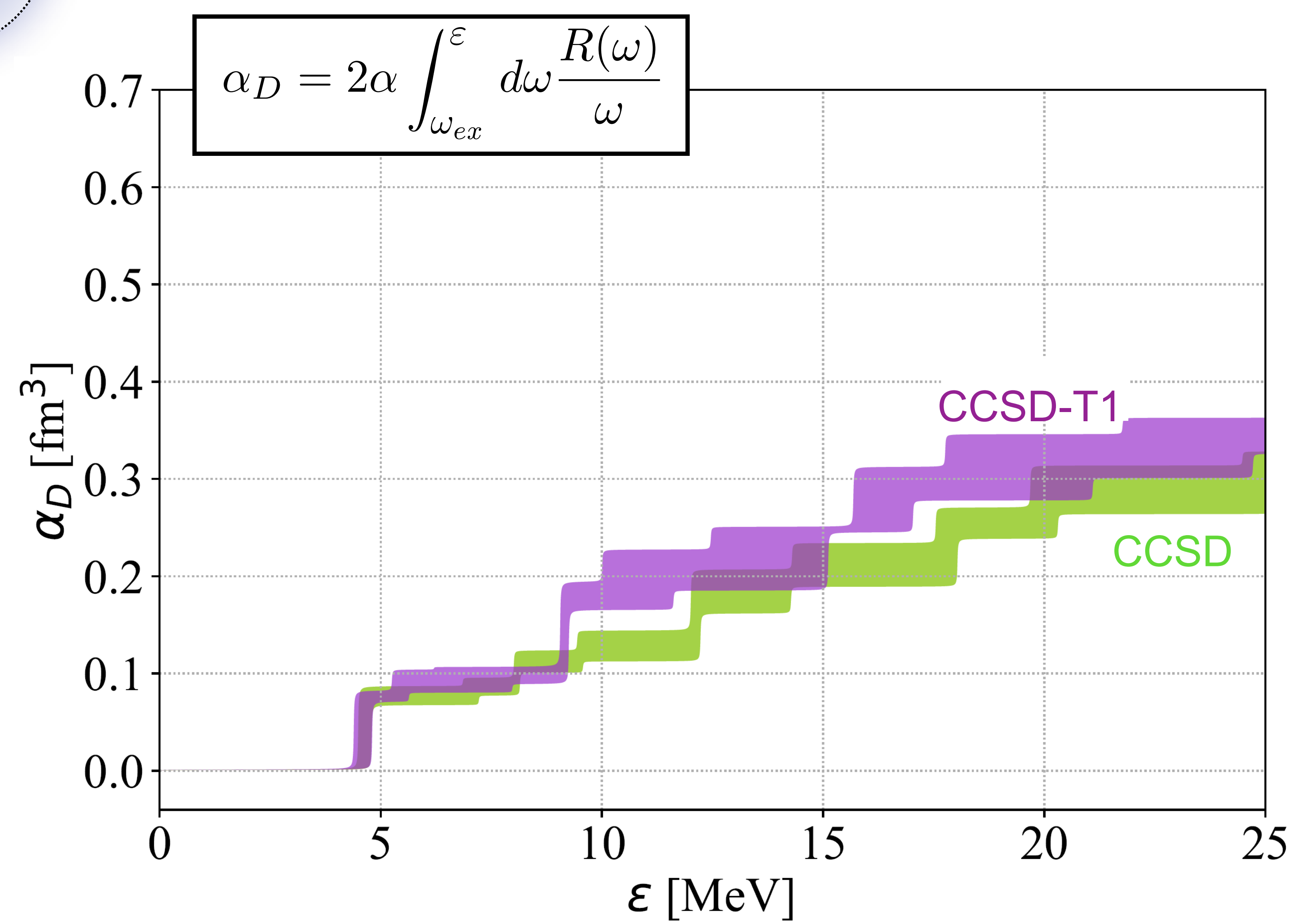


Electric dipole polarizability, running sum

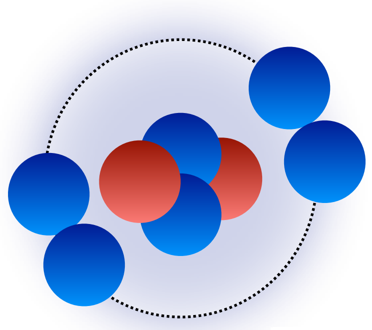
F. Bonaiti, SB, G.Hagen, PRC 105, 034313 (2022)



Theory:
brand spanning
$NNLO_{sat}$
$\Delta NNLO_{GO}$

The Halo ^8He nucleus

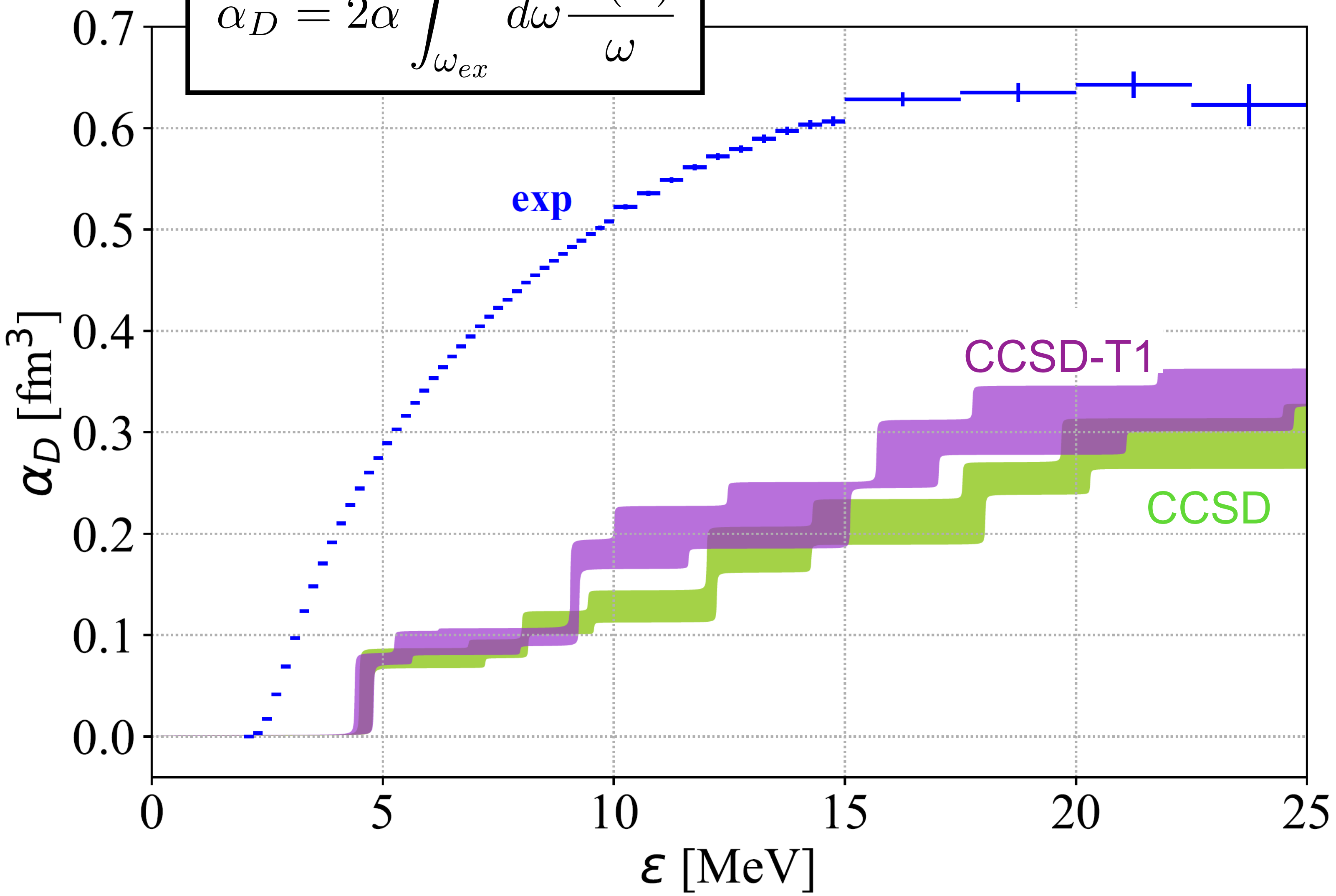
^{8}He



Electric dipole polarizability, running sum

F. Bonaiti, SB,G.Hagen, PRC 105, 034313 (2022)

$$\alpha_D = 2\alpha \int_{\omega_{ex}}^{\varepsilon} d\omega \frac{R(\omega)}{\omega}$$

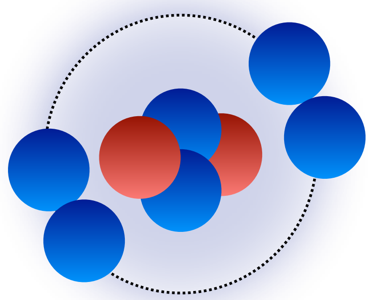


Exp (preliminary)
C. Lehr et al.
SAMURAI
collaboration

Theory:
brand spanning
NNLO$_{sat}$
ΔNNLO$_{GO}$

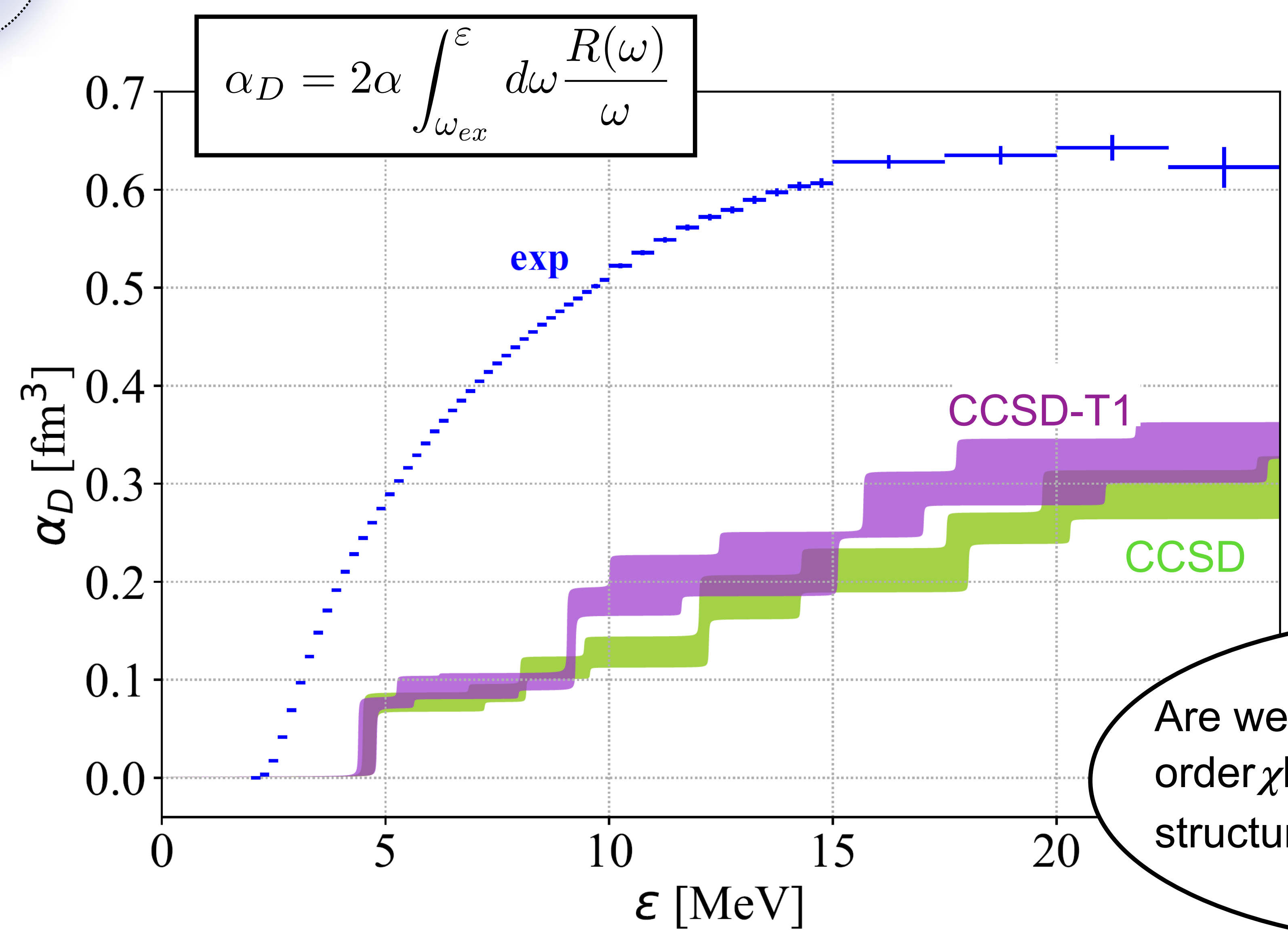
The Halo ^{8}He nucleus

^{8}He



Electric dipole polarizability, running sum

F. Bonaiti, SB,G.Hagen, PRC 105, 034313 (2022)



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C. Lehr et al.
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collaboration

Theory:
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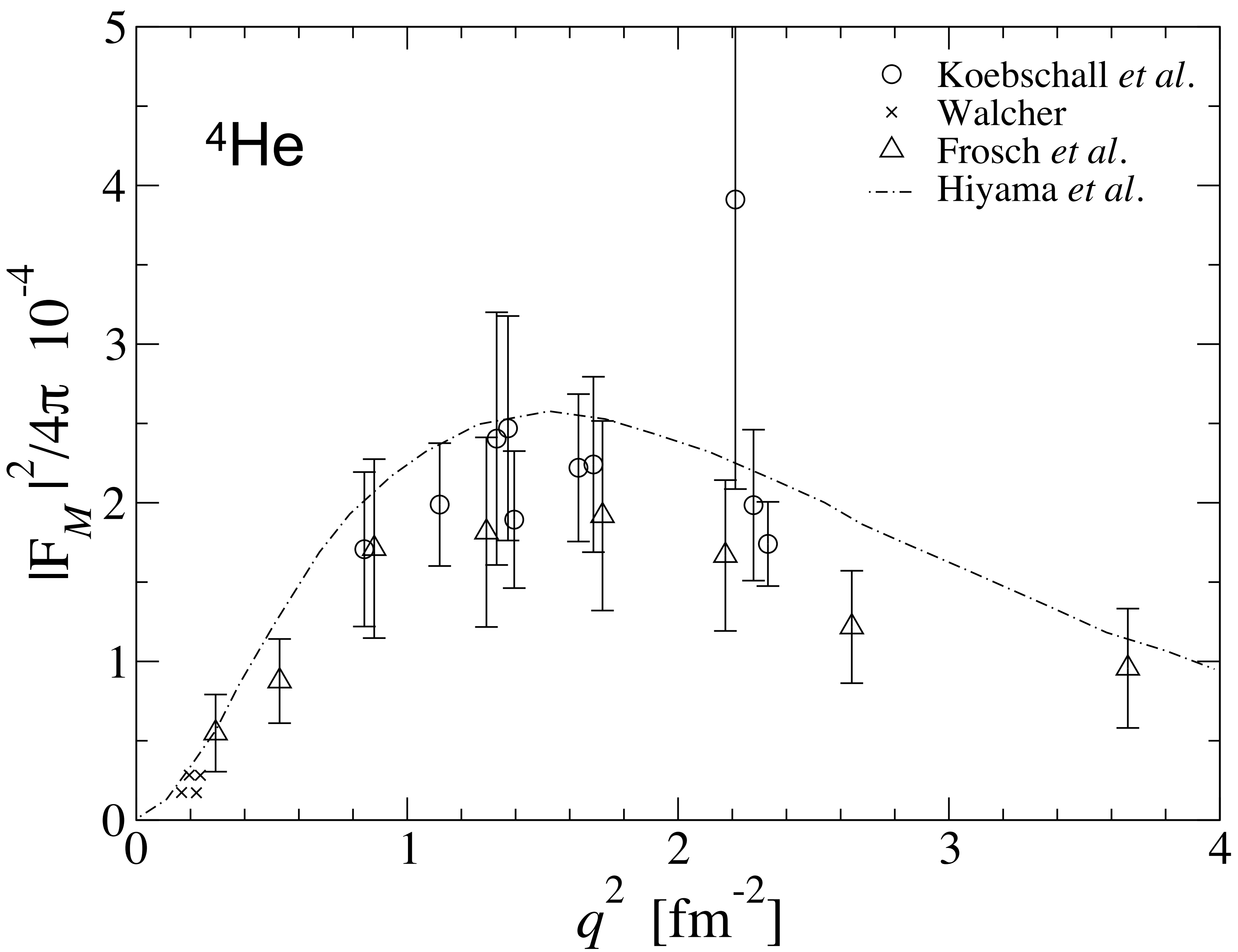
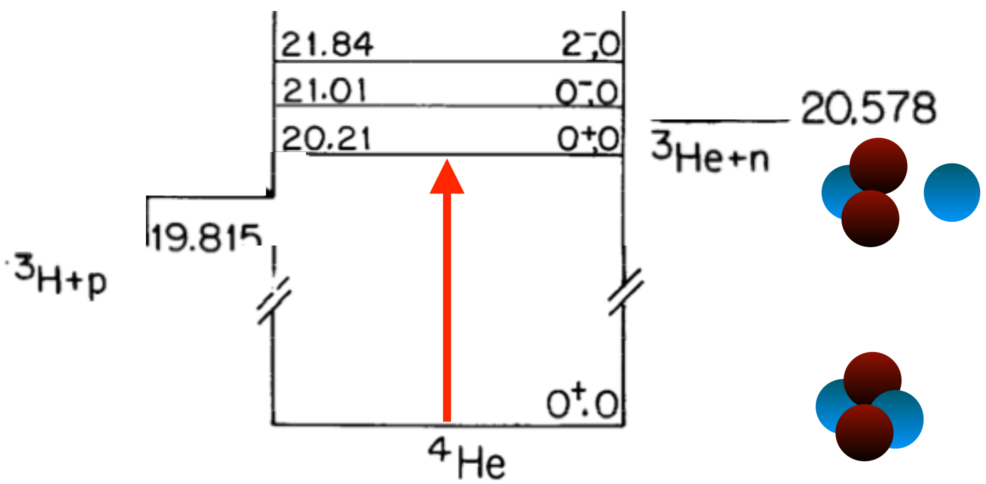
NNLO$_{sat}$
ΔNNLO$_{GO}$

Are we seeing missing higher order χEFT effects or is the halo structure challenging CC theory?

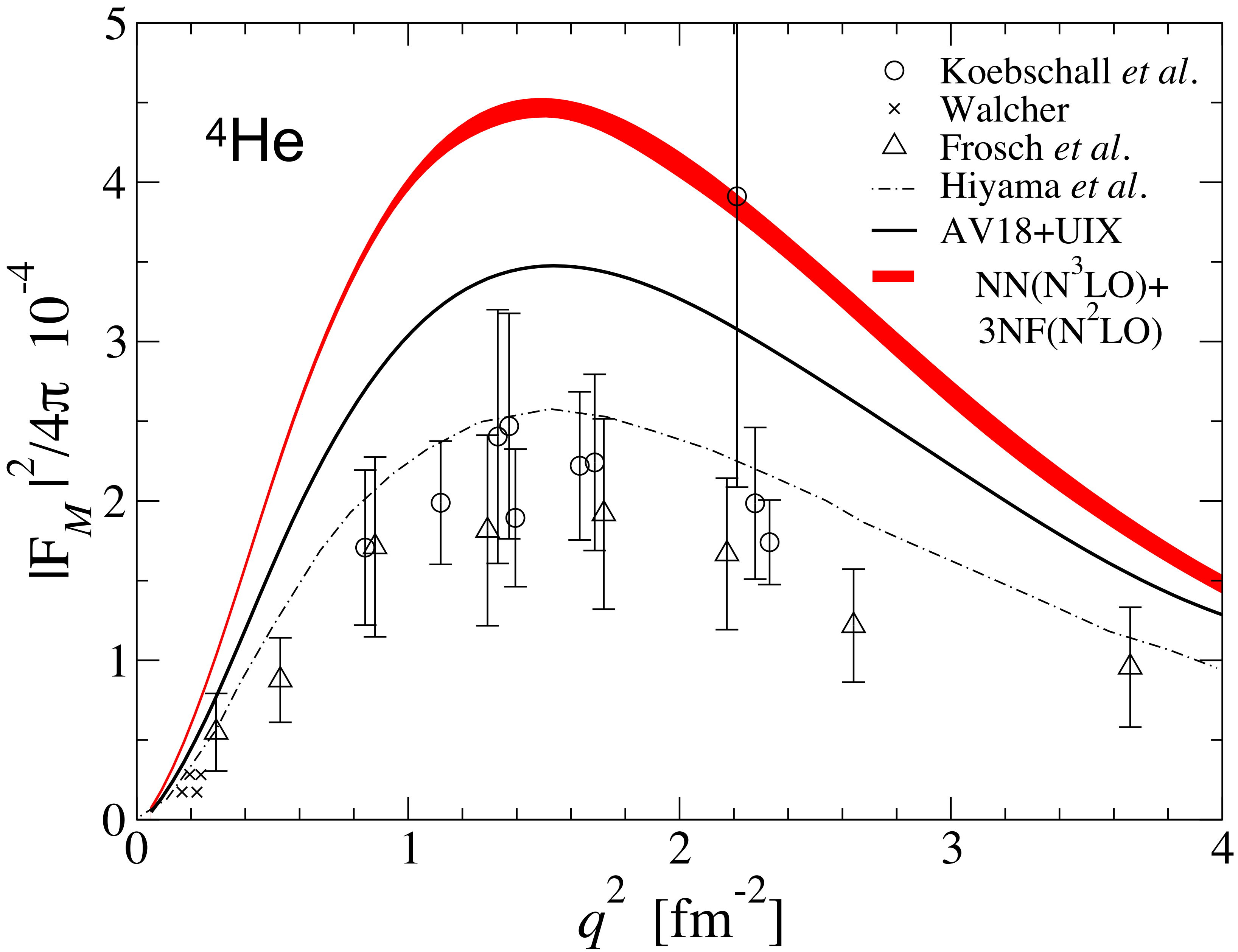
^{4}He monopole transition

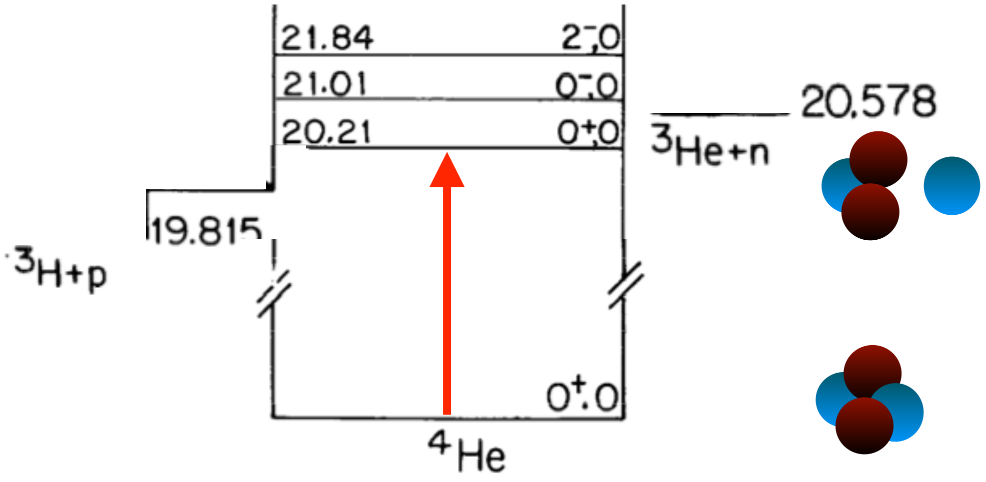




^{4}He monopole transition

SB et al., Phys. Rev. Lett. 110, 042503 (2013)

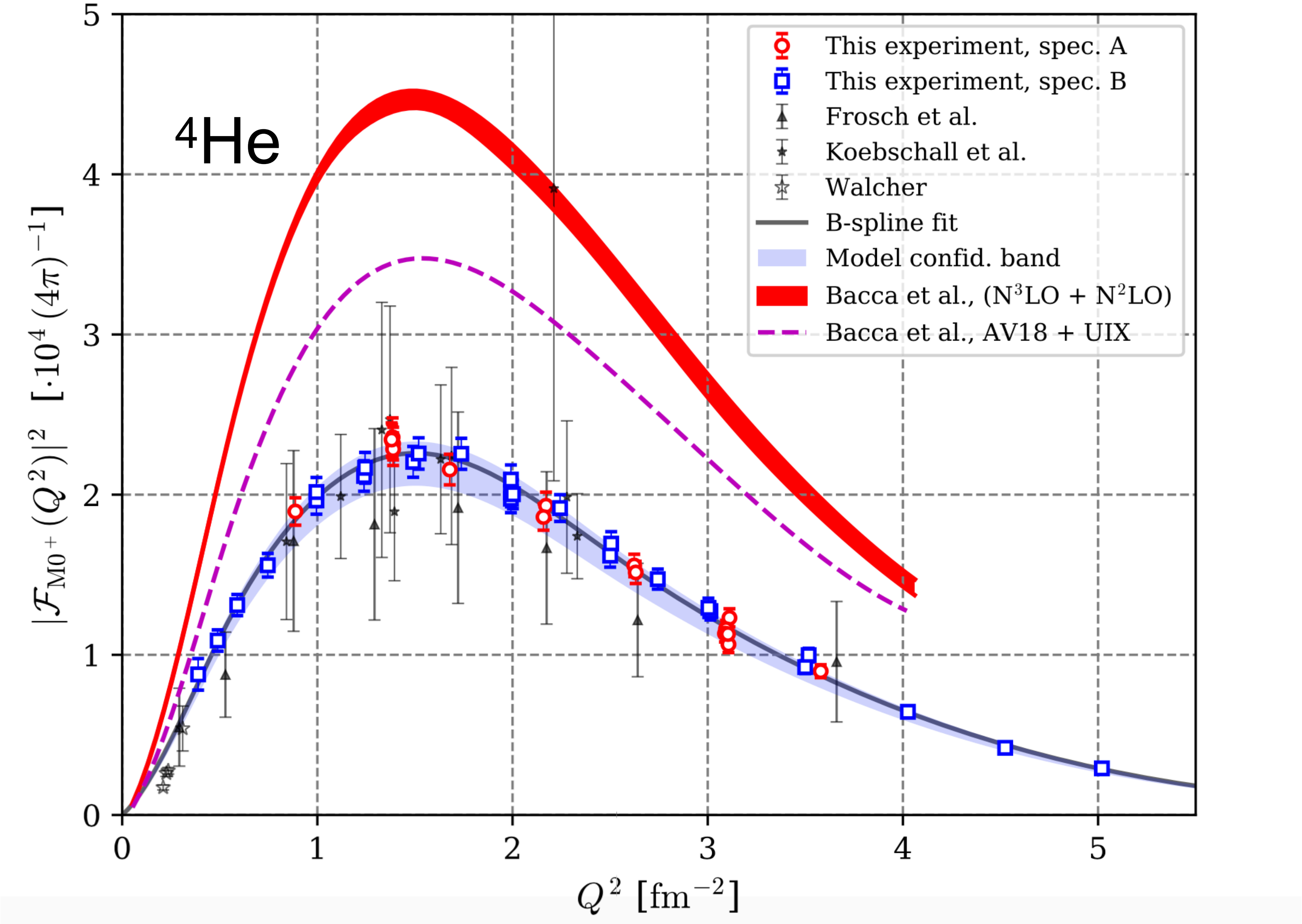


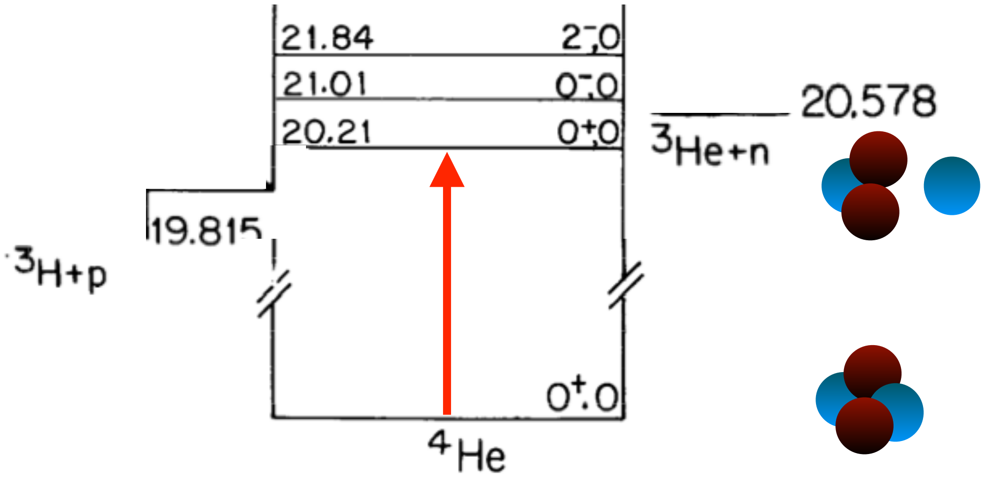


- Hiyama's calculation agree with data but our computation disagree
- Experimental data have large error bars

^{4}He monopole transition

Kegel et al., arXiv:2112.10582

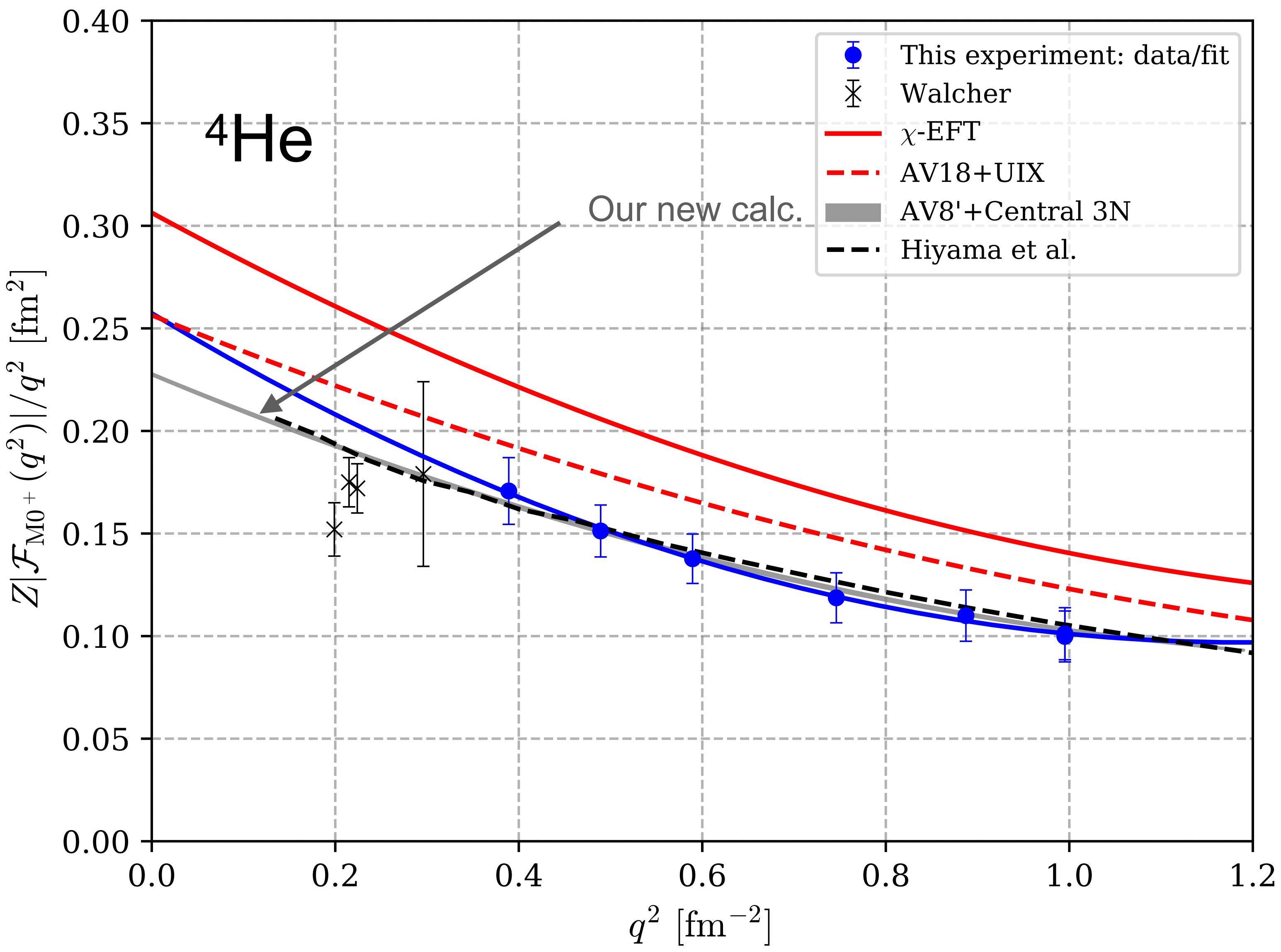


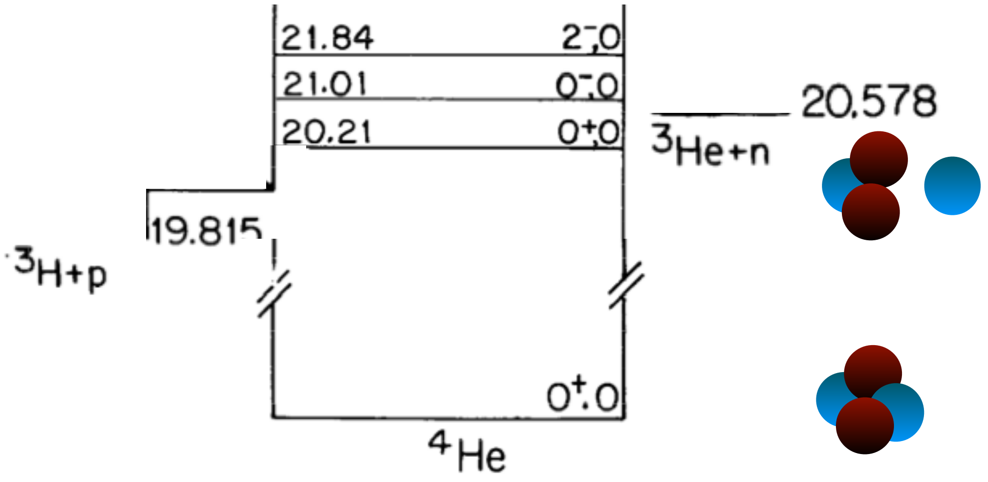


- New experiment in Mainz with dramatically improved ⇒ problem is in the theory
- Calculations done with different methods and different interactions.
  Can the different methods be the problem?

^{4}He monopole transition

Kegel et al., arXiv:2112.10582





- We perfectly reproduce Hiyama's results within error bars.
- Puzzle remains to be solved.

Conclusions

- Remarkable progress in first principle calculations of electromagnetic properties and more work is ahead of us

Thanks to all my collaborators:

B. Acharya, F. Bonaiti, S. Li Muli, W. Jiang, J.E.Sobczyk,
N. Barnea, G. Hagen, W. Leidemann, T. Papenbrock, G. Orlandini,
J. Simonis, C. Payne, et al.

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Thanks for your attention!

25th European Conference on Few-Body Problems in Physics

25 EFB

Mainz, 30 July - 4 Aug, 2023

Topics:

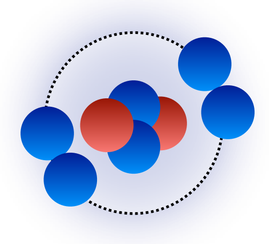
- Hadron physics
- Nuclei and hypernuclei
- Electroweak processes
- Nuclear astrophysics
- Cold atoms and quantum gases
- Atoms and molecules
- Few-body methods
- Few-body aspects of many-body systems

Strong overlap with this Symposium:
Keywords mentioned in Nakamura's introduction:
Three-body forces
Halo nuclei
Hoyle states
Universality
Effimov physics
Feshbach resonance

Backup

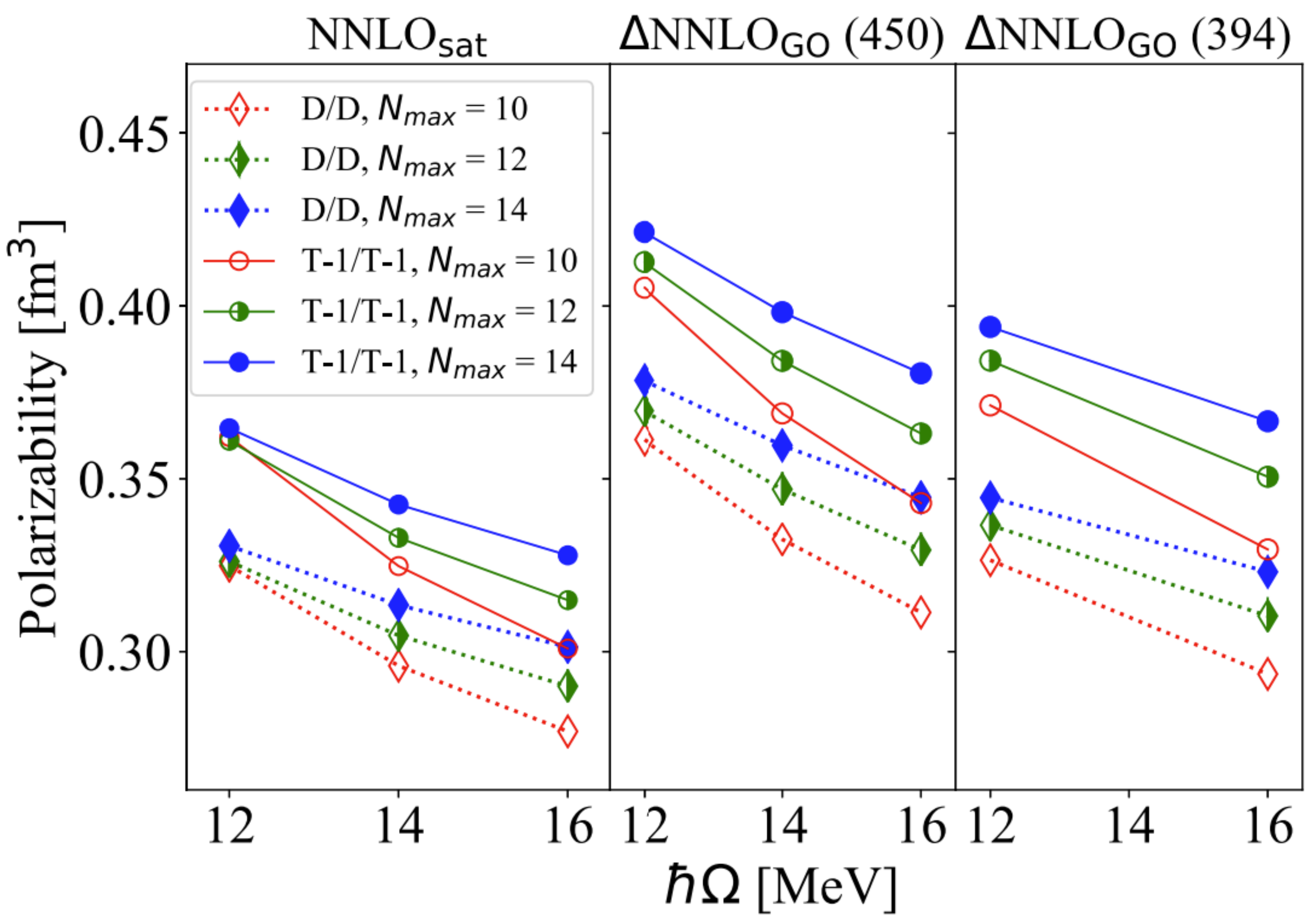
Halo nuclei

^{8}He



Halo nucleus

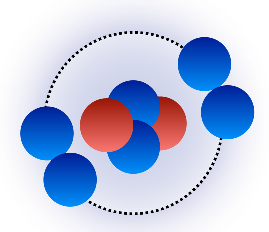
F. Bonaiti, SB,G.Hagen, PRC 105, 034313 (2022)



D → CCSD
T-1 → CCSDT-1

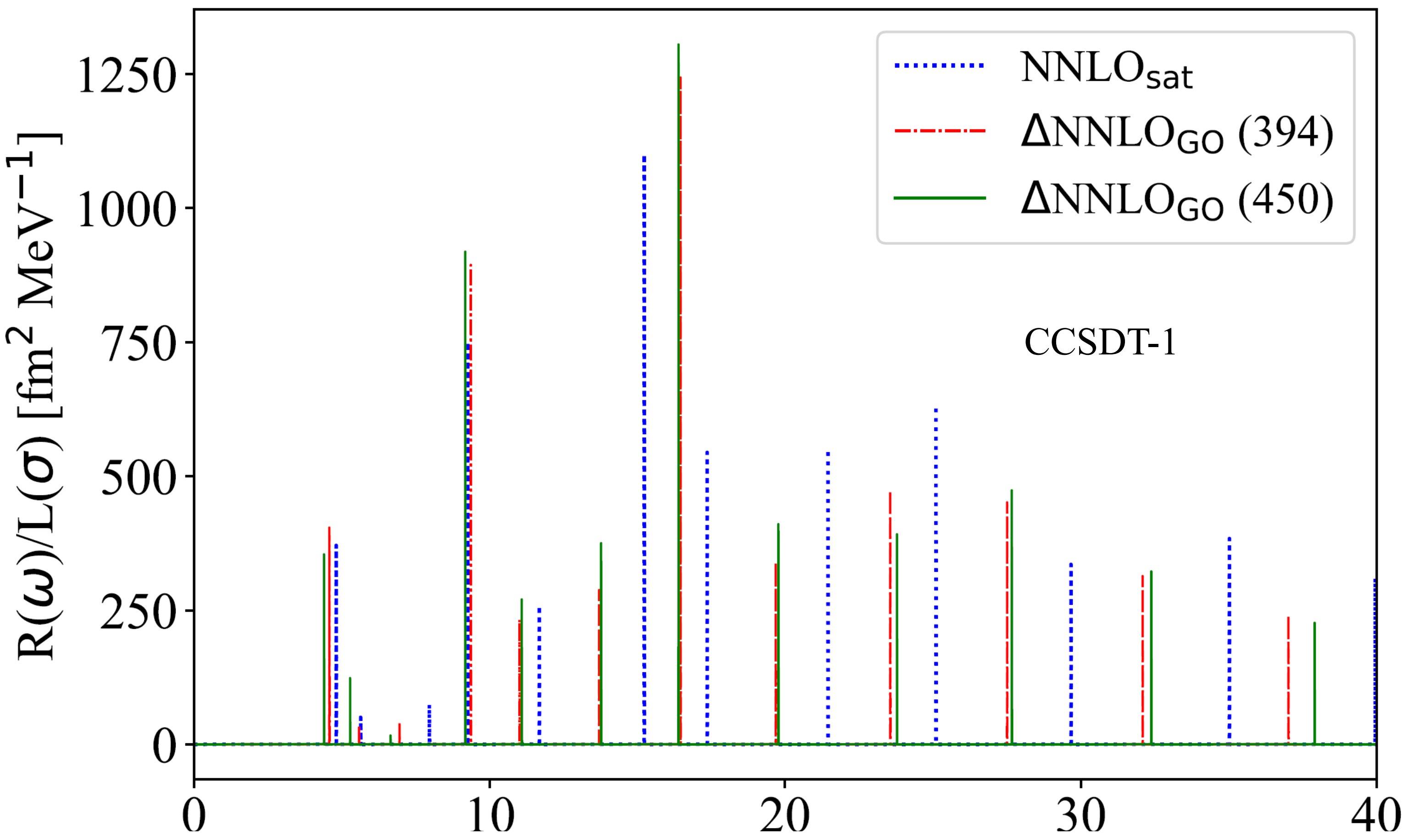
The Halo ^{8}He nucleus

^{8}He



Halo nucleus

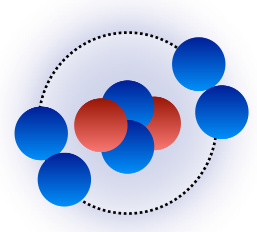
F. Bonaiti, SB, G.Hagen, PRC 105, 034313 (2022)



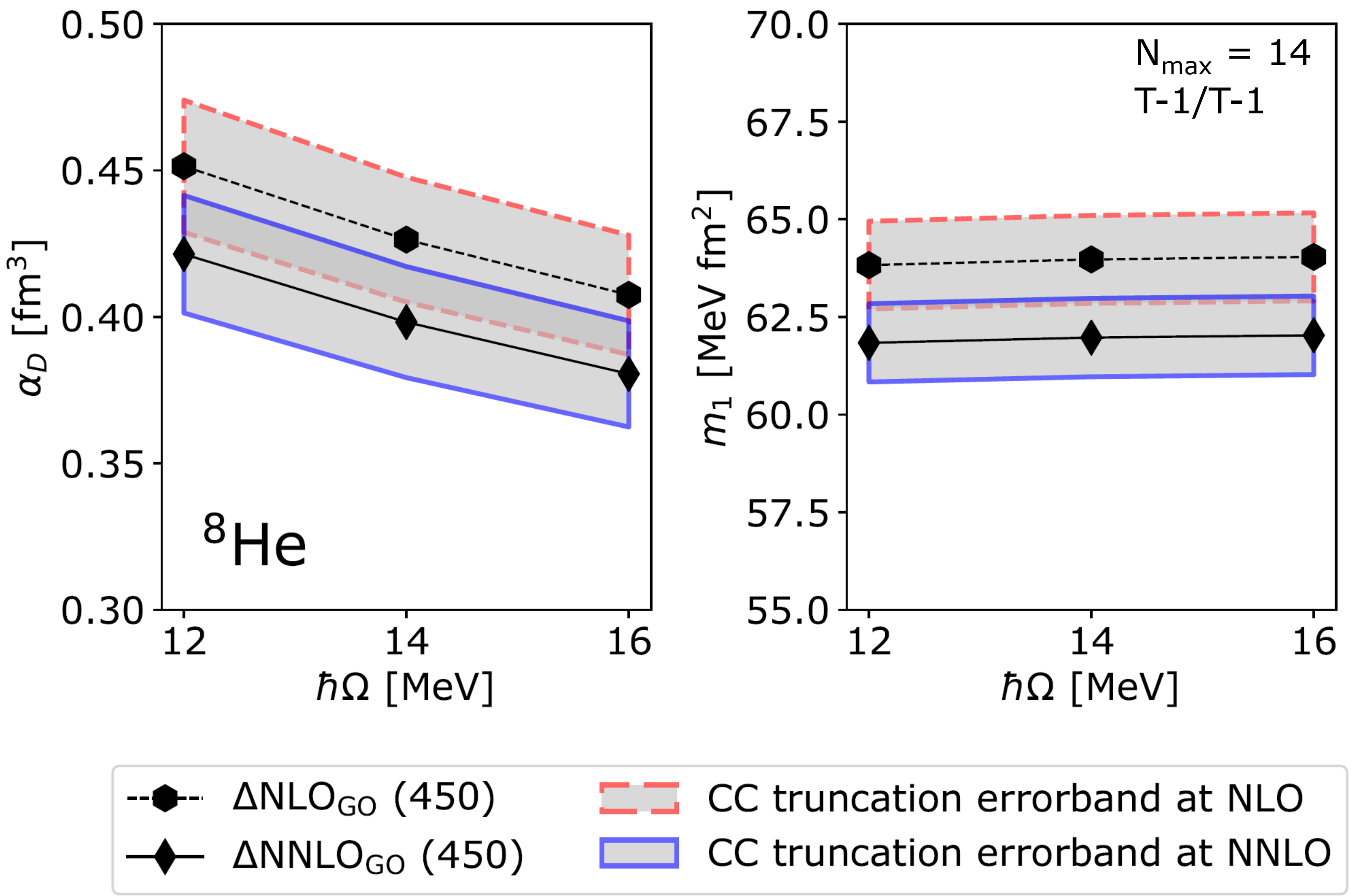
Interaction	α_D [fm^3]
$NNLO_{sat}$	0.37(3)
$\Delta NNLO_{GO}(450)$	0.42(3)
$\Delta NNLO_{GO}(394)$	0.39(2)

Halo nuclei

^{8}He



Acharya, SB, Bonaiti, Li Muli, Sobczyk, 2210.04632



Inversion of the LIT

The inversion is performed numerically with a regularization procedure (ill-posed problem)

Ansatz $$R(\omega) = \sum_i^{I_{\max}} c_i \chi_i(\omega, \alpha) \Longrightarrow L(\sigma, \Gamma) = \sum_i^{I_{\max}} c_i \mathcal{L}[\chi_i(\omega, \alpha)]$$

fit (c_i)

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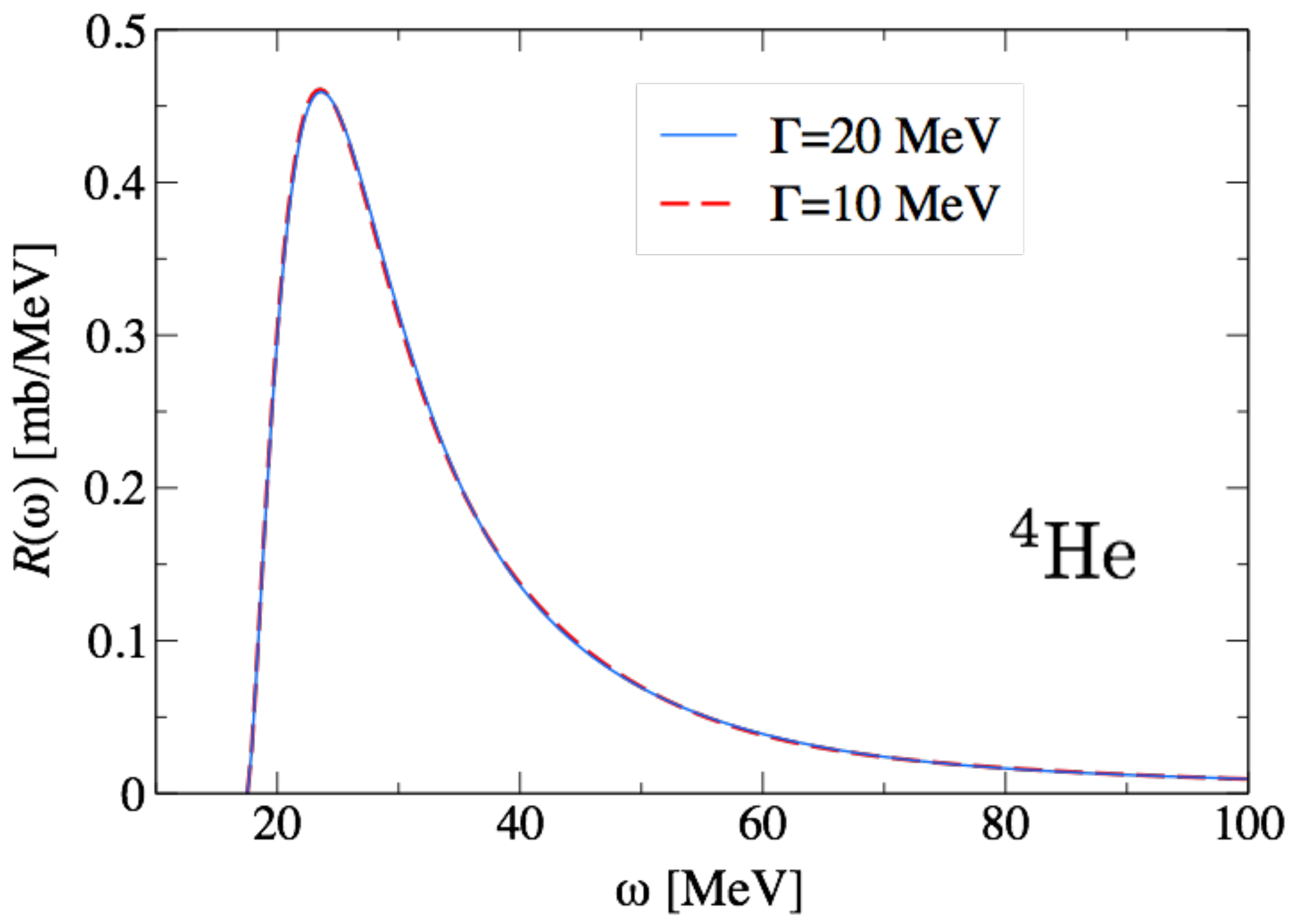
fit



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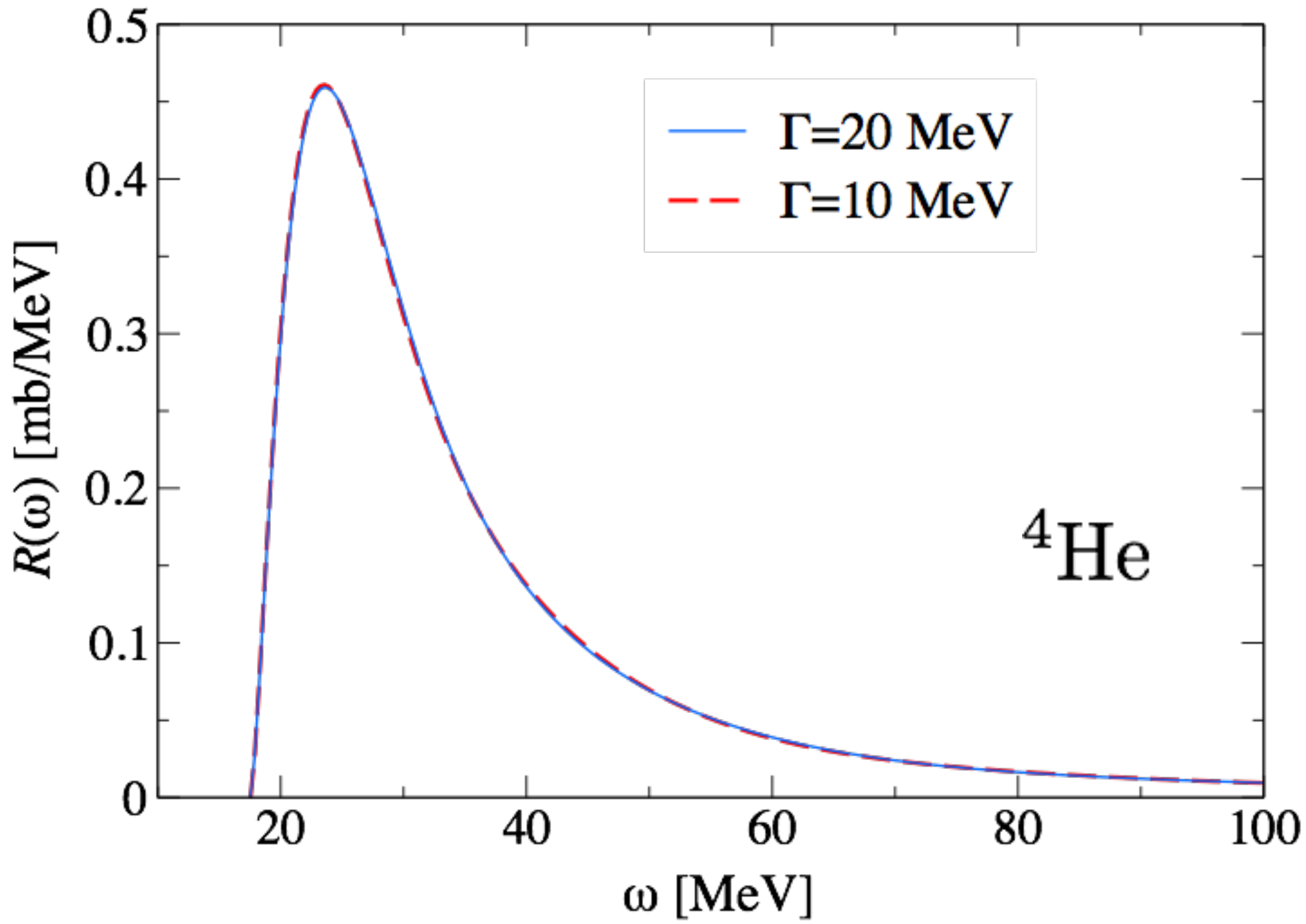
Ansatz $$R(\omega) = \sum_i^{I_{\max}} c_i \chi_i(\omega, \alpha) \;\Rightarrow\; L(\sigma, \Gamma) = \sum_i^{I_{\max}} c_i \mathcal{L}[\chi_i(\omega, \alpha)]$$

fit



Message: Inversions are stable if the LIT is calculated precisely enough

Sum Rules

$$m_n = \int_0^\infty d\omega\ \omega^n R(\omega) = \langle \Psi_0 | \hat{\Theta}^\dagger (\hat{H} - E_0)^n \hat{\Theta} | \Psi_0 \rangle$$

The polarizability is an inverse-energy weighted sum rule of the dipole response function

$$\alpha_D = 2\ \alpha\ m_{-1} = 2\ \alpha\ \langle \Psi_0 | \Theta^\dagger \frac{1}{(H - E_0)} \Theta | \Psi_0 \rangle$$

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Can be obtained from the Lorentz Integral Transform in the limit of $\Gamma \rightarrow 0$