International Symposium on Clustering as a Window on the Hierarchical Structure of Quantum Systems (CLUSHIQ2022)
31 Oct. -3 Nov., 2022

Quark-cluster aspects in baryons and baryon spectroscopy at J-PARC

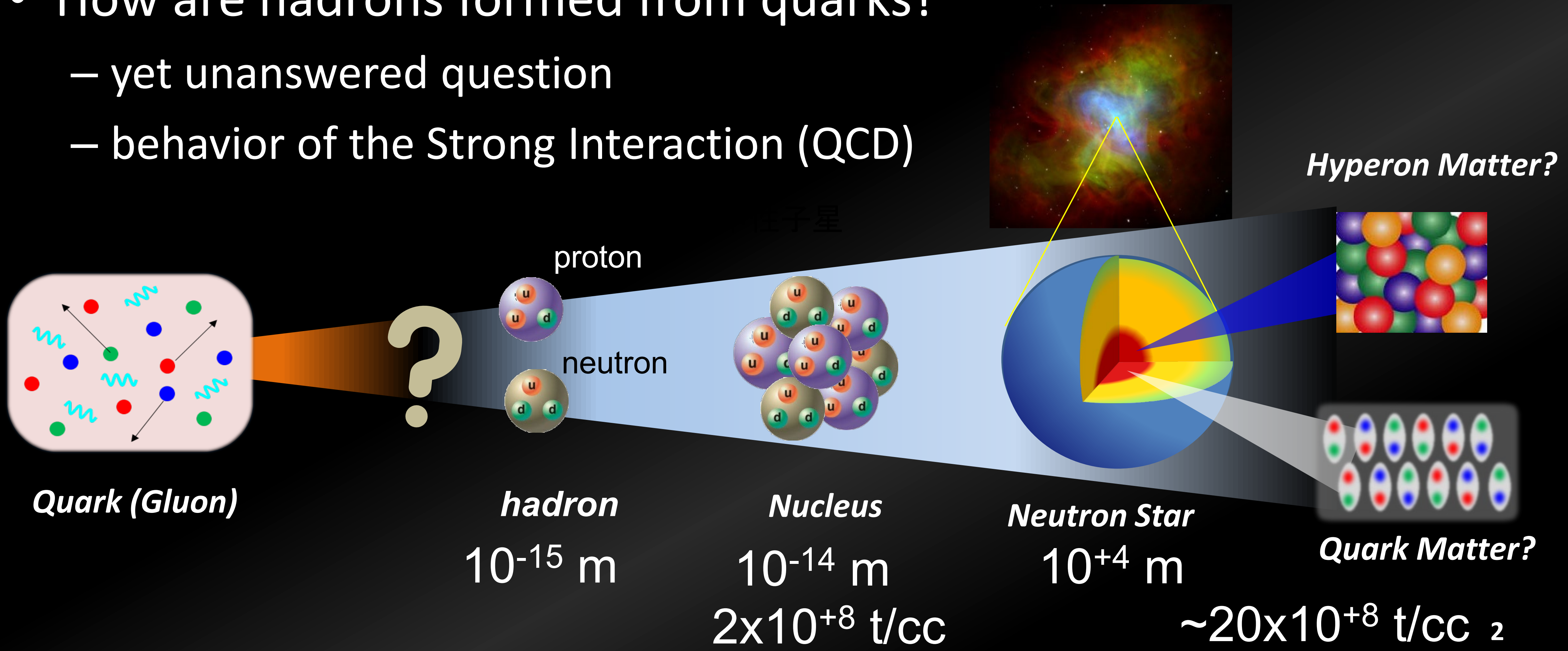
Hiroyuki NOUMI*,#
*Research Center for Nuclear Physics, Osaka University
#Institute of Particle and Nuclear Studies, KEK

CONTENTS

1. "Diquark" in baryons
2. Lambda(1405), a $\bar{K}N$ system

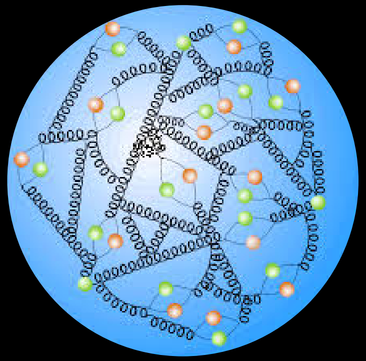
Matter Evolution in the Universe

- Hadrons: complex system of quarks (and gluons)
- How are hadrons formed from quarks?
  - yet unanswered question
  - behavior of the Strong Interaction (QCD)

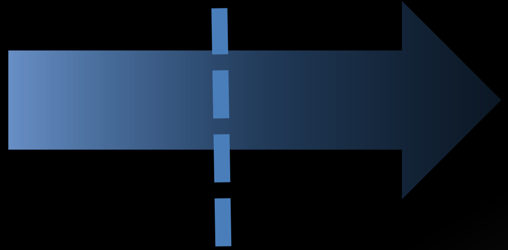


How does QCD build baryons?

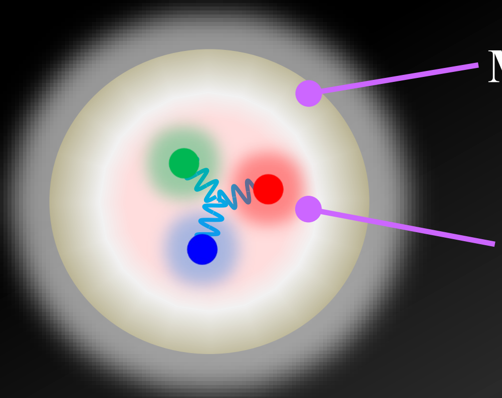
High E
perturbative



$\alpha_s = \infty$
at Λ_{QCD}



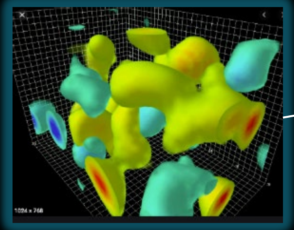
Low E
non-perturbative



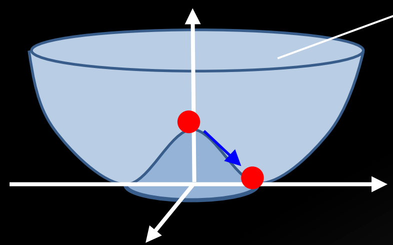
Meson Cloud

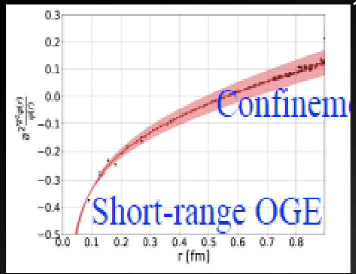
"Constituent" Quark (CQ)

Instanton
(LQCD demo.
by D. Leinweber)



- Non-trivial vacuum
- Spontaneous Breaking of Chiral Symmetry
- Confinement





**Q-Diquark Pot.
in LQCD cal.**

Eff. DoF dynamically emerge:

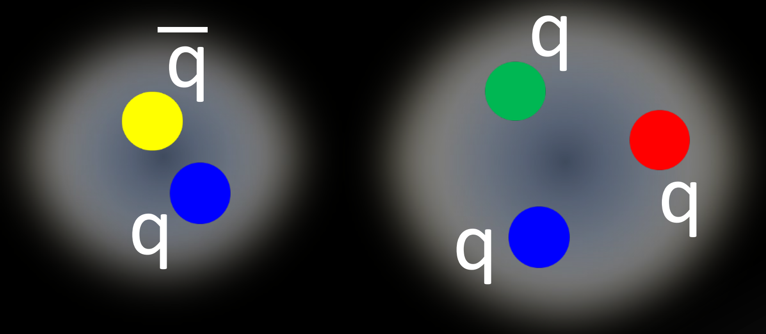
"Massive" CQ

NG bosons (pion, …)

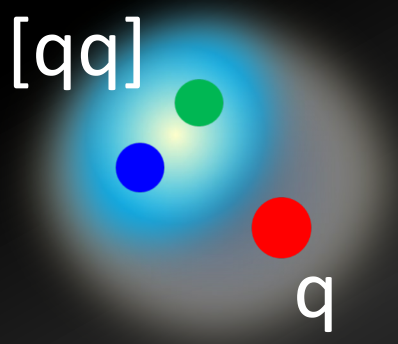
*Dynamics of CQ reflects
the nature of QCD in Low E*

Quark-cluster aspects in Hadrons

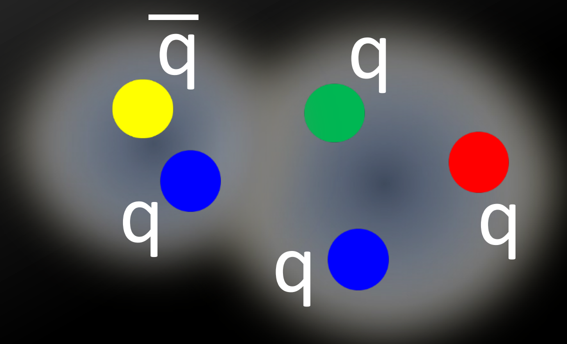
Constituent Quark



Quark-Diquark
(Colored cluster)



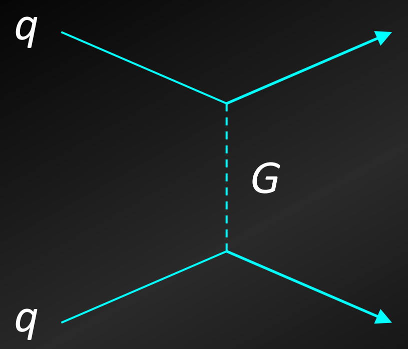
Hadronic Molecule
(colorless cluster)



Spectroscopy of Baryons

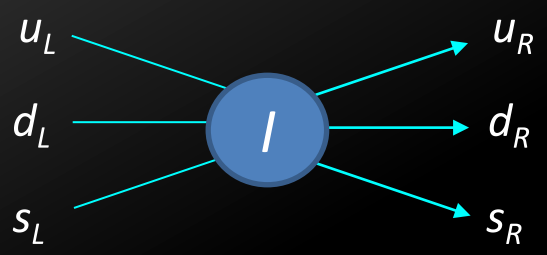
to reveal dynamics of Constituent Quarks

$$H = K + V^{Conf} + \underbrace{V^{Coul} + V^{SS} + V^{LS}}_{\text{“short-range” int.}} + \cdots$$

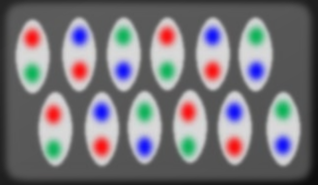


- Diquarks (DQs)
  - Color Magnetic Interaction ($\lesssim$0.5fm)
    - Origin of the SS and LS forces is an open question

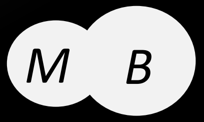
      i.e. OGE vs Instanton Induced Interaction (*III, KMT int.*)
  - may form “BE condensate” in high-density matter

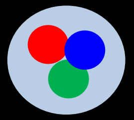


U_A(1) anomaly



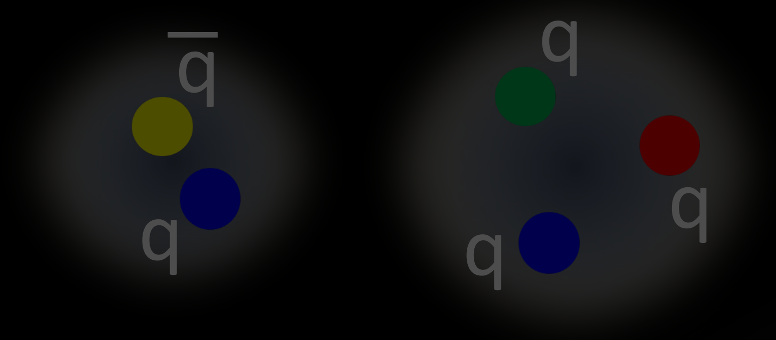
- Hadronic Molecule
  - Behavior of QCD in a long range region ($\gtrsim$1fm)



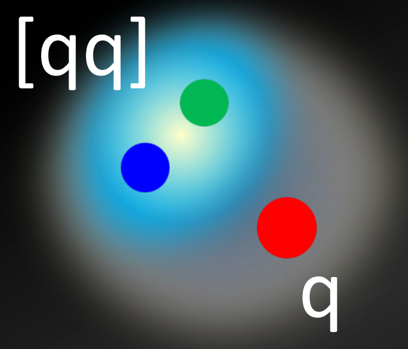


Quark-cluster aspects in Hadrons

Constituent Quark

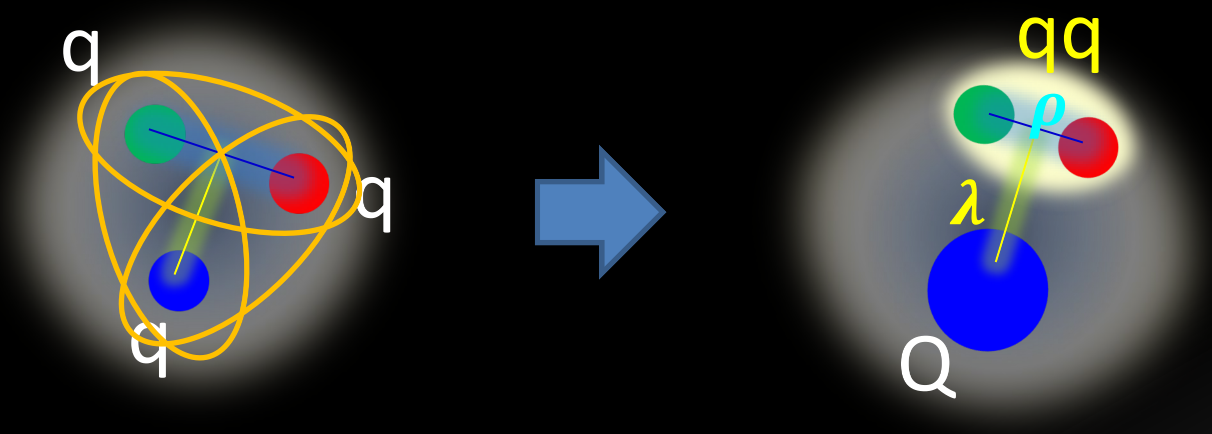


Quark-Diquark
(Colored cluster)



Hadronic Molecule
(colorless cluster)

Roles of Heavy Quark: *to see dynamics of EDoF*



$V_{CMI} \sim [\alpha_s/(m_i m_j)]*(\lambda_i \cdot \lambda_j)(\sigma_i \cdot \sigma_j)$

$\rightarrow 0$ if $m_{i,j} \rightarrow \infty$

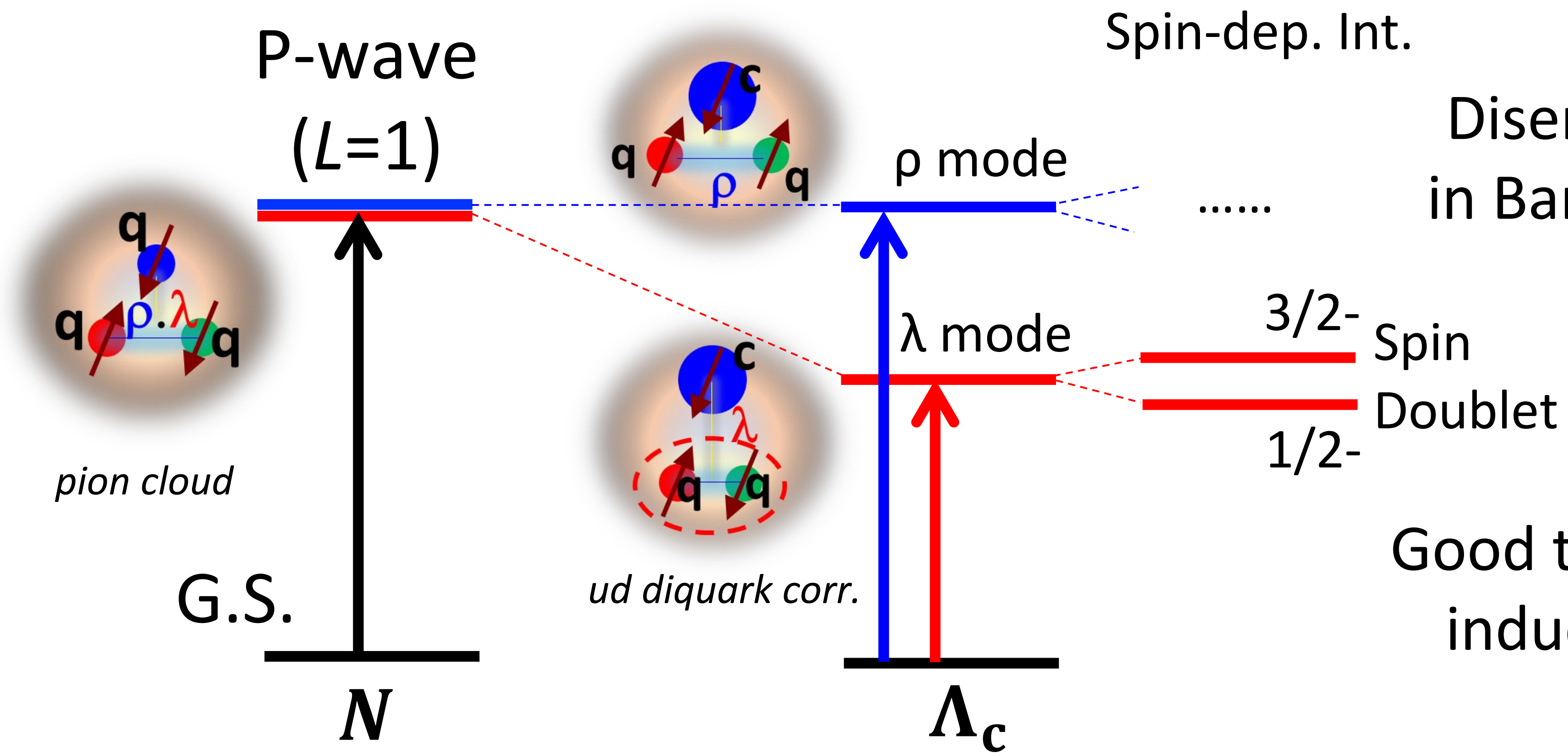
$V_{CMI}({}^1S_0, \bar{3}_c) = 1/2 * V_{CMI}({}^1S_0, 1_c)$

$[qq]$ $\quad$ $[\bar{q}q]$

- Motion of "*qq*" is singled out by a heavy Q
  - Disentangle Diquark correlation
- Level structure, Production rate, Decay properties
  - sensitive to the internal quark(diquark) WFs in baryons
- Properties are expected to depend on a Q mass.

Internal structure of baryons in terms of EDoF

※Disentangle motions of a *Diquark* by introducing different flavors



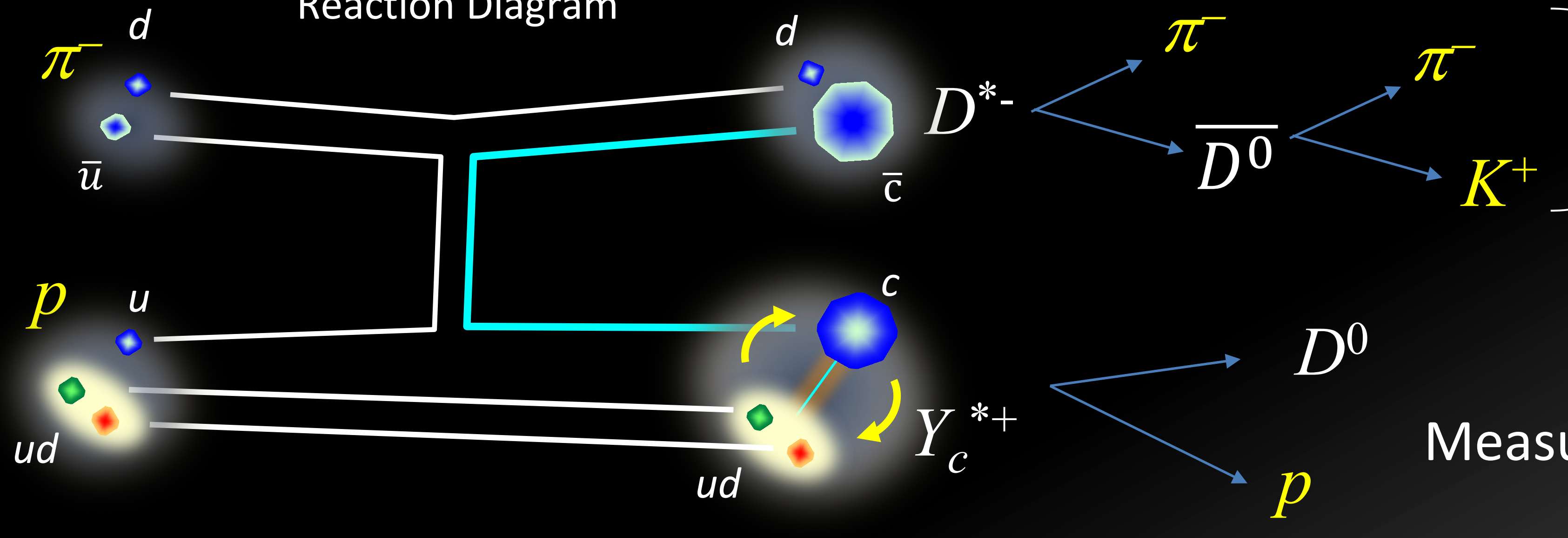
Disentangle "*qq*" motion in Baryon w/ Heavy Quark

Good to study *qq* correlation induced by Spin-Spin int.

T. Yoshida, E. Hiyama, A. Hosaka, M. Oka, K. Sadato, Phys. Rev. D**92** (2015) 114029

Production and Decay of Charmed Baryons

Reaction Diagram



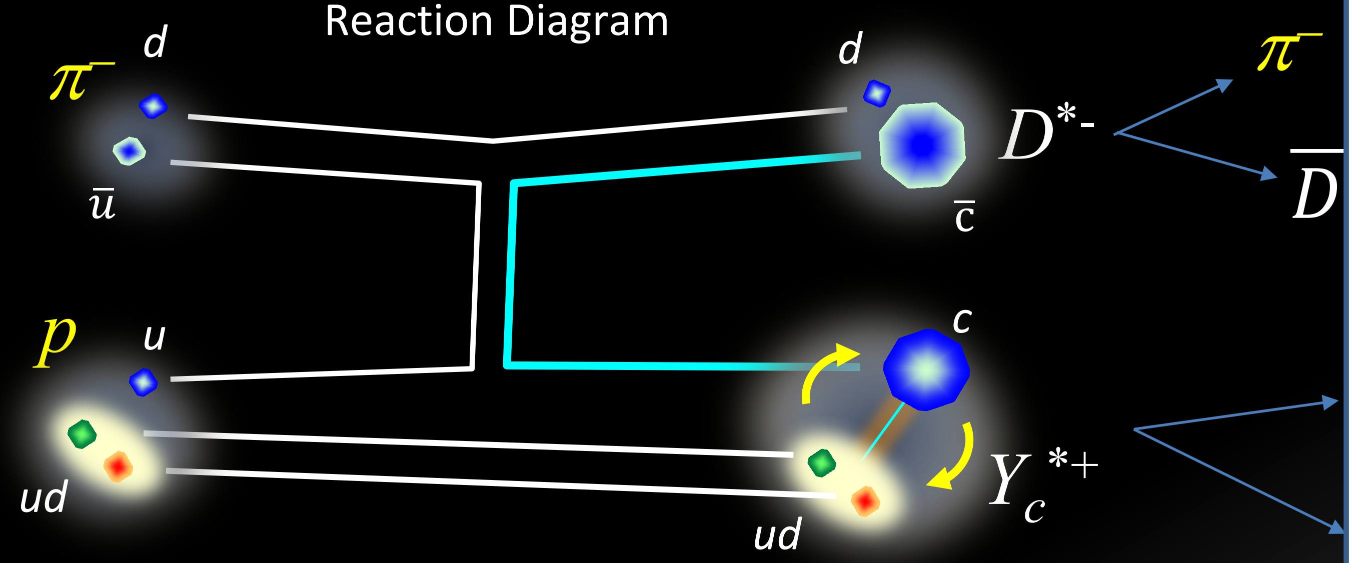
Measure D*- and Identify Y_c^{*+} in the Missing mass spectrum

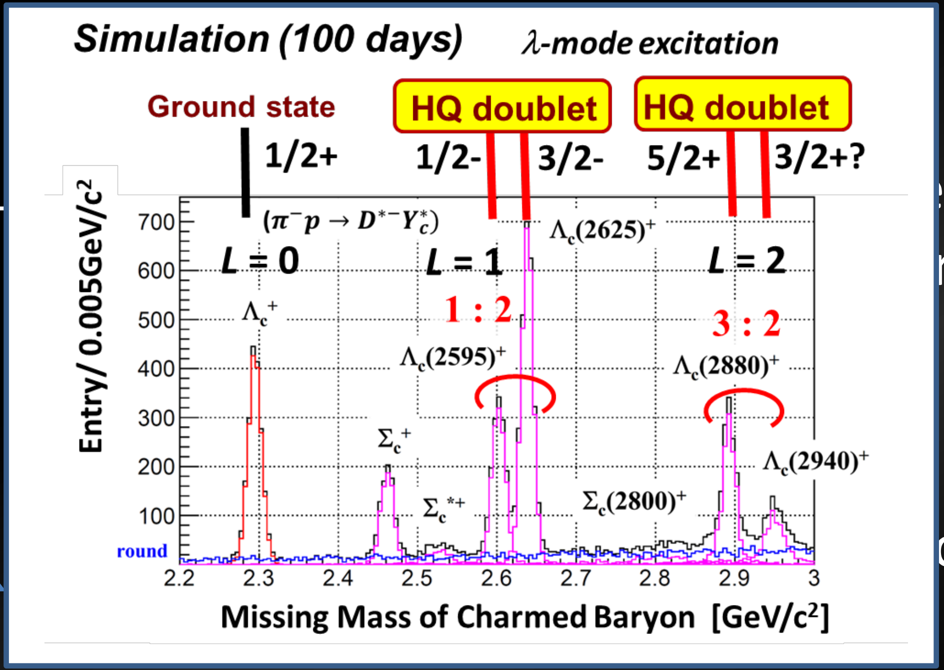
Measure Decay particles , too

References for estimations on production cross sections:

- Reggeon Exchange: S.H. Kim, A. Hosaka, H.C. Kim, and HN, Phys.Rev. D92 (2015) 094021
- Single-quark involved process: S.H. Kim, A. Hosaka, H.C. Kim, and HN, PTEP, (2014) 103D01
- Two-quark involved process: S.I. Shim, A. Hosaka, H.C. Kim, PTEP 2020, (2020) 5, 053D01

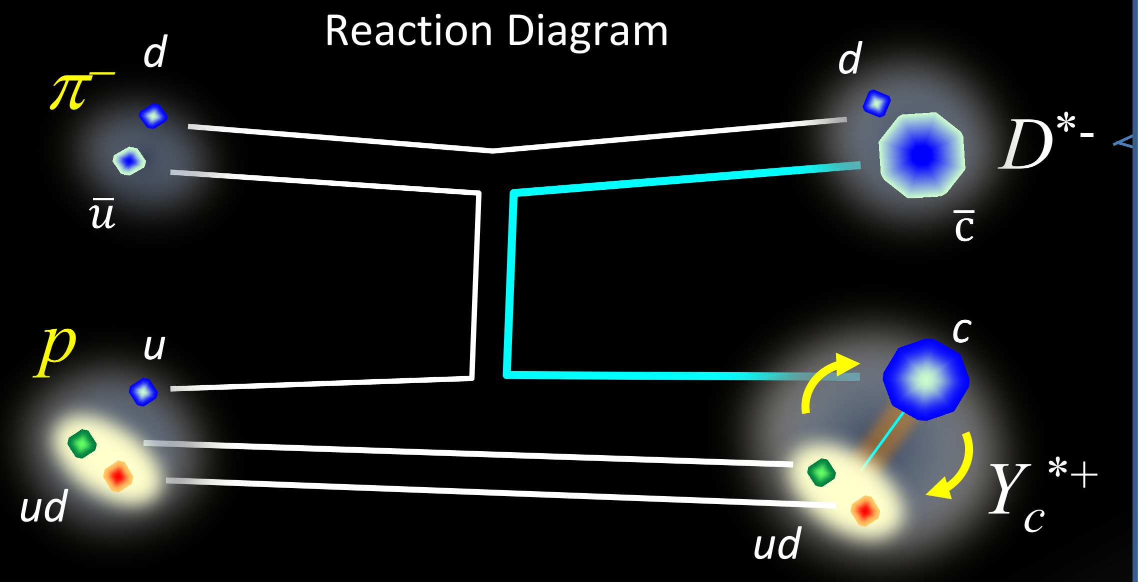
Production and Decay of Charmed Baryons

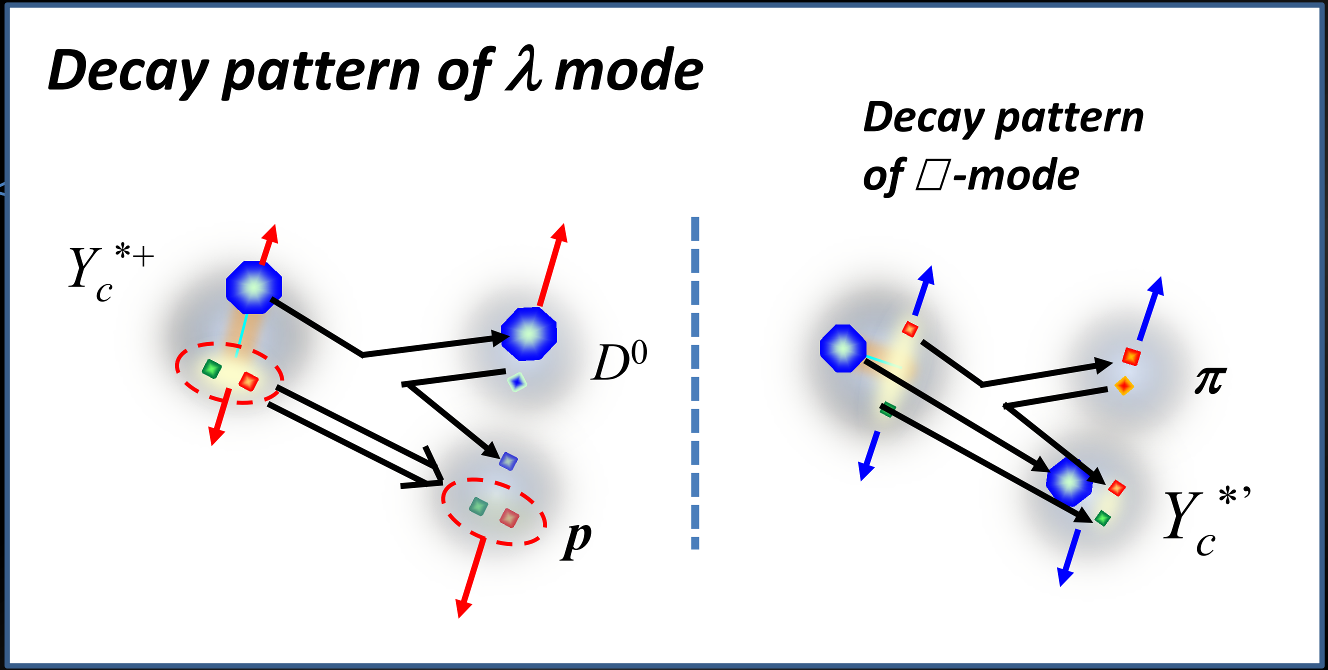




- Introducing a finite orbital angular momentum L ⇒ favor λ-mode excitations
  - Establish "*ud*" diquark motion in baryon
- Production ratio of the HQ doublet to be L:L+1 ⇒ Spin, Parity
  - The ratio would be a measure of how "*ud*" is correlated.
- Production and Decay measurement ⇒ Branching Ratio (partial width)
  - Decay rates would be a measure of how "*ud*" is firmly correlated

Production and Decay of Charmed Baryons





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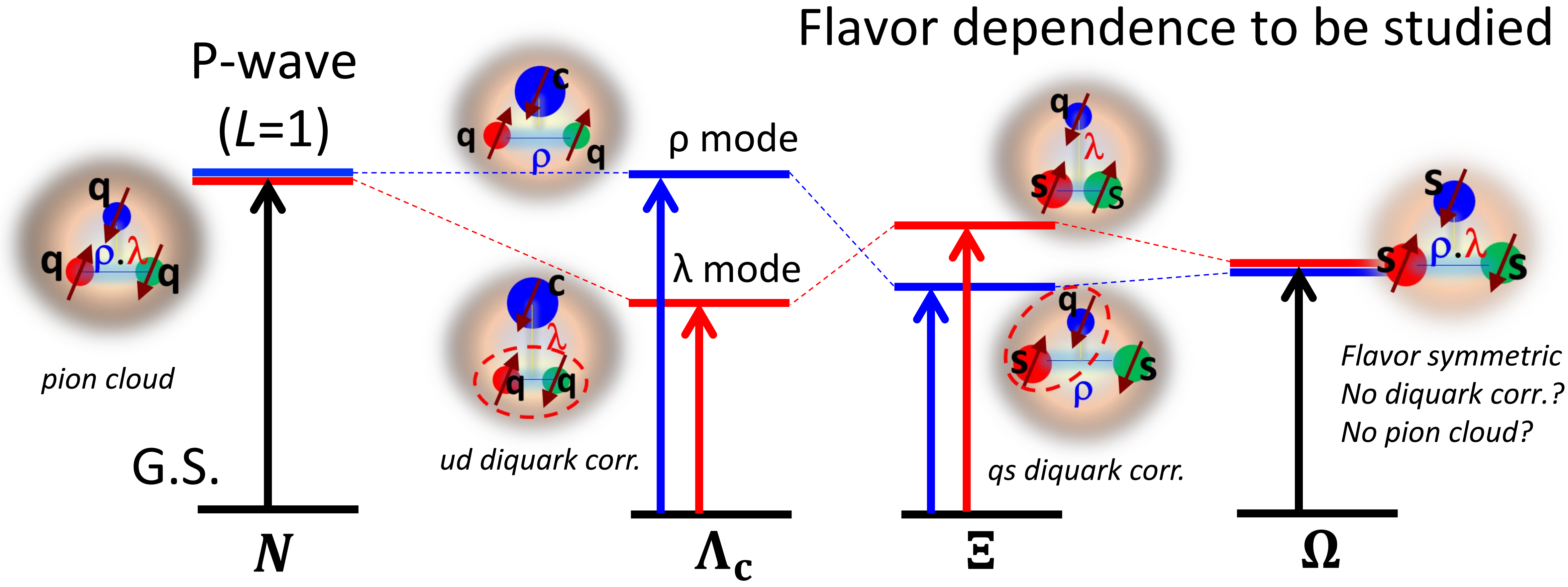
Extension to multi-strange baryons

※Disentangle motions of a quark pair by introducing different flavors

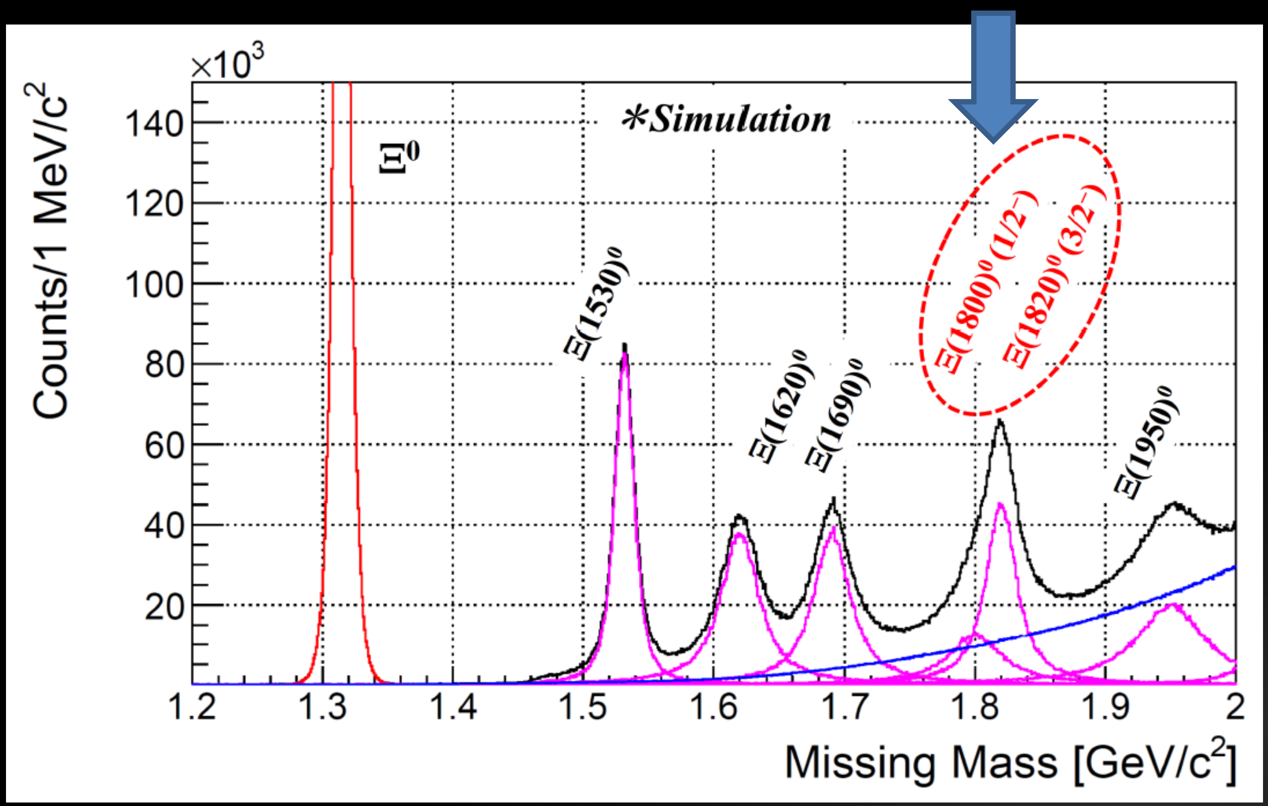


T. Yoshida, E. Hiyama, A. Hosaka, M. Oka, K. Sadato, Phys. Rev. D**92** (2015) 114029

Expected Spectra in $K^- p \to K^{*0}\Xi^{*0}$ at 8 GeV/c

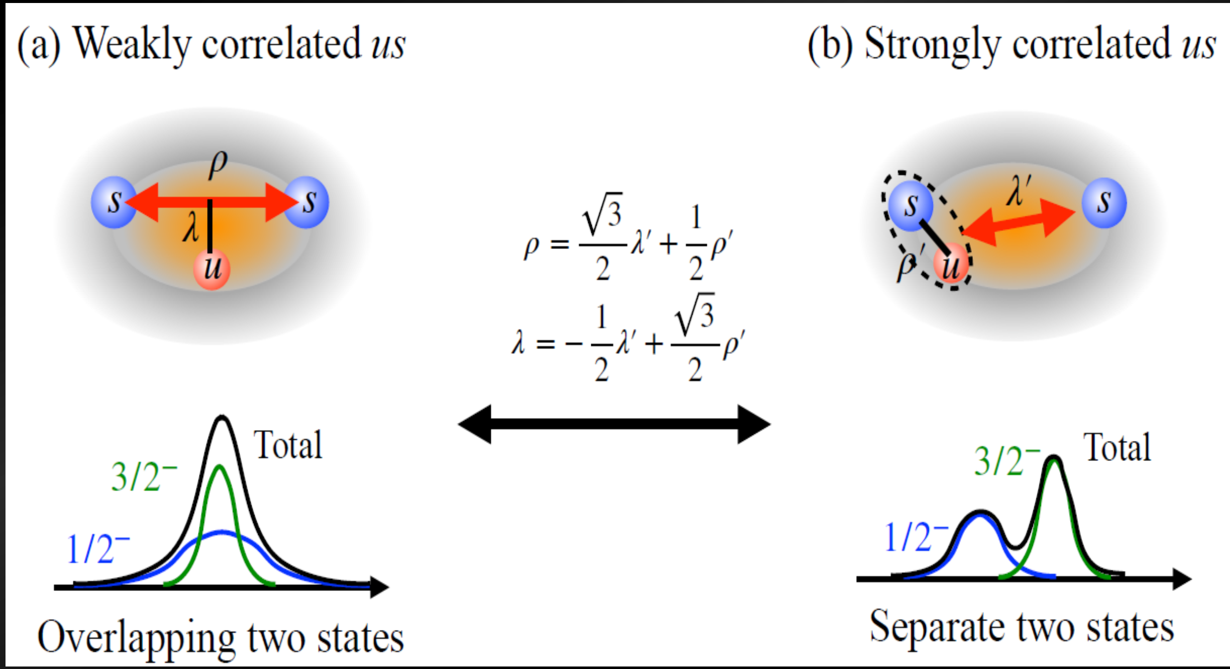
GS

$1P$ states (ρ-mode)



※Ξ(1800)1/2- assumed for demo.

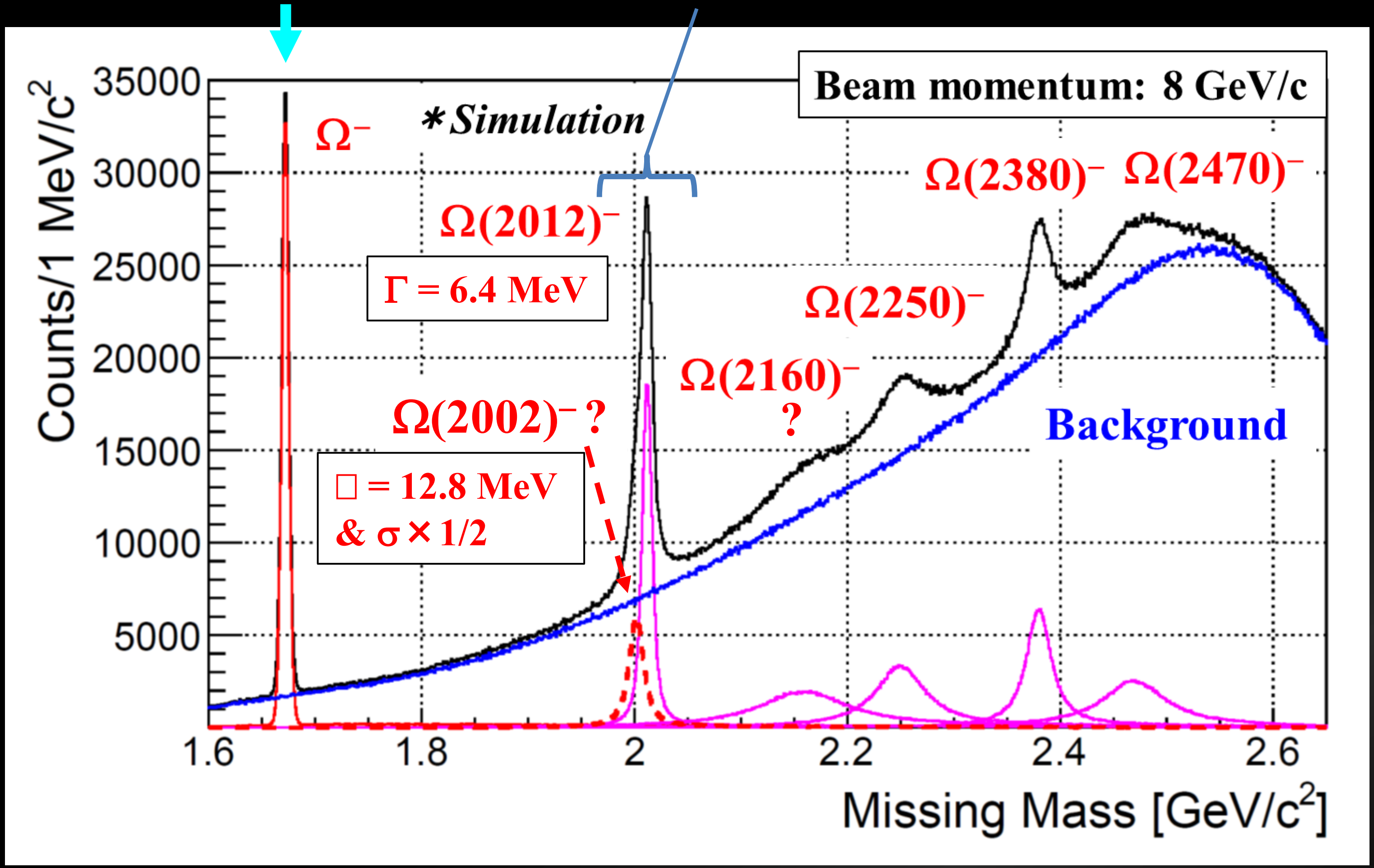
- Interest of ρ-mode excited states
  - Ξ(1820)3/2- to be confirmed
  - *LS* partner (1/2-) to be found
  - Reveal *us*-diquark correlation



Expected Spectra in $K^- p \rightarrow K^{*0} K^+ \Omega^{*-}$ at 8 GeV/c

63 nb assumed

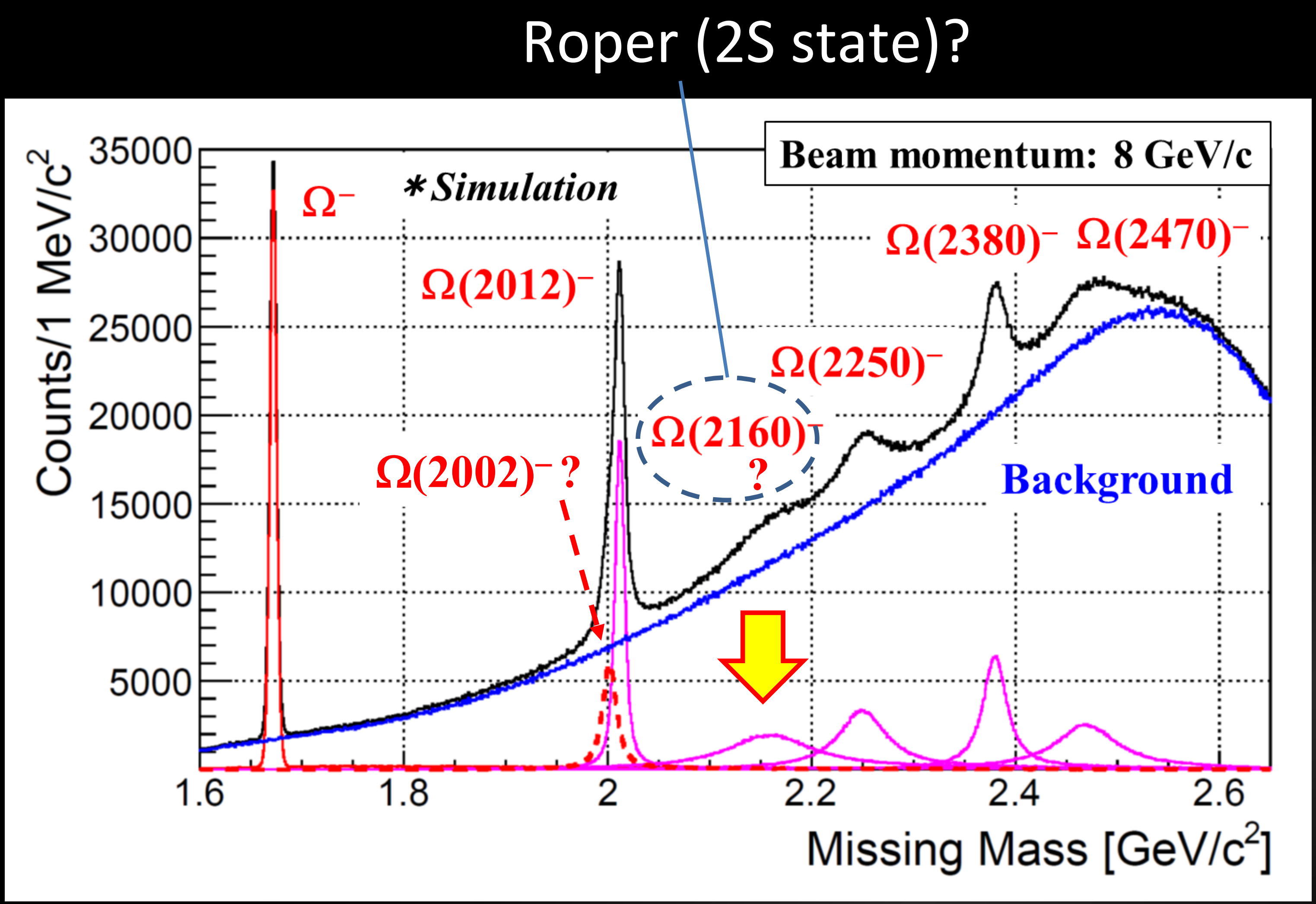
1P states?



Physics Highlight

- 1P excited states
  - $\Omega(2012)$ J^P to be measured
    - 3/2-?
  - *LS* partner (1/2-) to be found
    - No LS splitting by CQM due to flavor symmetry
  - If a Finite *LS* splitting, Relativistic effect in confinement force?
- Is $\Omega(2012)$: $\Xi^* \bar{K}$ Molecular?
  - PRD101, 094016(2020)

Expected Spectra in $K^- p \to K^{*0} K^+ \Omega^{*-}$ at 8 GeV/c

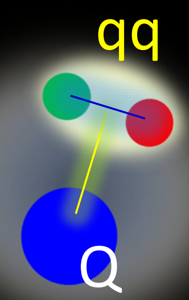


Physics Highlight

- 2S excited states
  - Radial excitation
  - So-called Roper-like state, yet to be found
    - $\Omega(2160), \Gamma\sim 100$ MeV assumed in the Sim.
  - The width
    - $\propto \langle p^2 \rangle$: "Quark core" size no pion cloud
  - The excitation energy
    - Universality if it is ~400 MeV.

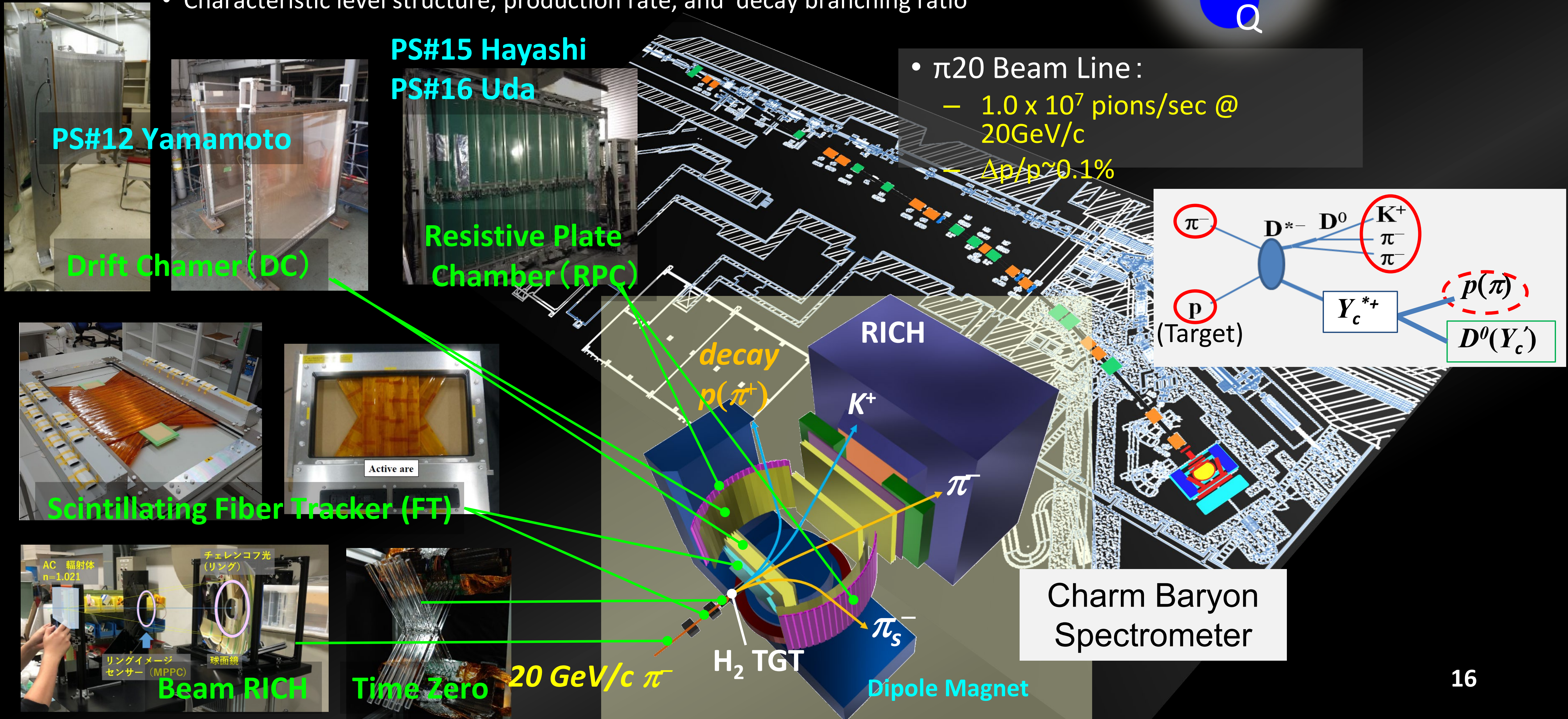
A. Jafar Arifi, A. Hosaka, et al., PRD103, 094003(2021)

Charm Baryon Spectroscopy at High-p (π20)



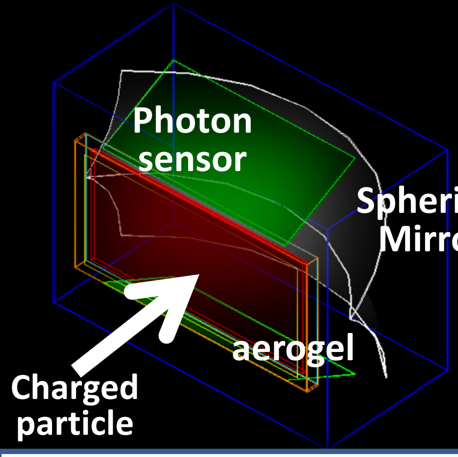
Diquark [qq]: an effective degree of freedom to describe hadrons

- [qq] would be singled out by Introducing a Heavy Quark
- Characteristic level structure, production rate, and decay branching ratio



Ring Imaging Cherenkov counter

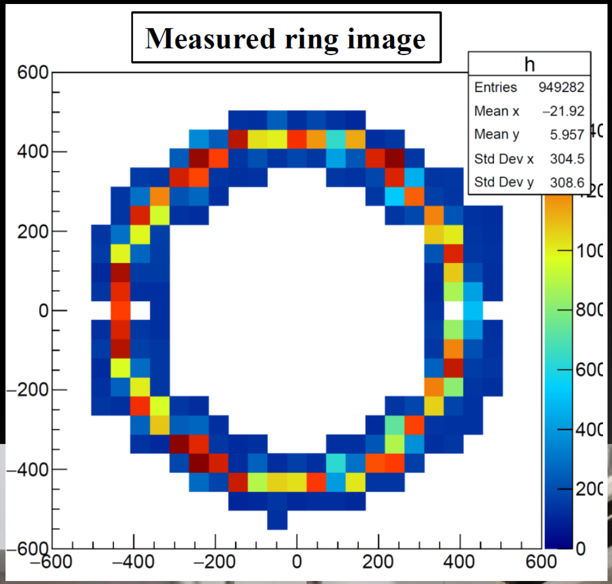
for π, K, p ID in 2～16GeV/c

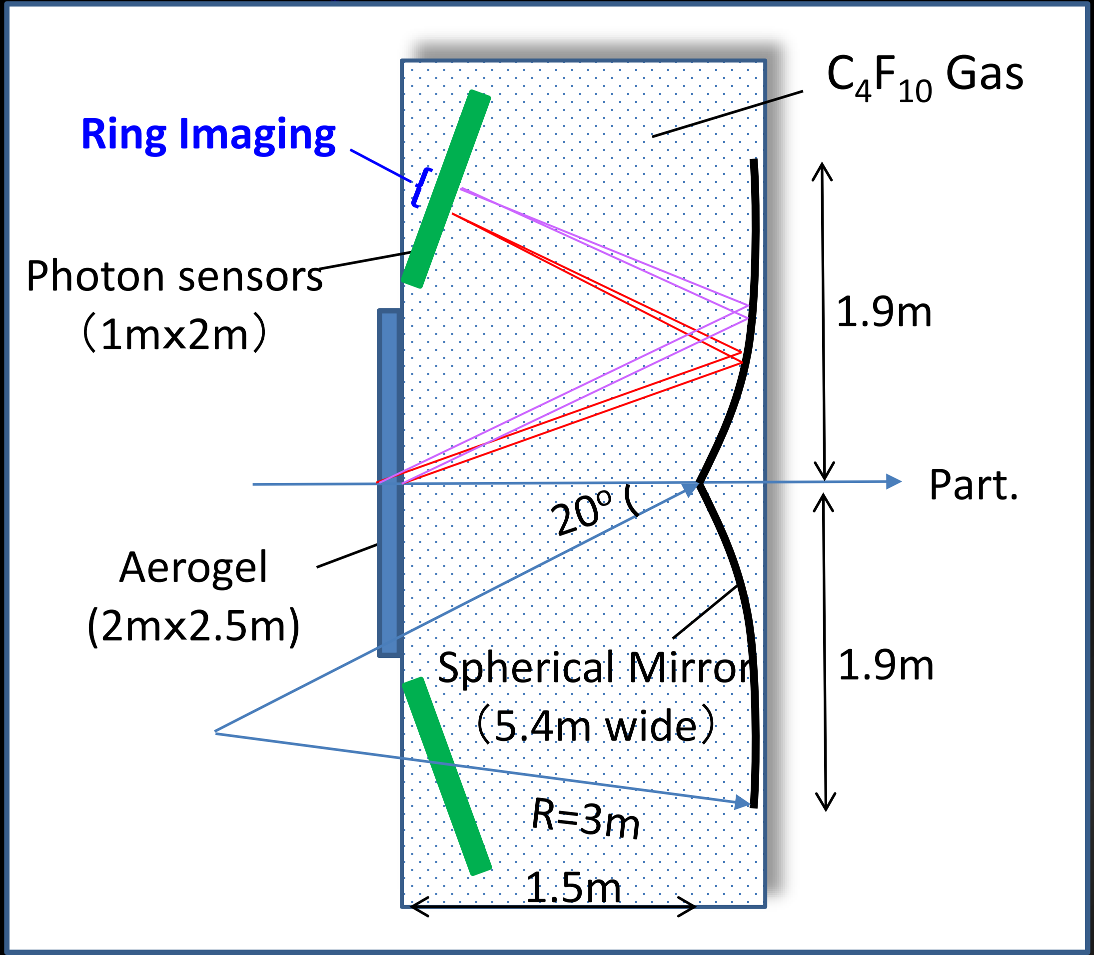


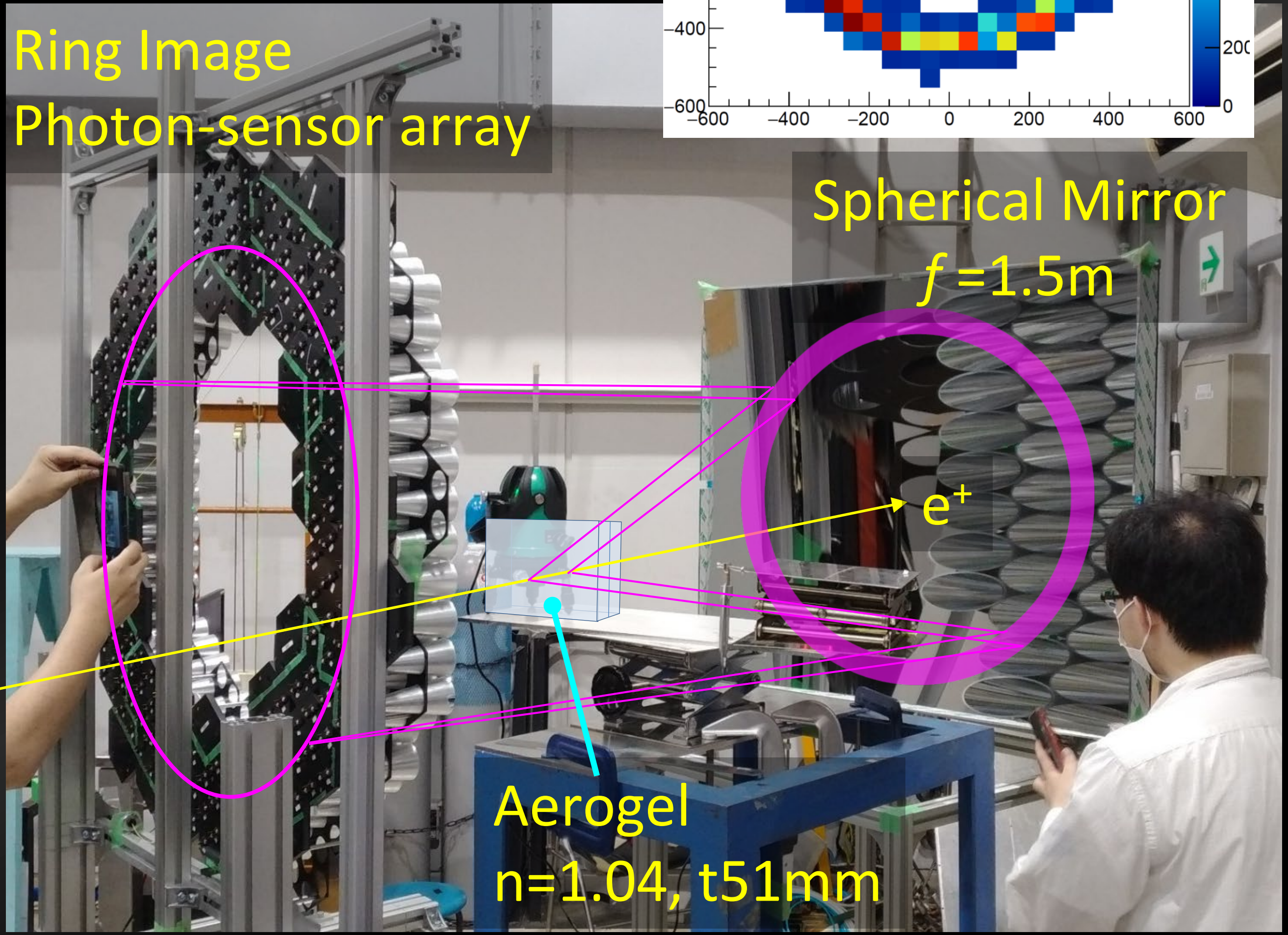
RICH

PS#13 Toda, PS#14 Tokuda

Test at ELPH, Tohoku U.





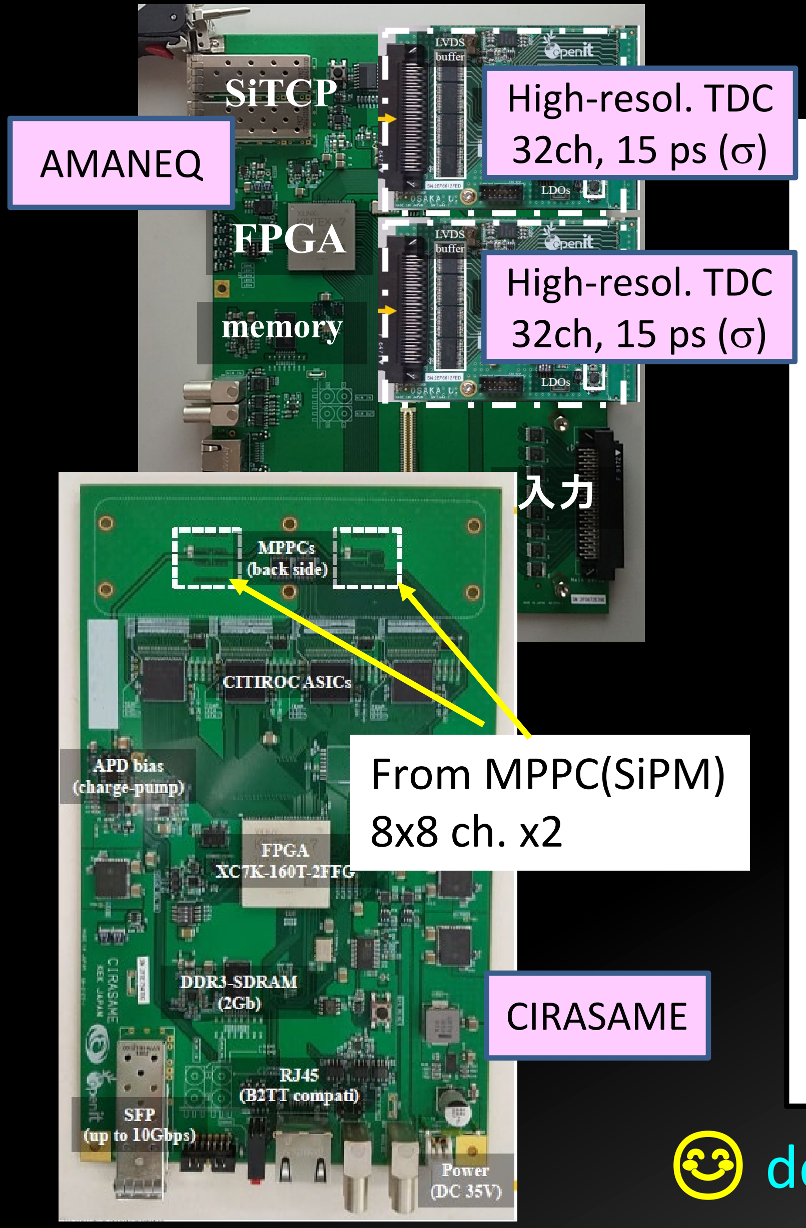


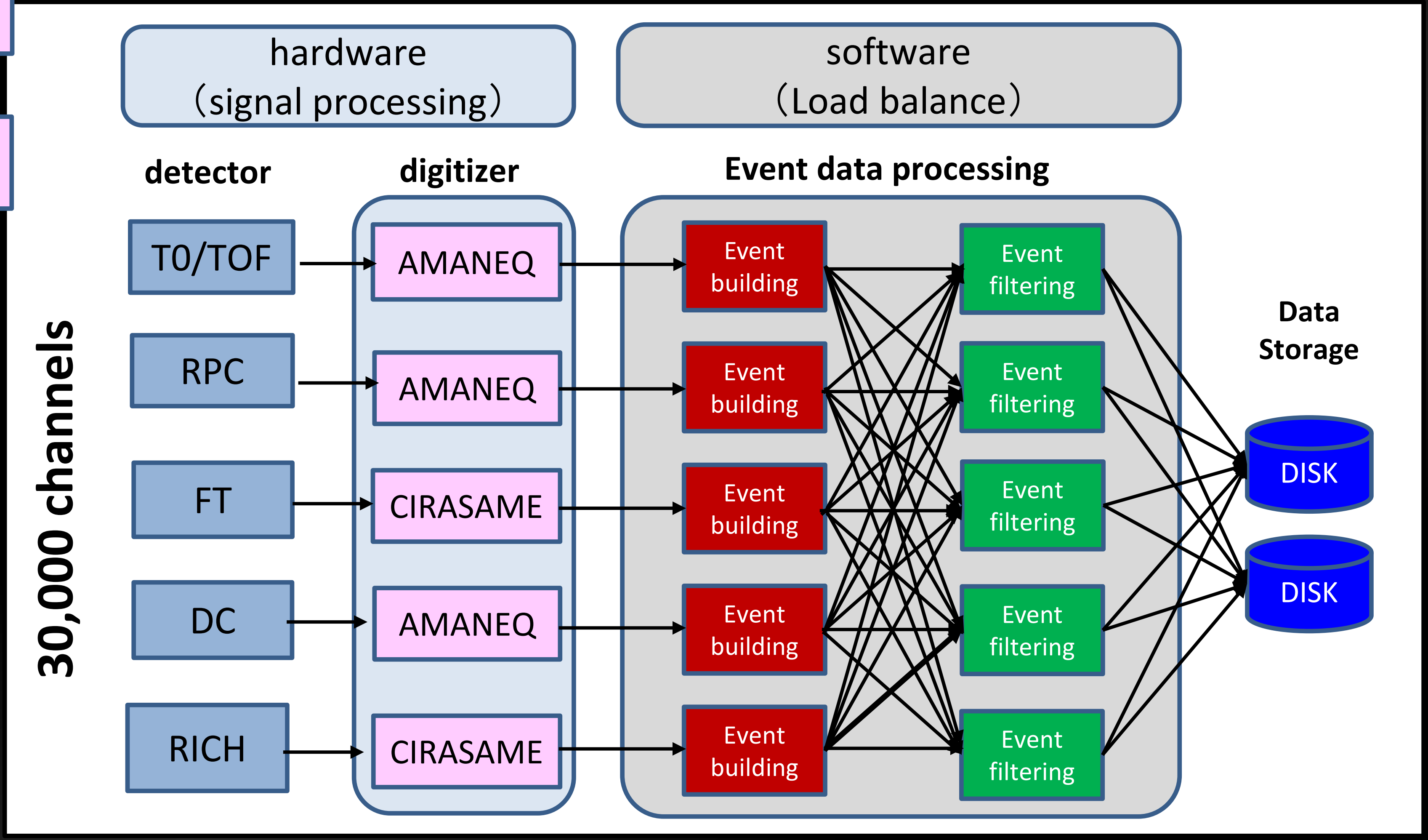
Streaming DAQ system: *a new standard brought you by*

SPADI Alliance

Efficiency 100%

new architecture w/o hardware trigger

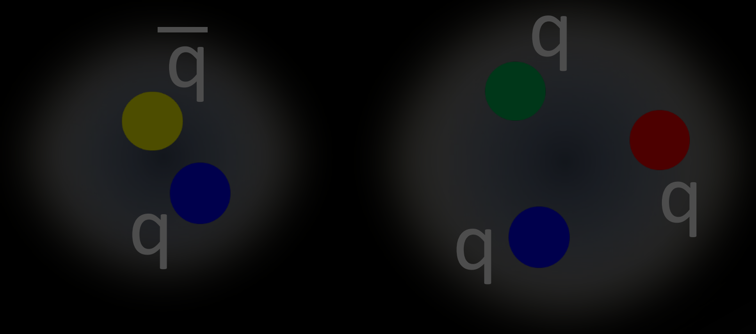




☺ demonstrated in test experiments：R. Honda, PTEP 2021, 123H01)

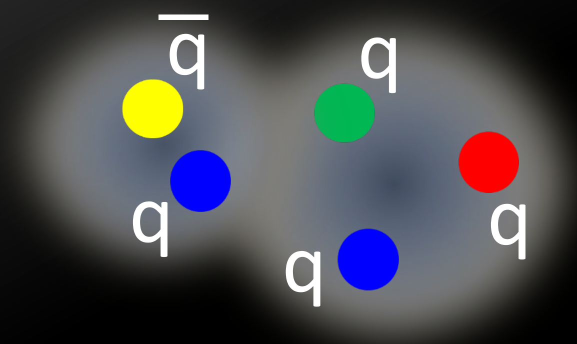
Quark-cluster aspects in Hadrons

Constituent Quark



Quark-Diquark
(Colored cluster)

Hadronic Molecule
(colorless cluster)

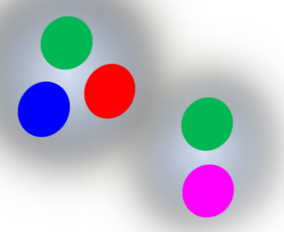


T. Yoshida, E. Hiyama, A. Hosaka, M. Oka, K. Sadato, Phys. Rev. D**92** (2015) 114029

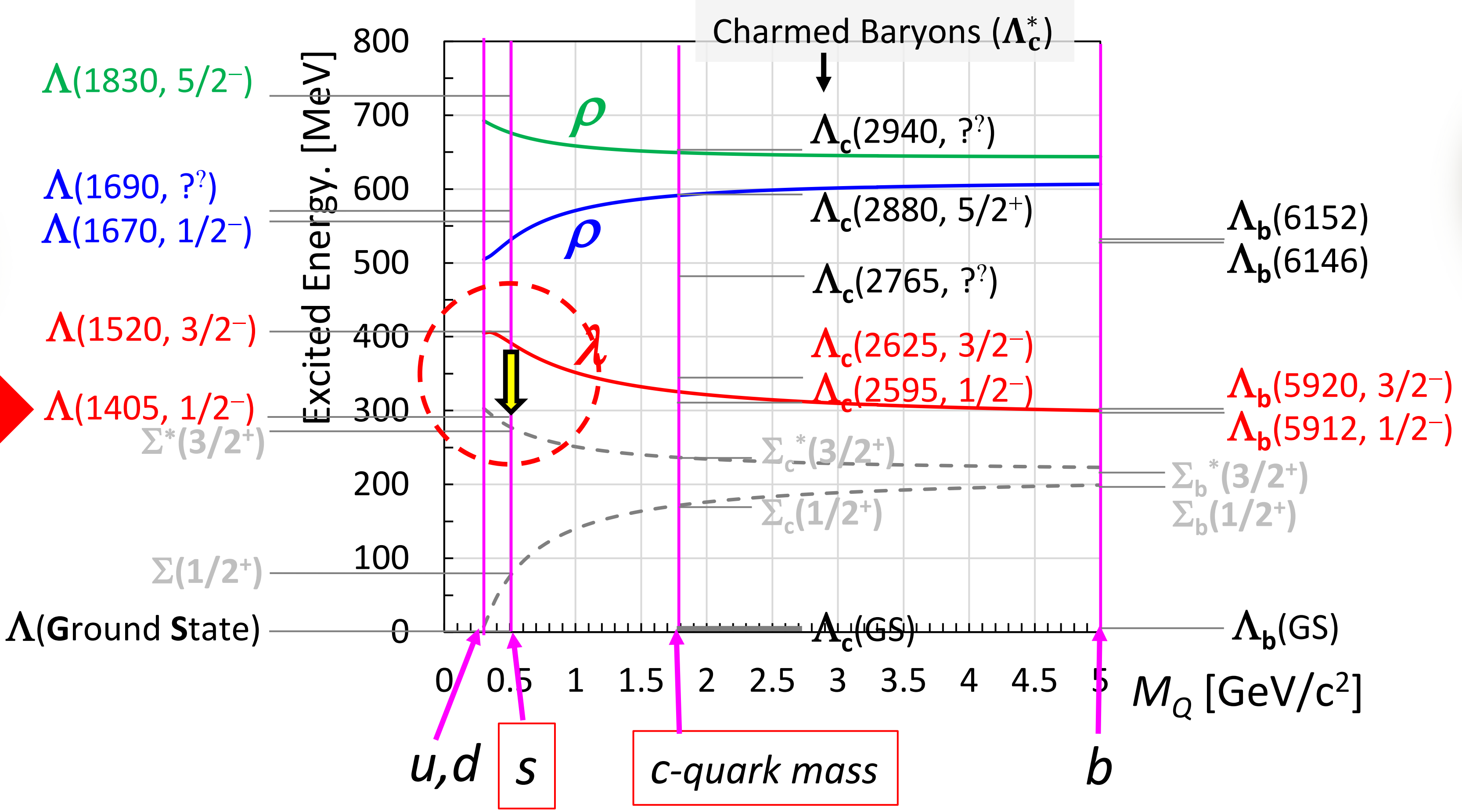
Level structure of Lambda baryons as a function of M_Q

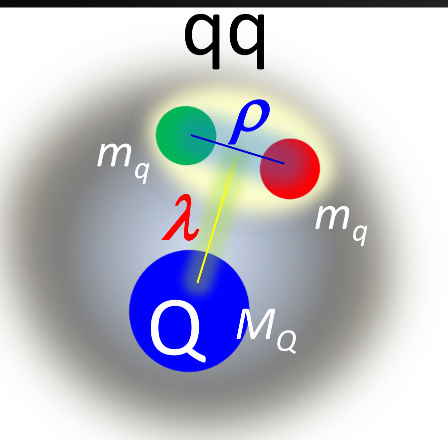
Quark Model Calculation (curves) for Excitation Energy Spectra as a function of Heavy quark mass (M_Q)

※Mass/spin/parity of Λ, Λ_c, Λ_b observed so far are shown below : Their excitation modes (internal structure) to be clarified



Exotic?





[Qqq] baryons

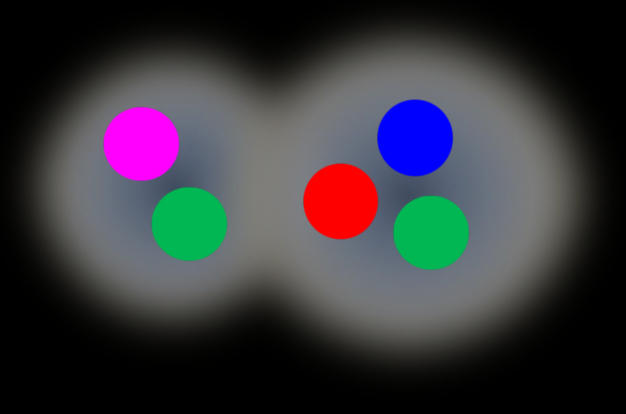
$m_q=m_u=m_d$

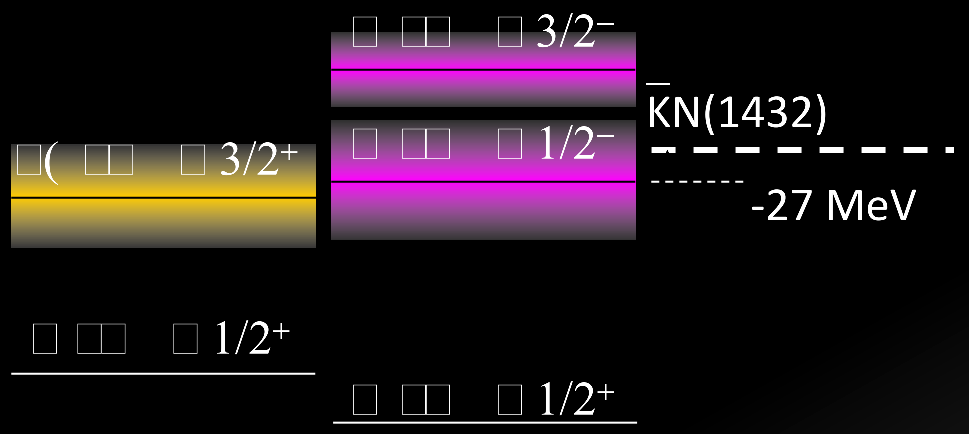
M_Q: m_q to ∞

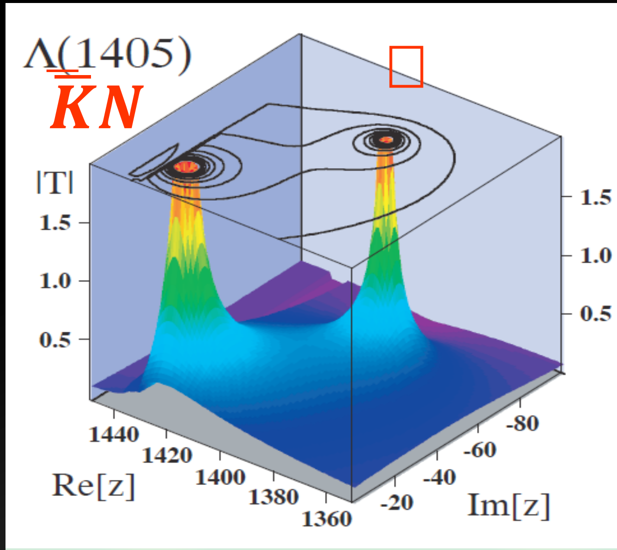
※Further understanding of baryon structure though systematic change of the excitation modes in different flavors

$\Lambda(1405)$: $1405.1^{+1.3}_{-0.9}$ MeV (PDG in 2022) ⇔ Double pole?

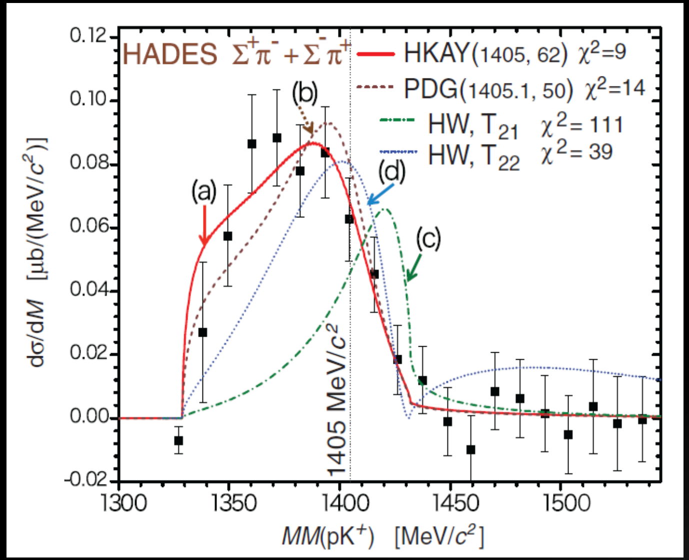
$J^P = \frac{1}{2}^-$, $I = 0$, $M_{\Lambda(1405)} < M_{\mathrm{KbarN}}$, lightest in neg. parity baryons



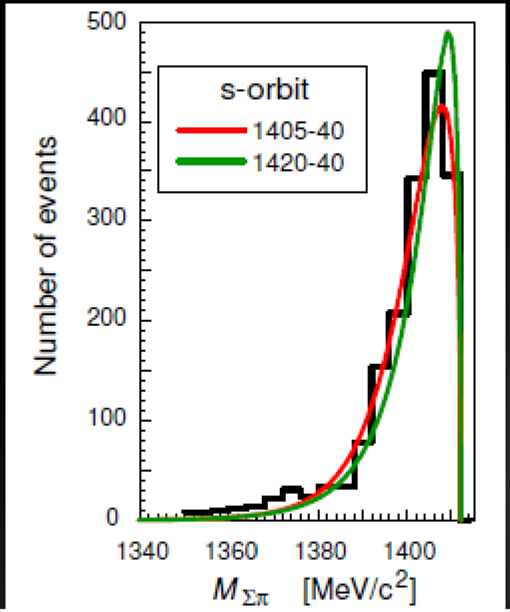




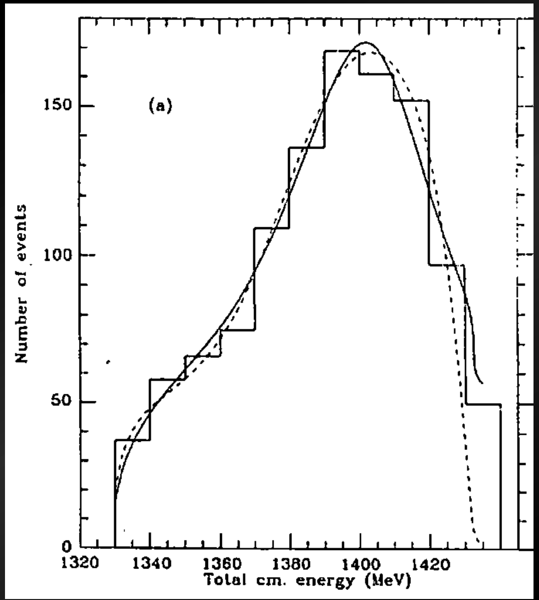
ChU model, T. Hyodo



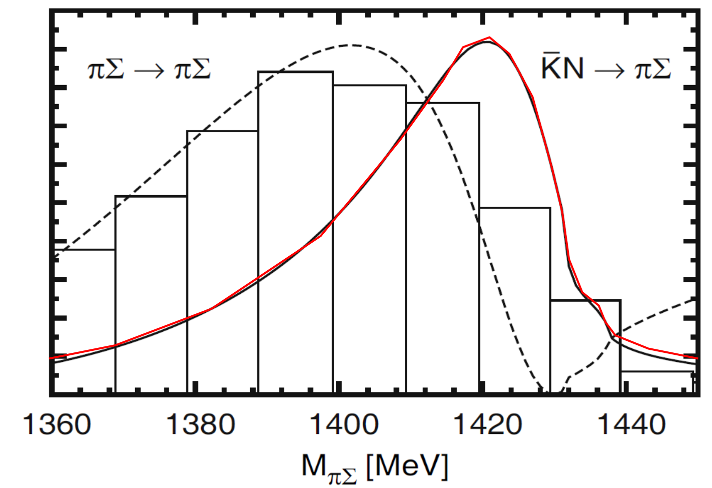
M. Hassanvand et al: IM Spec. of pp→K⁺



J. Esmaili et al: IM Spec. of Stopped K⁻ on ⁴He

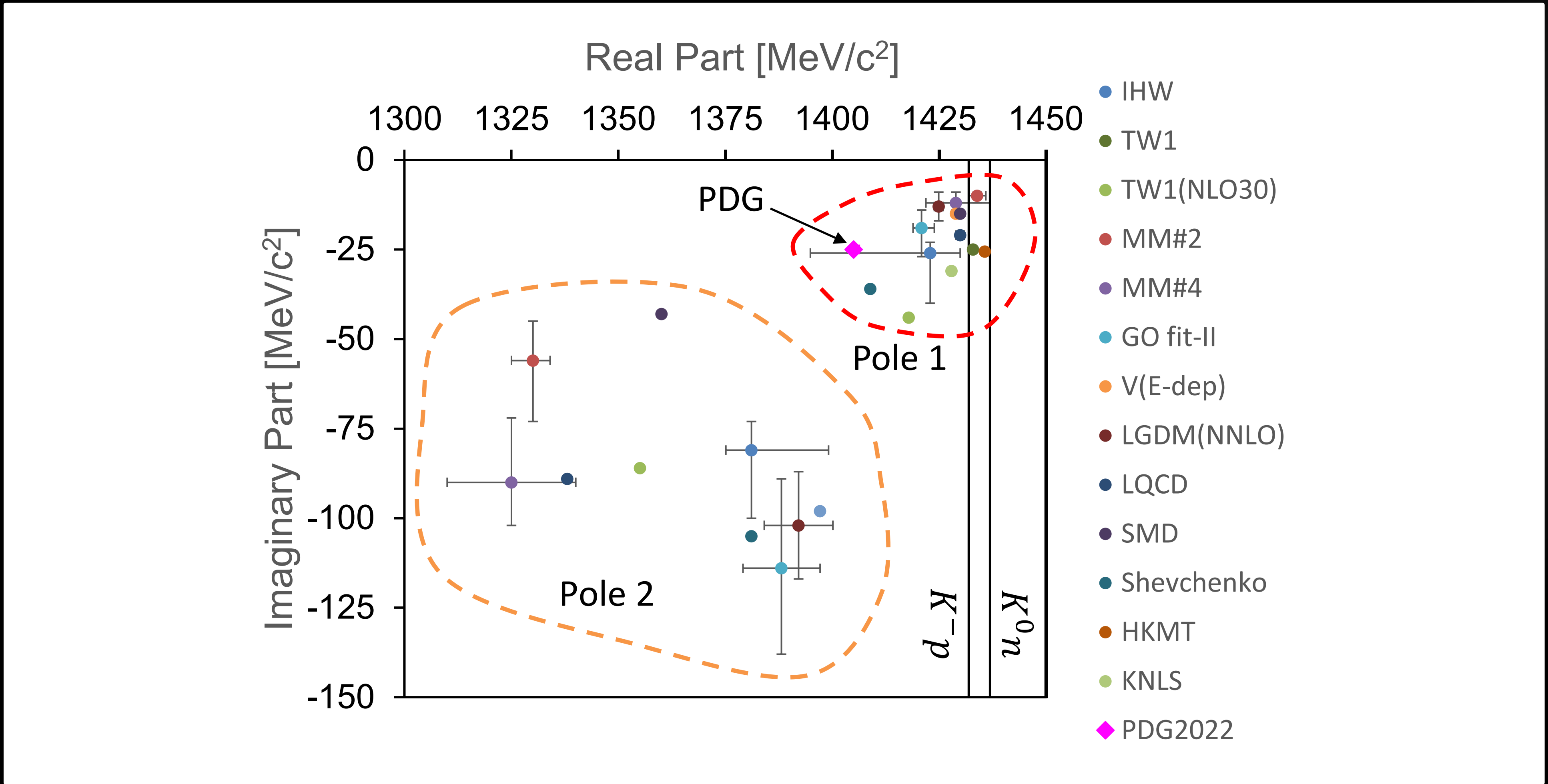


R.H. Dalitz et al: IM Spec. in K-p→ππΣ w/ M-matrix



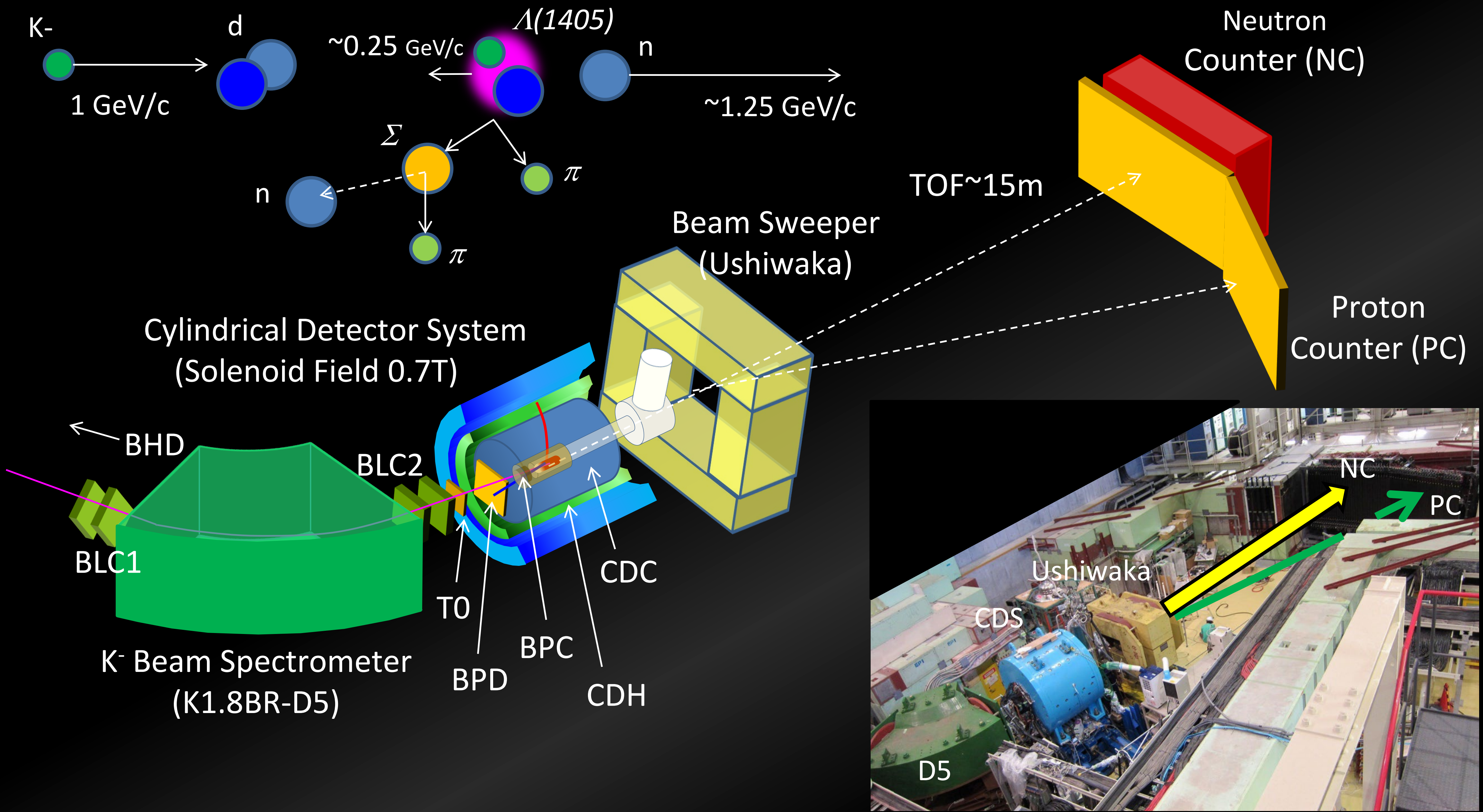
Chiral Unitary Model:
D. Jido et al., NPA725(03)181

Two-pole structure of $\Lambda(1405)$ in Meson-Baryon dynamics
(theoretical analyses constraint by $\bar{K}N$ scat., Kaonic X-ray data, etc)

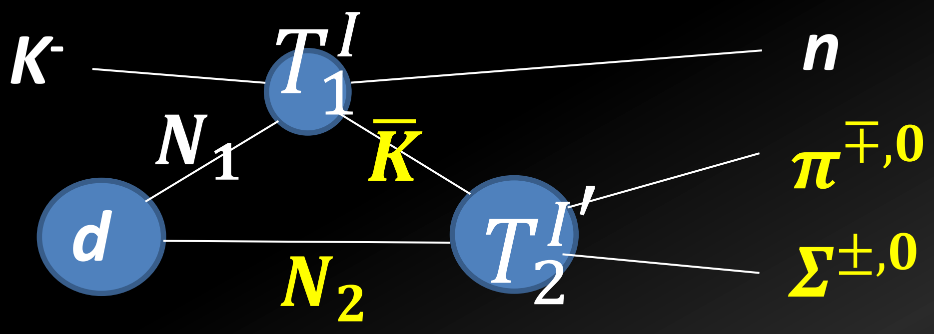


Need direct access to the $\bar{K}N$ Scat. Amp. and pole position.

Experimental Setup for E31 at J-PARC K1.8BR



Description of the $\pi\Sigma$ Spectrum w/ $\bar{K}N$ Scattering Amplitude



$$\frac{d\sigma}{dM_{\pi\Sigma}}\Big|_{\theta_n=3^\circ}$$

$$\sim\left|\langle n\pi\Sigma\middle|T_2^{I'}(\bar{K}N_2\to\pi\Sigma)G_0T_1^I(K^-N_1\to\bar{K}n)\middle|K^-\Phi_d\rangle\right|^2\sim\left|T_2^{I'}(\bar{K}N\to\pi\Sigma)\right|^2F_{\mathrm{res}}(M_{\pi\Sigma})$$

Factorization Approximation

$$T_2^{I'}(\bar{K}N\to\bar{K}N)=\frac{A}{1-iAk_2+\frac{1}{2}ARk_2^2}$$

$$T_2^{I'}(\bar{K}N\to\pi\Sigma)=\frac{1}{\sqrt{k_1}}e^{i\delta_0}\frac{\sqrt{ImA-\frac{1}{2}|A|^2ImRk_2^2}}{1-iAk_2+\frac{1}{2}ARk_2^2}$$

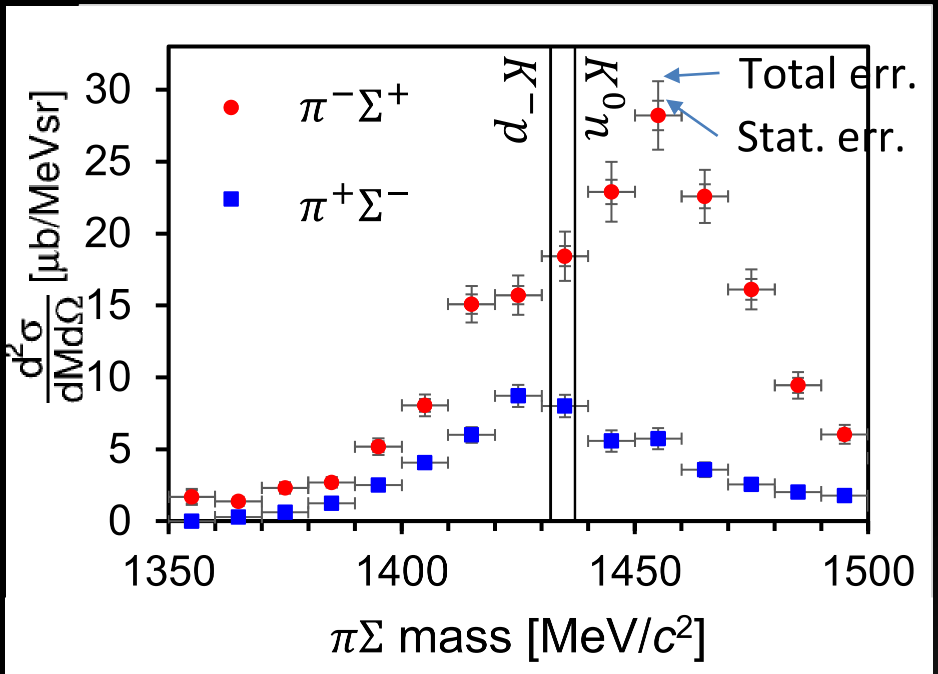
A: scattering length

R: effective range

δ_0: phase

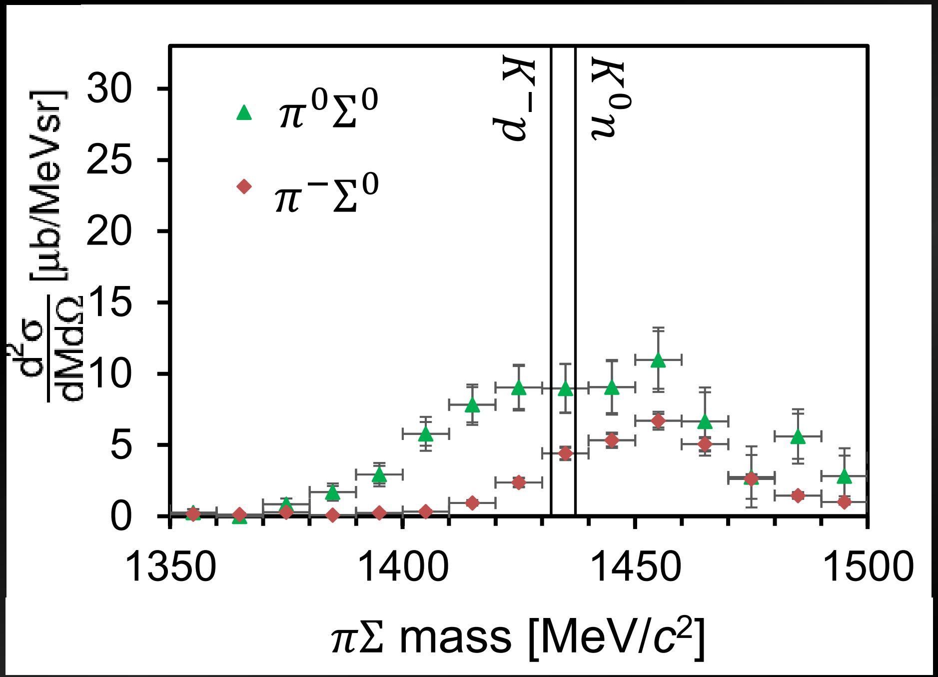
$$F_{\mathrm{res}}(M_{\pi\Sigma})\sim\left|\int_0^\infty dq_{N_2}^3\,T_1^I\frac{1}{E_{\bar{K}}-E_{\bar{K}}(q_{\bar{K}})+i\epsilon}\Phi_d(q_{N_2})\right|^2,\quad q_{\bar{K}}+q_{N_2}=q_{\pi\Sigma}$$

$$\pi^+\Sigma^-/\pi^-\Sigma^+ \\ (I'=0,1)$$



$$\frac{d\sigma}{d\Omega}(\pi^-\Sigma^+/\pi^+\Sigma^-) \propto \left|\frac{3T_1^{I=0}-T_1^{I=1}}{4\sqrt{3}}\boldsymbol{T_2^{I'=0}} \pm \frac{T_1^{I=0}+T_1^{I=1}}{4\sqrt{2}}\boldsymbol{T_2^{I'=1}}\right|^2$$

$$\pi^0\Sigma^0(I'=0) \\ \pi^-\Sigma^0(I'=1)$$

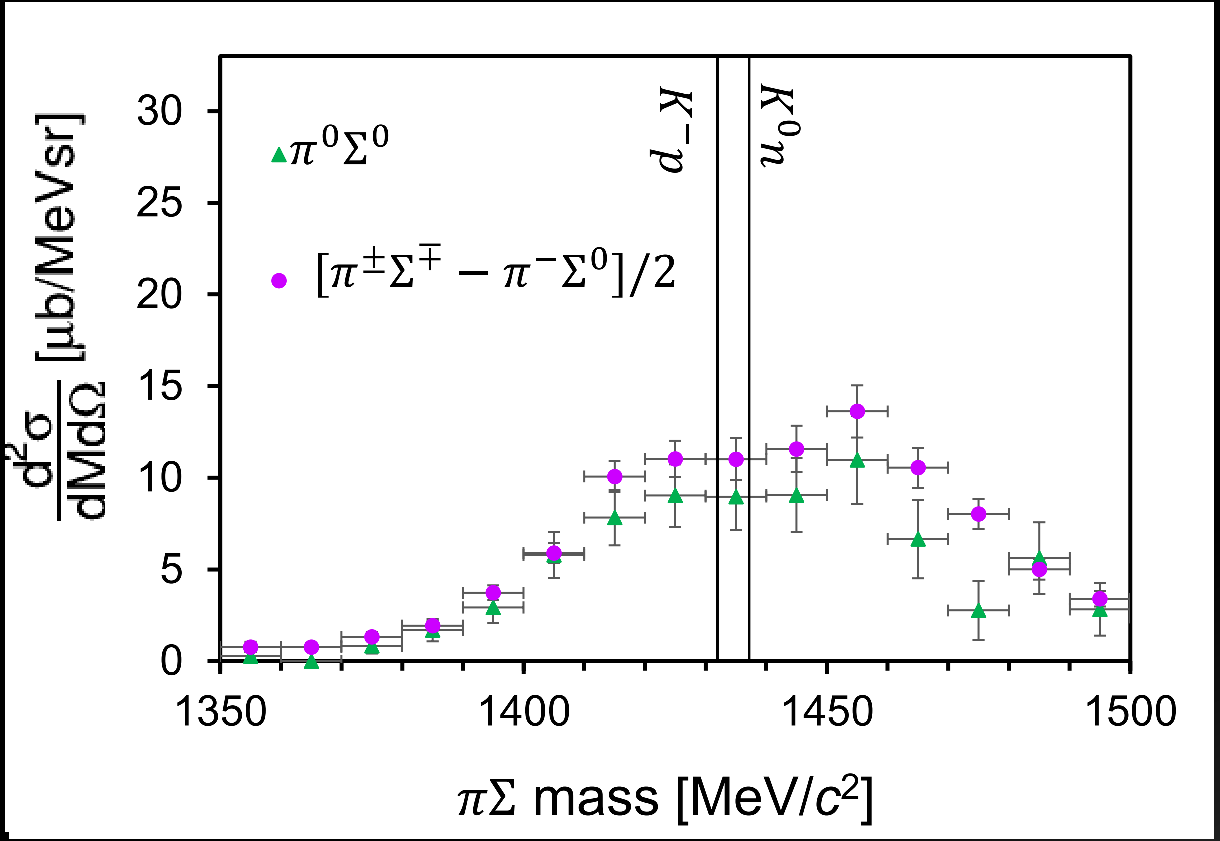


$$\frac{d\sigma}{d\Omega}(\pi^0\Sigma^0) \propto \left|-\frac{3T_1^{I=0}-T_1^{I=1}}{4\sqrt{3}}\boldsymbol{T_2^{I'=0}}\right|^2$$

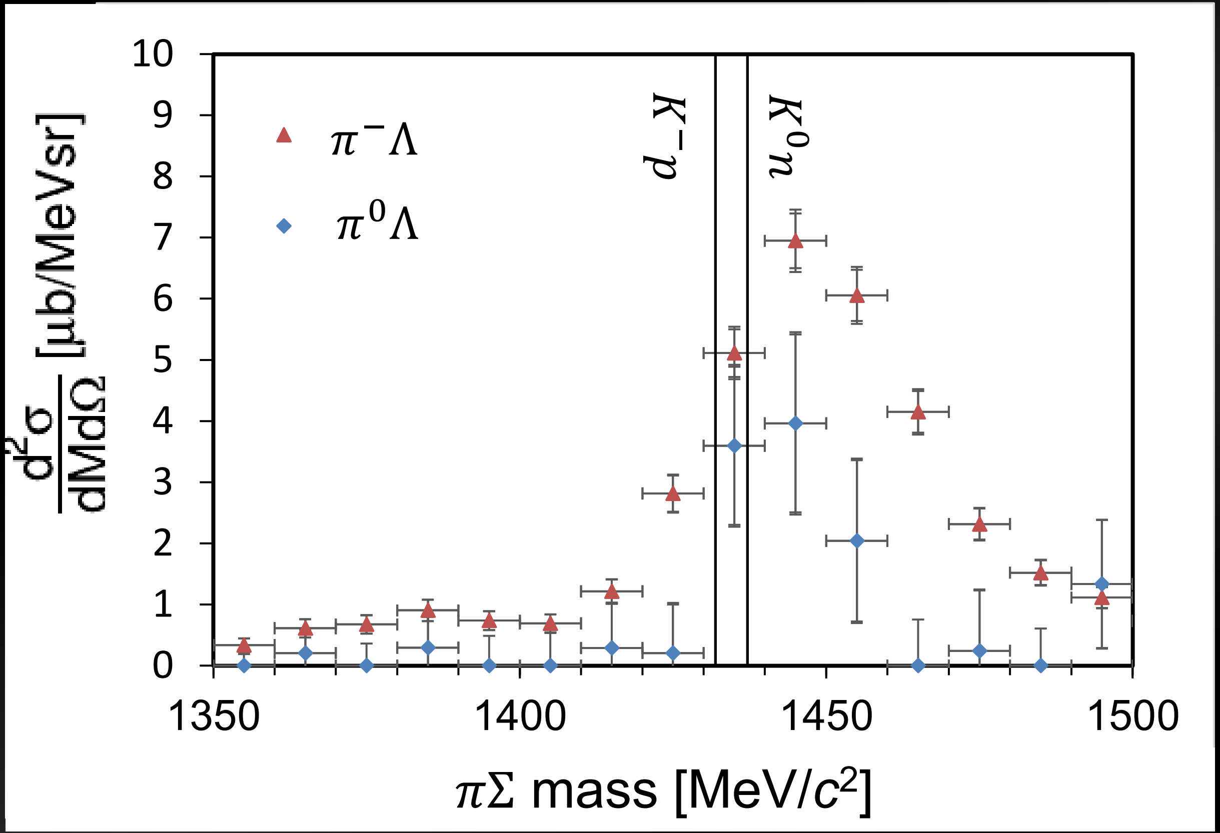
$$\frac{d\sigma}{d\Omega}(\pi^-\Sigma^0) \propto \left|-\frac{T_1^{I=0}+T_1^{I=1}}{4}\boldsymbol{T_2^{I'=1}}\right|^2$$

Isospin relations seem to be satisfied well.

$[\pi^{\pm}\Sigma^{\mp} - \pi^{-}\Sigma^{0}]/2$ vs $\pi^{0}\Sigma^{0} (I' = 0)$



$\pi^{-}\Lambda$ vs $\pi^{0}\Lambda$ $(I' = 1)$



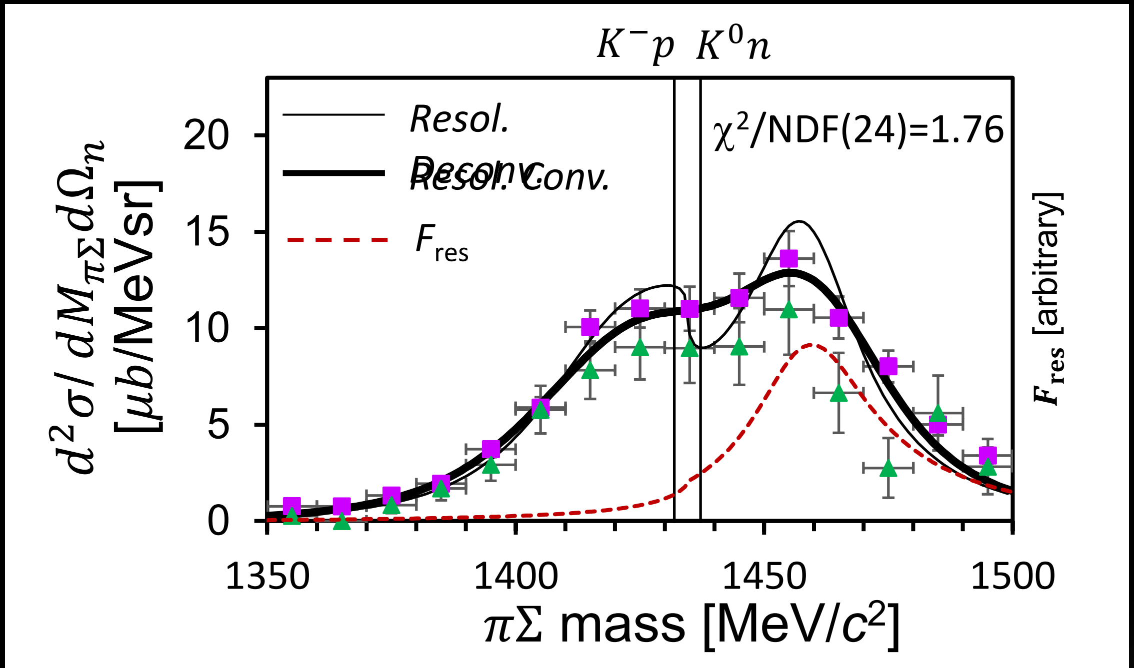
$$\frac{d\sigma}{d\Omega}\left([\pi^{\pm}\Sigma^{\mp} - \pi^{-}\Sigma^{0}]/2\right) \propto \left|-\frac{3T_1^{I=0} - T_1^{I=1}}{4\sqrt{3}} \boldsymbol{T_2^{I'=0}}\right|^2$$

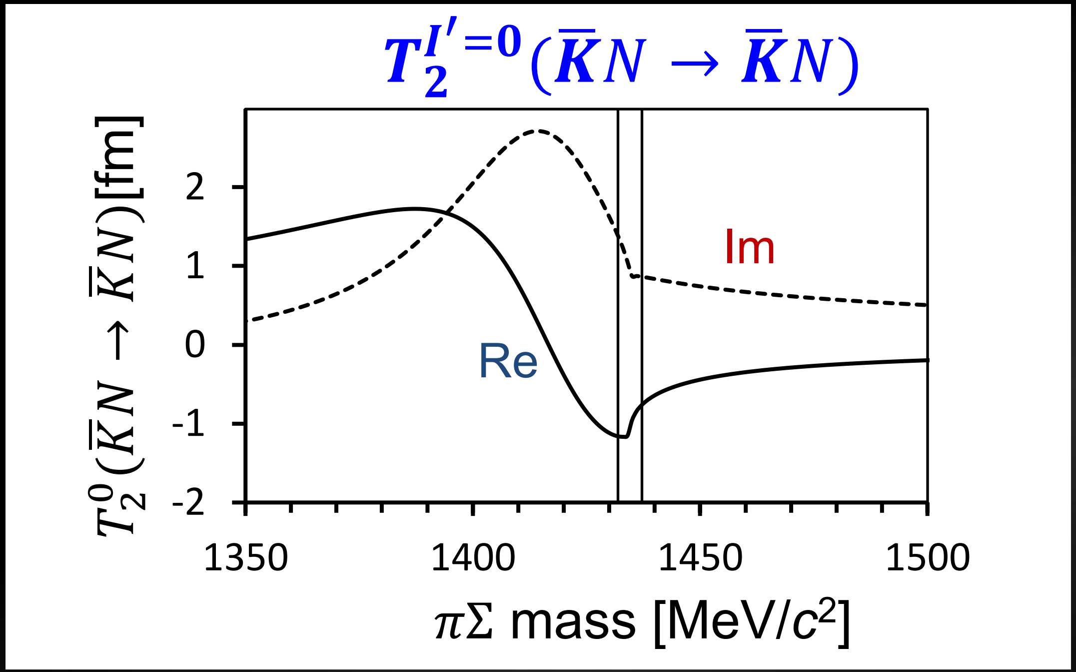
$$\approx \quad \frac{d\sigma}{d\Omega}(\pi^0\Sigma^0) \propto \left|-\frac{3T_1^{I=0} - T_1^{I=1}}{4\sqrt{3}} \boldsymbol{T_2^{I'=0}}\right|^2$$

$$\frac{d\sigma}{d\Omega}(\pi^-\Lambda) \propto \left|\frac{T_1^{I=0} + T_1^{I=1}}{2\sqrt{2}} \boldsymbol{T'^{I'=1}_2}\right|^2$$

$$\approx 2 \times \frac{d\sigma}{d\Omega}(\pi^0\Lambda) \propto \left|-\frac{T_1^{I=0} + T_1^{I=1}}{4} \boldsymbol{T'^{I'=1}_2}\right|^2$$

Best fit $\bar{K}N$ scattering amplitude





A pole at $(1417.7^{+6.0+1.1}_{-7.4-1.0}) + (-26.1^{+6.0+1.7}_{-7.9-2.0})i$ MeV/c^2

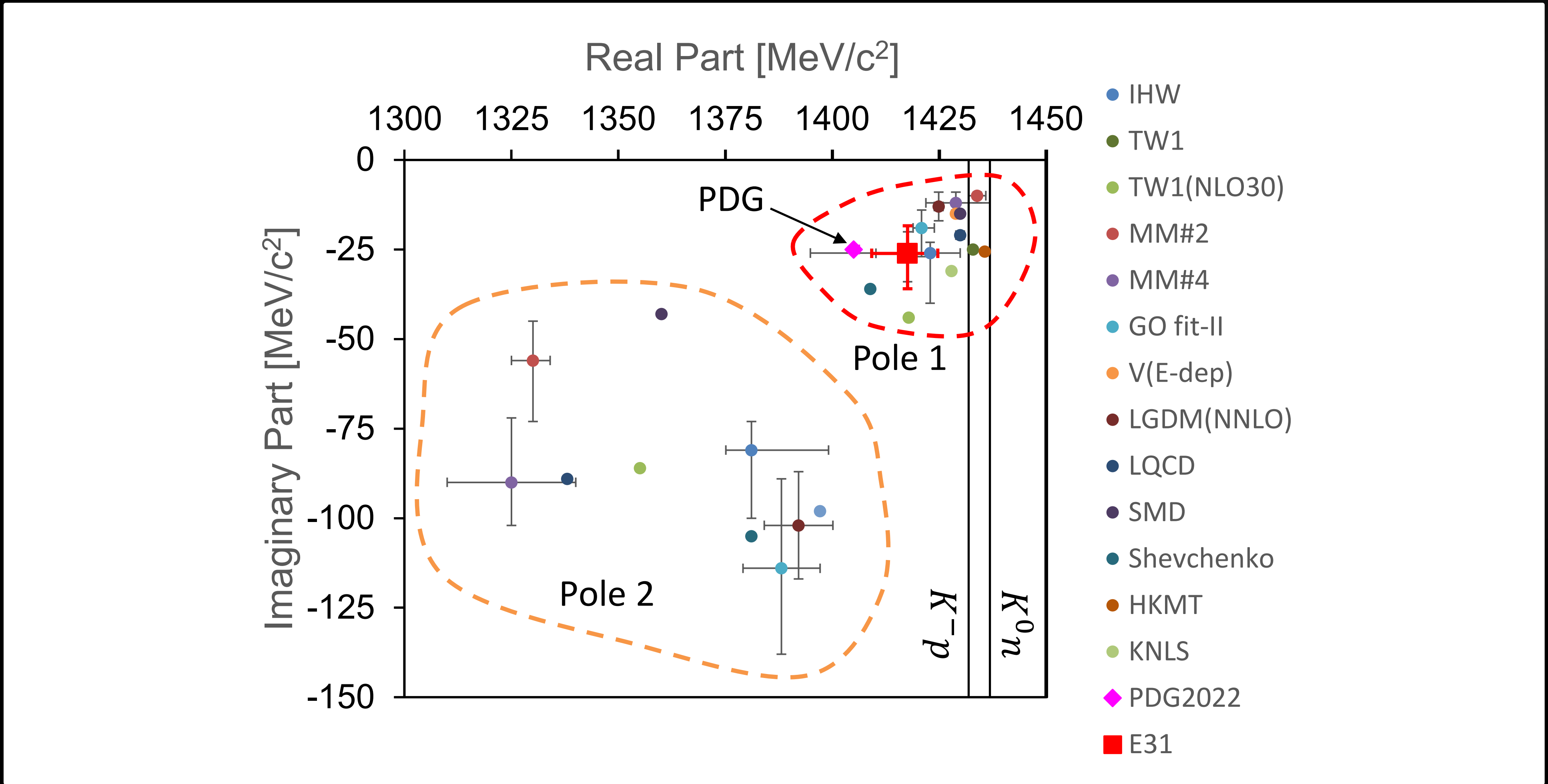
$$\left|T_2^{I'=0}(\bar{K}N \to \bar{K}N)\right|^2 / \left|T_2^{I'=0}(\bar{K}N \to \pi\Sigma)\right|^2 = 2.2^{+1.0+0.3}_{-0.6-0.3}$$

$$\boldsymbol{A}^{I'=0} = (-1.12 \pm 0.11^{+0.10}_{-0.07}) + i(0.84 \pm 0.12^{+0.08}_{-0.07})\ \text{fm}$$

$$\boldsymbol{R}^{I'=0} = (-0.18 \pm 0.31^{+0.08}_{-0.06}) + i(0.41 \pm 0.13^{+0.09}_{-0.09})\ \text{fm}$$

*best fit value ± fitting error ± systematic error
systematic errors assuming the K^-p/K^0n mass threshold

Two-pole structure of $\Lambda(1405)$ in Meson-Baryon dynamics
(theoretical analyses constraint by $\bar{K}N$ scat., Kaonic X-ray data, etc.)



E31 result → arXiv:2209.08254, Submitted to PLB

Summary

- Spectroscopy of baryons with heavy flavors provides unique opportunities to investigate dynamics of quarks, which reflects the nature of QCD in low energy.
  - "Diquarks" play a key role.
- A spectrometer system to conduct charmed baryon spectroscopy via the $\pi^- p \to D^{*-} Y_c^{*+}$ reaction is under developing at J-PARC.
  - To be a new platform for hadron physics at J-PARC
- A pole position of Lambda(1405) in the $\bar{K}N$ scattering amplitude measured via the $K^- d \to n(\pi\Sigma)^{I=0}$ reaction at J-PARC is found to be $1417.7 - 26.1i$ MeV.
  - Consistent with the so-called "two-pole" picture deduced from meson-baryon dynamics based on the chiral perturbation theory.

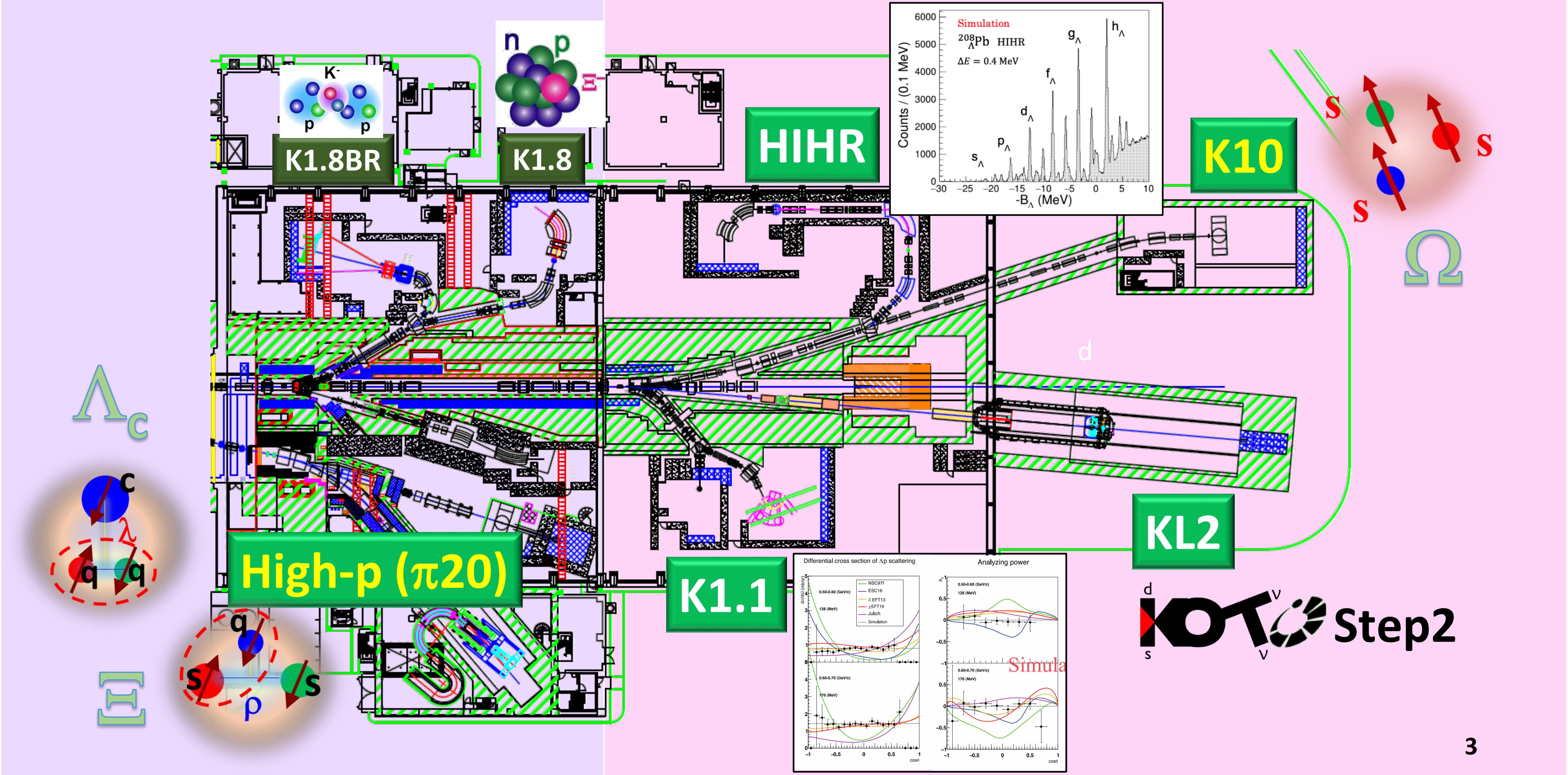
arXiv:2110.04462

Extension of the J-PARC Hadron Experimental Facility
- Third White Paper -

Taskforce on the extension of the Hadron Experimental Facility,

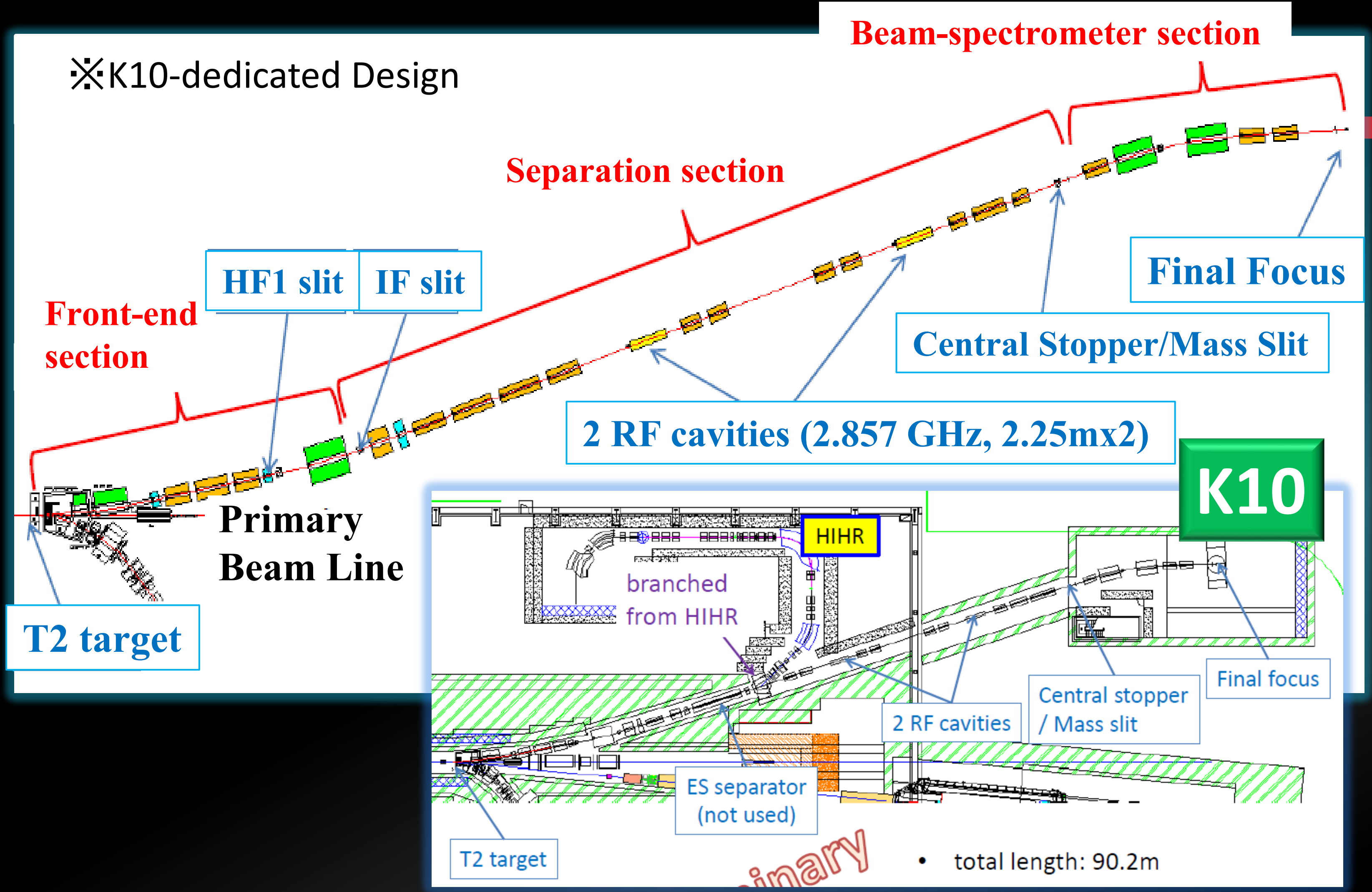
Contents

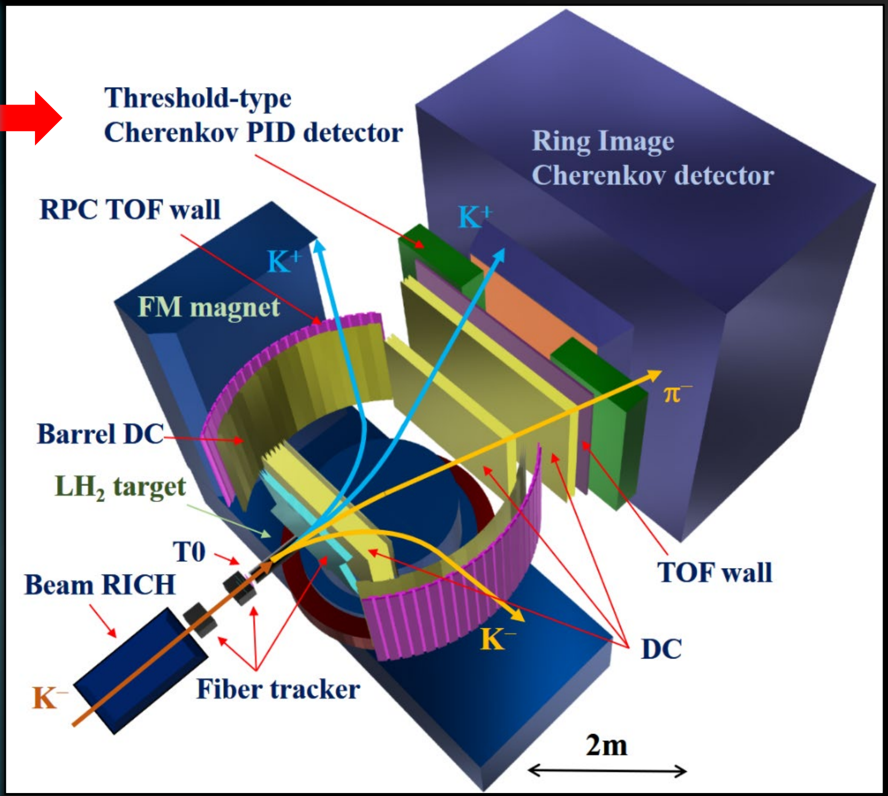
Current and Extended Hadron Experimental Facility



Xi/Omega Baryon Spectroscopy at K10

- Intense Kaon Beam: K- 7.9M/spill@8 GeV/c (50-kW p on T2 [Au 66mm])
- RF-separated Kaon Beam: $K^{-}/\pi^{-} \sim$ 1:2.1@8 GeV/c (1:2.5@10 GeV/c)

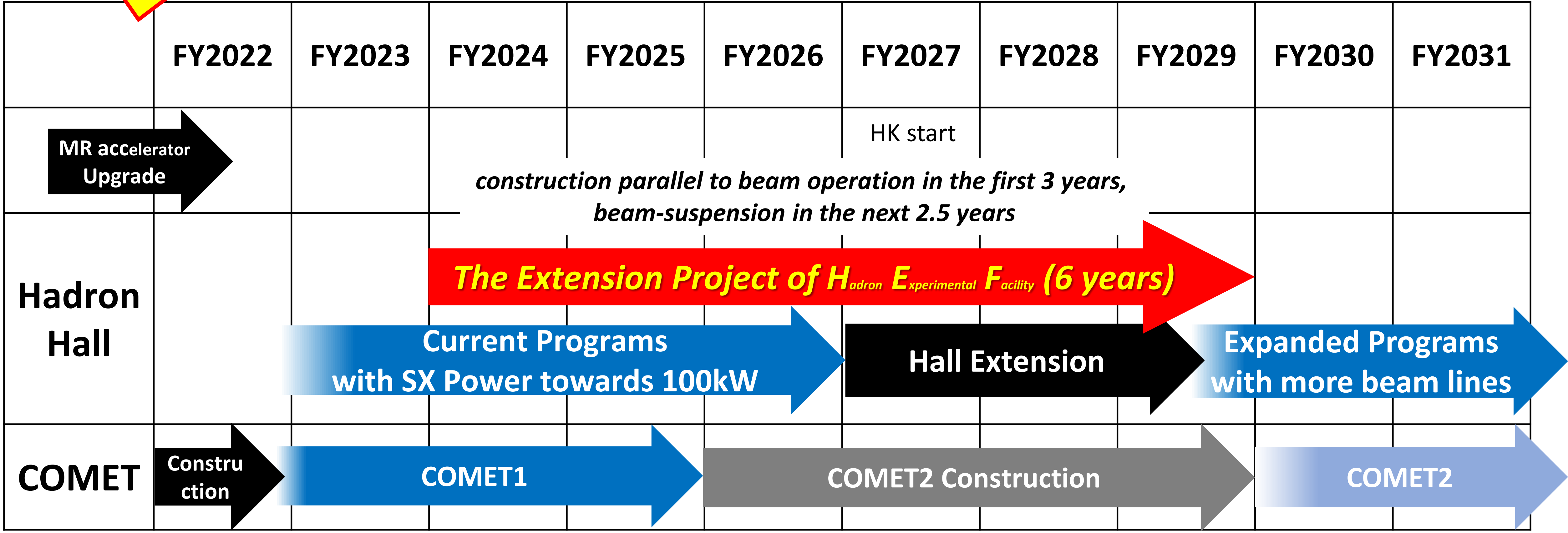




※Design to share upstream w/ HIRH is in progress

FAQ

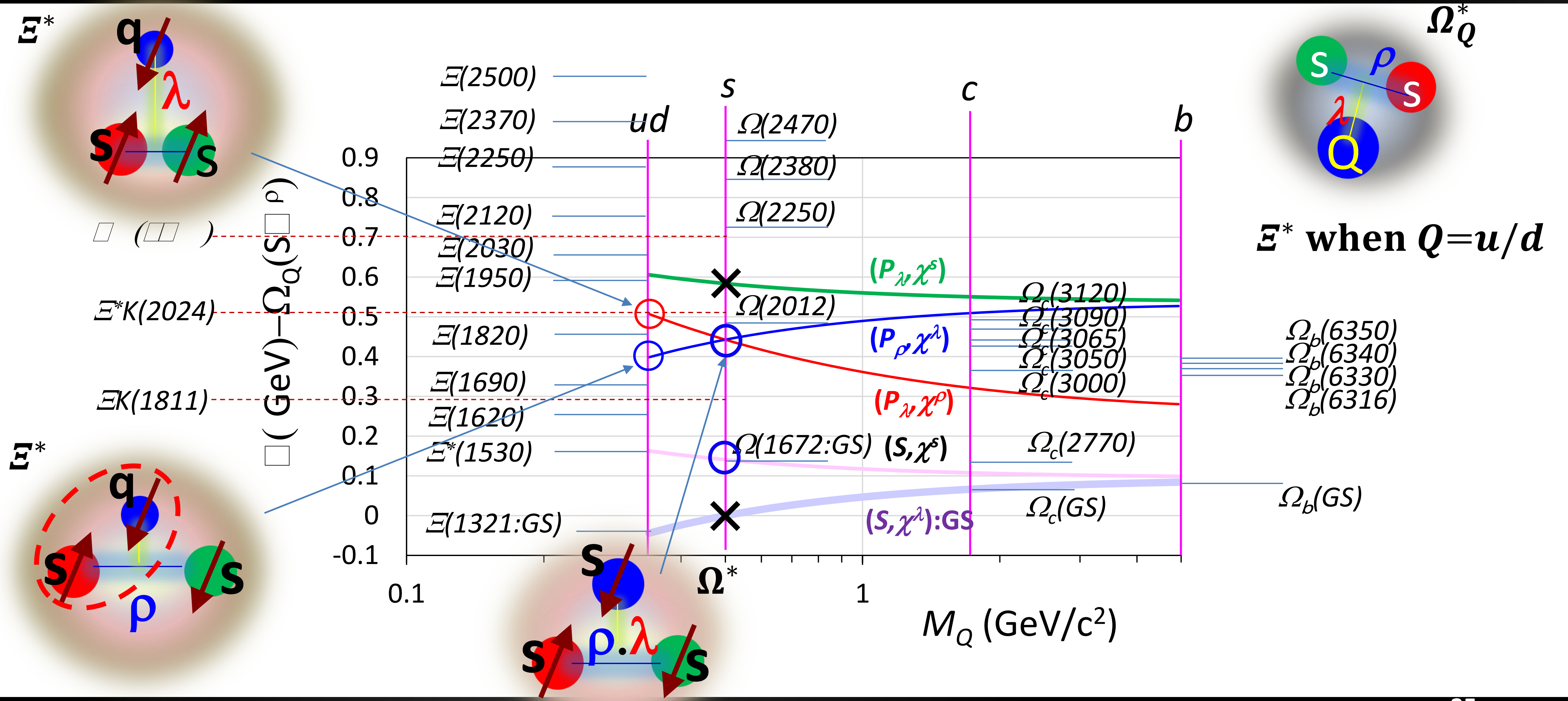
Timeline of the Project



We will start the project in FY2024

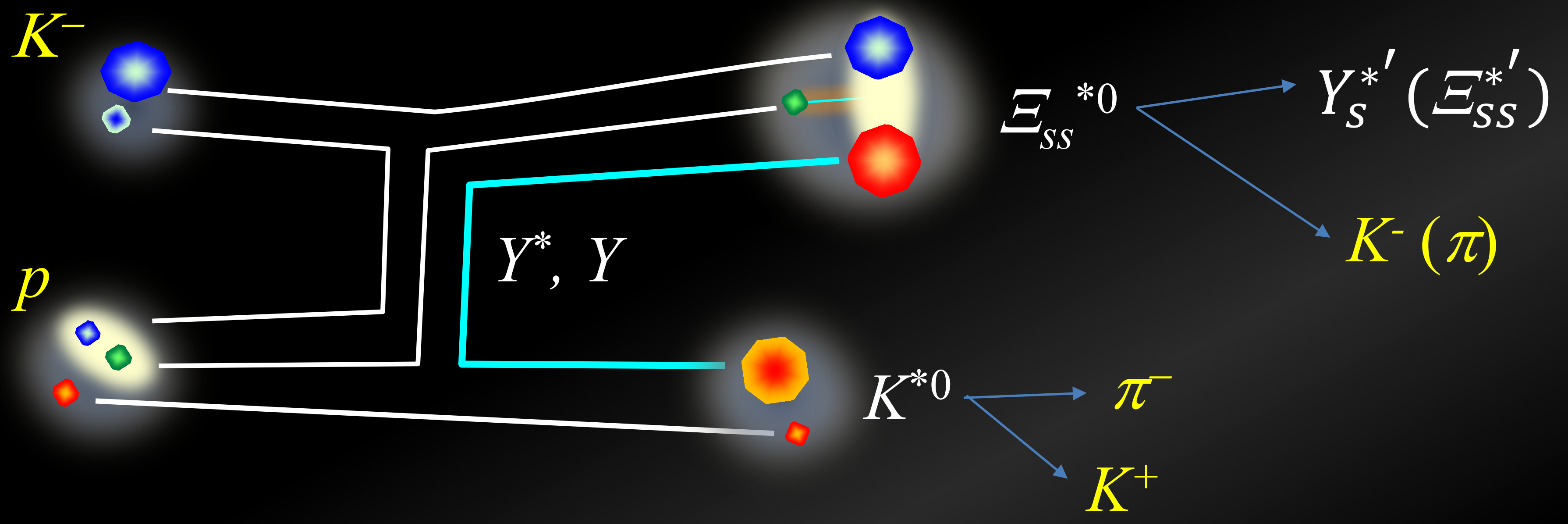
→ We are working on getting the timeline consistent with current programs

Systematics of Qss Baryons – Ξ and Ω –



Multi-Strangeness Baryon Spectroscopy Using Missing Mass Techniques

M. Naruki and K. Shirotori, LoI submitted to the 18th J-PARC PAC in May, 2014(KEK/J-PARC-PAC 2014-4)

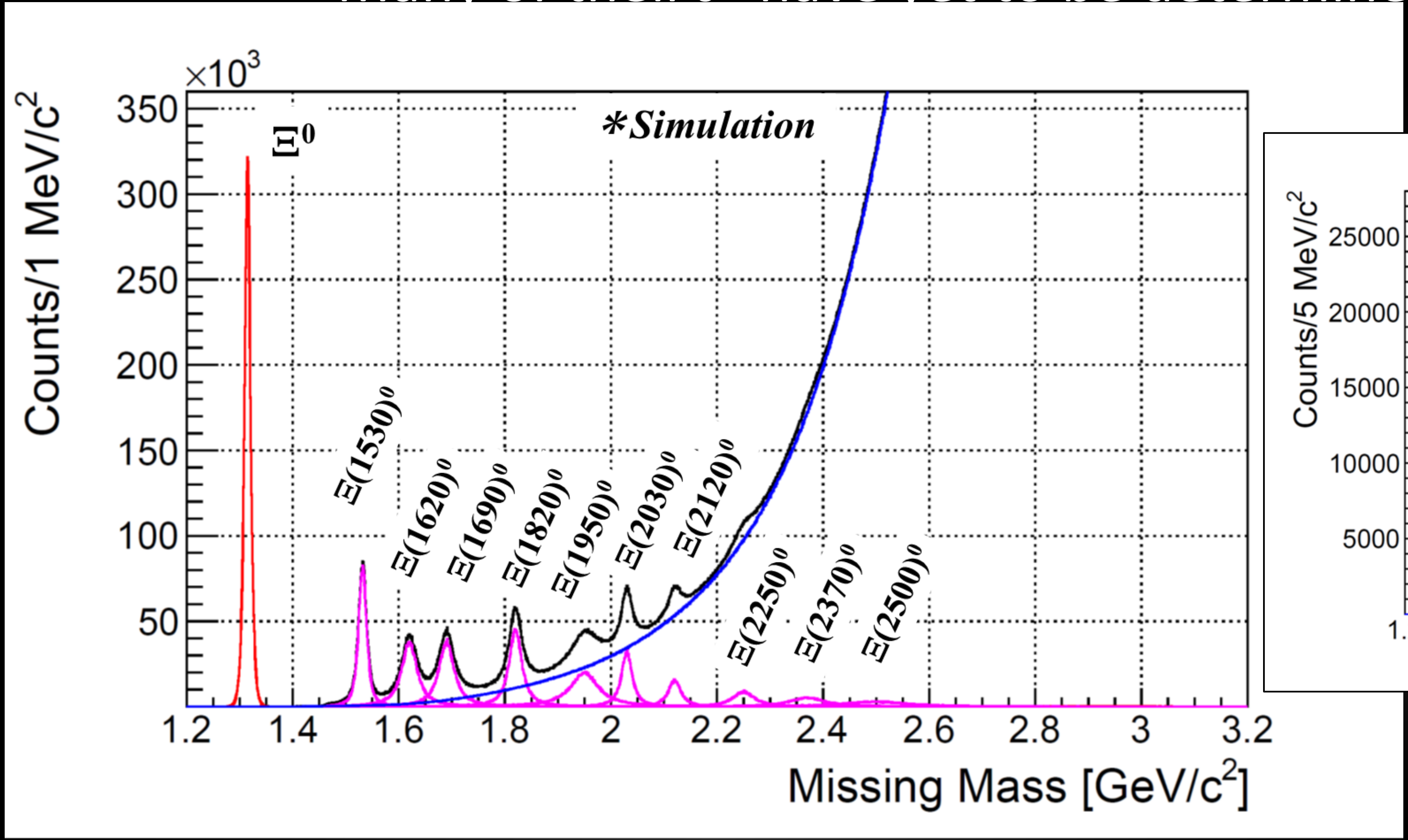


- ✓ Production and Decay reflect [QQ] correlation
- ✓ *Two-quark-involved reaction →Both ρ/λ mode excitations*

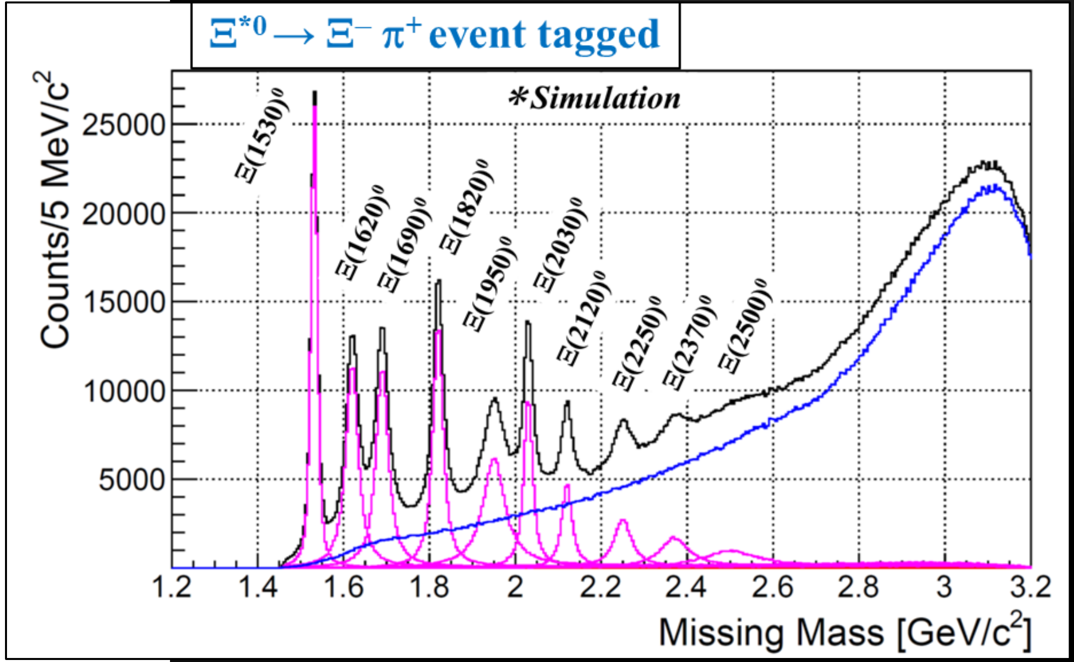
Expected Spectra in $K^- p \rightarrow K^{*0}\Xi^{*0}$ at 8 GeV/c

※simulation: known states are included w/ BG estimated by JAM.

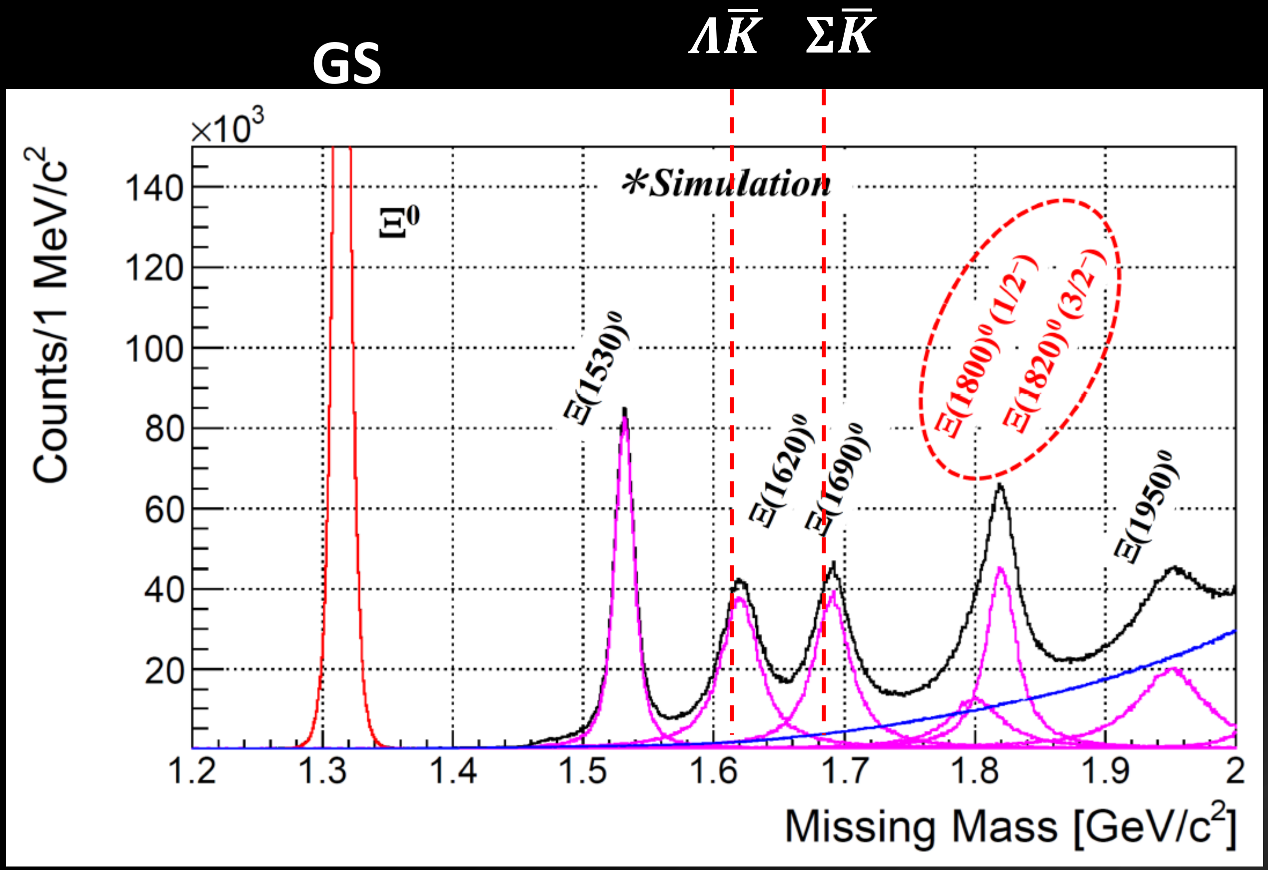
Many of their J^P have yet to be determined.



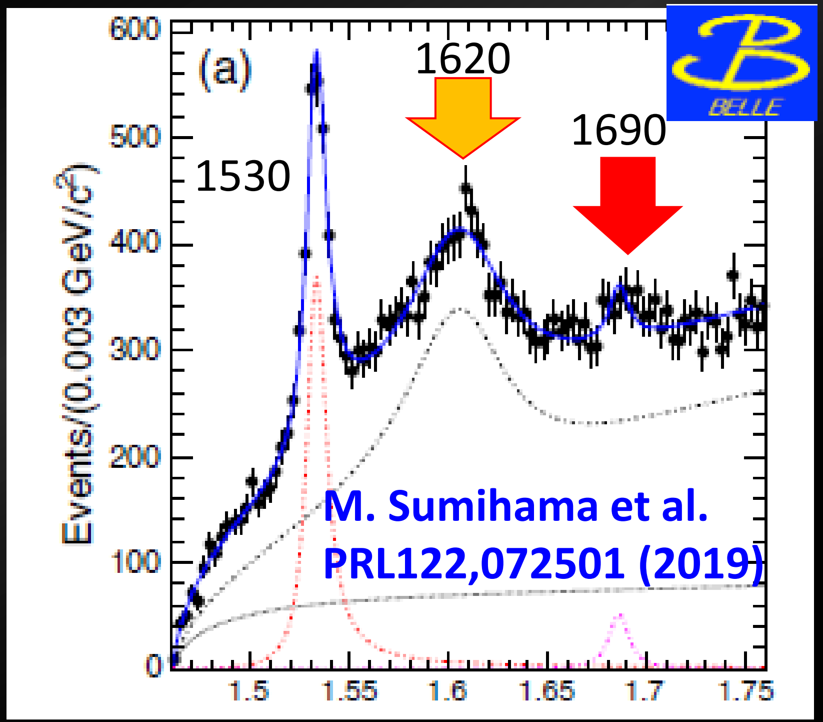
Tagged spectrum showing BG reduction.



Closed-up the low-lying Ξ states



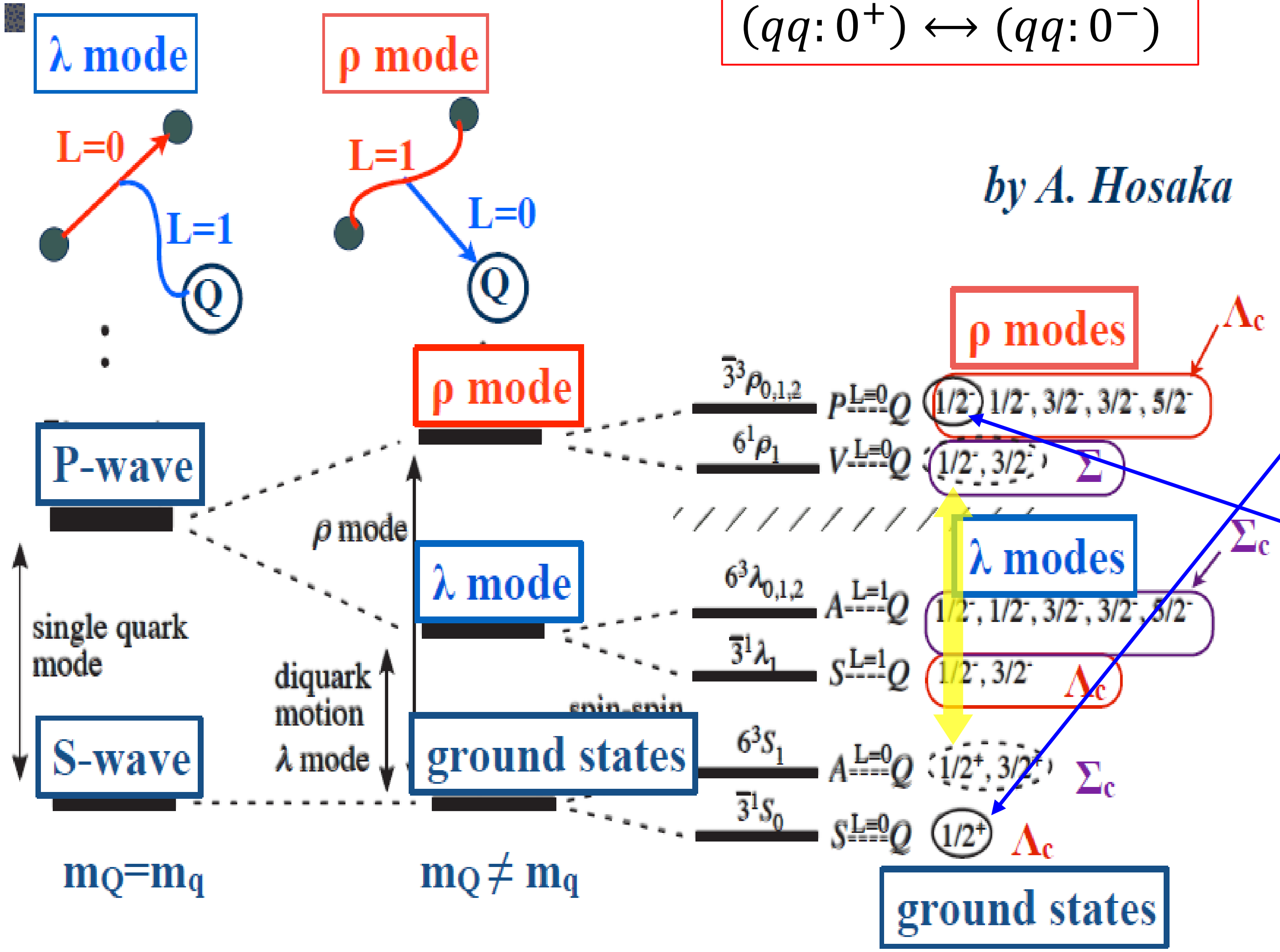
- Unexpected states in CQM?
  - Ξ(1620) nearby $\Lambda\bar{K}$ threshold
  - Ξ(1690) nearby $\Sigma\bar{K}$ threshold
- $Y\bar{K}$ molecular state or casp?
  - Prod. rates would be reduced.



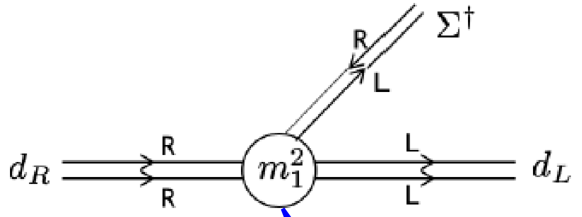
Diquark in Heavy Baryons

$U_A(1)$ anomalous singlet current in Chiral diquark effective theory

Chiral partner
$(qq{:}\,0^+) \leftrightarrow (qq{:}\,0^-)$



by A. Hosaka



by M. Oka

Scalar diquark

$$S_i^a = \frac{1}{\sqrt{2}}(d^a_{R,i} - d^a_{L,i})$$

$$\longrightarrow M(0^+) = \sqrt{m_0^2 - m_1^2 - m_2^2},$$

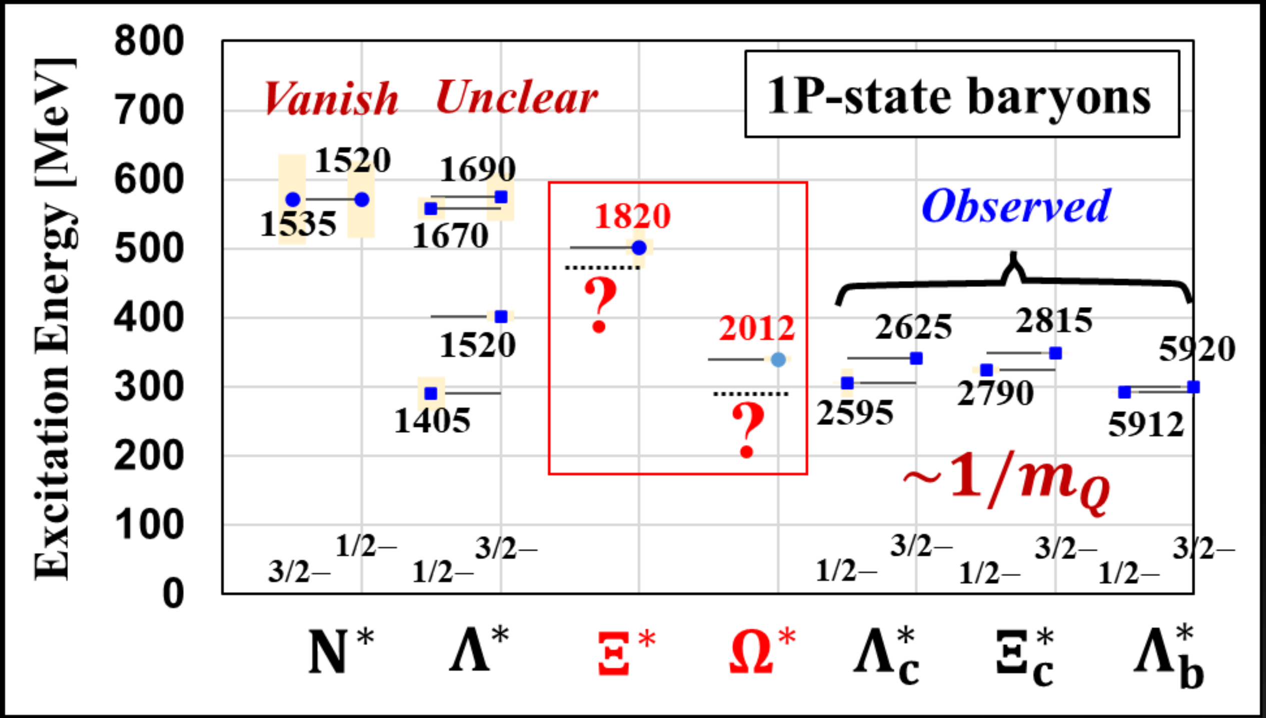
Pseudo-scalar diquark

$$P_i^a = \frac{1}{\sqrt{2}}(d^a_{R,i} + d^a_{L,i})$$

$$\longrightarrow M(0^-) = \sqrt{m_0^2 + m_1^2 + m_2^2},$$

Kim et al, Phys.Rev.D 102 (2020) 014004

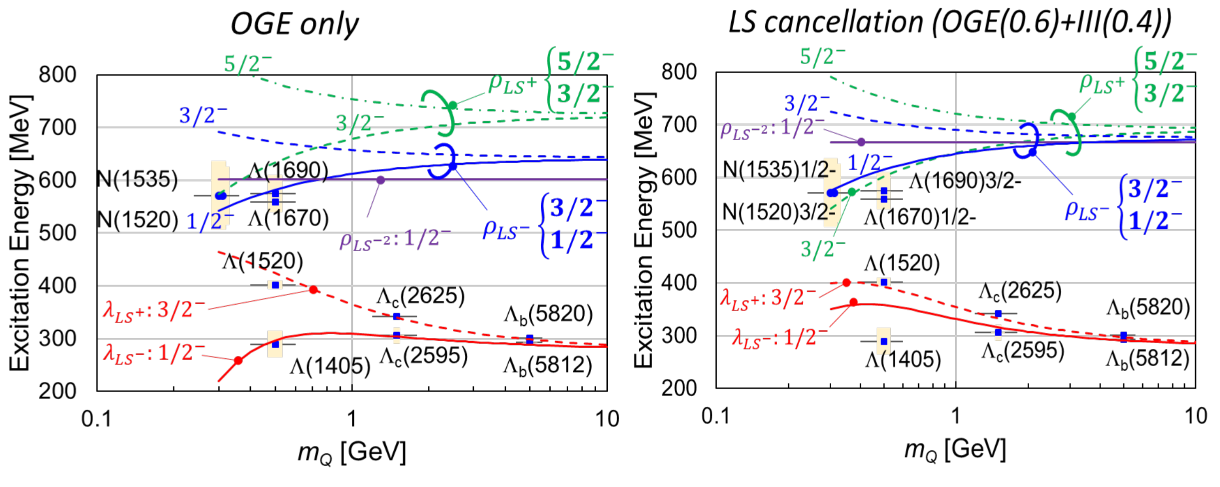
Systematic behaviors



LS int.
(naïve demo.)

$$R_{OGE} - R_{III} \sim 0.6 - 0.4\exp\left(-\frac{m_Q - m_q}{\Lambda_\chi}\right)$$

Destructive for *LS*

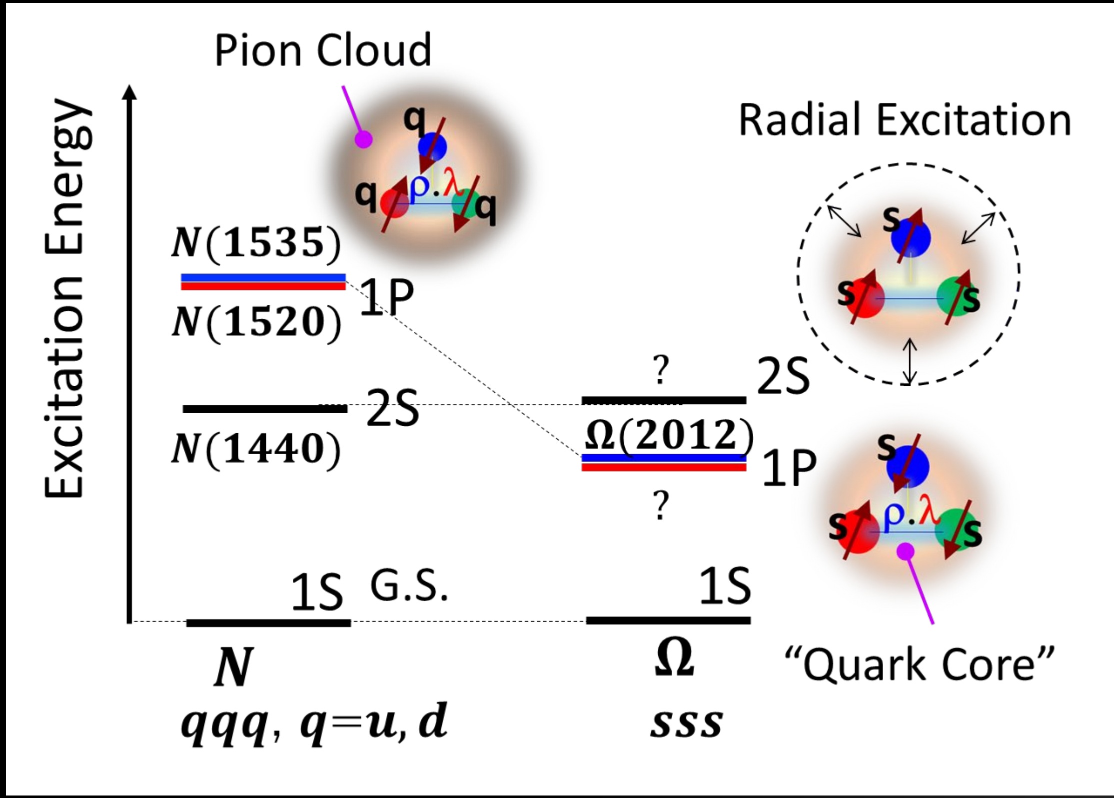


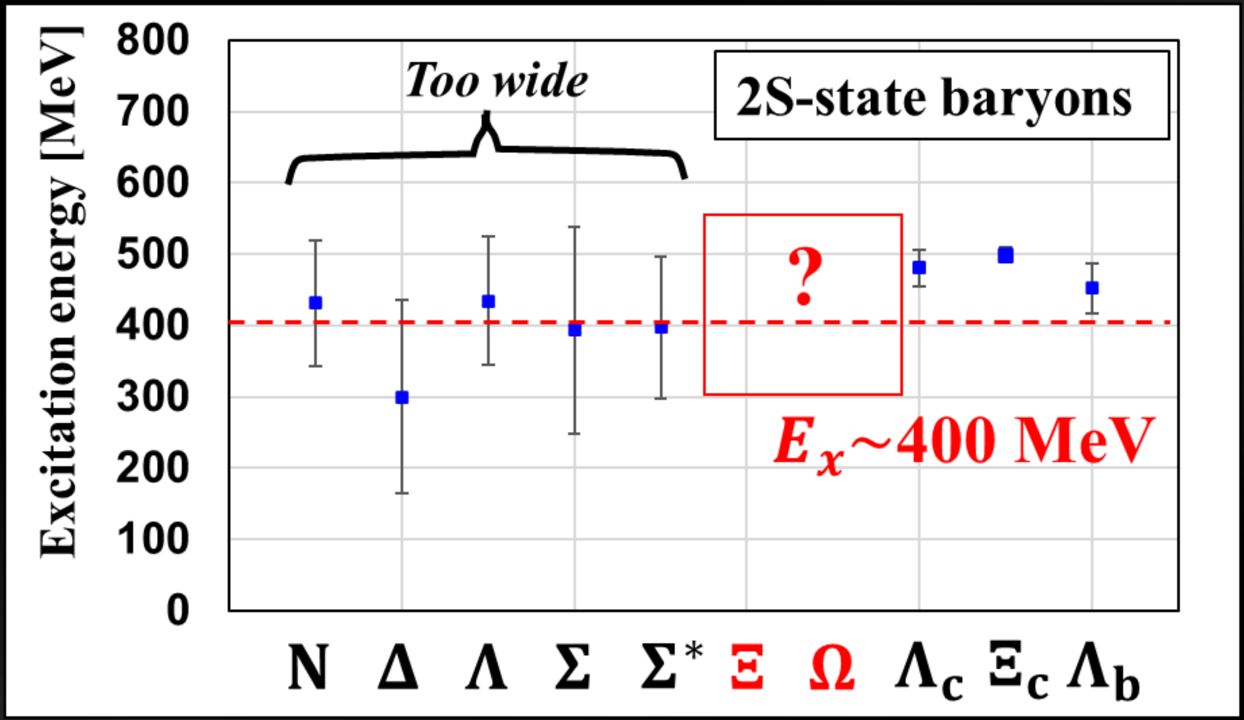
- A cancellation scenario in LS
  - Instanton Induced Int. (III) [KMT int.]
    - Same structure as OGE
    - Effective for only light quarks
    - $U_A(1)$ anomaly in III introduces $\Delta M(\eta' - \eta)$
  - OGE + Instanton Induced Int. (III) in SS
    - No affect in SS splitting both in Baryons and Mesons
  - OGE – III in LS in lighter baryons

※Origin of LS forces:
Vanished in Light System
Observed in Heavy System

S. Takeuchi, Nucl. Phys. A **642**, 543 (1998).

Systematic behaviors in Excited Baryons





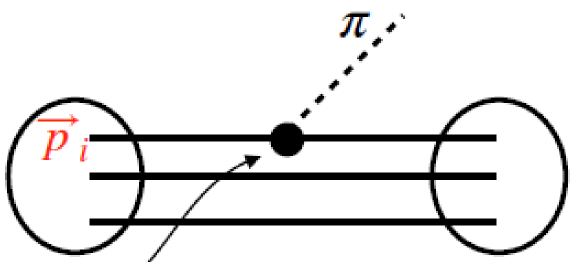
※Universality of "Roper Like" states:
By chance or Mechanism behind them?

Prof. Hosaka’s slide

(3) The Roper like states

Another method to look at the internal quark motion

Decay of Roper like stats



G.S.

NR expansion of ME

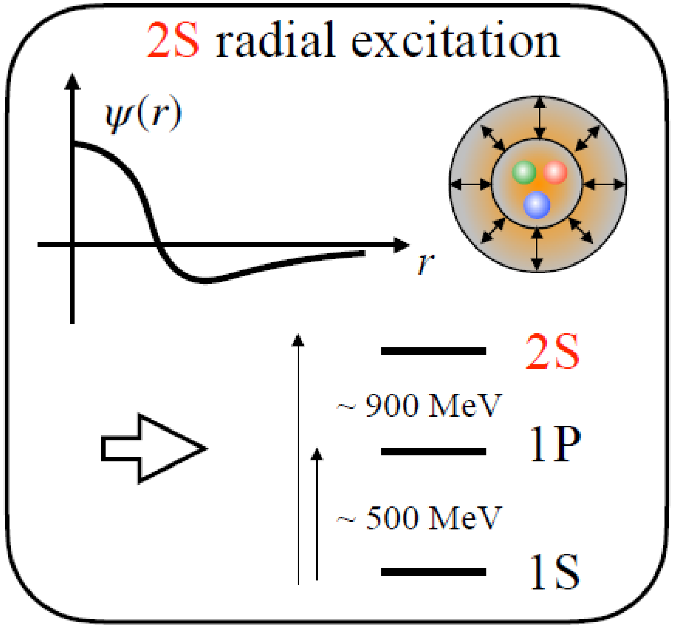
$$\langle \text{Roper} | \mathcal{O} | \text{G.S.} \rangle \sim \langle \vec{\sigma} \cdot \vec{q} \rangle (\cancel{a_0} + a_2 \vec{p}_i^{\,2} + \cdots)$$

Leading order (LO) suppressed
Unique for radial excitations

Next to leading order (NLO)

$$\langle p_i^2 \rangle \sim \frac{1}{\langle r^2 \rangle} \quad \sim \text{Size}$$

Arifi et al, *PRD* 103 (2021) 9, 094003



- Previous calculations (LO): too small
- Inclusion of NLO: 50 - 100 MeV for $\Omega^*(3/2^+)$

Pole Structure of the Lambda(1405) Region

PDG Reviews: Ulf-G. Meissner and T. Hyodo (since Nov. 2015)

Table 1: Comparison of the pole positions of $\Lambda(1405)$ in the complex energy plane from next-to-leading order chiral unitary coupled-channel approaches including the SIDDHARTA constraint.

approach	pole 1 [MeV]	pole 2 [MeV]
Refs. 11,12, NLO	$1424^{+7}_{-23} - i\,26^{+3}_{-14}$	$1381^{+18}_{-6} - i\,81^{+19}_{-8}$
Ref. 14, Fit II	$1421^{+3}_{-2} - i\,19^{+8}_{-5}$	$1388^{+9}_{-9} - i\,114^{+24}_{-25}$
Ref. 15, solution #2	$1434^{+2}_{-2} - i\,10^{+2}_{-1}$	$1330^{+4}_{-5} - i\,56^{+17}_{-11}$
Ref. 15, solution #4	$1429^{+8}_{-7} - i\,12^{+2}_{-3}$	$1325^{+15}_{-15} - i\,90^{+12}_{-18}$

$\Lambda(1405)$: $1405.1^{+1.3}_{-1.0}$ MeV (Part. Listing in '22)

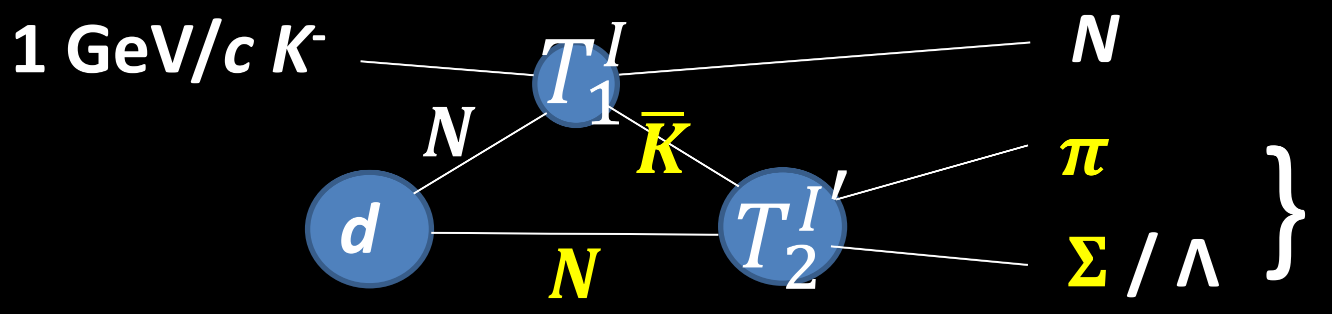
$J^P = \frac{1}{2}^-$, $I = 0$, $M_{\Lambda(1405)} < M_{\mathrm{KbarN}}$, lightest in neg. parity baryons

M. Hassanvand et al: $\pi\Sigma$ IM Spec. of $pp \rightarrow K^+\pi\Sigma$

J. Esmaili et al: □ IM Spec. of Stopped K^- on ^{4}He

R.H. Dalitz et al: □ IM Spec. in K-$p \rightarrow \pi\pi\Sigma$ w/ M-matrix

To measure $\bar{K}N$ Scattering Amp. below the $\bar{K}N$ mass thres. (J-PARC E31)



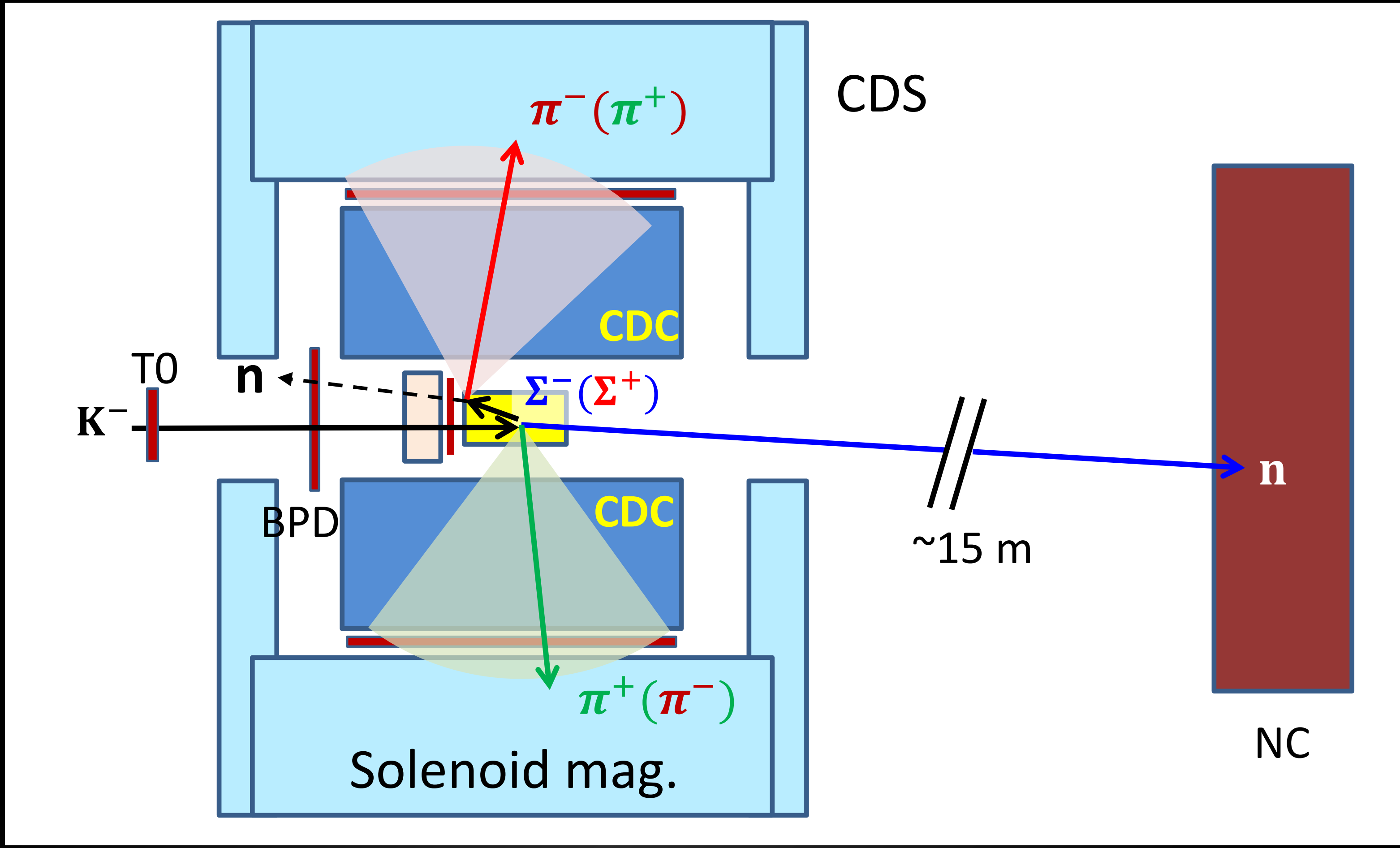
← forward, knocked out *N*

← backward,
relative S-wave

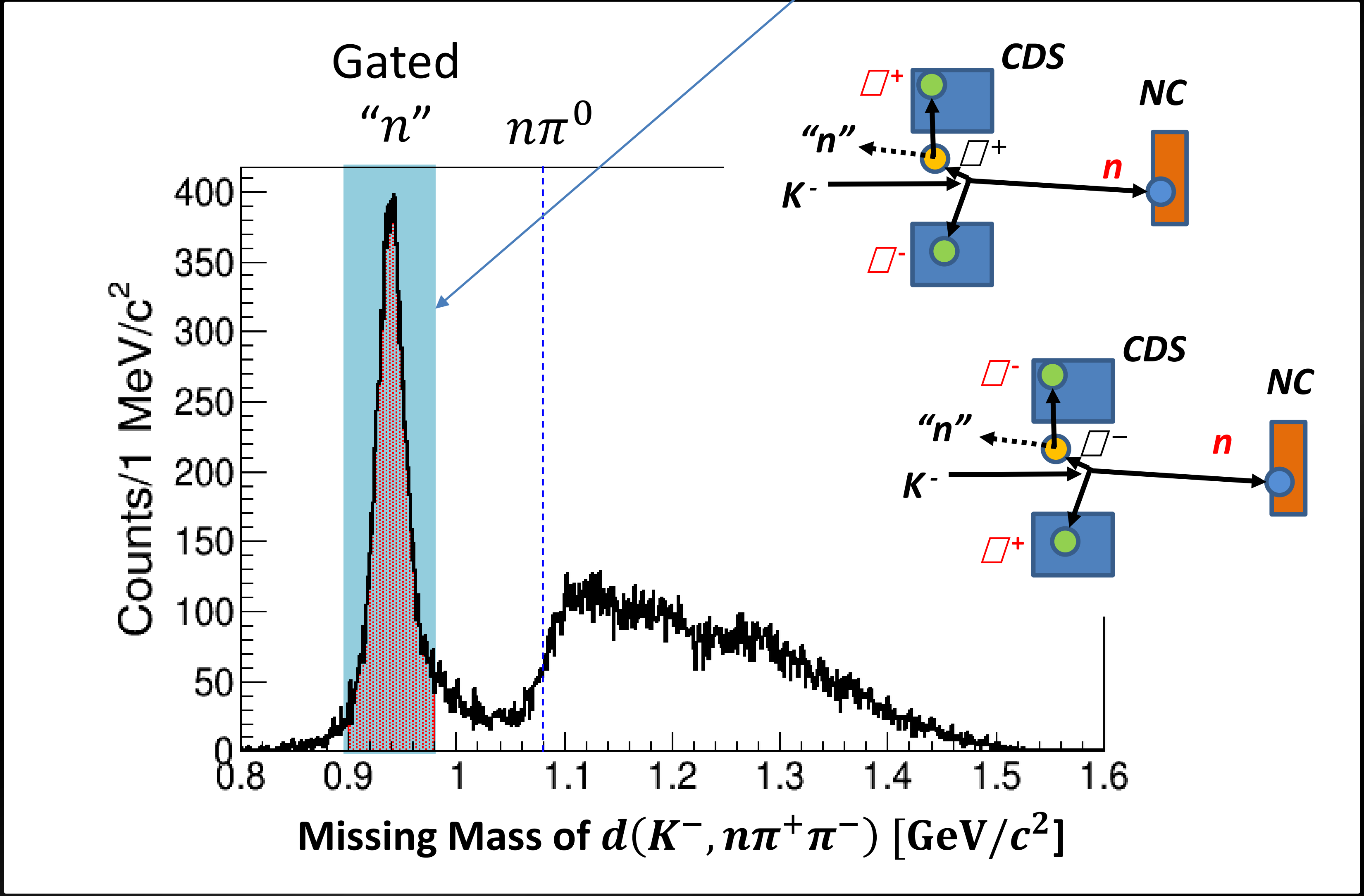
- measuring an **S-wave $\bar{K}N \to \pi\Sigma$** scattering below the $\bar{K}N$ threshold in the $d(K^-,n)$□ reactions at a forward angle of *N*.
- ID's all the final states to decompose the I=0 and 1 amplitudes.

Fwd *N*	$\pi\Sigma$ mode	Isospin	Expected resonance
n	$\pi^\pm\Sigma^\mp$	**0**, 1	$\Lambda(1405)$ interference btw I=0 and 1 ampl's.
p	$\pi^-\Sigma^0$	1	P-wave $\Sigma^*(1385)$ to be suppressed
n	$\pi^0\Sigma^0$	**0**	$\Lambda(1405)$
n, p	$\pi^-\Lambda, \pi^0\Lambda$	1	For reference to confirm Isospin Relation

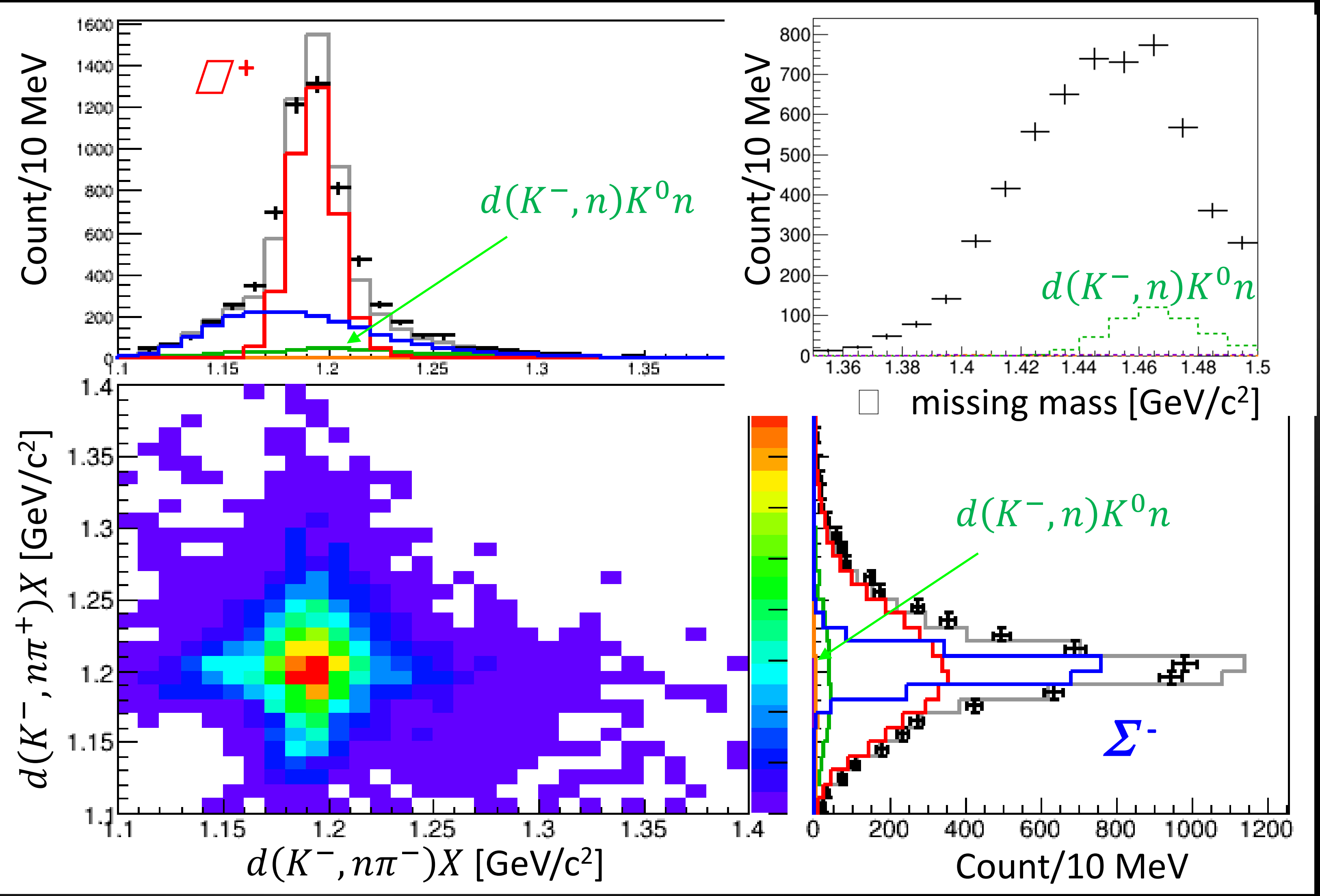
Event topology of $d(K^-, n)X_{\pi^\pm\Sigma^\mp}$



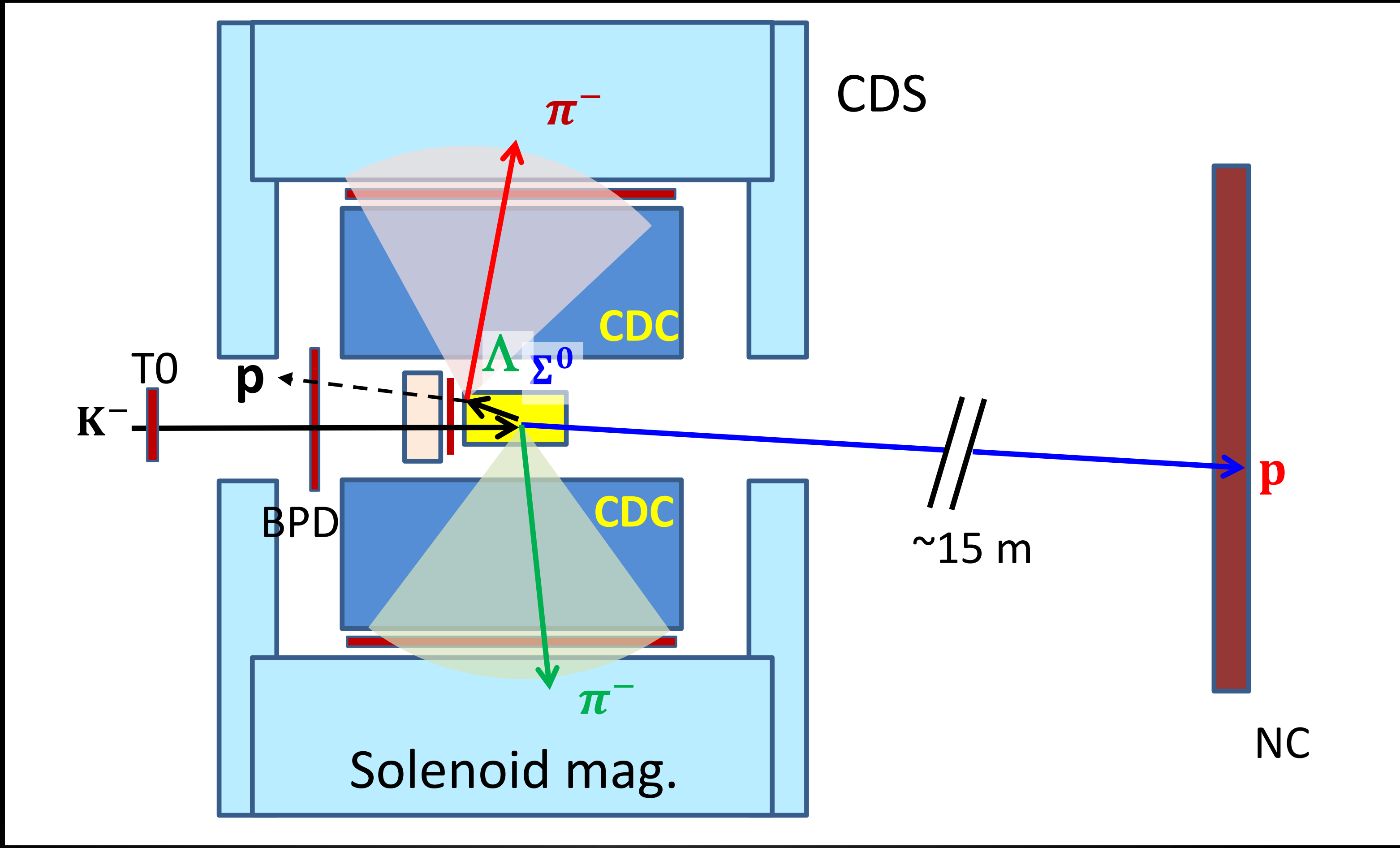
$d(K^-, n\pi^+\pi^-)n_{missing}$



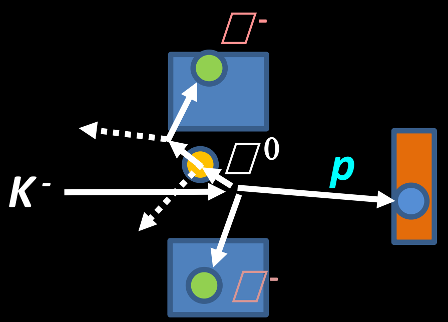
$\pi^+\Sigma^-/\pi^-\Sigma^+$ Mode separation (template fitting, Run78)



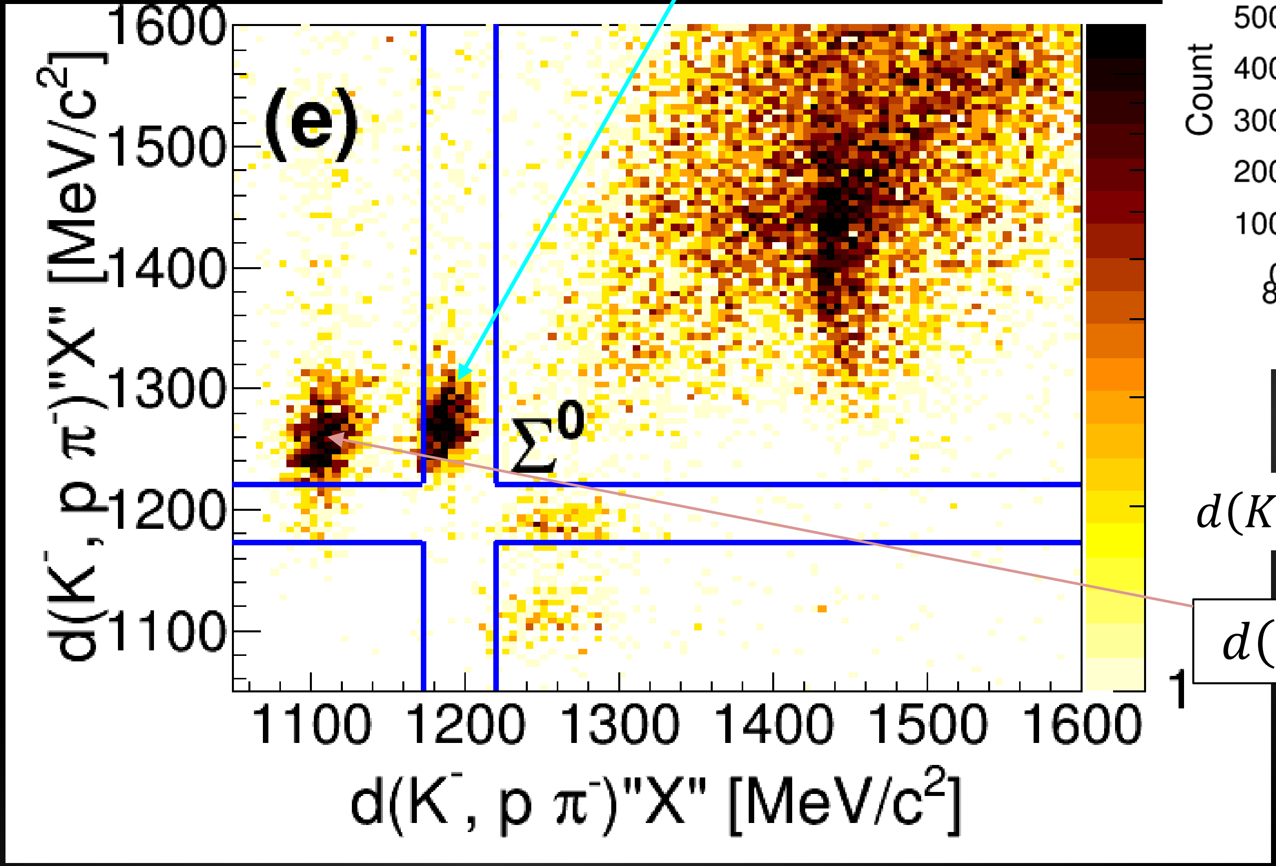
Event topology of $d(K^-, p)X_{\pi^-\Sigma^0}$

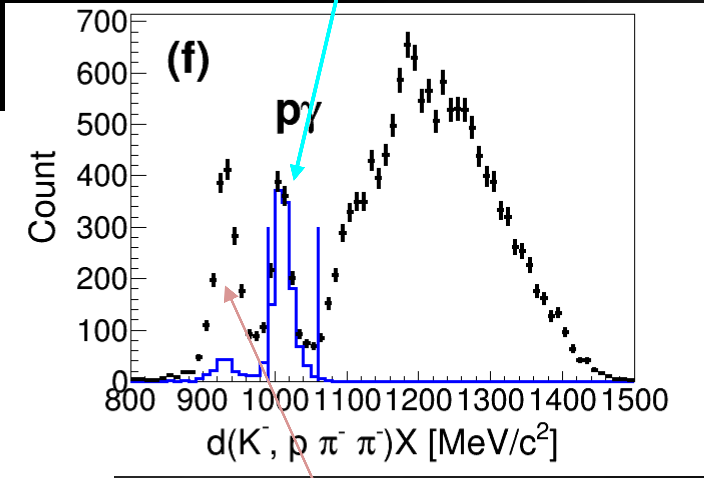


$d(K^-, p)X_{\pi^-\Sigma^0}$ Mode ($I = 1$)



From $d(K^-, p\pi^-\pi^-)$"$p\gamma$" sample

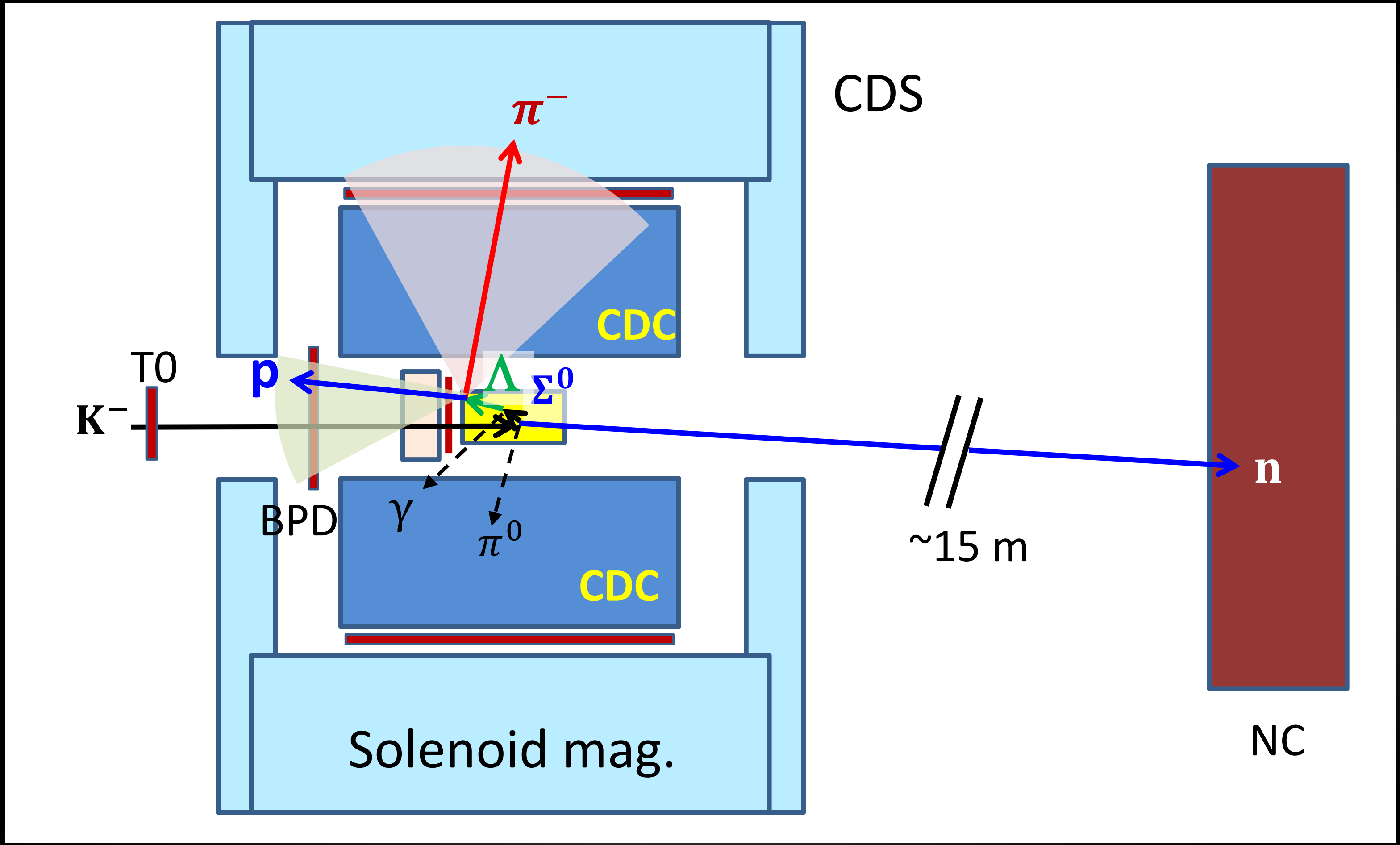




$d(K^-, p\pi^-\pi^-)$"p"

$d(K^-, p)X_{\pi^-\Lambda}$

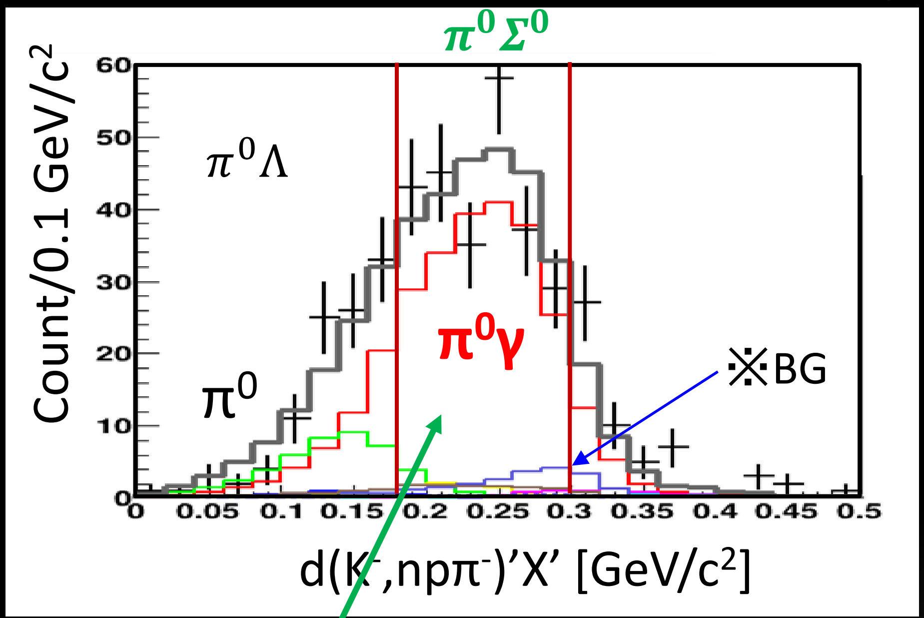
Event topology of $d(K^-,n)X_{\pi^0\Sigma^0}$

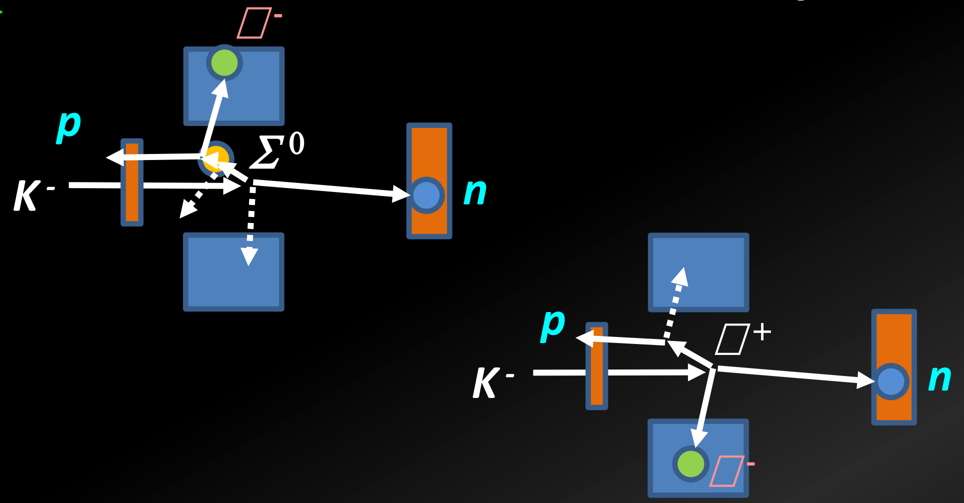


Other major process: $d(K^-,n)X_{\pi^0\Lambda}$, $d(K^-,n)X_{\pi^-\Sigma^+}$,
Minor processes: $d(K^-,n)X_{\pi^0\pi^0\Lambda}$, $d(K^-,Yp)X$, …

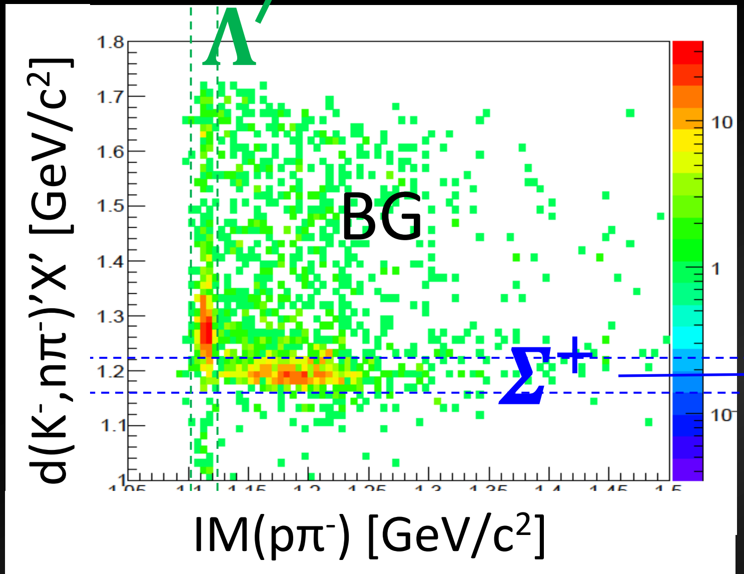
$d(K^-,n)\underline{\pi^0\Sigma^0}$ vs $d(K^-,n)\underline{\pi^-\Sigma^+}$

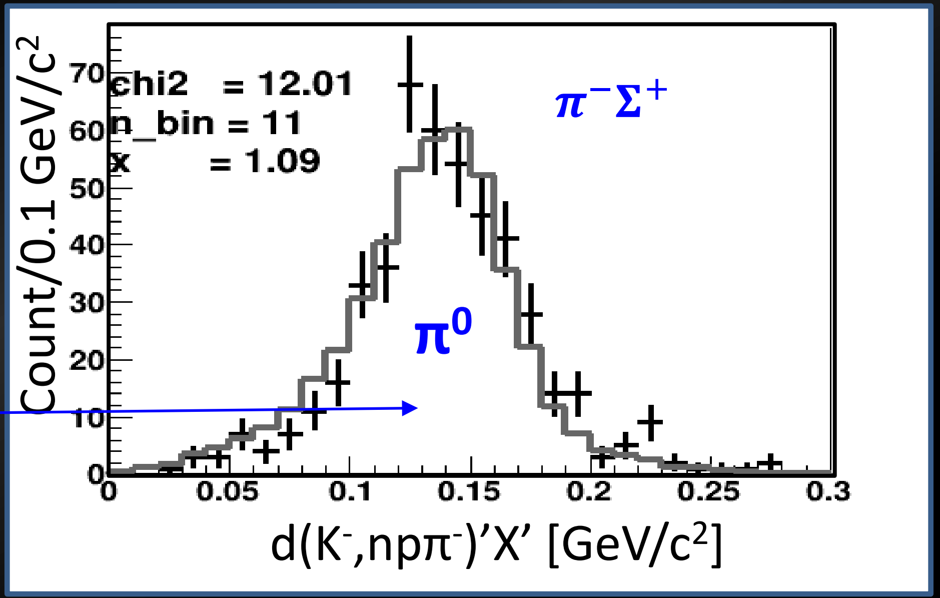
$\hookrightarrow \pi^0\gamma\Lambda$ $\hookrightarrow \pi^- p \pi^0$





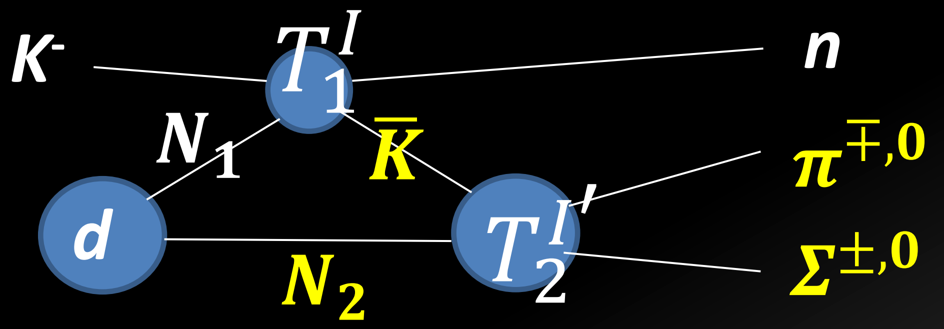
※BG:(K-d→p(Yπ)-, QF-K induced Y prod.)





Extracting Scattering Amplitude

- 2-step process



$$\left.\frac{d\sigma}{dM_{\pi\Sigma}}\right|_{\theta_n=3^\circ} \sim \left|\left\langle n\pi\Sigma\right| T_2^{I'}(\bar{K}N_2 \to \pi\Sigma) G_0 T_1^I(K^- N_1 \to \bar{K}n)\left|K^-\Phi_d\right\rangle\right|^2$$

$$\sim \left|T_2^{I'}(\bar{K}N \to \pi\Sigma)\right|^2 F_{\mathrm{res}}(M_{\pi\Sigma})$$

Factorization Approximation

$$F_{\mathrm{res}}(M_{\pi\Sigma}) \sim \left|\int_0^\infty dq_{N_2}^3\, T_1^I \frac{1}{E_{\bar{K}} - E_{\bar{K}}(q_{\bar{K}}) + i\epsilon} \Phi_d(q_{N_2})\right|^2, q_{\bar{K}} + q_{N_2} = q_{\pi\Sigma}$$

E31: Response Function, $F_{\text{res}}(M_{\pi\Sigma})$

- $F_{\text{res}}(M_{\pi\Sigma}) = \left| \int G_0(q_2, q_1) T_1 \Phi_d(q_2)\, d^3 q_2 \right|^2$
  - $G_0(q_2, q_1) = \frac{1}{q_0^2 - q'^2 + i\varepsilon} f(q_0, q') \frac{\left(\sqrt{P_{\pi\Sigma}^2 + M_{\pi\Sigma}^2} + \sqrt{P_{\pi\Sigma}^2 + W(q')^2}\right)}{M_{\pi\Sigma} + W(q')}$,
    $f(q_0, q')^{-1} = [E_1(q_0) + E_1(q')]^{-1} + [E_2(q_0) + E_2(q')]^{-1}$
    Miyagawa and Haidenbauer, PRC85, 065201(2012)
  - $T_1 : K^- n \to K^- n\ (I = 1), K^- p \to \bar{K}^0 n (I = 0{,}1)$ amplitude,
    Gopal et al., NPB119, 362(1977)
    - $T_1(K^- n \to K^- n) = f(I = 1)$
    - $T_1(K^- p \to \bar{K}^0 n) = [f(I = 1) - f(I = 0)]/2$

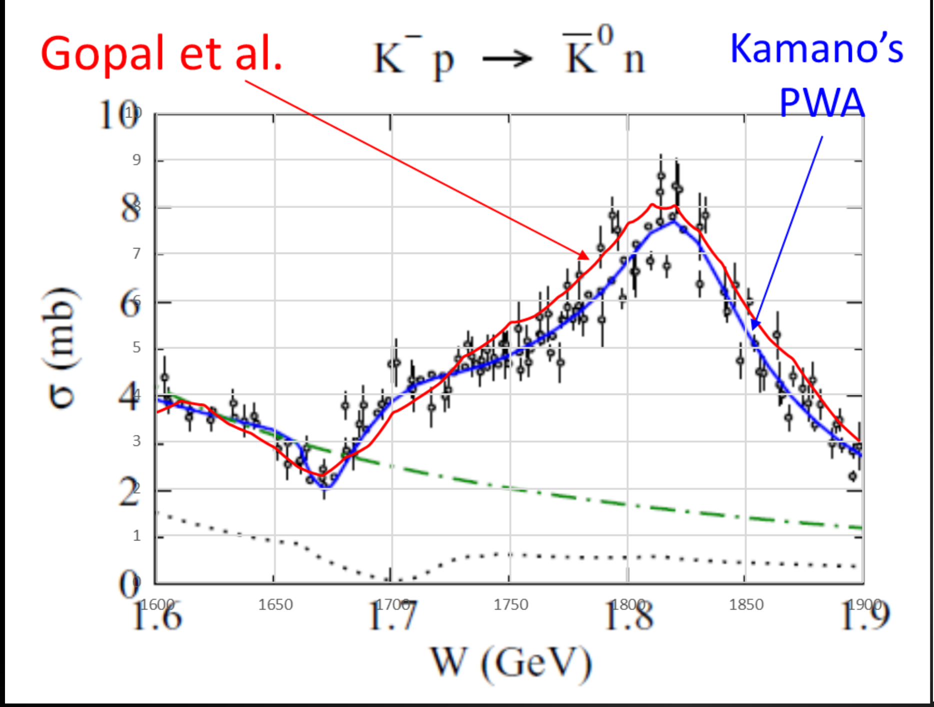
Off-shell treatment :See eq.(17) in PRC94, 065205

  - $\Phi_d(q_2)$: deuteron wave function, PRC63, 024001(2001)

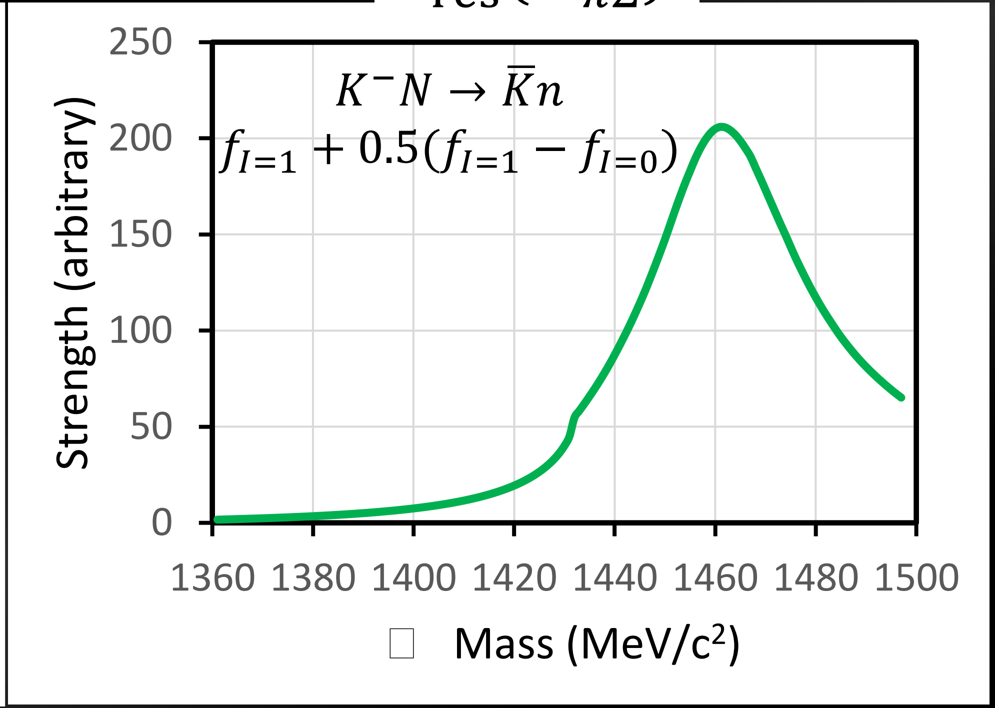
E31: Response Function, $F_{\rm res}(M_{\pi\Sigma})$

$$F_{\rm res}(M_{\pi\Sigma}) \sim p_\pi^{cm} p_n^2 / |(E_{K^-} + m_d)\beta_n - p_{K^-}\cos\theta\,| \times$$
$$\int d\Omega_\pi^{cm} E_\pi E_\Sigma \left| \int q_2 T_1^I(p_{K^-}, q_N, p_n, q_{\bar{K}}, \cos\theta_{n\bar{K}}\,; M_{\pi\Sigma}) G_0(q_2, q_1) \Phi_d(q_2)\, d^3 q_2 \right|^2$$

Elementary Cross Section for T_1^I



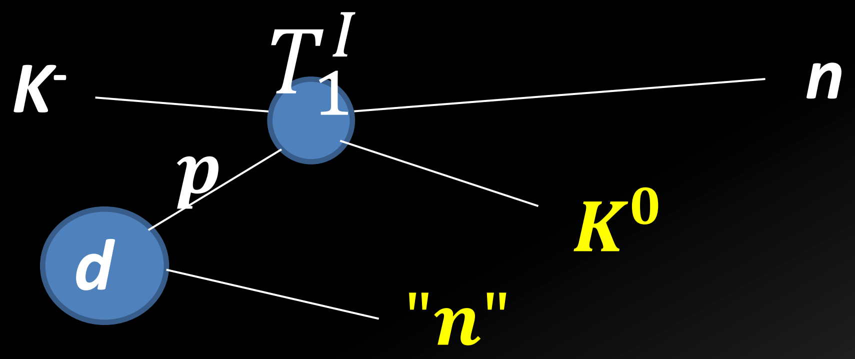
$F_{\rm res}(M_{\pi\Sigma})$



Gopal et al., NPB119, 362(1977)

Demonstration of the T_1^I amplitude

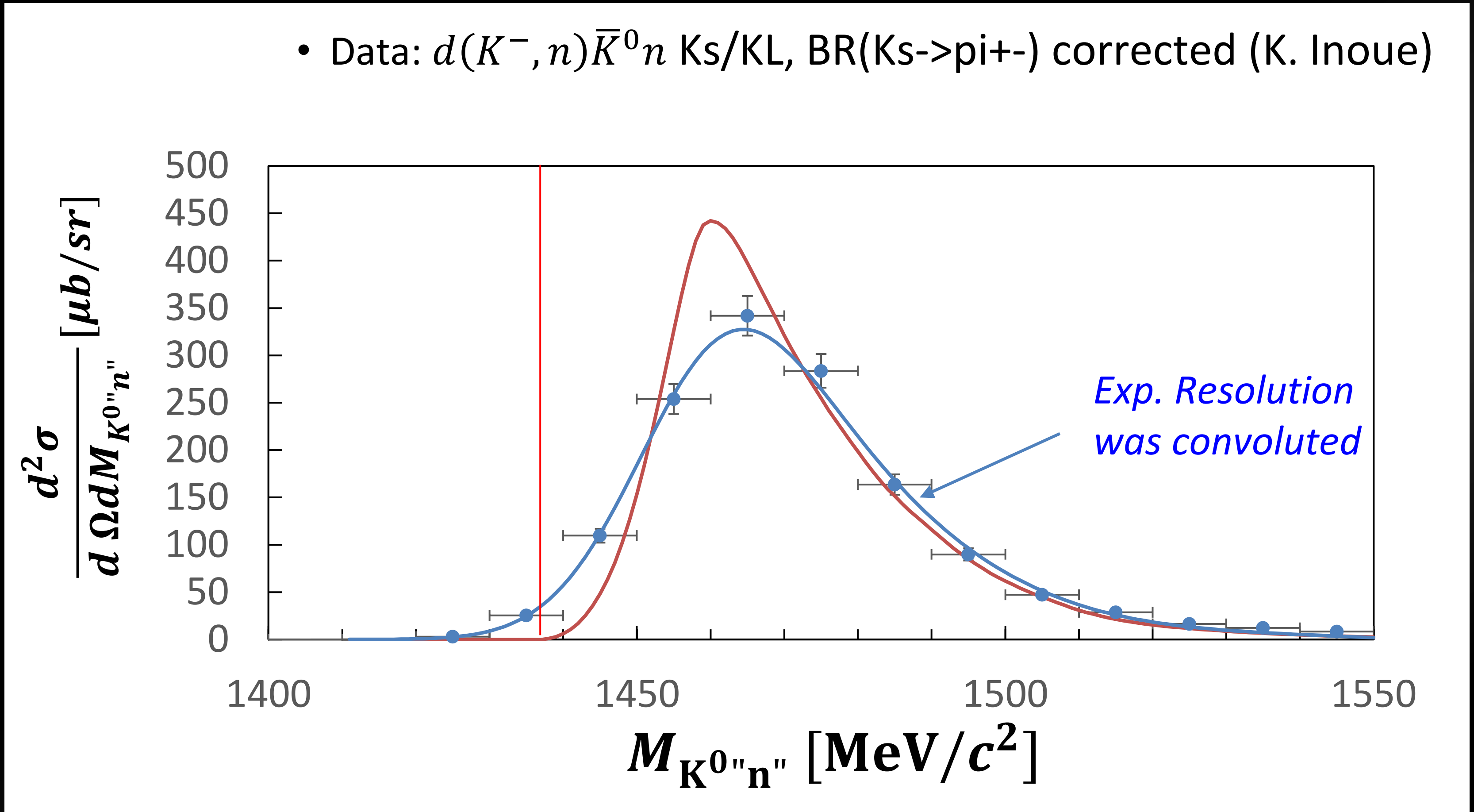
- 1-step process



$$\left.\frac{d\sigma}{dM_{\pi\Sigma}}\right|_{\theta_n=3^\circ} \sim \left|\langle nK^0 n \middle| T_1^I(K^- p \to \overline{K^0} n) \middle| K^- \Phi_d \rangle\right|^2$$

$$\frac{d\sigma}{dM_{\pi\Sigma}} \sim \left|\int_0^\infty dq_{N_2}^3\, T_1^I \delta(p_{K^-} + p_p - p_n - p_{K^0}) \Phi_d(q_{N_2})\right|^2$$

Demonstration for fitting data with the 1-step $K^- d \to nK^0"n"$ reaction calculation



$\bar{K}N$ Scattering Amplitude

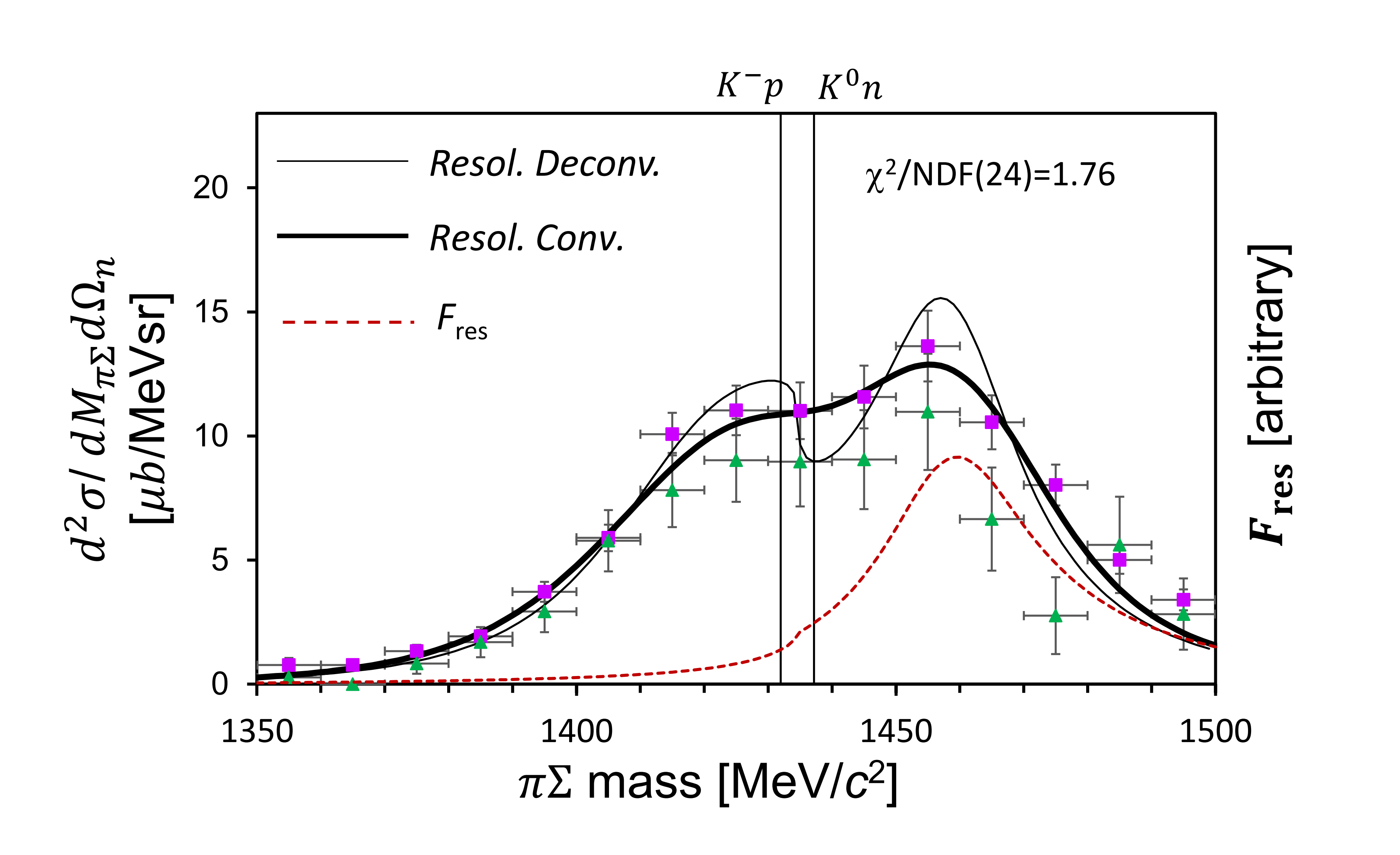
L. Lensniak, arXiv:0804.3479v1(2008)

- $T_{\frac{1}{2}}^{I'}(\bar{K}N \to \bar{K}N) = \dfrac{A}{1-iAk_2+\frac{1}{2}ARk_2^2}$
- $T_{\frac{1}{2}}^{I'}(\bar{K}N \to \pi\Sigma) = \dfrac{1}{\sqrt{k_1}} e^{i\delta_0} \dfrac{\sqrt{ImA-\frac{1}{2}|A|^2 ImRk_2^2}}{1-iAk_2+\frac{1}{2}ARk_2^2}$
- $T_{\frac{1}{2}}^{I'}(\pi\Sigma \to \pi\Sigma)$

$$= \frac{e^{i\delta_0}}{k_1} \frac{\left(\sin\delta_0 + iIm\left(e^{-i\delta_0}A\right)k_2 - \frac{1}{2}Im\left(e^{-i\delta_0}AR\right)k_2^2\right)}{1-iAk_2+\frac{1}{2}ARk_2^2}$$

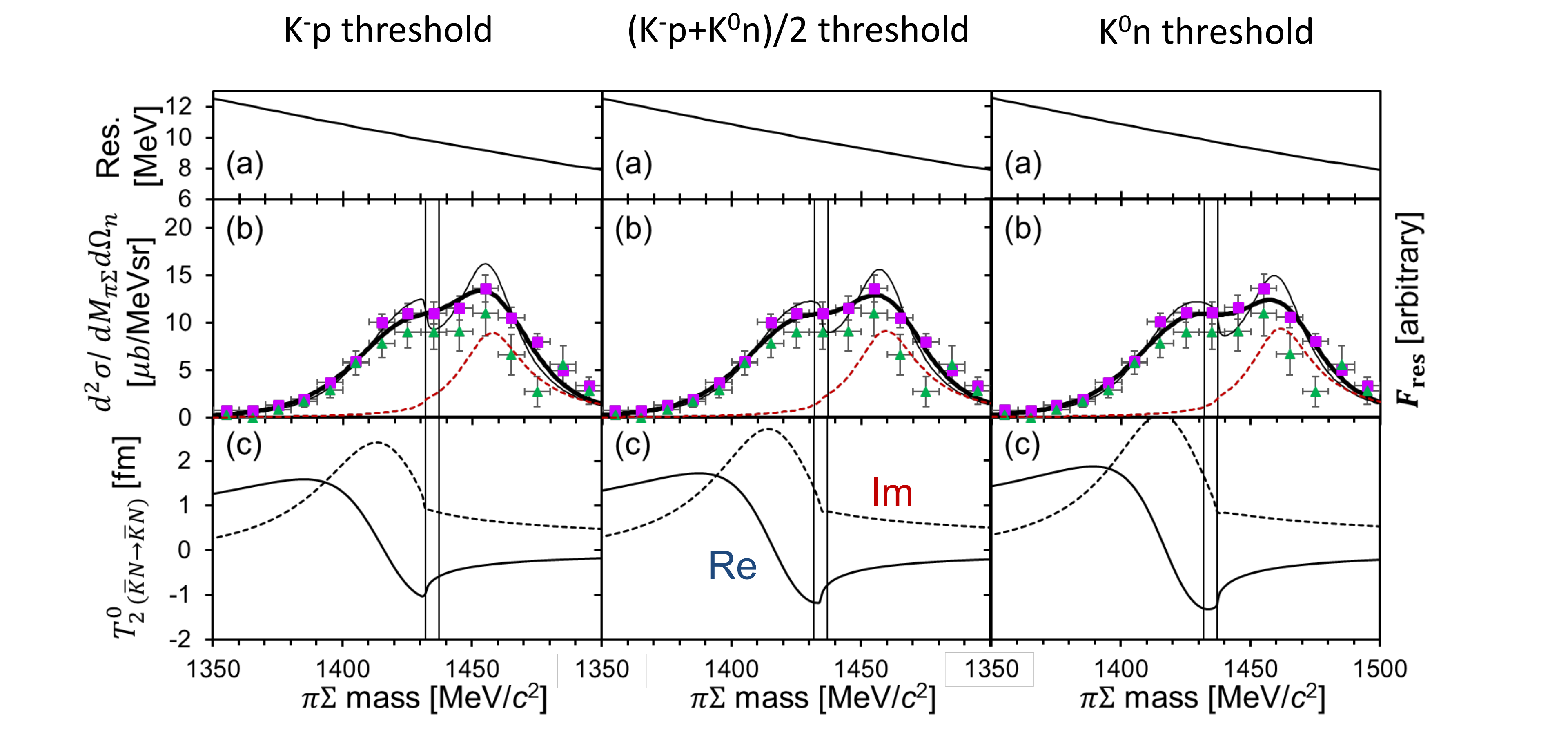
- 5 real number parameters (effective range expansion)
  - A: scattering length, R: effective range, δ_0: phase

Fit the spectra to deduce $\bar{K}N$ scattering amplitude



Systematics of the fitting result by the assumed $\bar{K}N$ mass threshold

$$\left.\frac{d\sigma}{dM_{\pi\Sigma}}\right|_{\theta_n=0} \sim \left|T_2^{I'}(\bar{K}N \to \pi\Sigma)\right|^2 F_{\mathrm{res}}(M_{\pi\Sigma})$$



Fin.