Clustering as a Window on the Hierarchical Structure of Quantum Systems
--CLUSHIQ2022 (EMMI Workshop)--

Toward complete of the phase diagram of the BCS-BEC crossover

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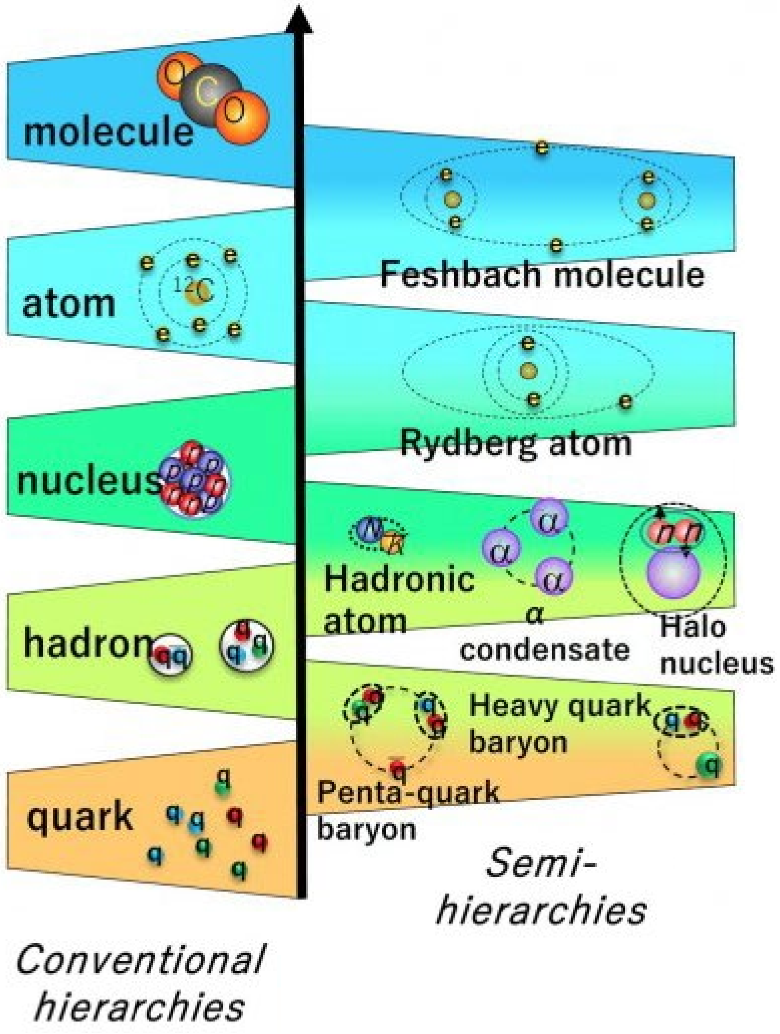
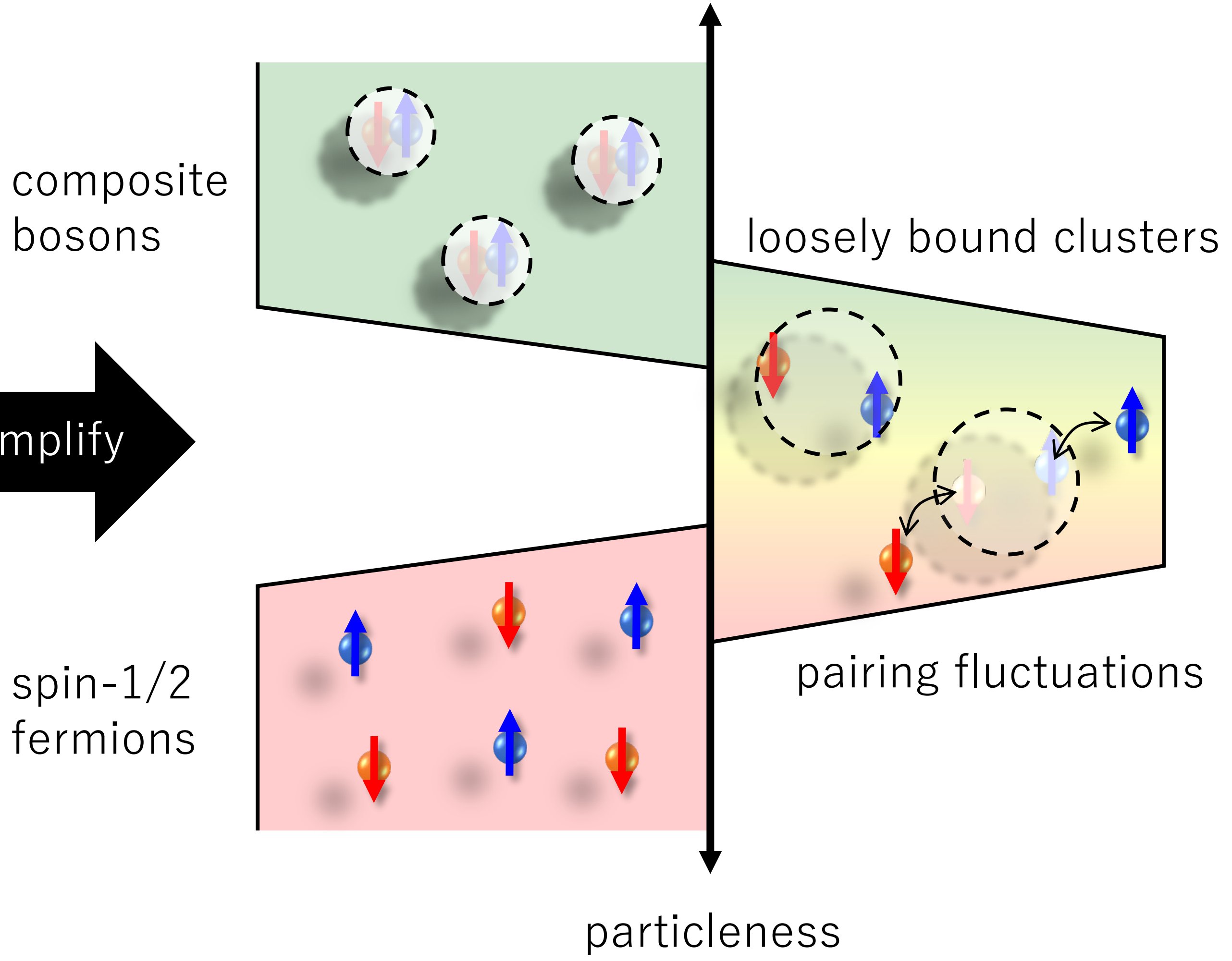
Sendai, Japan, Oct 31- Nov 3, 2022

Introduction

Simplify the Hierarchical Structure of Quantum Systems



simplify



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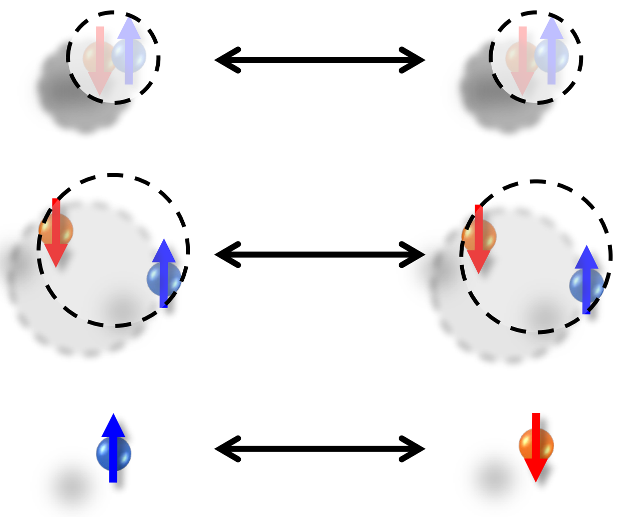
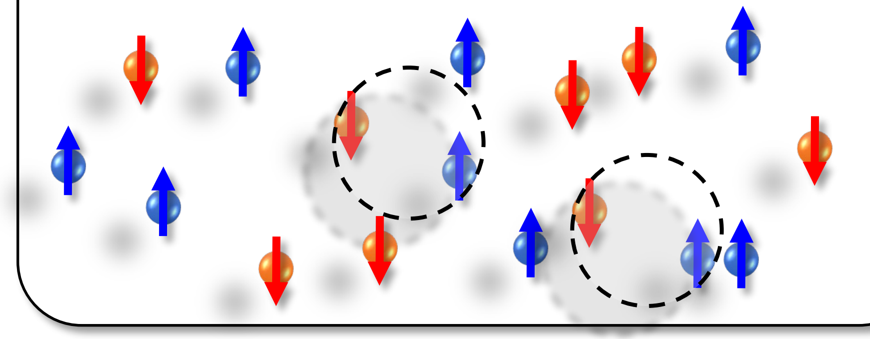
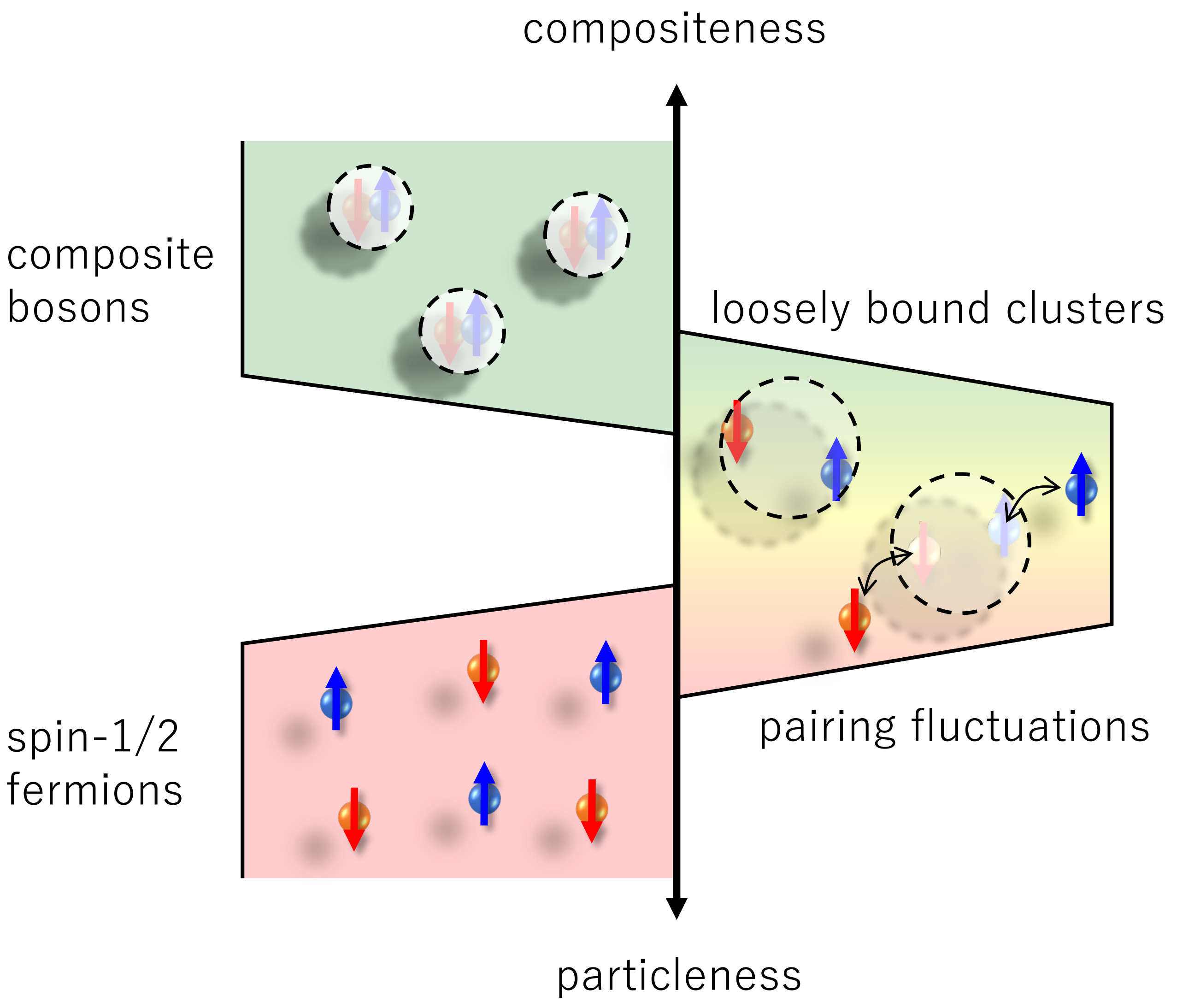
How does the interaction change?



Cluster formation mechanism in the many-body quantum system





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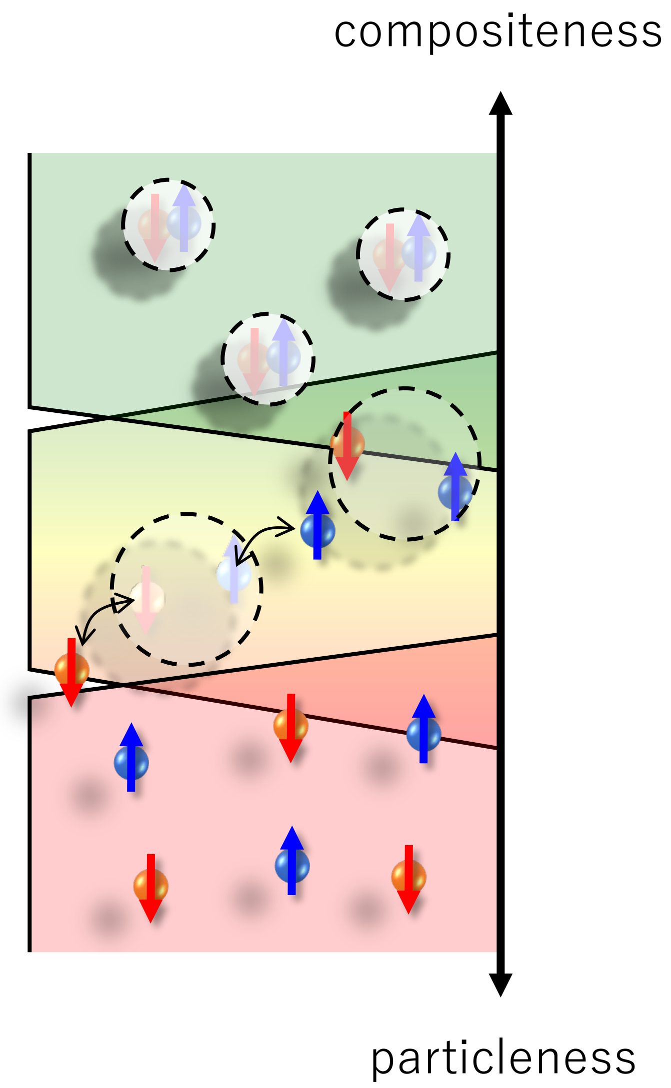

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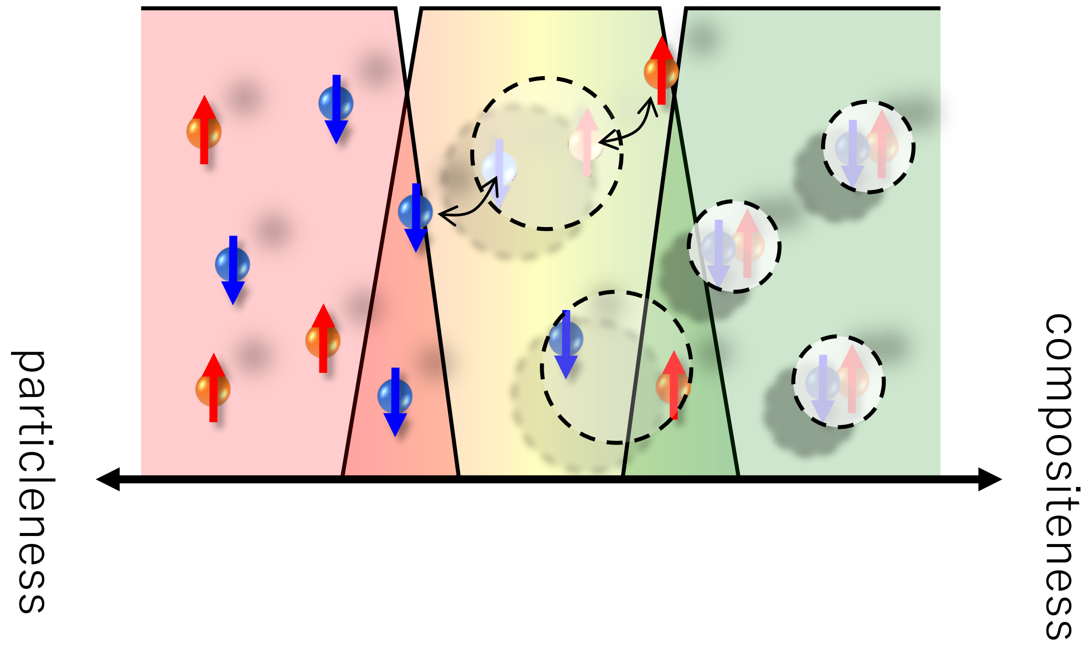
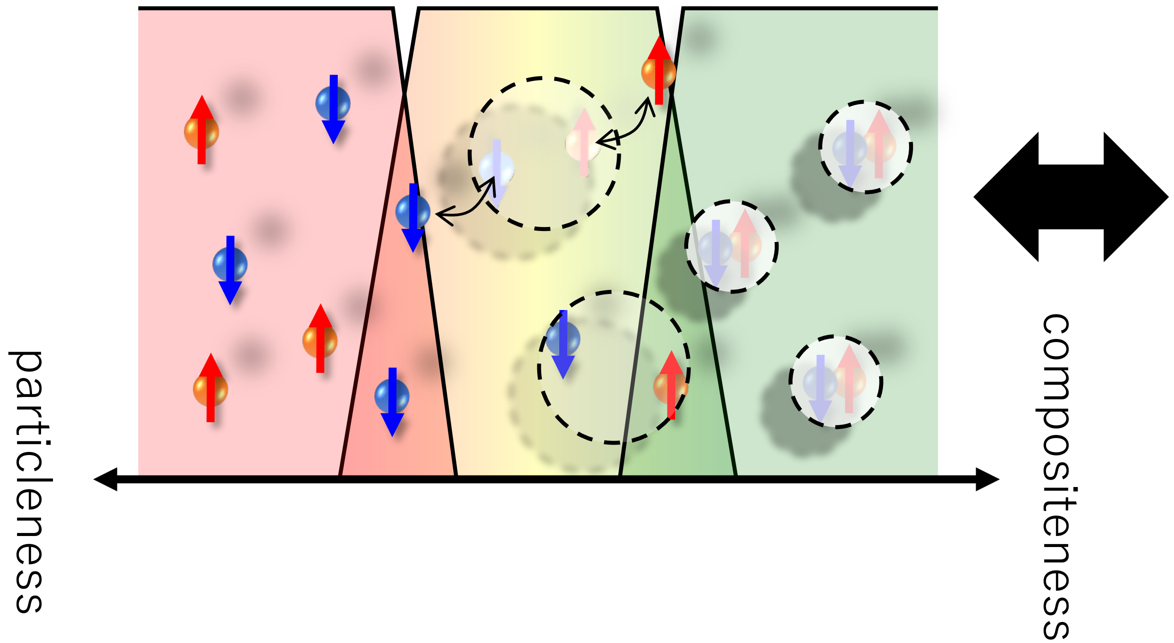
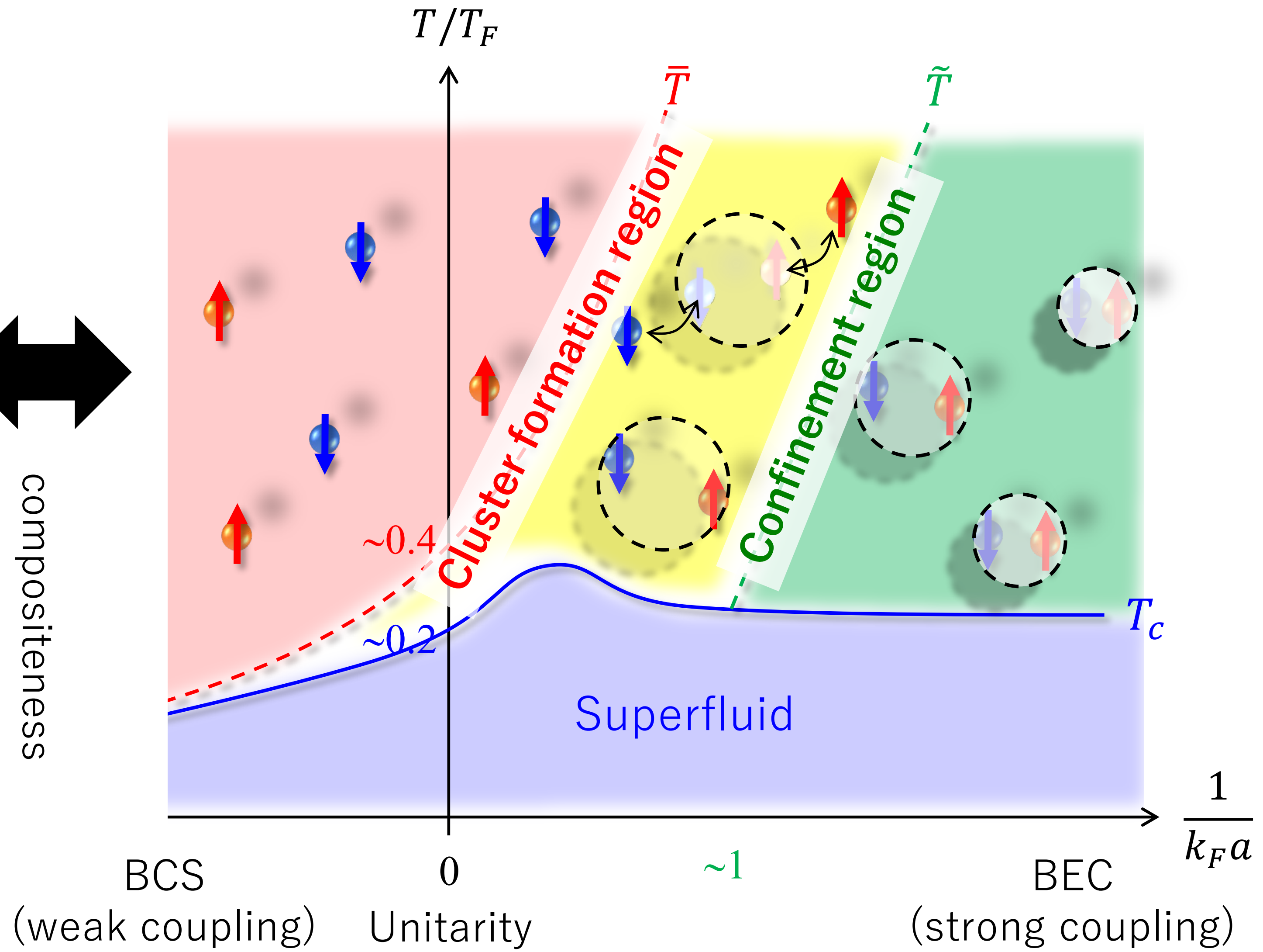


The phase diagram of spin-1/2 fermions with s-wave interactions

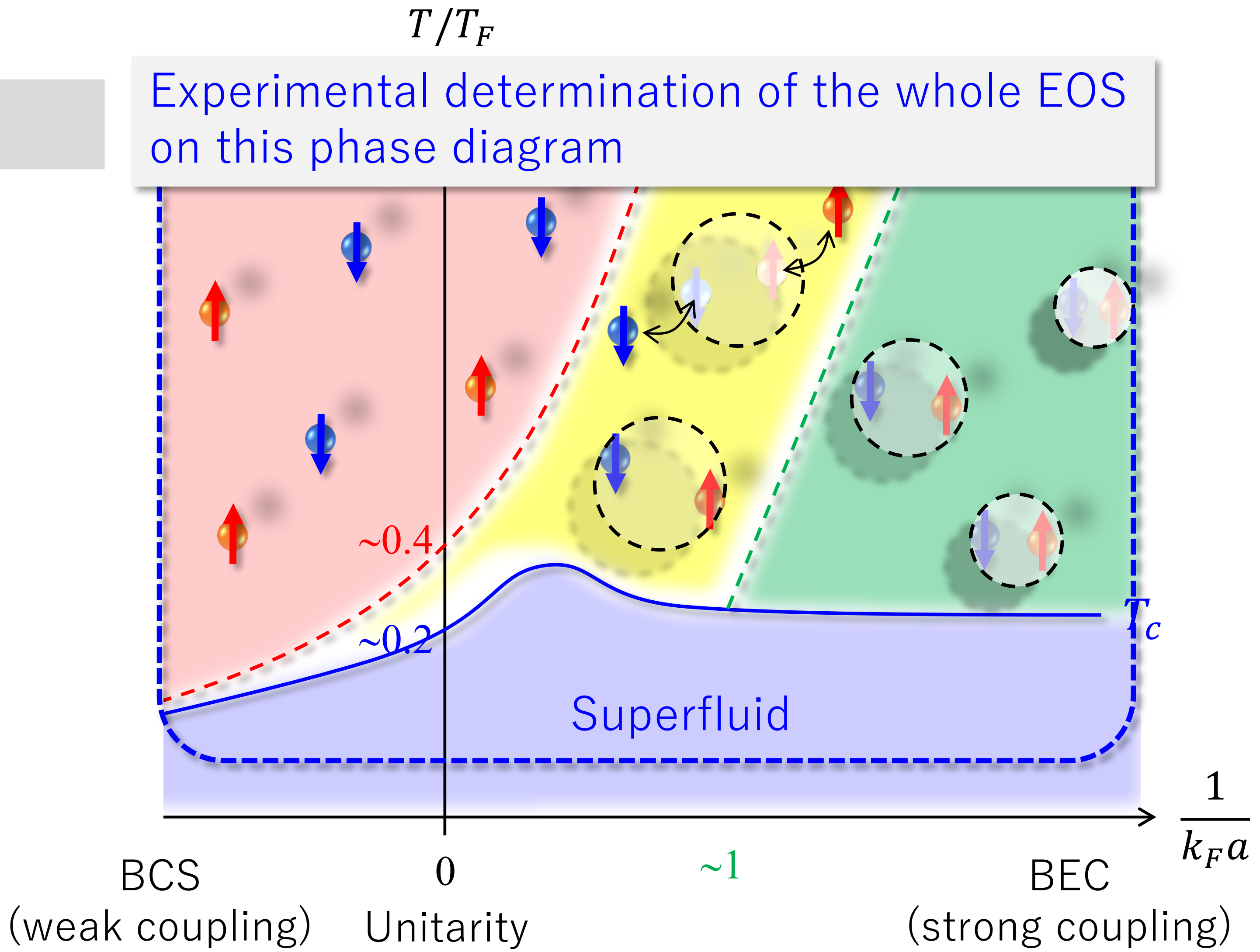




Our approach to the cluster physics

- **Cluster formation region**
- **Confinement region**
- **Cluster-Cluster interactions**
- **Viscosity**

Universal quantum cluster physics in the many-body (matter) system



Why EOS is necessary?

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because the EOS tells us many physics

- **Cluster formation region**
- **Confinement region**

  specific heat

  [Pieter van Wyk, et al., Phys. Rev. A 93, 013621 (2016)]

- **Cluster-Cluster interactions**

  compressibility, Virial coefficients

  Poster 29 by K. Yoneda   Poster 25 by H. Funaba

  [D. Kagamihara, et al., Phys. Rev. A 106, 033308 (2022)]

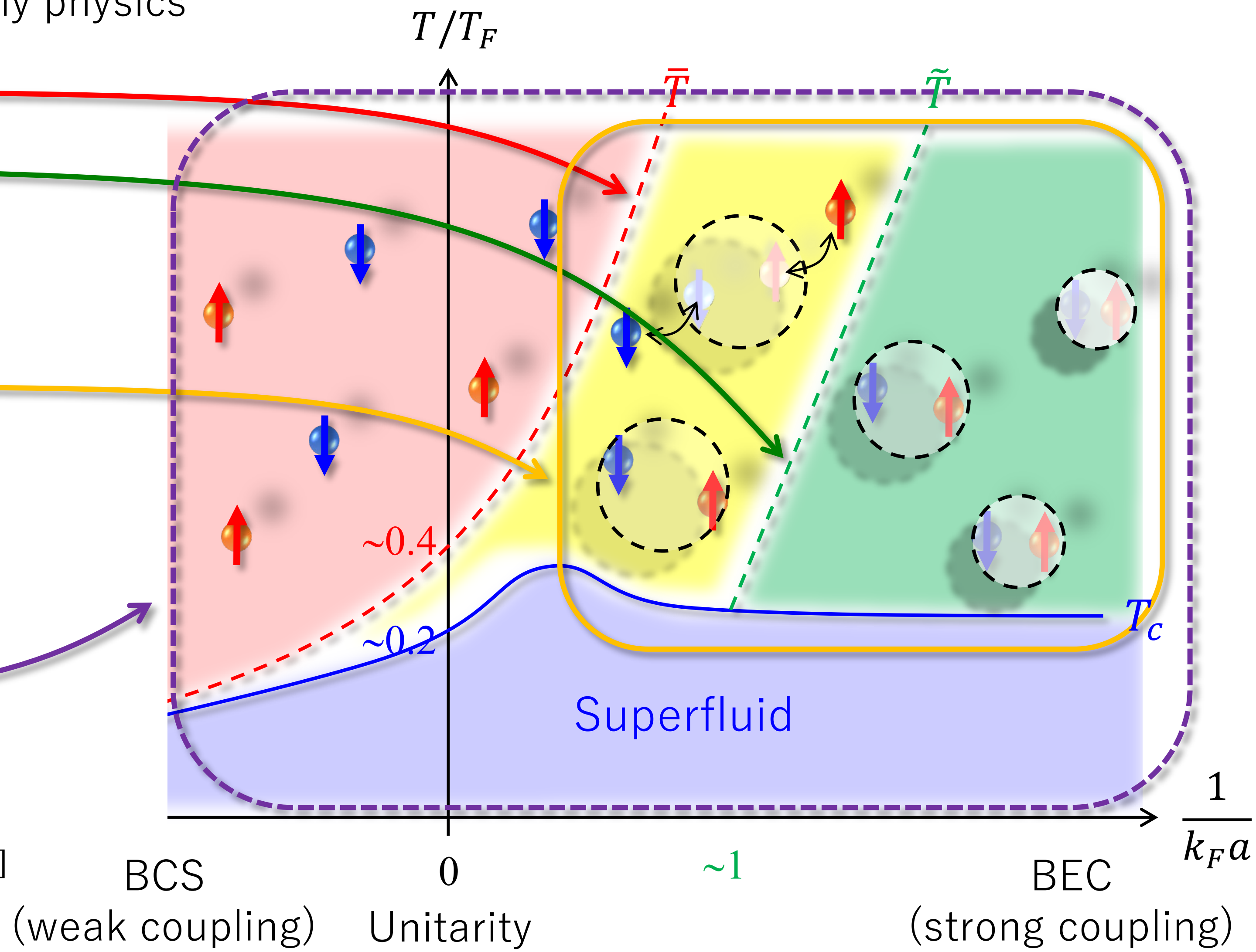
  [S. Endo and Y. Castin, Phys. Rev. A 92, 053624 (2015)]

- **Viscosity**

  The EOS is required

  [K. Fujii and Y. Nishida, Phys. Rev. A 98, 063634 (2018)]

  [D. Kagamihara, et al., J. Phys. Soc. Jpn. 89, 044005 (2020)]



Why EOS is necessary?

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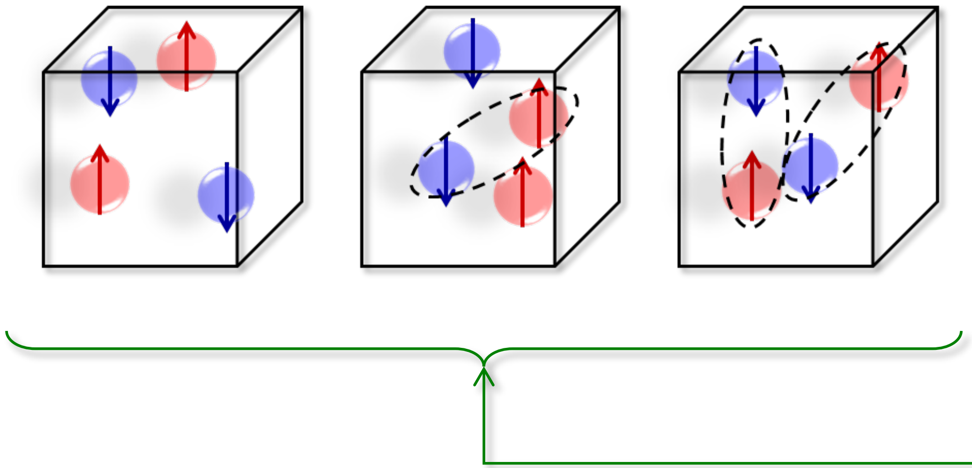
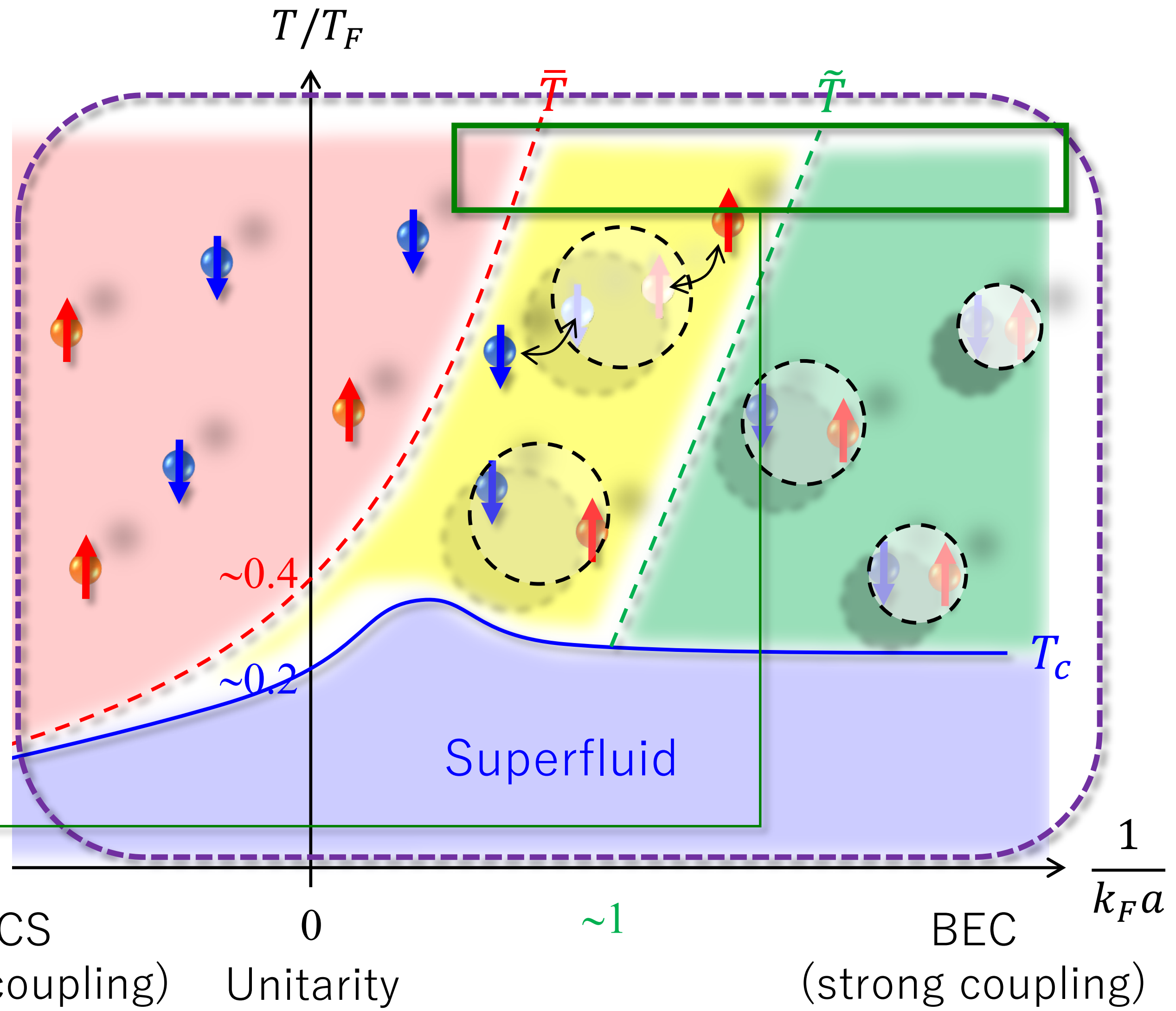
- **composite boson interactions in vacuum**

Virial expansion of the EOS

$$P = 2\frac{k_B T}{\Lambda_T^3}(z + B_2 z^2 + B_3 z^3 + B_4 z^4 + \cdots)$$





Why EOS is necessary?

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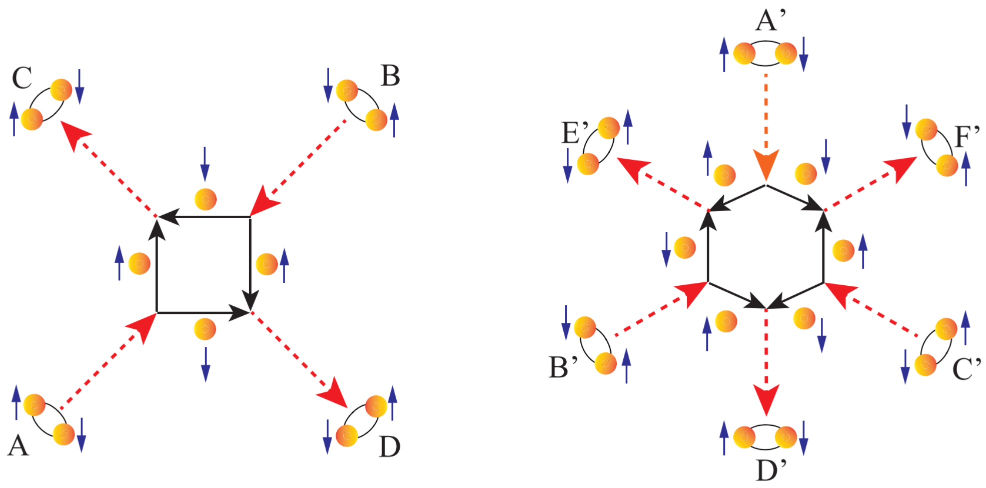
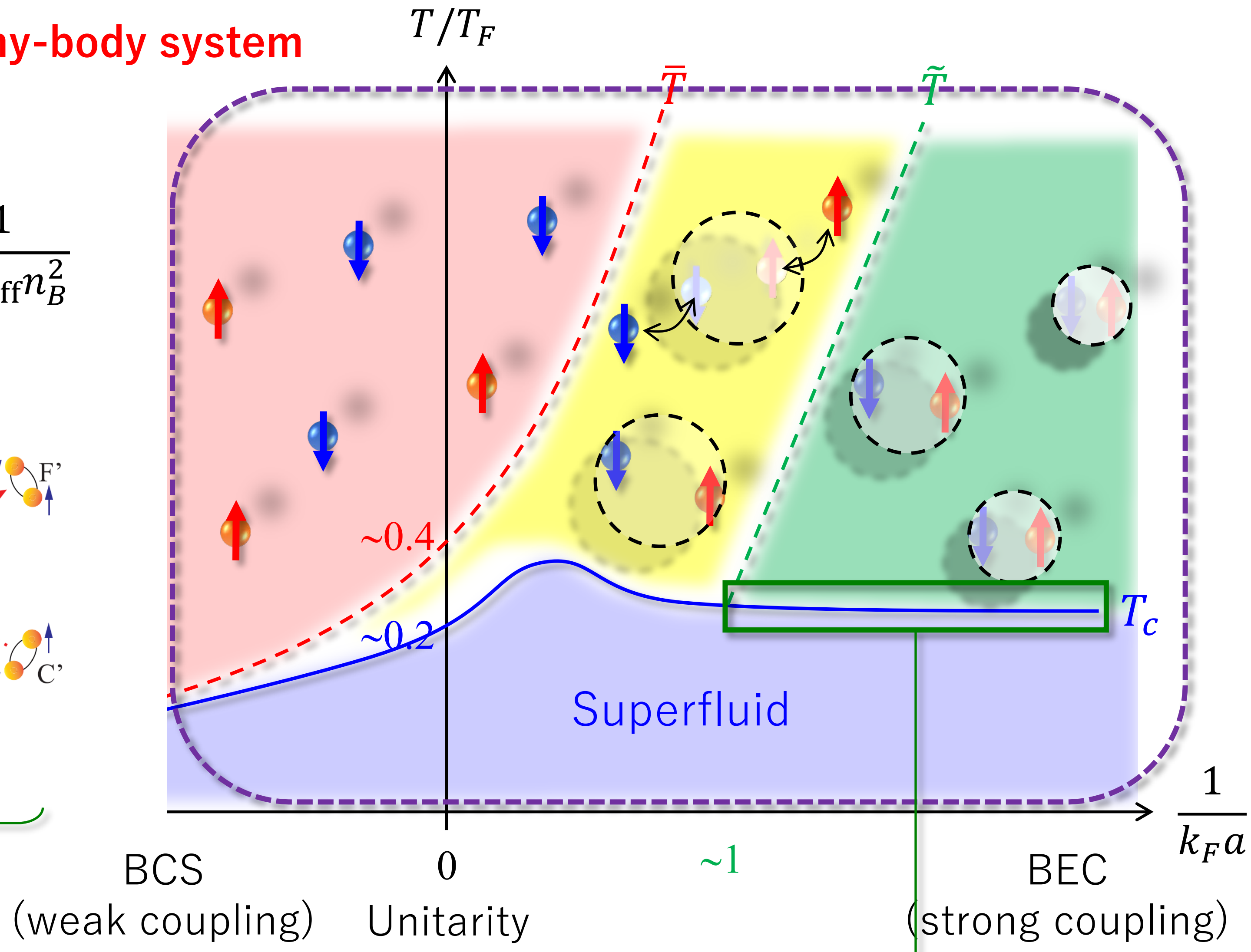
- composite boson interactions **in the many-body system**

[D. Kagamihara, et al., Phys. Rev. A 106, 033308 (2022)]

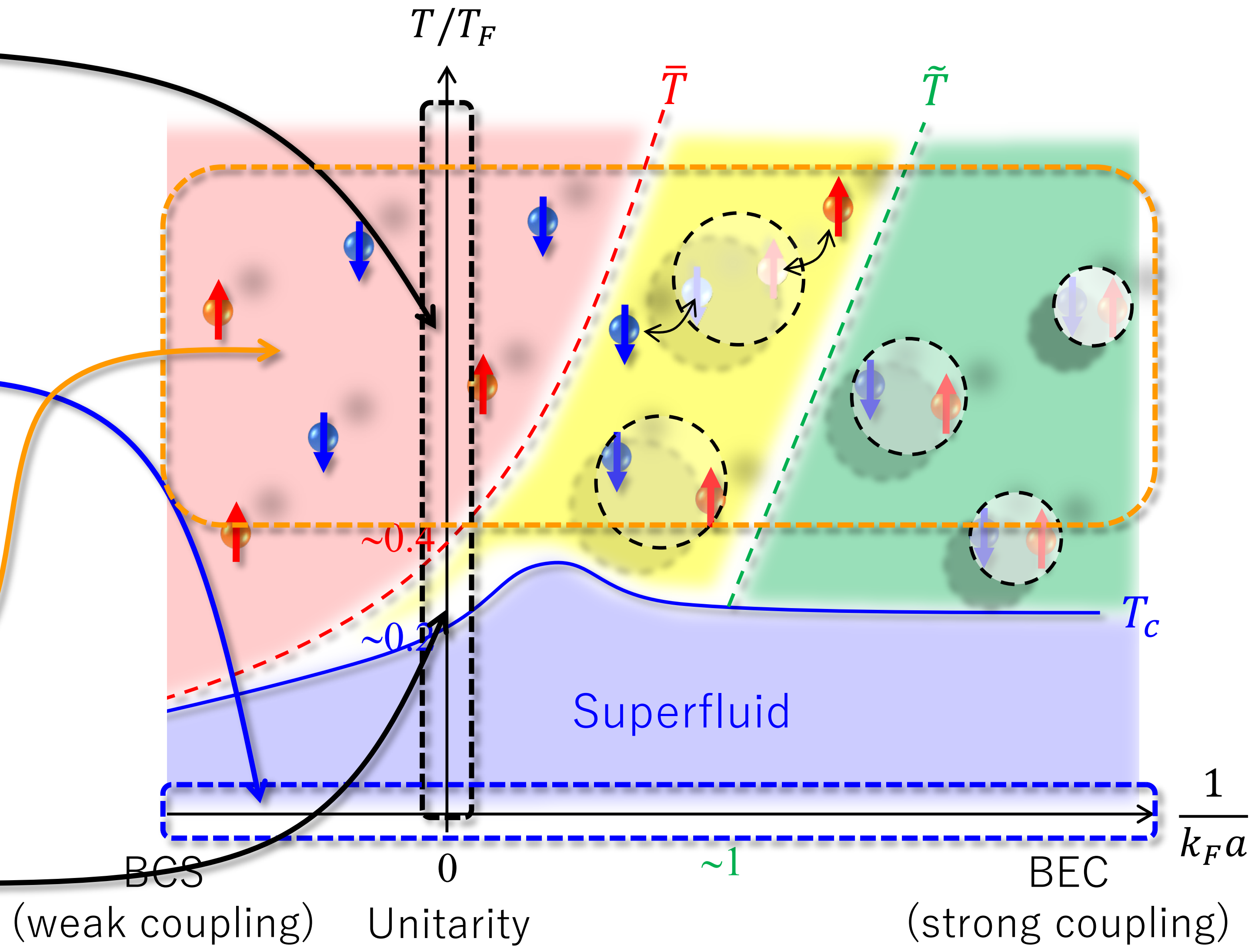
$$\kappa_T(T_c) = \frac{1}{2\,(U_B^{2-body} + U_B^{3-body} + \cdots)n_B^2} = \frac{1}{2U_{\mathrm{eff}}n_B^2}$$





Previous experimental researches on the Phase diagram

- **EOS at the unitarity limit**
  [MH, et al., Science 327, 442 (2010)]
  [S. Nascimbène, et al., Nature 463, 1057 (2010)]
  [M. JH Ku, et al., Science 335, 563 (2012)]
  [C. Carcy, et al., Phys. Rev. Lett. 122, 203401 (2019)]
- **EOS in the BCS-BEC crossover at almost zero temperature**
  [S. Nascimbène, et al., Nature 463, 1057 (2010)]
  [C. Sanner, et al., Phys. Rev. Lett. 106, 010402 (2011)]
  [MH, et al., Phys. Rev. X 7, 041004 (2017)]
- **Share viscosity in the BCS-BEC crossover above T_c**
  [E. Elliott, et al., Phys. Rev. Lett. 113, 020406 (2014)]
- **Share viscosity at the unitarity limit**
  [J. A. Joseph, et al., Phys. Rev. Lett. 115, 020401 (2015)



Method to construct the EOS

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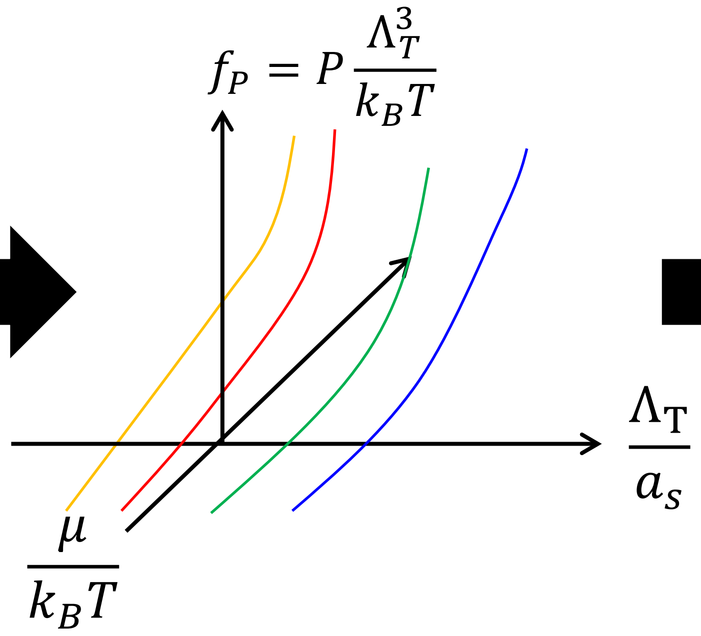
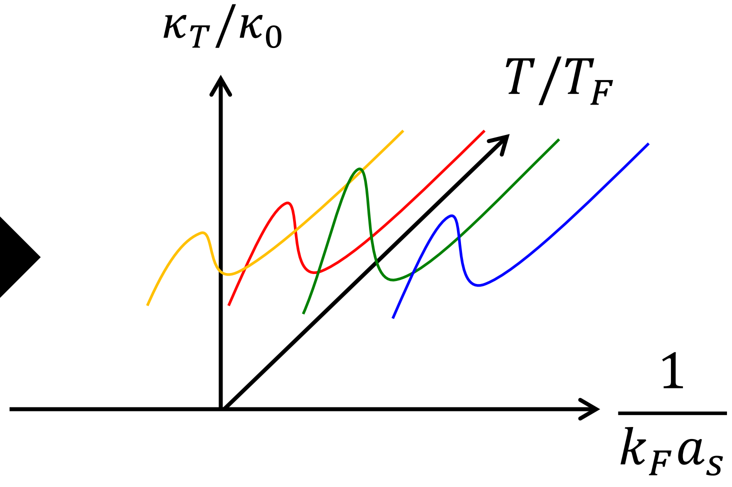
This function is what we want to determine

EOS (pressure) : $P(T,\mu_\uparrow,\mu_\downarrow,a_s^{-1}) = \frac{k_BT}{\Lambda_T^3} f_P\left(\frac{\mu_\uparrow}{k_BT},\frac{\mu_\downarrow}{k_BT},\frac{\Lambda_T}{a_s}\right)$, $\Lambda_T \equiv \sqrt{\frac{2\pi\hbar^2}{mk_BT}}$

for spin-balanced system
$(\mu_\uparrow = \mu_\downarrow = \mu)$

$f_P = P\frac{\Lambda_T^3}{k_BT}$	$\frac{\mu}{k_BT}$	$\frac{\Lambda_T}{a_s}$
⋮	⋮	⋮





Method to construct the EOS

experimental observables

local density : $n_\sigma(\boldsymbol{r})$

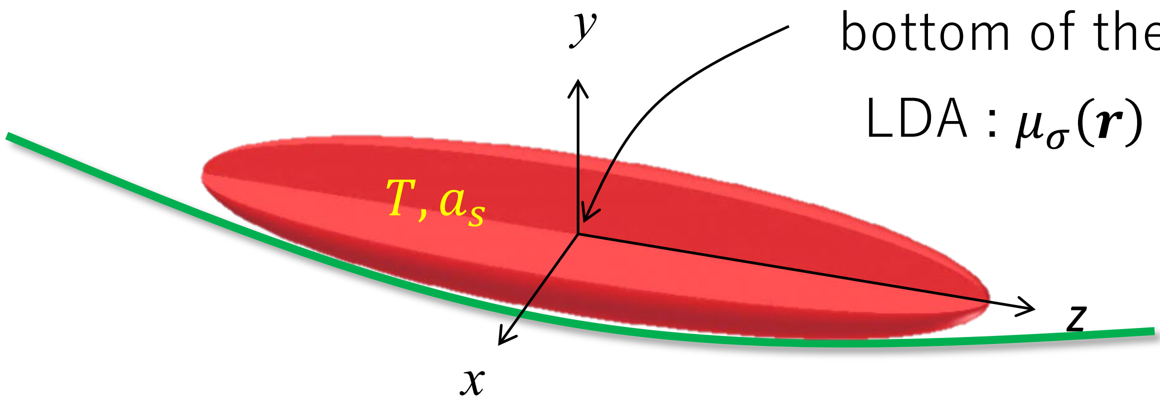
trapping potential : $V(\boldsymbol{r})$

scattering length : $a_s(B)$

LDA

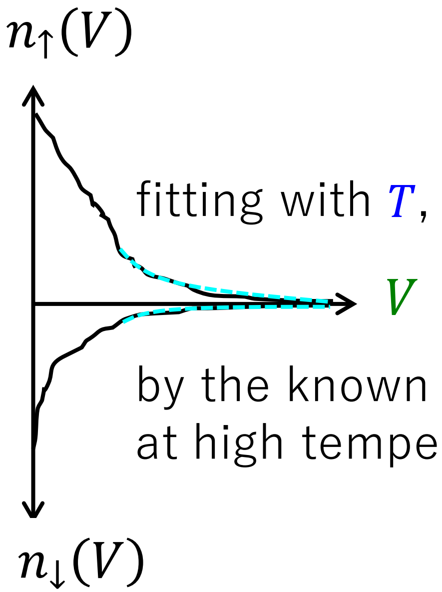
$n_\uparrow(V), n_\downarrow(V)$

$P(V) = -\int_{+\infty}^{V} \left(n_\uparrow(V) + n_\downarrow(V)\right) dV$



Chemical potential at the bottom of the potential : $\mu_{0\sigma}$

LDA : $\mu_\sigma(\boldsymbol{r}) = \mu_{0\sigma} - V(\boldsymbol{r})$



fitting with T, $\mu_{0\uparrow}$, $\mu_{0\downarrow}$

by the known EOS
at high temperature region

extension of the EOS

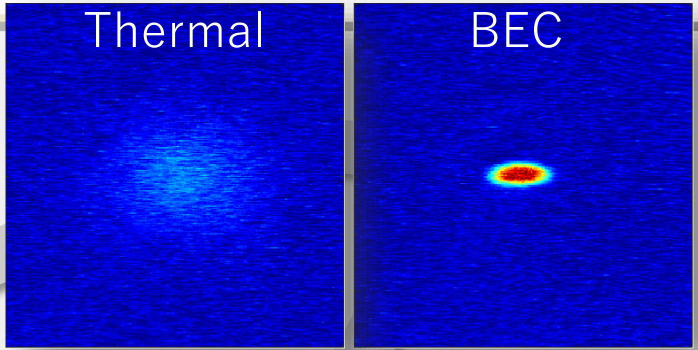
$$\left\{ P(V)\frac{\Lambda_T^3(T)}{k_B T},\ \frac{\mu_{0\uparrow} - V}{k_B T},\ \frac{\mu_{0\downarrow} - V}{k_B T},\ \frac{\Lambda_T(T)}{a_s} \right\}$$

to lower temperature

Inspired by [S. Nascimbène, et al., Nature 463, 1057 (2010)]

Ultracold ^{6}Li experiment @ Osaka

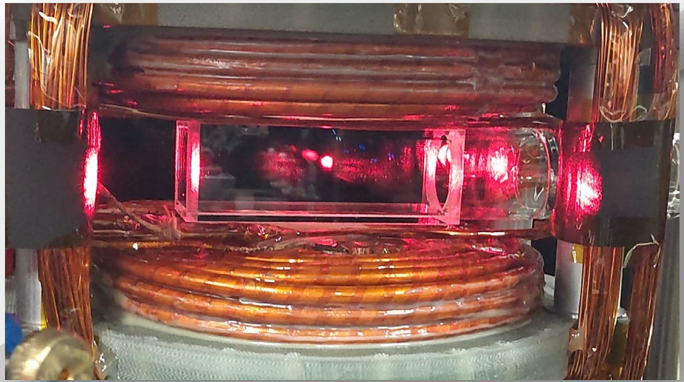
momentum distribution
of molecules



First BEC
(2021/9/3)

Move to Osaka
(2020/2/17)

Construction



First laser cooling
(2020/8/7)

Stabilization and calibration
of the apparatus

Test measurement of the equation of state

Experimental apparatus

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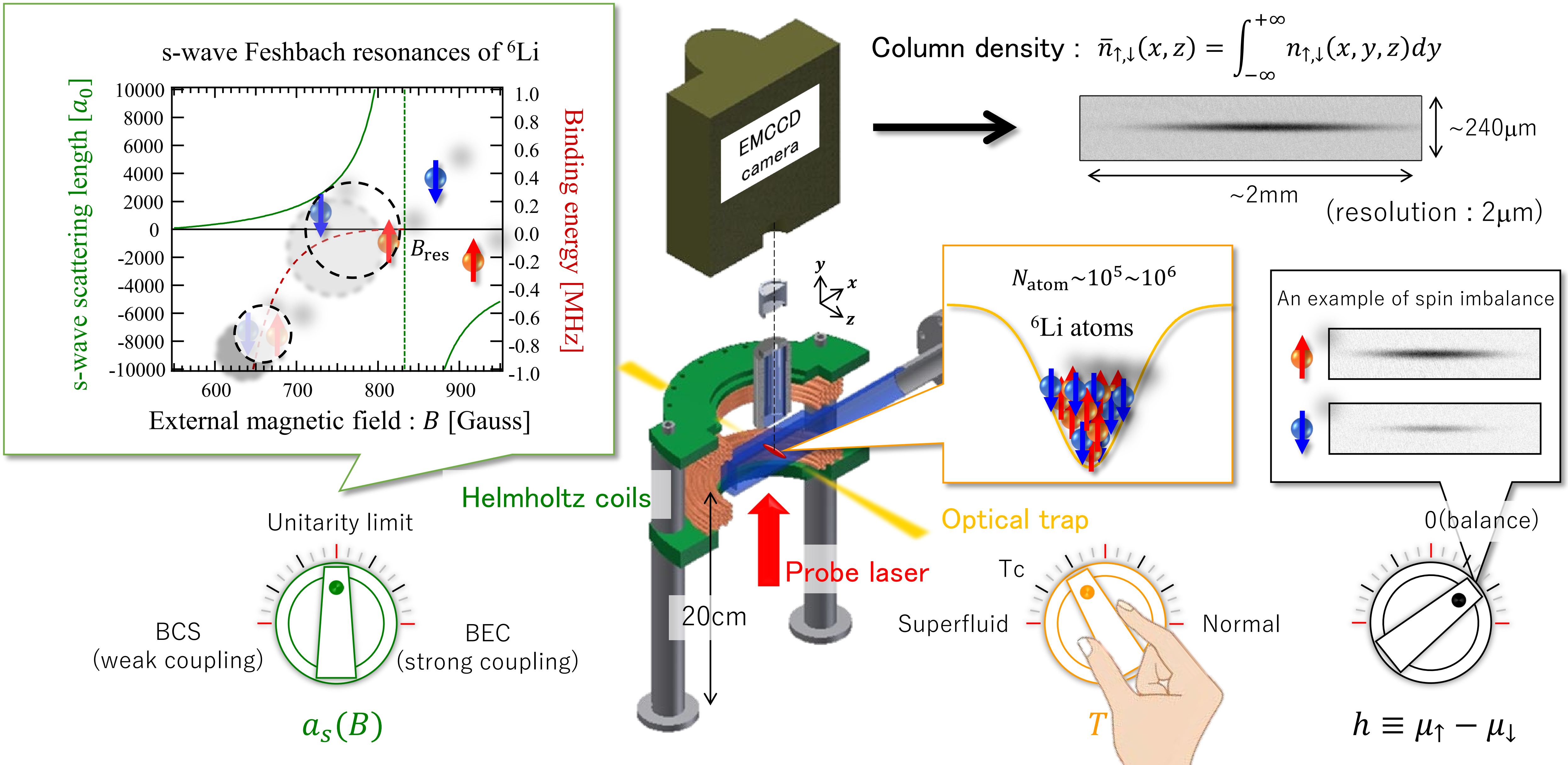
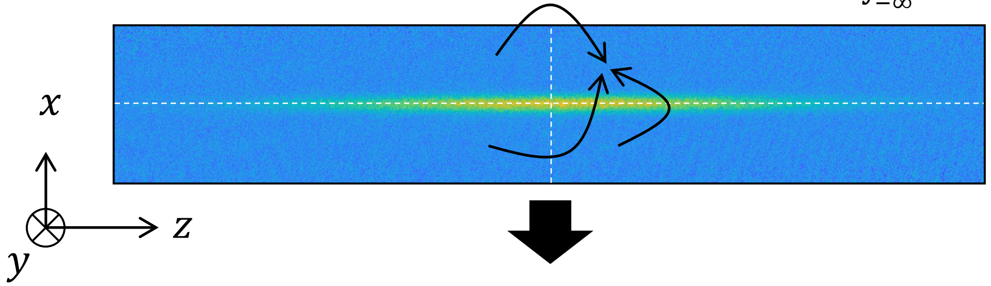
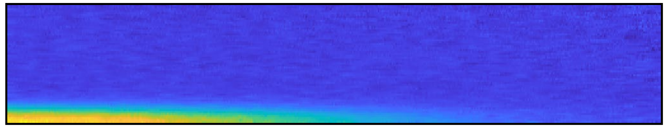


Typical data for a spin balanced gas

Column density : $\bar{n}(x,z) = \int_{-\infty}^{+\infty} n(x,y,z)dy$



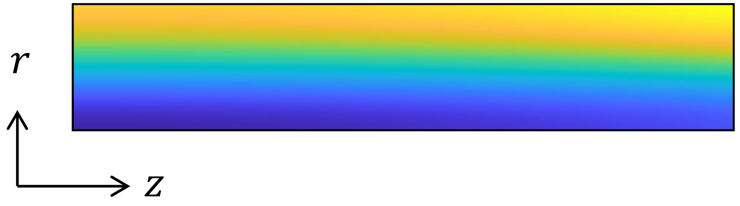
Averaged column density



Inverse Abel

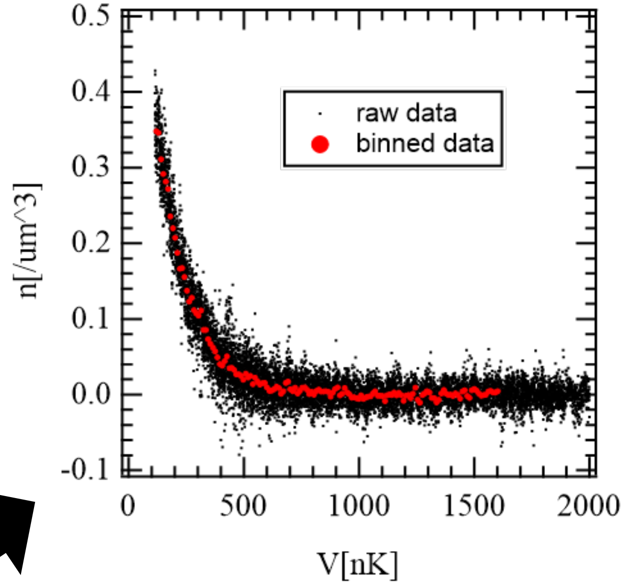
Local density : $n(r,z)$

Trapping potential : $V(r,z)$



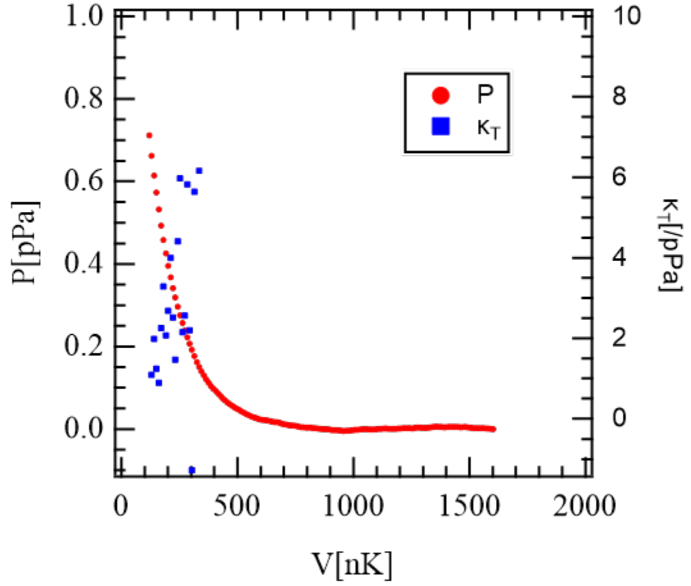
LDA

Number density



$n(V)$

Pressure and compressibility



$$P(V) = -\int_{+\infty}^{V} n(V)dV$$

$$\kappa_T = \frac{1}{n}\left(\frac{\partial n}{\partial P}\right)_T = \frac{1}{n^2}\left(\frac{dn}{dV}\right)$$

Test of data reliability for balanced unitary and non-interacting Fermi gases

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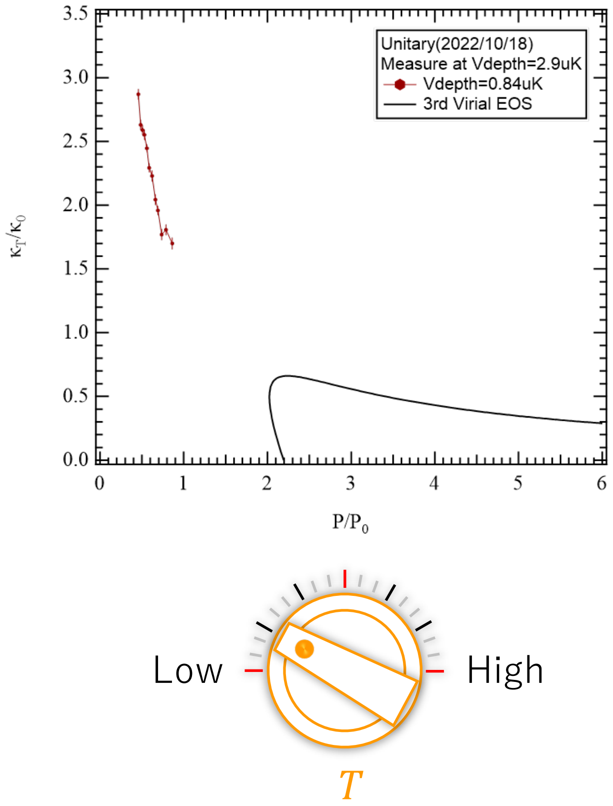
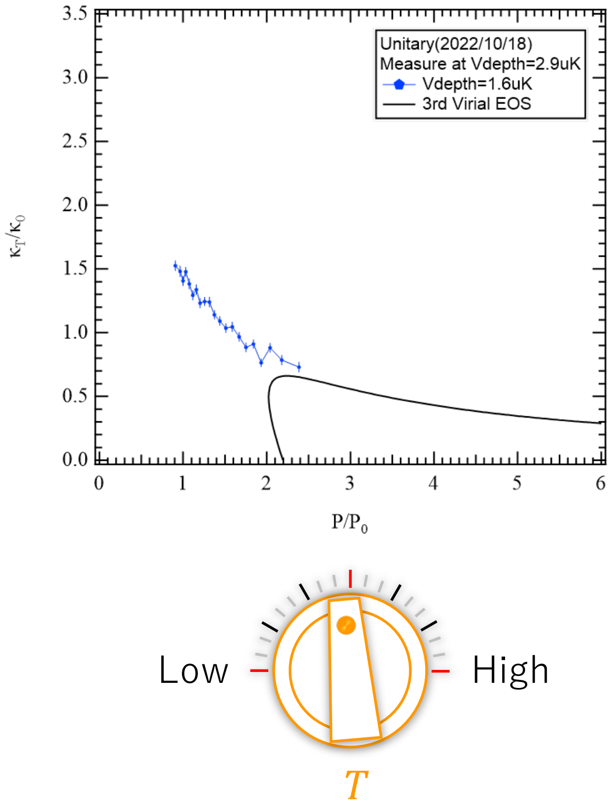
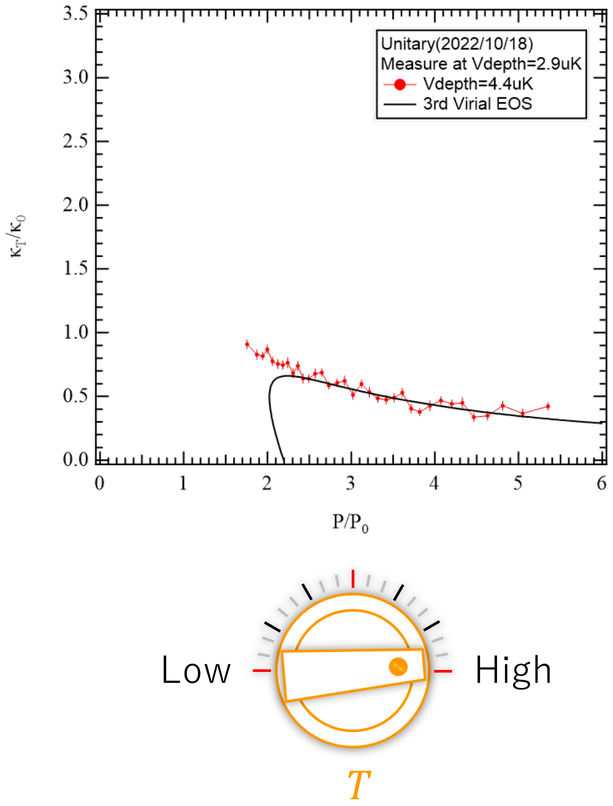
From thermodynamic relations, unitary Fermi gases and non-interacting Fermi gases obey

$$\frac{\kappa_T}{\kappa_0} = g\left(\frac{P}{P_0}\right) ,\left(P_o = \frac{2}{5} n\varepsilon_F(n), \kappa_o = \frac{3}{2}\frac{1}{n\varepsilon_F(n)}\right) \quad \text{for any temperature}$$







Test of data reliability for balanced unitary and non-interacting Fermi gases

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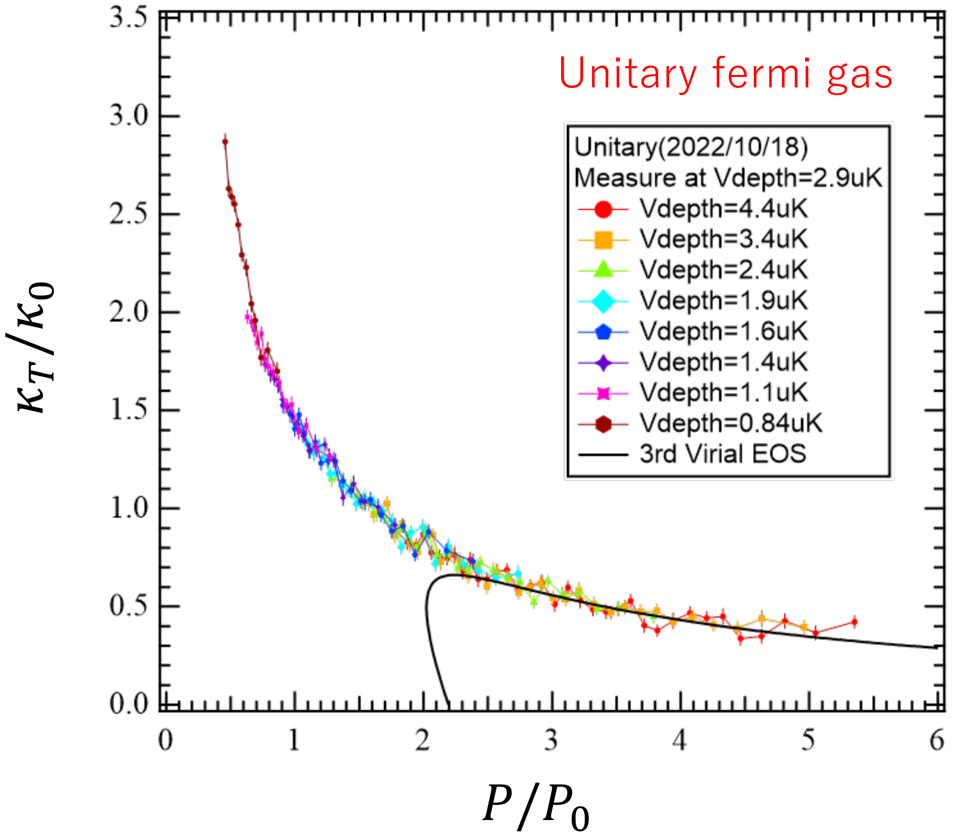
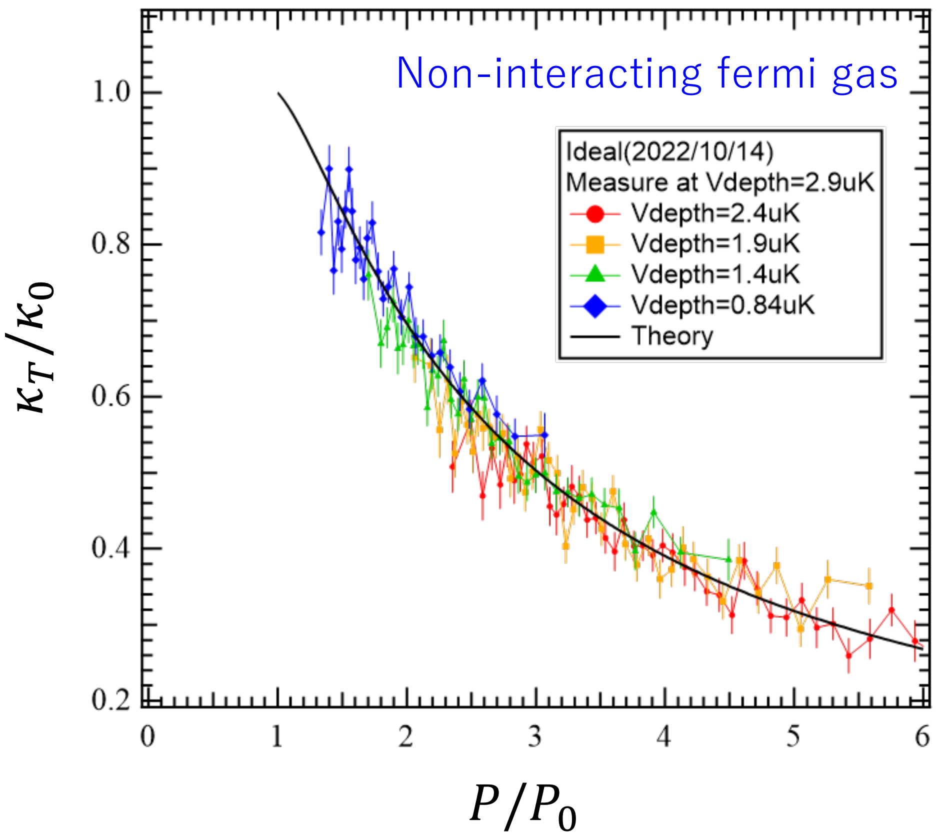
From thermodynamic relations,

$$\frac{\kappa_T}{\kappa_0} = g\left(\frac{P}{P_0}\right),\left(\kappa_T = \frac{1}{n}\left(\frac{\partial n}{\partial P}\right)_T, P_o = \frac{2}{5}n\varepsilon_F(n), \kappa_o = \frac{3}{2}\frac{1}{n\varepsilon_F(n)}\right)$$

for any temperature





Behavior around Tc

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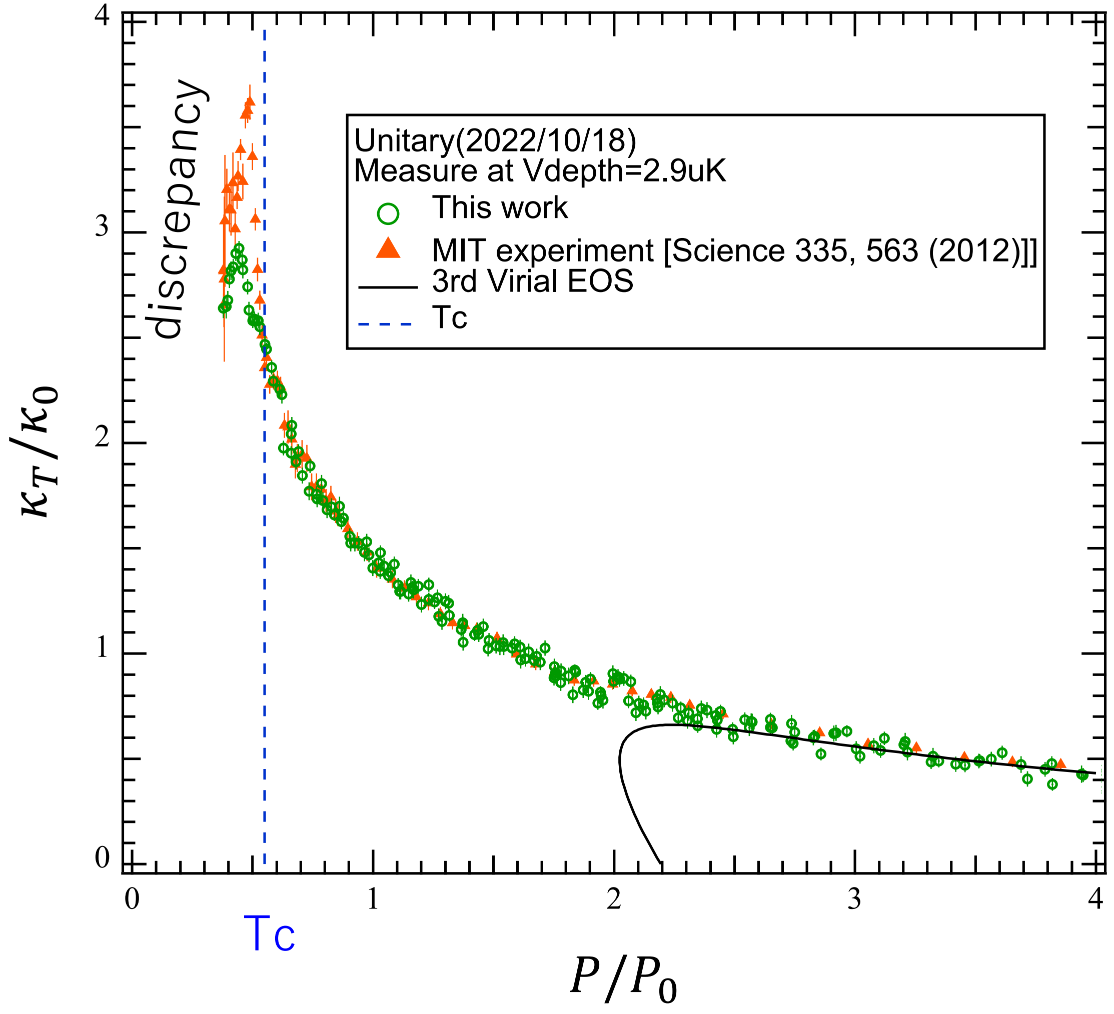

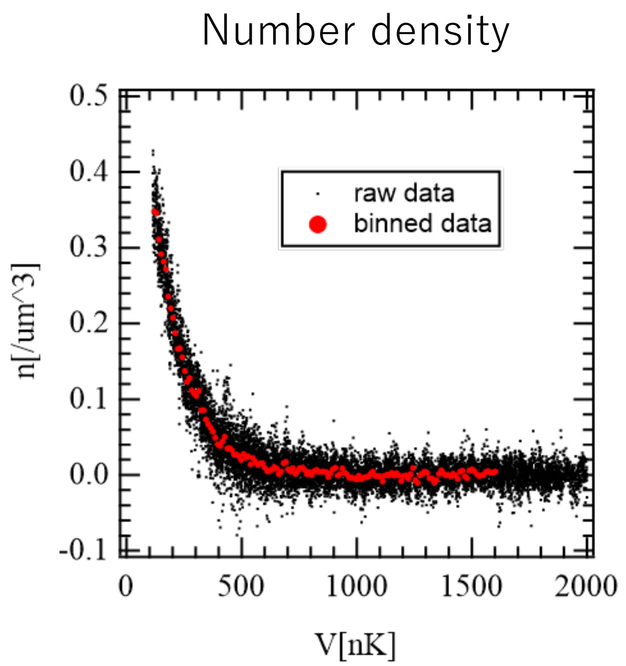

$$\kappa_T = \frac{1}{n}\left(\frac{\partial n}{\partial P}\right)_T = \frac{1}{n^2}\left(\frac{dn}{dV}\right)$$

We need numerical derivative of the data

Summary

Toward complete of the phase diagram of the BCS-BEC crossover

The **Quantum Simulator** is ready

Three remained subjects

1. Discrepancy around Tc
2. Robust EOS construction process
3. Imaging condition for molecules

We will show you cluster physics within this simplified model from the EOS in near future.