2022.11.2 CLUSHIQ2022

Feasibility of the non-uniform FFLO superfluid Fermi atomic gas

Taira Kawamura and Yoji Ohashi, Phys. Rev. A **106**, 033320 (2022)

Taira Kawamura (Keio University)

Yoji Ohashi (Keio University)

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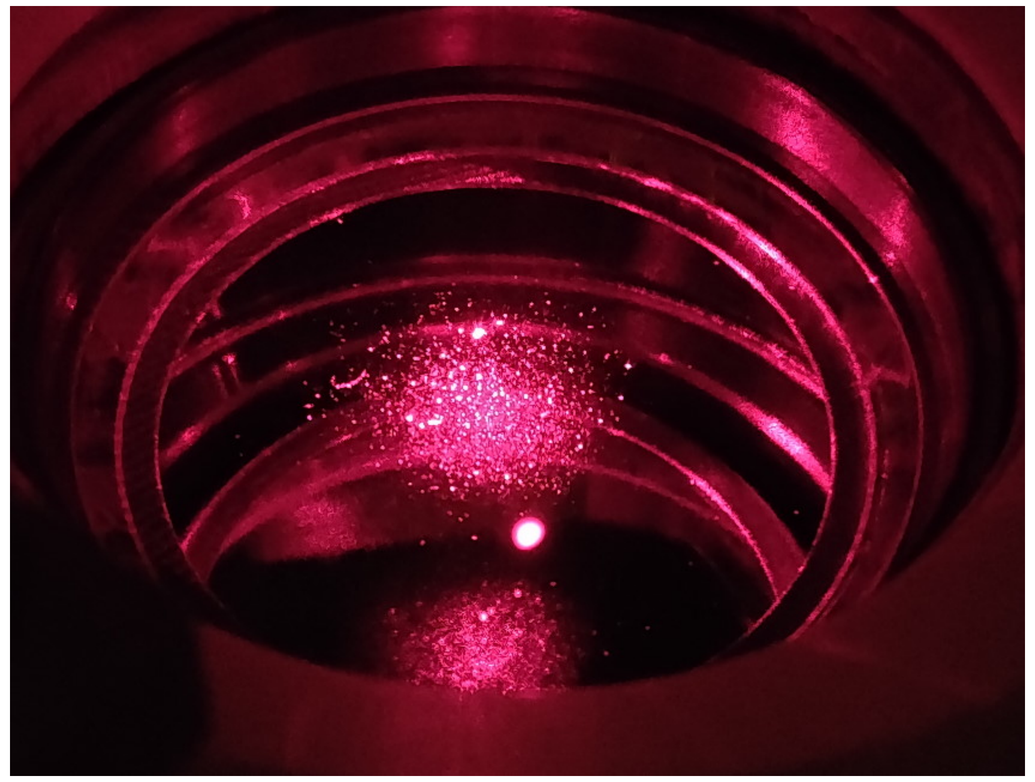
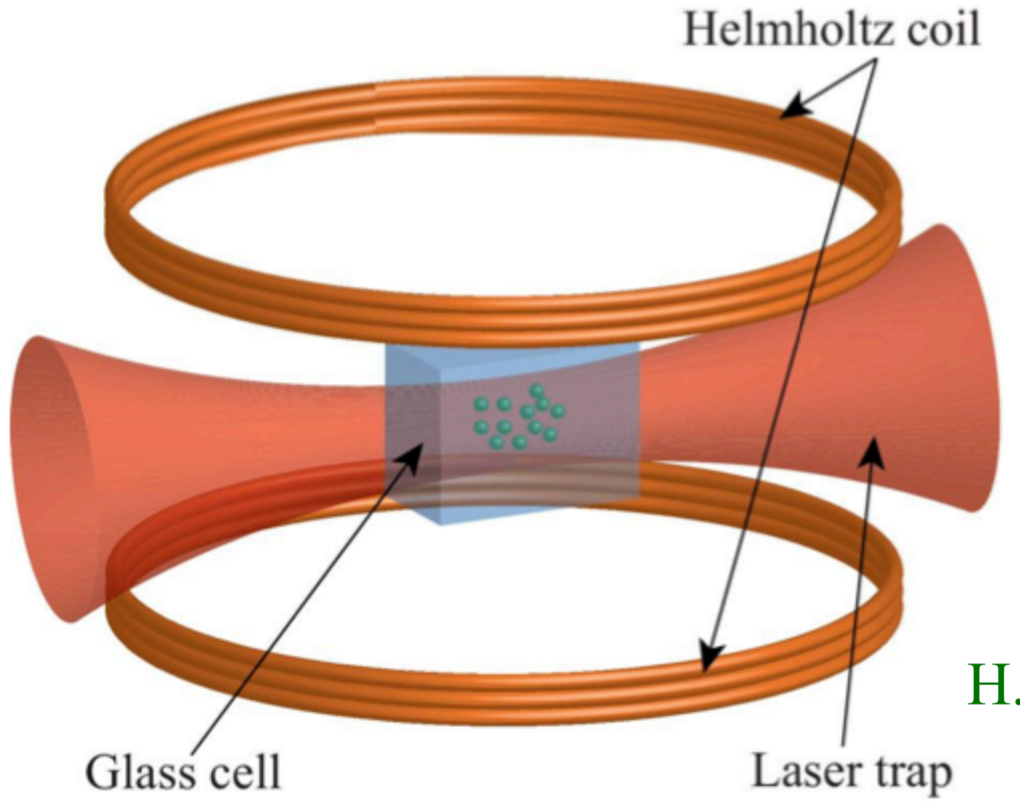
Ultracold Fermi Gas

Powerful tool to experimentally study strongly correlated quantum many-body systems

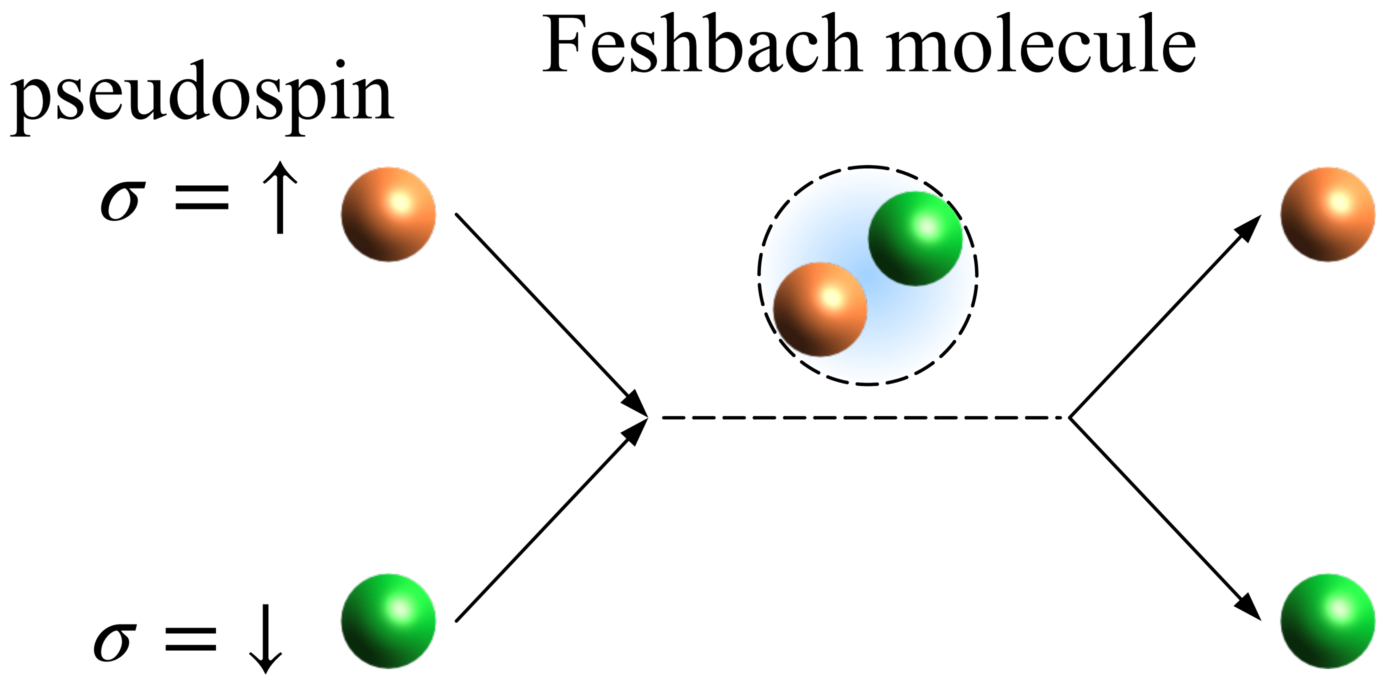


https://ultracold.phys.virginia.edu/public_html/

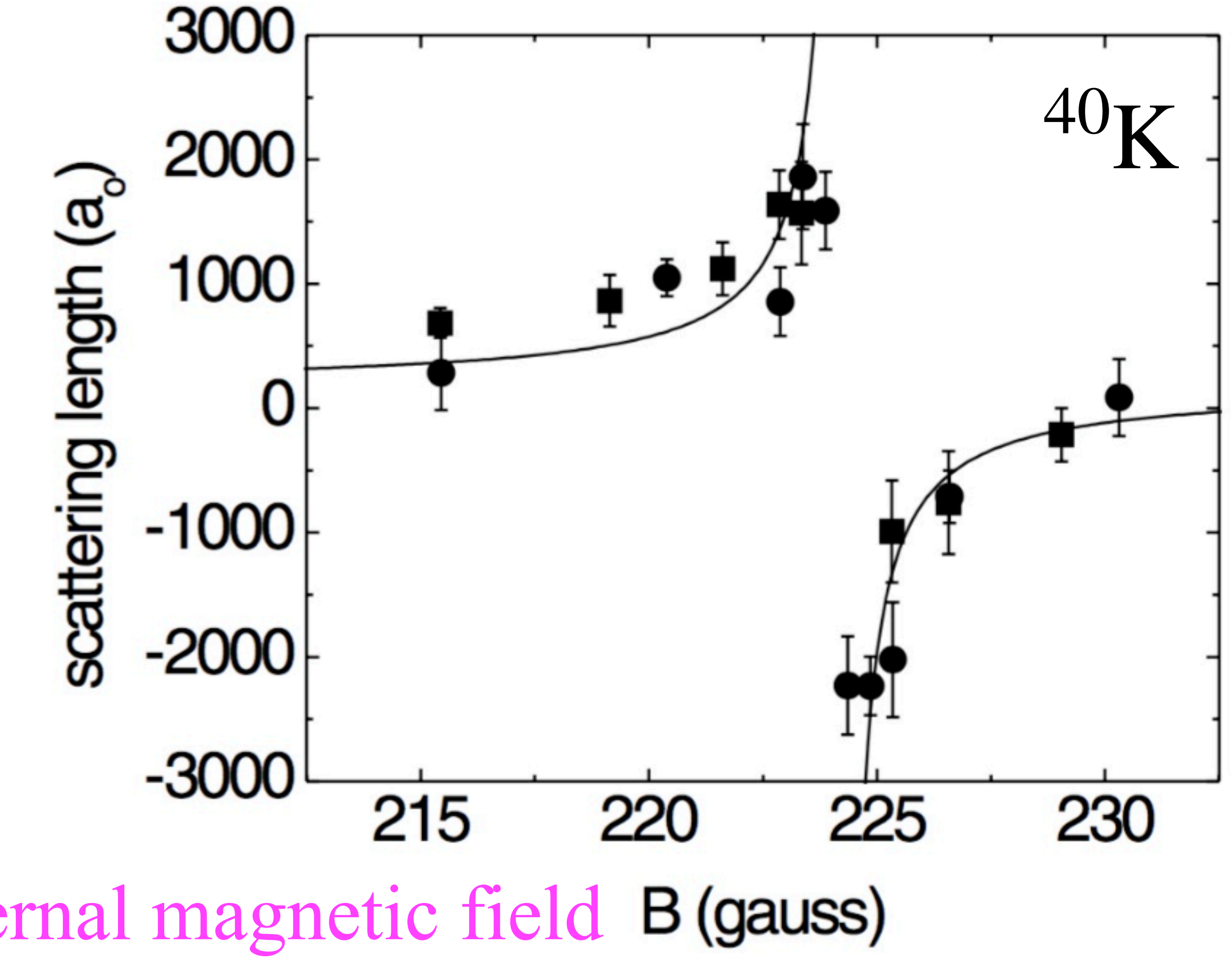


H. Tajima (doctor thesis)

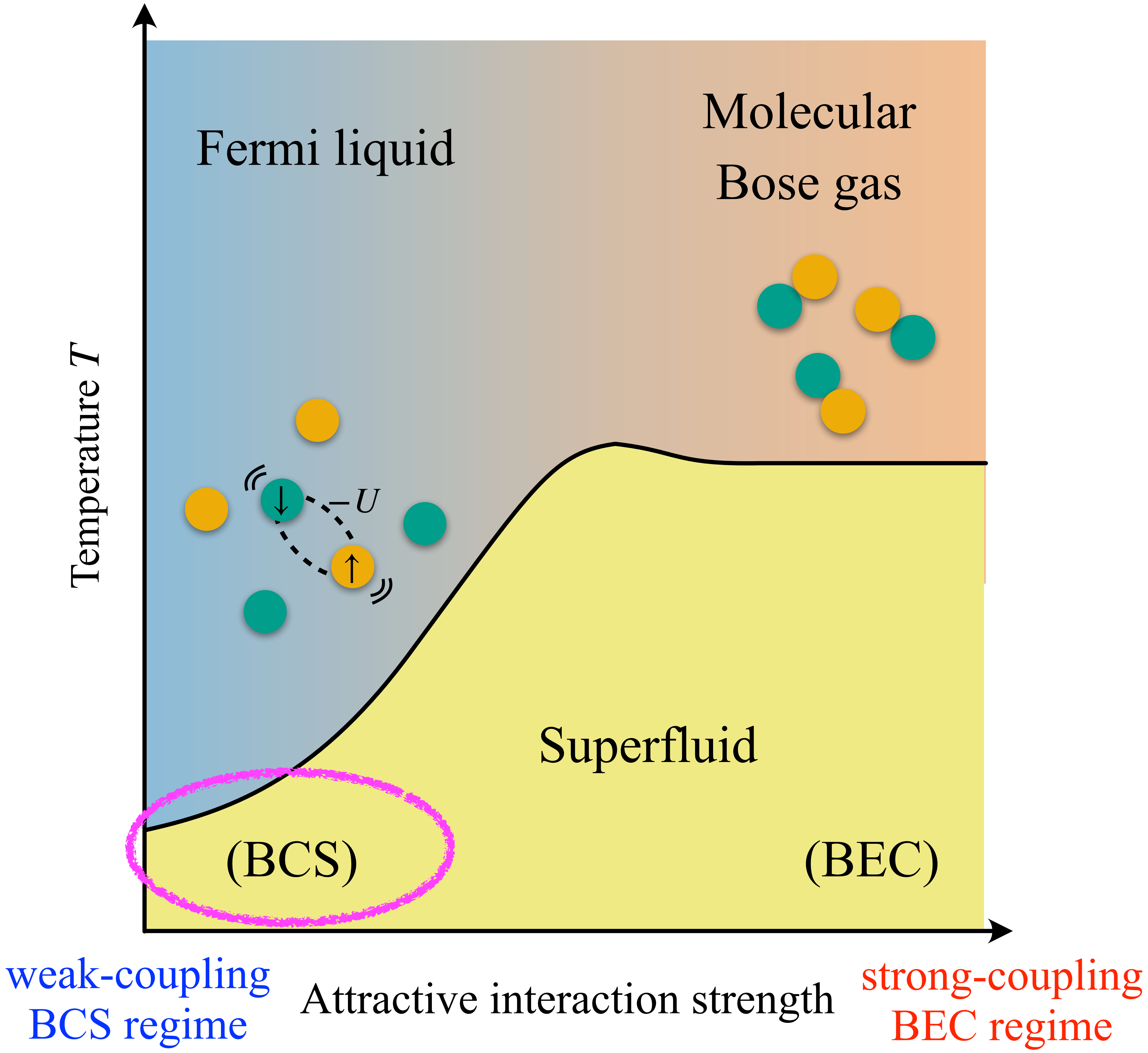
Tunable interaction associated with a Feshbach resonance



C. A. Regal, *et al.*, PRL **90**, 230404 (2003)

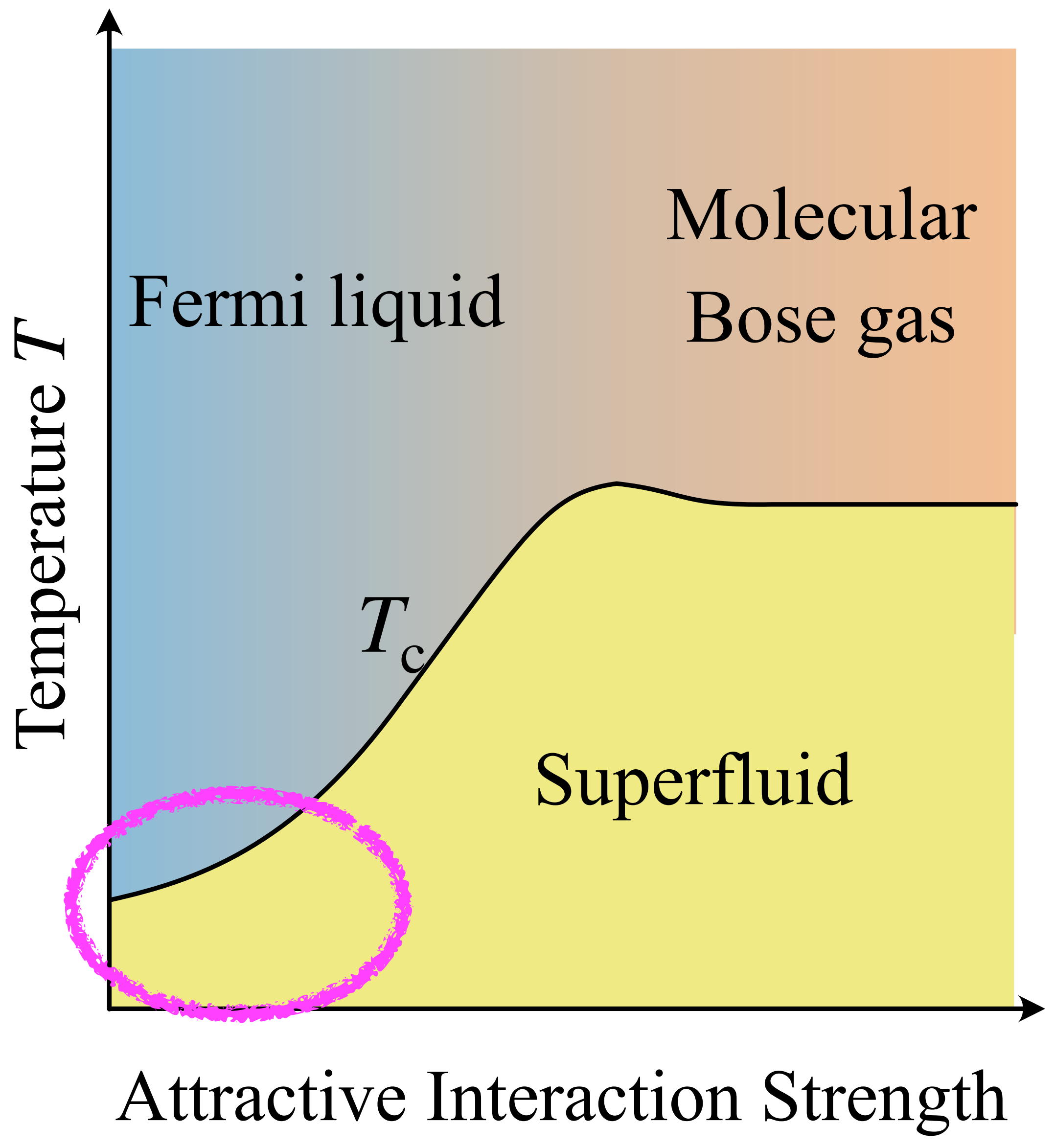


External magnetic field



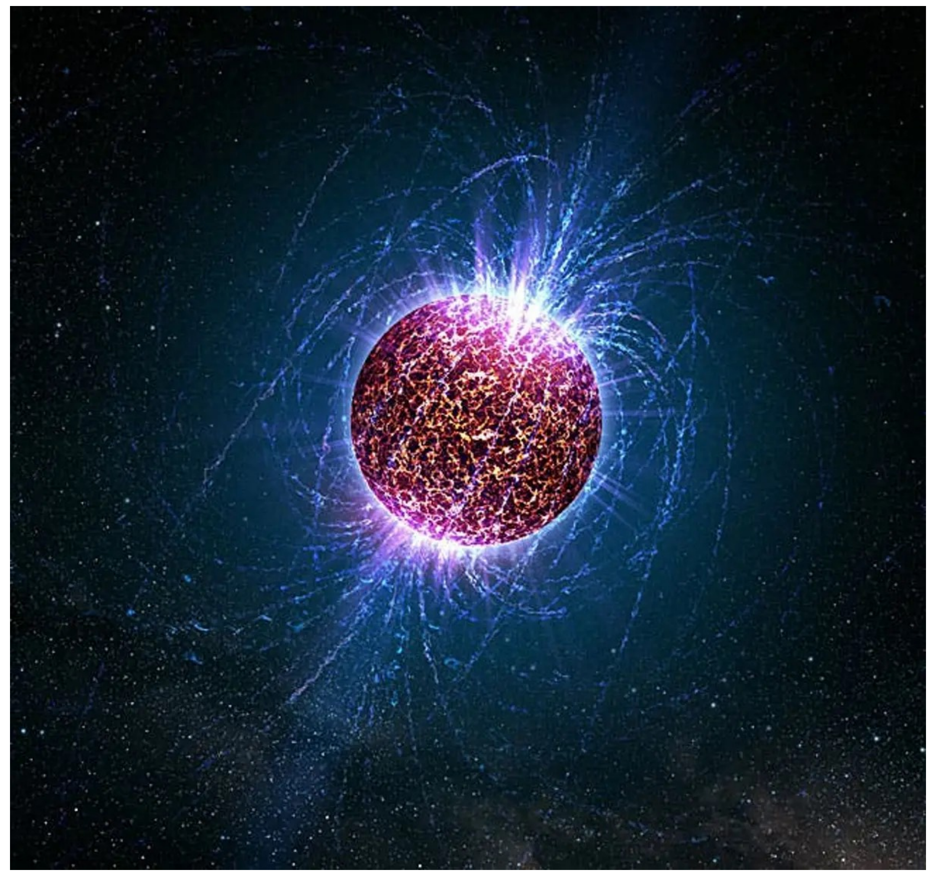
Superconductivity in Different Hierarchies

Ultracold Fermi Gas



Nuclear Matter

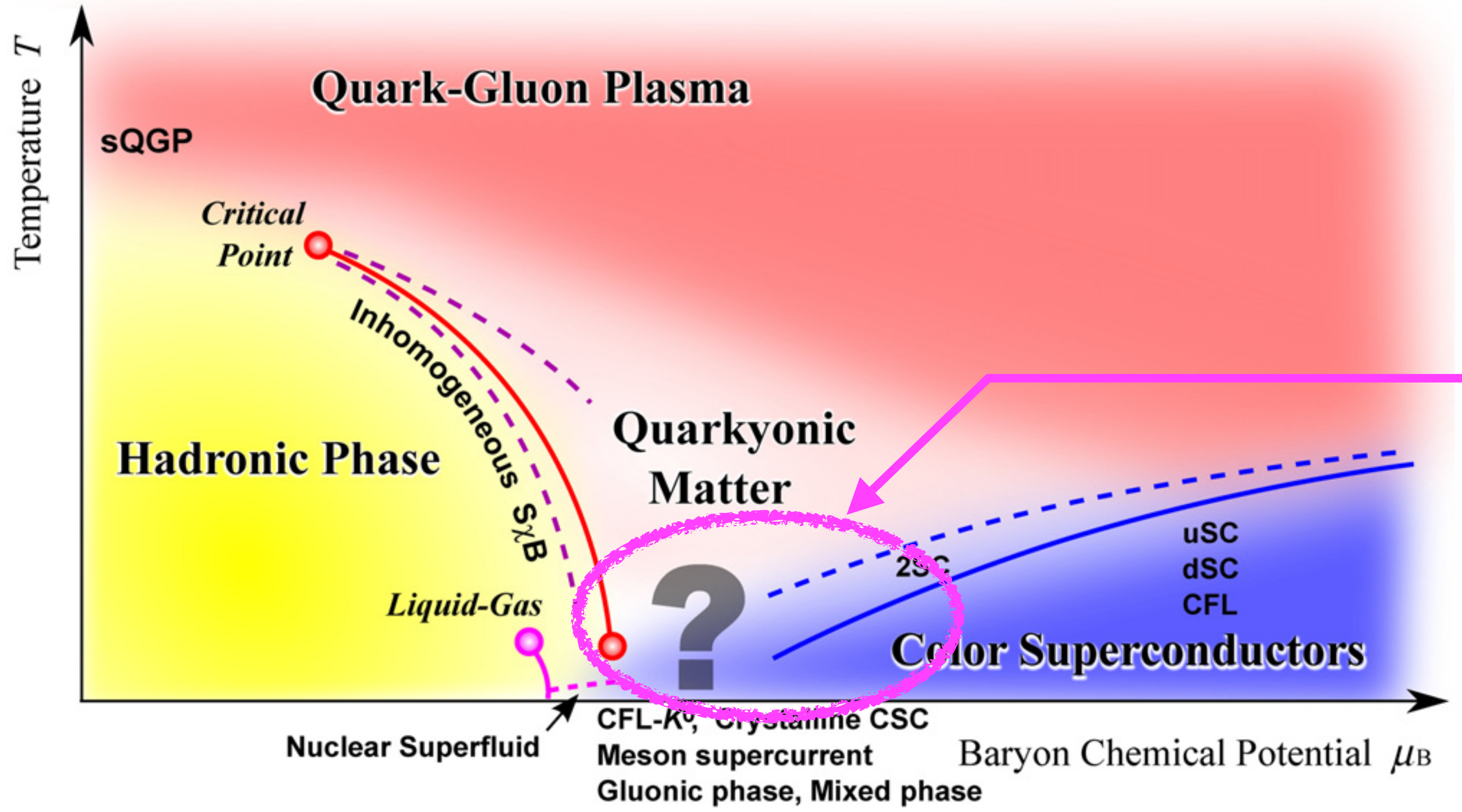
Neutron Star



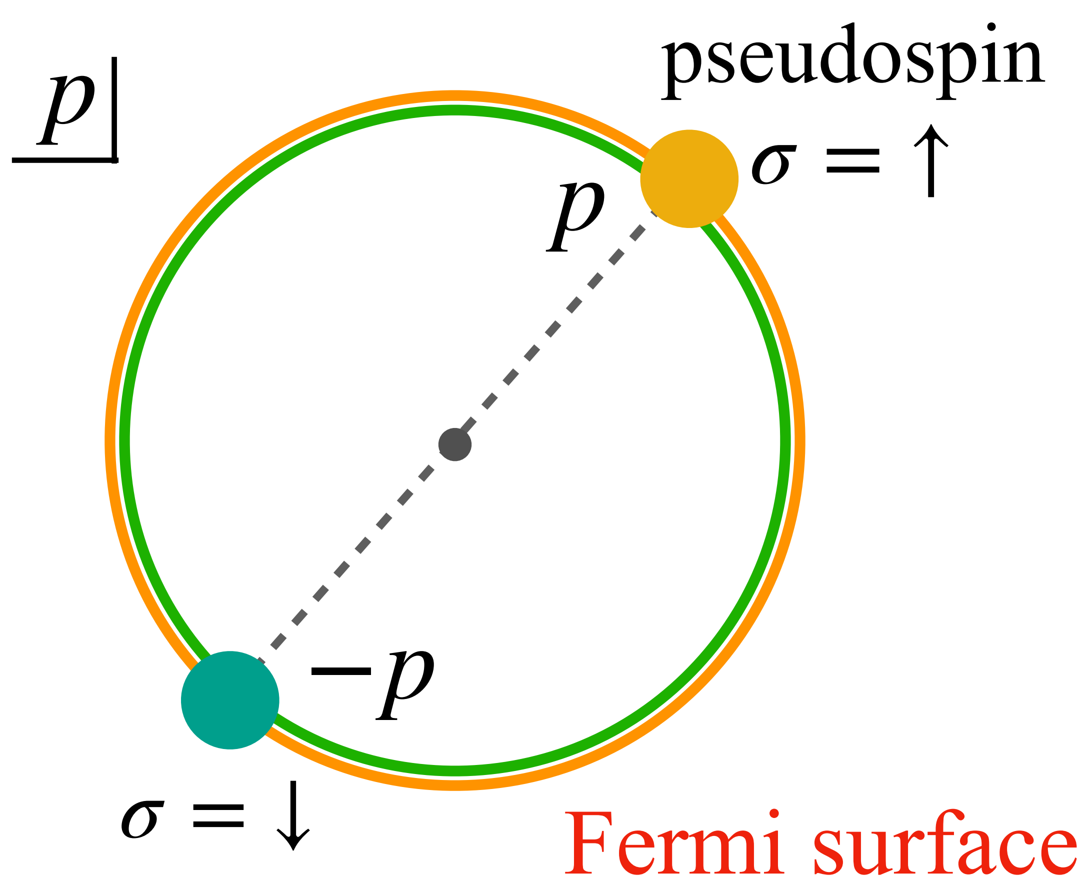
https://astronomy.com/news/2020/03/how-big-are-neutron-stars

Quark Matter

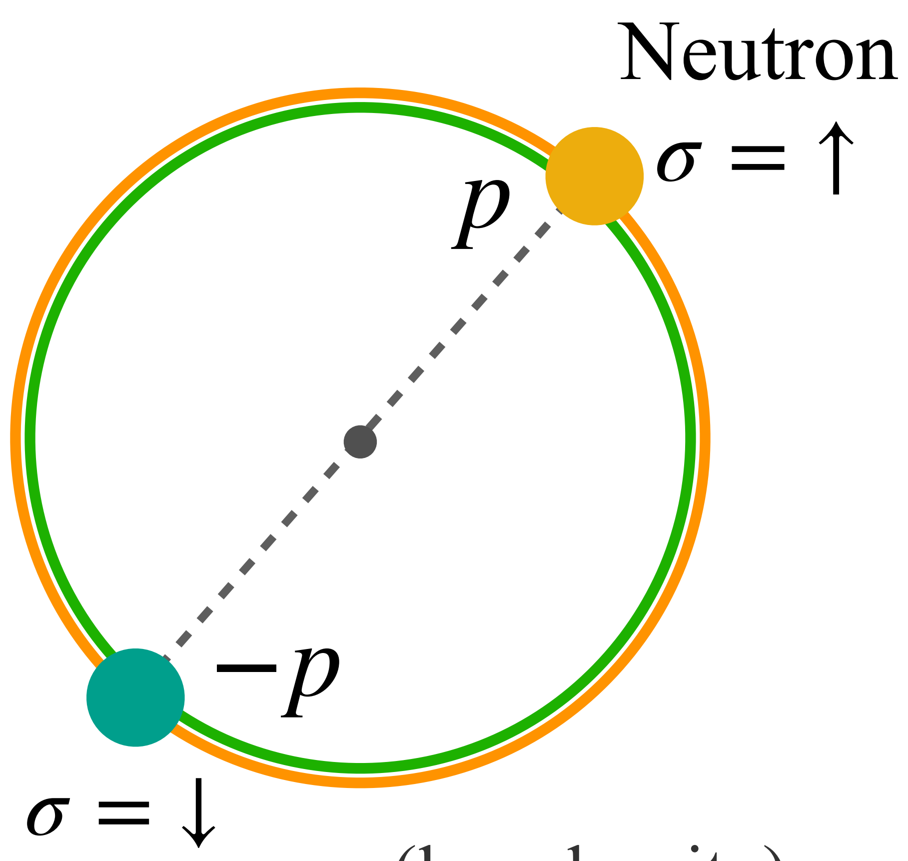
K. Fukushima and T. Hatsuda, *Rep. Prog. Phys.* **74**, 014001 (2011)



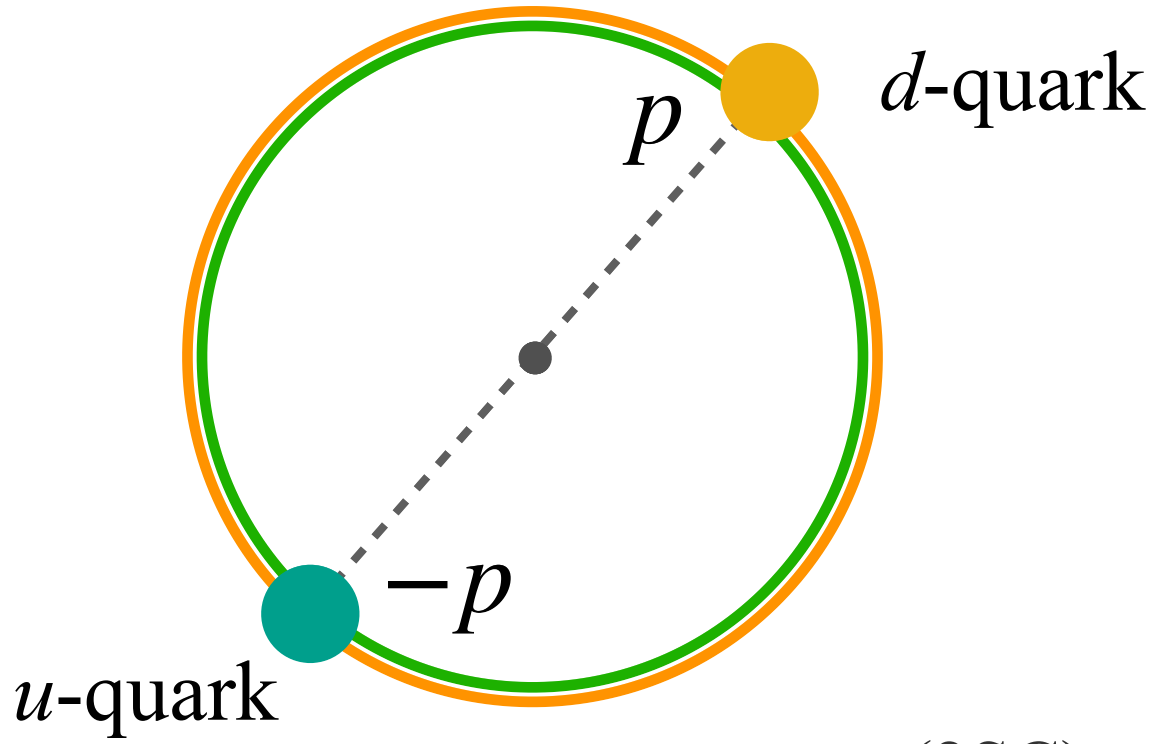
2SC
(2-flavor superconductor)



Fermi surface



(low density)



(2SC)

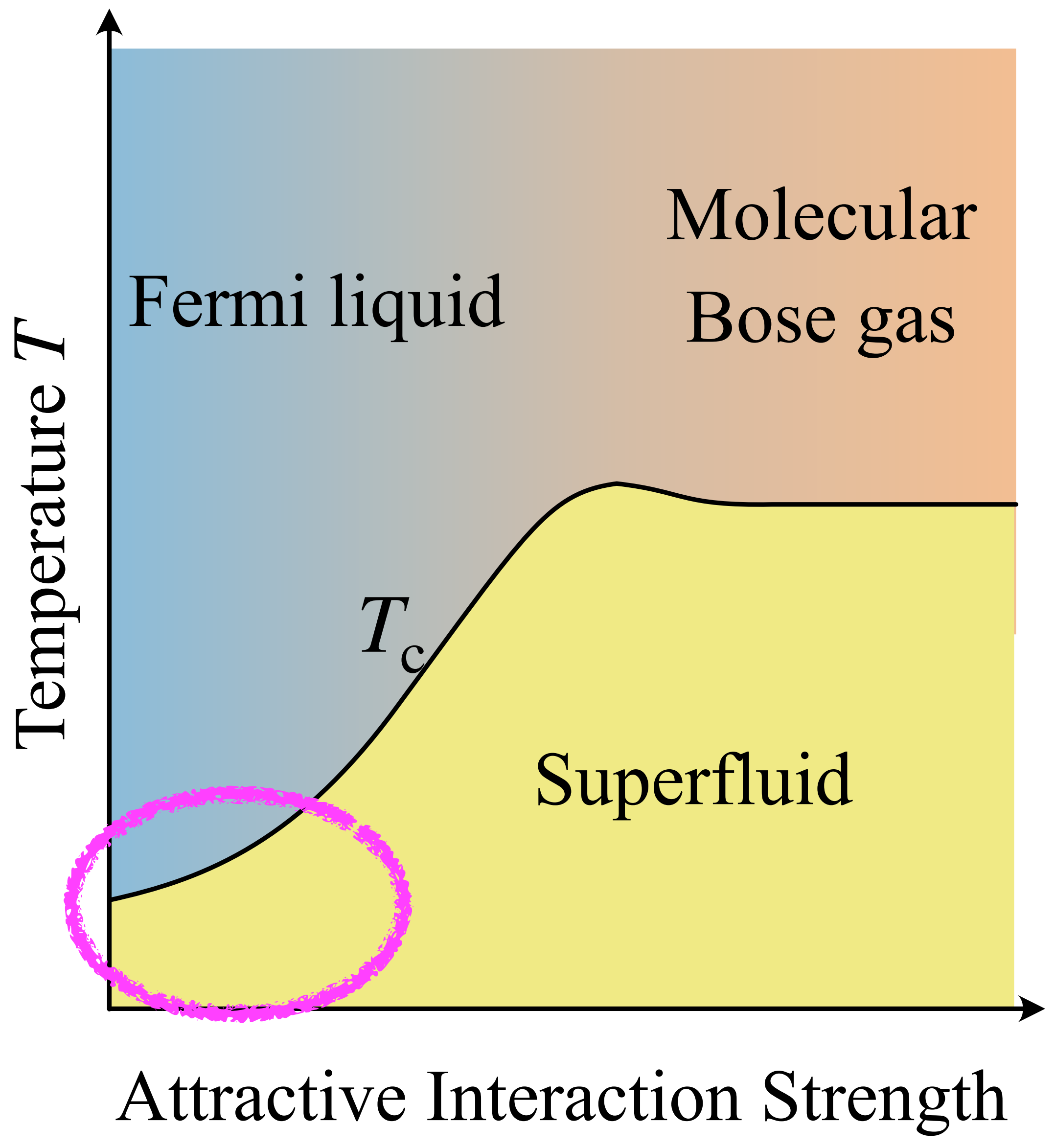
$$\Delta(x) = \Delta_0$$

Bardeen, Cooper, Schrieffer (1957)

Uniform Fermi superfluid (BCS state)

Non-uniform Superconductivity in Different Hierarchies

Ultracold Fermi Gas



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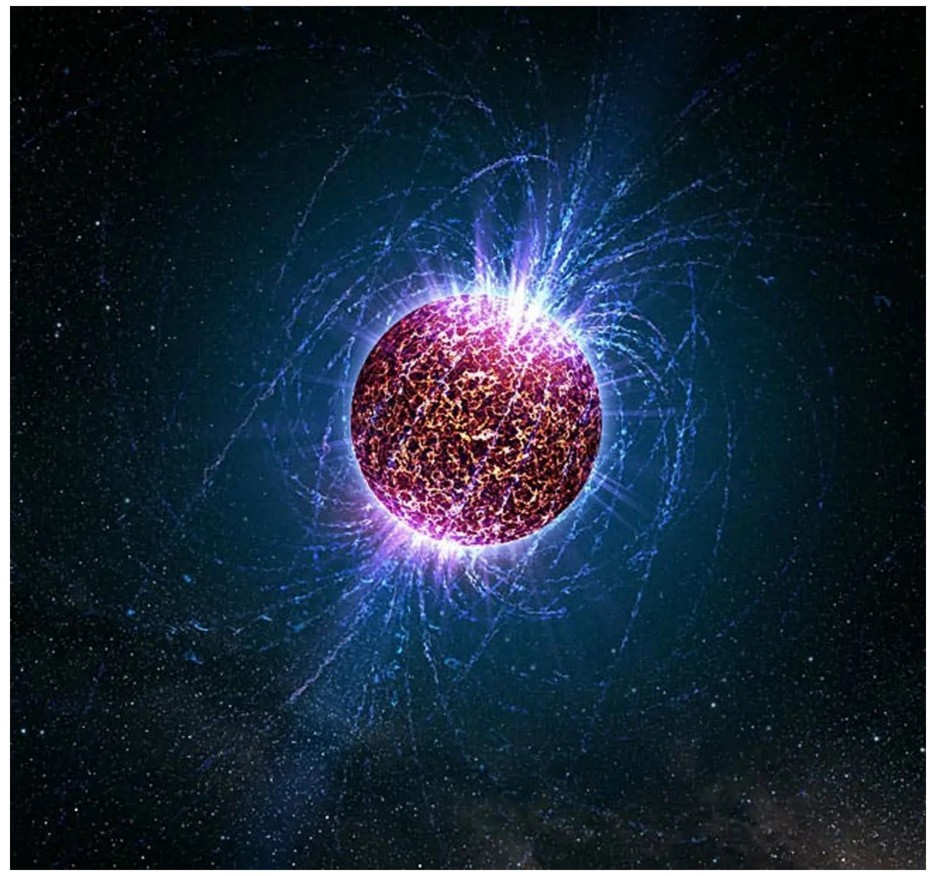

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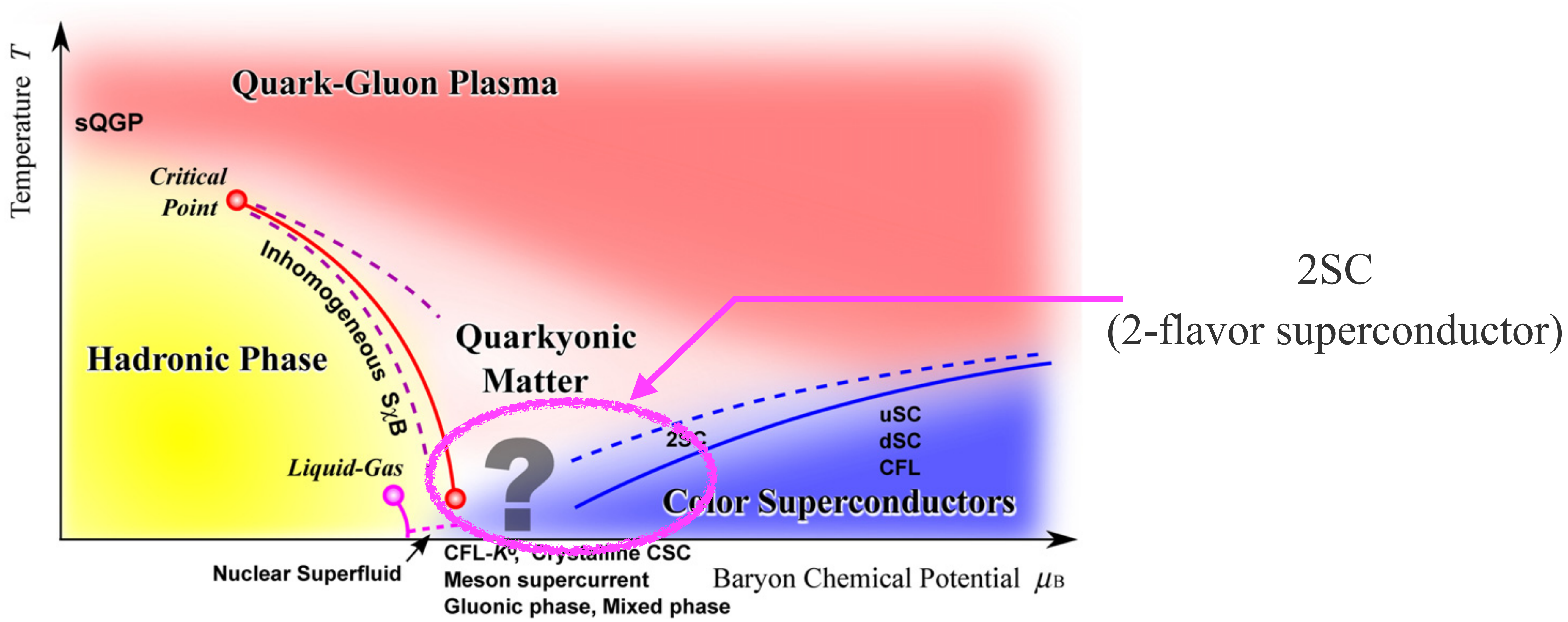
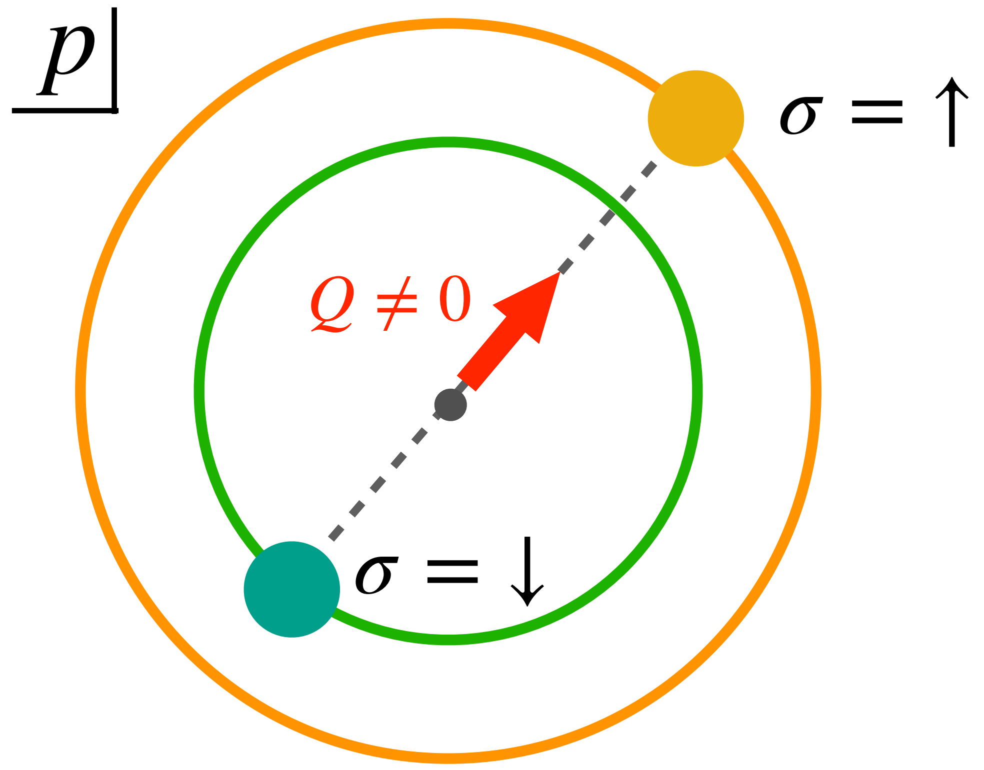
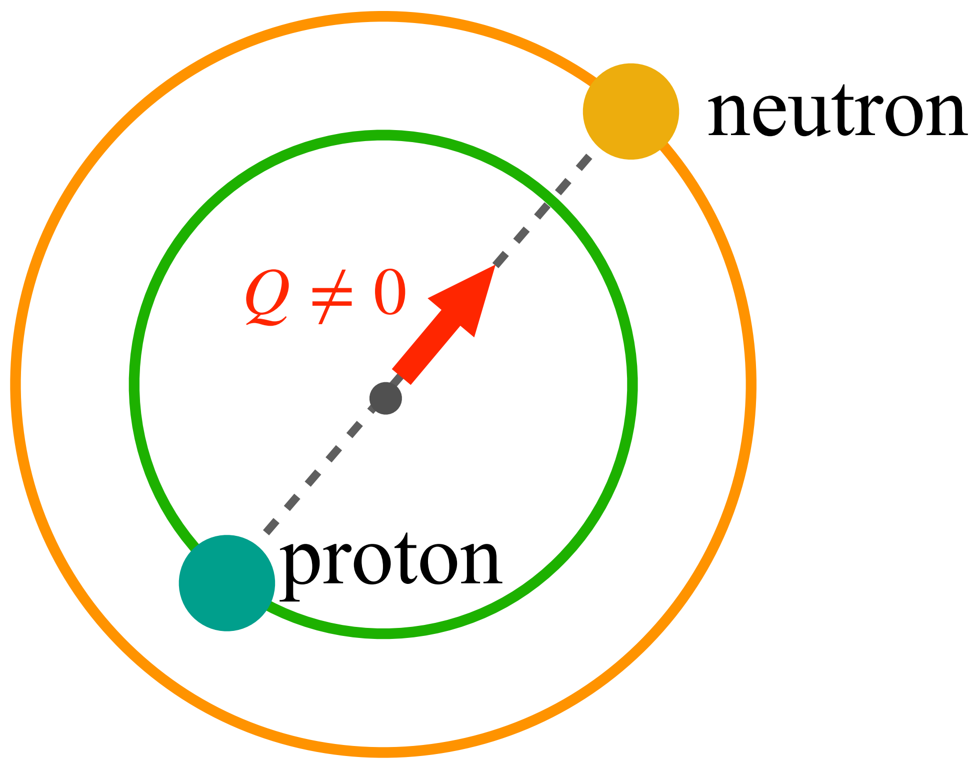
K. Fukushima and T. Hatsuda, *Rep. Prog. Phys.* **74**, 014001 (2011)

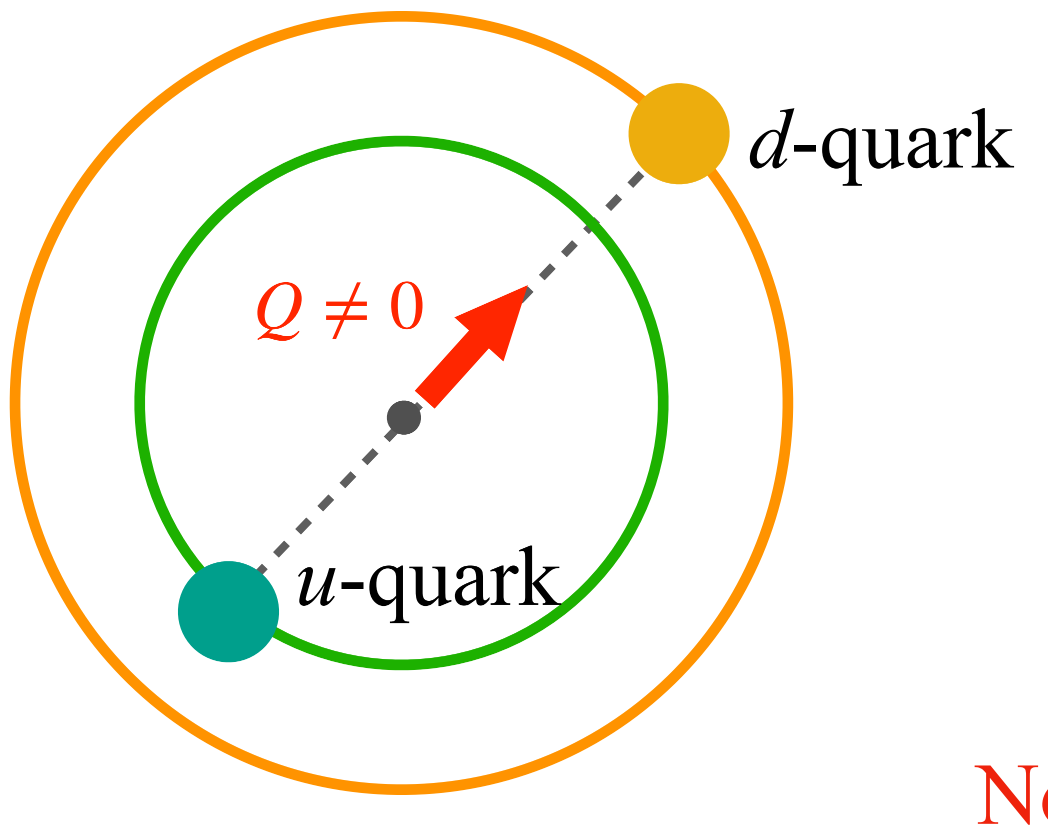




Spin imbalance



Isospin asymmetry



Electrical and color neutrality

$\boxed{\text{Mismatched Fermi surfaces}}$

$$\Delta(x) = \Delta_0 e^{iQx}$$

Fulde, Ferrell (1964)

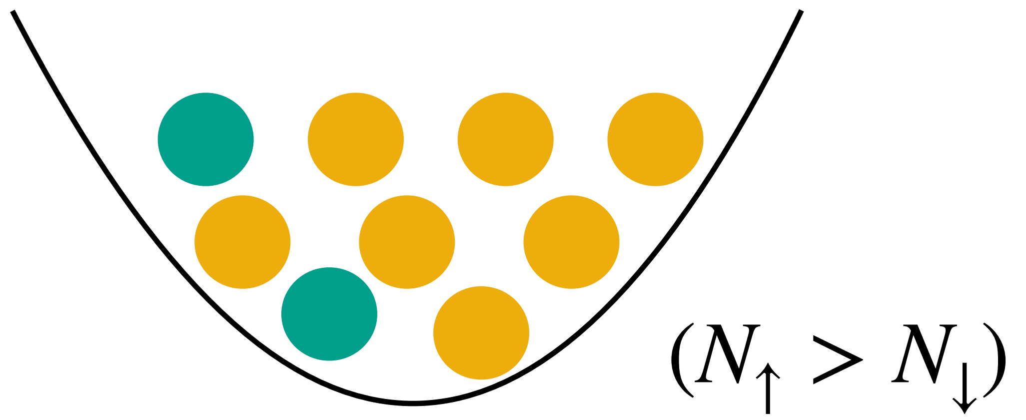
$$\Delta(x) = \Delta_0 \cos(Qx)$$

Larkin, Ovchinnikov (1965)

Non-uniform Fermi superfluid (FFLO state)

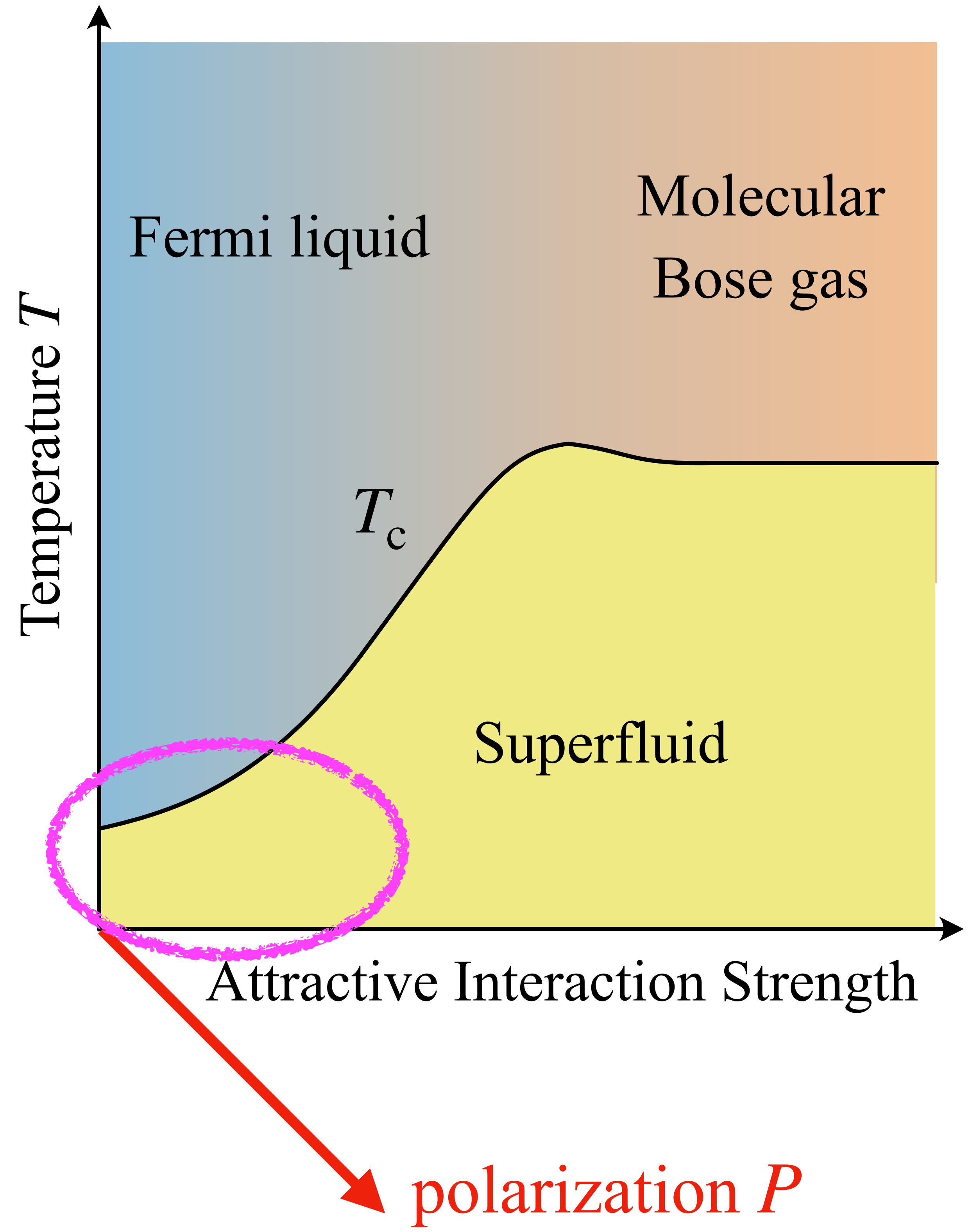
FFLO State in a Spin-imbalanced Ultracold Fermi Gas

Spin-imbalanced Fermi gas ($N_\uparrow \neq N_\downarrow$)



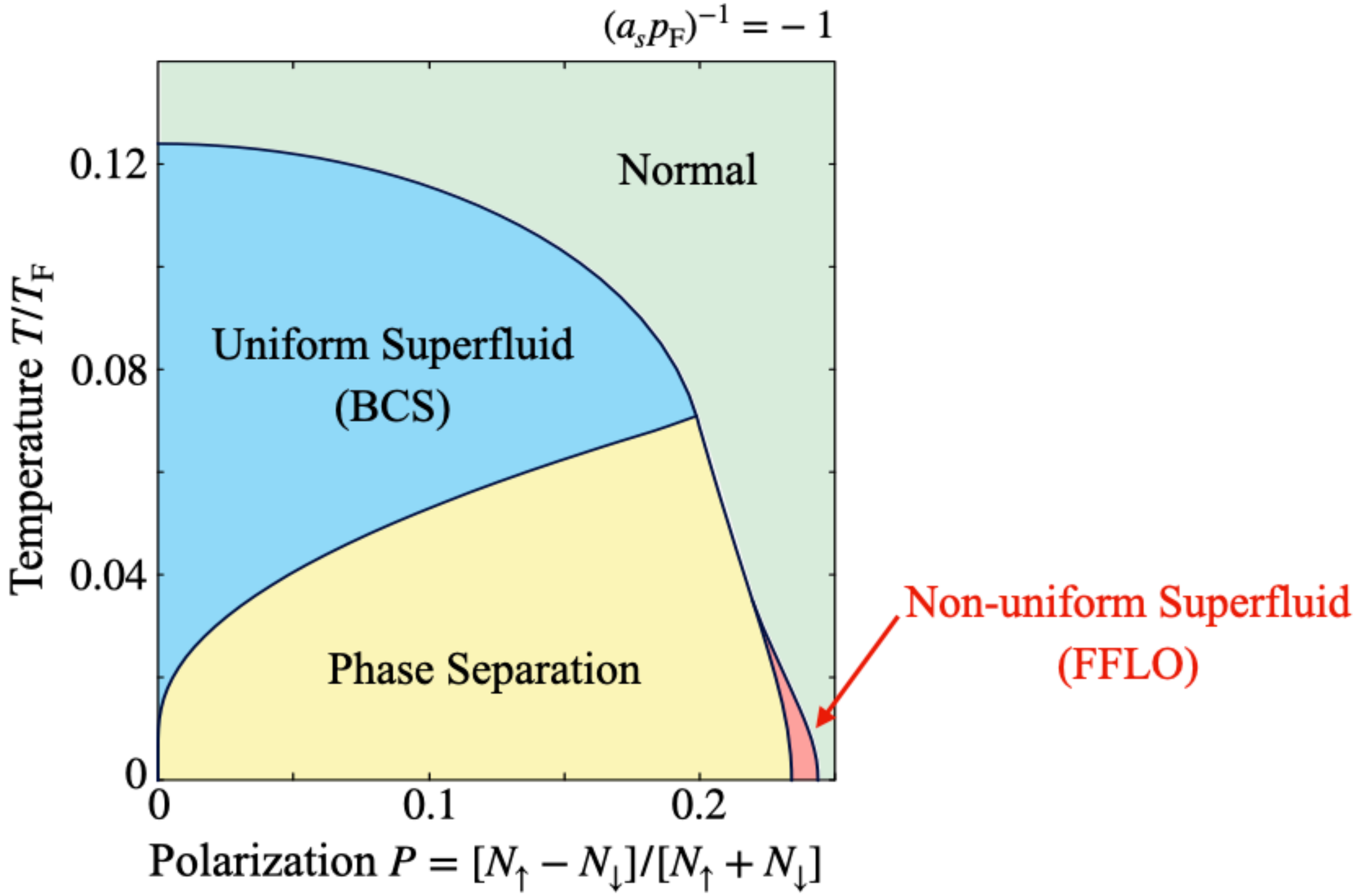
polarization

$P = [N_\uparrow - N_\downarrow]/[N_\uparrow + N_\downarrow]$



Theory (Mean-field approximation)

M. M. Parish *et. al.*, Nature Physics **3**, 124 (2007)
G. C. Strinati *et. al.*, Phys. Rep. **738**, 1 (2018)
⋮



The FFLO state has not been observed experimentally in a 2D and 3D Fermi gas.

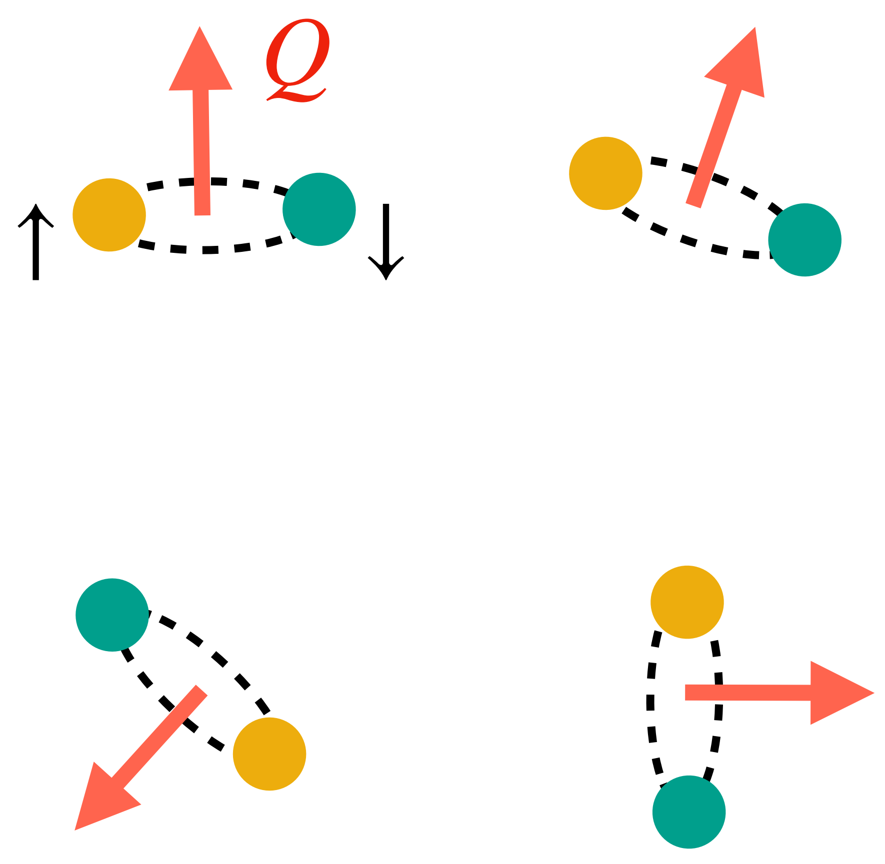
Difficulty (I) : Destruction of the FFLO state by pairing fluctuations

In isotropic systems, the FFLO-type non-uniform superfluid state is unstable against pairing fluctuations at nonzero temperature even in a 3D system.

- H. Shimahara, J. Phys. Soc. Jpn. **67**, 1872 (1998)
- Y. Ohashi, J. Phys. Soc. Jpn. **71**, 2625 (2002)
- J. Wang *et. al.*, Phys. Rev. B **97**, 134513 (2018)
- P. Zdybel *et. al.*, Phys. Rev. A **104**, 063317 (2021)

⋮

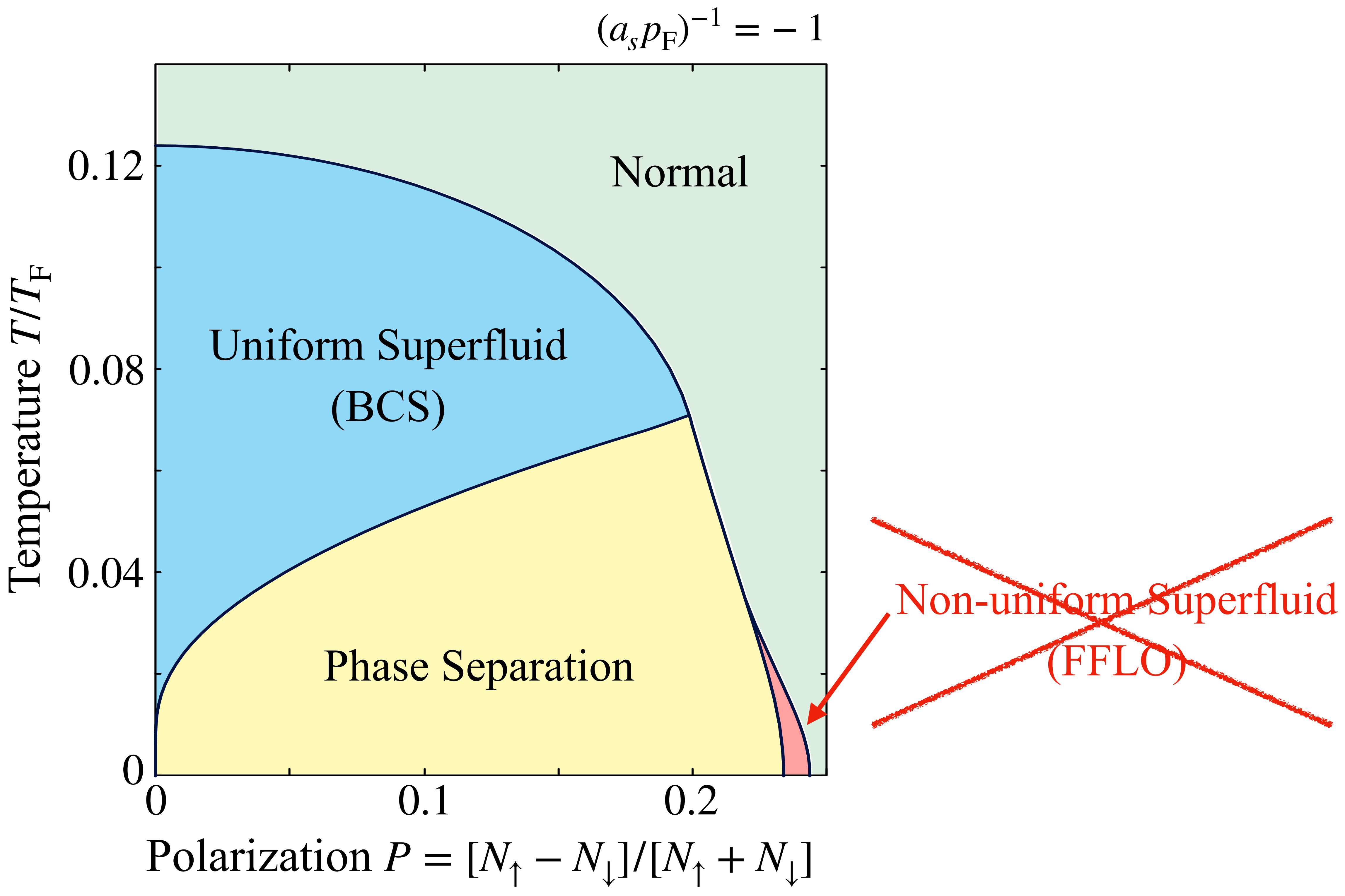
gas system
(continuous rotational symmetry)



infinite degeneracy

Large fluctuations caused by an infinite degeneracy with respect to the direction of the FFLO Q vector destroy the FFLO-type long-range order.

This mean-field phase diagram is completely wrong !



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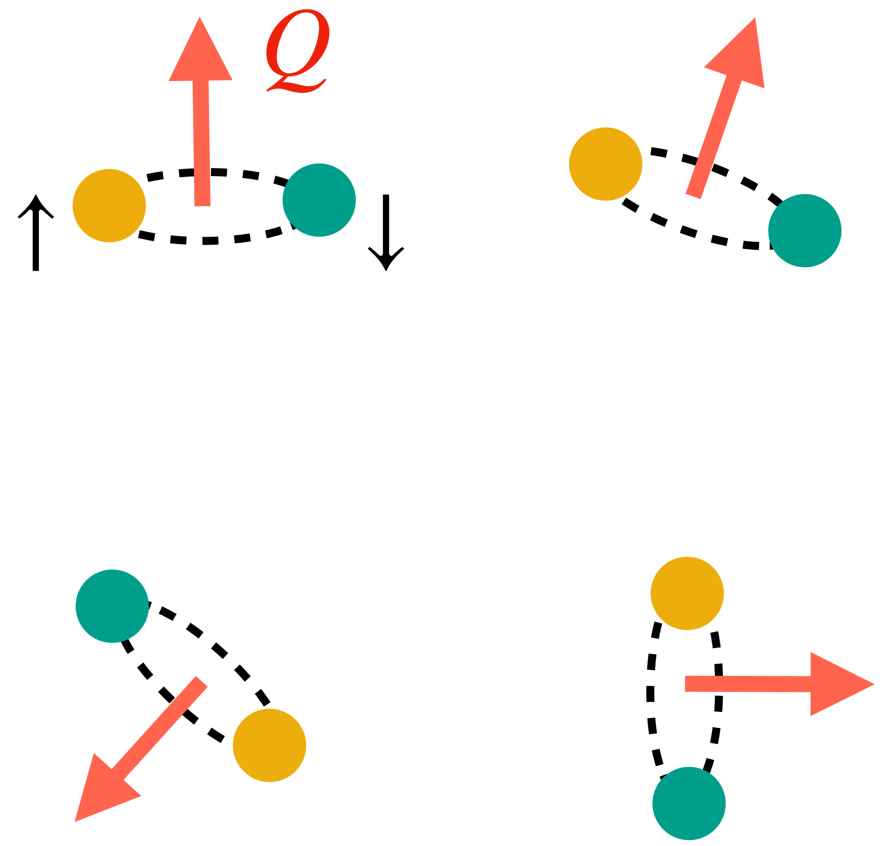
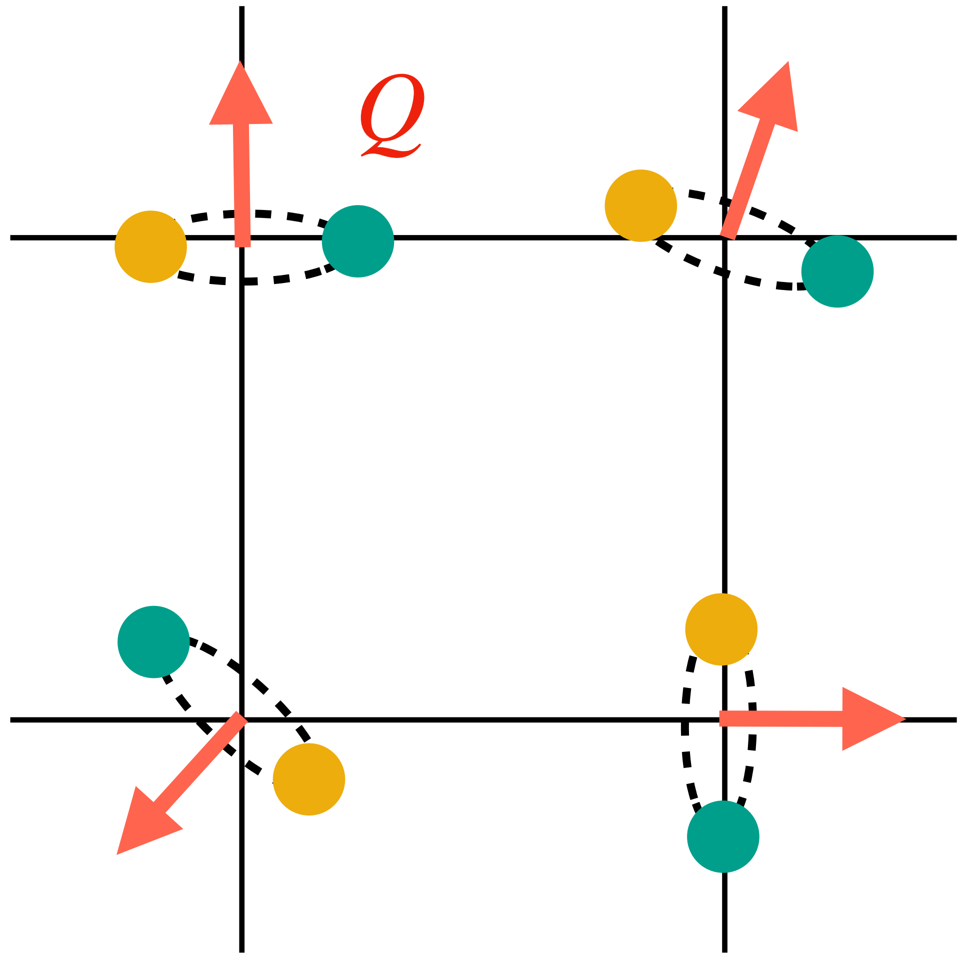


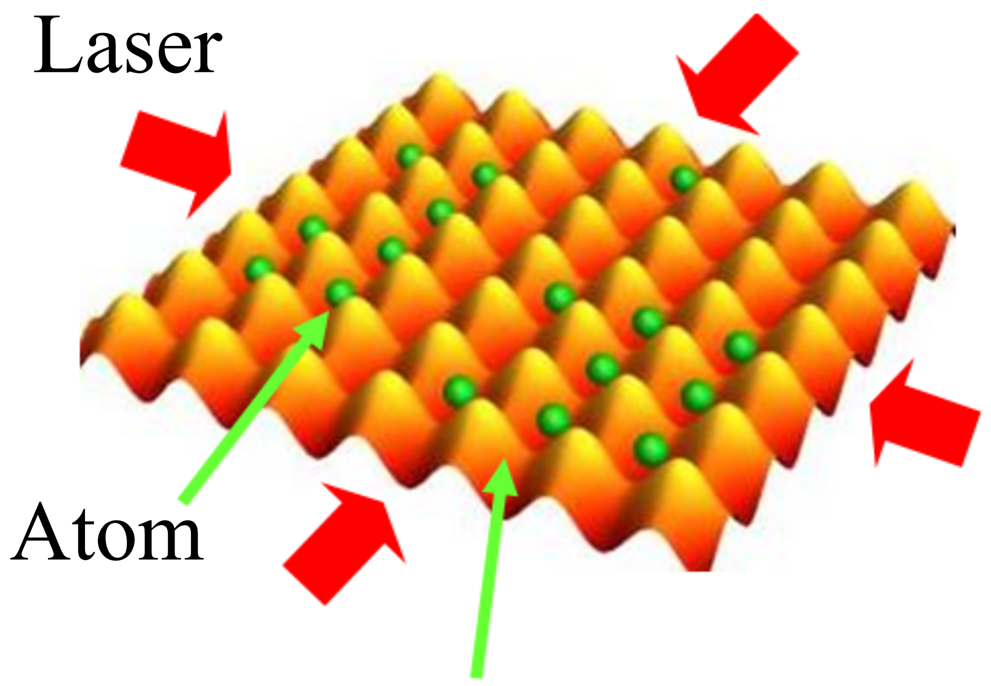
infinite degeneracy

Explicitly breaking
the continuous rotational symmetry

Optical lattice system
(discrete rotational symmetry)



finite degeneracy



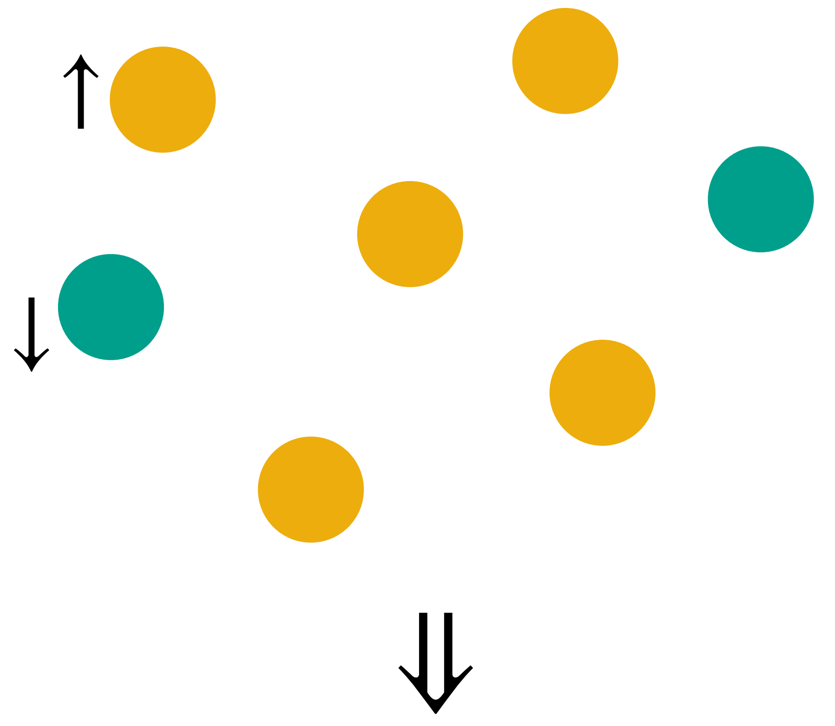
Periodic potential made by optical interference

http://www.kozuma.phys.titech.ac.jp/research_category/entry7.html

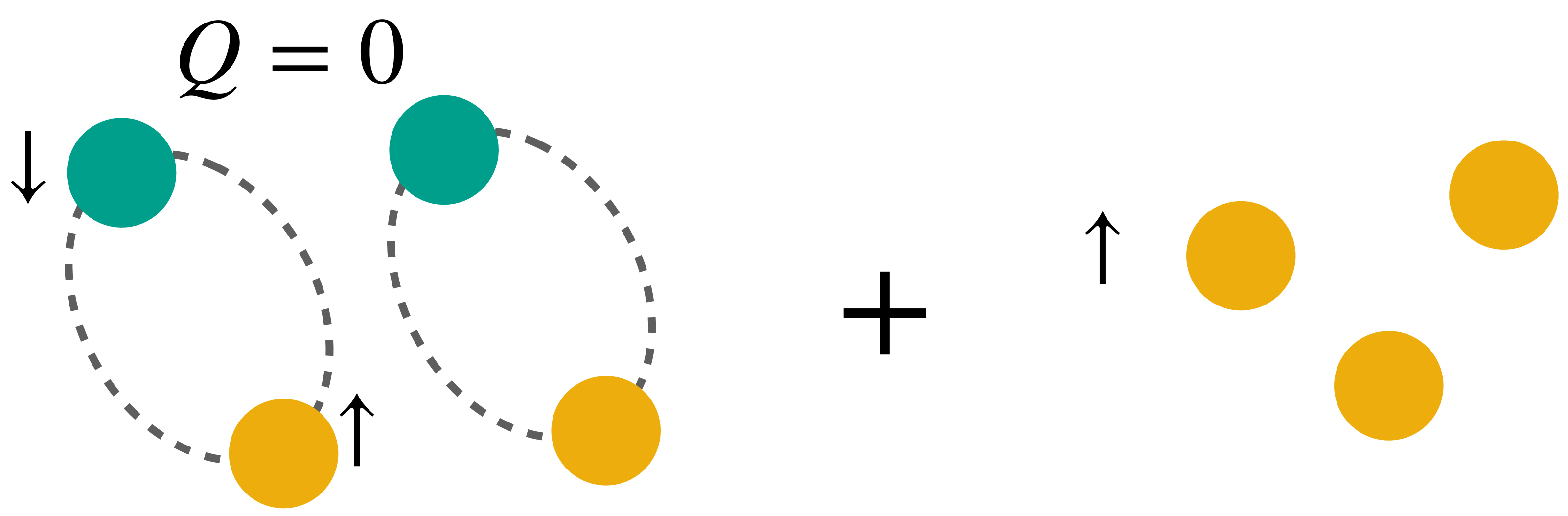
Difficulty (II) : Competition with phase separation (BCS+Normal)

In a spin-imbalanced Fermi gas, phase separation into the BCS uniform superfluid and the polarized normal gas can occur.

P. F. Bedaque *et. al.*, PRL **91**, 247002 (2003)
⋮



Phase separation (BCS + Normal)

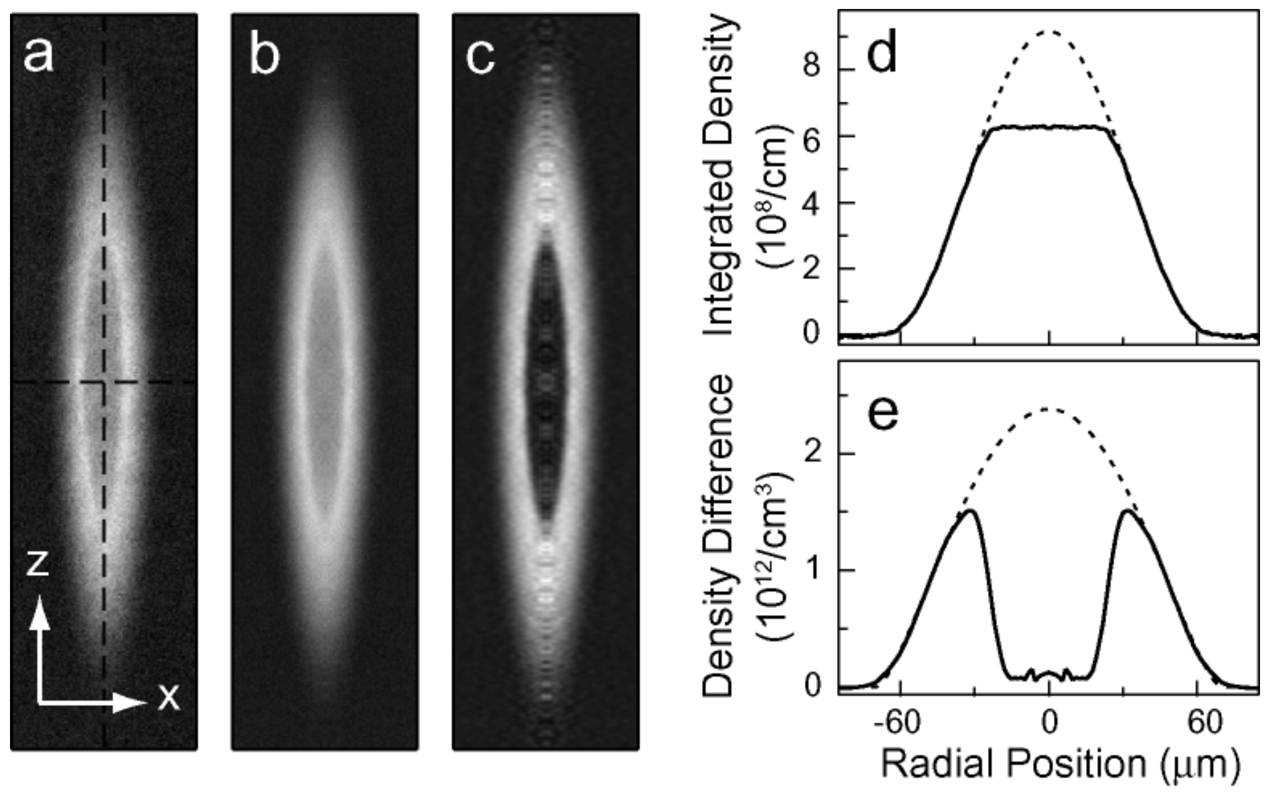


BCS-type Cooper pairs

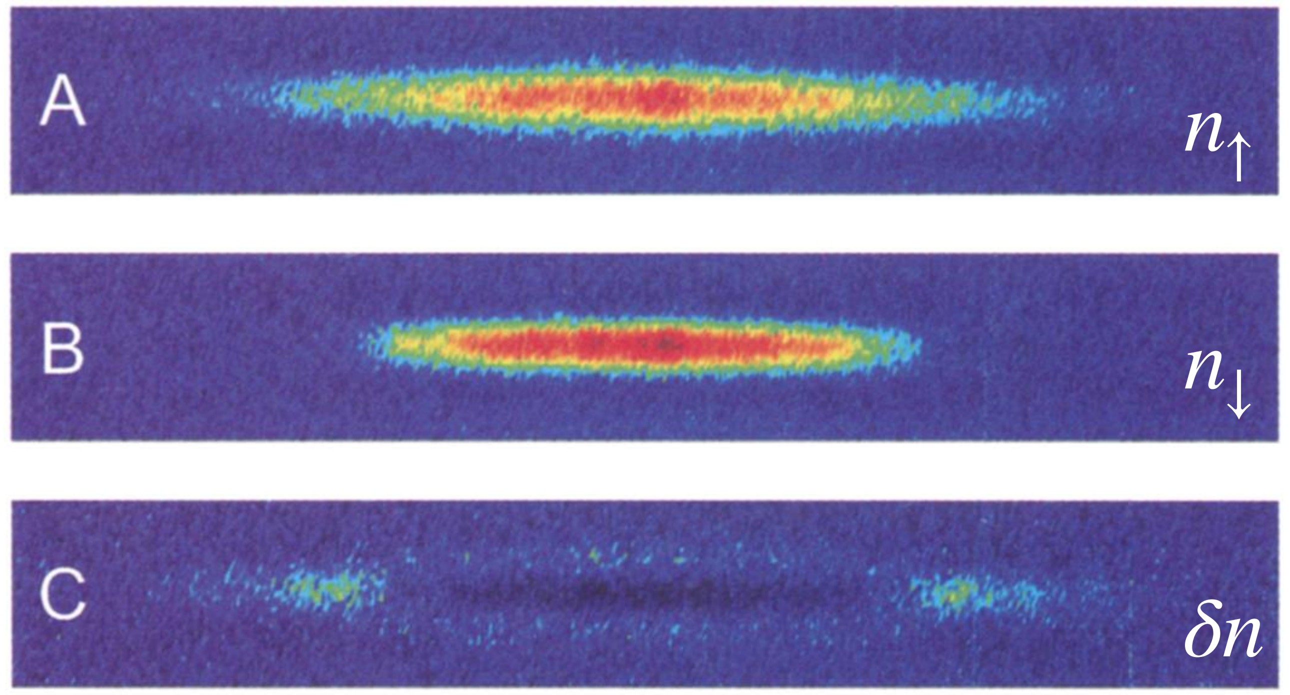
Polarized normal gas

Experiments

- MIT (2006)
  Y. Shin *et.al.*, PRL **97**, 030401 (2006)



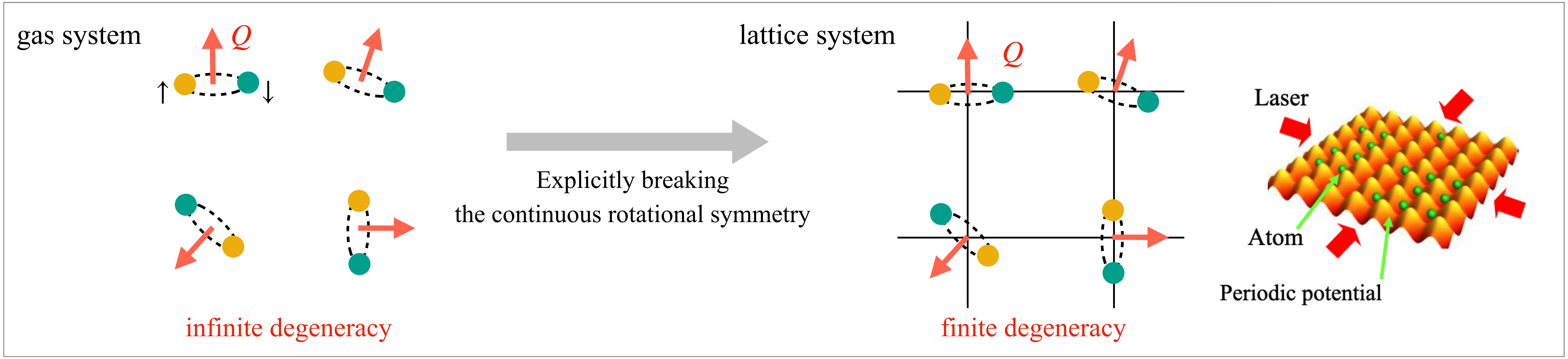
- Rice (2006)
  G. B. Partridge *et.al.*, Science **311**, 5760 (2006)



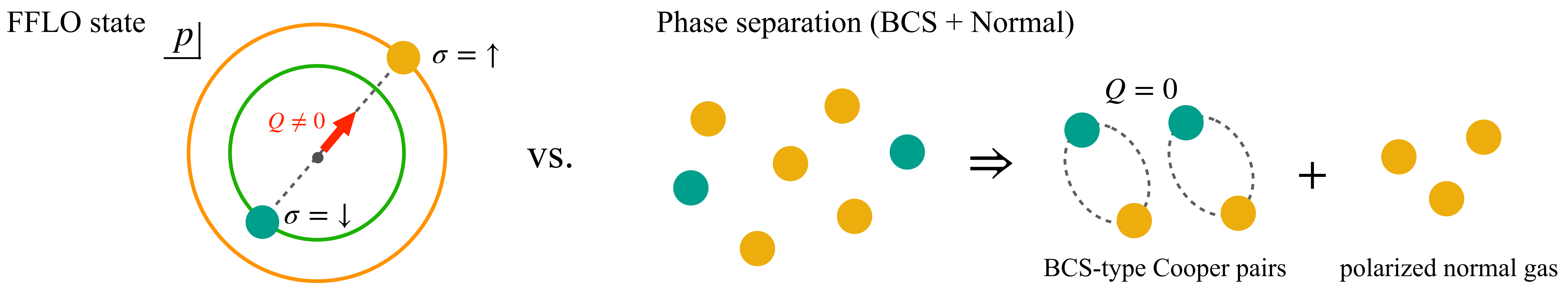
Thus, even if the destruction of the FFLO state by pairing fluctuations can be removed in an optical lattice, we still need to overcome the competition between the FFLO state and the unwanted phase separation.

This work

(1) Is it possible to stabilize the FFLO state against pairing fluctuations by introducing an optical lattice ?



(2) Can the FFLO state be stabilized against the phase separation (BCS + Normal) in an optical lattice ?



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Nozières and Schmitt-Rink (NSR) Theory

Hamiltonian (3D attractive Hubbard model)

$$H = -\sum_{i,j,\sigma} t_{i,j}\hat{c}^\dagger_{i,\sigma}\hat{c}_{j,\sigma} - U\sum_{i}\hat{n}_{i,\uparrow}\hat{n}_{i,\downarrow} - \sum_{i,\sigma}\mu_\sigma\hat{n}_{i,\sigma}$$

$$\Rightarrow H = \underbrace{\sum_{\boldsymbol{k}}\hat{\Psi}^\dagger_{\boldsymbol{k}}\left[\tilde{\xi}^{\rm s}_{\boldsymbol{k},\boldsymbol{Q}}\tau_3 + \tilde{\xi}^{\rm a}_{\boldsymbol{k},\boldsymbol{Q}} - \Delta\tau_1\right]\hat{\Psi}_{\boldsymbol{k}} + \sum_{\boldsymbol{k}}\left[\tilde{\xi}_{-\boldsymbol{k}+\boldsymbol{Q}/2,\downarrow} + \frac{\Sigma^{\rm H}_\uparrow\Sigma^{\rm H}_\downarrow}{U} + \frac{\Delta^2}{U}\right]}_{\text{Mean-field Hamiltonian } H_{\rm FL}} \underbrace{-U\sum_{\boldsymbol{q}}\hat{\rho}_+(\boldsymbol{q})\hat{\rho}_-(-\boldsymbol{q})}_{\text{Fluctuations in the Cooper channel } H_{\rm FL}}$$

- Superfluid order parameter

$$\Delta_i = U\langle\hat{c}_{i\downarrow}\hat{c}_{i\uparrow}\rangle = \Delta e^{i\boldsymbol{Q}\cdot\boldsymbol{R}_i}$$

Fulde-Ferrell-type order parameter

- Nambu field

τ_i : Pauli matrix in Nambu space

$$\hat{\Psi}_{\boldsymbol{k}} = \begin{pmatrix}\hat{c}_{\boldsymbol{k}+\boldsymbol{Q}/2,\uparrow} \\ \hat{c}^\dagger_{-\boldsymbol{k}+\boldsymbol{Q}/2,\downarrow}\end{pmatrix}$$

$$\hat{\rho}_\pm(\boldsymbol{q}) = [\hat{\rho}_1(\boldsymbol{q}) \pm i\hat{\rho}_2(\boldsymbol{q})]/2$$

$$\hat{\rho}_{\alpha=1,2}(\boldsymbol{q}) = \sum_{\boldsymbol{k}}\hat{\Psi}^\dagger_{\boldsymbol{k}+\boldsymbol{q}/2}\tau_\alpha\hat{\Psi}_{\boldsymbol{k}-\boldsymbol{q}/2}$$

- Hartree energy ・effective chemical potential

$$\Sigma^{\rm H}_\sigma = -U\langle\hat{c}^\dagger_{i,-\sigma}\hat{c}_{i,-\sigma}\rangle$$

$$\tilde{\mu}_\sigma = \mu_\sigma - \Sigma^{\rm H}_\sigma$$

$$\tilde{\xi}_{\boldsymbol{k},\sigma} = \varepsilon_{\boldsymbol{k}} - \tilde{\mu}_\sigma$$

$$\tilde{\xi}^{\rm s}_{\boldsymbol{k},\boldsymbol{Q}} = \frac{1}{2}\left[\tilde{\xi}_{\boldsymbol{k}+\boldsymbol{Q}/2,\uparrow} + \tilde{\xi}_{-\boldsymbol{k}+\boldsymbol{Q}/2,\downarrow}\right]$$

$$\tilde{\xi}^{\rm a}_{\boldsymbol{k},\boldsymbol{Q}} = \frac{1}{2}\left[\tilde{\xi}_{\boldsymbol{k}+\boldsymbol{Q}/2,\uparrow} - \tilde{\xi}_{-\boldsymbol{k}+\boldsymbol{Q}/2,\downarrow}\right]$$

- Single-particle energy

$$\varepsilon_{\boldsymbol{k}} = -2t\sum_{\alpha=x,y,z}\cos(k_\alpha)$$ (nearest-neighbor hopping)

Nozières and Schmitt-Rink (NSR) Theory

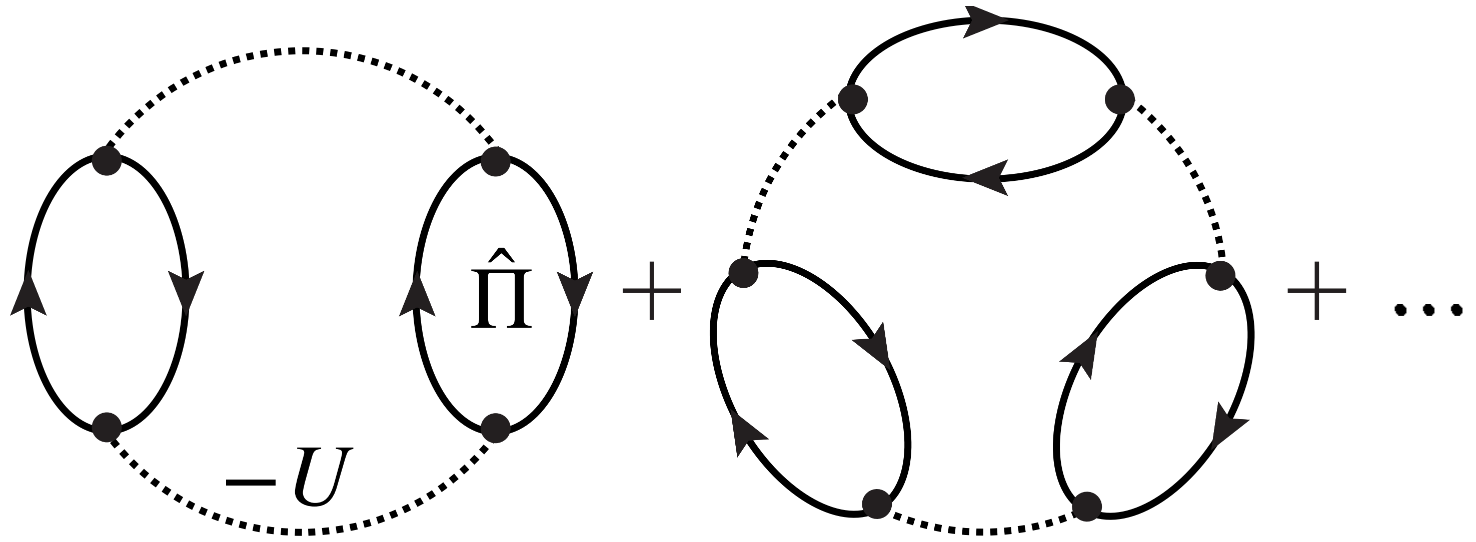
Hamiltonian (3D attractive Hubbard model)

$$H = \underbrace{\sum_{\boldsymbol{k}} \hat{\Psi}^\dagger_{\boldsymbol{k}} \left[\tilde{\xi}^{\mathrm{s}}_{\boldsymbol{k},\boldsymbol{Q}}\tau_3 + \tilde{\xi}^{\mathrm{a}}_{\boldsymbol{k},\boldsymbol{Q}} - \Delta\tau_1\right] \hat{\Psi}_{\boldsymbol{k}} + \sum_{\boldsymbol{k}} \left[\tilde{\xi}_{-\boldsymbol{k}+\boldsymbol{Q}/2,\downarrow} + \frac{\Sigma^{\mathrm{H}}_{\uparrow}\Sigma^{\mathrm{H}}_{\downarrow}}{U} + \frac{\Delta^2}{U}\right]}_{\text{Mean-field hamiltonian } H_{\mathrm{MF}}} \underbrace{-U\sum_{\boldsymbol{q}} \hat{\rho}_+(\boldsymbol{q})\hat{\rho}_-(-\boldsymbol{q})}_{\text{Fluctuations in the Cooper channel } H_{\mathrm{FL}}}$$

Thermodynamic potential $\quad \Omega = \Omega_{\mathrm{MF}} + \Omega_{\mathrm{FL}}$

$$\Omega_{\mathrm{MF}} = -T\ln\left[\mathrm{Tr}e^{-\beta H_{\mathrm{MF}}}\right]$$

$$\Omega_{\mathrm{FL}} =$$

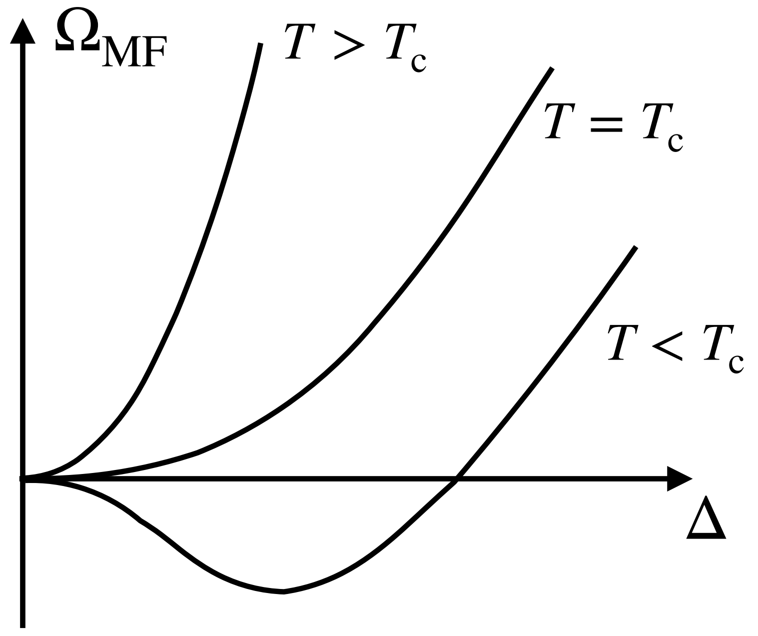


Number equation

$$N = -\left(\frac{\partial}{\partial\mu}\left[\Omega_{\mathrm{MF}} + \Omega_{\mathrm{FL}}\right]\right)_T = N_0 + N_{\mathrm{FL}}$$

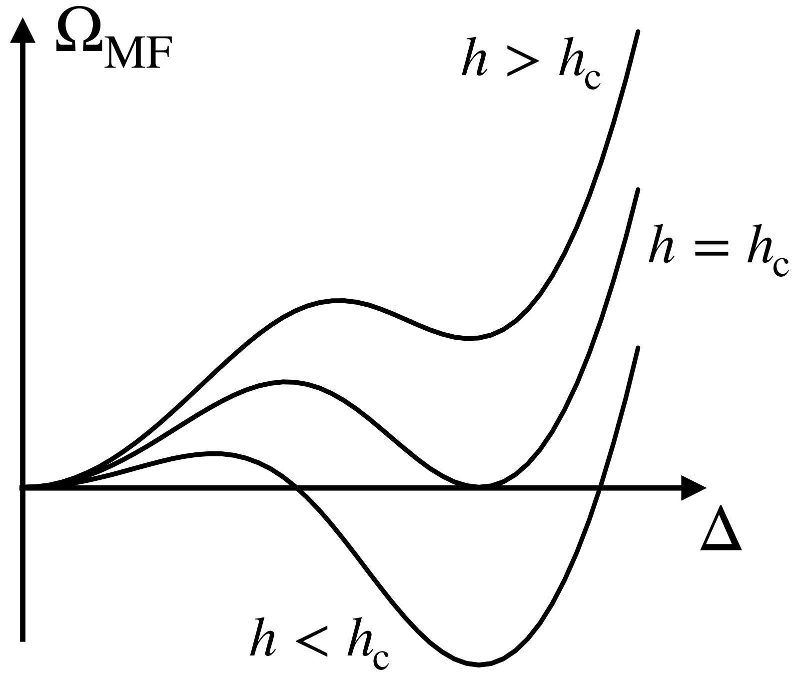
Fluctuation correction

Phase transition



2nd order phase transition (Thouless criterion)

$$\min_{\boldsymbol{Q}} \left.\frac{\partial^2\Omega_{\mathrm{MF}}(\boldsymbol{Q})}{\partial|\Delta|^2}\right|_{\Delta=0} = 0 \quad \begin{cases} \boldsymbol{Q} = 0 & \text{BCS} \\ \boldsymbol{Q} \neq 0 & \text{FFLO} \end{cases}$$



1st order phase transition

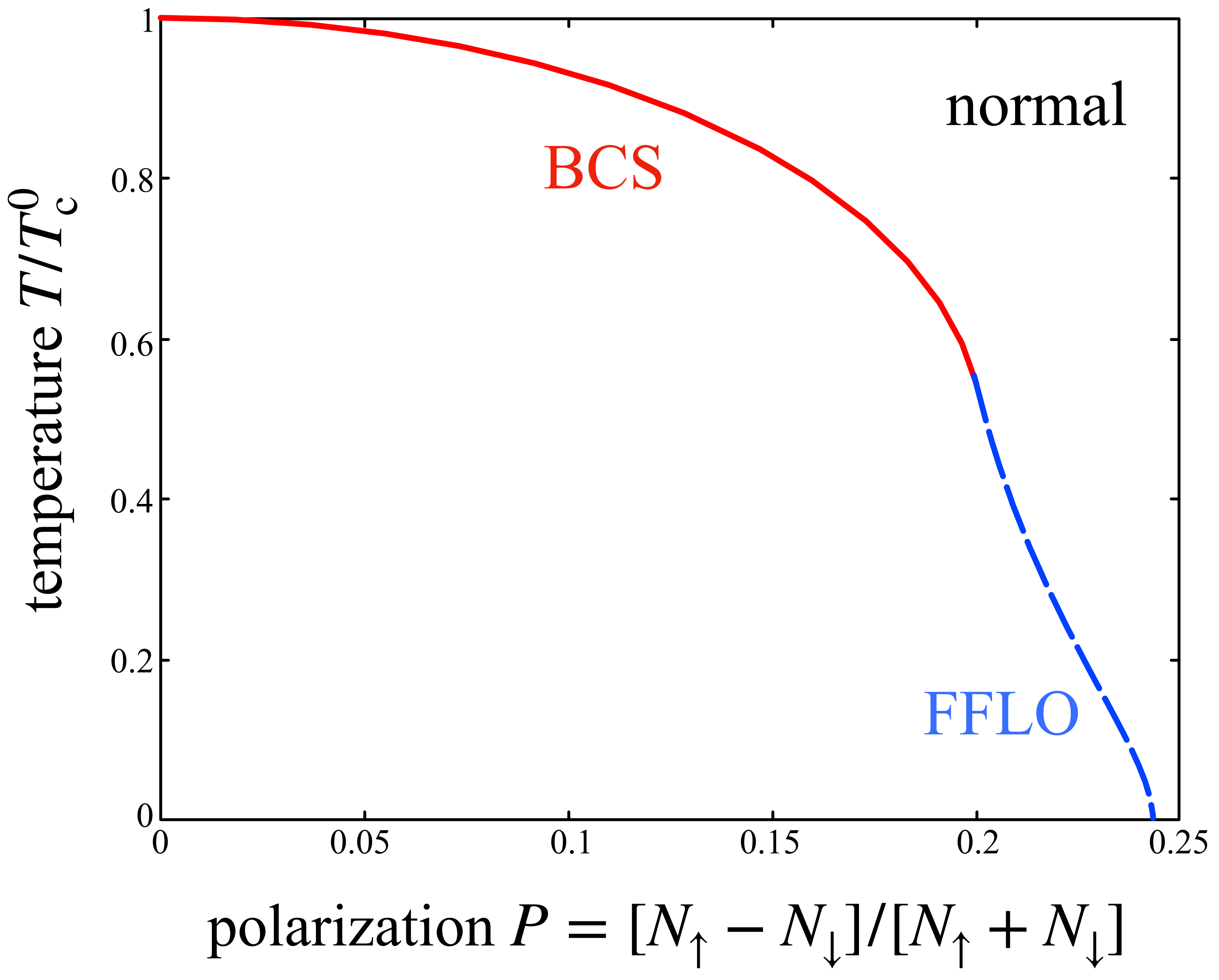
$$\Omega_{\mathrm{MF}}(0) = \Omega_{\mathrm{MF}}(\Delta_0)$$

Contents

FFLO State in a Gas System (without an optical lattice)

Mean-field approximation

(neglecting pairing fluctuations)



$(a_s p_F)^{-1} = -1$

(weak-coupling BCS regime)

FFLO State in a Gas System (without an optical lattice)

Mean-field approximation

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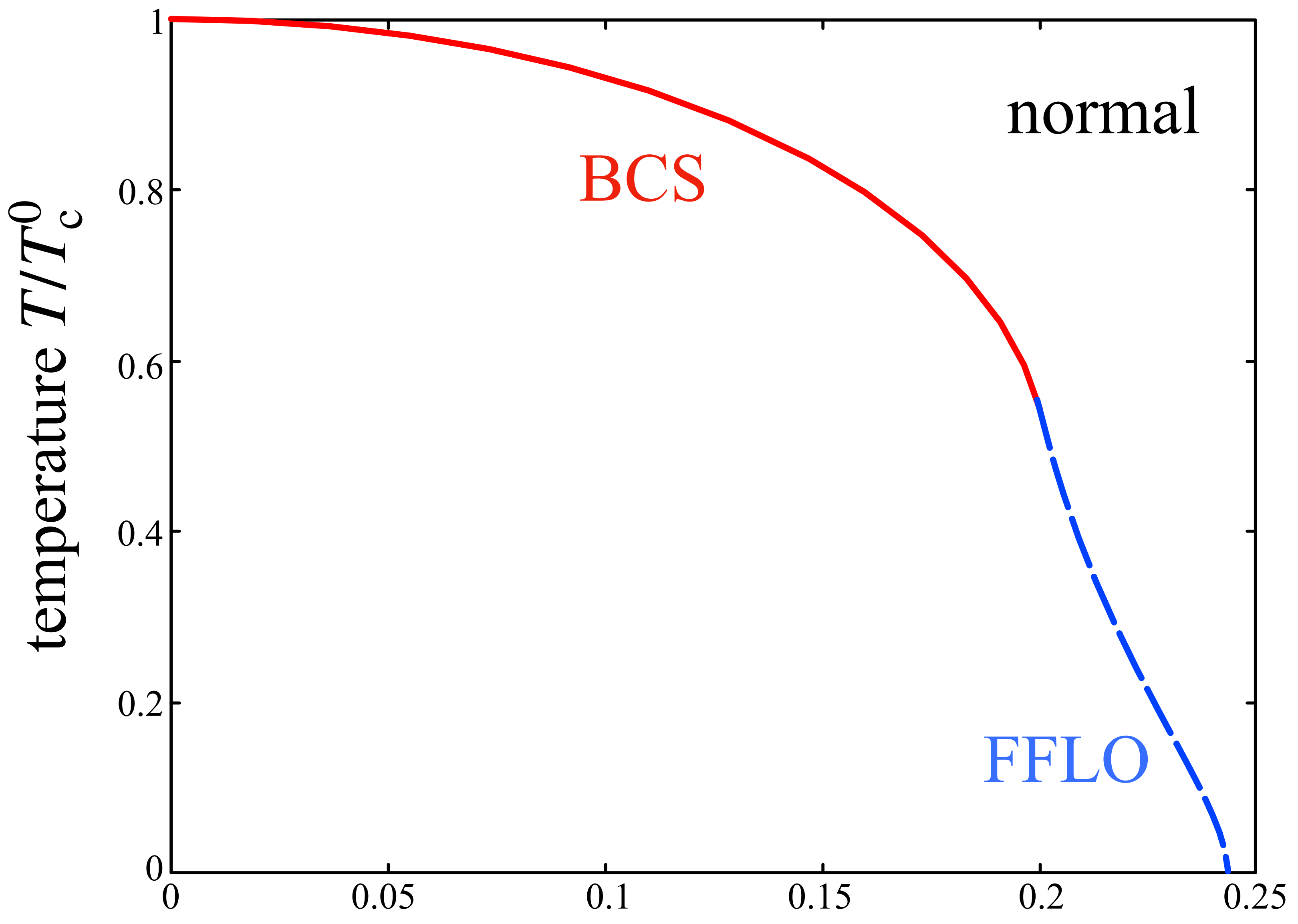

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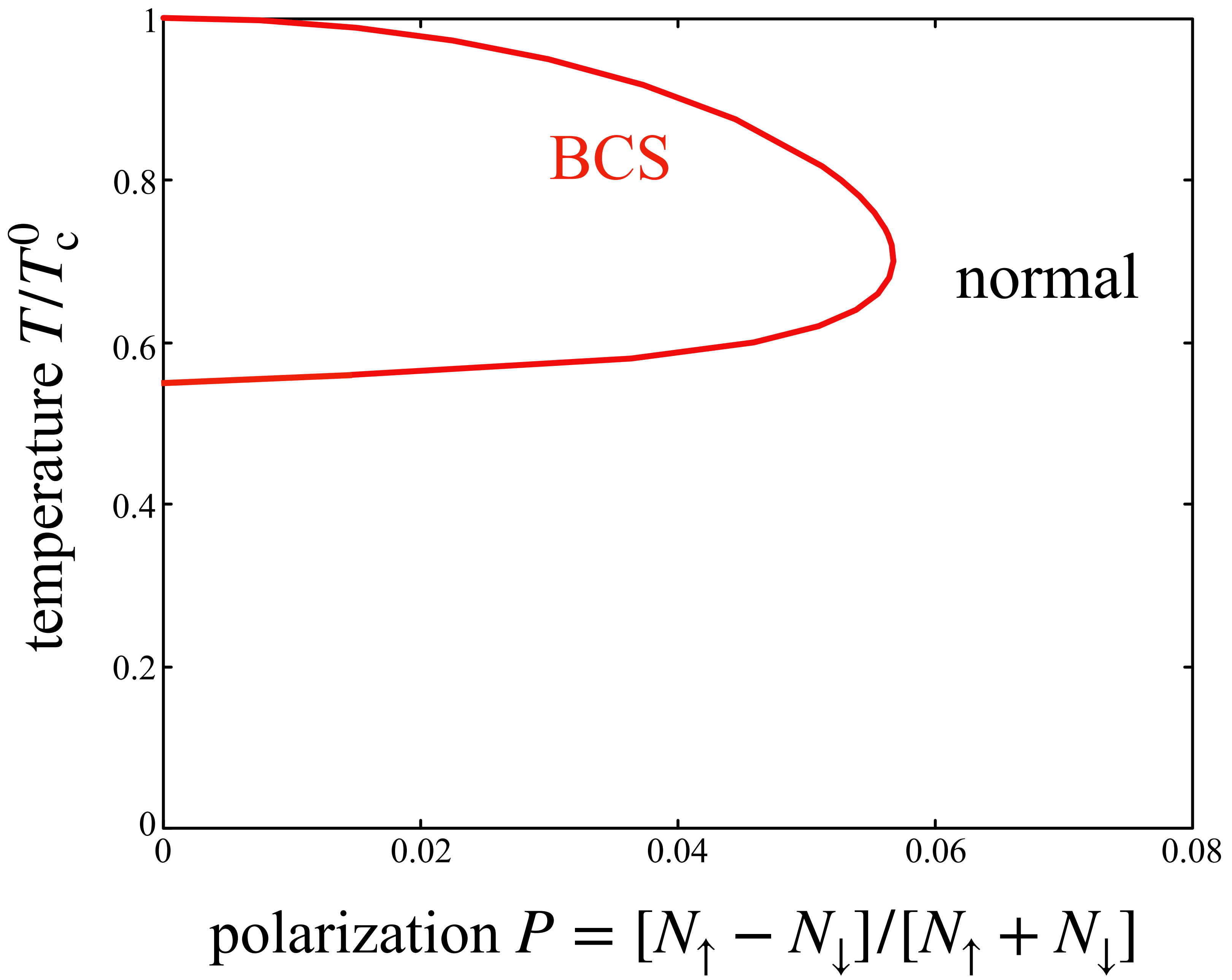
(weak-coupling BCS regime)

NSR theory

(including pairing fluctuations)



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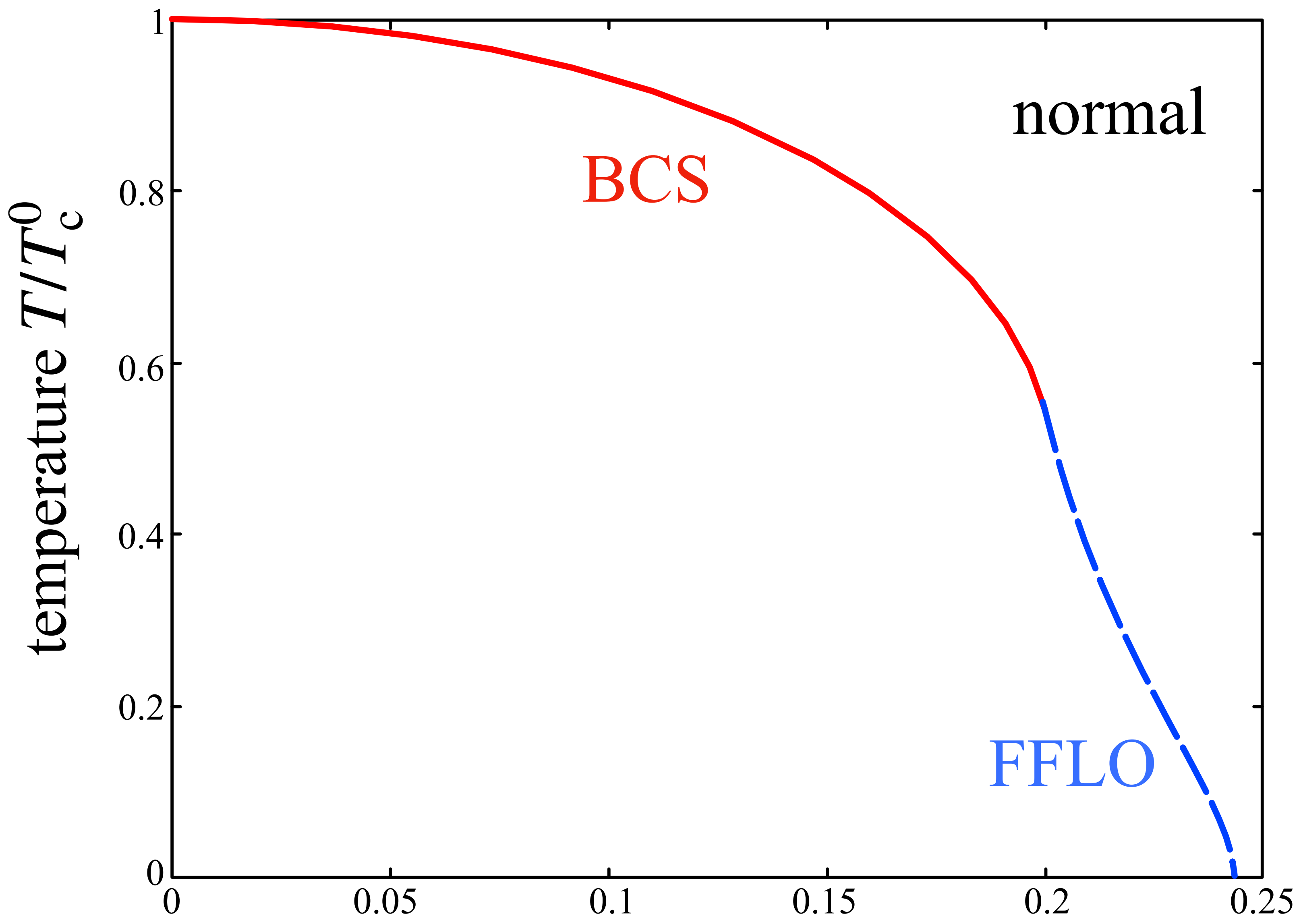

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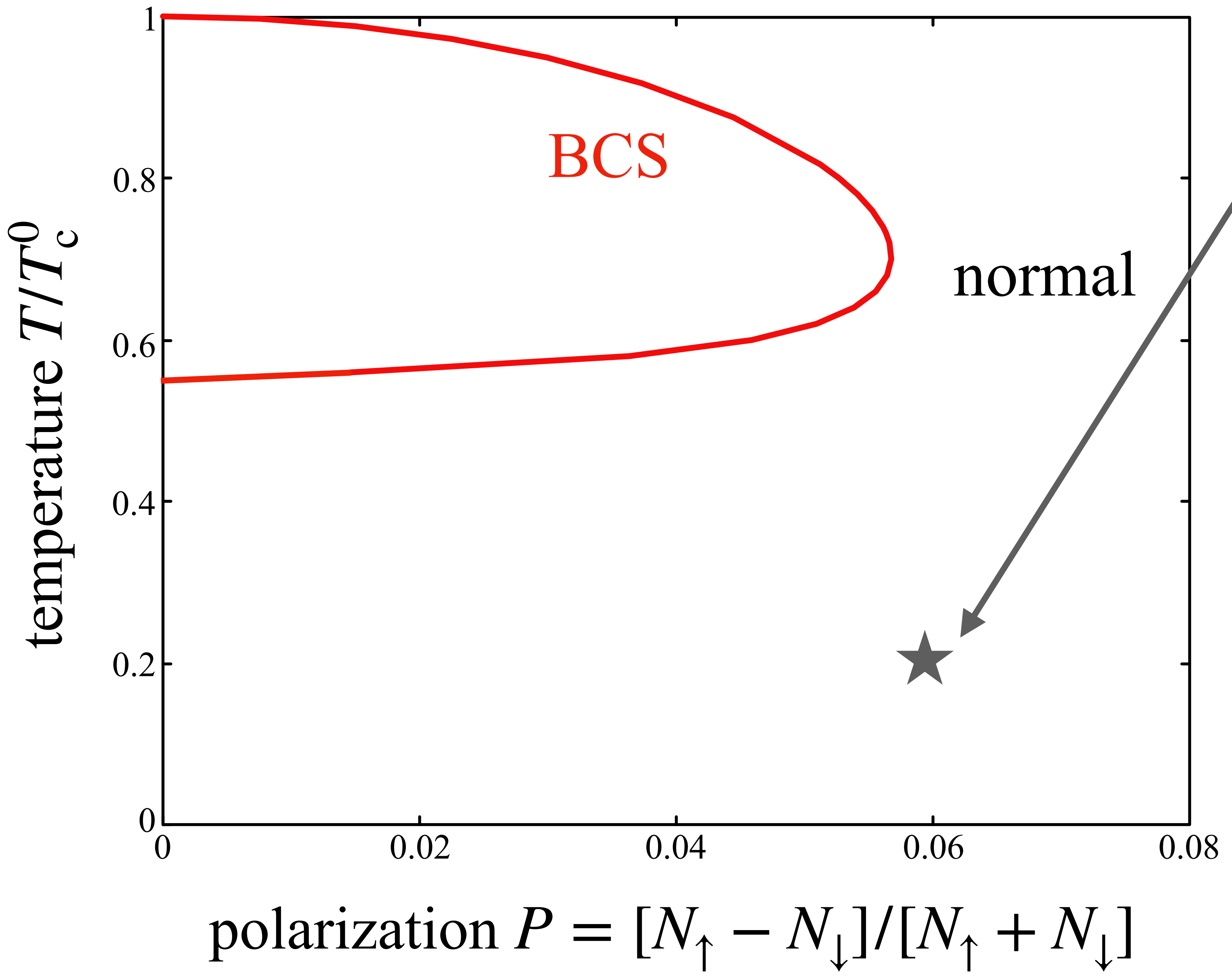
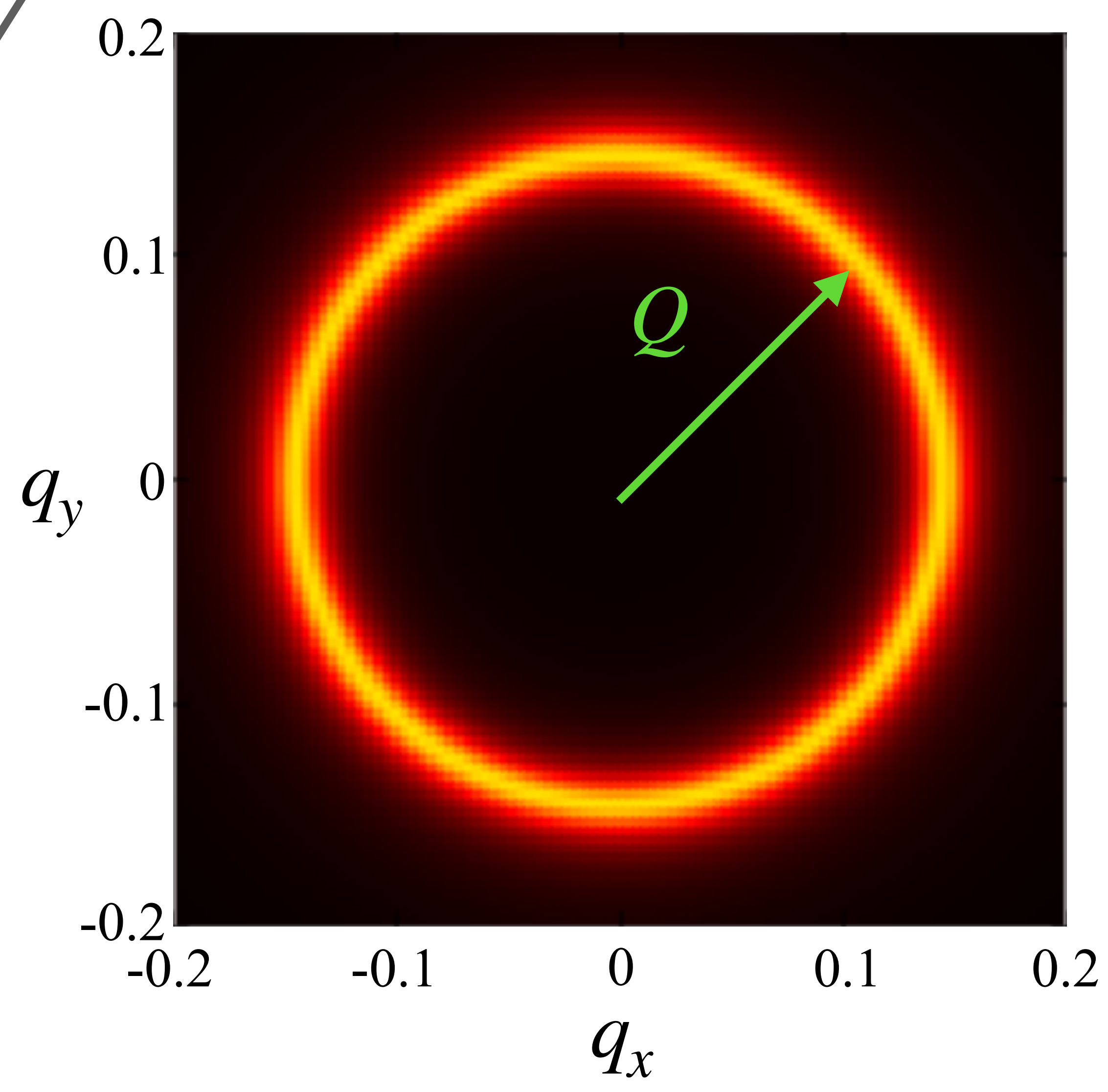
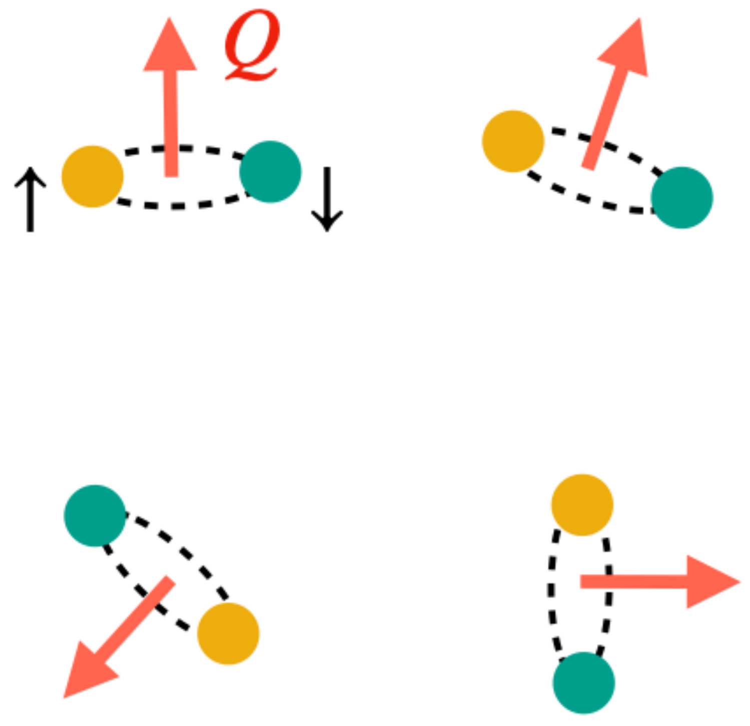


Intensity of pairing fluctuations





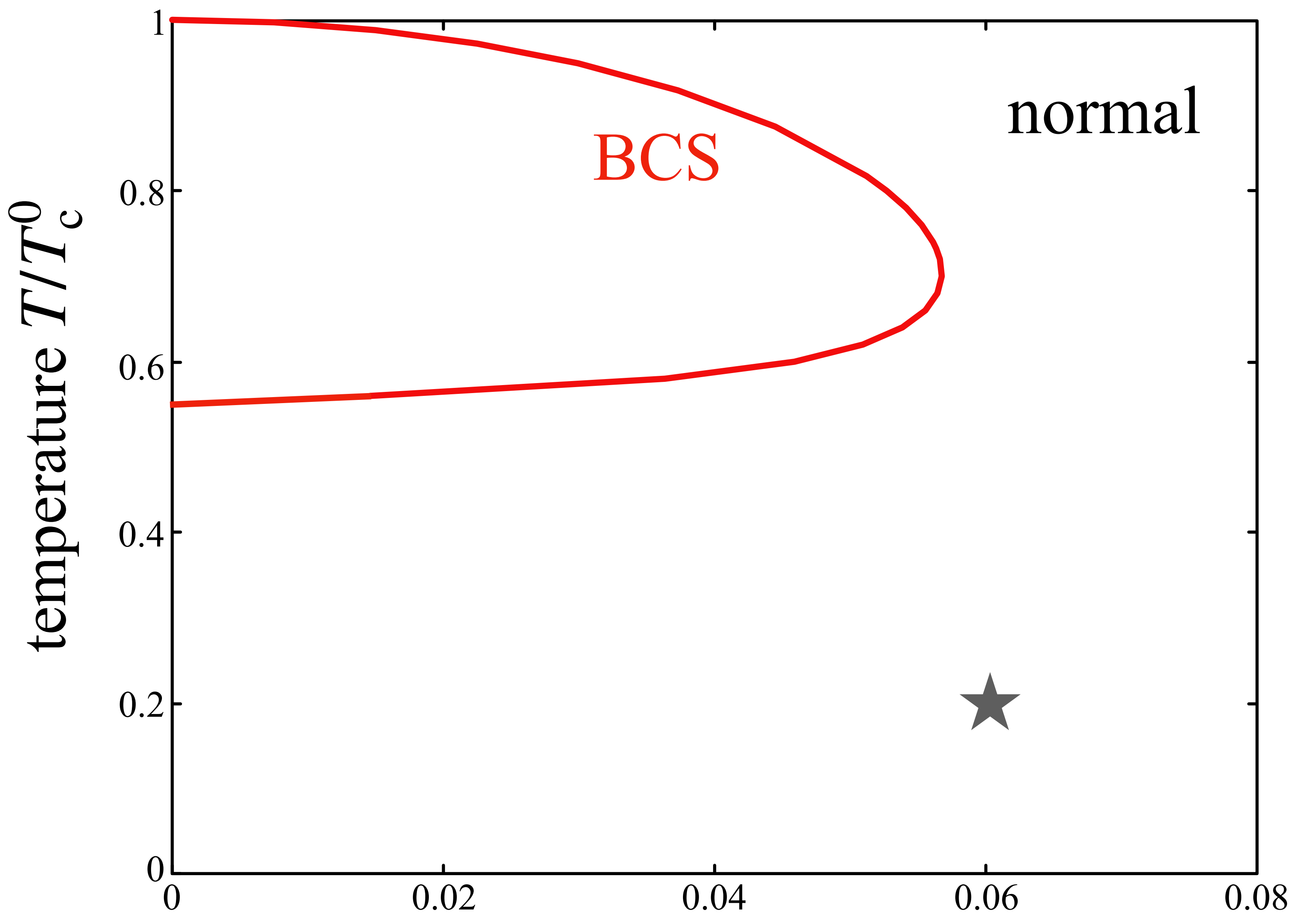
infinite degeneracy

FFLO State in a Lattice System (Stabilization of the FFLO state)

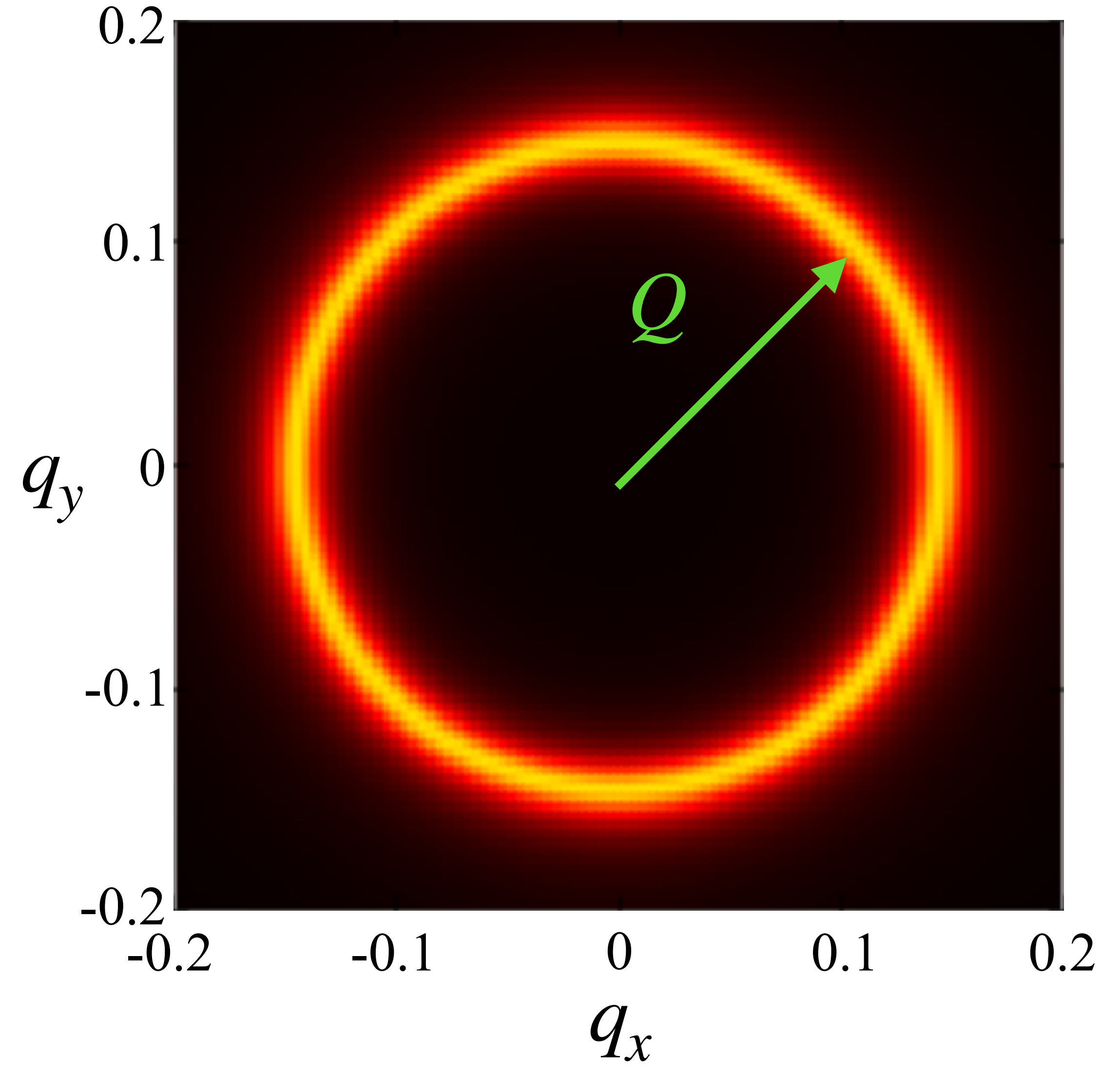
Gas system

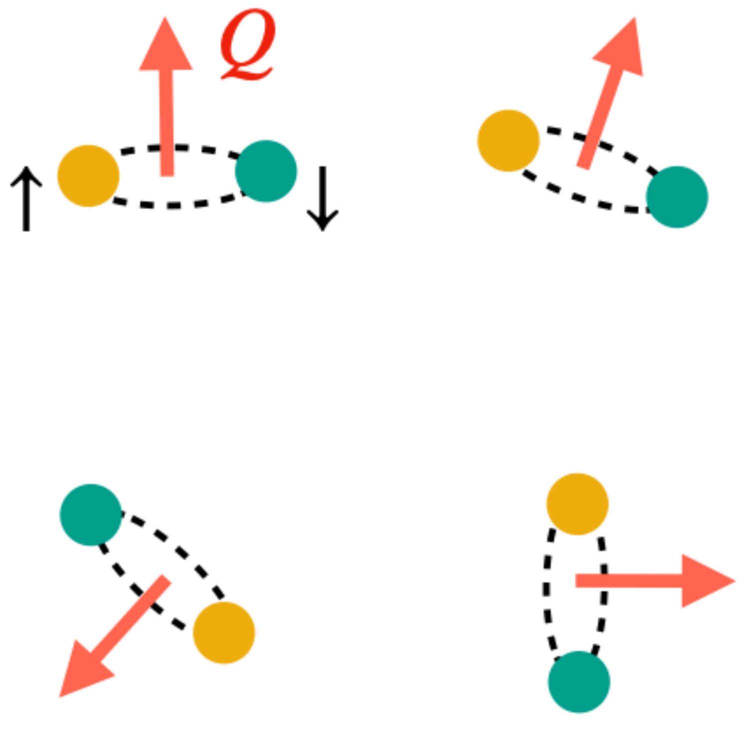
(continuous
rotational symmetry)

$(a_s p_F)^{-1} = -1$ (BCS regime)



Intensity of pairing fluctuations





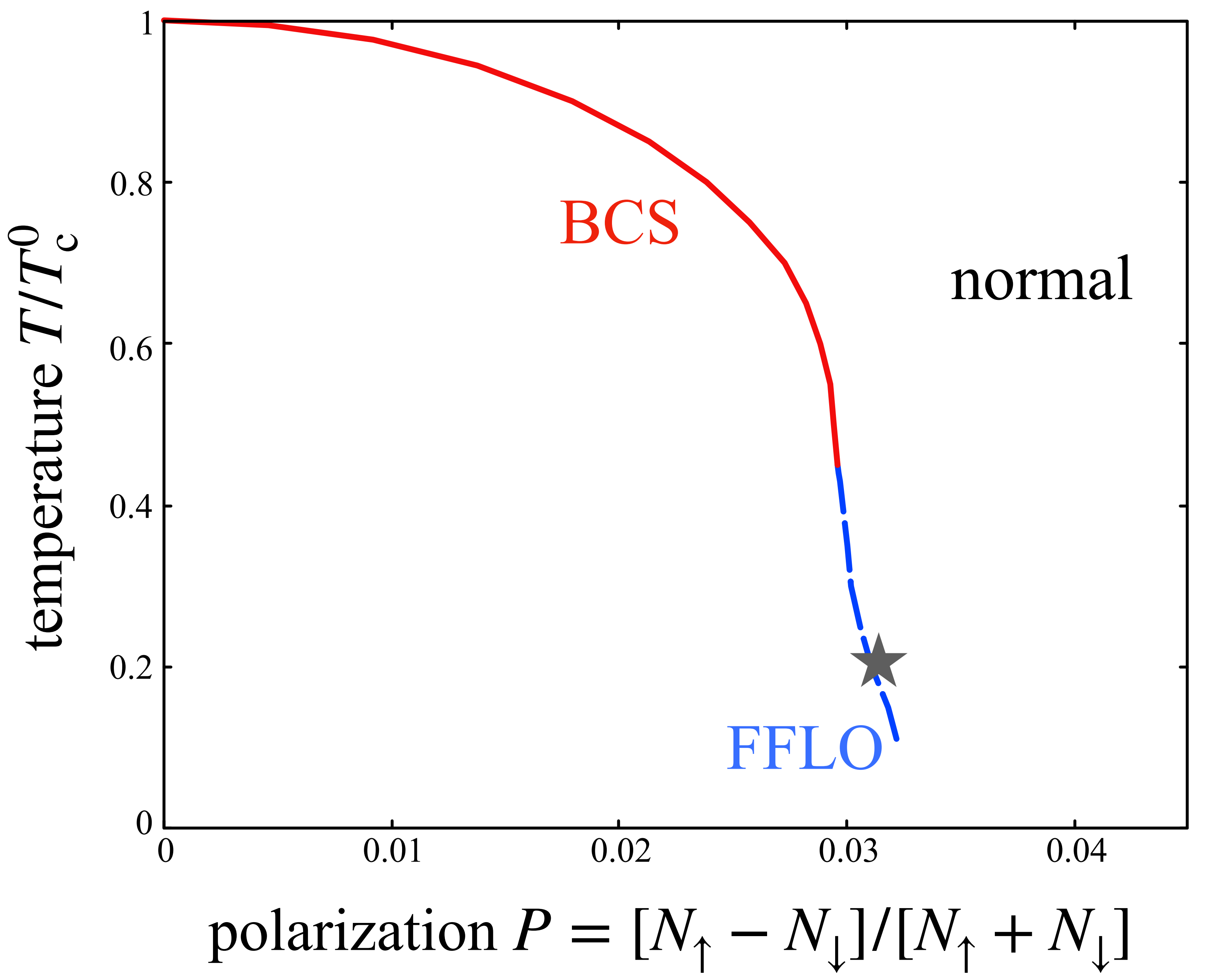
infinite degeneracy

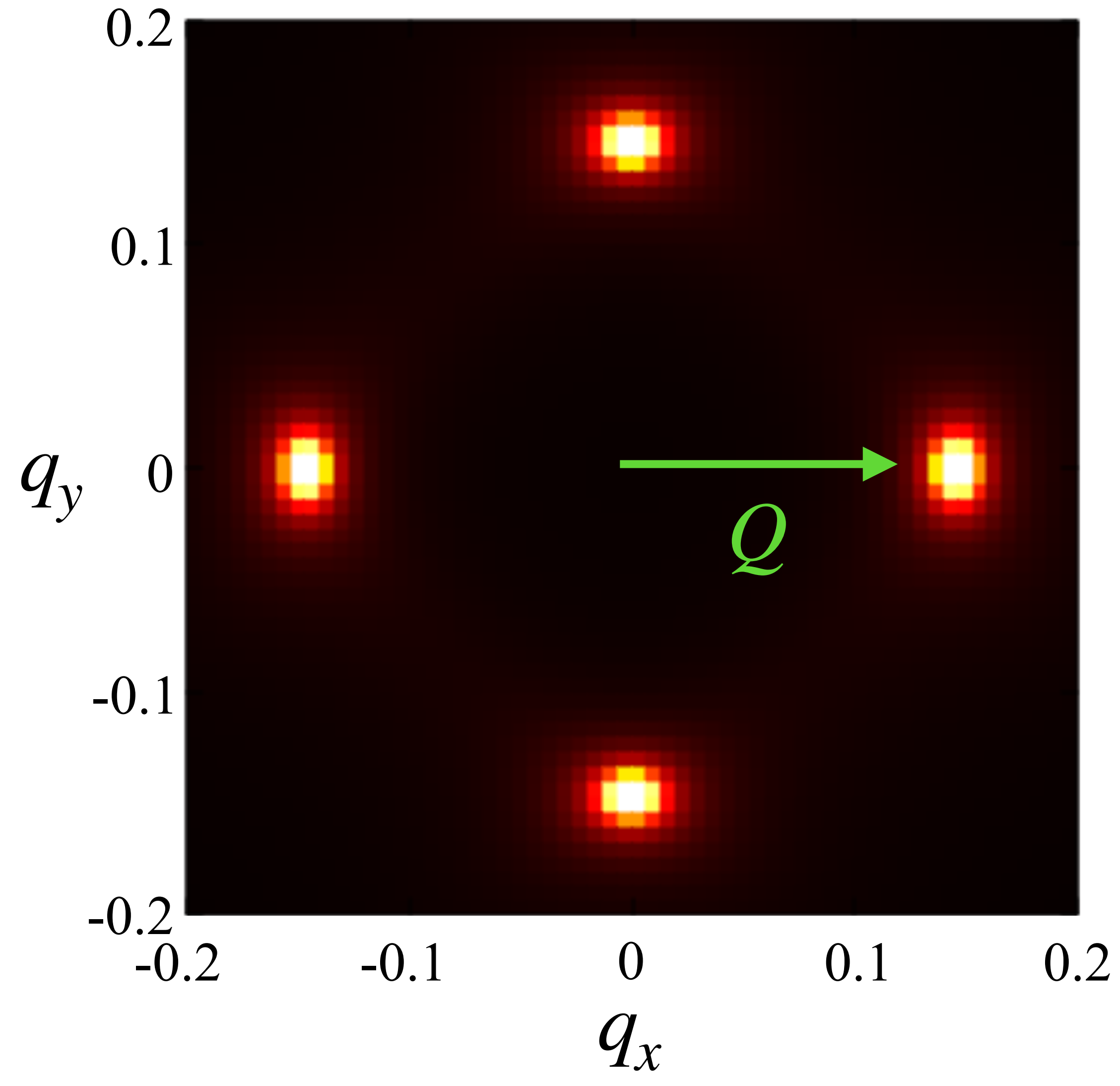
Optical lattice system

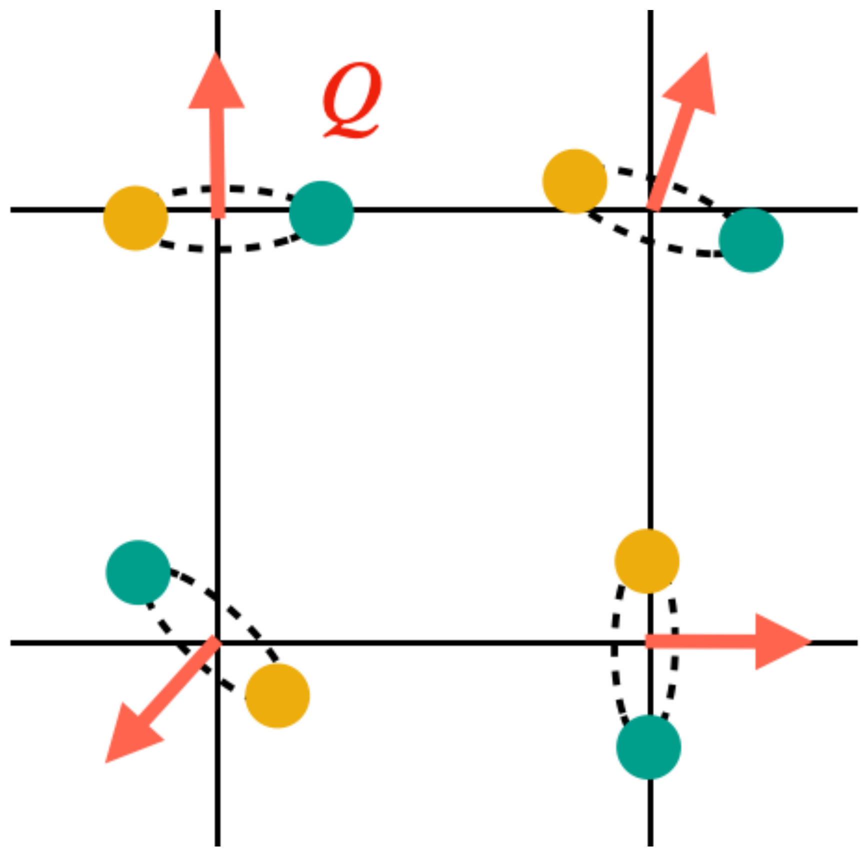
(discrete fourfold
rotational symmetry)

$U/6t = 0.4$ (BCS regime)

$n = n_\uparrow + n_\downarrow = 0.3$



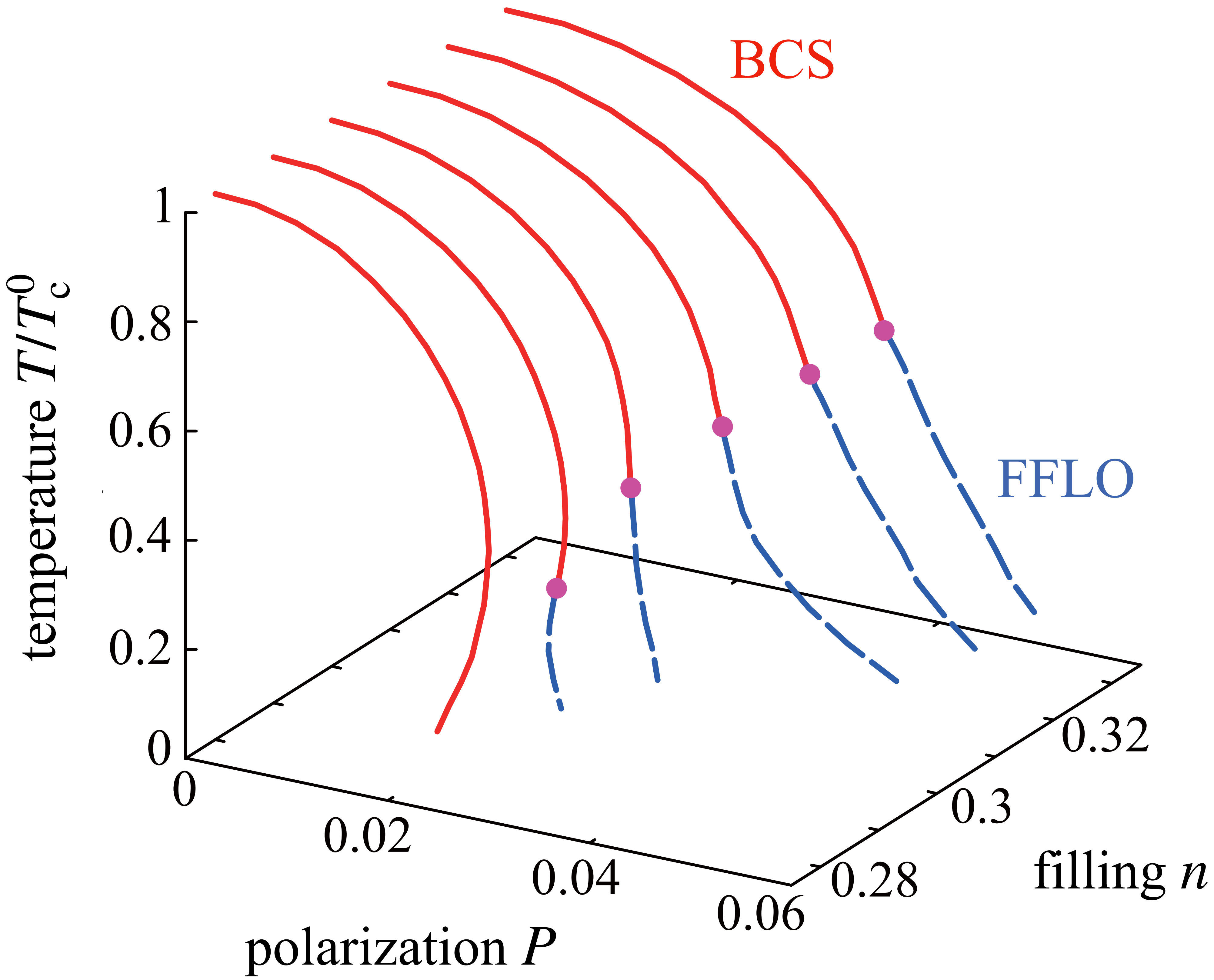




finite degeneracy

FFLO State in a Lattice System (Fermi Surface shape)

Filling n dependence



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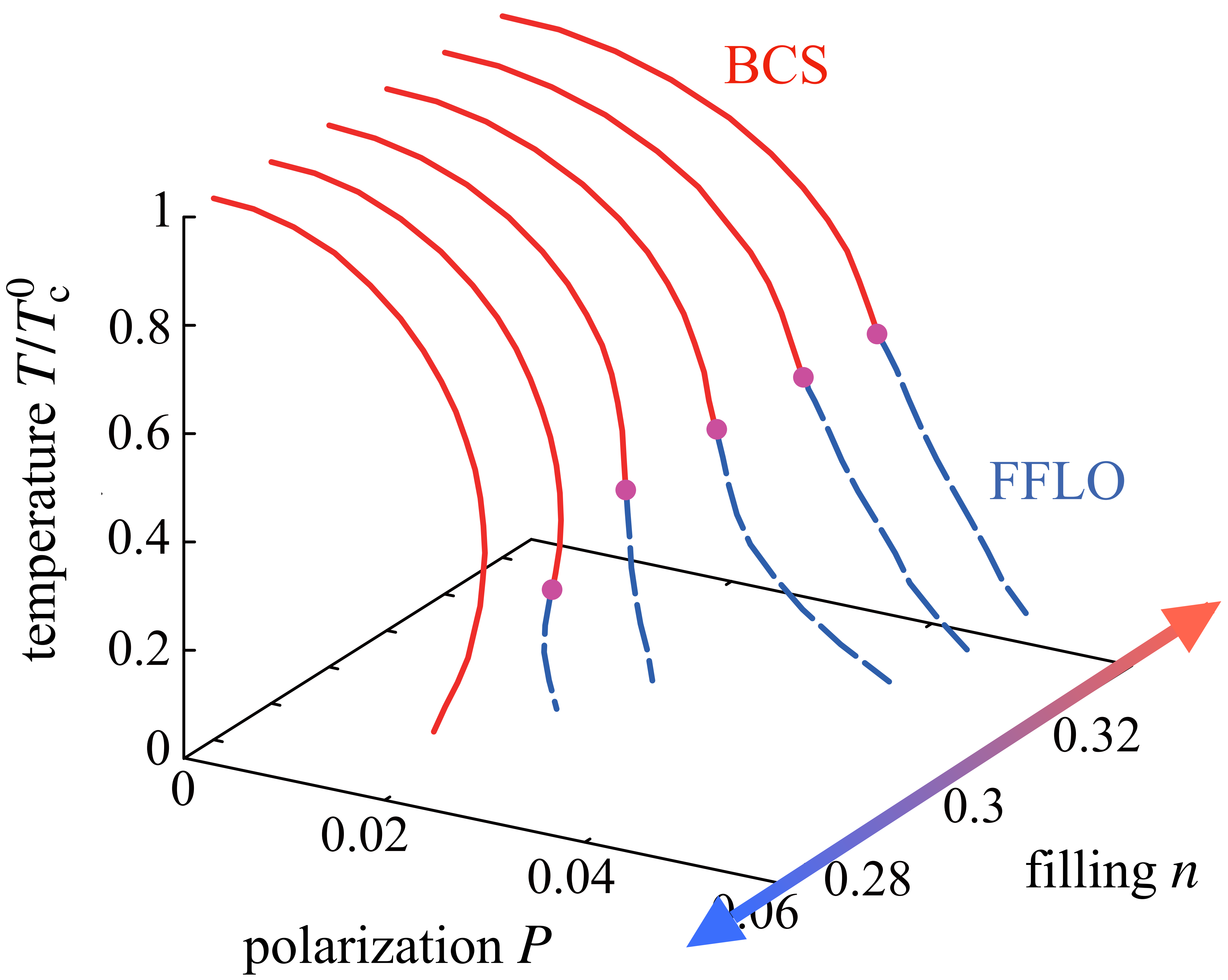
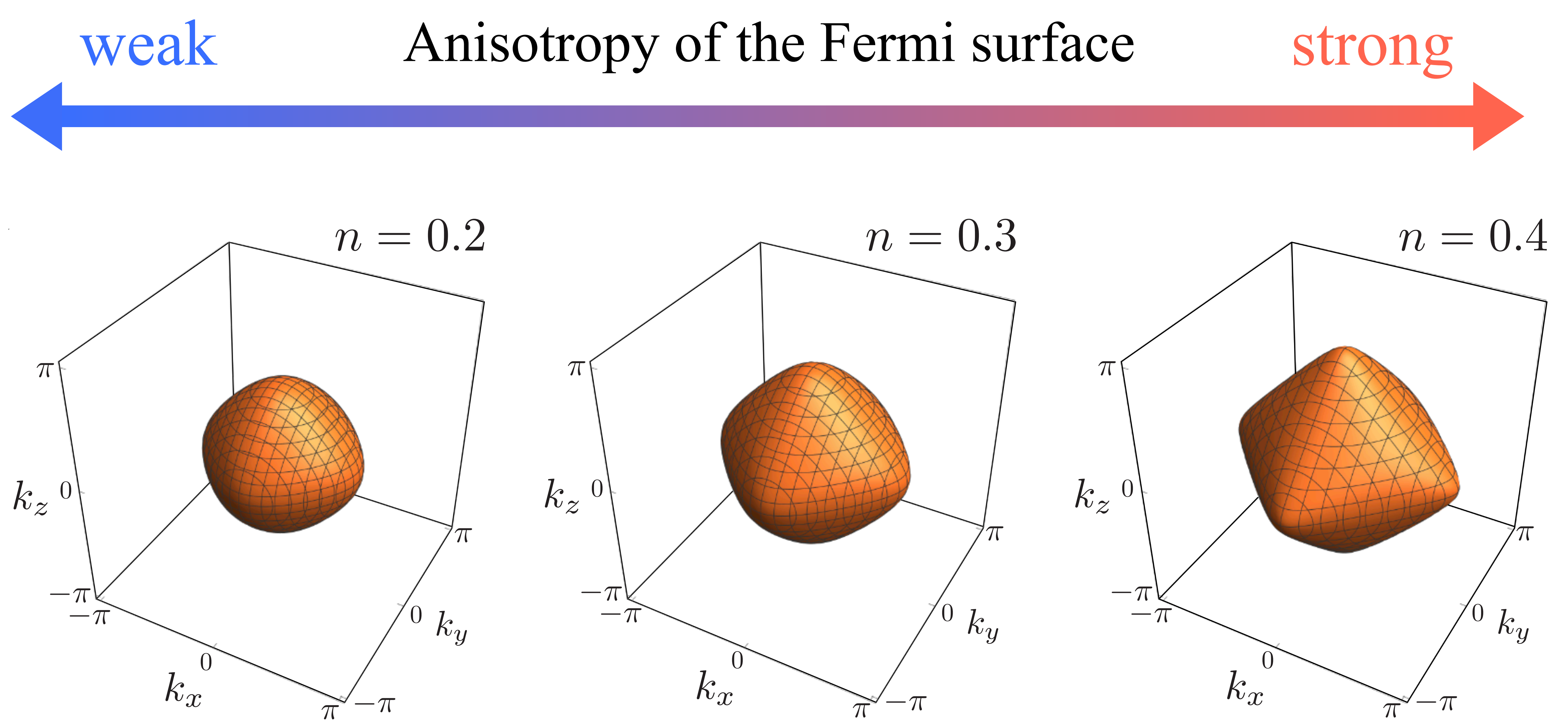
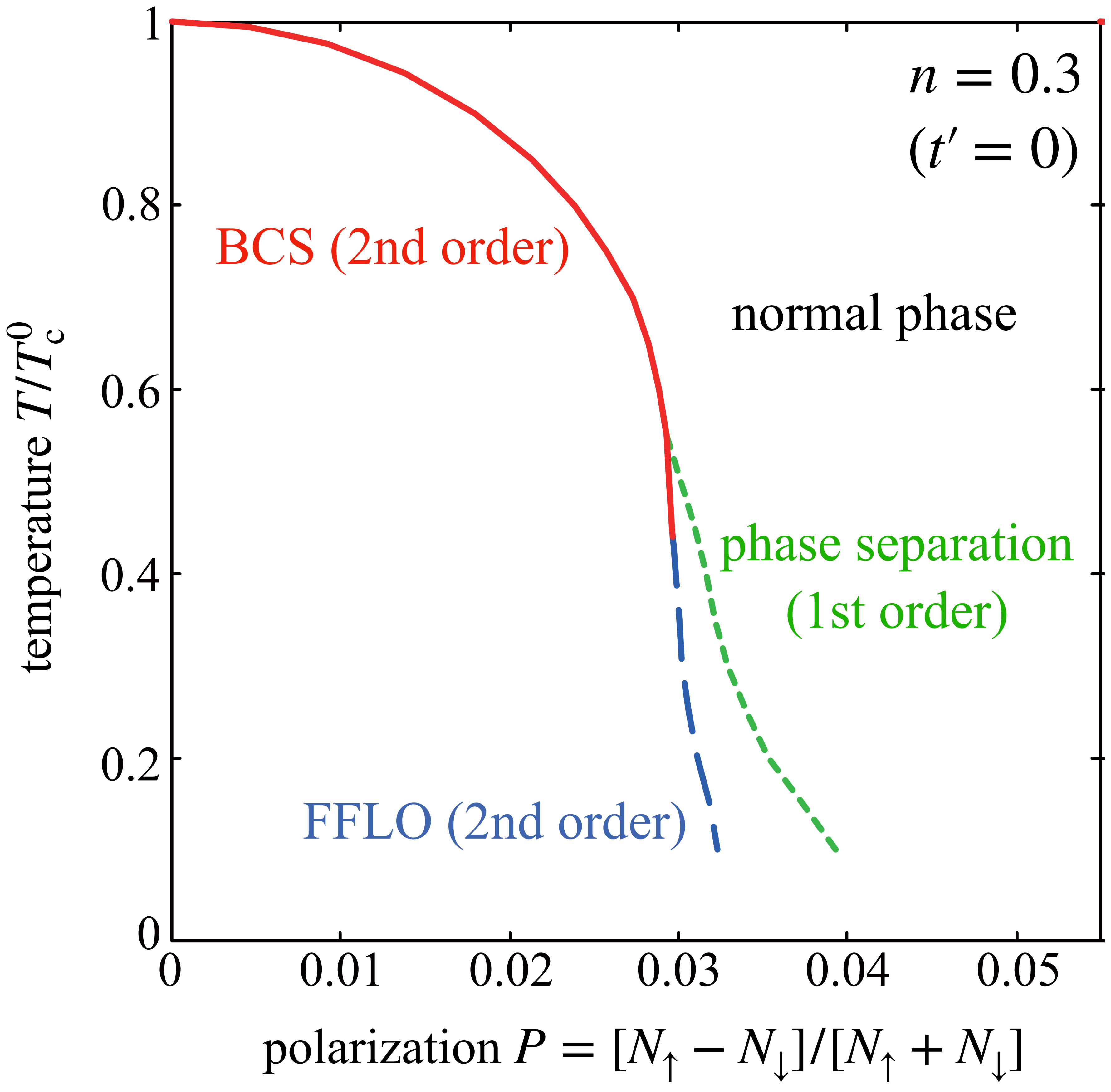
Filling n dependence





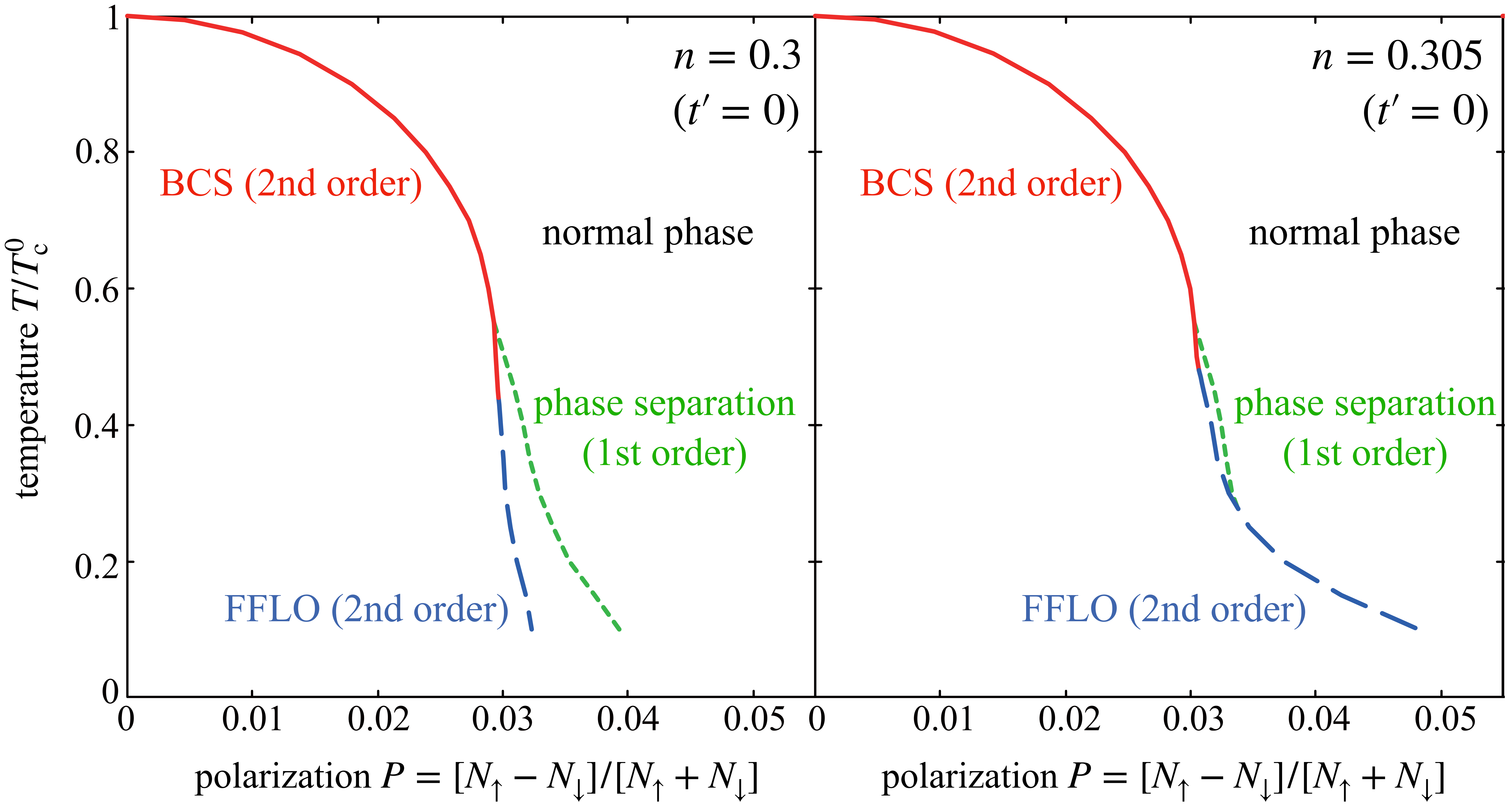
Competition between the FFLO and the Phase Separation (BCS+Normal)



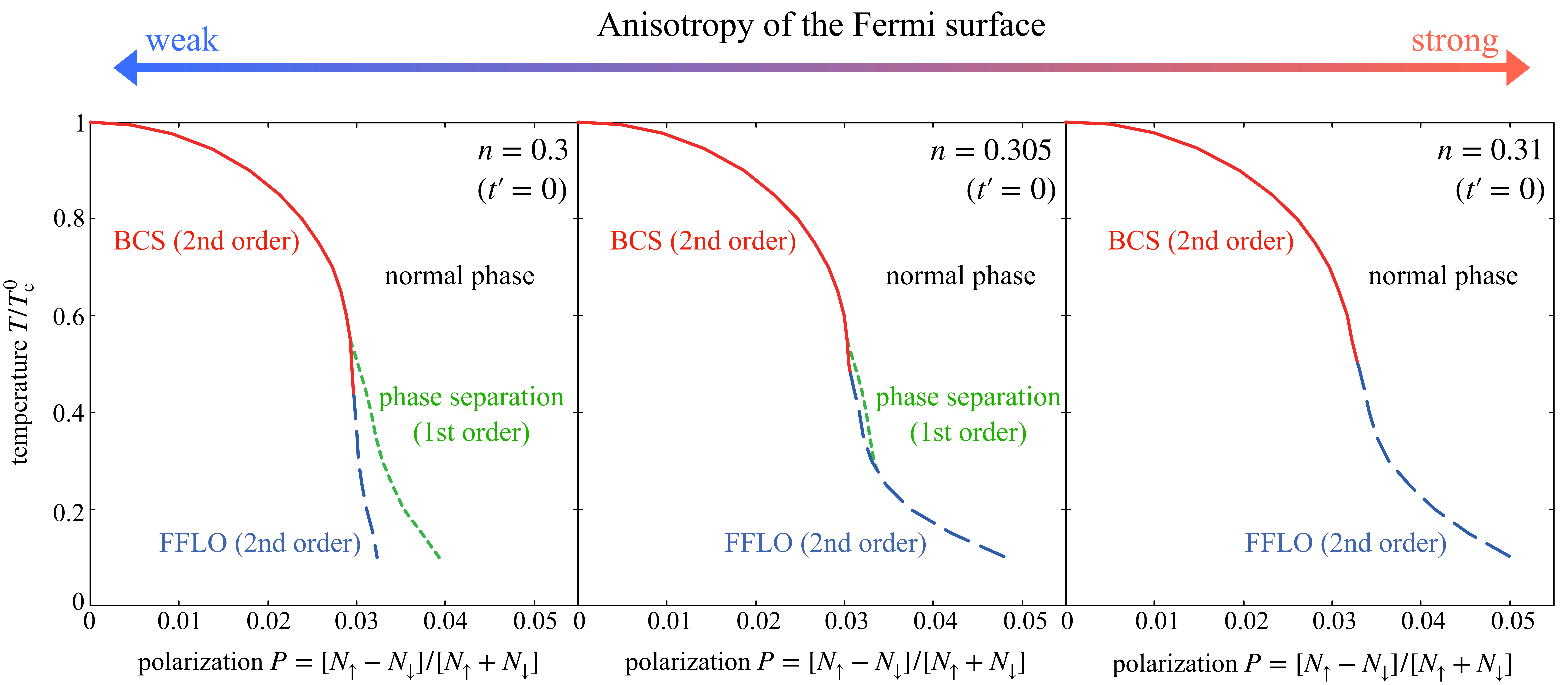
Competition between the FFLO and the Phase Separation (BCS+Normal)

Anisotropy of the Fermi surface

weak ⟷ strong



Competition between the FFLO and the Phase Separation (BCS+Normal)



Contents

Summary

Taira Kawamura and Yoji Ohashi, Phys. Rev. A **106**, 033320 (2022).

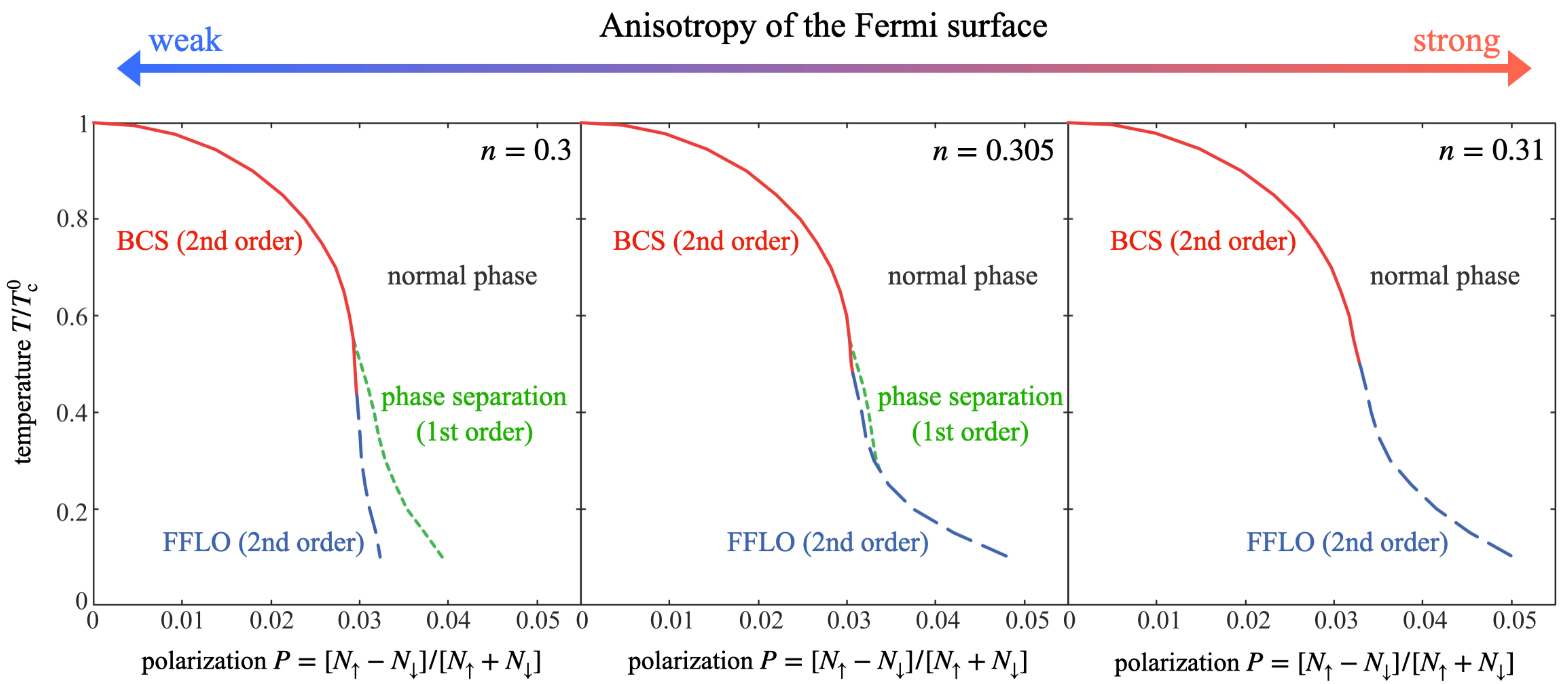
- We theoretically explored a promising route to achieve the Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) state in a spin-imbalanced ultracold Fermi gas.

- We have found that the anisotropy of the Fermi surface is crucial to the realization of the FFLO state. As the Fermi surface deviates from spherical with increasing the filling fraction, the FFLO pairing fluctuations become weak, which promotes the stabilization of the FFLO-type long-range order.

Future work

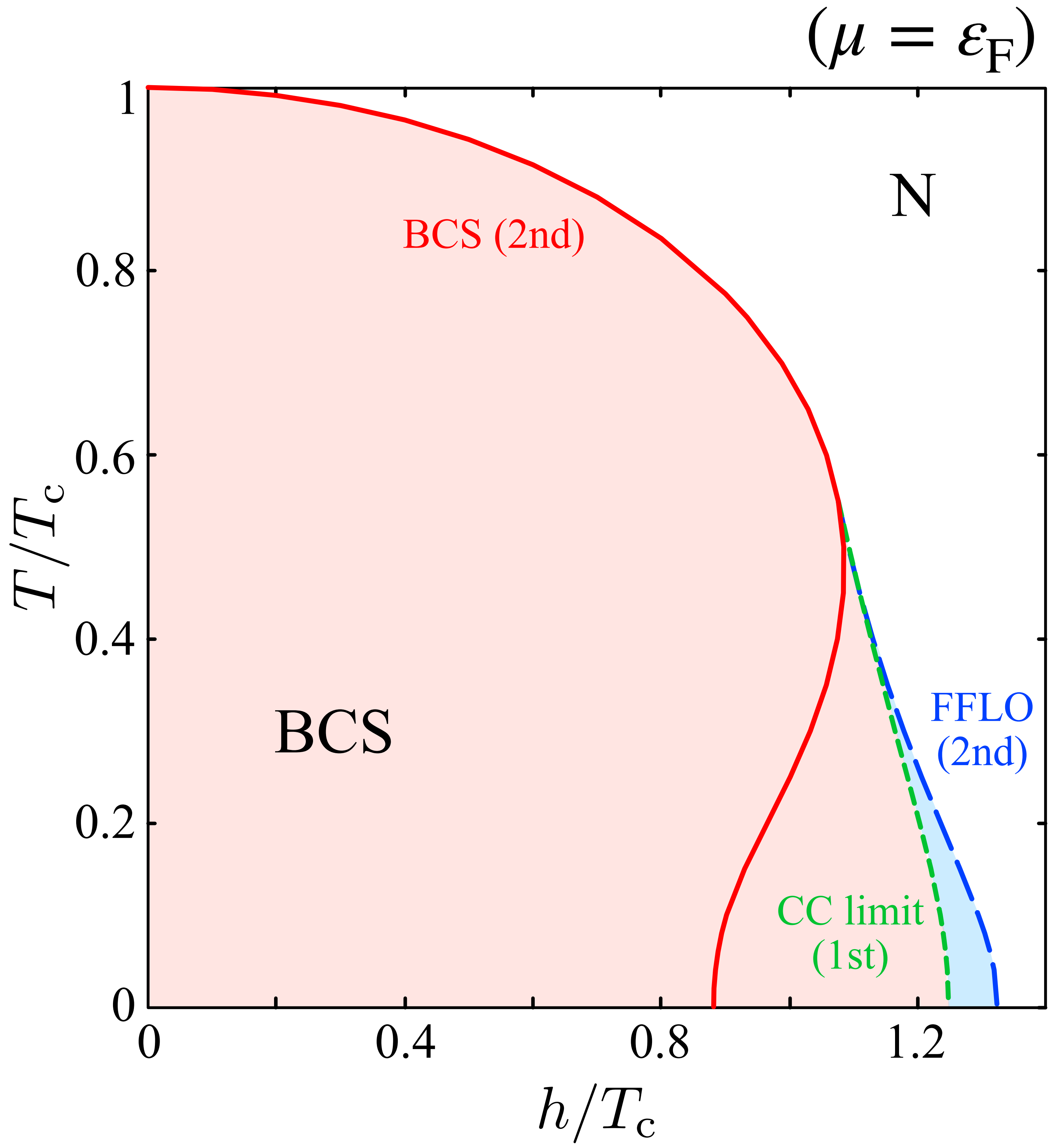
- Fluctuations in other channels
- Crystalline (multiple-Q) FFLO state

$$\Delta(\boldsymbol{r}) = \sum_l \Delta_l e^{i\boldsymbol{Q}_l\cdot\boldsymbol{r}}$$



Phase diagram

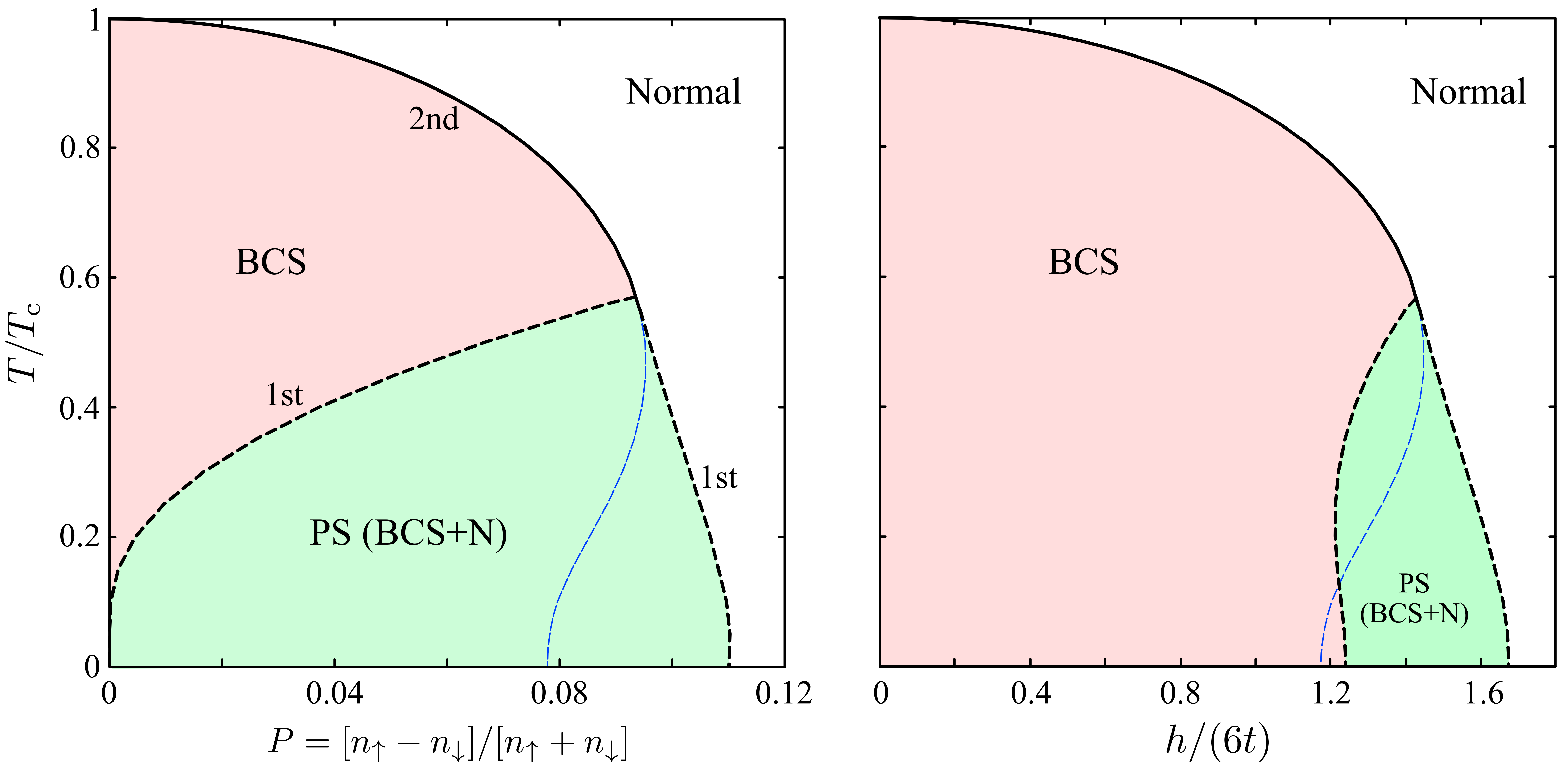
Chemical potential is fixed



Total particle number is fixed

$n = n_\uparrow + n_\downarrow = 0.5$

$U/(6t) = 0.4$



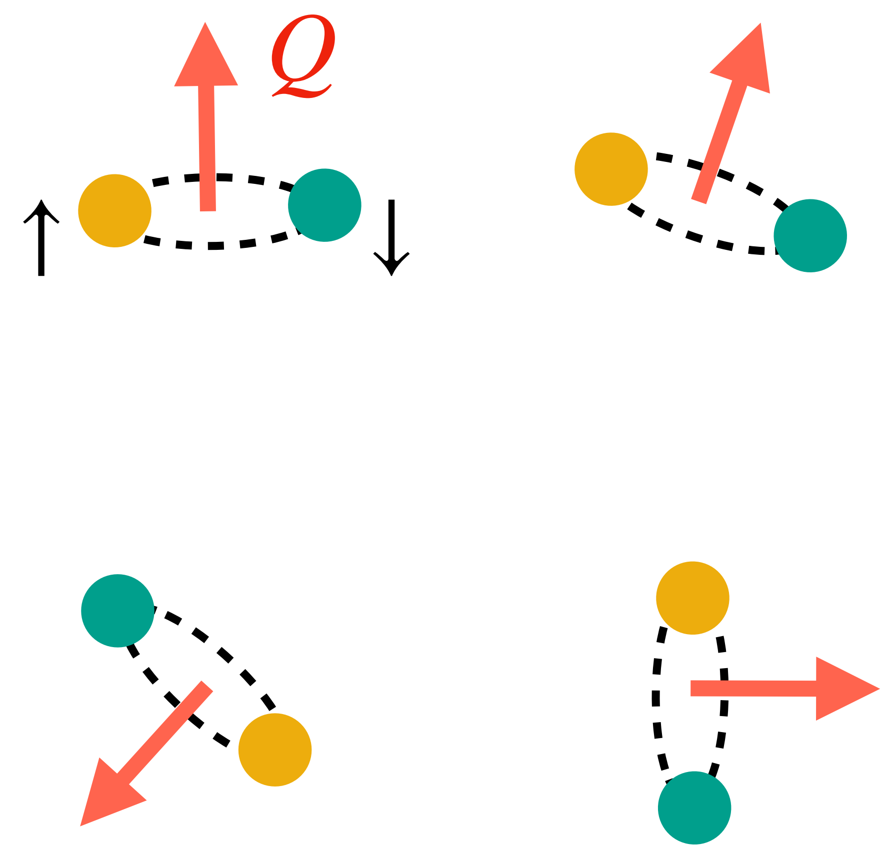
Destruction of the FFLO long-range order by FFLO pairing fluctuations

In isotropic systems, the FFLO-type non-uniform superfluid state is unstable against pairing fluctuations at nonzero temperature even in a 3D system.

- H. Shimahara, J. Phys. Soc. Jpn. **67**, 1872 (1998)
- Y. Ohashi, J. Phys. Soc. Jpn. **71**, 2625 (2002)
- J. Wang *et. al.*, Phys. Rev. B **97**, 134513 (2018)
- P. Zdybel *et. al.*, Phys. Rev. A **104**, 063317 (2021)

⋮

gas system
(continuous rotational symmetry)



Large fluctuations caused by an infinite degeneracy with respect to the direction of the FFLO Q vector destroy the FFLO-type long-range order.

(cf.) Hohenberg-Mermin-Wagner theorem

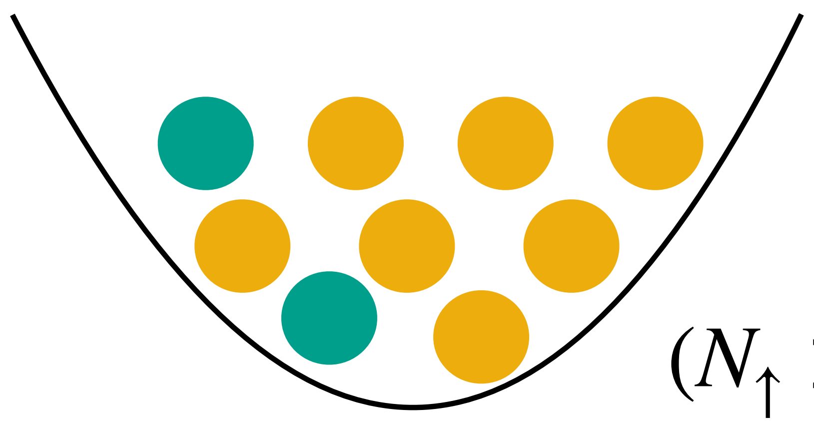
2D XY model
O(2) symmetry
continuous symmetry

Long-range order can be stabilized!

2D Ising model
Z_2 symmetry
discrete symmetry

FFLO State in a Spin-imbalanced Ultracold Fermi Gas

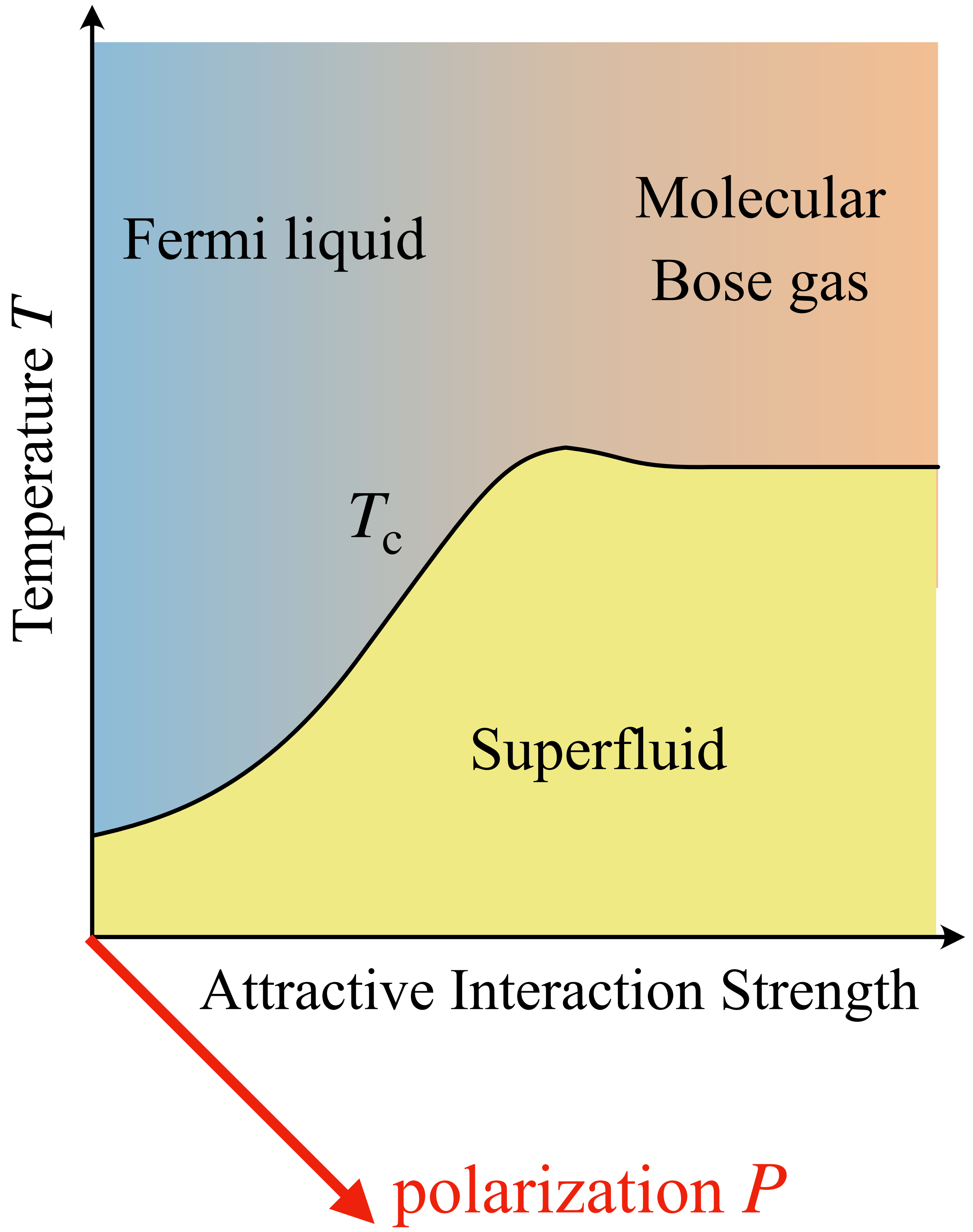
Spin-imbalanced Fermi gas ($N_\uparrow \neq N_\downarrow$)



$(N_\uparrow > N_\downarrow)$

polarization

$P = [N_\uparrow - N_\downarrow]/[N_\uparrow + N_\downarrow]$

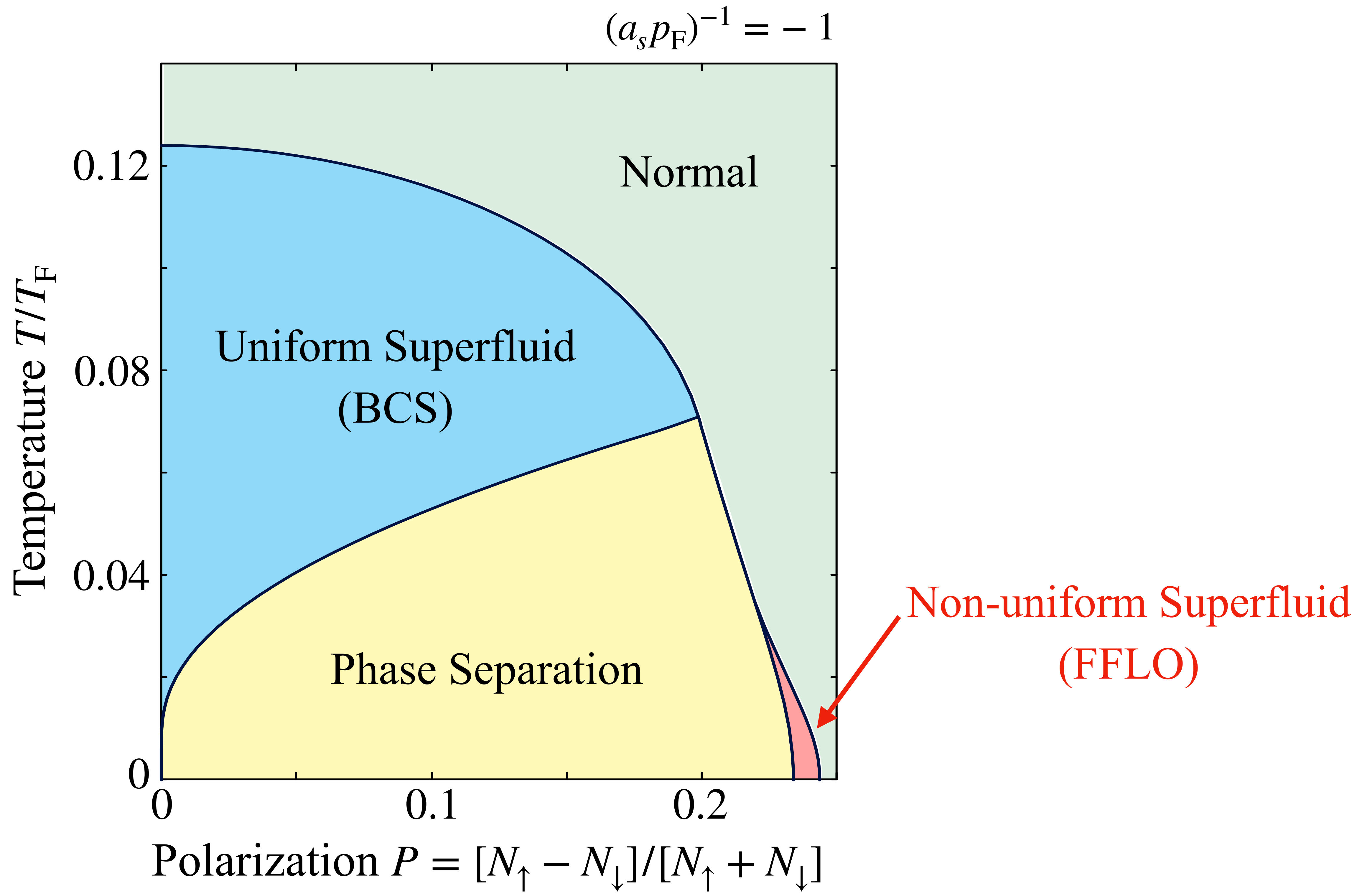


Theory (Mean-field approximation)

M. M. Parish *et. al.*, Nature Physics **3**, 124 (2007)

G. C. Strinati *et. al.*, Phys. Rep. **738**, 1 (2018)

⋮



The FFLO state has not been observed experimentally in a 2D and 3D Fermi gas.

Destruction of the FFLO long-range order by FFLO pairing fluctuations

In isotropic systems, the FFLO-type non-uniform superfluid state is unstable against pairing fluctuations at nonzero temperature even in a 3D system.

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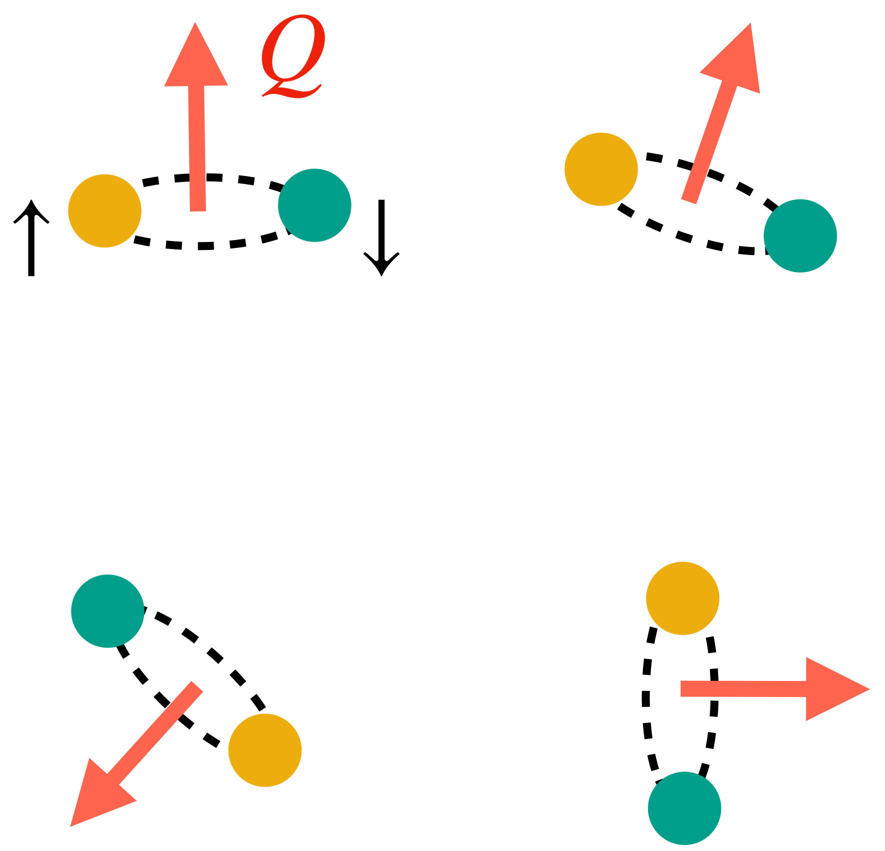
⋮

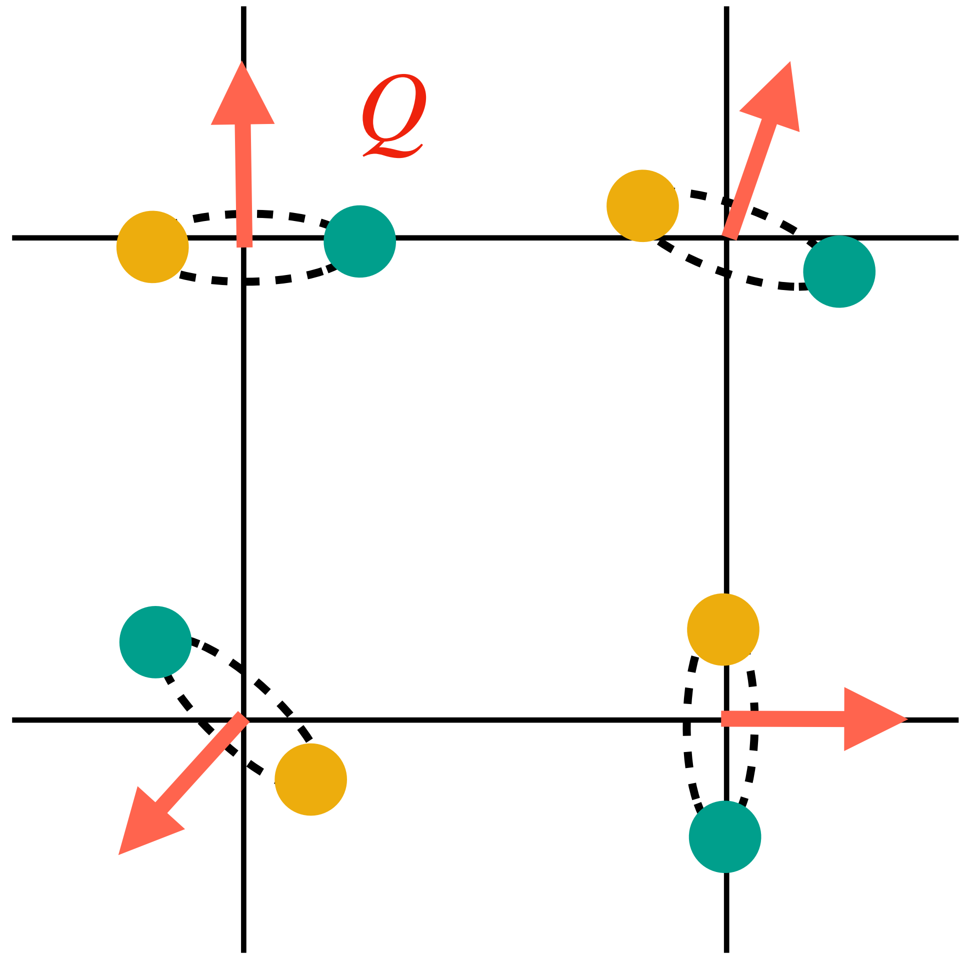
gas system
(continuous rotational symmetry)



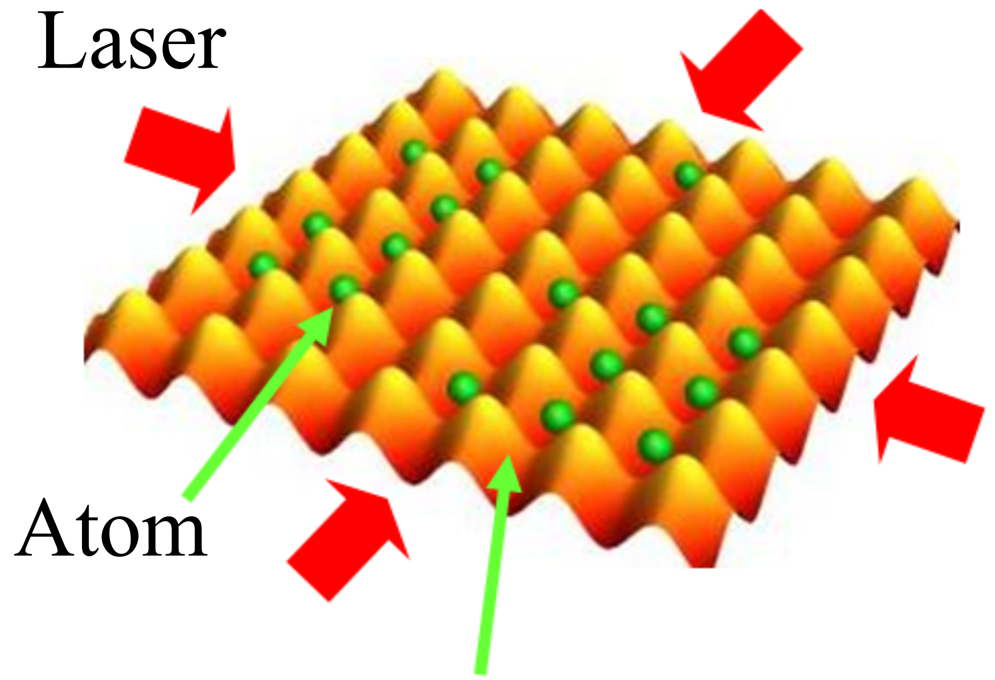
infinite degeneracy

Explicitly breaking
the continuous rotational symmetry

Optical lattice system
(discrete rotational symmetry)



finite degeneracy



Periodic potential made by optical interference

http://www.kozuma.phys.titech.ac.jp/research_category/entry7.html

(cf.) Hohenberg-Mermin-Wagner theorem

2D XY model
O(2) symmetry
continuous symmetry

2D Ising model
Z_2 symmetry
discrete symmetry

Landau-Peierls instability

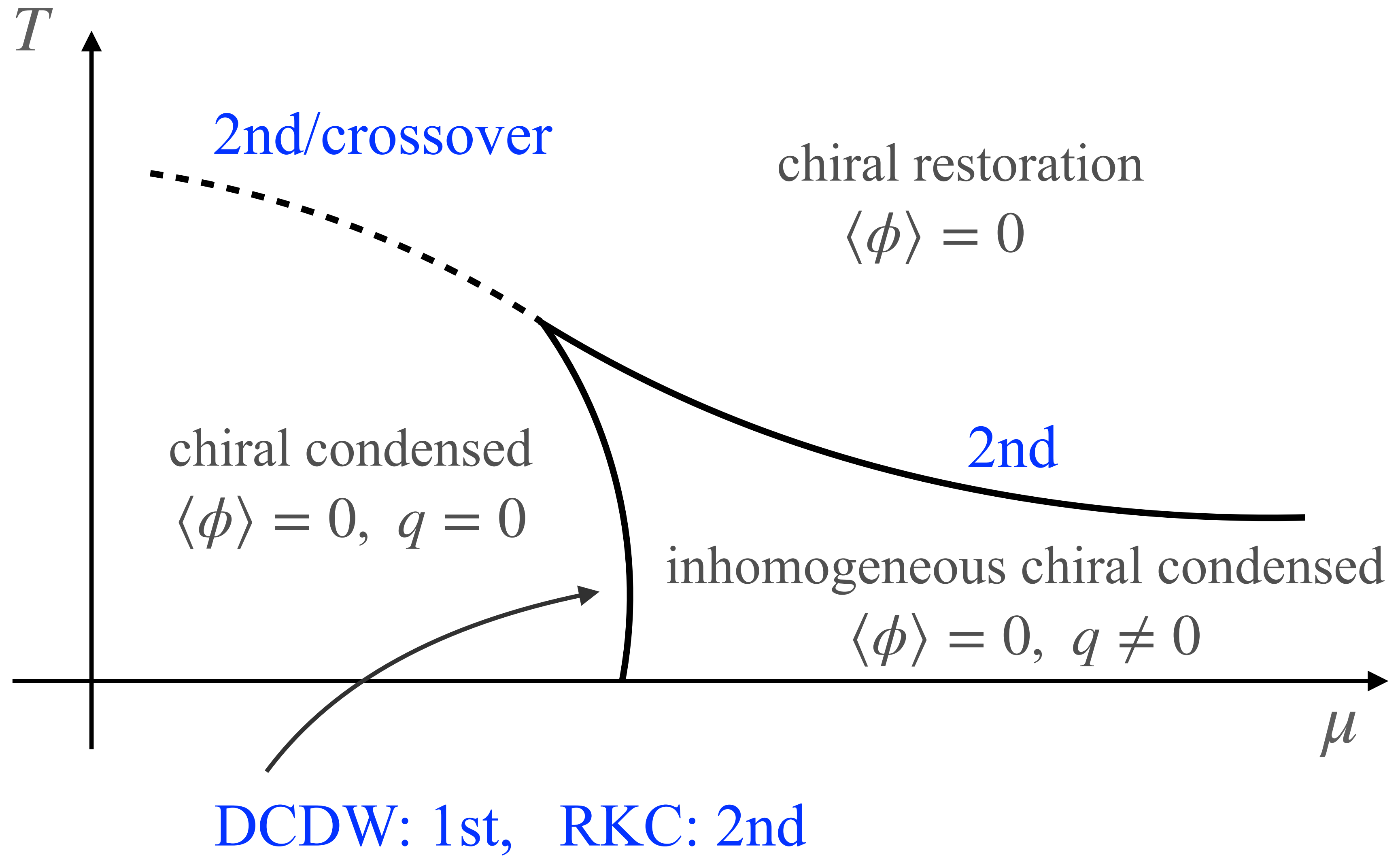
QCD (chiral condensate)

GL expansion for the (3+1)-dimensional NJL model in the chiral limit [Nickel (2009)]

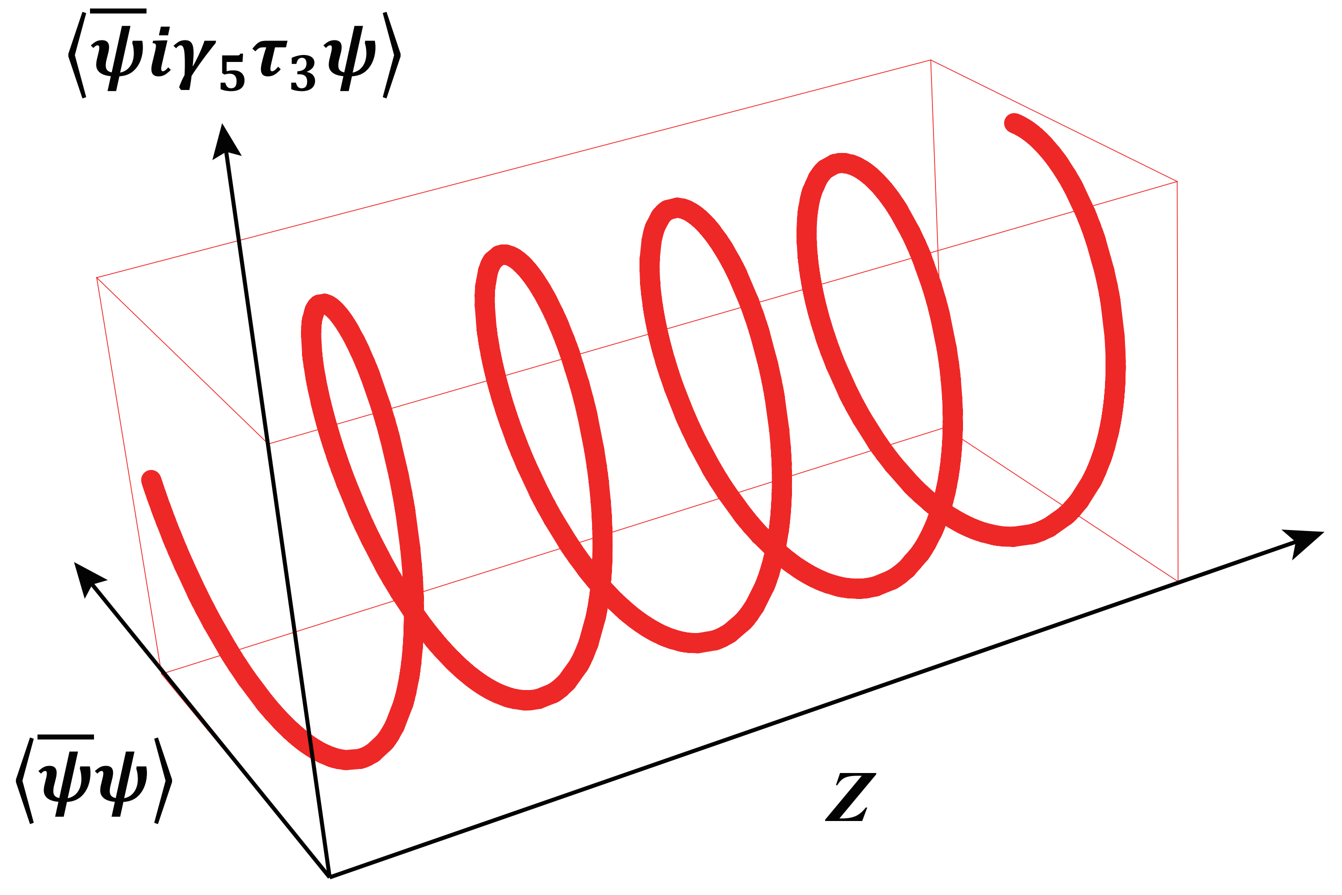
$$\Omega_{\mathrm{GL}} = \alpha_2 M^2 + \alpha_4 \left[M^4 + (\nabla M)^2\right] + \alpha_6 \left[2M^6 + 10M^2(\nabla M)^2 + (\nabla^2 M)^2\right]$$

Dual Chiral Density Wave $M(x) = me^{iQx}$ Fulde-Ferrell type

real kink crystal $M(x) = m\sqrt{\nu}\,\mathrm{sn}(x;\nu)$ Larkin-Ovchinikov type

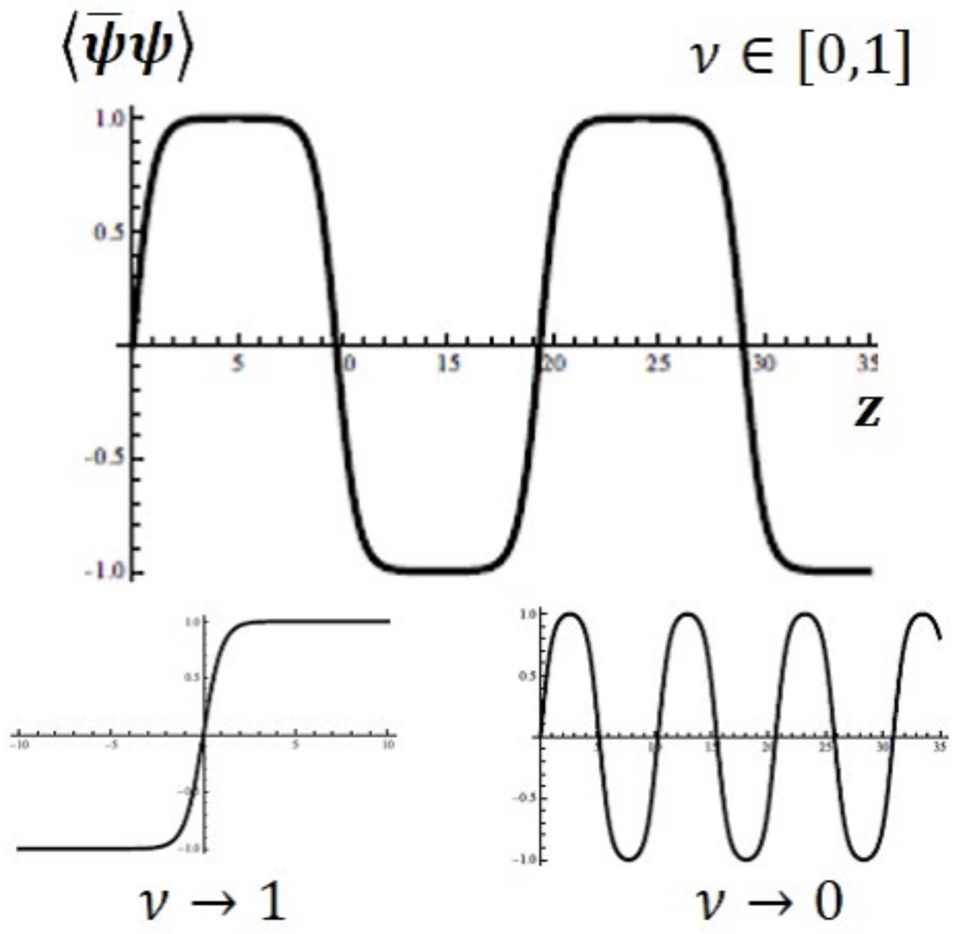


DCDW



(chiral spirals in 3+1D systems)

RKC

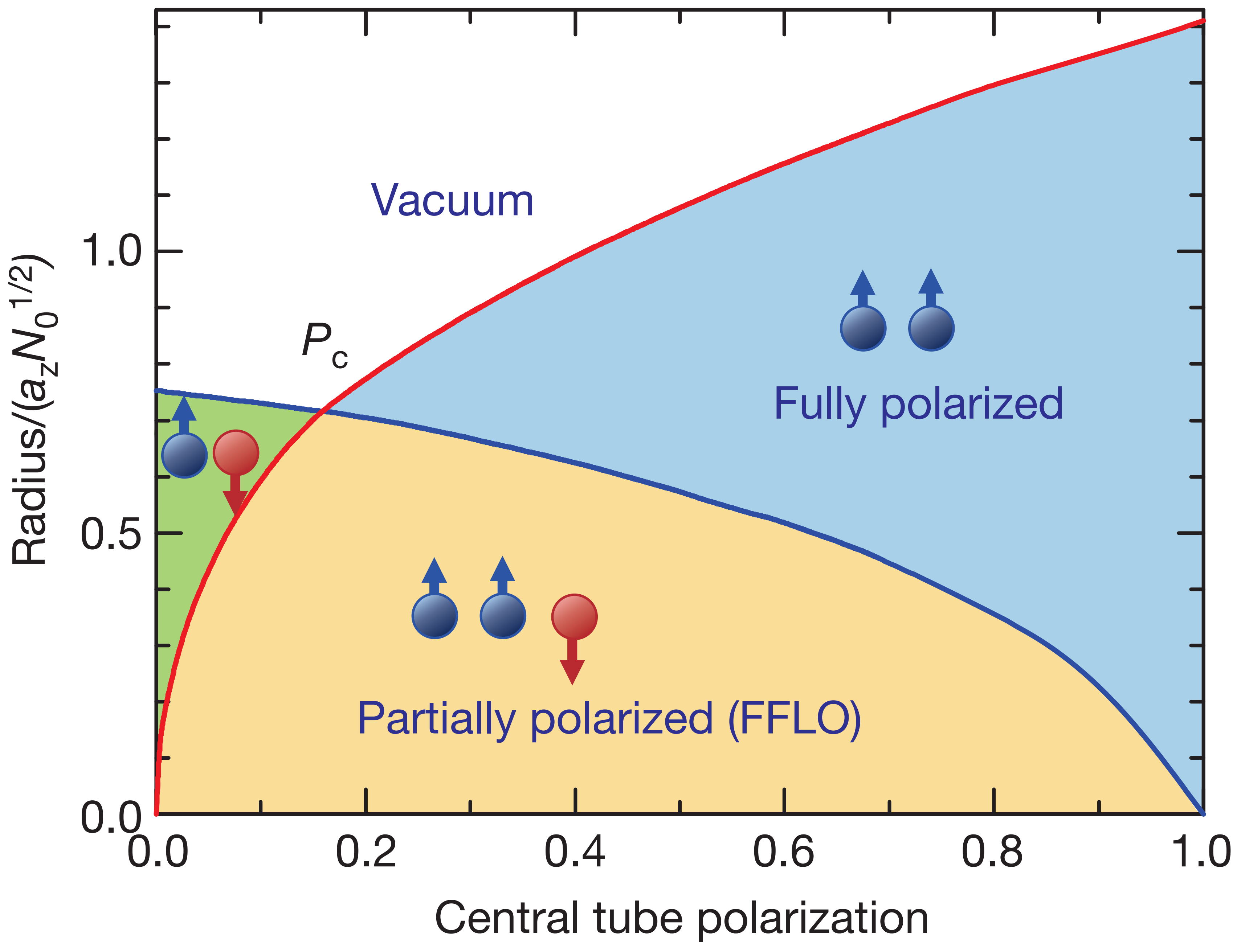


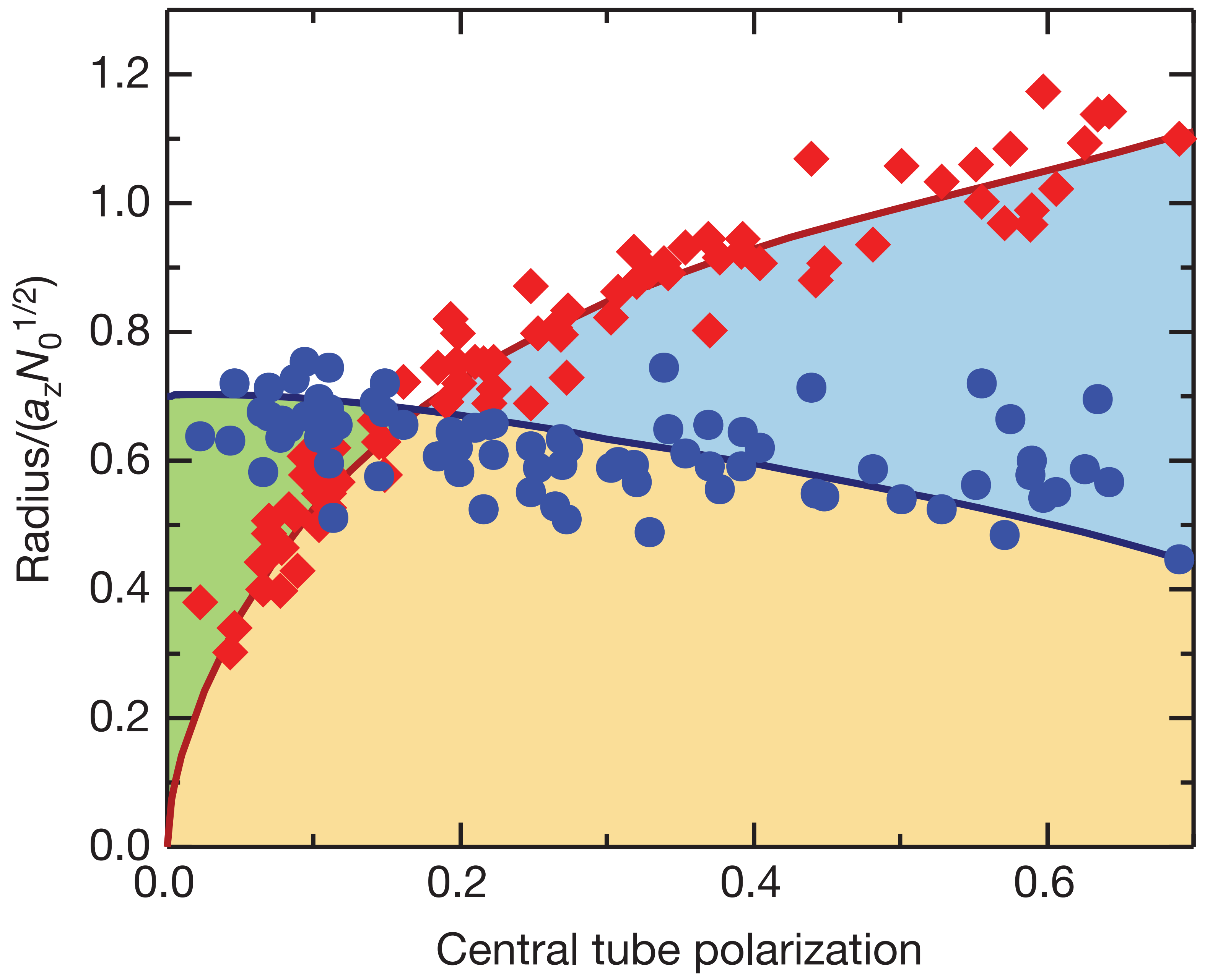
(periodic domain walls in 3+1D systems)

FFLO state in a 1D spin-imbalanced ultracold Fermi gas

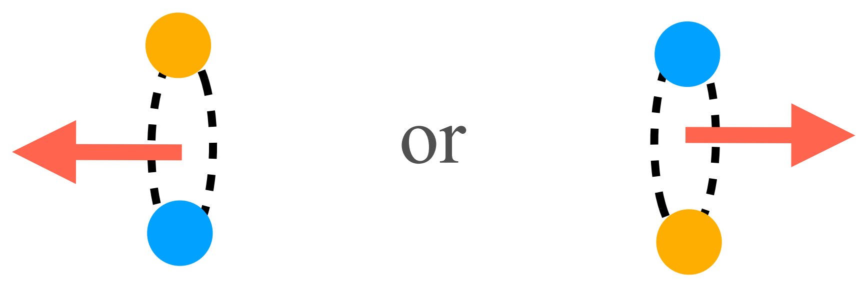
1D spin-imbalanced Fermi gas Y. Liao, *et. al.*, Nature (London) **467**, 567 (2010).

Theory: exactly solvable Gaudin–Yang model





- The density profile of population imbalanced 1D Fermi gas was found to qualitatively agree with a theoretical prediction, exhibiting the FFLO state.
- In a 1D system, there is no infinite degeneracy associated with the Cooper pair’s momentum.



Observation of the FFLO state ? (superconductor under an external magnetic field)

- Heavy fermion superconductor

$CeCoIn_5$ - Bianchi *et.al.*, PRL **89**, 137002 (2002)
- Kumagi *et.al.*, PRL **97**, 227002 (2006)

$CeCu_2Si_2$ - Kitagawa *et.al.*, PRL **121**, 157004 (2018)

- Iron-based superconductor

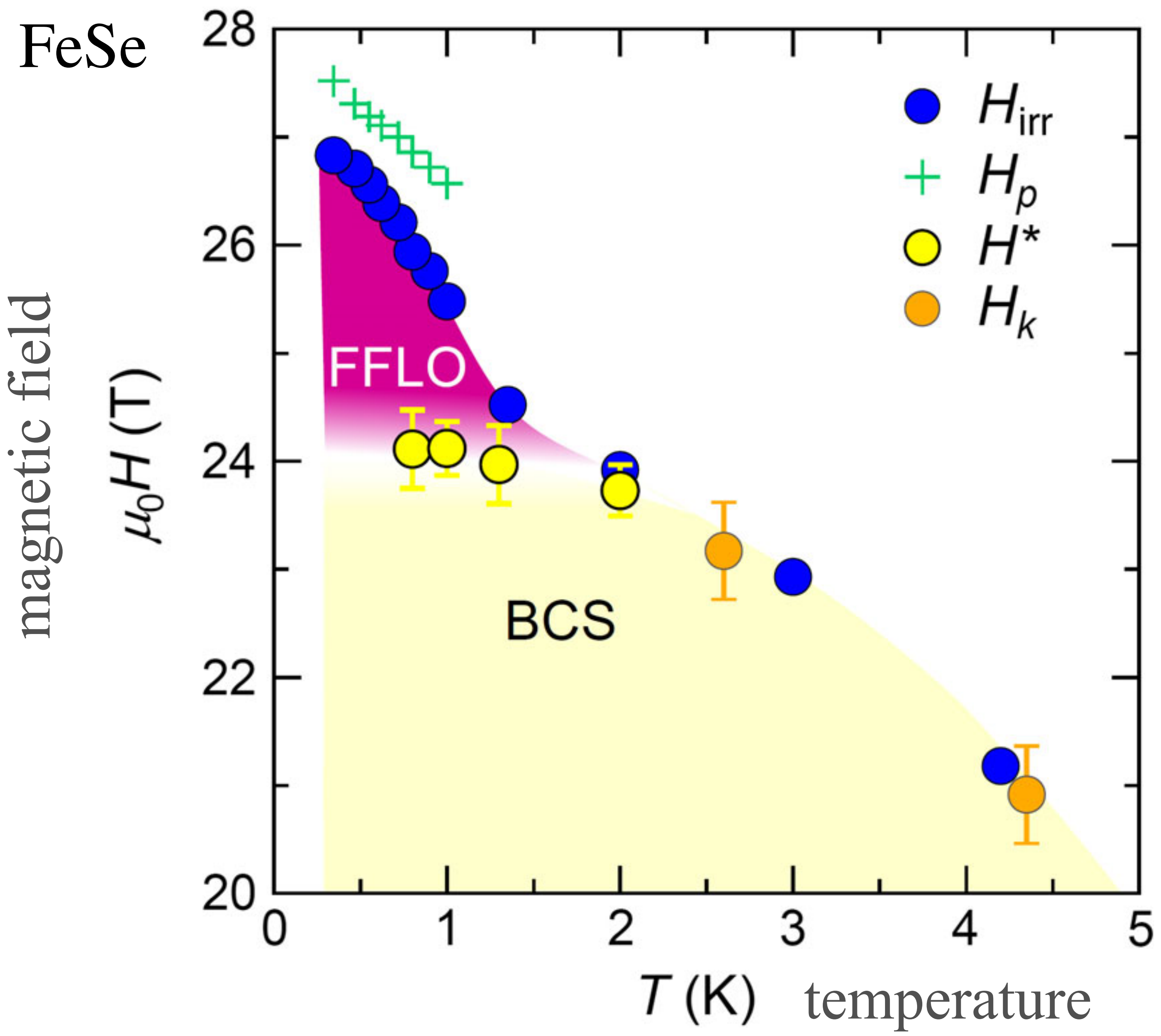
FeSe - Ok *et.al.*, PRB **101**, 224509 (2020)
- Kasahara *et.al.*, PRL **124**, 107001 (2020)

KFe_2As_2 - Kumagi *et.al.*, PRL **119**, 217002 (2017)

- Organic superconductor

$\kappa-(\mathrm{BEDT-TTF})_2\mathrm{Cu(NCS)}_2$
- Wright *et.al.*, PRL **107**, 087002 (2011)
- Mayaffre *et.al.*, Nat. Phys. **10**, 928 (2014)

$\beta''-(\mathrm{BEDT-TTF})_2\mathrm{SF_5CH_2CF_2SO_3}$
- Beyer *et.al.*, PRL **109**, 027003 (2012)
- Koutroulakis *et.al.*, PRL **116**, 067003 (2016)



Kasahara *et.al.*, PRL(2020)

Conditions for realizing the FFLO state

(1) Pauli pair-breaking effect > orbital pair-breaking effect

(2) ultra-clean system

Boundary between the normal and phase-separated phase

Phase separation (BCS superfluid + Normal)

$$F^{\rm mix}_{\rm MF} = xF^{\rm SF}_{\rm MF} + (1-x)F^{\rm N}_{\rm MF} \qquad \left(n^{\rm MF}_{\sigma} = xn^{\rm MF}_{{\rm SF},\sigma} + [1-x]n^{\rm MF}_{{\rm N},\sigma}\right)$$

$$= x\left[\Omega^{\rm SF}_{\rm MF} + \mu_{\uparrow}n_{{\rm SF},\uparrow} + \mu_{\downarrow}n_{{\rm SF},\downarrow}\right] + (1-x)\left[\Omega^{\rm N}_{\rm MF} + \mu_{\uparrow}n_{{\rm N},\uparrow} + \mu_{\downarrow}n_{{\rm N},\downarrow}\right]$$

$$= x\Omega^{\rm SF}_{\rm NF} + (1-x)\Omega^{\rm N}_{\rm MF} + \mu_{\uparrow}n^{\rm MF}_{\uparrow} + \mu_{\downarrow}n^{\rm MF}_{\downarrow}$$

We need to minimize $F^{\rm mix}_{\rm MF}$ with respect to $\mu_{\uparrow}$, $\mu_{\downarrow}$, Δ_0, and x

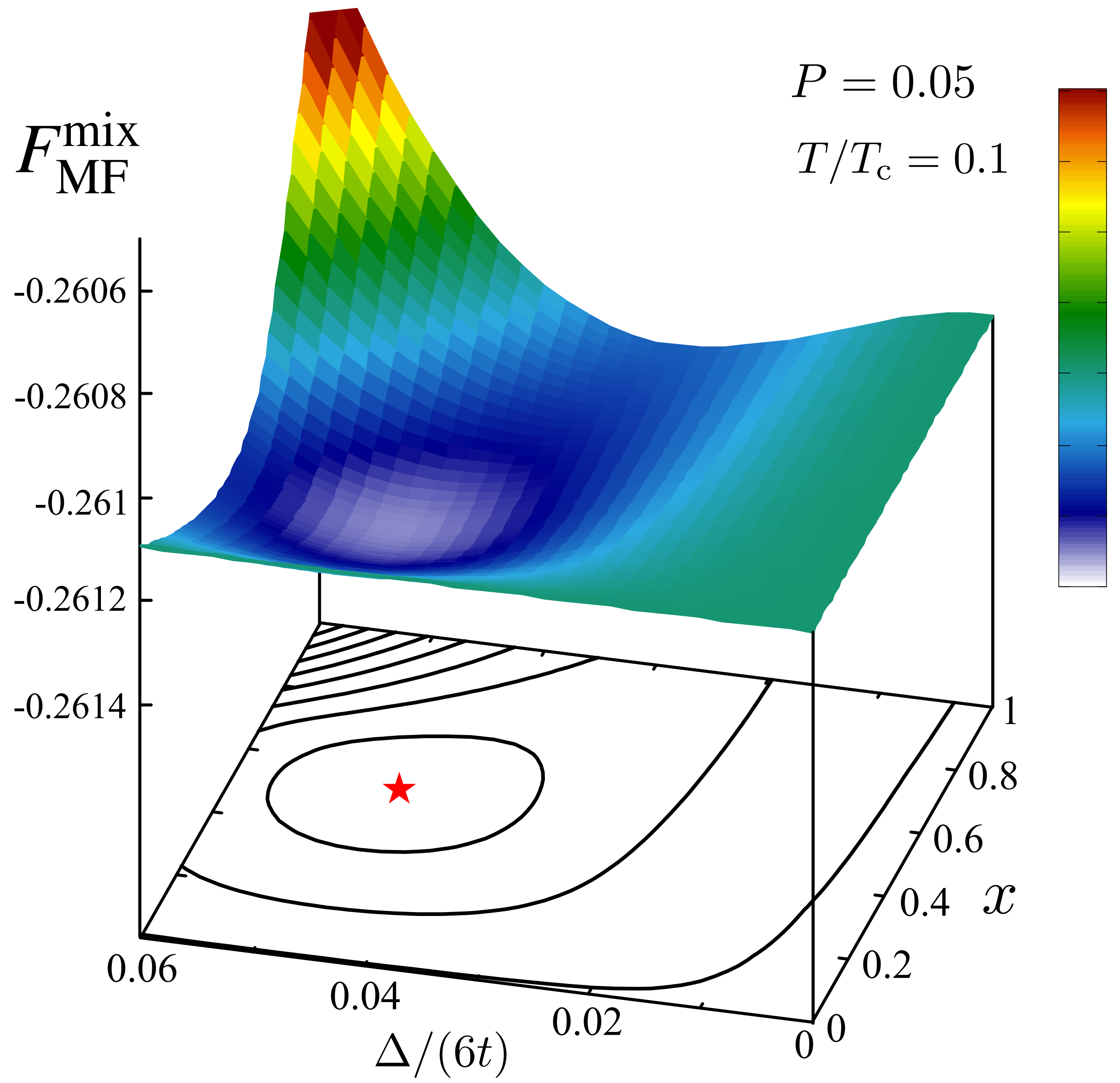
$$0 = \frac{\partial F^{\rm mix}_{\rm MF}}{\partial x} = \Omega^{\rm SF}_{\rm MF} - \Omega^{\rm N}_{\rm MF}. \Leftrightarrow$$ mechanical equilibrium condition

- Hartree density equations $n^{\rm MF}_{\sigma} = xn^{\rm MF}_{{\rm SF},\sigma} + [1-x]n^{\rm MF}_{{\rm N},\sigma}$
- gap equation $\Delta_0 = U\sum_{\boldsymbol{k}} \frac{1}{2\tilde{E}_{\boldsymbol{k},\boldsymbol{Q}}}\left[1 - f(\tilde{E}^{+}_{\boldsymbol{k},\boldsymbol{Q}}) - f(\tilde{E}^{-}_{\boldsymbol{k},\boldsymbol{Q}})\right]$
- mechanical equilibrium condition $\Omega^{\rm SF}_{\rm MF} = \Omega^{\rm N}_{\rm MF}$

We neglect the interface energy between the normal and superfluid components

Equilibrium conditions

- thermal equilibrium $T_{\rm SF} = T_{\rm N}$
- chemical equilibrium $\mu_{{\rm SF},\sigma} = \mu_{{\rm N},\sigma}$
- mechanical equilibrium $P_{\rm SF} = P_{\rm N}$

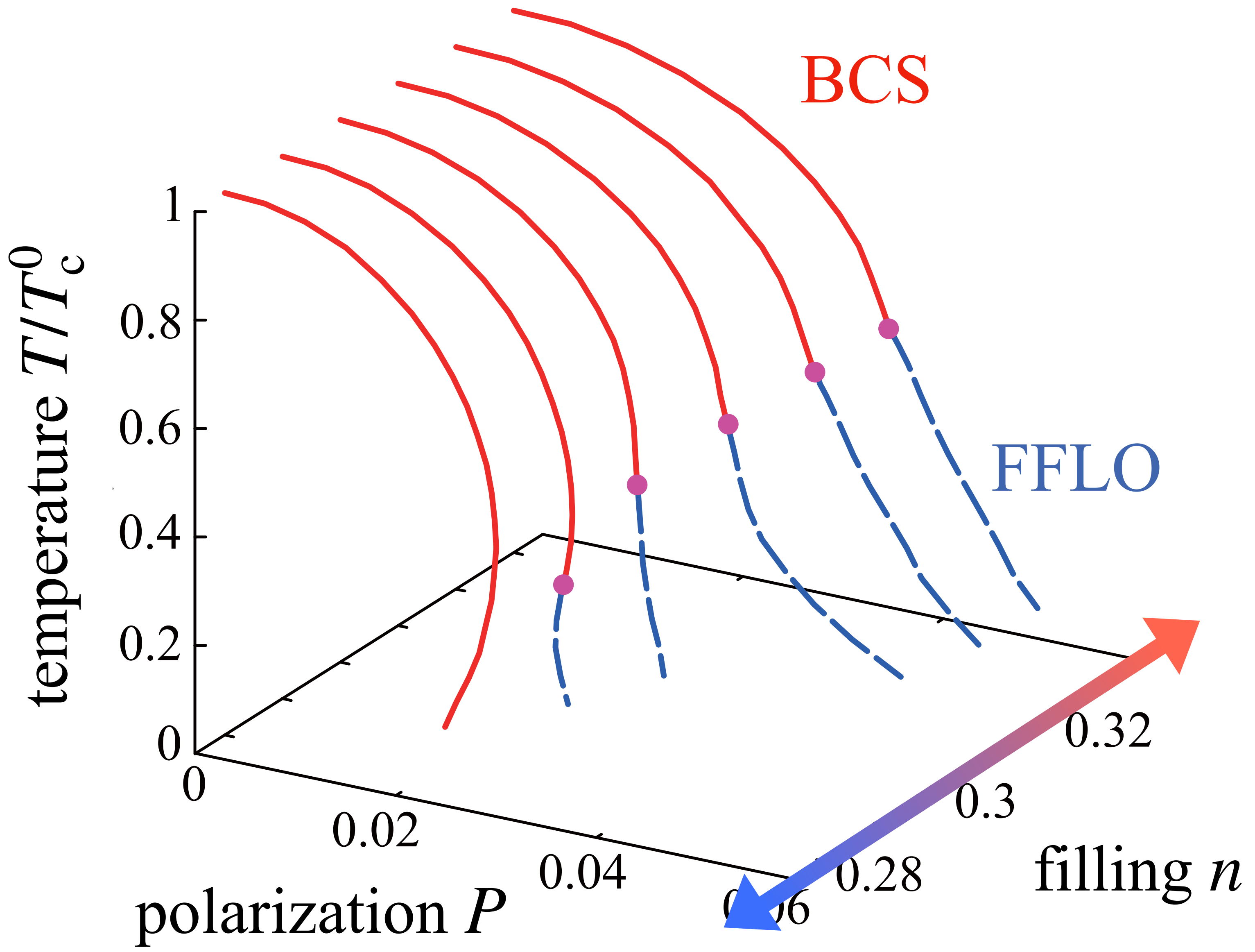


(BCS + N)

FFLO State in a Lattice System (Fermi Surface shape)

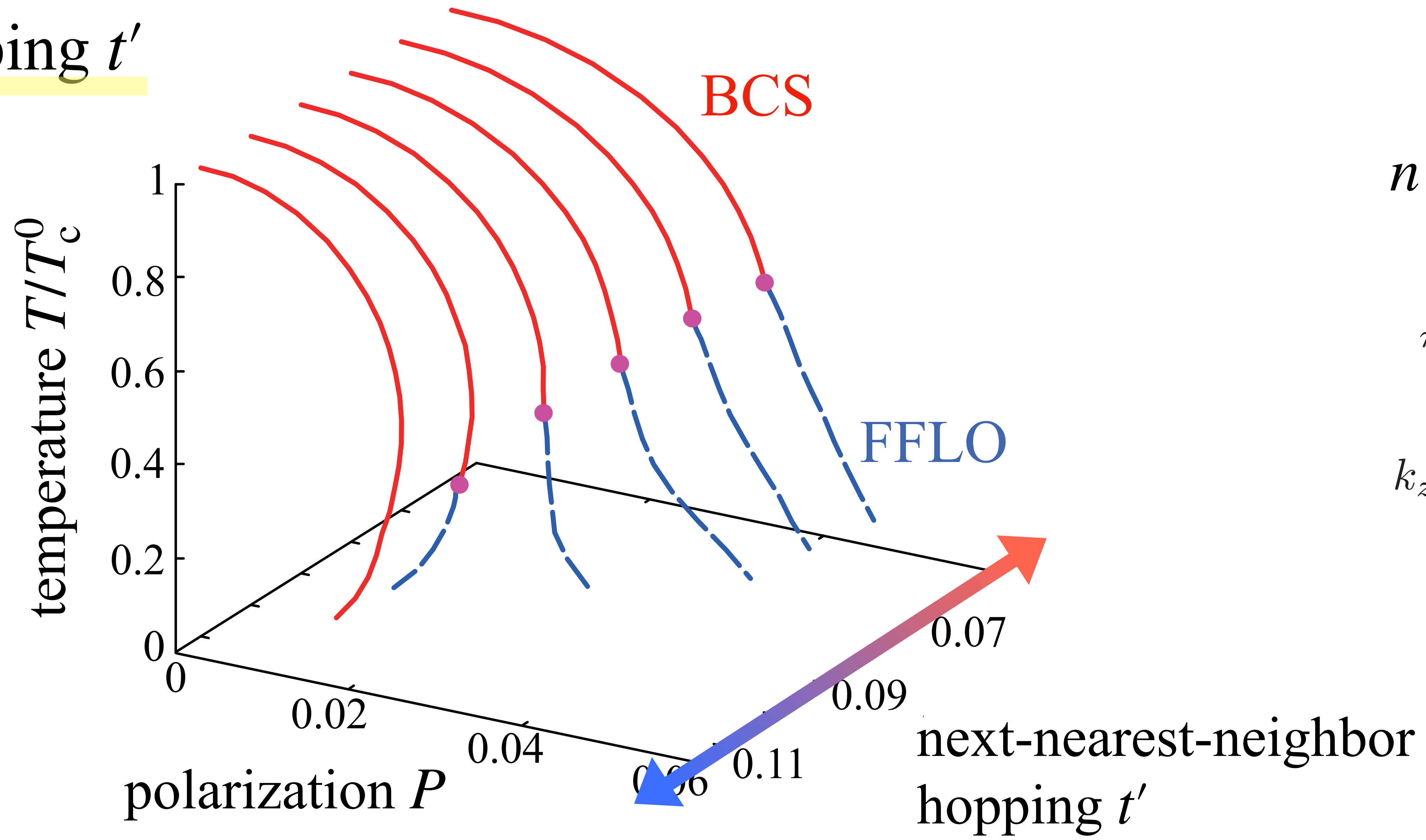
Filling n

($t' = 0$)



NNN hopping t'

($n = 0.4$)



Fermi surface