The 2nd International symposium on
Clustering as a Window on the Hierarchical Structure of Quantum Systems
--CLUSHIQ2022 (EMMI Workshop)--

Sendai, Japan, Nov 2, 2022

# Ultracold atom study of exotic phenomena bridging different hierarchies

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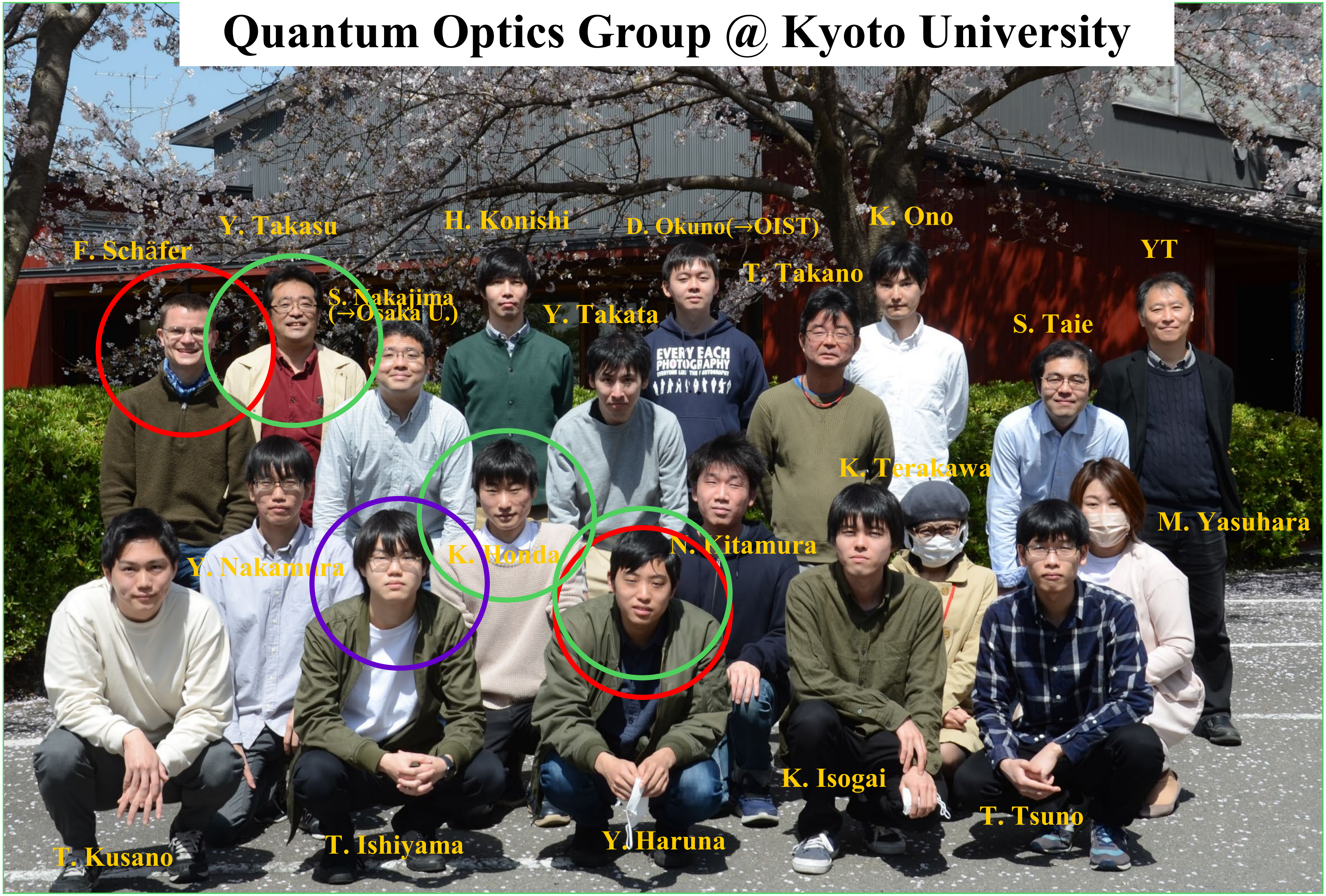
# Quantum Optics Group @ Kyoto University



# Outline

1. Introduction: three-body problem for ultracold atoms

2. Three-body force for atoms in an optical lattice

   occupancy-resolved high-resolution spectroscopy

   prospects for non-perturbative regime

3. Efimov trimmer with unequal mass

   creation of ultracold Er-Li mixture

   Feshbach resonances between Er and Li

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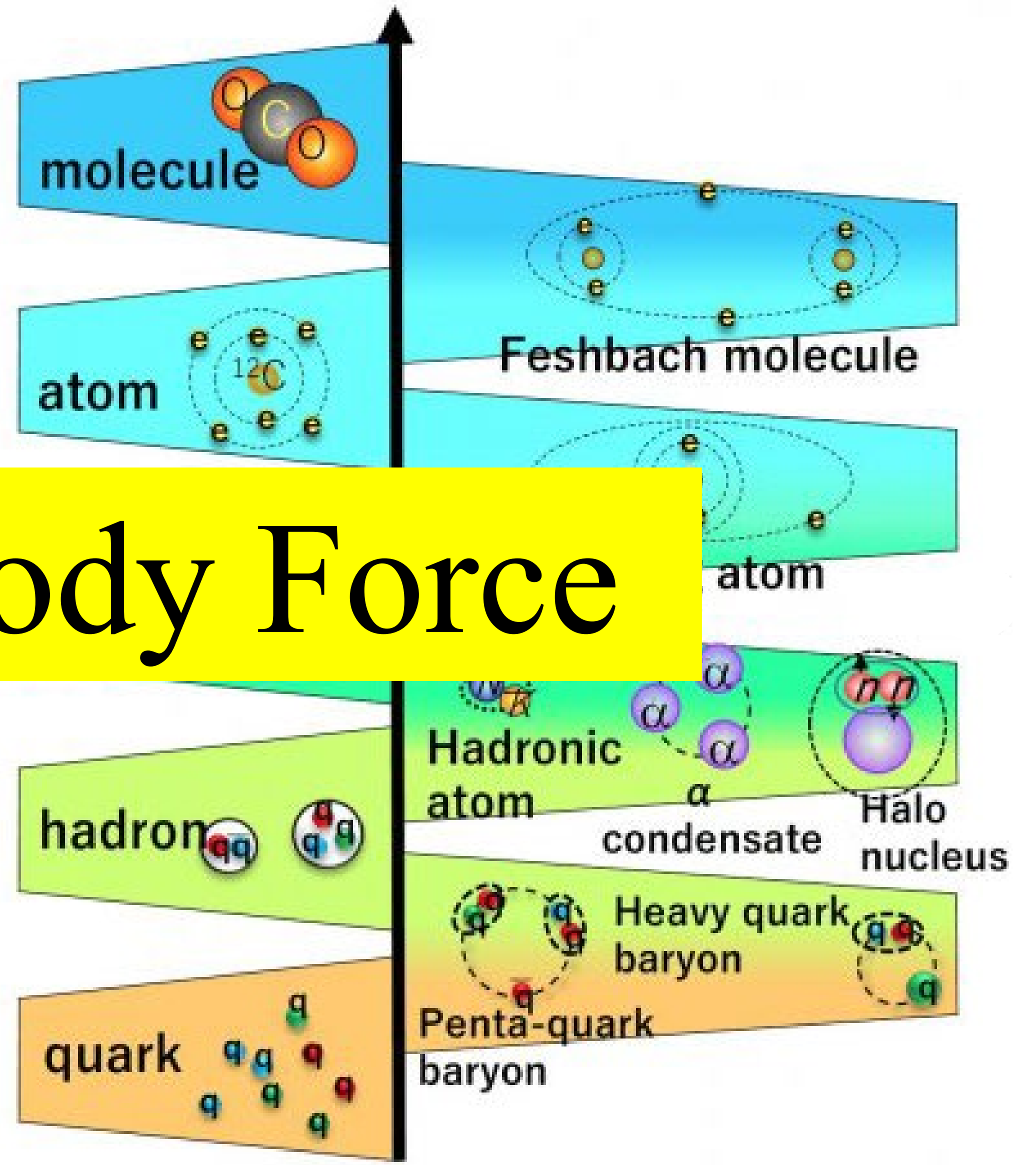
# Quantum Simulation

quantum systems in various hierarchies and between them

(quarks, hadron, nucleus, atom, molecules,…)

theoretical mo

**Three-Body Force**

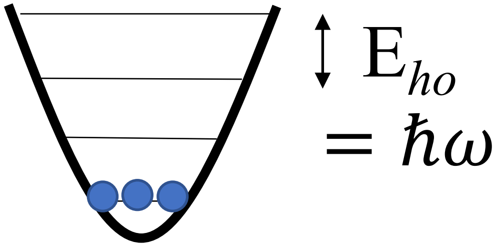



ultracold atoms

"highly controllable"

(Feshbach resonance, mass ratio, dimensionality, quantum statistics,…)

# Three ultracold atoms in a harmonic trap




S. Jonssel, *et al.,* PRL, 89, 250401 (2002)
M. Stoll and T. Köller PRA 72, 022714 (2005)
F. Werner and Y. Castin, PRL 97, 150401 (2006)
D. Blume, *et al.*, PRA 97, 033621 (2018)
…

$$\sum_{i=1}^{3}\left[-\frac{\hbar^2}{2m}\Delta_{\mathbf{r}_i}+\frac{1}{2}m\omega^2 r_i^2\right]\psi = E\psi$$

$$\partial \ln(r_{ij}\Psi)/\partial r_{ij}|_{r_{ij}\to 0} = -1/a$$

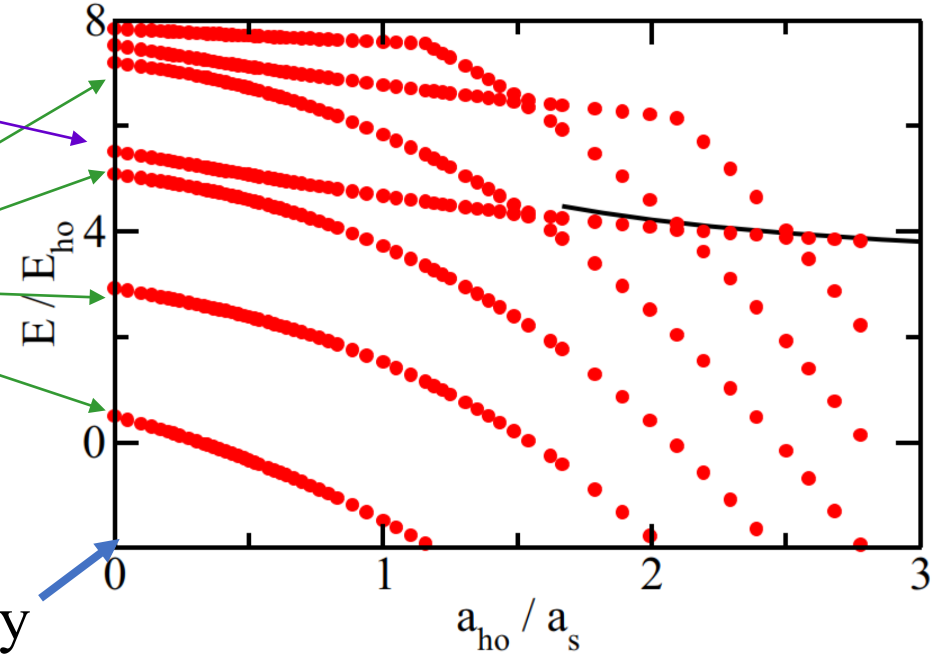
$s_1$=4.46529… **“atomic state”**
**“Fujita-Miyazawa type 3-body force”**

$s_0$=$i$1.00624 **“Efimov state”**
$\longrightarrow -(|s_0|^2+1/4)R^{-2}$

$$E_{\nu q}^{\mathrm{unit}} = (2q+s_\nu+1)E_{\mathrm{ho}}$$

F. Werner and Y. Castin,
PRL 97, 150401 (2006)

3-body pot. $V_0$ =97700 $E_{ho}$




unitarity
($a_s = \infty$)

D. Blume, *et al.*, PRA 97, 033621 (2018)

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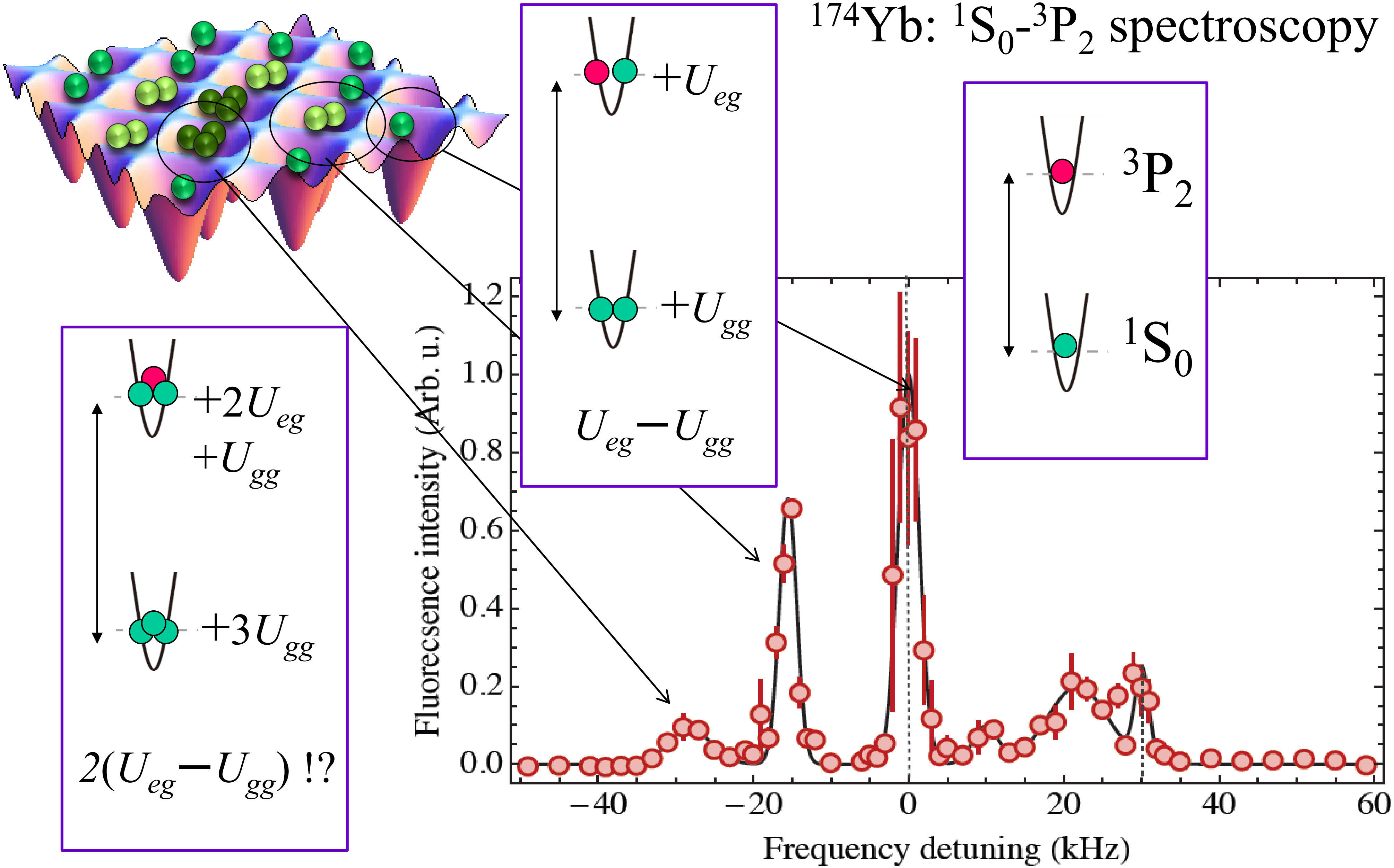
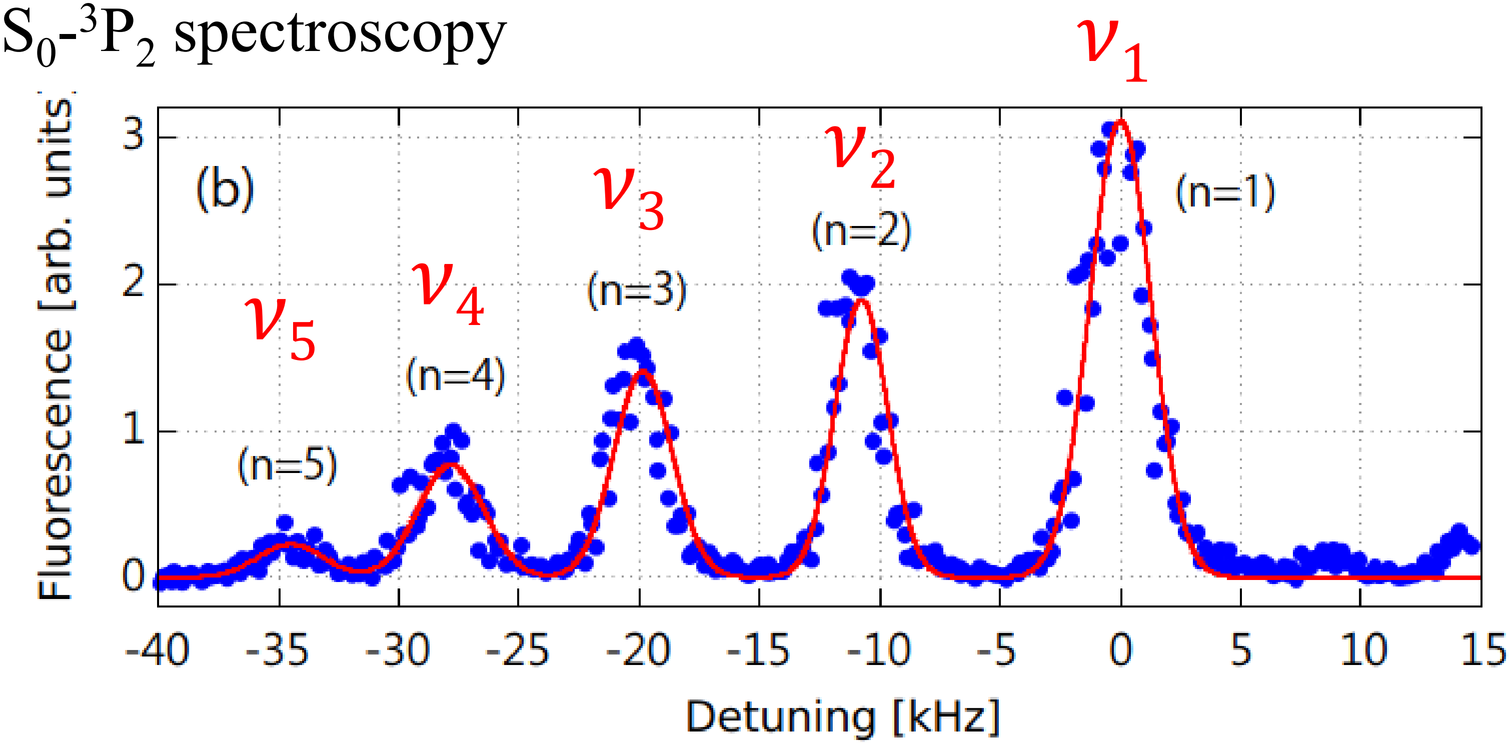
# Spectroscopy of Atoms in an Optical Lattice



# "Effective 3-body interaction"

$^{174}$Yb: $^1S_0$-$^3P_2$ spectroscopy




[Y. Nakamura, *et al.*, PRA 99, 033609 (2019)]

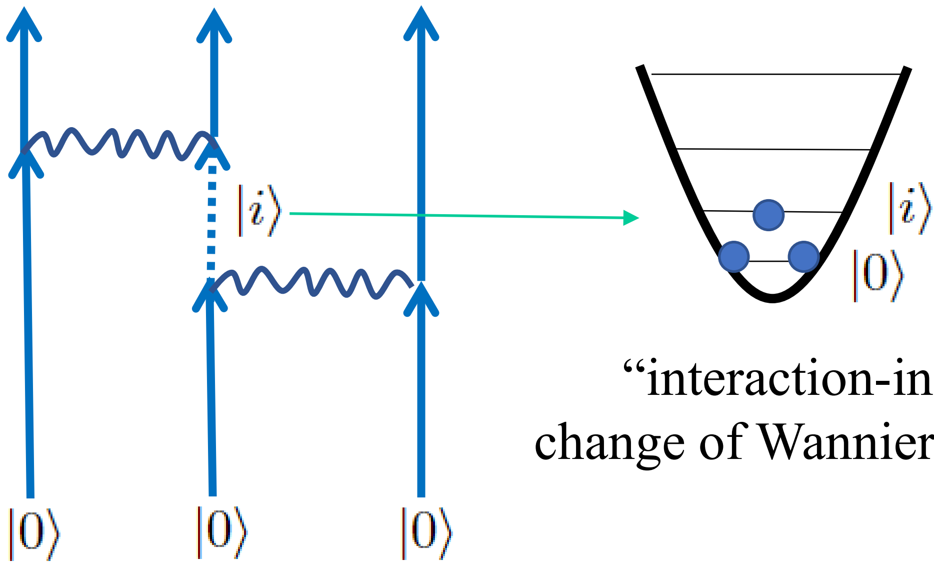
| | |
|---|---|
| $\nu_2 - \nu_1$ | $-10.82(9)$ kHz |
| $\nu_3 - \nu_1$ | $-20.0(1)$ kHz |
| $\nu_4 - \nu_1$ | $-28.0(3)$ kHz |
| $\nu_5 - \nu_1$ | $-34.7(4)$ kHz |

$$\nu_n - \nu_1 \neq (n-1)(\nu_2 - \nu_1)$$

simple 2-body physics unapplicable

# "Effective 3-body interaction"

P. R. Johnson, E. Tiesinga, J. V. Porto, and C. J. Williams, NJP 11, 093022 (2009)
"Effective three-body interactions of neutral bosons in optical lattices"
P. R. Johnson, D. Blume, X. Y. Yin, W. F. Flynn, and E. Tiesinga, NJP 14, 053037 (2012)
"Effective renormalized multi-body interactions of harmonically confined ultracold neutral bosons"

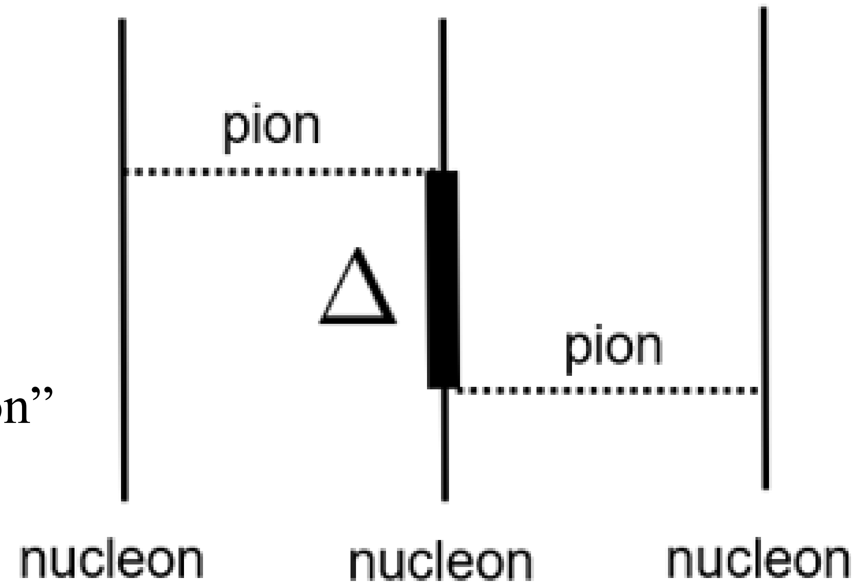



"interaction-induced change of Wannier function"

Small scattering length: $a_s << \sigma = a_{ho}$

$$E = \frac{3}{2}\hbar\omega + 3U_2 + U_3$$

Evaluated upto 2nd & 3rd order of $a_s / a_{ho}$



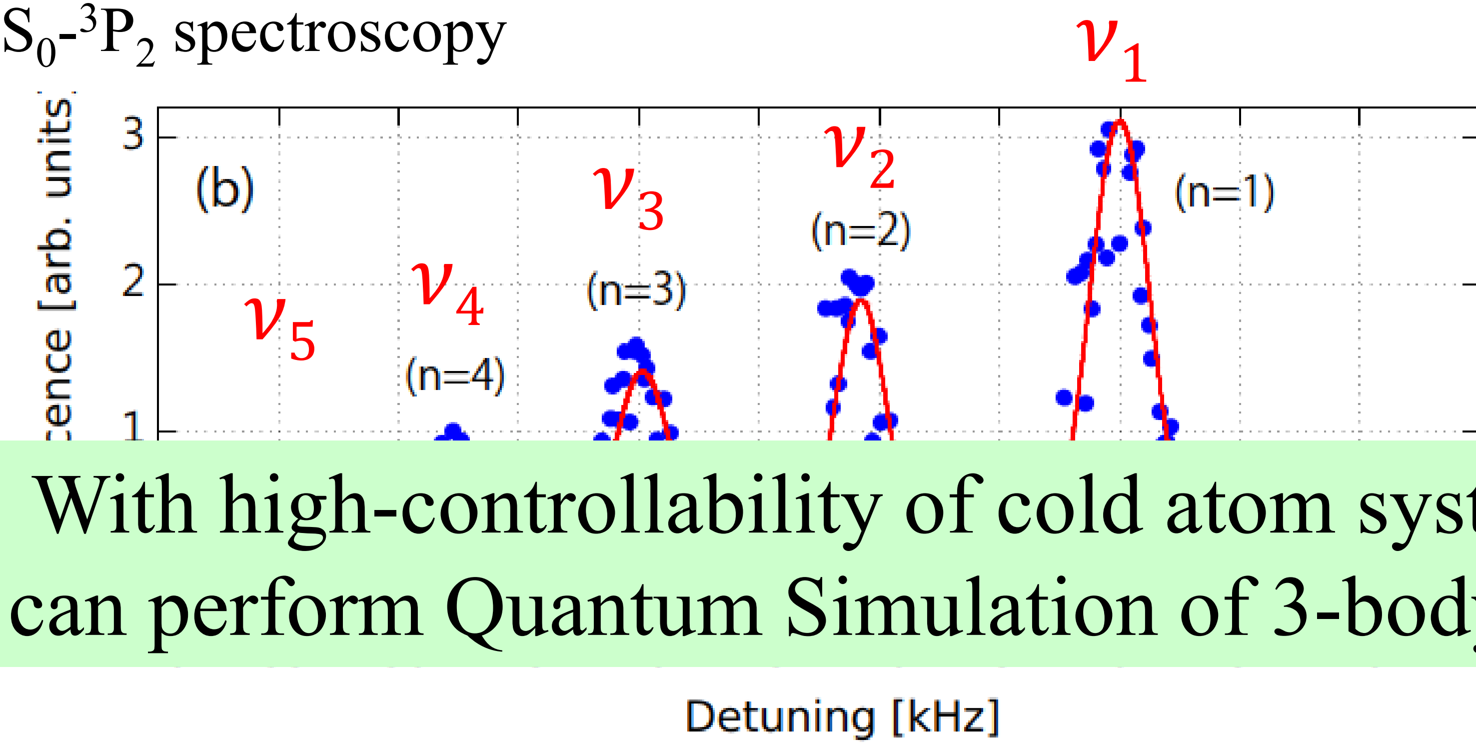

J. Fujita *et al.,* Prog. Theor. Phys. 17, 360 (1957).

Effective 3-body force in an optical lattice

Fujita-Miyazawa-type 3-body force

# "Effective 3-body interaction"

$^{174}Yb$: $^1S_0$-$^3P_2$ spectroscopy




With high-controllability of cold atom system, we can perform Quantum Simulation of 3-body force !

[Y. Nakamura, *et al.*, PRA 99, 033609 (2019)]

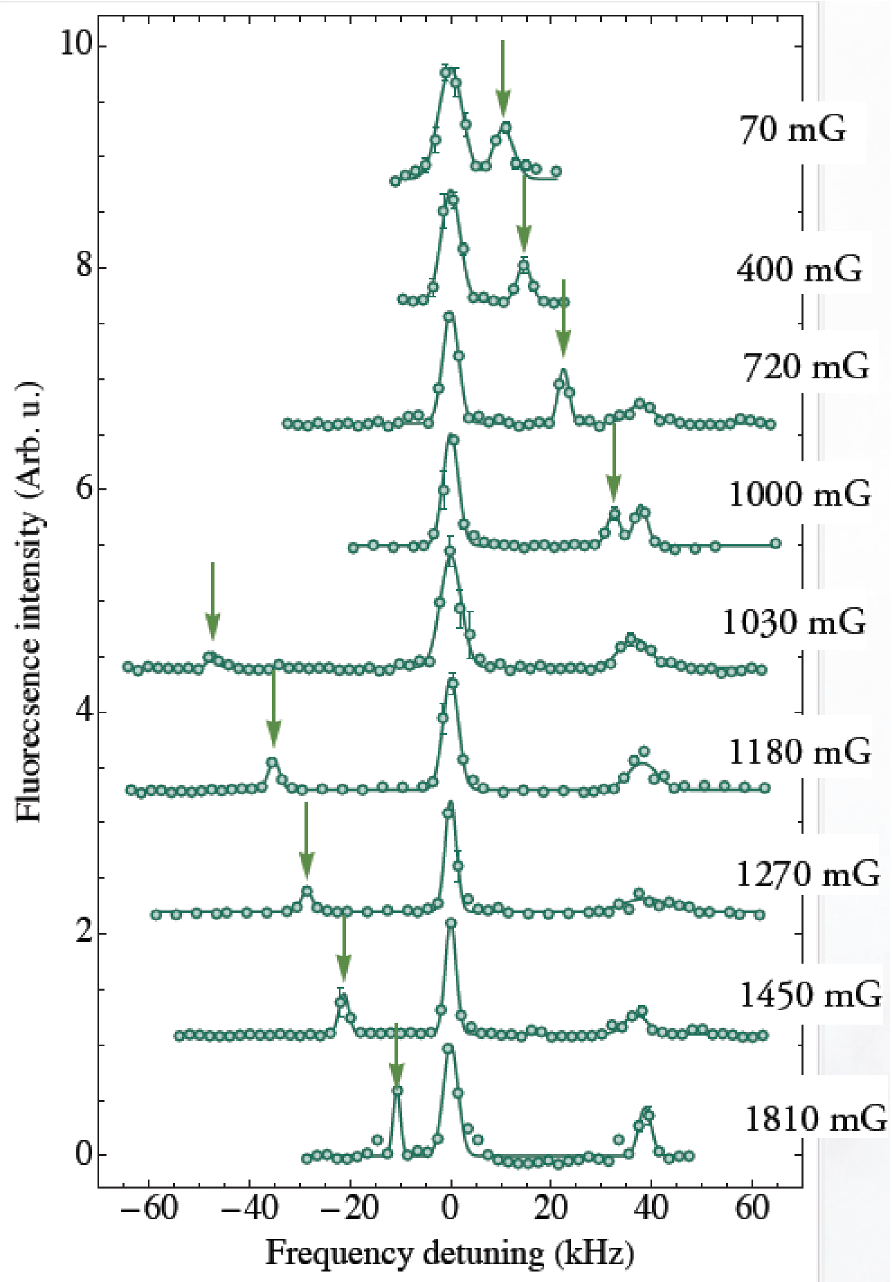
| | |
|---|---|
| $\nu_2 - \nu_1$ | $-10.82(9)$ kHz |
| $\nu_3 - \nu_1$ | $-20.0(1)$ kHz |
| $\nu_4 - \nu_1$ | $-28.0(3)$ kHz |
| $\nu_5 - \nu_1$ | $-34.7(4)$ kHz |

Calculation:Y. Haruna, T. Ishiyama *et al*., (unpublished)

$$\nu_3 - \nu_1 = -20.2(2)\text{kHz}$$
$$\nu_4 - \nu_1 = -28.1(2)\text{kHz}$$
$$\nu_5 - \nu_1 = -34.6(4)\text{kHz}$$

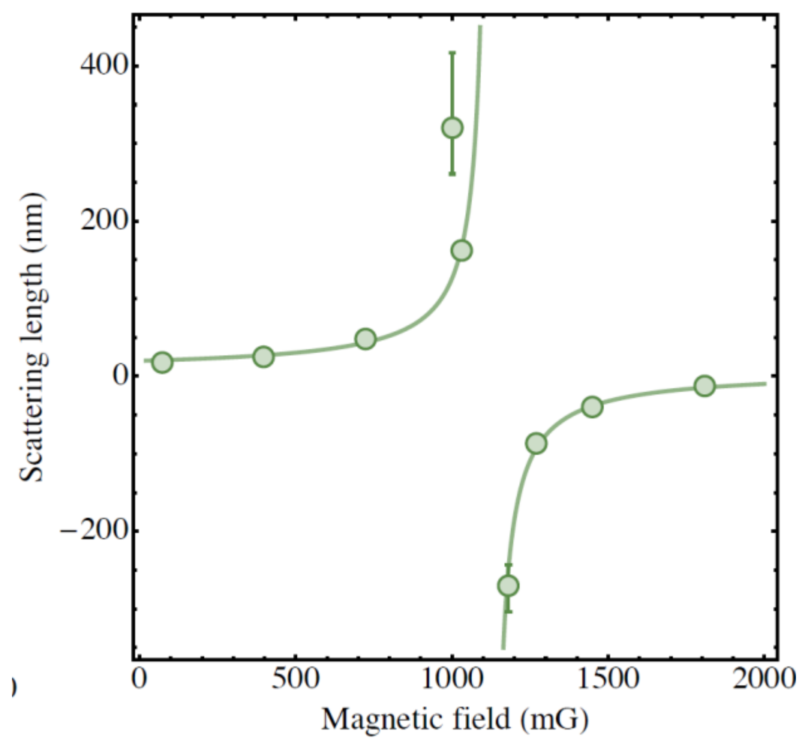
Similar work: L. Franchi, *et al*., NJP 19, 103037 (2017)

# Magnetic Feshbach Resonance ($^{170}$Yb)




S. Kato *et al.*, PRL 110, 173201 (2013)

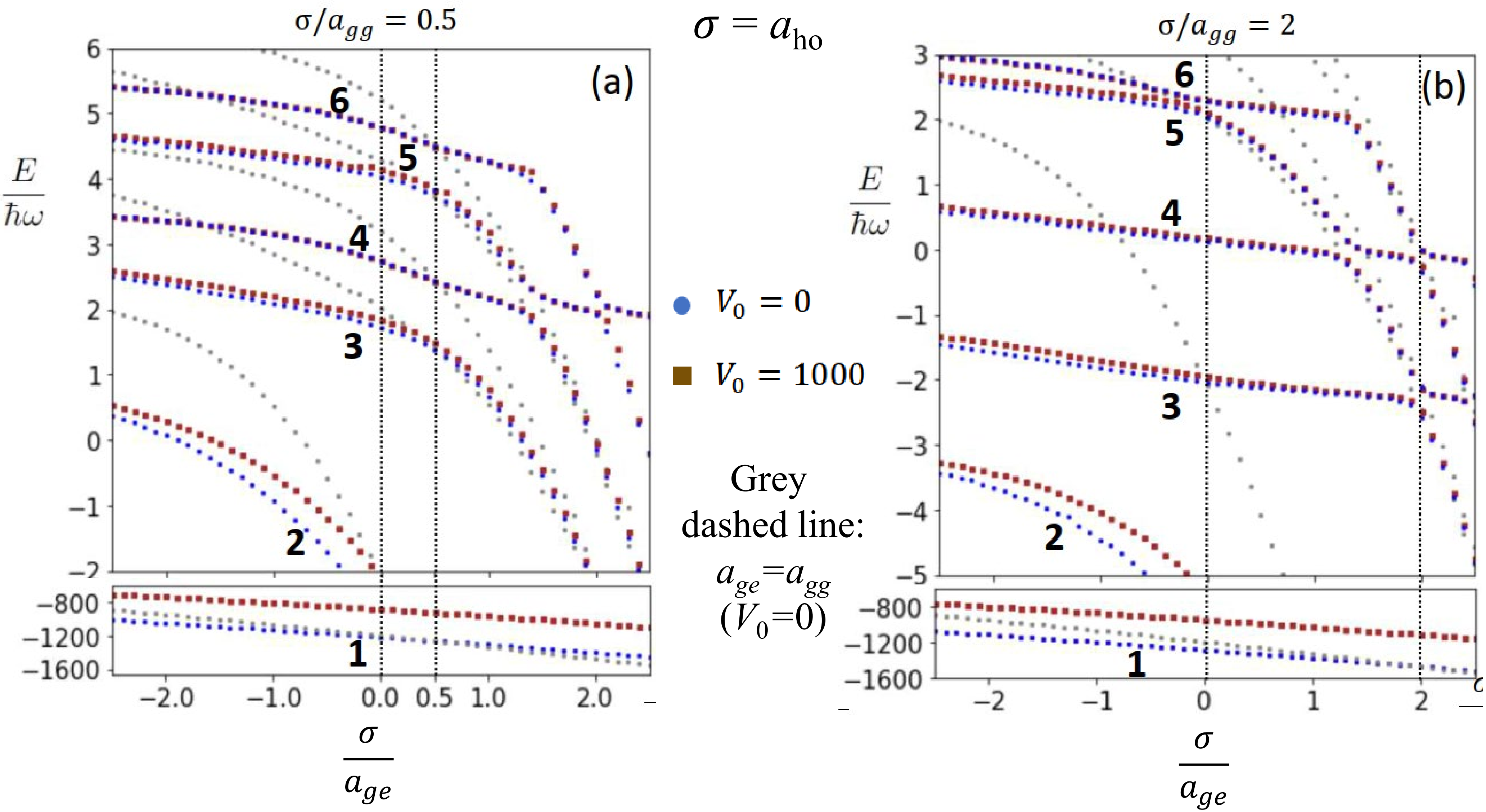
"$^1S_0 \longleftrightarrow {}^3P_2(m=-2)$": $^{170}$Yb




$$a(B) = a_{bg}(1 - \Delta/(B - B_0))$$

$$a_{bg} = 119\, a_0 \quad \Delta = 2.1\text{G} \quad B_0 = 1.1\text{ G}$$

# Calculation of energies of 3 bosons with large $U_{eg}$ (poster presentation by Y. Haruna)



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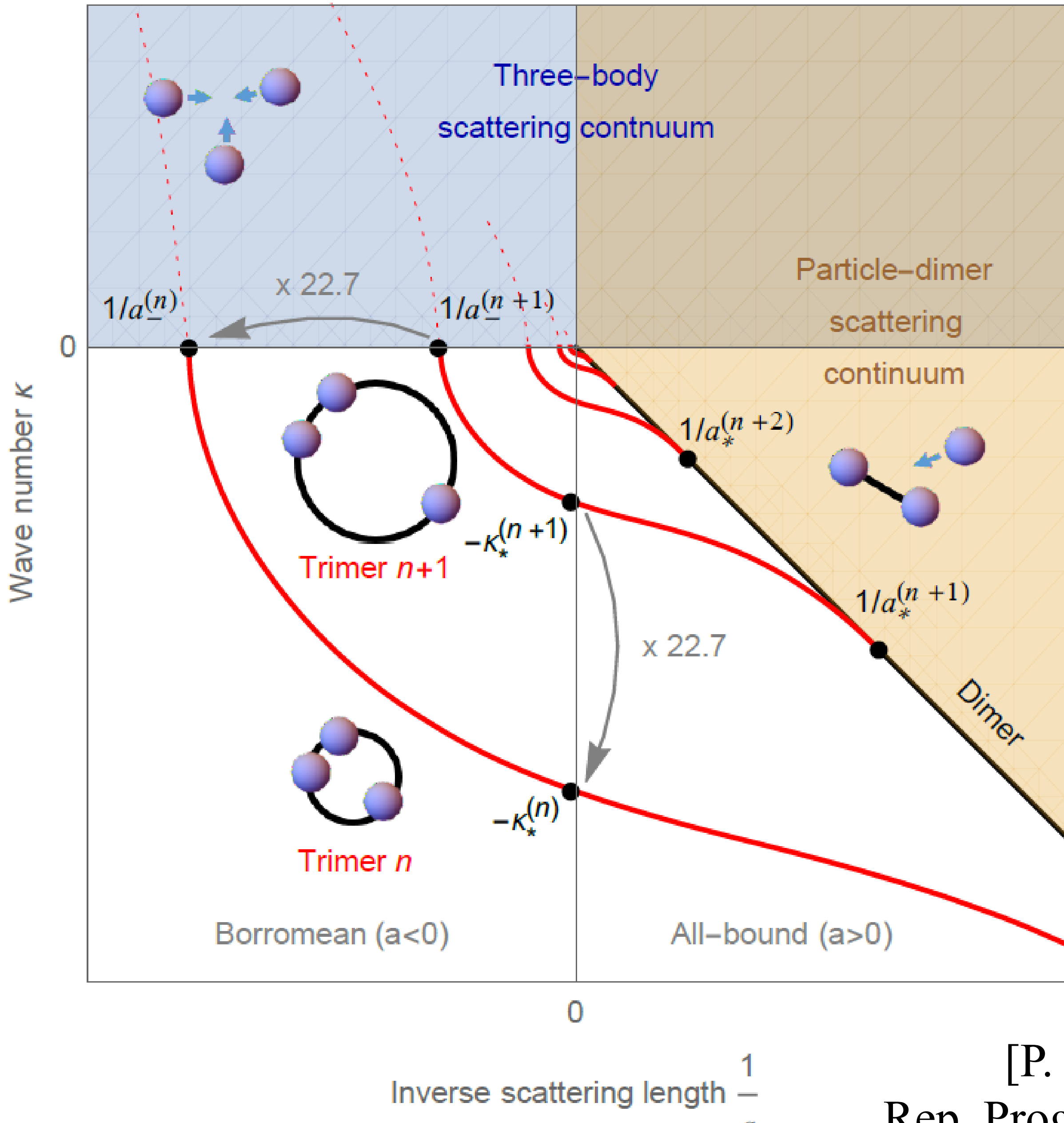
# Efimov Trimer

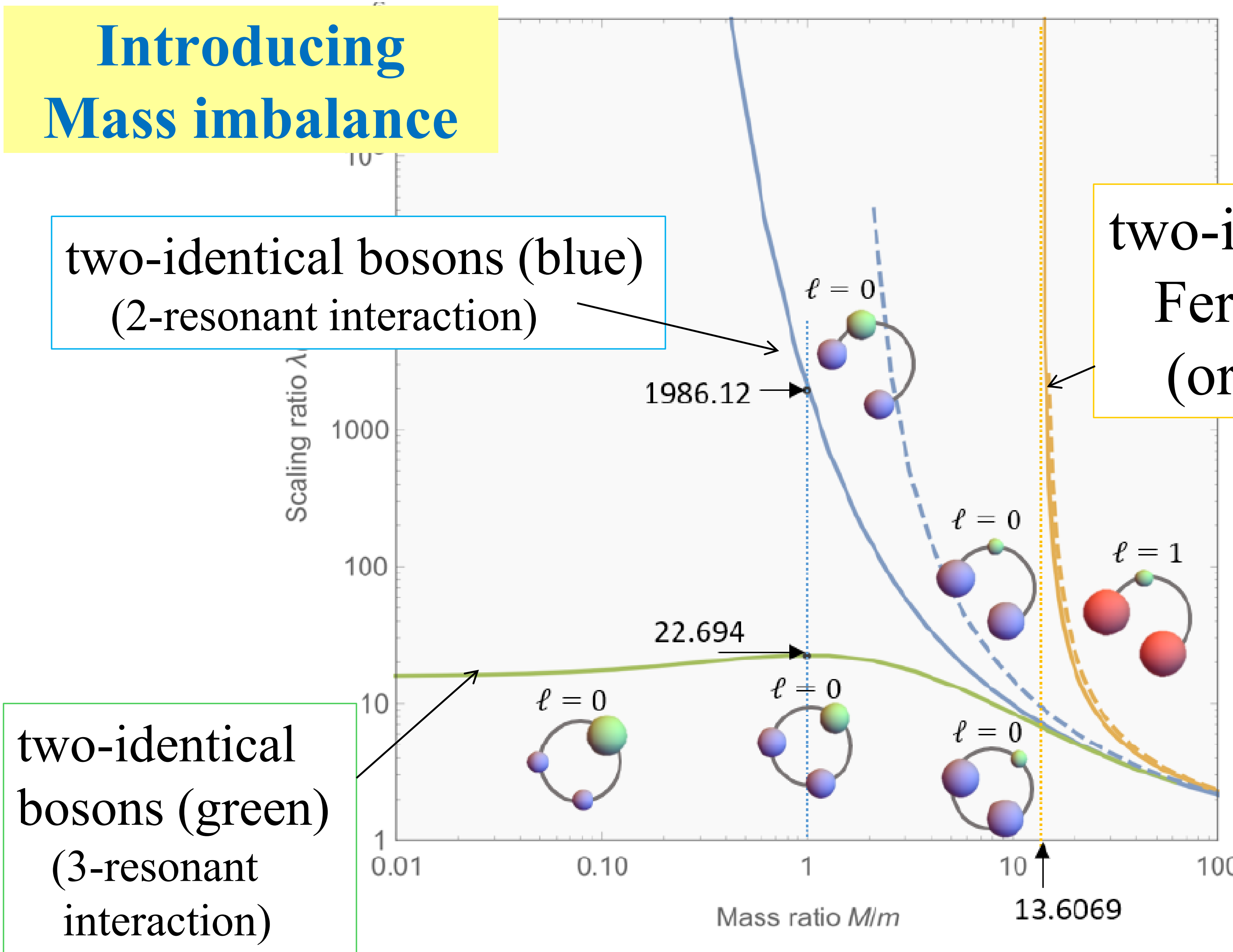



Identical Boson:

$$e^{\pi/s_0} \cong 22.7$$

[P. Naidon and S. Endo, Rep. Prog. Phys. 80, 056001 (2017)]

# Introducing Mass imbalance




**LiErEr**

$e^{\pi/s_0} \cong 5$

$\frac{M_{Er}}{M_{Li}} \cong 28$

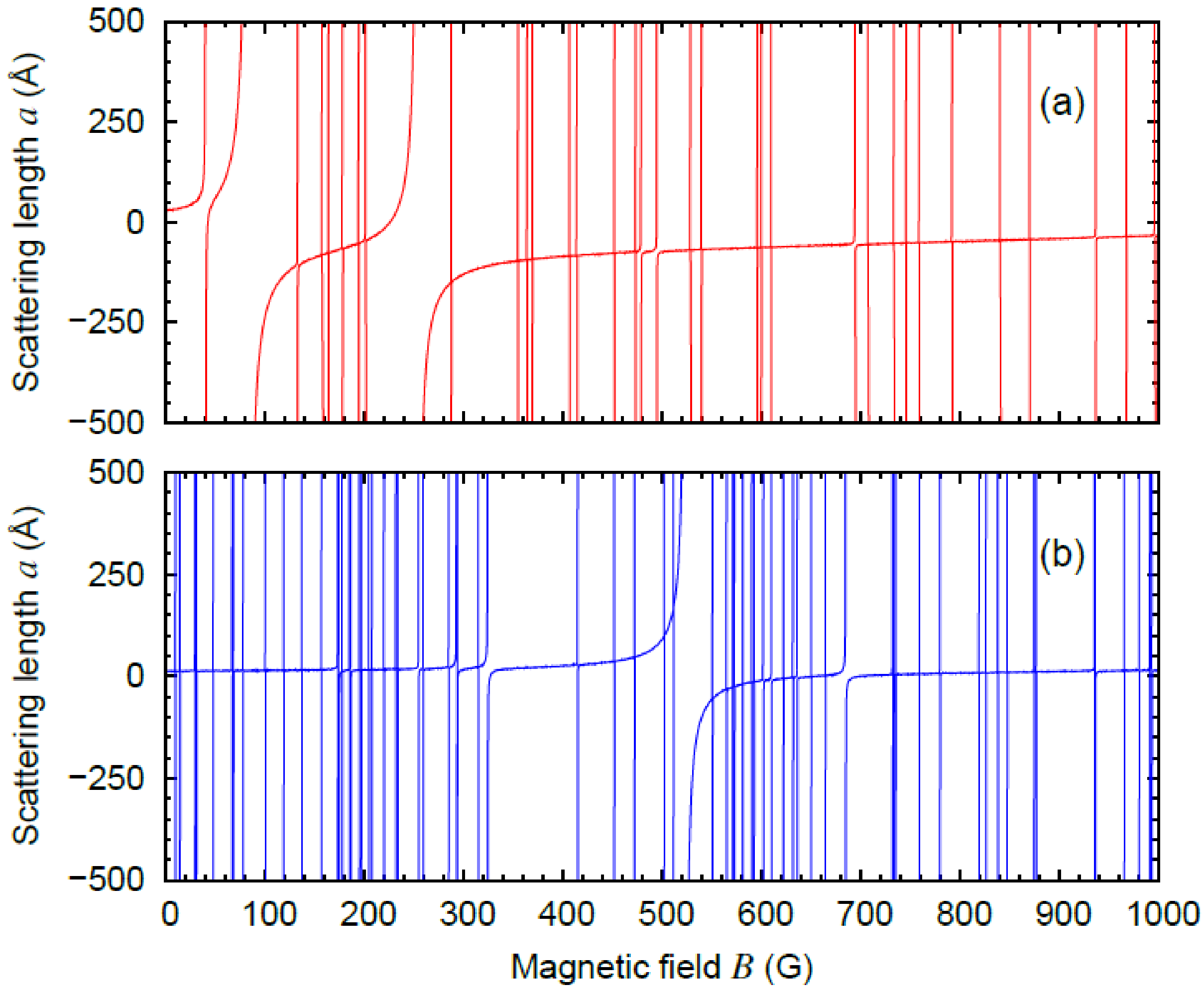
[P. Naidon and S. Endo, Rep. Prog. Phys. 80, 056001 (2017)]

# Magnetically tunable Feshbach resonances in **Li+Er**

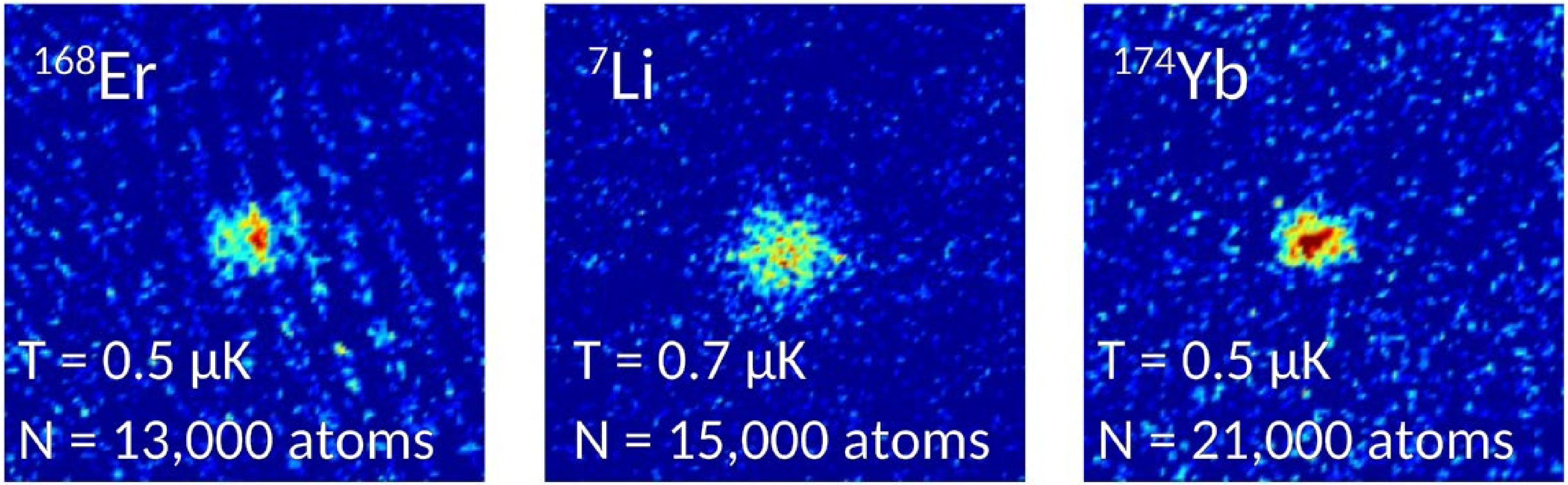
[M. L. Gonzalez-Martinez and Piotr S. Zuchowski, PRA 92, 022708 (2015)]

$^{166}$Er(b) +$^{6}$Li(f)

$^{166}$Er(b) +$^{7}$Li(b)

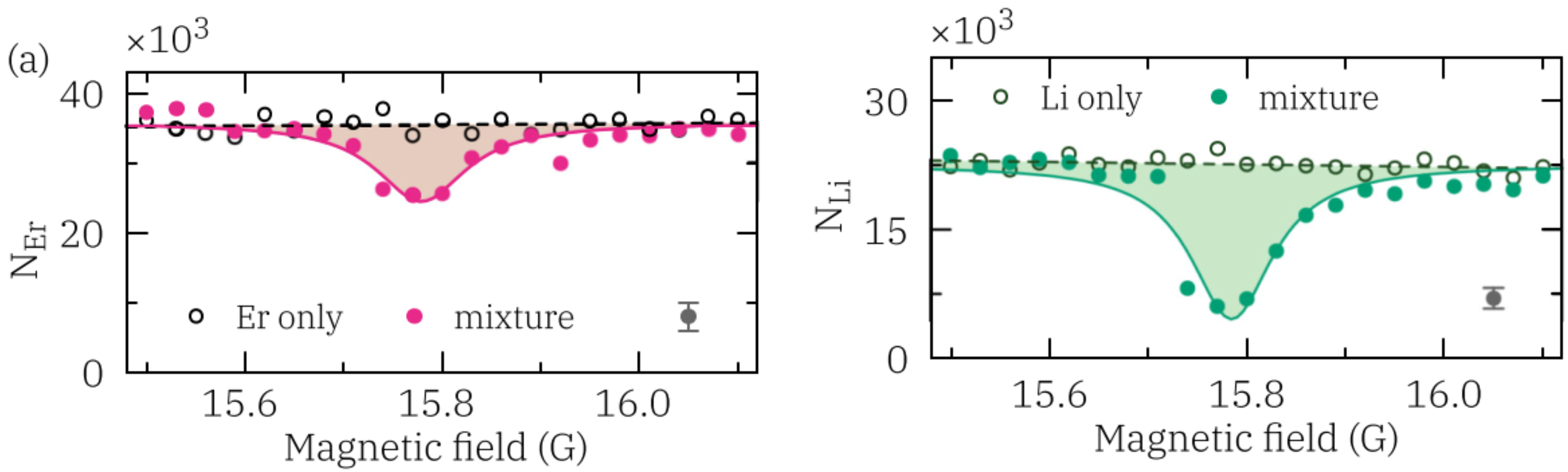


# Realization of ultracold Er-Li (& Yb) mixtures



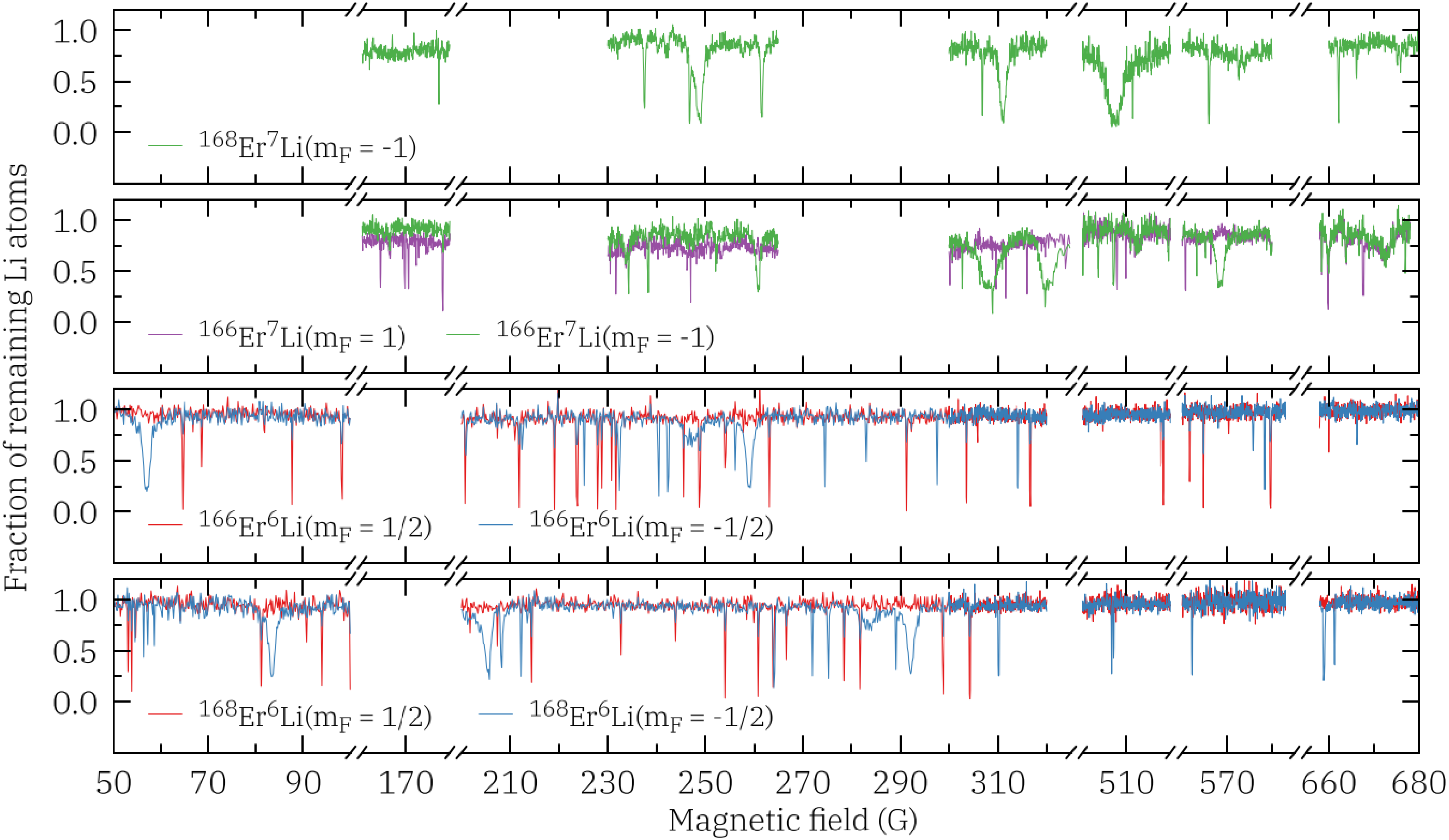

# Observation of $^{168}$Er-$^{7}$Li(F=1, $m_F$=−1) Feshbach resonances

F. Schäfer *et al.*, PRA 105, 012816 (2022)



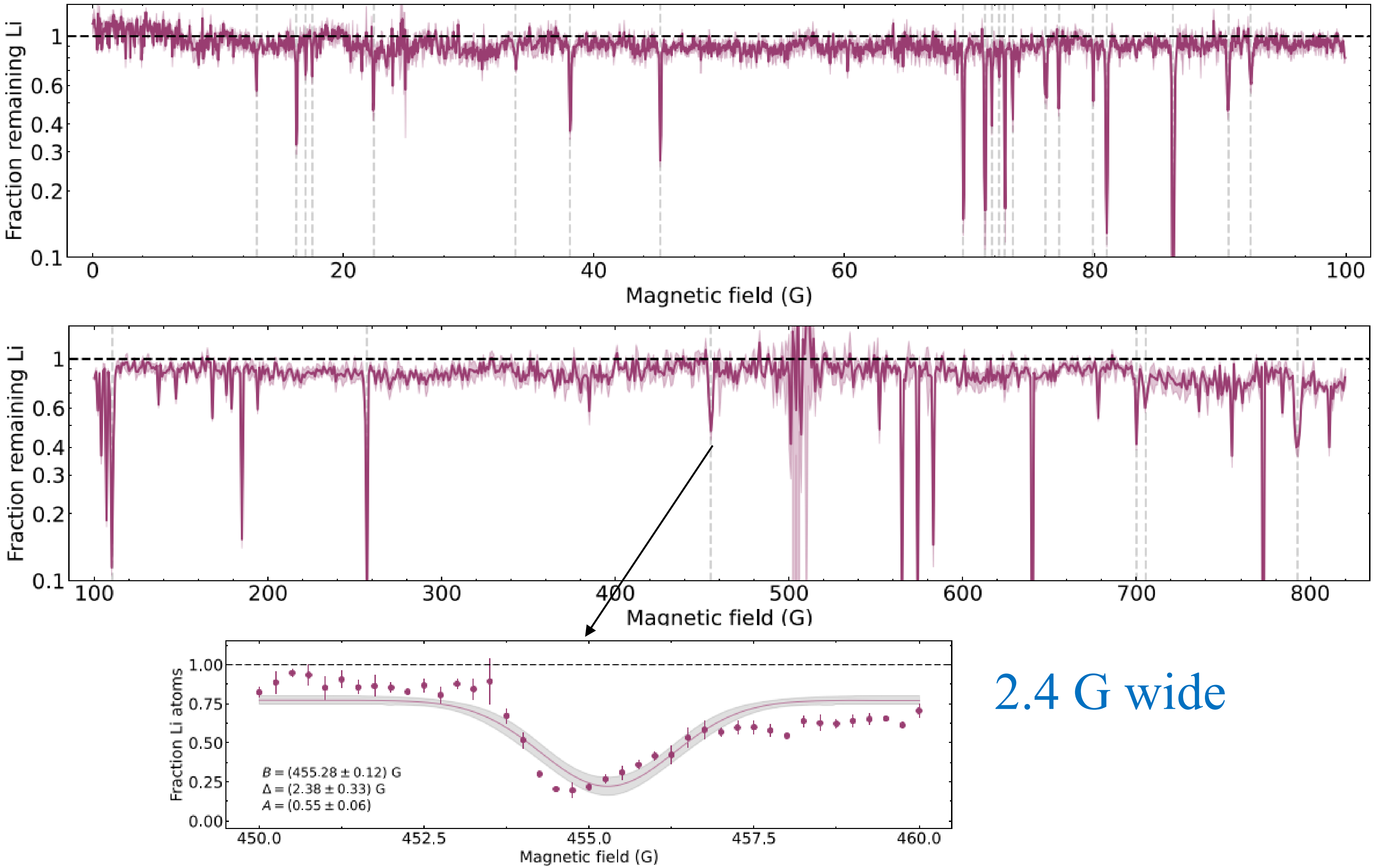
# Observation of $^{168,166}$Er - $^{6,7}$Li Feshbach resonances

F. Schäfer *et al.*, PRA 105, 012816 (2022)



# $^{167}$Er ($m_F = -19/2$) - $^{6}$Li (F=1/2, $m_F$ = 1/2) Feshbach resonances

T= 2~ 4 μK




2.4 G wide

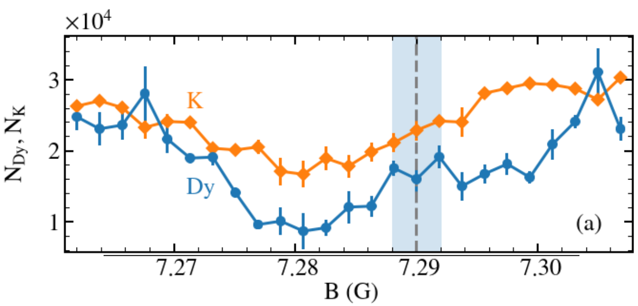
# Ultracold mixture of spin-polarized fermions

## → **Suppression of atom loss due to Pauli exclusion**

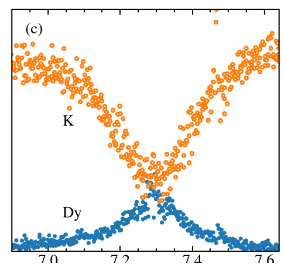
## $^{161}$Dy and $^{40}$K Feshbach resonances (R. Grimm group)

arXiv:2207.03407v3 & PRL 124, 203402 (2020)

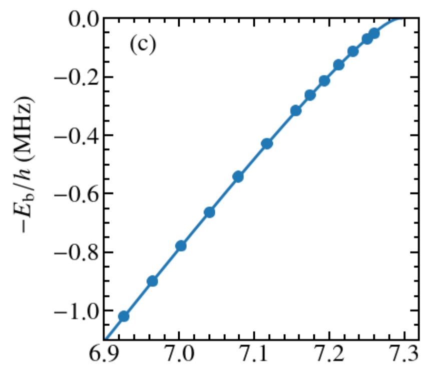
loss spectroscopy



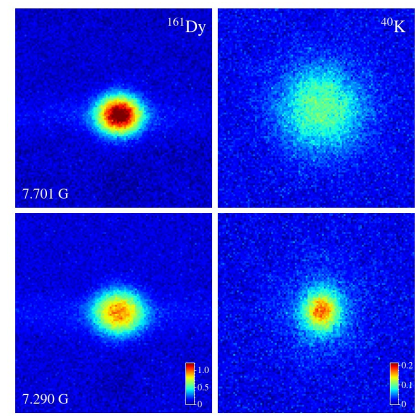

interspecies thermalization




binding energy measurement




hydrodynamic expansion




## Characterization of narrow Feshbach resonances

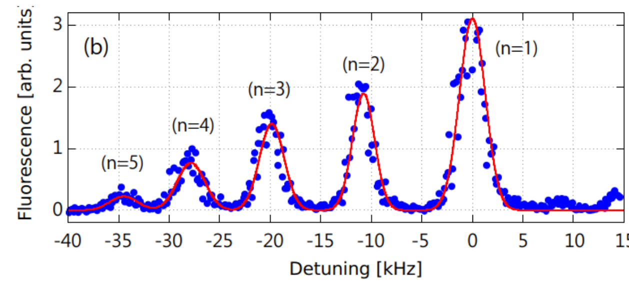
D. S. Petrov, PRL 93, 143201 (2004)

$$a(B) = a_{\rm bg} - \frac{A}{B - B_0}\, a_0 \qquad R^* = \frac{\hbar^2}{2 m_{\rm r} a_0\, \delta\mu\, A} \qquad E_{\rm b} = \frac{\hbar^2}{8\,(R^*)^2\, m_{\rm r}} \left( \sqrt{1 - \frac{4R^*(B - B_0)}{a_0 A}} - 1 \right)^2$$

# Summary

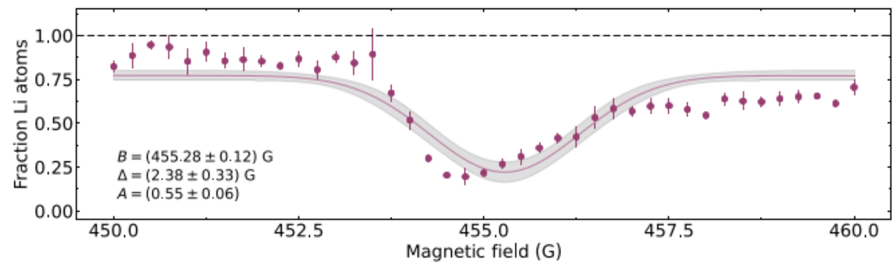
1. Three-body force for atoms in an optical lattice

   occupancy-resolved spectroscopy done
   interpreted by effective 3-body force
   prospects for quantum simulation




2. Feshbach resonances of Er-Li mixture

   successful creation of ultracold Er-Li (&Yb) mixture
   observation of many "narrow" Feshbach resonances
   prospects for careful characterization



# *Thank you very much for attention*

16 August Mount Daimonji at Kyoto

# p-wave Superfluidity

## fermion attraction mediated via BEC in mixed dimension

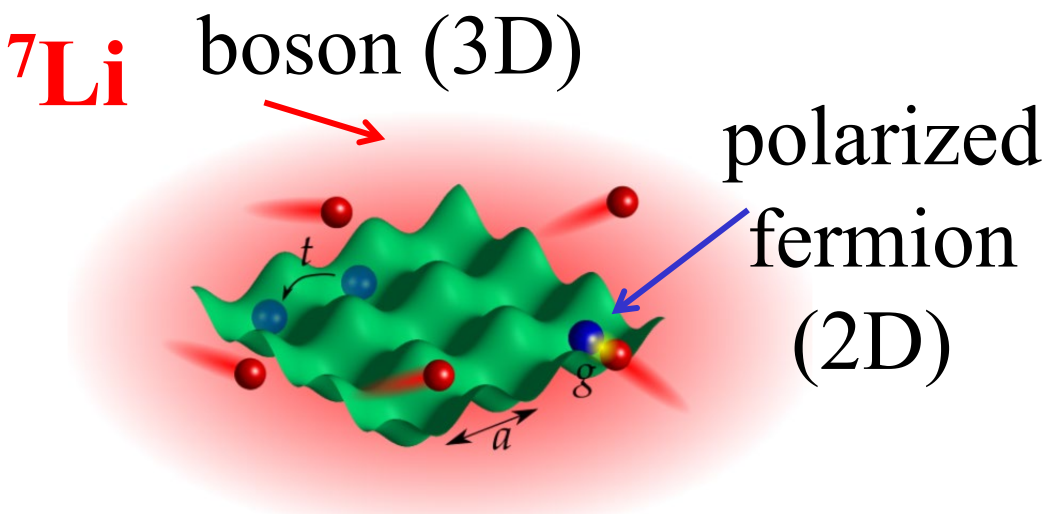
”PRL **117**,245302 (2016) , PRA **94**, 063631(2016), NJP**19**, 115011 (2017), arXiv:1809.04812”

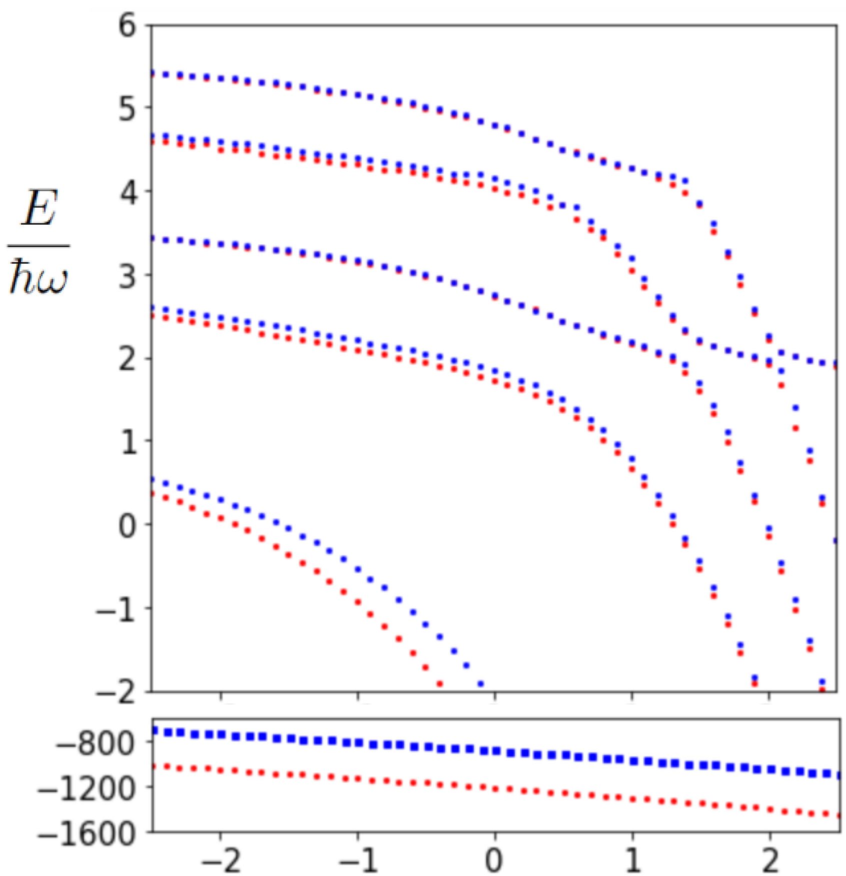
$$V_{\rm ind}(i,j) = -g^2 \frac{n_0 m_B}{\pi} \frac{e^{-\sqrt{2}|\mathbf{r}_i - \mathbf{r}_j|/\xi_{\rm B}}}{|\mathbf{r}_i - \mathbf{r}_j|}$$

$a_{\rm BF}$

$$g = 2\pi a/m_r$$

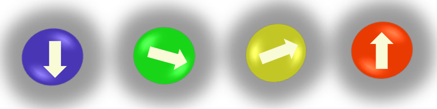
$$m_r = m m_{\rm B}/(m + m_{\rm B})$$

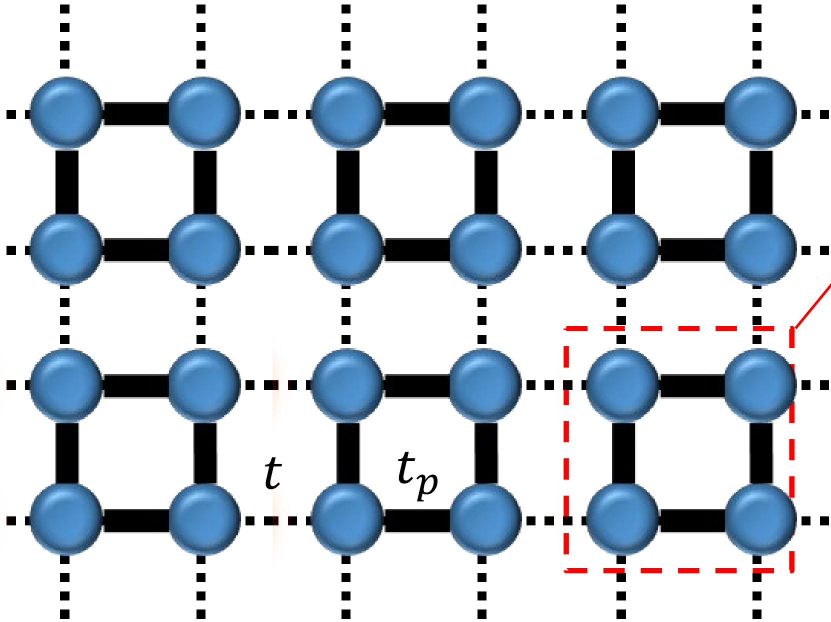




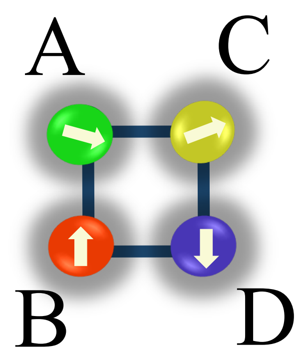
$\frac{E}{\hbar\omega}$
6
5
4
3
2
1
0
−1
−2
−800
−1200
−1600
−2
−1
0
1
2

# SU(4) singlet: 4-body entangled state

**SU(4) in plaquette: $^{173}$Yb** 



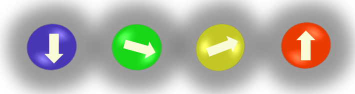

$t/t_p = 0.043$




**SU(4) singlet:** $|SU(4)S\rangle$

$$= \frac{1}{\sqrt{24}} \sum_{\{ijkl\}} c^\dagger_{1i} c^\dagger_{2j} c^\dagger_{3k} c^\dagger_{4l} |0\rangle$$

$i,j,k,l$ = A,B,C,D

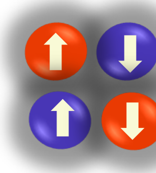
1,2,3,4 = 

$$H_{SU2} = J_{i,j}(\mathbf{S}^i \cdot \mathbf{S}^j - 1/4)$$

$$J_{i,j} = \frac{4t^2}{U}, \quad U \gg t$$

$$H_{SU4} = \sum_{\langle i,j \rangle} J_{i,j} S^n_m(i) S^m_n(j)$$

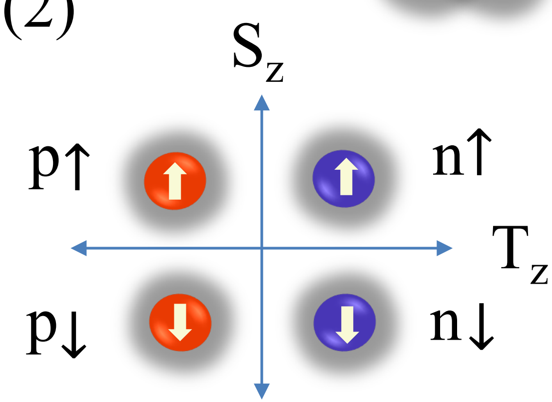
**Alpha particle**: $\boldsymbol{\alpha}$ ($^4He^{2+}$)



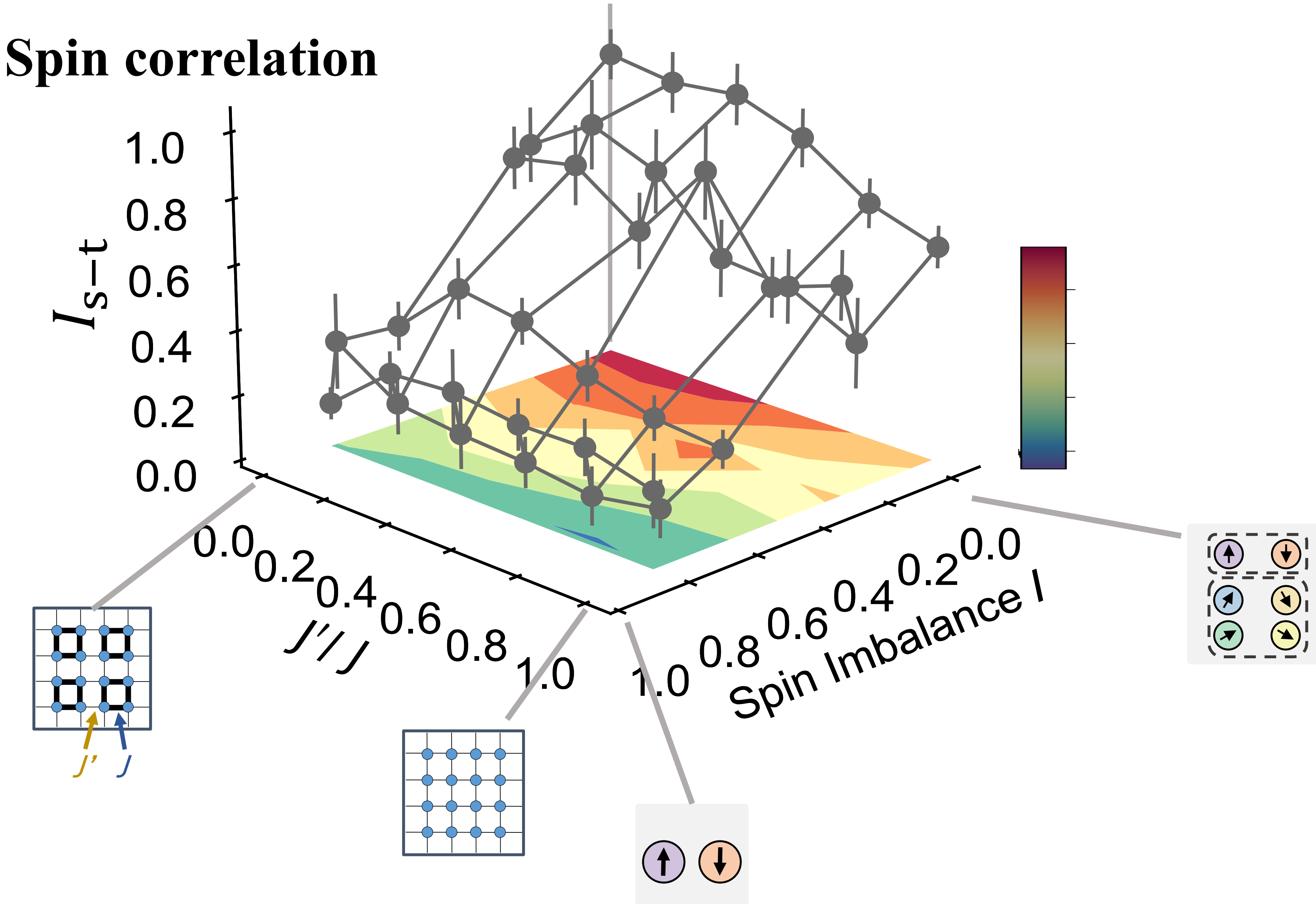
SU(4):SU(2)× SU(2)

S(Spin)=0

T(Isospin)=0



# Robustness of SU(4)-Singlet State against Interaction



# Isotope Shift and King Linear Relation

Delaunay *et al.*, PRD **96** 093001 (2017),
Berengut *et al.*, PRL **120** 091801 (2018)

Isotope shift between isotope pairs ($A'$, $A$) for the transition $\lambda$

$$\nu_\lambda^{A'A} = K_\lambda \delta\mu^{A'A} + F_\lambda \delta\langle r^2\rangle^{A'A} + H_\lambda \delta\eta^{A'A}$$

Mass shift — Field shift — New Particle shift

Higher-order Isotope shift $H_\lambda \delta\eta^{A'A}$

New Particle shift (beyond SM) $\alpha_{\rm NP} X_\lambda (A' - A)$

$$\alpha_{\rm NP} = (-1)^{s+1} y_e y_n / (4\pi\hbar c)$$

$$X_\lambda = \frac{c}{2\pi}\int_0^\infty dr\,\delta\rho_\lambda(r)\frac{e^{-\frac{mcr}{\hbar}}}{r}$$

Higher-Order Field shift (within SM)

Next-leading-Order Seltzer moment: $G_\lambda^{(4)} \delta\langle r^4\rangle^{A'A}$

$$\delta\langle r^4\rangle^{A'A} = \langle r^4\rangle^{A'} - \langle r^4\rangle^{A}$$

Quadratic Field Shift (QFS): $G_\lambda^{(2)} [\delta\langle r^2\rangle^2]^{A'A}$

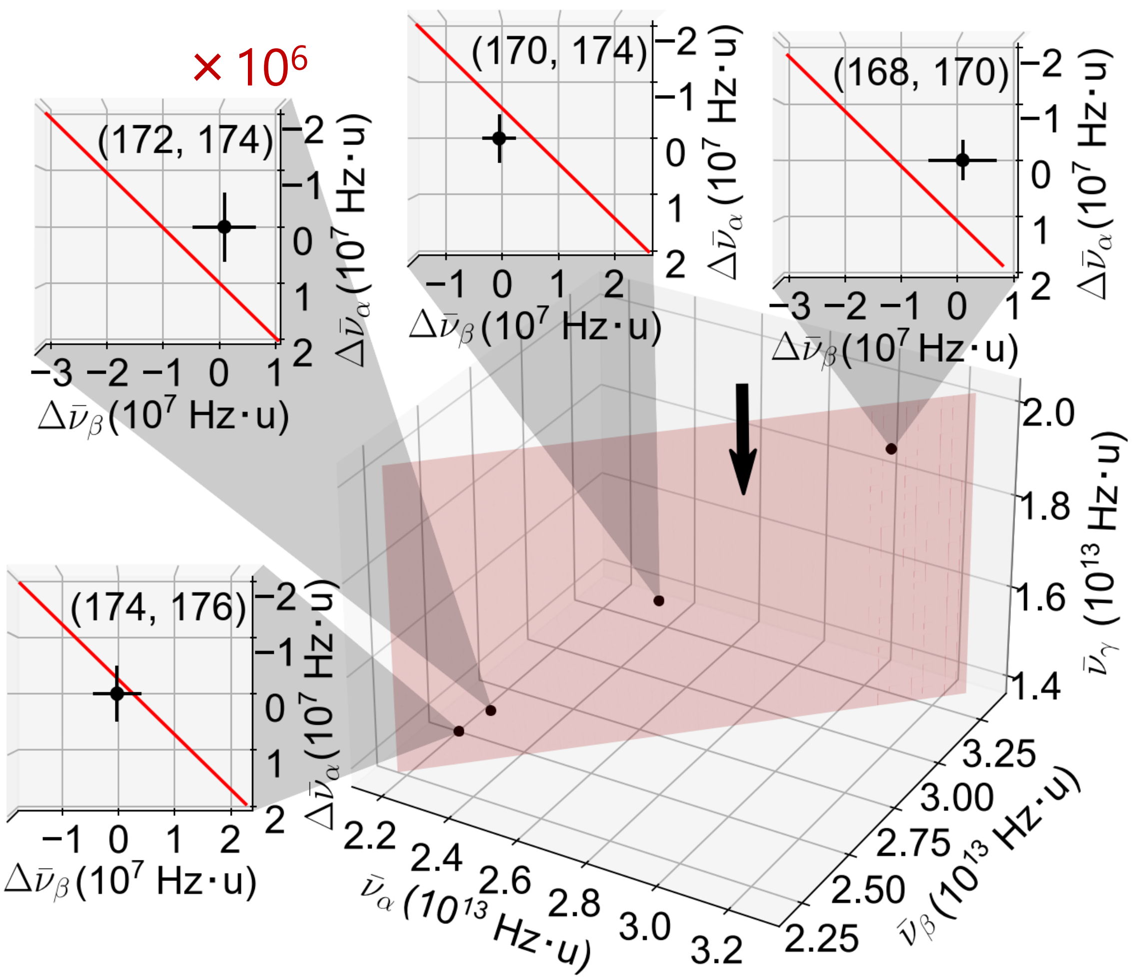
$$[\delta\langle r^2\rangle^2]^{A'A} = (\delta\langle r^2\rangle^{A'A_0})^2 - (\delta\langle r^2\rangle^{AA_0})^2$$

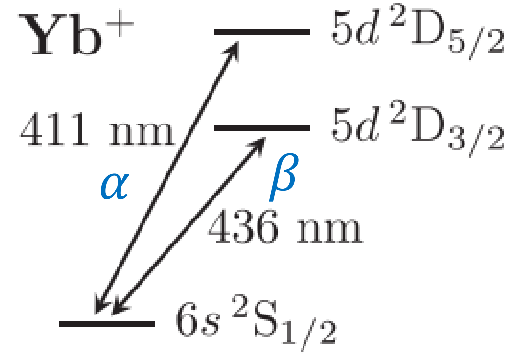
nonlinearity of King relation $\xrightarrow{?}$ signature of physics beyond SM

# Generalized 3D King Plot Analysis

W. H. King, *J. Opt. Soc. Am* **53,** 638-639 (1963) K. Mikami et al., Eur. Phys. J. C 77, 896 (2017)

$\alpha$ : Yb$^+$ $^2S_{1/2} - ^2D_{5/2}$ $\beta$ : Yb$^+$ $^2S_{1/2} - ^2D_{3/2}$ $\gamma$ : Yb $^1S_0 - ^3P_0$




$$\chi^2_{[\gamma,\alpha,\beta]} = 15 \quad (p = 2.3 \times 10^{-3})$$

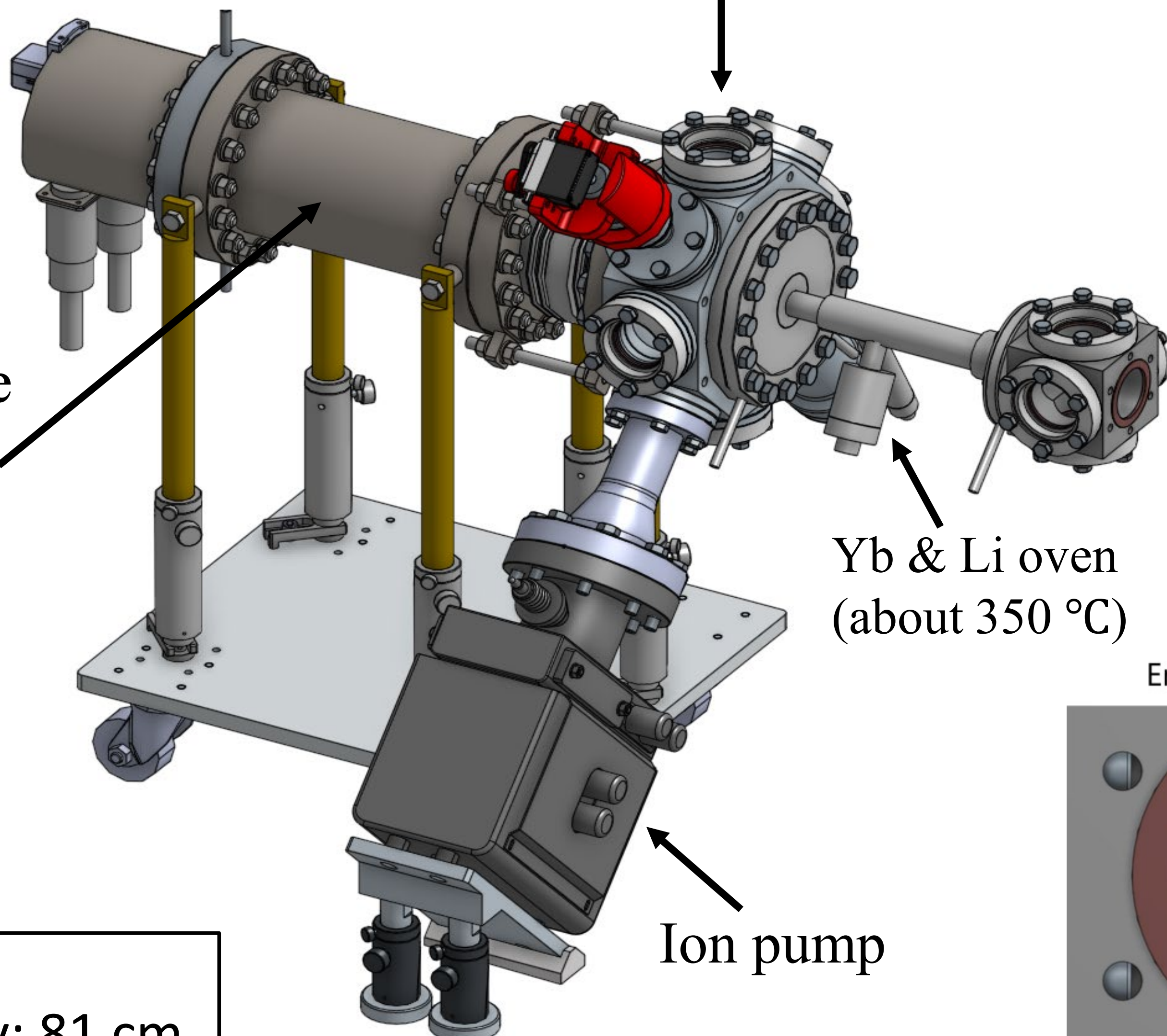
(dof = 3)

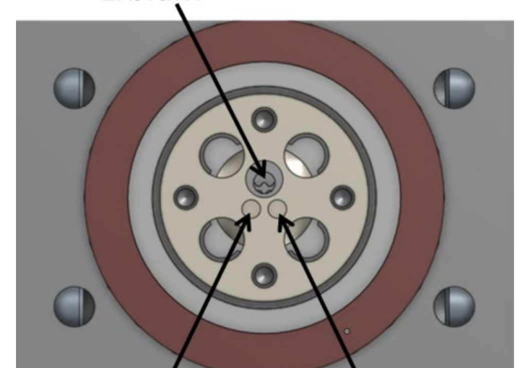
→ Nonlinearity @ $3\sigma$

Determine the upper-bound of new boson

[arXiv:2110.13544, K. Ono, *et al.,to appear in Phys. Rev. X*]

# Er-Li-Yb Oven

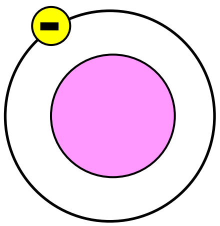



total length of
oven assembly: 81 cm

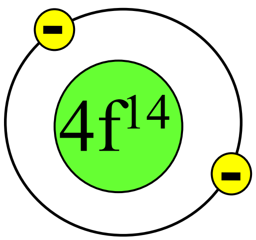
# Quantum Gas Mixtures with Unequal Mass

**Lithium(Li)** : [He] $2s^1$ : light



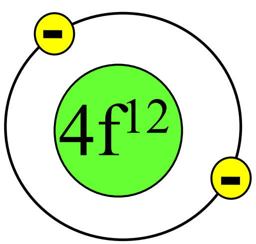
| Mass number | 6 | 7 |
|---|---|---|
| Abundance[%] | 7.5 | 92.5 |
| Bose or Fermi | F | B |

**Ytterbium(Yb)** : [Xe]$4f^{14}6s^2$ : heavy




| Mass number | 168 | 170 | 171 | 172 | 173 | 174 | 176 |
|---|---|---|---|---|---|---|---|
| Abundance[%] | 0.13 | 3.1 | 14.3 | 21.9 | 16.1 | 31.8 | 12.7 |
| Bose or Fermi | B | B | F | B | F | B | B |

**Erbium(Er)** : [Xe]$4f^{12}6s^2$ : heavy



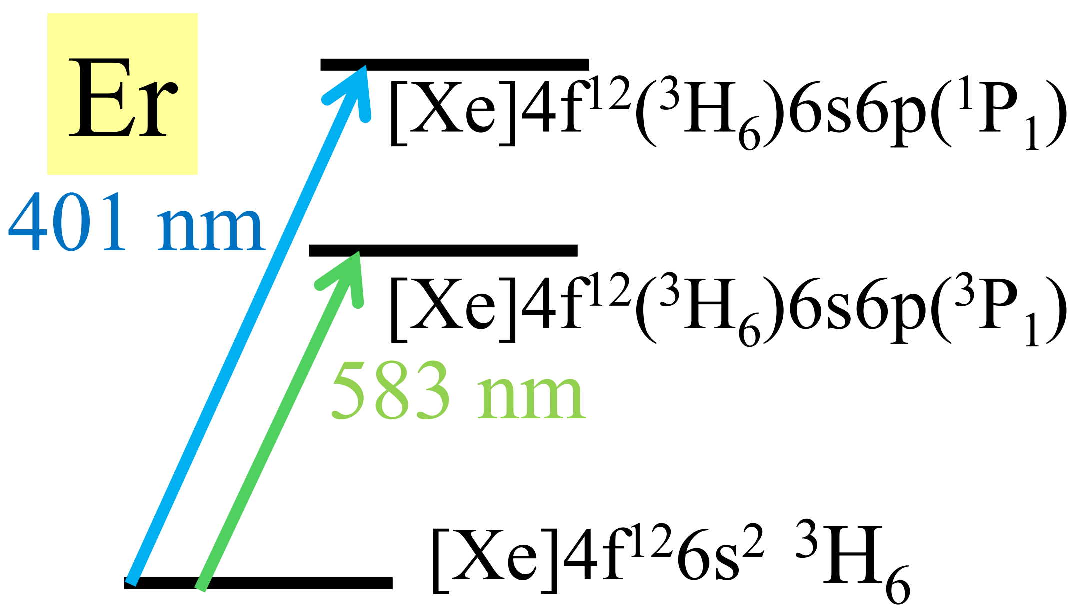

| Mass number | 162 | 164 | 166 | 167 | 168 | 170 |
|---|---|---|---|---|---|---|
| Abundance[%] | 0.1 | 1.6 | 33.5 | 22.9 | 27.0 | 14.9 |
| Bose or Fermi | B | B | B | F | B | B |

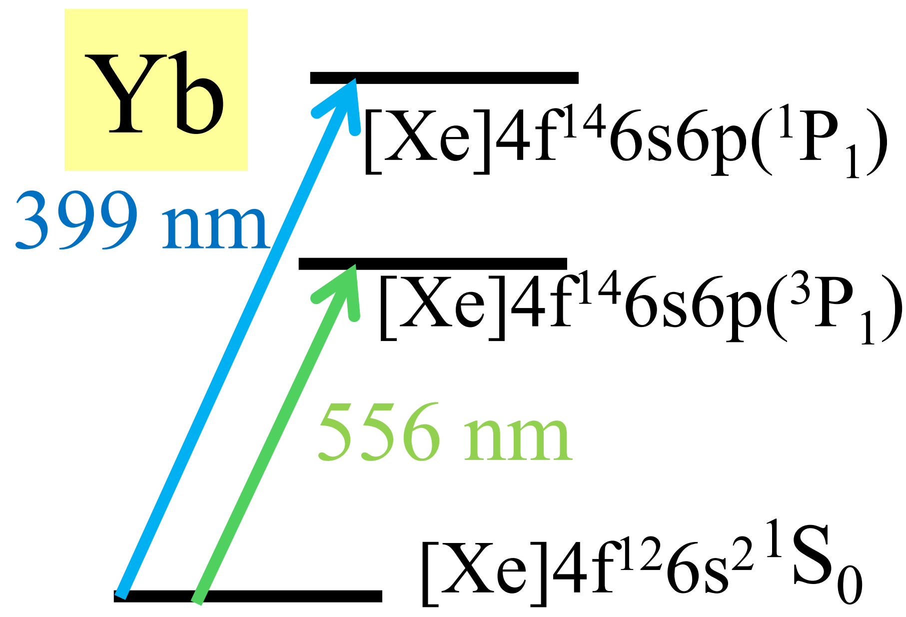
# Er: $[Xe]4f^{12}6s^2$ (Yb: $[Xe]4f^{14}6s^2$)

| mass $m$ [amu] | abundance [%] | nuclear spin $I$ | statistic |
|---|---|---|---|
| 162 | 0.1 | 0 | boson |
| 164 | 1.6 | 0 | boson |
| 166 | 33.5 | 0 | boson |
| 167 | 22.9 | 7/2 | fermion |
| 168 | 27.0 | 0 | boson |
| 170 | 14.9 | 0 | boson |

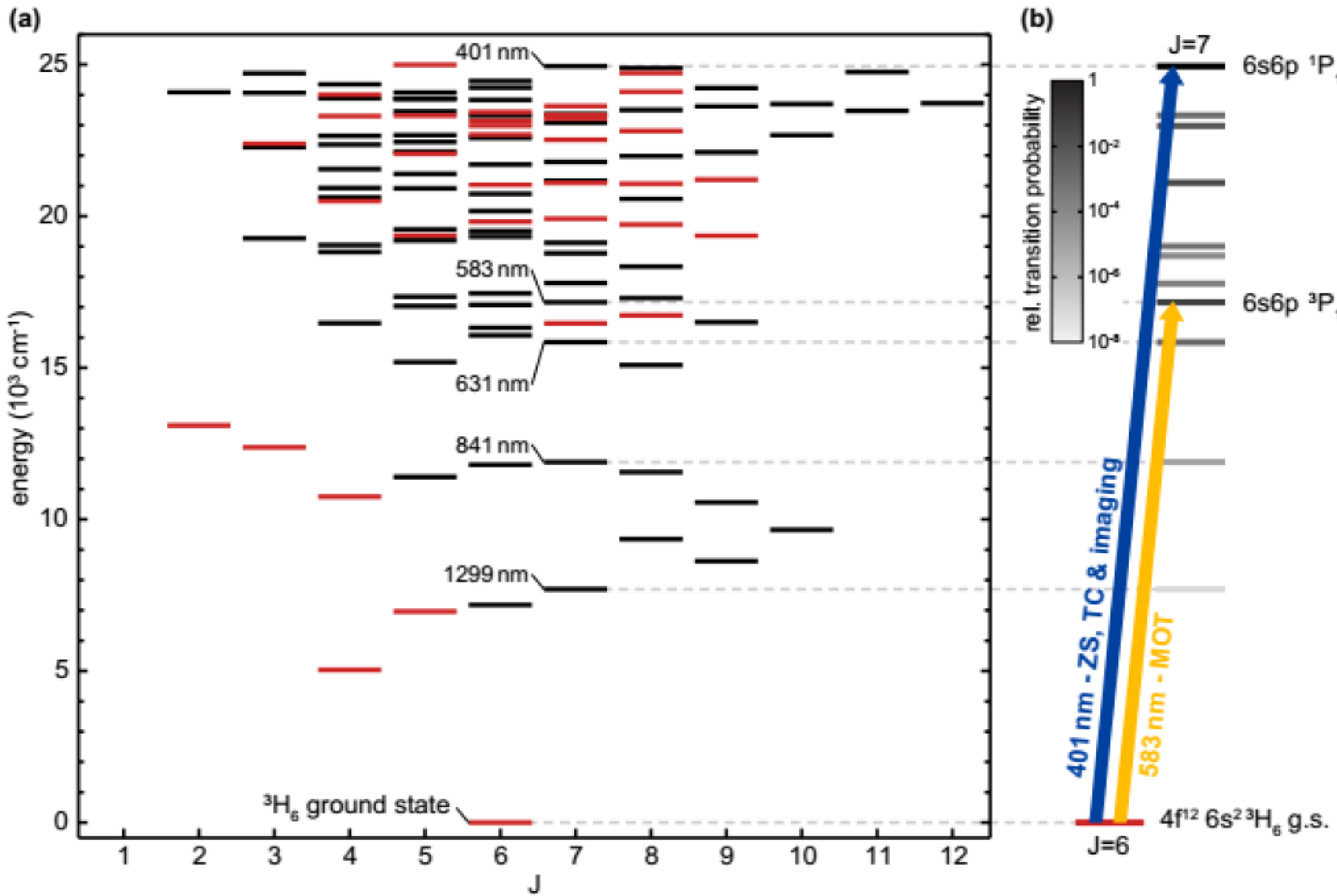
from
J. Schindler

M(Er)/m(Li)= 23.14 ~ 28.3



# Er level scheme




PhD thesis A. Frisch, Innsbruck (2014)