Composite-boson approach to ultracold interacting Fermi gases

Ana Majtey

Clusters & Hierarchies

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Composite-boson approach to ultracold interacting Fermi gases

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Clusters & Hierarchies

Is it experimentally possible to generate multiparticle entangled states by splitting an ultracold interacting Fermi gas?

How strong is the generated entanglement?

Does this entanglement have observable consequences?

Which are the effects of quantum depletion on the phase estimation of two interfering mBECs?

PHYSICAL REVIEW A **99**, 063601 (2019)

Entanglement between two spatially separated ultracold interacting Fermi gases

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PAPER

Molecular interferometers: effects of Pauli principle on entangled-enhanced precision measurements

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Keywords: molecular BEC, fermion interference, quantum metrology

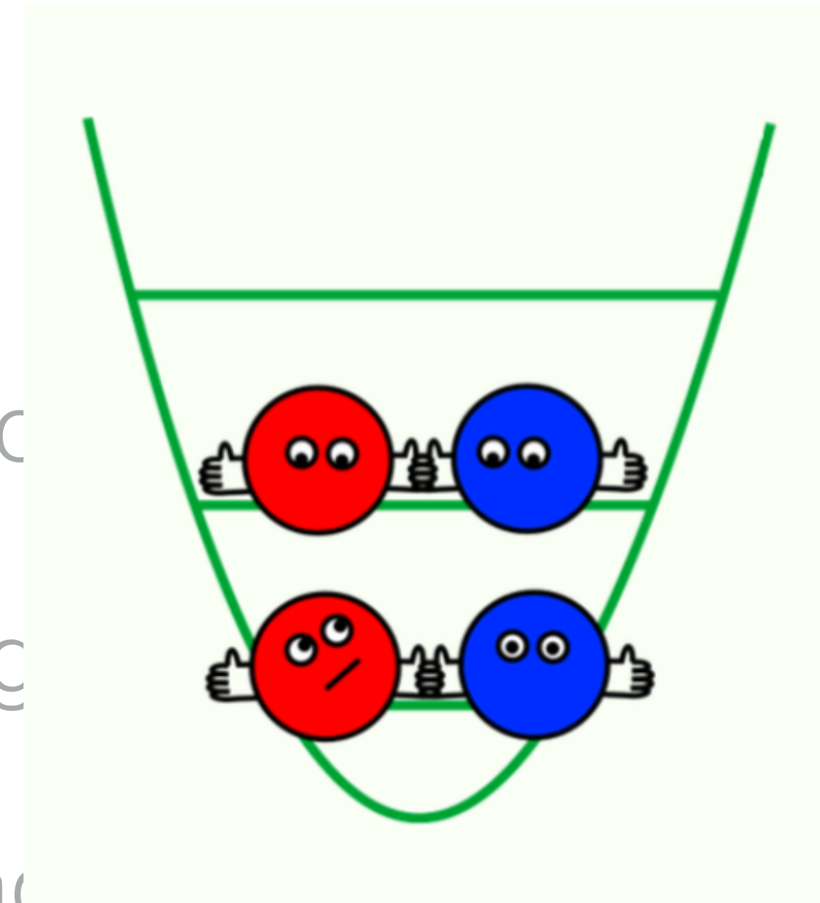
Outline

- Composite boson formalism
- Composite-boson approach to molecular BECs
- Entanglement between interacting Fermi gases
- Effects of Pauli principle on entangled- enhanced precision measurements

Clusters & Hierarchies

Outline

- **Composite boson formalism**
- Composite-boson approach to mo
- Entanglement between interacting
- Effects of Pauli principle on entang enhanced precision measurements



Clusters & Hierarchies

Elementary particles are fermions

- Take composite particles with even number of fermions. Exchange two of these particles, and the wave function must be symmetric.
- How can we have Bose-Einstein condensation if particles can't really occupy the same state

How can bosons made of fermions condensate?

Towards a rigorous treatment of composite bosons

Composite boson formalism

Combescot , Tanguy, Europhys. Lett. **55**, 390 (2001)
fermions

Leggett, Rev. Mod. Phys. **73**, 307 (2001)
bosons

Law, Pays. Rev. A **71**, 034306 (2005)
quantum information approach

For a composite particle,

“boson-like behaviour” is a **state-dependent** property

Clusters & Hierarchies

Towards a rigorous treatment of composite bosons

[Combescot and Tanguy, Europhys. Lett. 55, 390 (2001)]

- Take a "bosonic" exciton with creation operator $c^\dagger$, so that $|\psi\rangle = c^\dagger|0\rangle$ is the single-exciton state with $|0\rangle$ the vacuum.
- To see how bosonic an exciton is, analyze
  - boson departure: $1 - [c, c^\dagger]$
  - boson number: $c^\dagger c$
  - normalization of the N-exciton state $(c^\dagger)^N|0\rangle$
- Pauli exclusion is an intrinsically N-body phenomenon - the density dependence of corrections is non-trivial.

Towards a rigorous treatment of composite bosons

PHYSICAL REVIEW A **71**, 034306 (2005)

Quantum entanglement as an interpretation of bosonic character in composite two-particle systems

C. K. Law
Department of Physics, The Chinese University of Hong Kong, Shatin, Hong Kong SAR, China
(Received 31 October 2004; published 21 March 2005)

We consider a composite particle formed by two fermions or two bosons. We discover that composite behavior is closely related to the quantum entanglement between the constituent particles. By analyzing the properties of creation and annihilation operators, we show that bosonic character emerges if the constituent particles become strongly entangled. Such a connection is demonstrated explicitly in a class of two-particle wave functions.

We focus on a particle made of two distinguishable fermions:

$$|1\rangle = \sum_{\alpha=1}^{S} \sqrt{\lambda_\alpha}\, |\alpha\rangle_a \otimes |\alpha\rangle_b \quad \Rightarrow \quad c^\dagger = \sum_{\alpha=1}^{S} \sqrt{\lambda_\alpha}\, a_\alpha^\dagger b_\alpha^\dagger$$

$$|N\rangle = \frac{\left(c^\dagger\right)^N}{\sqrt{N!\,\chi_N}}|0\rangle.$$

$\chi_2 = 1 - P$, with P the purity of the reduced density matrix of one of the constituent particles of a pair in the state $c^\dagger|0\rangle$.

Clusters & Hierarchies

Entanglement and ideal boson behaviour

Law, Pays. Rev. A **71**, 034306 (2005)

Chudzicki, Oke, and Wootters, Phys. Rev. Lett. **104**, 070402 (2010)

- The origin of composite behavior relies on entanglement

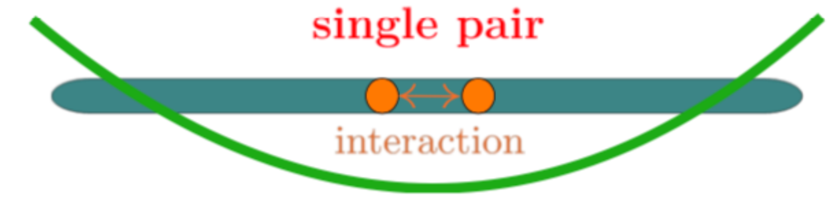
- The composite particles can be treated as bosons when they are sufficiently entangled

- The effective number of available single-particle states >> total number of composite particles

- The available space to each pair >> pair size

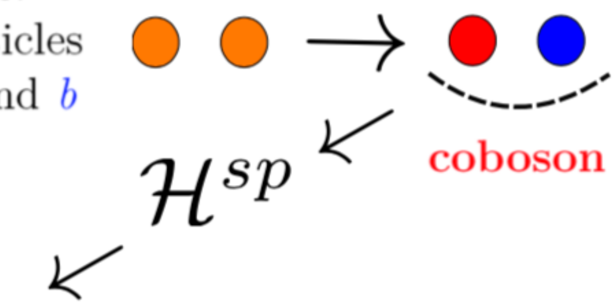
- Binding forces are not essential, they constitute the physical means to enforce quantum correlations

Composite boson (coboson) ansatz – C. K. Law's formulation



- The unpolarized state of an even number of particles of distinct species is built upon the two-particle state
- The description reveals the quantum correlations by showing explicitly the pairing structure
- The observables are obtained in terms of the single-particle states (intuitive)
- The particle distribution among the single-particle states follows the Pauli exclusion principle

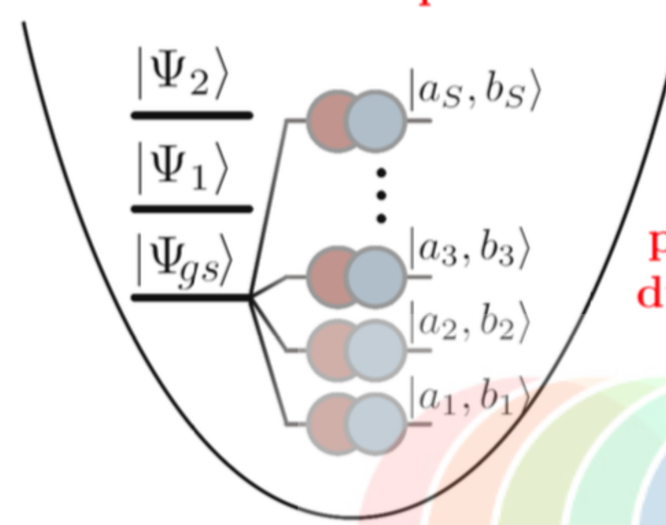
Feshbach molecule:
distinguishable particles
distinct species a and b



Ground state decomposition

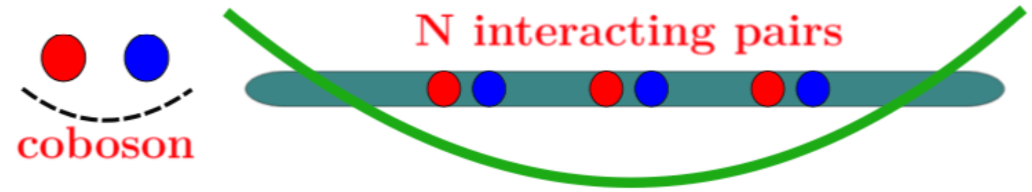
$$\Psi\left(x_a, x_b\right) = \sum_{j}^{\infty} \sqrt{\lambda_j}\,\phi_j^{(a)}(x_a)\,\phi_j^{(b)}(x_b)$$

occupations NO



pairing structure due to interaction (mixing in the single particle states)

Composite boson (coboson) ansatz – C. K. Law's formulation



- The 2N- particle state is built upon the two-particle state

$$c^{\dagger} = \sum_{j}^{\infty} \sqrt{\lambda_j} a_j^{\dagger} b_j^{\dagger}$$ coboson creation operator

- The description reveals the quantum correlations by showing explicitly the pairing structure

$$|\Psi_{gs}\rangle = c^{\dagger}|0\rangle = \sum_{j}^{\infty} \sqrt{\lambda_j} a_j^{\dagger} b_j^{\dagger}|0\rangle$$

- The observables are obtained in terms of the single-particle states (intuitive)

$$|N\rangle = \frac{1}{\sqrt{N! \chi_N^F}} \left(\hat{c}^{\dagger}\right)^N |0\rangle$$

- The particle distribution among the single-particle states follows the Pauli exclusion principle

$$\chi_N^F = N! \sum_{\{j_1, j_2, \ldots, j_N\}'} \lambda_{j_1} \lambda_{j_2} \ldots \lambda_{j_N}$$

normalization constant

↓

The many-particle physics arises from the state of a single pair and from the fermionic exchange interactions

$$[c, c^{\dagger}] = 1 \qquad \frac{\chi_{N+1}}{\chi_N} \to 1$$

Cobosons behave like ideal independent and non-interacting bosons

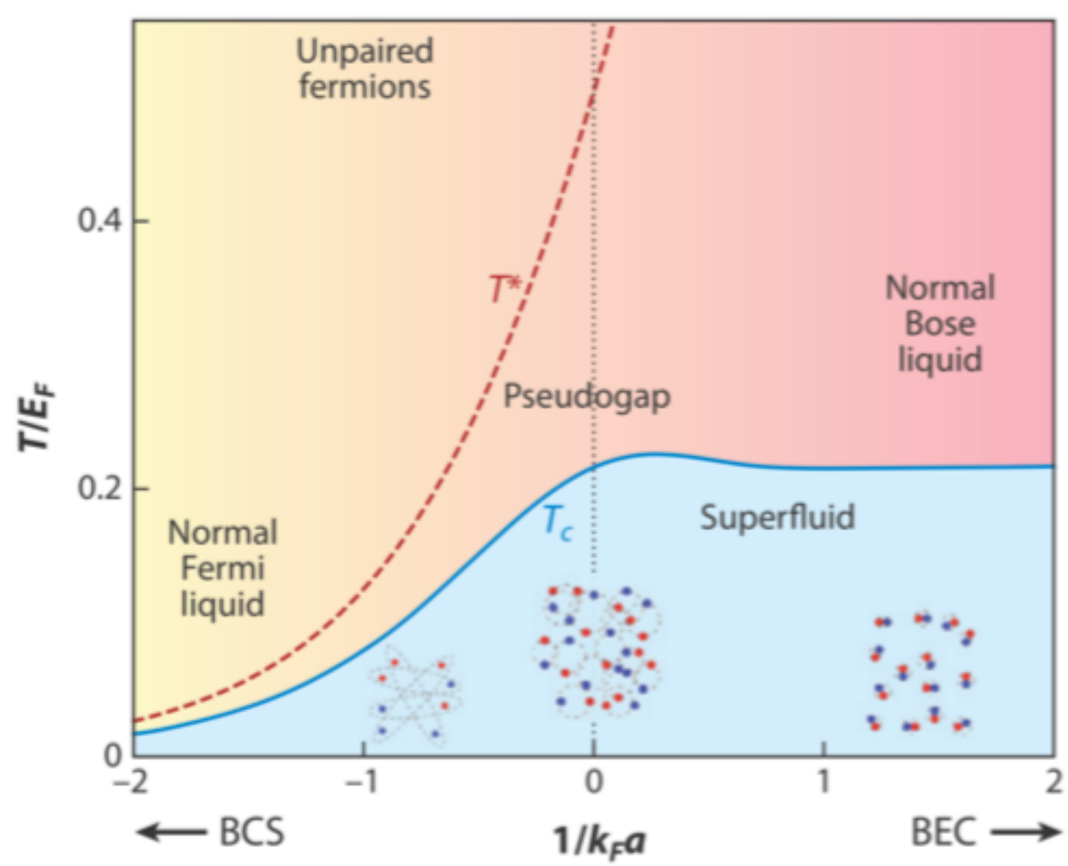
Outline

- Composite boson formalism
- **Composite-boson approach to molecular BECs**
- Entanglement between interacting Fermi gases
- Effects of Pauli principle on entangled- enhanced precision measurements

Clusters & Hierarchies

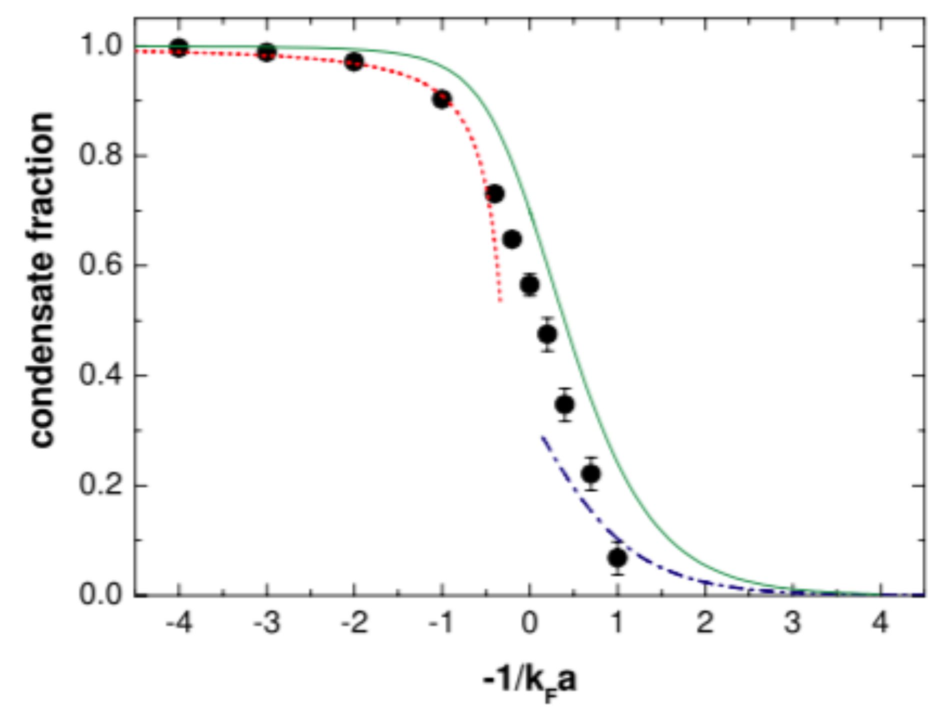
Ultracold Fermi gases

Annu. Rev. Condens. Matter Phys. 5, 209 (2014)



$$k_F = (6\pi^2 n)^{1/3} \text{ with } n = N/V$$

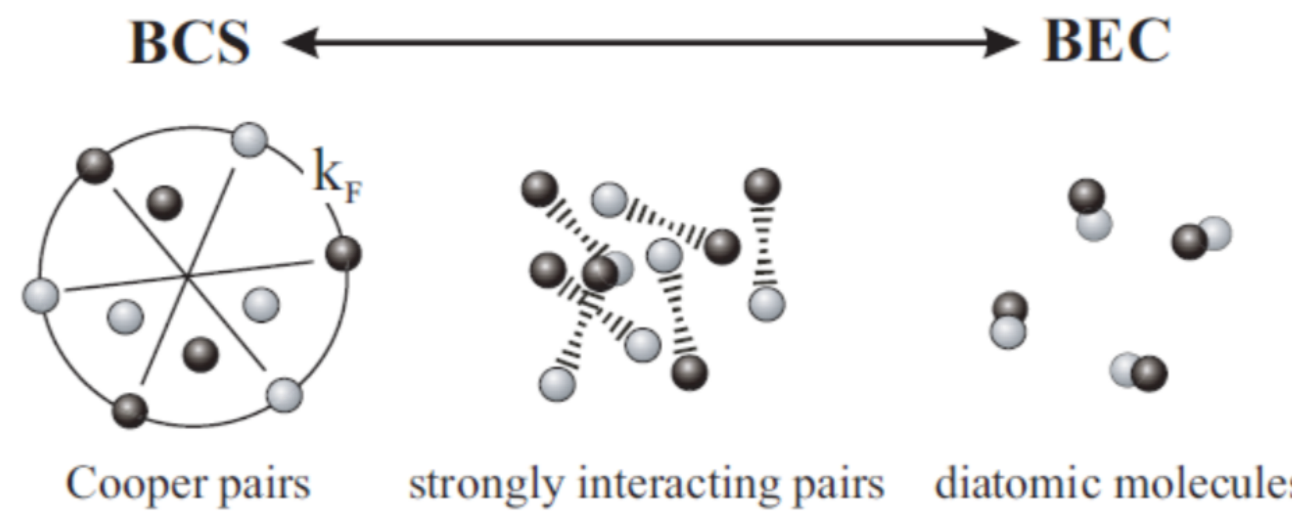
Phys. Rev. Lett. 95, 230405 (2005)



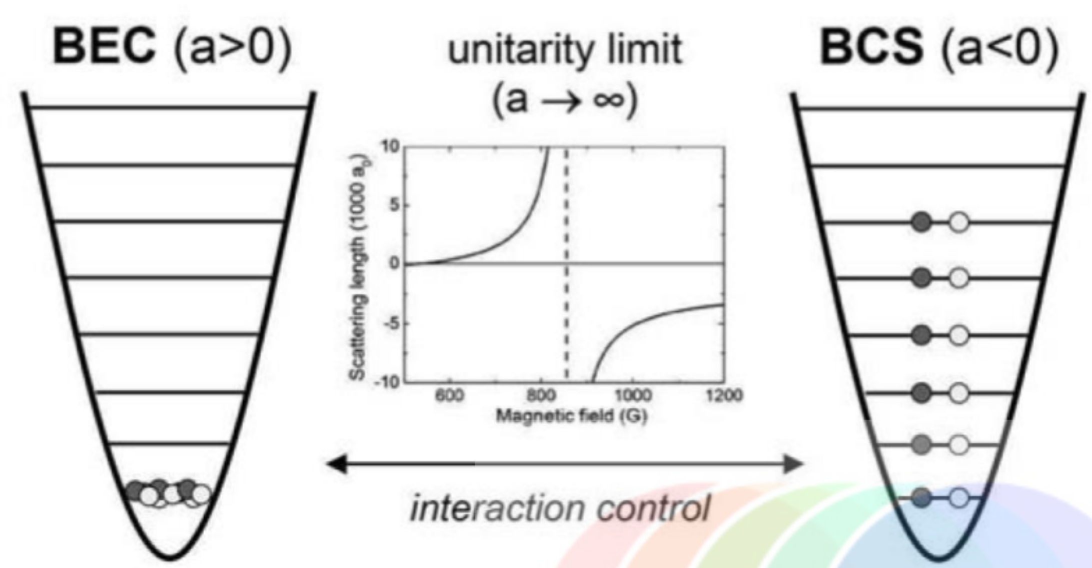
Condensation of molecules at T=0

Ultracold Fermi gases and the BEC-BCS Crossover

- Ultracold gases: tunable interactions (magnetic fields, Feshbach Resonances) and highly controllable confinement
- Fermi gases: ^{6}Li and ^{40}K, the fermions can pair to form composite bosons
- BEC-BCS Crossover: smooth change from small bosonic molecules whose ground state at *T= 0* is a BEC to large and strongly overlapping Cooper pairs
- The crossover can be “passed” in an adiabatic and reversible way by controlling the scattering length a by means of the magnetic field



(D.S. Jin and C. Regal, *Ultracold Fermi Gases, Enrico Fermi School 2006*)



(R. Grimm, *Ultracold Fermi Gases, Enrico Fermi School 2006*)

Feshbach molecule

$$H = -\frac{\hbar^2}{2m}(\vec{\nabla}_1^2 + \vec{\nabla}_2^2) + \frac{m\omega^2}{2}(r_1^2 + r_2^2) + \hat{V}_{\text{int}}(\vec{r}_1 - \vec{r}_2),$$

$$\hat{V}'_{\text{int}}(r) = \begin{cases} -\begin{pmatrix} q_o^2 & \Omega \\ \Omega & q_c^2 - \epsilon_c - \mu b \end{pmatrix} & \text{for} \quad r < r_0 \\ \begin{pmatrix} 0 & 0 \\ 0 & \infty \end{pmatrix} & \text{for} \quad r > r_0 \end{cases},$$

C. Chin, arXiv:cond-mat/0506313.

P. A. Boubrie, M. C. Tichy, and I. Roditi, Phys. Rev. A **95**, 023617 (2017)

E. Cuestas, A. P. Majtey, J. Phys. Condens. Matter **33**, 255601 (2021)

Feshbach molecule

$$H = -\frac{\hbar^2}{2m}(\vec{\nabla}_1^2 + \vec{\nabla}_2^2) + \frac{m\omega^2}{2}(r_1^2 + r_2^2) + \hat{V}_{\rm int}(\vec{r}_1 - \vec{r}_2),$$

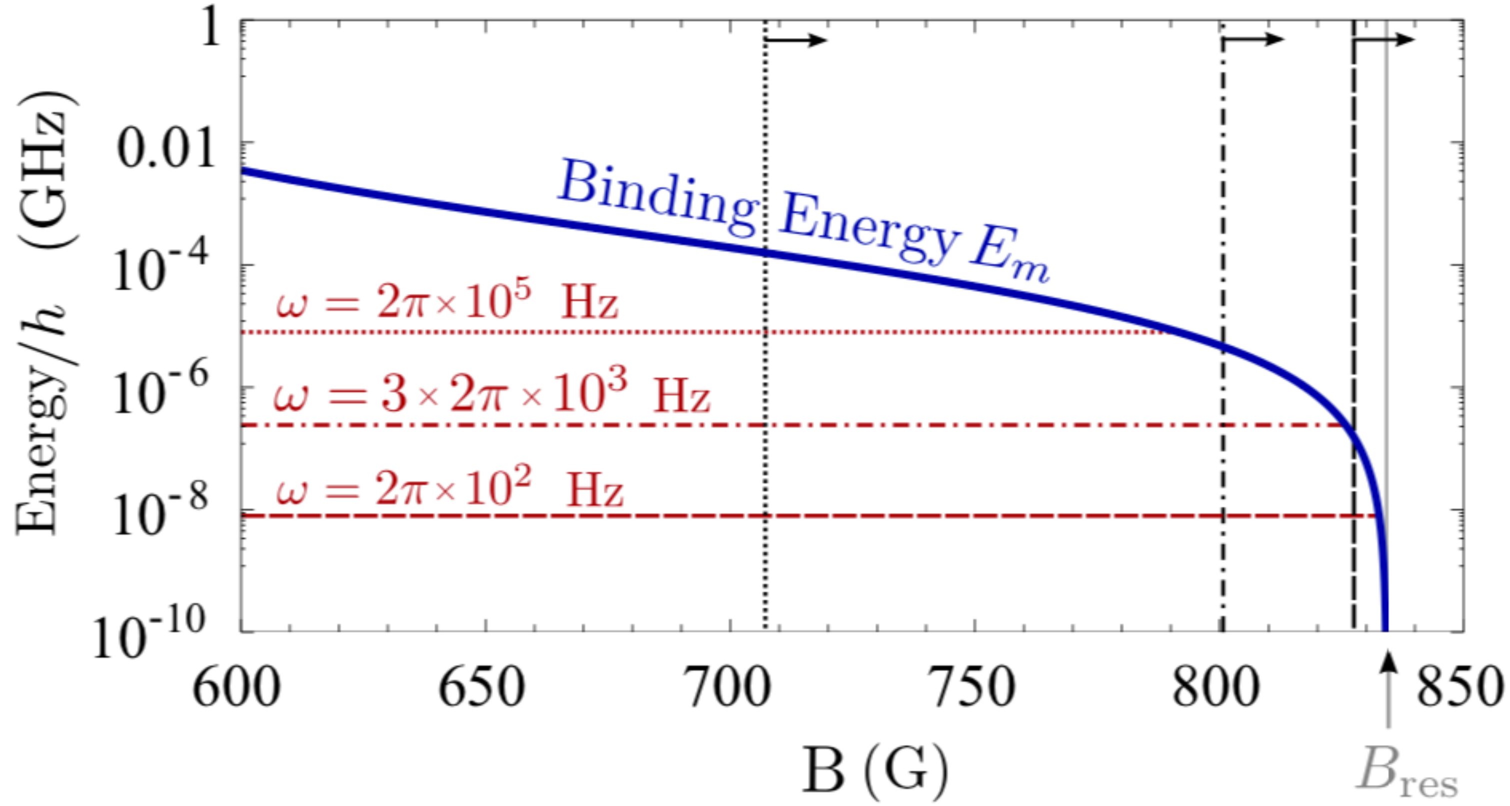


FIG. 1. Binding energy as a function of the magnetic field B (solid line). Horizontal lines are the confining energies $E_{\rm h.o.}/2$ for the frequencies $\omega = 2\pi \times 10^2$ Hz (dashed), $3 \times 2\pi \times 10^3$ Hz (dotted dashed), and $2\pi \times 10^5$ Hz (dotted). For these trapping frequencies, the BEC-BCS crossover extends to the strong-binding regime ($E_m > E_{\rm h.o.}/2$), i.e., from the vertical lines given by $k_F a = 1$ with $N = 1$ to the right, as the arrows indicate. The unitary limit is found in the resonant position $B_{\rm res}$ (vertical solid line).

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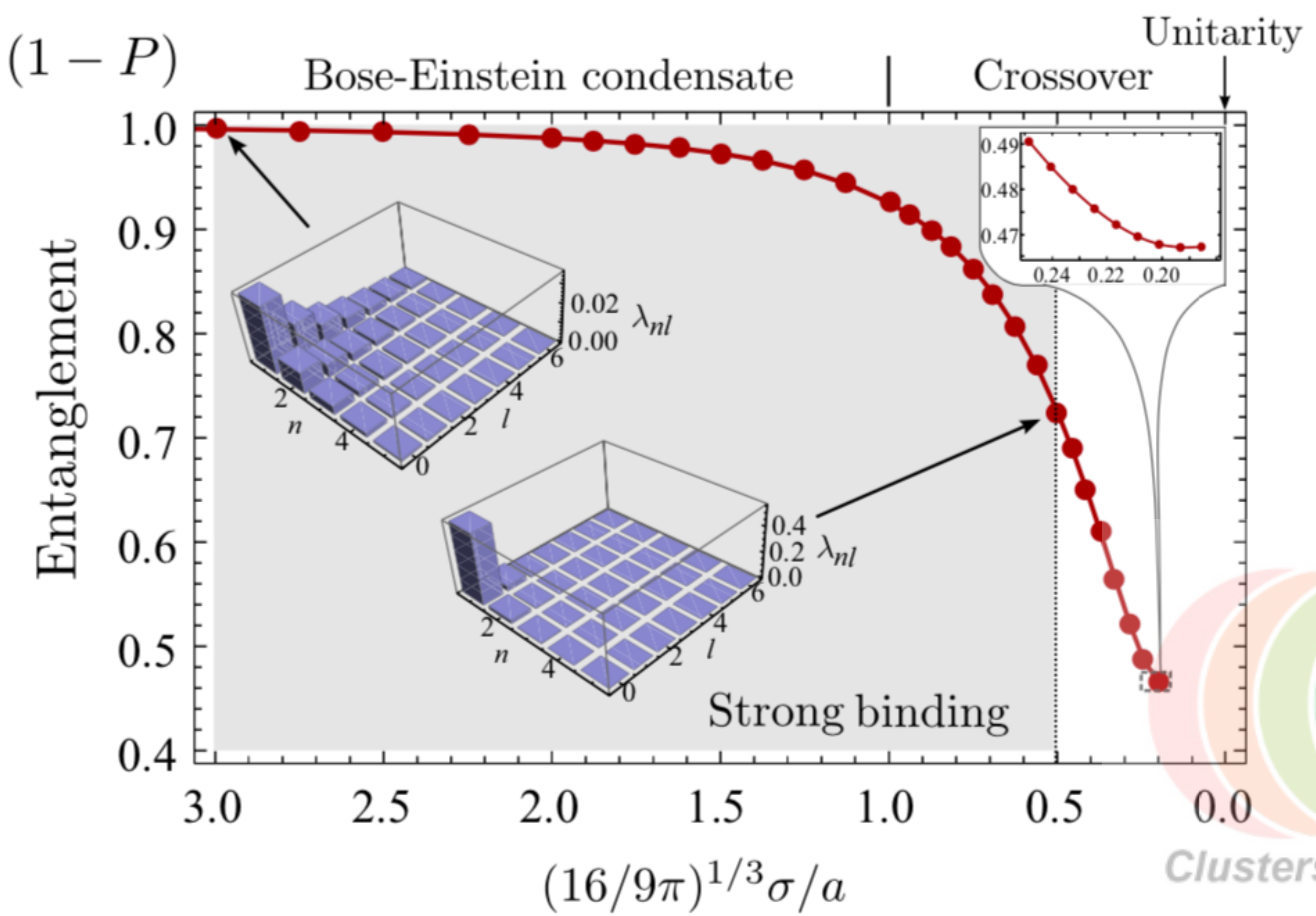
Feshbach molecule

- $|\Psi_{g.s.}\rangle = |\psi_{g.s.}(\vec{r}_A, \vec{r}_B)\rangle$ single-pair ground state
- Schmidt decomposition

$$|\psi_{g.s.}\rangle = \sum^{S} \sqrt{\lambda_j} |a_j\rangle |b_j\rangle,$$



Feshbach molecule

$$|\psi_{\mathrm{g.s.}}\rangle = \sum_{j=1}^{S} \sqrt{\lambda_j} |a_j\rangle |b_j\rangle,$$

Ansatz for N pairs

- Creation coboson operator $\hat{c}^\dagger = \sum_{j=1}^{S} \sqrt{\lambda_j} \hat{a}_j^\dagger \hat{b}_j^\dagger$

$$\hat{a}_j^\dagger \hat{b}_j^\dagger |0\rangle = |a_j, b_j\rangle \quad \longrightarrow \quad |\Psi_{\mathrm{g.s.}}\rangle = \hat{c}^\dagger |0\rangle$$

- Ground state of N composite bosons

$$|N\rangle = \frac{(\hat{c}^\dagger)^N}{\sqrt{N! \chi_N}} |0\rangle = \frac{1}{\sqrt{N! \chi_N}} \sum_{j_1 \neq j_2 \neq \cdots \neq j_N}^{1 \leq j_m \leq S} \prod_{k=1}^{N} \sqrt{\lambda_{j_k}} \hat{a}_{j_k}^\dagger \hat{b}_{j_k}^\dagger |0\rangle$$

$$\chi_N = N! \sum_{p_1 < p_2 < \cdots < p_N}^{S} \lambda_{p_1} \lambda_{p_2} \cdots \lambda_{p_N}$$

Composite-boson approach to molecular BECs

1.- $|N\rangle$ approximates the ground state (PRA 93, 013624 (2016))

$$H=\sum_{\mathbf{k}_\alpha}^{N}\varepsilon^{\alpha}_{\mathbf{k}_\alpha}\hat{a}^{\dagger}_{\mathbf{k}_\alpha}\hat{a}_{\mathbf{k}_\alpha}+\sum_{\mathbf{k}_\beta}^{N}\varepsilon^{\beta}_{\mathbf{k}_\beta}\hat{b}^{\dagger}_{\mathbf{k}_\beta}\hat{b}_{\mathbf{k}_\beta}-\sum_{\mathbf{q}}v_{\mathbf{q}}\sum_{\mathbf{k}_\alpha\mathbf{k}_\beta}\hat{a}^{\dagger}_{\mathbf{k}_\alpha+\mathbf{q}}\hat{b}^{\dagger}_{\mathbf{k}_\beta-\mathbf{q}}\hat{b}_{\mathbf{k}_\beta}\hat{a}_{\mathbf{k}_\alpha}$$

2.- Characteristic dimer-dimer scattering length $a^{(cob)}_{dd}=0.64\approx 0.6$

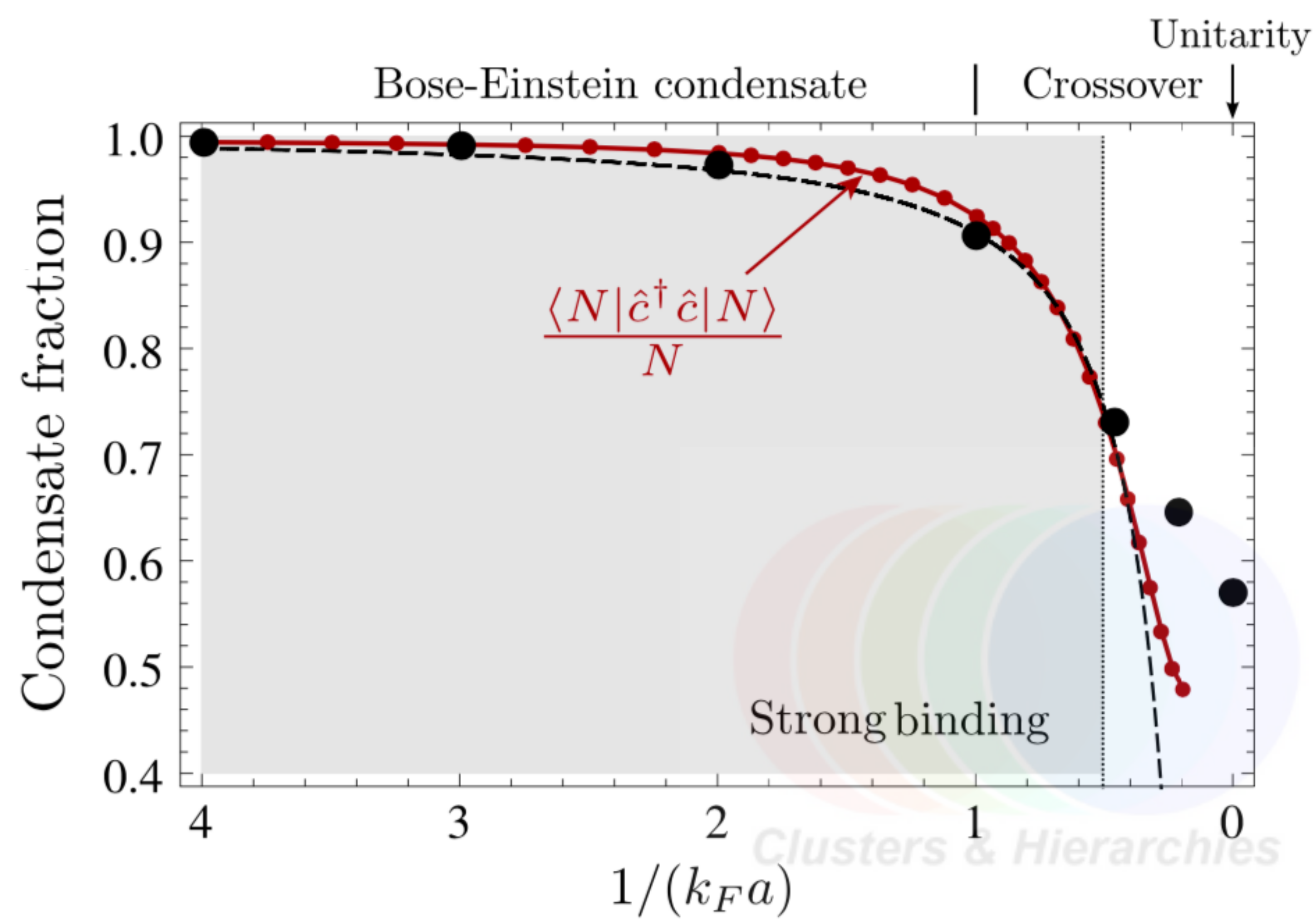
PRA 94, 052706 (2016)

3.- Condensate fractions

P.A. Bouvrie et al.

PRA 95, 023617 (2017)

$$\frac{\langle N|\,\hat{c}^{\dagger}\hat{c}\,|N\rangle}{N}$$



What happens in other systems?

Are there sufficient conditions for the validity of the coboson ansatz?

- Composite systems may behave very much like elementary bosons if:
  - Constituents are highly entangled
  - The system is dilute
  - Interactions are short ranged
- But this is not enough! Dimensionality can also play a role
- The coboson ansatz works well for a discrete model in 2D but fails in 1D even if all the previous conditions are met

1. Phys. Rev. A **100**, 012309 (2019)

- The coboson ansatz also fails for continuous 1D models

2. Phys. Rev. A **105**, 013302 (2022)

3. Composite-boson formalism applied to strongly bound fermion pairs in a one-dimensional trap, submitted (2022)

Outline

- Composite boson formalism
- Composite-boson approach to molecular BECs
- **Entanglement between interacting Fermi gases**
- Effects of Pauli principle on entangled- enhanced precision measurements

Clusters & Hierarchies

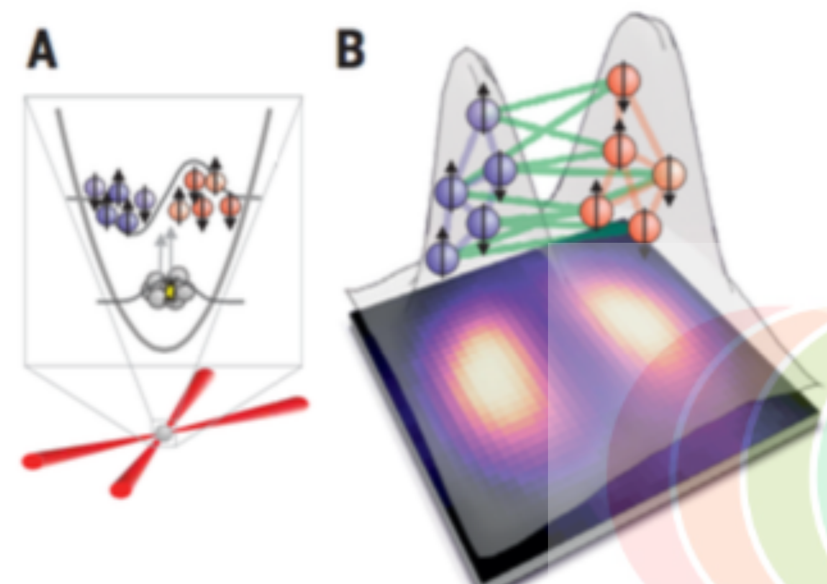
Entanglement between two spatially separated atomic modes

Karsten Lange,[1] Jan Peise,[1] Bernd Lücke,[1] Ilka Kruse,[1] Giuseppe Vitagliano,[2,3] Iagoba Apellaniz,[3] Matthias Kleinmann,[3,4] Géza Tóth,[3,5,6] Carsten Klempt[1]*

Modern quantum technologies in the fields of quantum computing, quantum simulation, and quantum metrology require the creation and control of large ensembles of entangled particles. In ultracold ensembles of neutral atoms, nonclassical states have been generated with mutual entanglement among thousands of particles. The entanglement generation relies on the fundamental particle-exchange symmetry in ensembles of identical particles, which lacks the standard notion of entanglement between clearly definable subsystems. Here, we present the generation of entanglement between two spatially separated clouds by splitting an ensemble of ultracold identical particles prepared in a twin Fock state. Because the clouds can be addressed individually, our experiments open a path to exploit the available entangled states of indistinguishable particles for quantum information applications.

Science 360, 416 (2018)

Fig. 1. Generation of entanglement between two spatially separated atomic clouds. (**A**) A BEC of atoms in the Zeeman level $m_F = 0$ is prepared in a crossed-beam optical trap. Collisions generate entangled pairs of atoms in the levels $m_F = \pm 1$ (spin up/down) in the first spatially excited mode. The created multiparticle-entangled ensemble is naturally divided into two clouds (red and blue). (**B**) The atomic density profile obtained from an average over 3329 measurements is shown in the background. The entanglement between the two clouds (indicated schematically with green lines) can be detected by analyzing spin correlations.



Splitting a molecular BEC

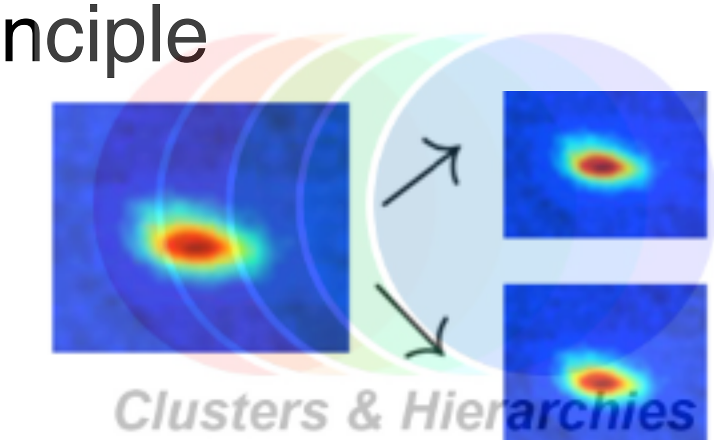
Splitting dynamics: $\hat{d}_j^\dagger \xrightarrow{BS} \frac{1}{\sqrt{2}}\left(\hat{d}_{1,j}^\dagger + \hat{d}_{2,j}^\dagger\right)$ $\qquad |N\rangle \xrightarrow{BS} |\Psi_N\rangle$

$$|\Psi_N\rangle = \frac{1}{\sqrt{2^N N! \chi_N}} \sum_{M=0}^{N} \binom{N}{M} \underbrace{\sum_{\substack{k_1,\ldots,k_N=1 \\ k_1 \neq \cdots \neq k_N}}^{S} \prod_{i=1}^{M} \sqrt{\lambda_{k_i}} \hat{d}_{1,k_i}^\dagger \prod_{j=M+1}^{N} \sqrt{\lambda_{k_j}} \hat{d}_{2,k_j}^\dagger |0,0\rangle}_{|\Phi_{N,M}\rangle}$$

- Fermion pairs are distributed binomially on the two modes of a perfect beam splitter, as for ideal bosons or distinguishable particles

- Pairs in each mode are correlated due to Pauli principle

P.A. Bouvrie, M.C. Tichy, K. Mølmer, Phys. Rev. A 94, 053624 (2016)



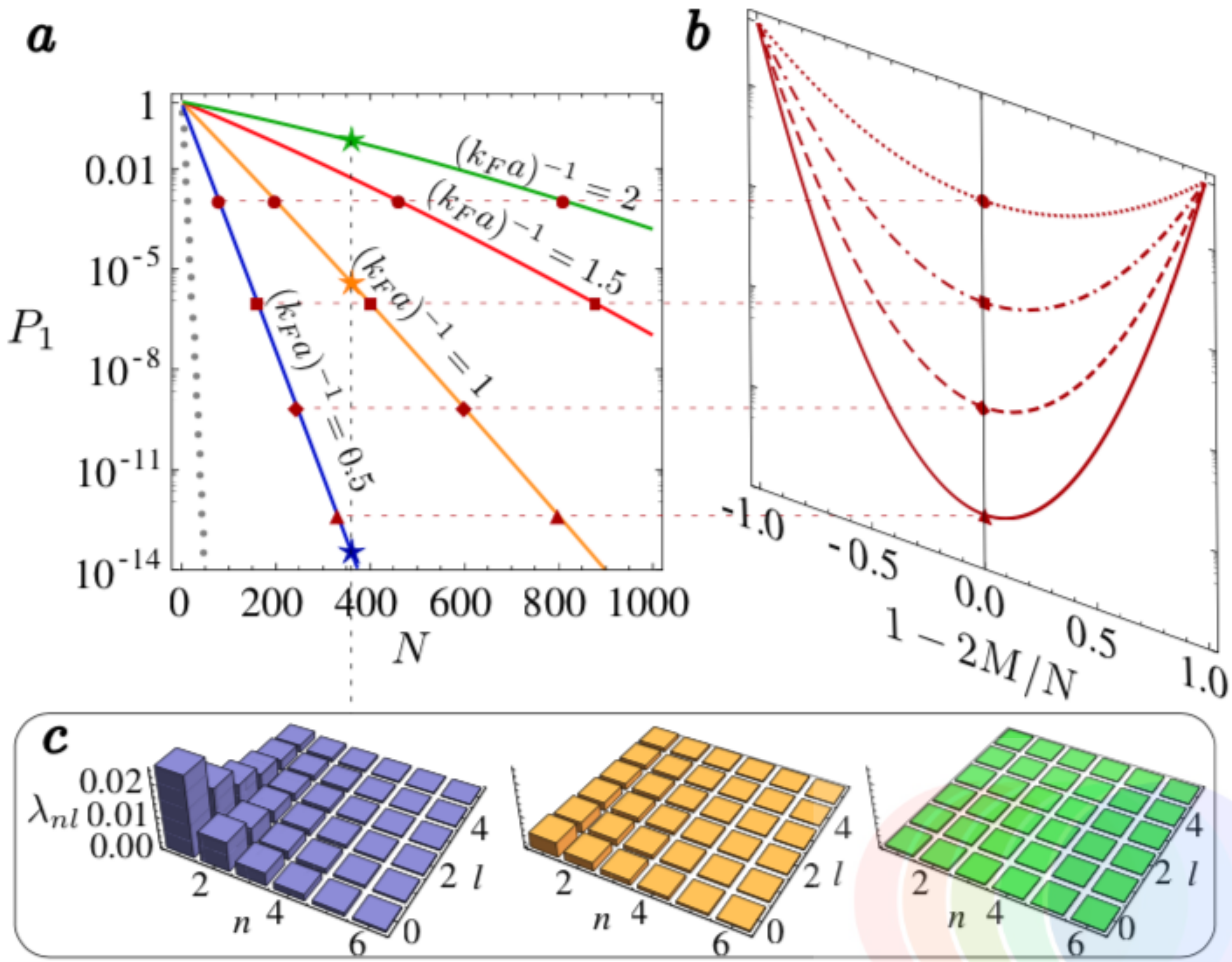
Entanglement between mBECS

$P_1 = \mathrm{Tr}[(\rho_1)^2]$

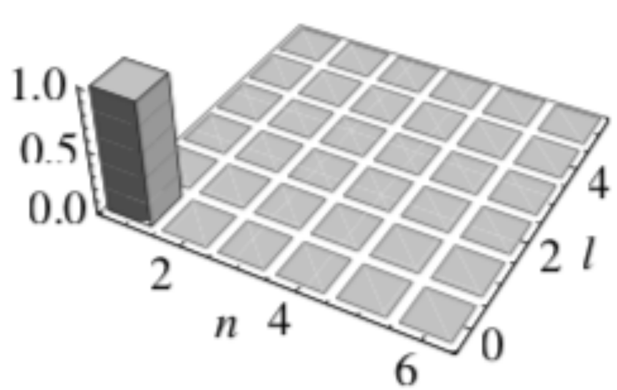
N indentical fermion systems

$P_1^f = \binom{N}{M}^{-1}$

$k \leq k_F$



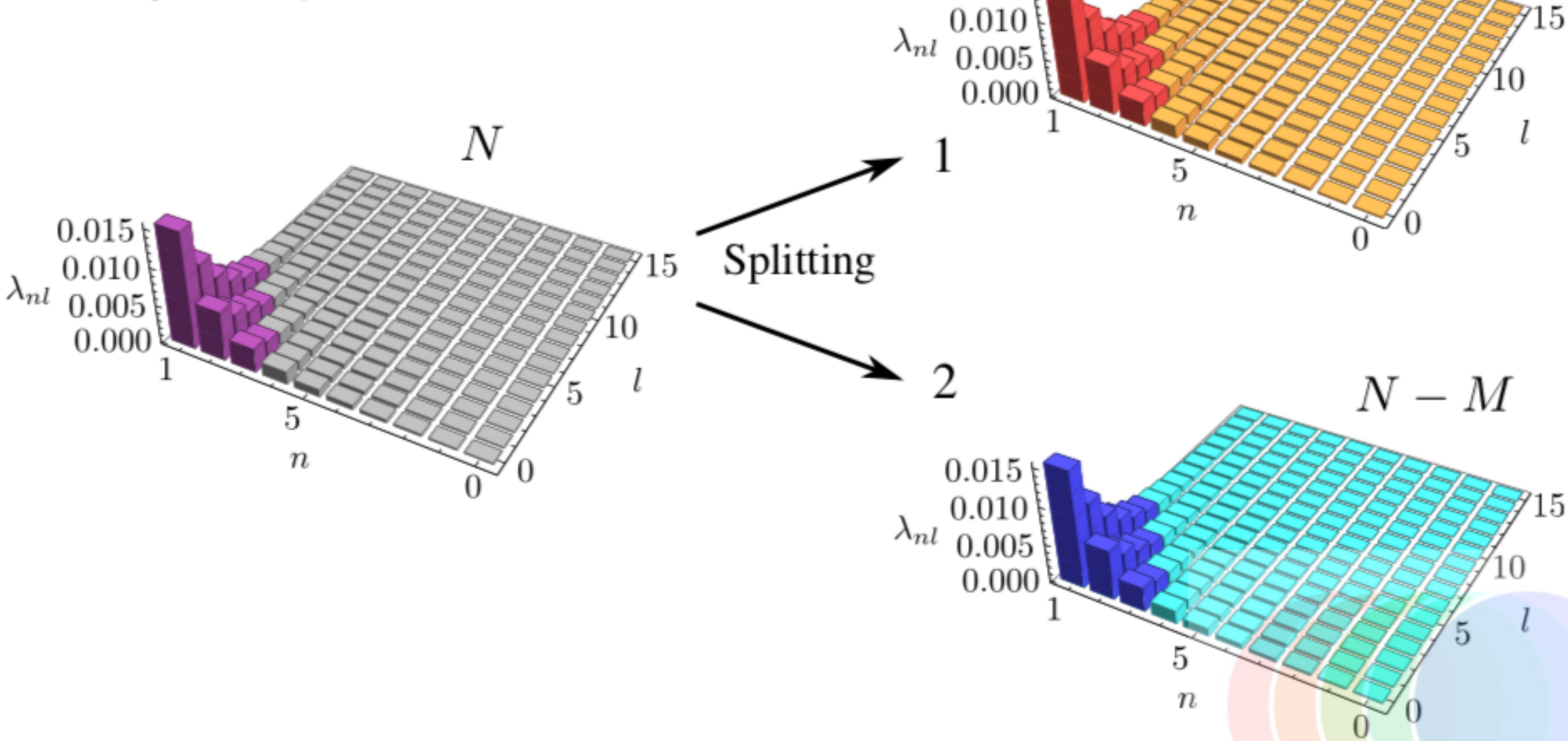
BCS



Fluctuations of single-particle espectral densities

$\Lambda = (\lambda_1, \ldots, \lambda_{56}, \lambda_{57}, \ldots, \lambda_S)$

Degeneracy of $\lambda_{nl} \to 2l + 1$



Fluctuations of single-particle espectral densities

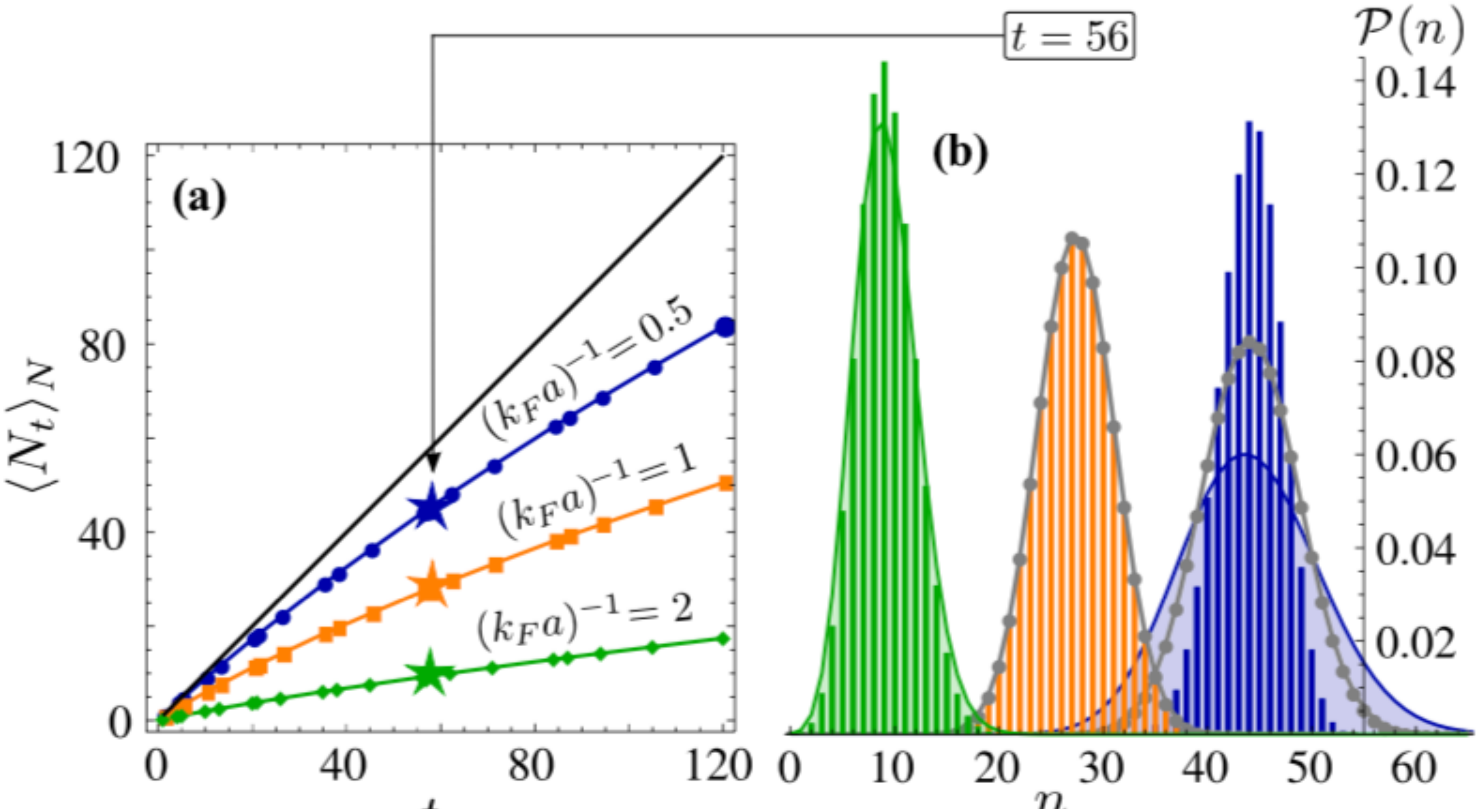
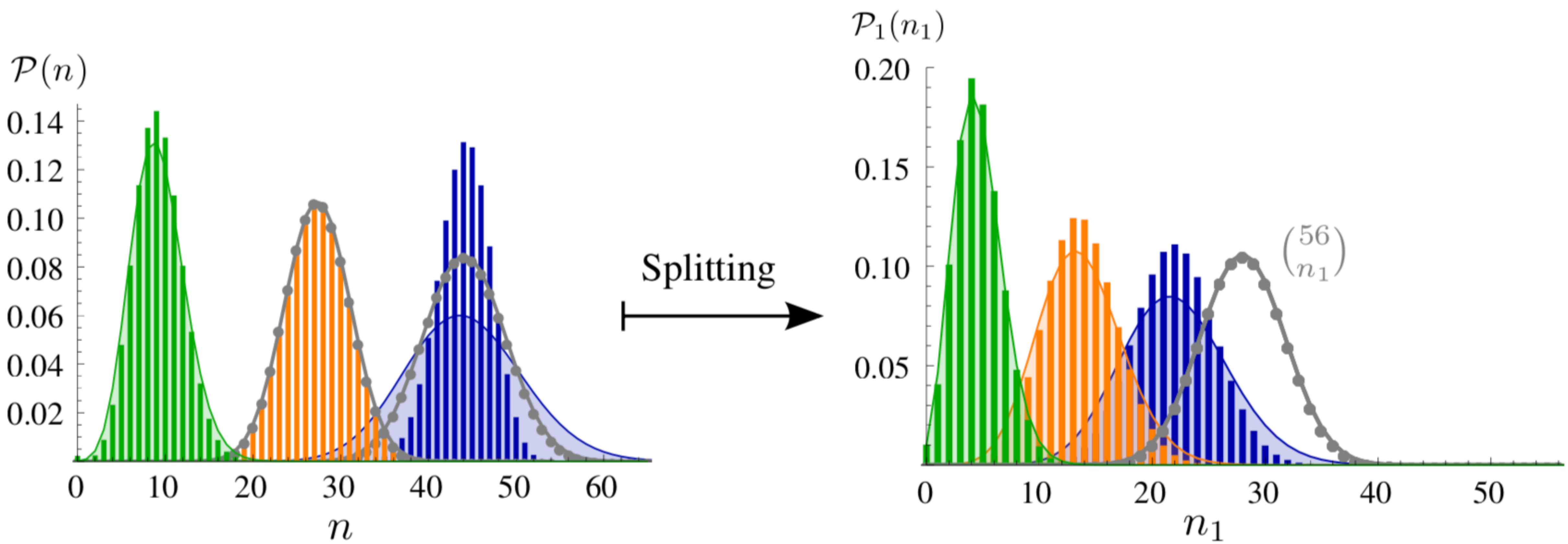


FIG. 2. (a) Mean population of the t lowest energetic states of the single-particle spectrum ($\langle N_t \rangle_N = N \sum_{j=1}^{t} D_j$) of an interacting Fermi gas with $N = 10^3$ fermion pairs and interaction parameter $(k_F a)^{-1} = 2$, 1, and 0.5 (green, orange, and blue, respectively). Suppression of particle fluctuations in this spectral region $\tilde{\Lambda}_t$ is shown in (b), where the probability $\mathcal{P}(n)$ is plotted for $t = 56$. Dashed areas are Poissonian distributions and connected gray dots are binomial distributions. All depicted quantities are dimensionless.

Fluctuations of single-particle espectral densities



Fluctuations of single-particle espectral densities

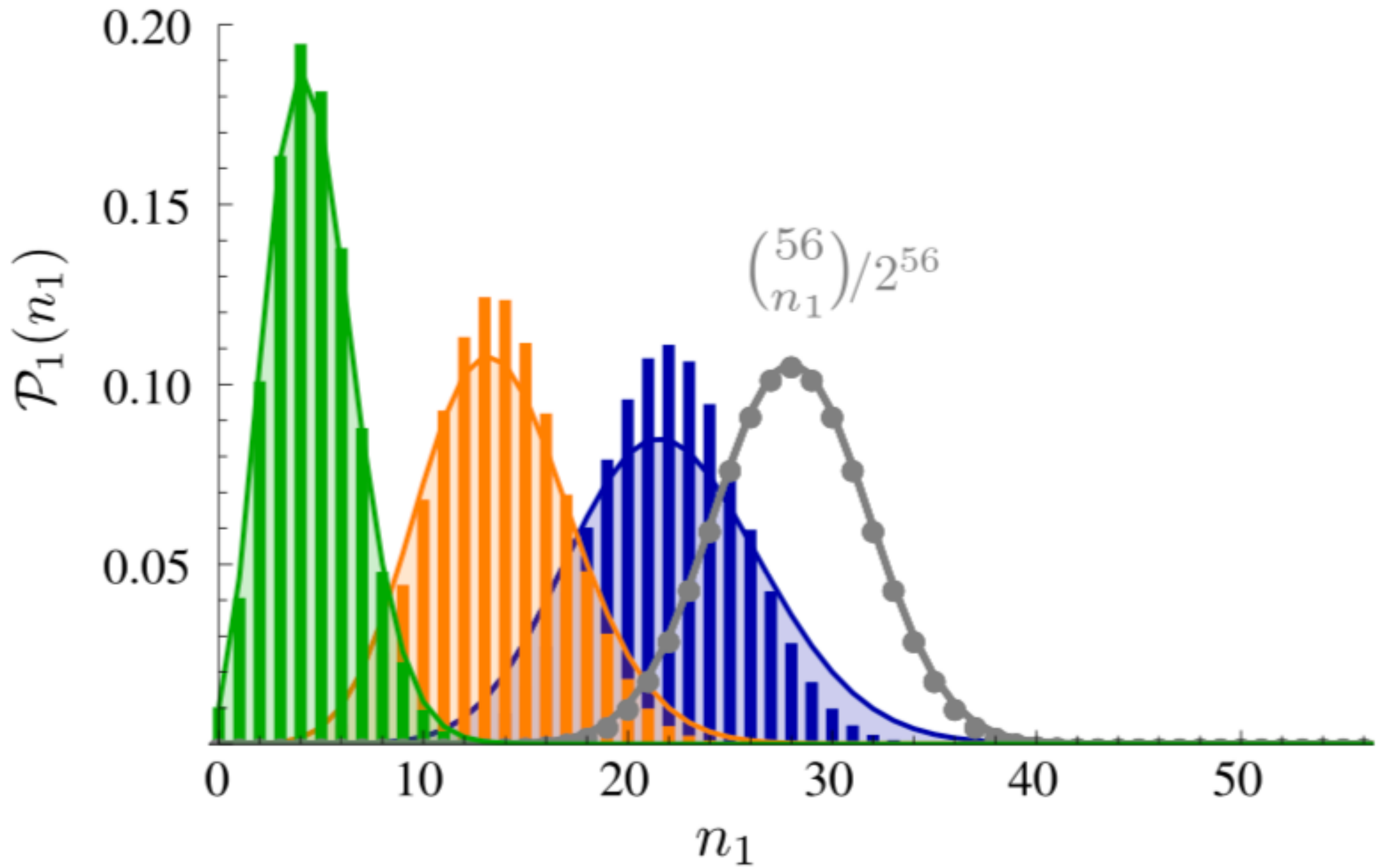


FIG. 3. Particle fluctuations in the states $\tilde{\Lambda}_t$ of the fermionic ensemble 1 [$\mathcal{P}_1(n_1)$] after splitting the system into two balanced ensembles of $M = N/2 = 0.5 \times 10^3$ fermion pairs. When decreasing $(k_F a)^{-1}$, $\mathcal{P}_1(n_1)$ approaches the binomial distribution $\binom{t}{n_1}/2^t$ of two maximally entangled fermionic ensembles with perfectly correlated fluctuations. All depicted quantities are dimensionless.

Bell-like nonlocal quantum correlations

$$
\begin{aligned}
\left|\Phi_{M,N-M}\right\rangle \;=\;& \sqrt{MD_j}\,\left|d_j\right\rangle_1 \left|0_j\right\rangle_2 \left|\Phi^{[\lambda_j]}_{M-1,N-M}\right\rangle + \\
&+ \sqrt{(N-M)D_j}\,\left|0_j\right\rangle_1 \left|d_j\right\rangle_2 \left|\Phi^{[\lambda_j]}_{M,N-M-1}\right\rangle + \\
&+ \sqrt{1-ND_j}\,\left|0_j\right\rangle_1 \left|0_j\right\rangle_2 \left|\Phi^{[\lambda_j]}_{M,N-M}\right\rangle
\end{aligned}
$$

Bell-like state when $ND_j \to 1$

CHSH inequality $\mathcal{M} = \langle QS\rangle + \langle RS\rangle + \langle RT\rangle - \langle QT\rangle \leq 2$

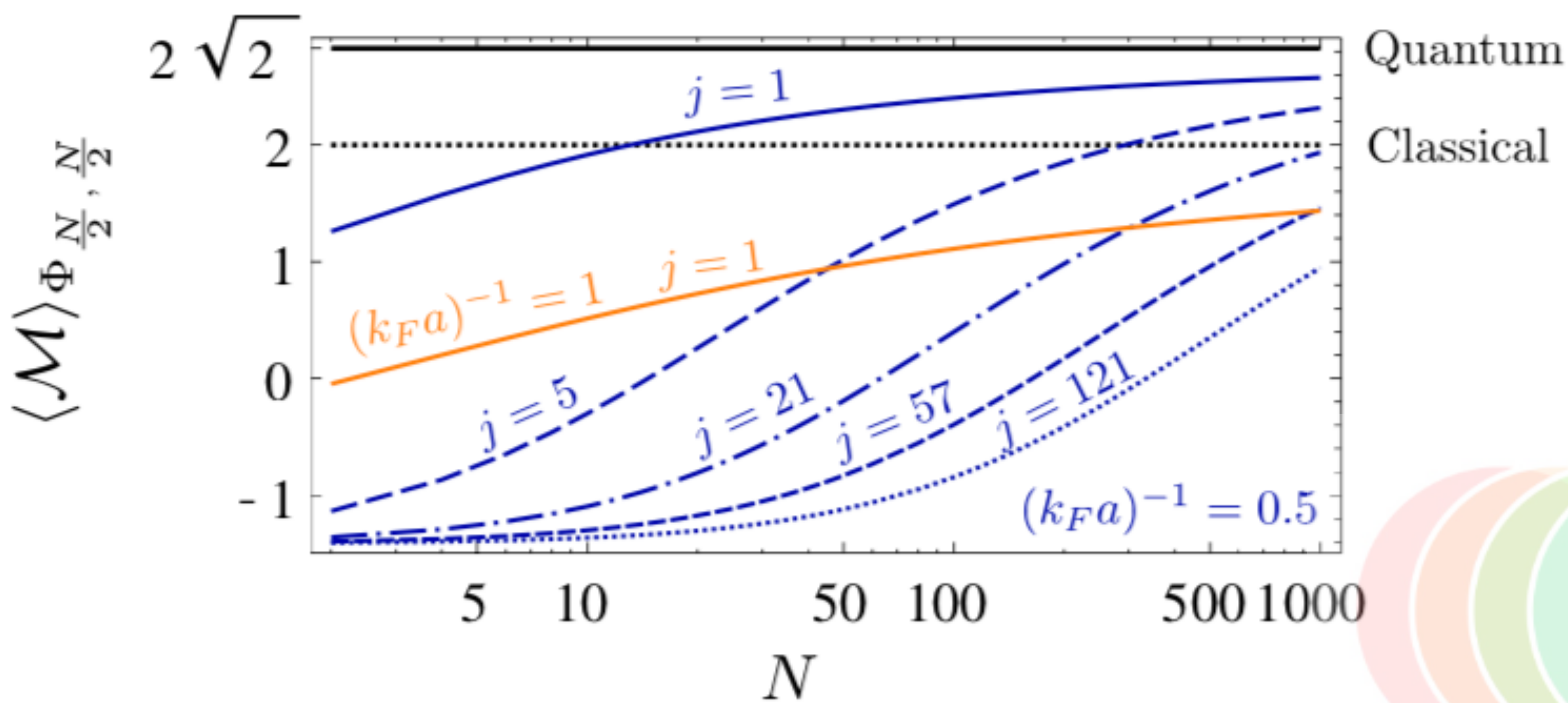
Clusters & Hierarchies

Bell-like nonlocal quantum correlations

$$Q = Z_1, \quad R = X_1, \quad S = \frac{(X_2 - Z_2)}{\sqrt{2}} \quad \text{and} \quad T = \frac{(X_2 + Z_2)}{\sqrt{2}}$$

$$Z_q = (|0\rangle_{q\,q}\langle 0| - |1\rangle_{q\,q}\langle 1|) \text{ and } X_q = (|0\rangle_{q\,q}\langle 1| + |1\rangle_{q\,q}\langle 0|)$$

$\mathcal{M}_{\mathrm{CHSH}} = \sqrt{2}(3ND_j - 1)$ para el estado energético más bajo $j = 1$:



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Quantum metrology with identical particles

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Sub-shot-noise quantum metrology with entangled identical particles

F. Benatti [a,b,*], R. Floreanini [b], U. Marzolino [a,b]

PHYSICAL REVIEW A **100**, 012308 (2019)

Indistinguishability-enabled coherence for quantum metrology

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Dissipative quantum metrology in manybody systems of identical particles

F Benatti[1,2,4], S Alipour[3] and A T Rezakhani[3]

Clusters & Hierarchies

Interferometers with uncorrelated atoms aire limited by the shot noise

The best possible accuracy is given by the Kramer Rao bound

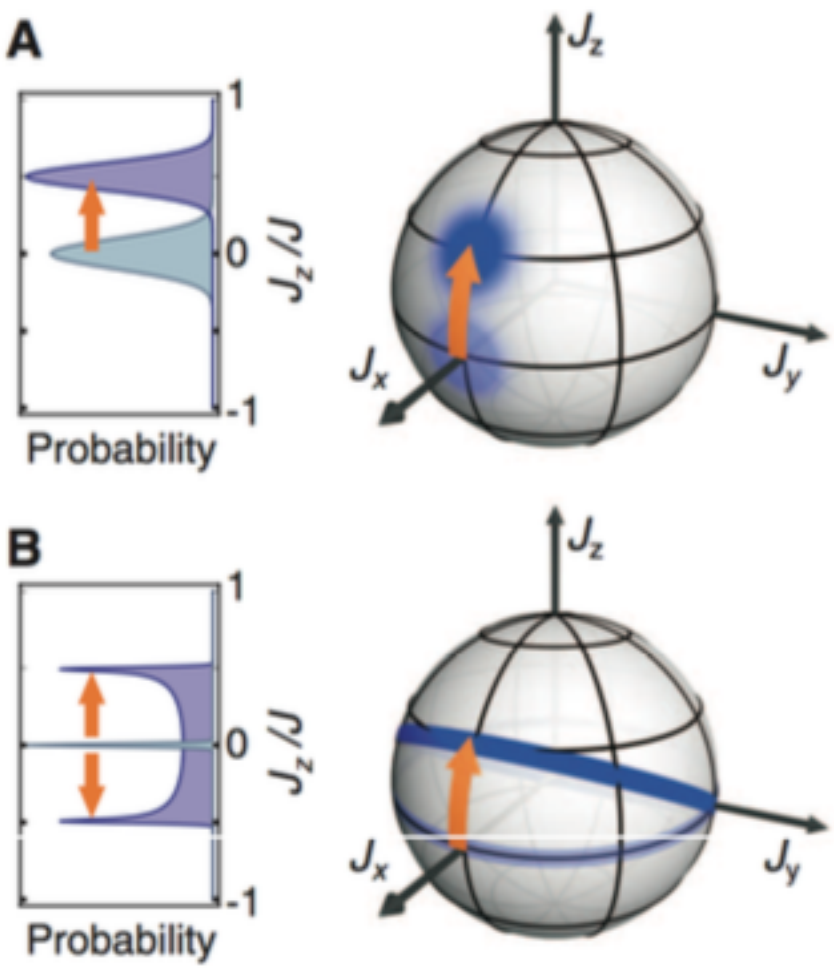
$$\Delta\theta_{SN} = 1/\sqrt{\nu N}$$

This limit can be exceeded with entangled atoms (sub-shot-noise precision).

This upper limit to the accuracy, the Heisemberg limit

$$\Delta\theta_{H} = 1/(\sqrt{\nu}N),$$

can be achieved using Twin Fock states.



$$\Delta\theta = \sigma(J_z)/\sqrt{\nu|d\langle J_z\rangle/d\theta|} \geq 1/\sqrt{\nu N}$$

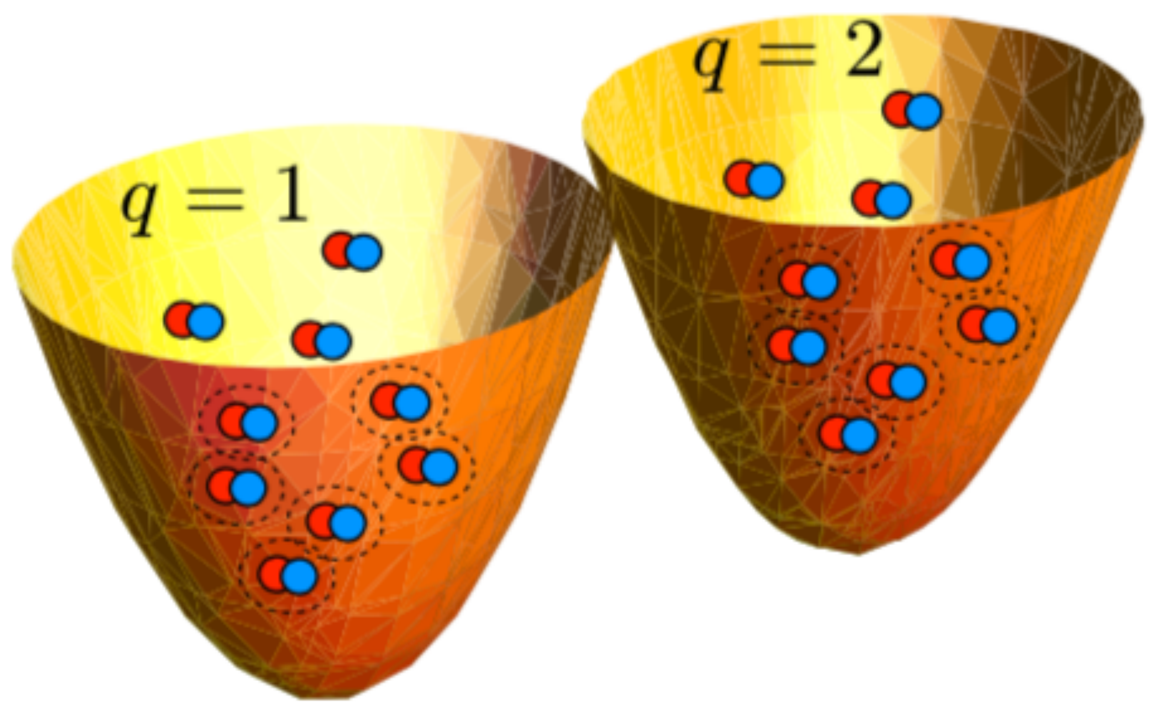
$$\Delta\theta = \sigma(J_z^2)/\sqrt{\nu|d\langle J_z\rangle/d\theta|} \geq 1/\sqrt{\nu}N$$

B. Lücke et al., Science 334, 773 (2011)

Twin-Fock state of fermions

$$|\Psi\rangle = |M\rangle_1 \, |M\rangle_2 = \frac{(\hat{c}_1^\dagger)^M}{\sqrt{M!\chi_M}} \frac{(\hat{c}_2^\dagger)^M}{\sqrt{M!\chi_M}} \, |0\rangle_1 \, |0\rangle_2 \, .$$

Two molecular modes 1 and 2 with M=N/2 pairs



Representation of the BEC regime of a interacting Fermi gas in a double well potential

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Dynamics of two fermions in a double well potential

In the strong binding regimen fermions A and B co-tunnel as pairs

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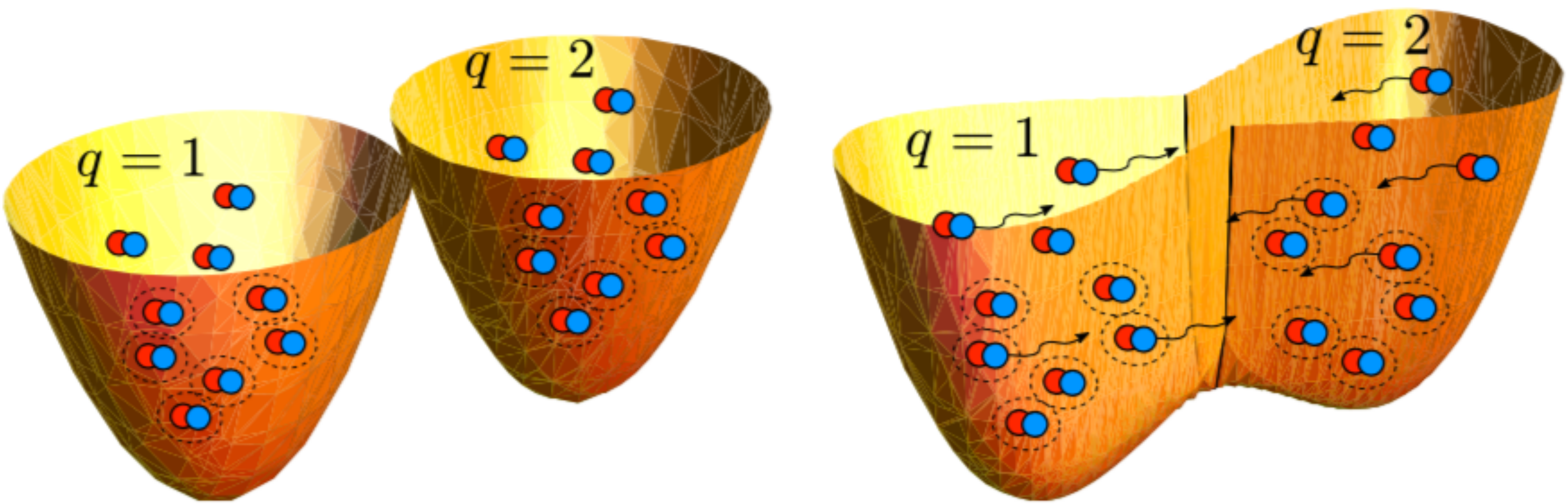
Selected for a Viewpoint in *Physics*

PRL **114,** 080402 (2015) PHYSICAL REVIEW LETTERS week ending 27 FEBRUARY 2015

Two Fermions in a Double Well: Exploring a Fundamental Building Block of the Hubbard Model

Simon Murmann,[*] Andrea Bergschneider, Vincent M. Klinkhamer, Gerhard Zürn, Thomas Lompe,[†] and Selim Jochim

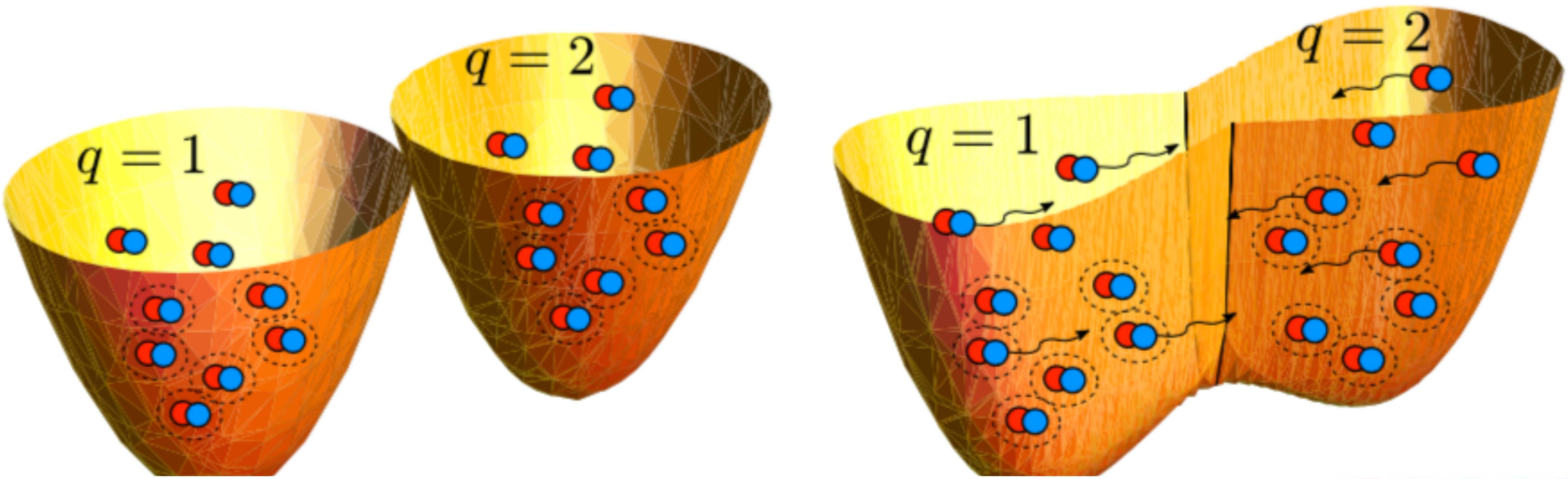
$$(k_F a)^{-1} > 1 \longrightarrow \hat{a}_j^\dagger \hat{b}_j^\dagger = \hat{d}_j^\dagger$$



A global phase can be obtained in the dynamics by adjusting the tunneling that couples the wells.

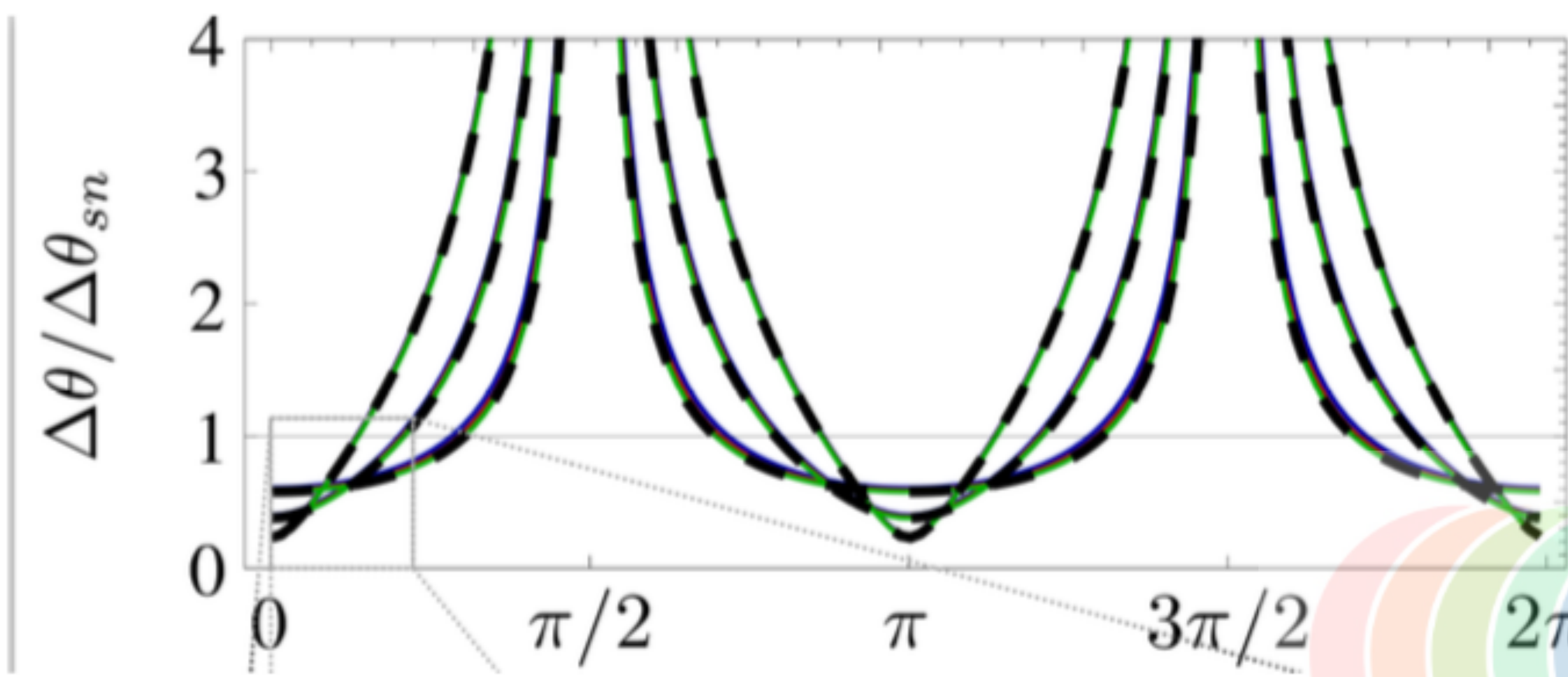
$$\hat{H} = \sum_{i,j=1}^{S} \frac{\alpha_{ij}^2}{2U_{\text{mol}}} (\hat{d}_{1,i}^\dagger \hat{d}_{2,j} + \hat{d}_{2,i}^\dagger \hat{d}_{1,j}).$$

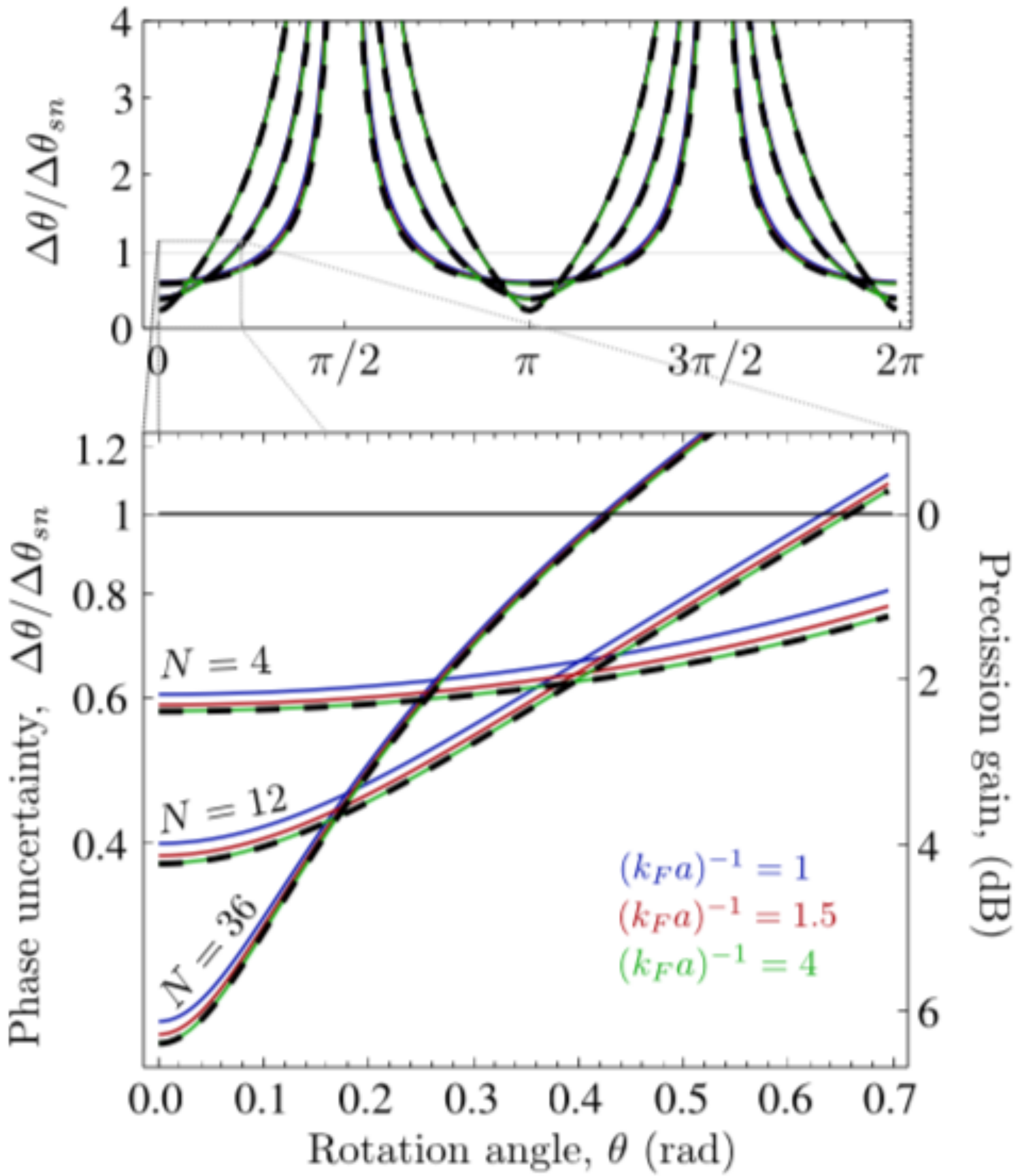
$$J = \alpha_{ii}^2 / U_{\text{mol}}$$



$$e^{-it\hat{H}} |M\rangle_1 |M\rangle_2 \approx \prod_{j=1}^{S} e^{-i\theta/2(\hat{d}_{1,j}^\dagger \hat{d}_{2,j} + \hat{d}_{2,j}^\dagger \hat{d}_{1,j})} |M\rangle_1 |M\rangle_2$$

$$\Delta\theta = \frac{\sqrt{\langle \hat{J}_z^4 \rangle - \langle \hat{J}_z^2 \rangle^2}}{\sqrt{\nu} \left| \frac{d\langle \hat{J}_z^2 \rangle}{d\theta} \right|},$$





Phase uncertainty Δθ with respect to shot noise Δθsn of a twin Fock interferometry realized with molecular Bose–Einstein condensates, as a function of the interference phase θ and for N=4,12, and 36

Summarizing

- Bosonic behavior ⟷ entanglement
- Ansatz of composite bosons that takes into account all exchange interactions.
- Fermi gases ✓
- 1-D systems ✗

- Generation of entangled molecular condensates.
- CHSH inequality violation.

- Deviation of bosonic behavior in interference is a consequence of condensate fraction loss
- Noise in detection favors the Pauli impact which affects the precision gain.
- With efficient detectors, metrological accuracy comparable to atomic interferometers can be achieved.

Thanks!

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