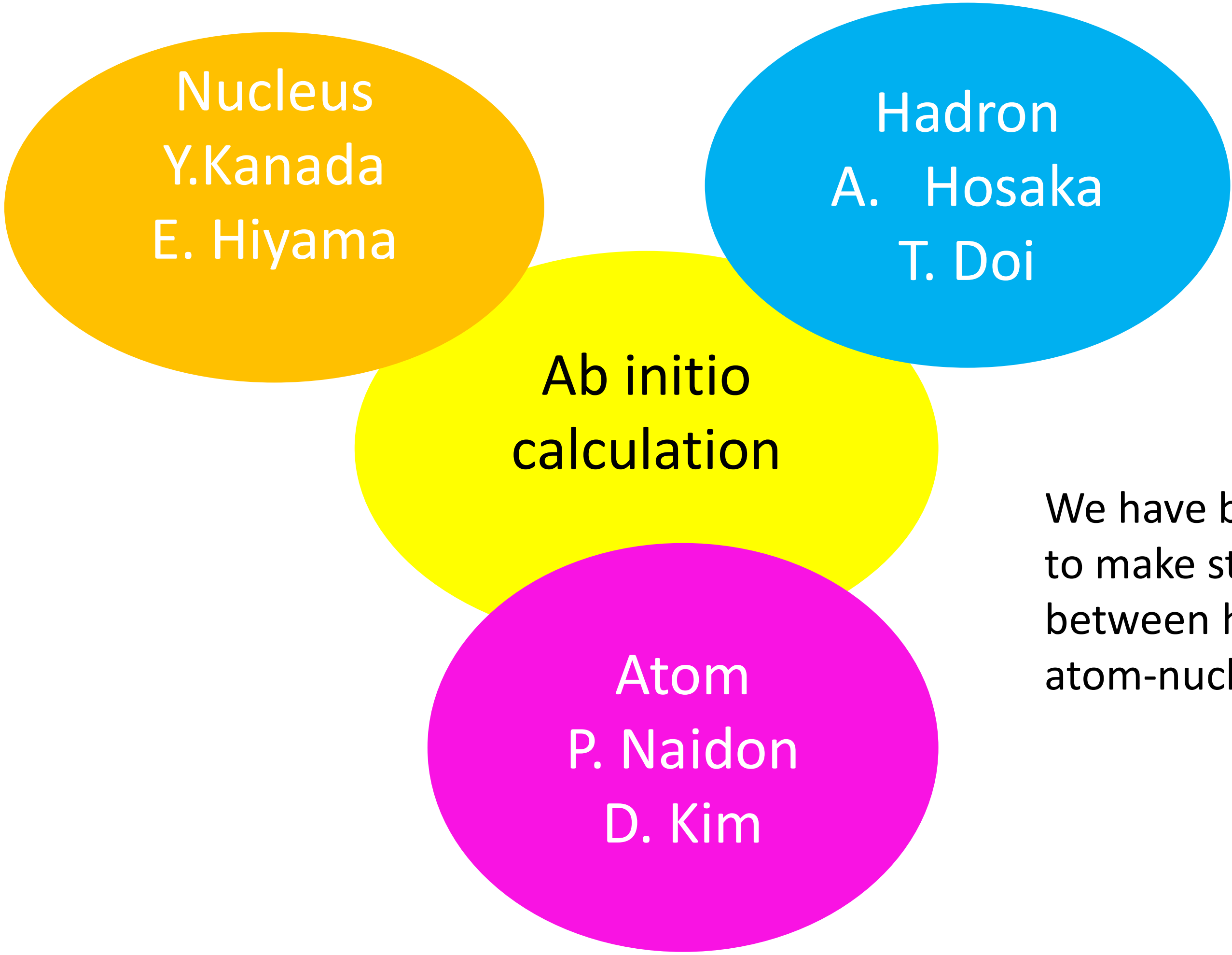
Clusters & Hierarchies

Structure of nuclei and hadron systems from view point of few-body problem

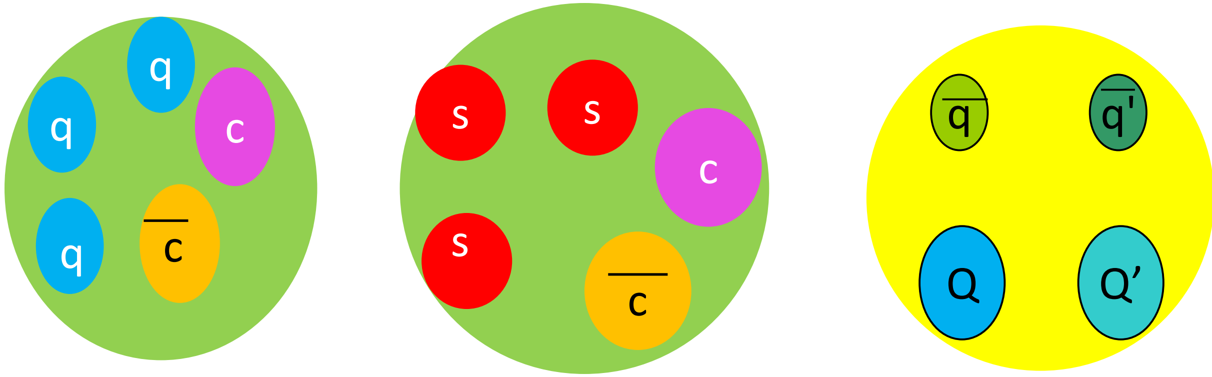
Emiko Hiyama (Tohoku Univ./RIKEN)



We have been trying
to make strong connection
between hadron-nucleus,
atom-nucleus etc.

Key word:
How do we form ‘cluster’ from view point of problem?
hadron, nuclei, atom・・・

▪Hadron



▪E. Hiyama, A. Hosaka, M. Oka and J−M. Richard,
PRC98, 045208 (2018).

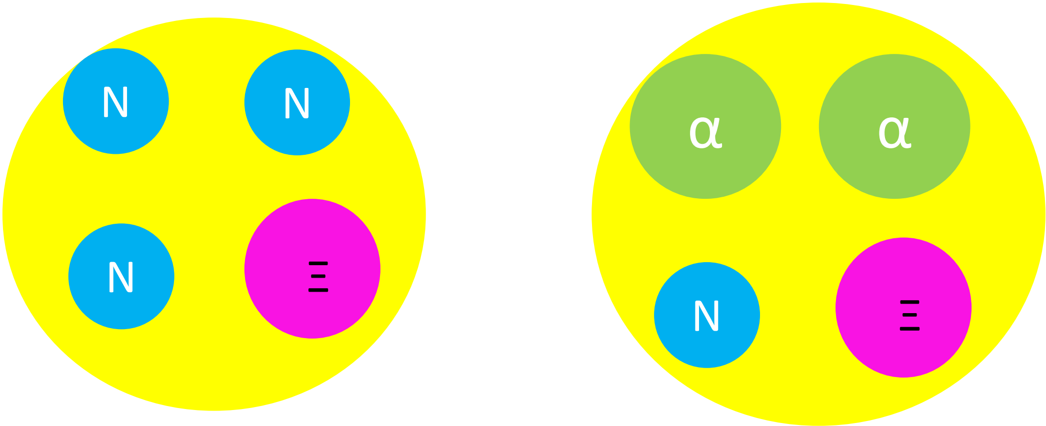
・Qi Meng, E. Hiyama, Kadir Utku Can, Philipp Gubler,
M. Oka, A. Hosaka, Hongshi Zong,
PLB 798, 135028 (2019).

・Meng, Q., Hiyama, E., Hosaka, A., Oka, M., Gubler, P., Can, K.U., Takahashi, T.T., Zong, H.S.,
PLB　814 ,136095 (2021).

・Qi Meng, M. Harada, E. Hiyama, A. Hosaka, M. Oka,
PLB 824, 136800 (2021)

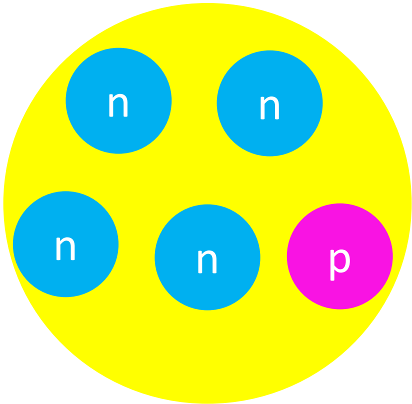
Collaboration with Hosaka san

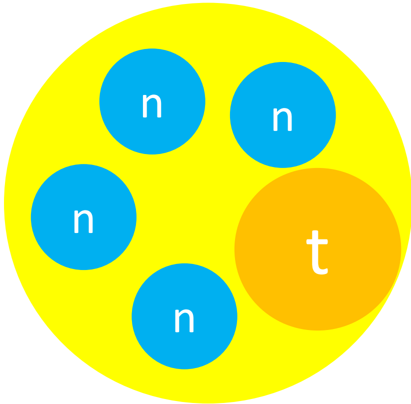
・Hypernuclear Physics



・Hiyama, E., Sasaki, K., Miyamoto, T., Doi, T., Hatsuda, T., Yamamoto, Y., Rijken, T.A., Physical Review Letters 124(9), 092501-1-092501-5 (2020)
・E. Hiyama, M. Isaka, T. Doi, and T. Hatsuda,　Physical Review C 106(6), (2022)

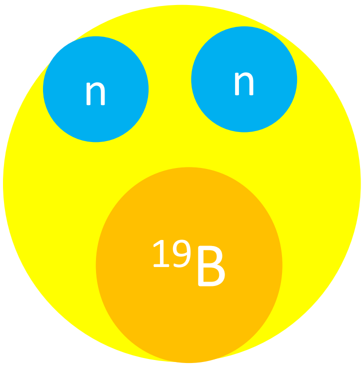
Unstable nuclear physics





^{5}H ^{7}H

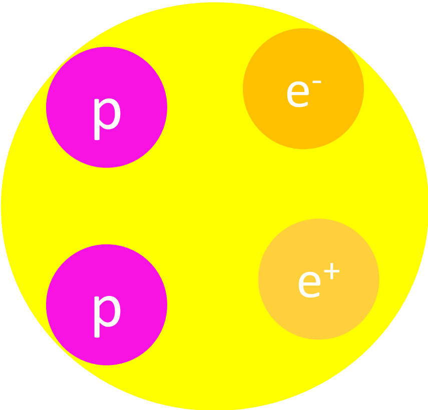
R. Lazauskas, E. Hiyama, J. Carbonell, PLB 791, 335 (2019).
E. Hiyama, R. Lazauskas, J. Carbonell, PLB 833, 137367 (2022)
France-Japan collaboration



Universality in neutron-rich nuclei=bridge with ultra cold physics

E. Hiyama, R. Lazauskas, F. M. Marques, and J. Carbonell,
Physical Review C100, 011603(R) (2019).
E. Hiyama, R. Lazauskas, J. Carbonell, T. Frederico,
Phys. Rev. C 106, 064001 (2022)

・Atomic physics



T. Yamashita, E. Hiyama, D. Yoshida, M. Tachikawa,
Physical Review A 105(1), (2022)

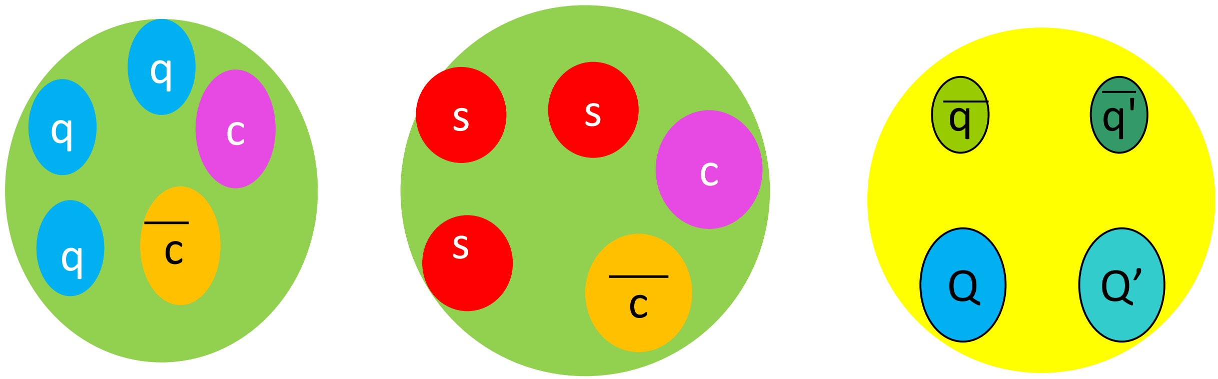
D班の活動

・階層を横断する会：11回（半日で、specificなトピックを深く議論をする）

・国際レクチャーシリーズ：12回

Today's talk

▪Hadron



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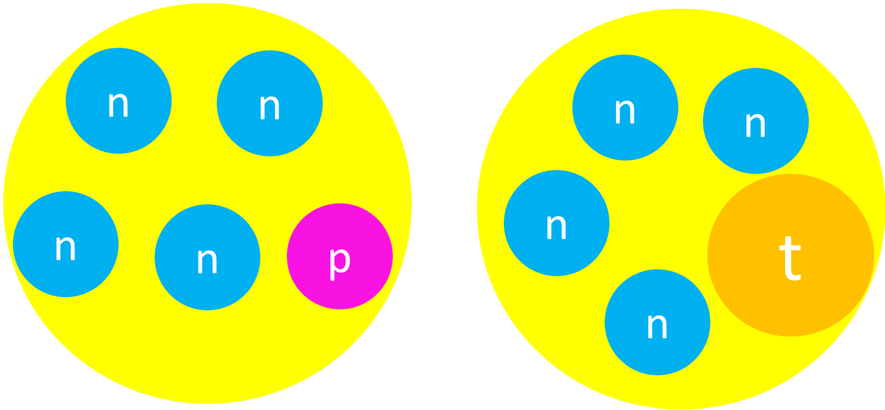
PRC98, 045208 (2018).

・Qi Meng, E. Hiyama, Kadir Utku Can, Philipp Gubler,
M. Oka, A. Hosaka, Hongshi Zong,
PLB 798, 135028 (2019).

・Meng, Q., Hiyama, E., Hosaka, A., Oka, M., Gubler, P., Can, K.U., Takahashi, T.T., Zong, H.S.,
PLB 814 ,136095 (2021).

・Qi Meng, M. Harada, E. Hiyama, A. Hosaka, M. Oka,
PLB 824, 136800 (2021)

Unstable nuclear physics



^{5}H ^{7}H

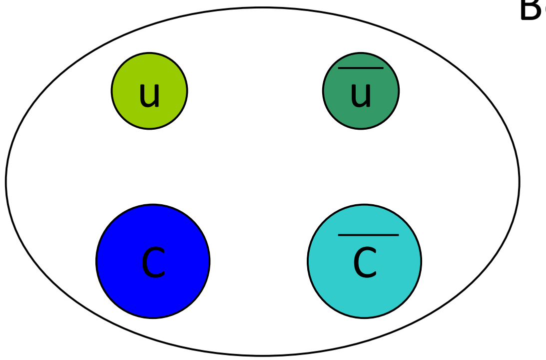
R. Lazauskas, E. Hiyama, J. Carbonell, PLB 791, 335 (2019).
E. Hiyama, R. Lazauskas, J. Carbonell, PLB 833, 137367 (2022)
France-Japan collaboration

時間があれば･･。

In Hadron physics,

Search for multi exotic quarks systems such as tetra quark systems, penta quark systems, and di-baryon systems have a long history.

Tetra quark systems:



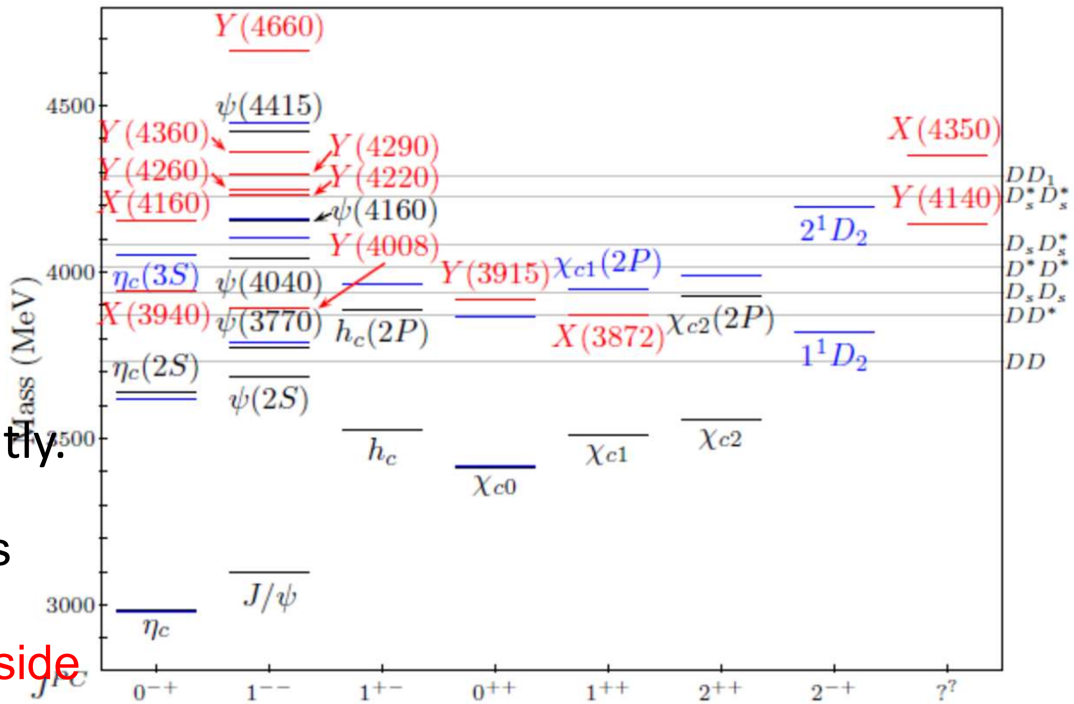
Phys. Rev. Lett. 91, 262001 (2003)
Belle Group

X(3872):3871.2±0.5 MeV

1^{++} Γ<2.3MeV

After observation of X(3872), there are many observed exotic state candidates shown in red color.

Z(4430) ± have been observed , recently.



Black: Observed conventional *cc* states

Blue: Predicted conventional *cc* states

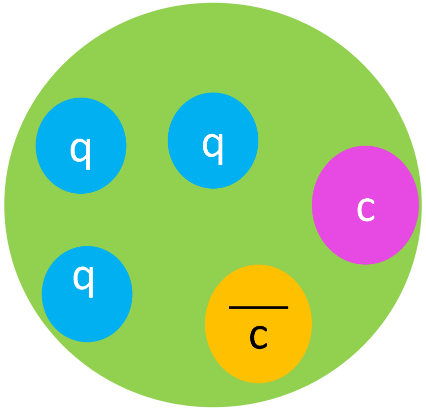
Red: Exotic state candidates with *cc* inside

Red states are very closed to some threshold.=> the state are compact states or meson-meson cluster states?=>there are many discussion for this issue.

PRL **115,** 072001 (2015) Selected for a Viewpoint in *Physics* PHYSICAL REVIEW LETTERS week ending 14 AUGUST 2015

Observation of $J/\psi p$ Resonances Consistent with Pentaquark States in $\Lambda_b^0 \to J/\psi K^- p$ Decays

R. Aaij *et al.**
(LHCb Collaboration)
(Received 13 July 2015; published 12 August 2015)



Observations of exotic structures in the $J/\psi p$ channel, which we refer to as charmonium-pentaquark states, in $\Lambda_b^0 \to J/\psi K^- p$ decays are presented. The data sample corresponds to an integrated luminosity of 3 fb^{-1} acquired with the LHCb detector from 7 and 8 TeV pp collisions. An amplitude analysis of the three-body final state reproduces the two-body mass and angular distributions. To obtain a satisfactory fit of the structures seen in the $J/\psi p$ mass spectrum, it is necessary to include two Breit-Wigner amplitudes that each describe a resonant state. The significance of each of these resonances is more than 9 standard deviations. One has a mass of $4380 \pm 8 \pm 29$ MeV and a width of $205 \pm 18 \pm 86$ MeV, while the second is narrower, with a mass of $4449.8 \pm 1.7 \pm 2.5$ MeV and a width of $39 \pm 5 \pm 19$ MeV. The preferred J^P assignments are of opposite parity, with one state having spin 3/2 and the other 5/2.

DOI: 10.1103/PhysRevLett.115.072001 PACS numbers: 14.40.Pq, 13.25.Gv

State	Mass (MeV)	Width (MeV)	Fit fraction (%)	Significanc
$P_c(4380)^+$	4380±8±29	205±18±86	8.4±0.7±4.2	9
$P_c(4450)^+$	4449.8±1.7±2.5	39± 5±19	4.1±0.5±1.1	12

- Best fit has J^P=($3/2^-$, $5/2^+$), also ($3/2^+$, $5/2^-$) & ($5/2^+$, $3/2^-$) are preferred

To describe the data of $P_c(4380)^+$ and $P_c(4459)^+$ state, there are theoretical effort.

- Cusp?
  Phys. Rev. D92 071502 (2015), Phys. Lett. B751 59 (2015)

- Meson-Baryon state?
  Phys. Rev. Lett. 115 172001(2015), Phys. Rev. D92 094003 (2015)
  Phys. Rev. Lett. 132002 (2015), Phys. Rev. D92 114002 (2015)
  Phys. Lett. B753 547 (2016)

- Baryoncharmonnia
  Phys. Rev. D92 031502 (2015)

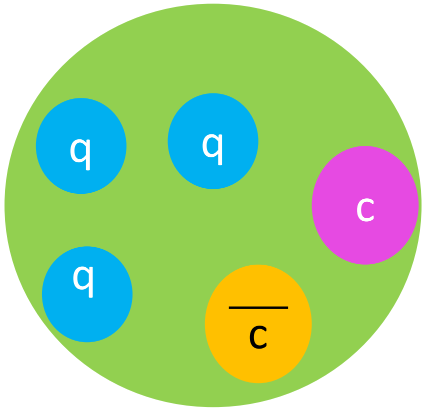
- Tightly bound pentaquark states
  Eur. Phys. J. A48 61 (2012), Phys. Lett. B 749 454 (2015),
  Phys. Lett. B749 289 (2015) , Phys. Lett. B764 254 (2017) etc.

In this way, we have the following question:
Tetra-quark and penta-quark systems observed so far are
Meson-meson cluster-like states(molecular systems), meson-baryon cluster –like states, or compact states?

To answer this question; with use of quark model, we have been studying penta-quark and tentra-quark systems.

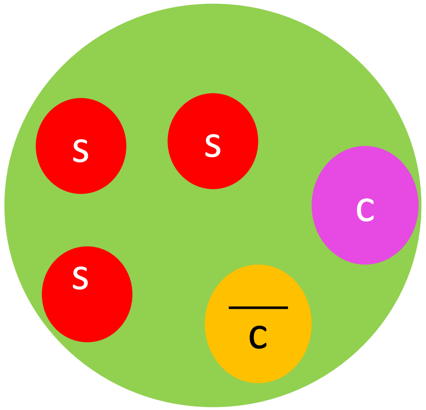
Collaborators: Hosaka san , Oka san
PhD student: Qi Meng (Nanjing Univ.)

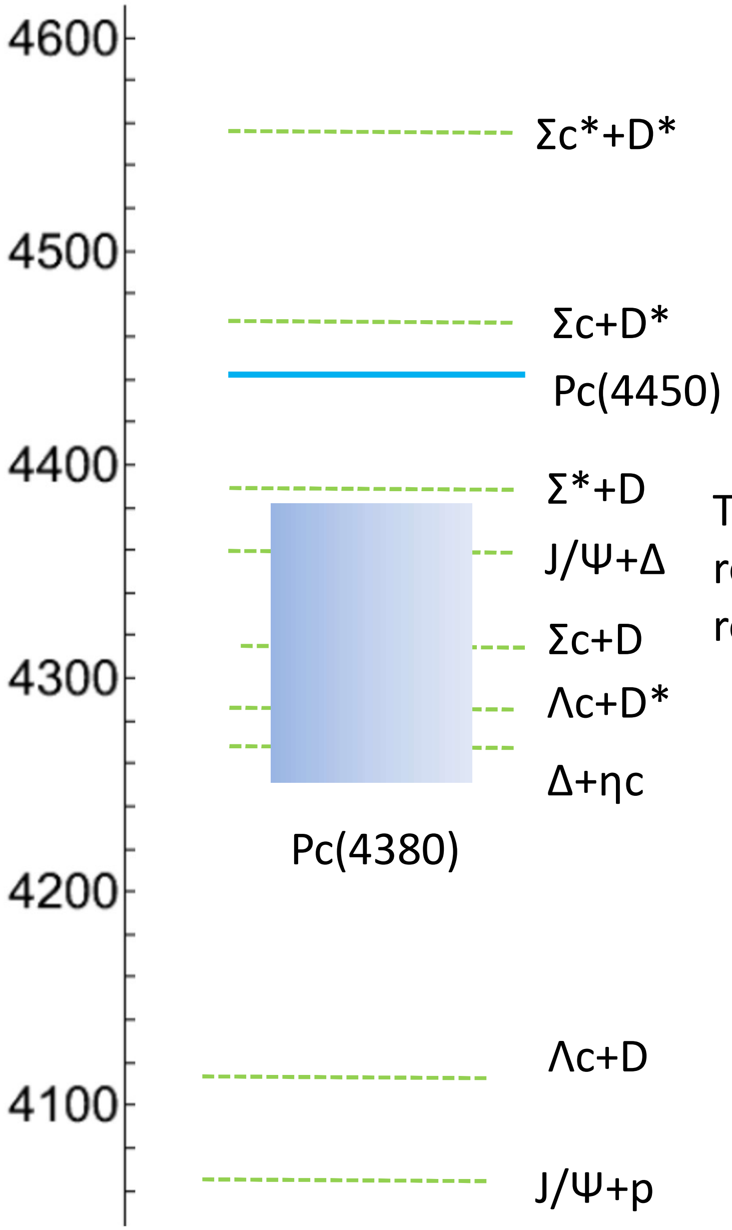
Penta-quark systems

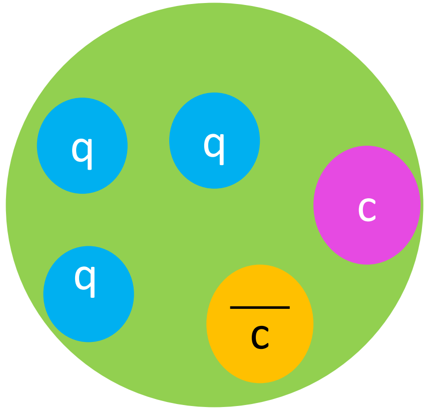


Observed at LHCb

No observation







This is 5-body problem and it requested to calculate resonant state. Then, we should develop our method ror resonant state.

To describe the experimental data,
It is necessary to reproduce the observed threshold.

The Hamiltonian is important
to reproduce the low-lying energy spectra of meson and baryon system.

Hamiltonian

$$H = \sum_i \left(m_i + \frac{\mathbf{p}_i^2}{2m_i} \right) - T_G + V_{\mathrm{Conf}} + V_{\mathrm{CM}} - \Lambda/r \qquad \Lambda=0.1653\,\mathrm{GeV}^2$$

$$V_{\mathrm{Conf}} = -\sum_{i<j} \sum_{\alpha=1}^{8} \frac{\lambda_i^\alpha}{2} \frac{\lambda_j^\alpha}{2} \left[\frac{k}{2} (\mathbf{x}_i - \mathbf{x}_j) \ + v_0 \right], \qquad K=0.5069$$

$$V_{\mathrm{CM}} = \sum_{i<j} \sum_{\alpha=1}^{8} \frac{\lambda_i^\alpha}{2} \frac{\lambda_j^\alpha}{2} \frac{\xi_\sigma}{m_i m_j} e^{-(\mathbf{x}_i - \mathbf{x}_j)^2/\beta^2} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j .$$

$\xi_\alpha=(2\pi/3)kp$ $\beta=A((2m_i m_j)/(m_i+m_j))^{(-B)}$

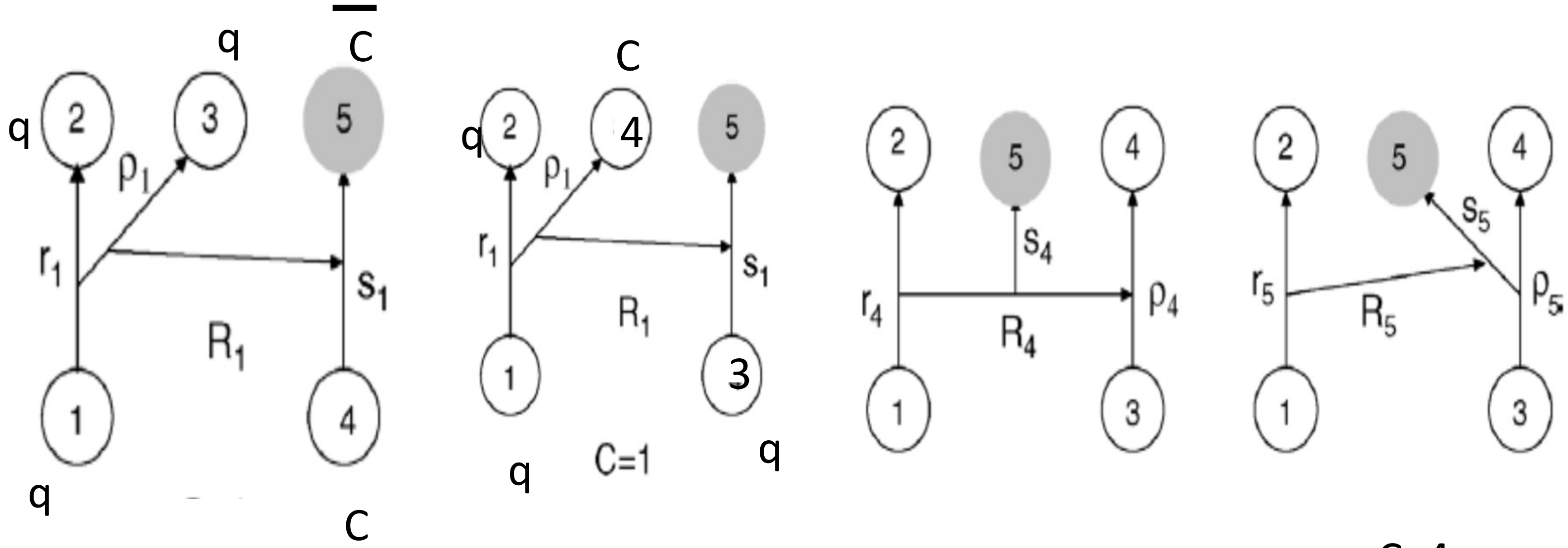
Kp=1.8609 A=1.6553 B=0.2204

m_q=315 MeV, m_c=1836 MeV

B. Silvestre-Brac and C. Semay, Z. Phys. C 61 (1994) 271

Cal.	Exp.
Baryon	
N: 953 MeV	939 MeV
Δ: 1265 MeV	1232
Λc: 2276 MeV	2286
Σc:2451 MeV	2465
Σc*:2531 MeV	2545
Meson	
D: 1862 MeV	1870
D*:2016 MeV	2010
J/Ψ:3102 MeV	3094
ηc :3007 MeV	2984
χ_c l=1,s=0: 3462.4 MeV	hc:3525 MeV
L=1,S=1 :3486.5 MeV	3530 MeV

Calculated energy spectra for meson and baryon systems are in good agreement with the observed data.

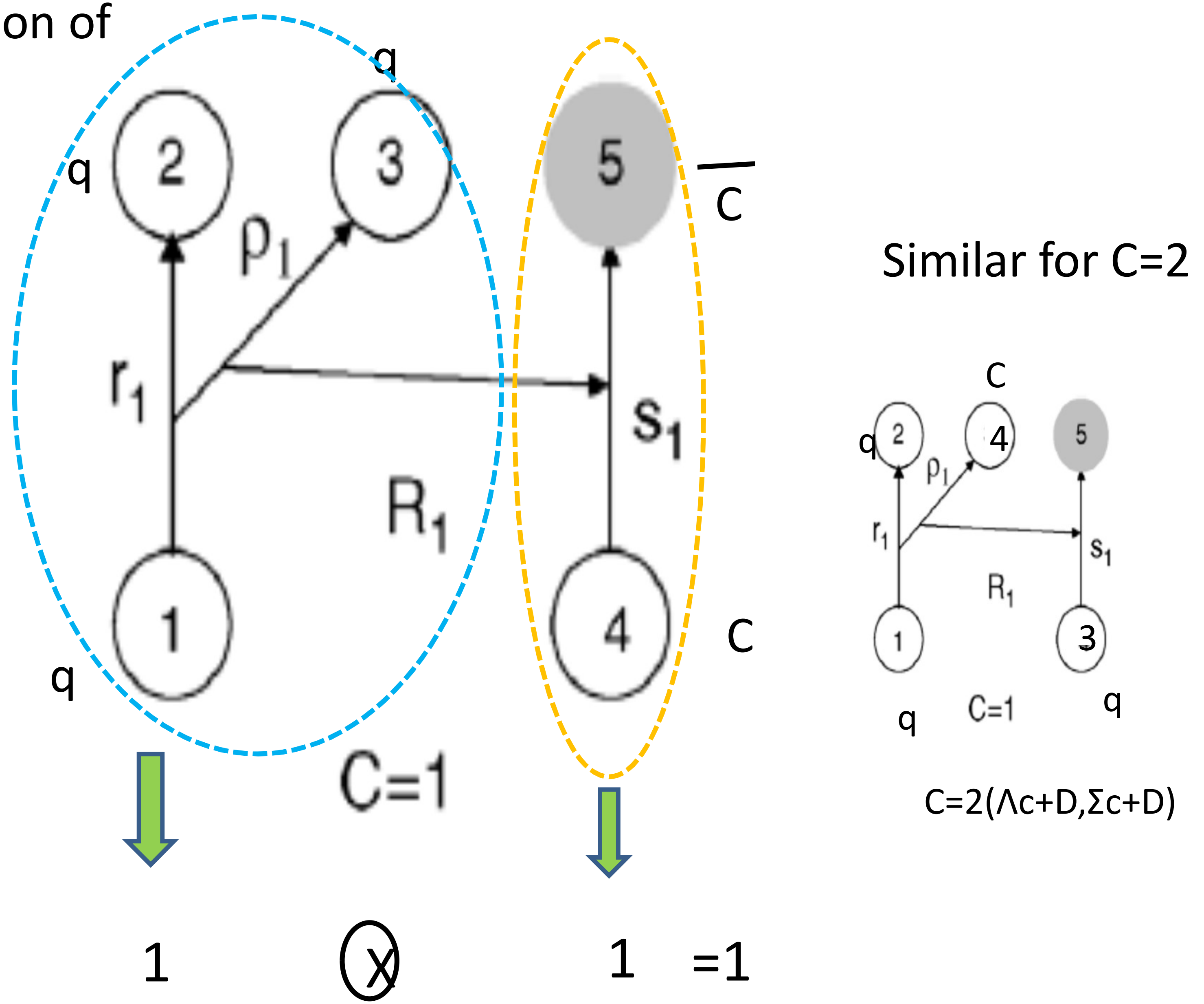


C=1(J/Ψ+p,ηc+p) C=2(Λc+D,Σc+D) C=3 C=4

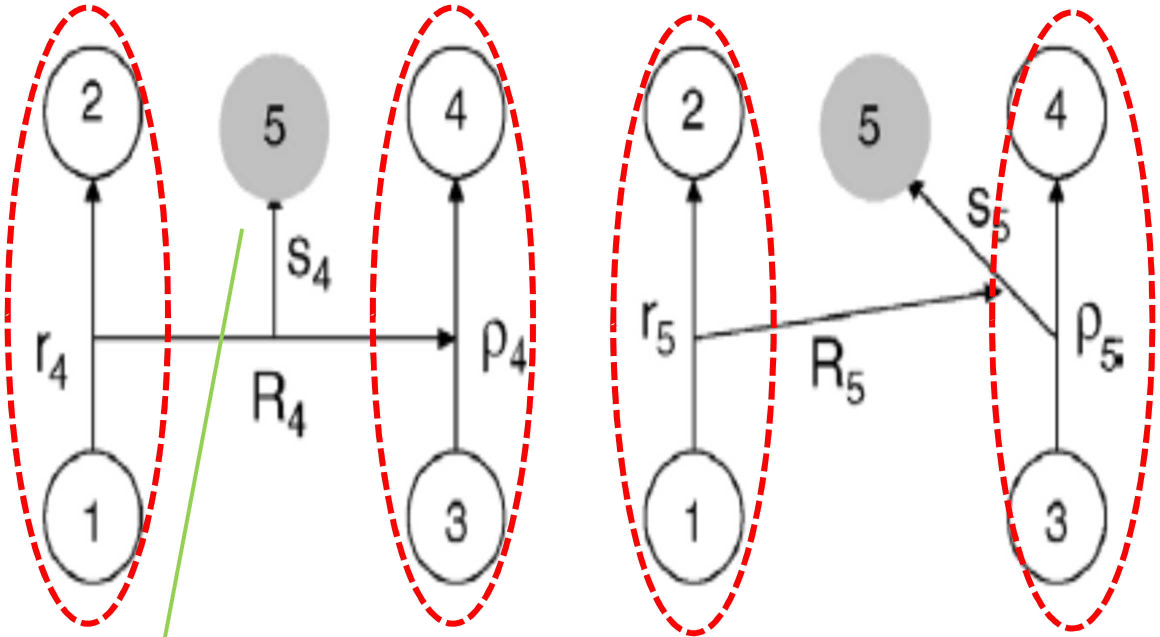
$$\Psi_{JM}(qq\bar{q}cc)= \quad \Phi_{JM}^{(C=1)} + \Phi_{JM}^{(C=2)} + \phi_{JM}^{(c=3)} + \Phi_{JM}^{(C=4)}$$

$$\Phi_{\alpha JM}(q\bar{q}qcc)=\mathcal{A}_{qqqq}\{[(\mathrm{color})^{(c)}_{\alpha} \quad (\mathrm{isospin})^{(c)}_{\alpha} \quad (\mathrm{spin})^{(C)}_{\alpha} \quad (\mathrm{spatial})^{(c)}_{\alpha}]_{JM}\}$$

Wavefunction of
Color part



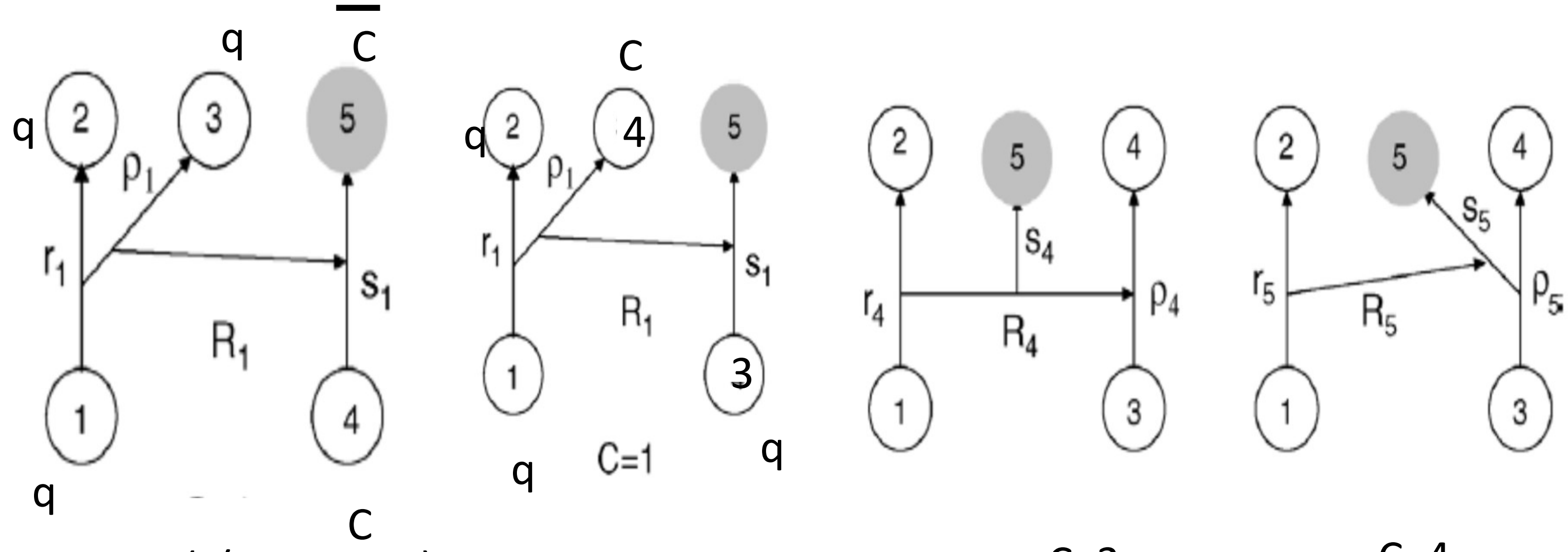
Confining channels



$\bar{3} \otimes \bar{3} = 3$ $\qquad$ $\bar{3} \otimes \bar{3} = 3$

$\bar{3} \otimes 3 = 8 + 1$ $\qquad$ $\bar{3} \otimes 3 = 8 + 1$

I take color singlet. $\qquad$ I take color singlet.



C=1(J/Ψ+p,ηc+p) C=2(Λc+D,Σc+D) C=3 C=4

$$\Psi_{JM}(qq\bar{q}cc)= \Phi_{JM}^{(C=1)}+\Phi_{JM}^{(C=2)}+\phi_{JM}^{(c=3)}+\Phi_{JM}^{(C=4)}$$

$$\Phi_{\alpha JM}(q\bar{q}qcc)=\mathcal{A}_{qqqq}\{[(\text{color})^{(c)}_{\alpha}\quad(\text{isospin})^{(c)}_{\alpha}\quad(\text{spin})^{(C)}_{\alpha}\quad(\text{spatial})^{(c)}_{\alpha}]_{JM}\}$$

$$(\text{spatial})^{(c)}_{\alpha}=\varphi_{nl}^{(c)}(r_c)\psi_{\nu\lambda}^{(c)}(\rho_c)\varphi_{R}^{(c)}{}_{kj}(s_c)\Phi_{n\,LM}^{(c)}(R_c)$$

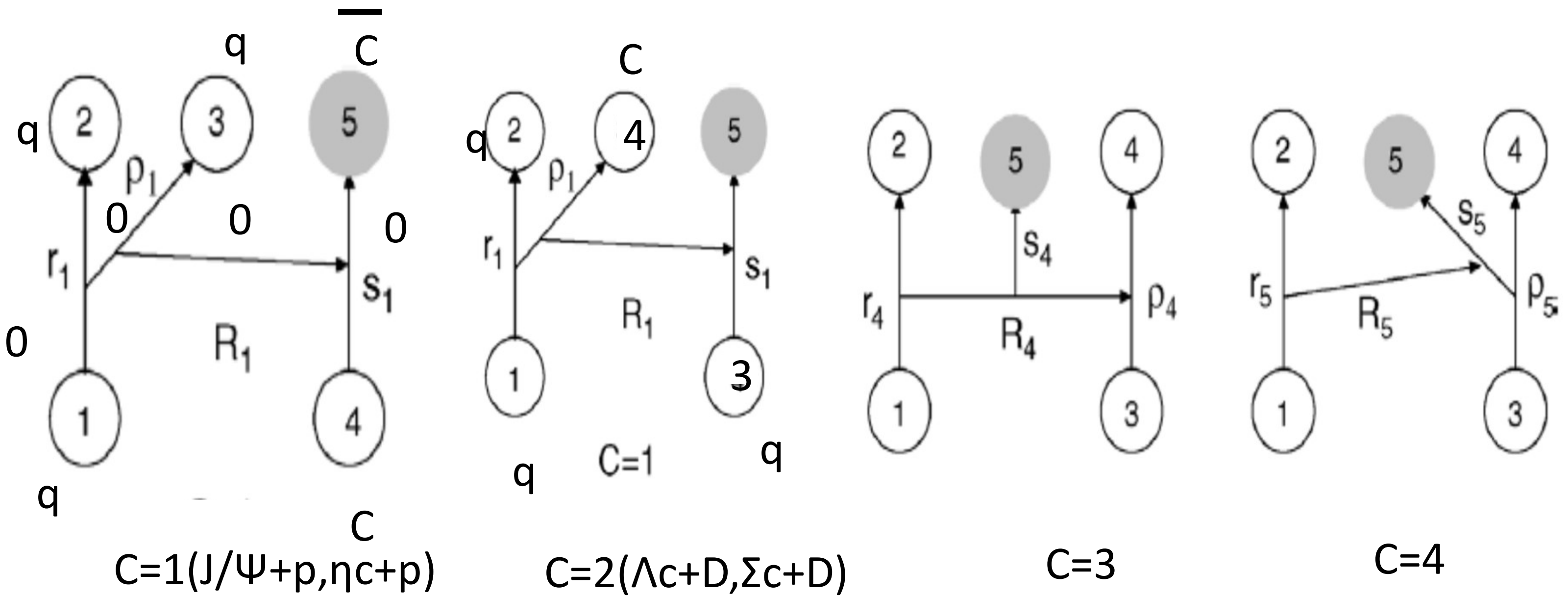
$$\phi_{n_R L_c M}(\mathbf{R})=R^{L_c}e^{-(R/\bar{R}_{n_R})^2}Y_{L_c M}(\hat{\mathbf{R}})\qquad \bar{R}_{n_R}=\bar{R}_1 a^{n_R-1}\quad\left(n_R=1-n_R^{\max}\right)$$

Same procedure is taken for r,ρ, and s.

For the Pc(4380) and (4450), we consider the following 9 candidates s

Total orbital angular momentum: L=0, 1, 2
Total Spin : S=1/2, 3/2, 5/2

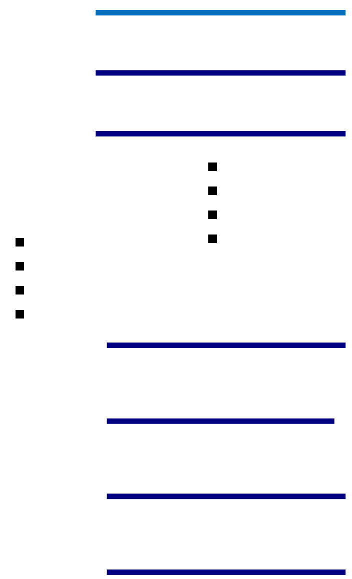
For example, in the case of total orbital angular momentum L=0, S=1/2, 3/2, 5/2, $J^{\pi}=1/2^{-},3/2^{-},5/2^{-}$
We take s-waves for all coordinates.



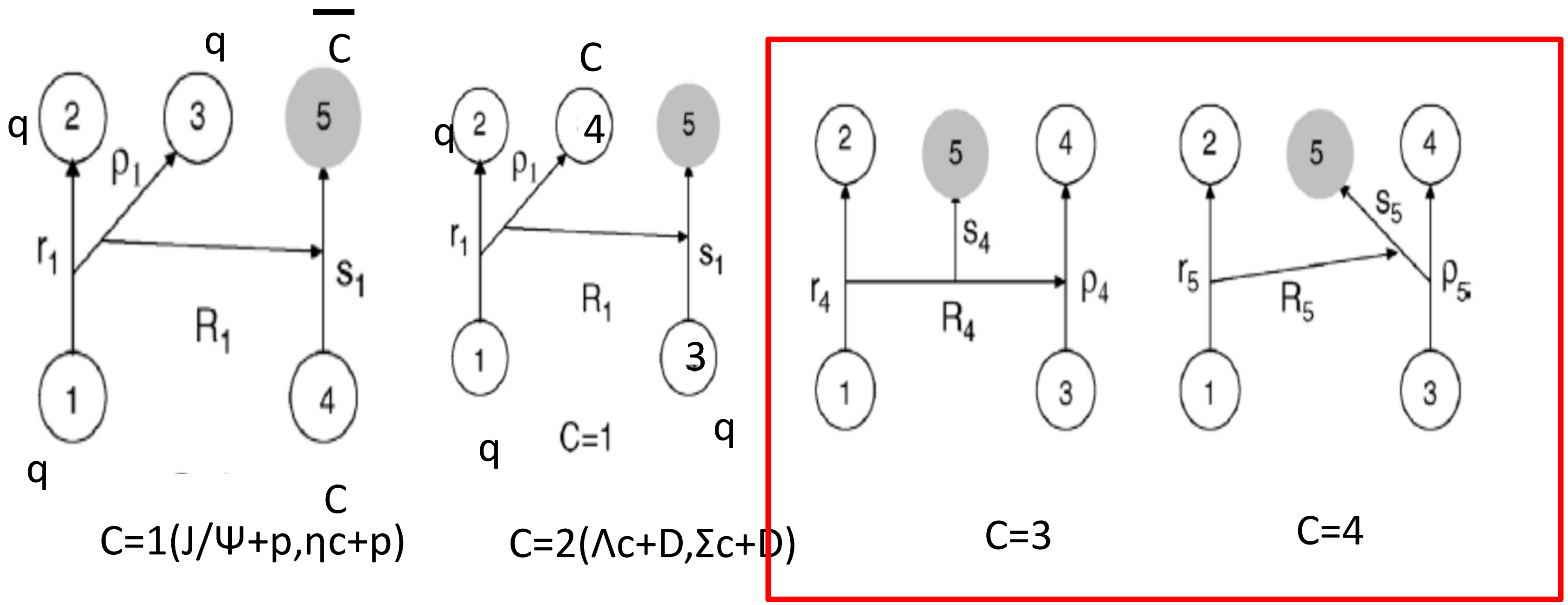
$(H-E)\Psi=0$

By the diagonalization of Hamiltonian, we obtain N eigenstates for each J^{π}.

Here, we use about 40,000 basis functions.
Then, we obtained 40,000 eigenfunction for each J^{π}.
First, we investigate J=1/2-, namely, L(total angular momentum)=0, S(total spin)=1/2.

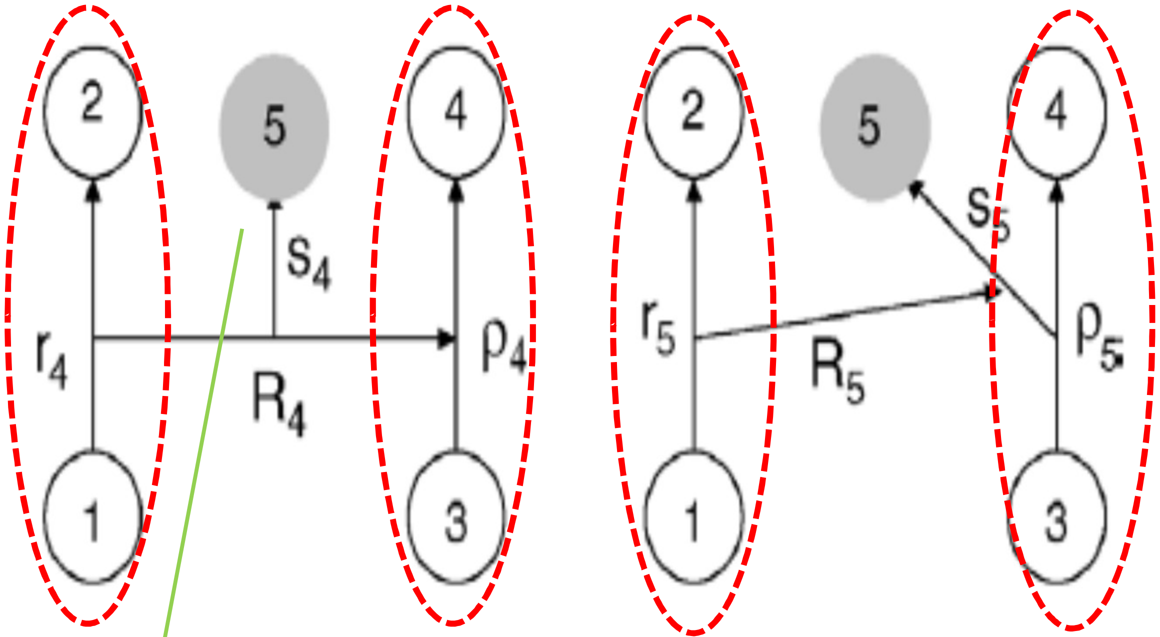


L=0,S=1/2 for example



First, we take two channels.

Confining channels



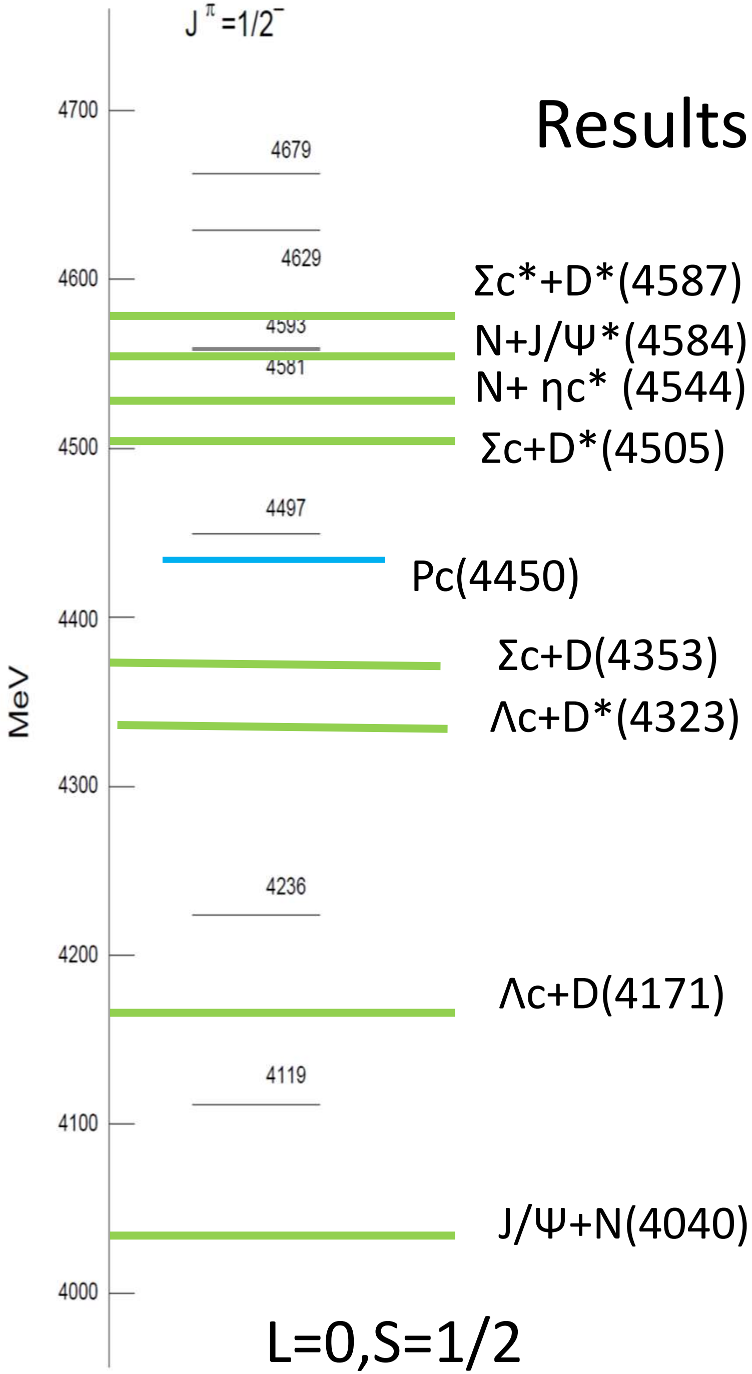
$\bar{3} \otimes \bar{3} = 3 \qquad \bar{3} \otimes \bar{3} = 3$

$\bar{3} \otimes 3 = 8 + 1 \qquad \bar{3} \otimes 3 = 8 + 1$

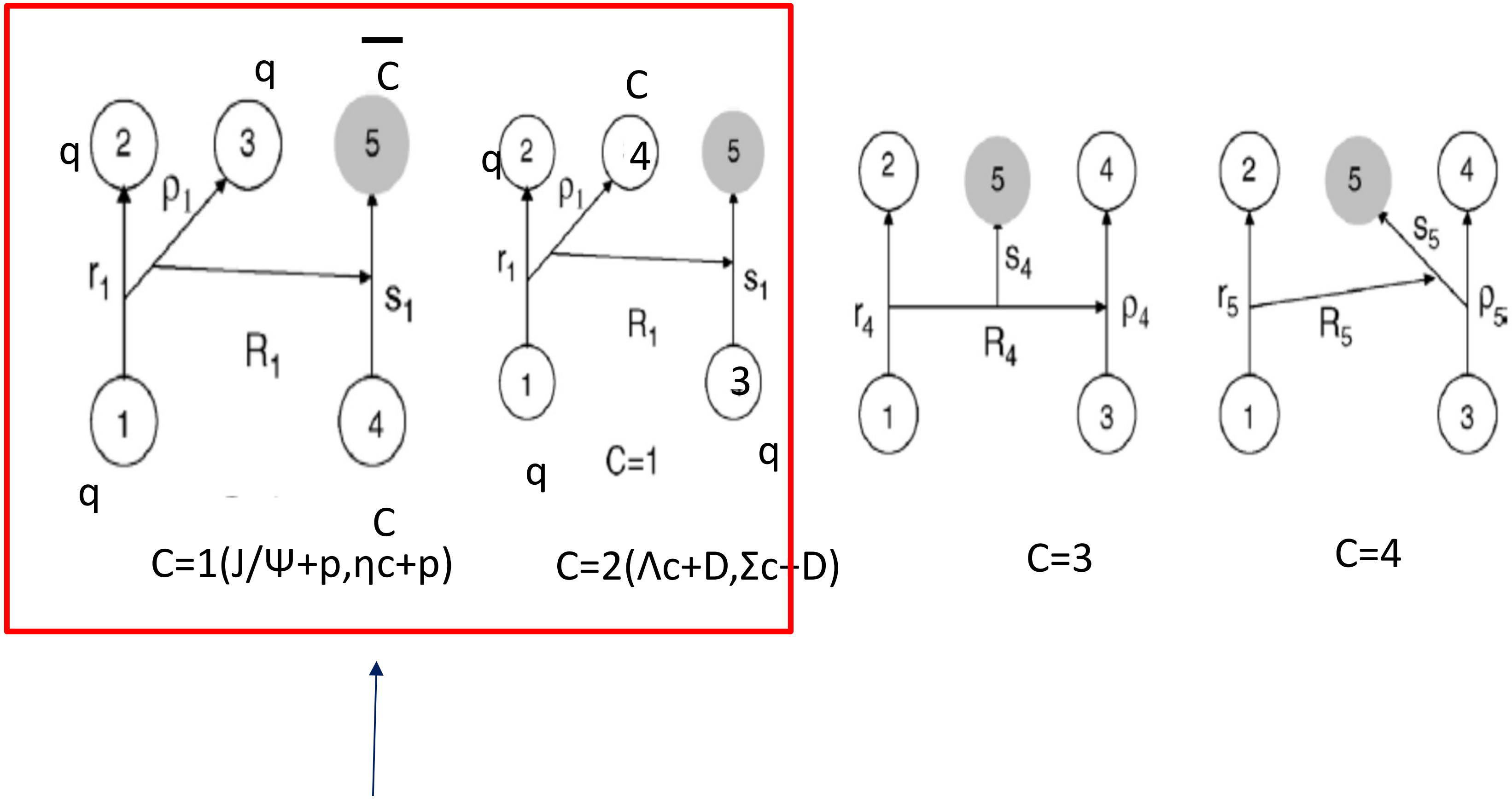
I take color singlet.

I take color singlet.

Results by connected channels only



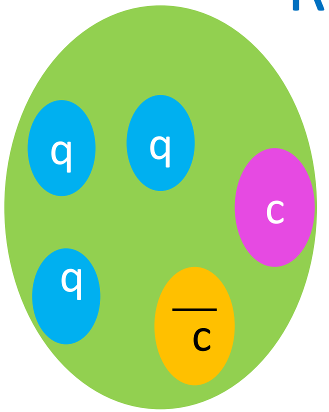
L=0,S=1/2



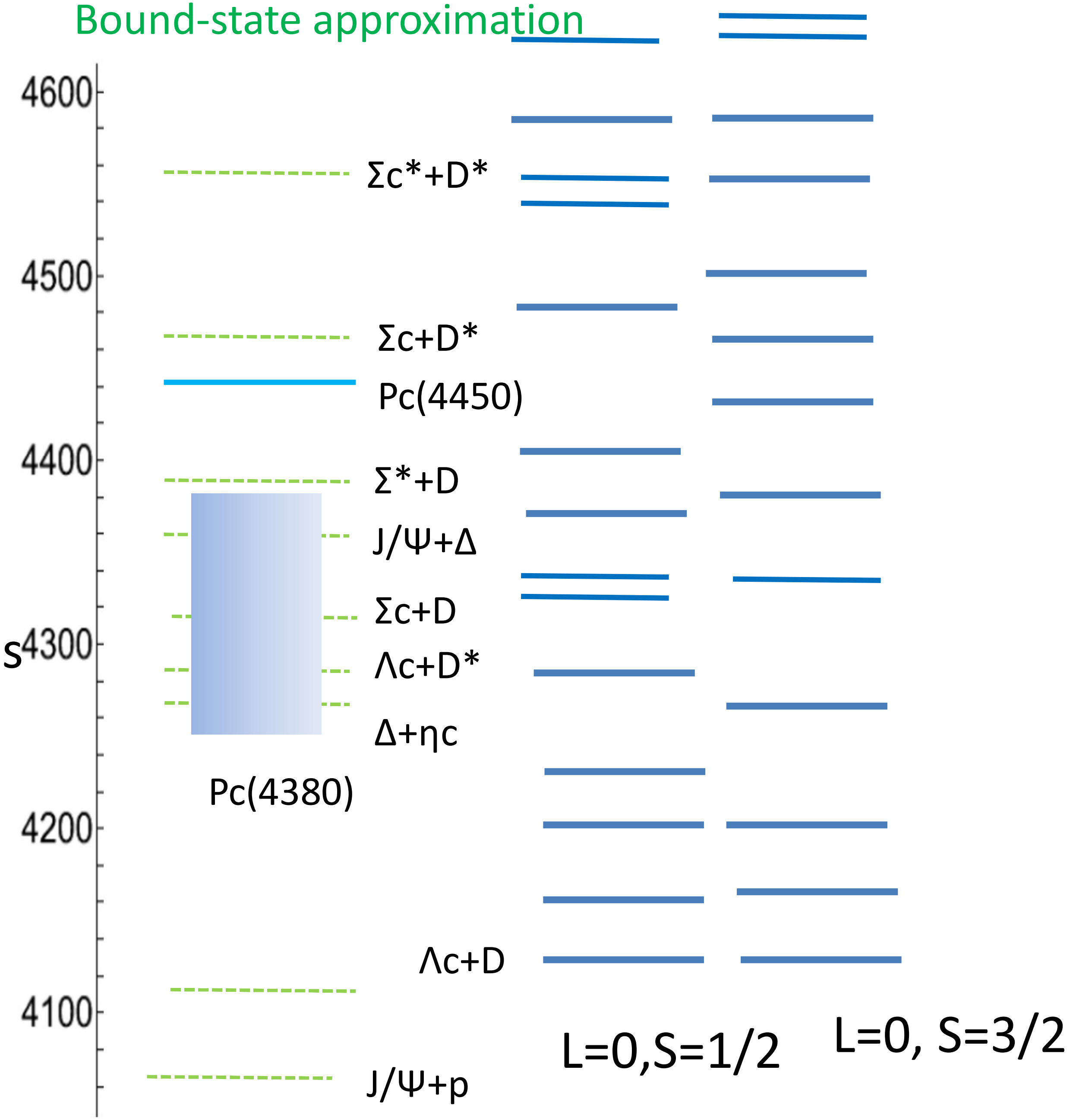
Next, we take two scattering channels.

Results before doing the scattering calculation

Bound-state approximation



Do these states correspond
to resonance states
or
discrete non-resonance
continuum states?



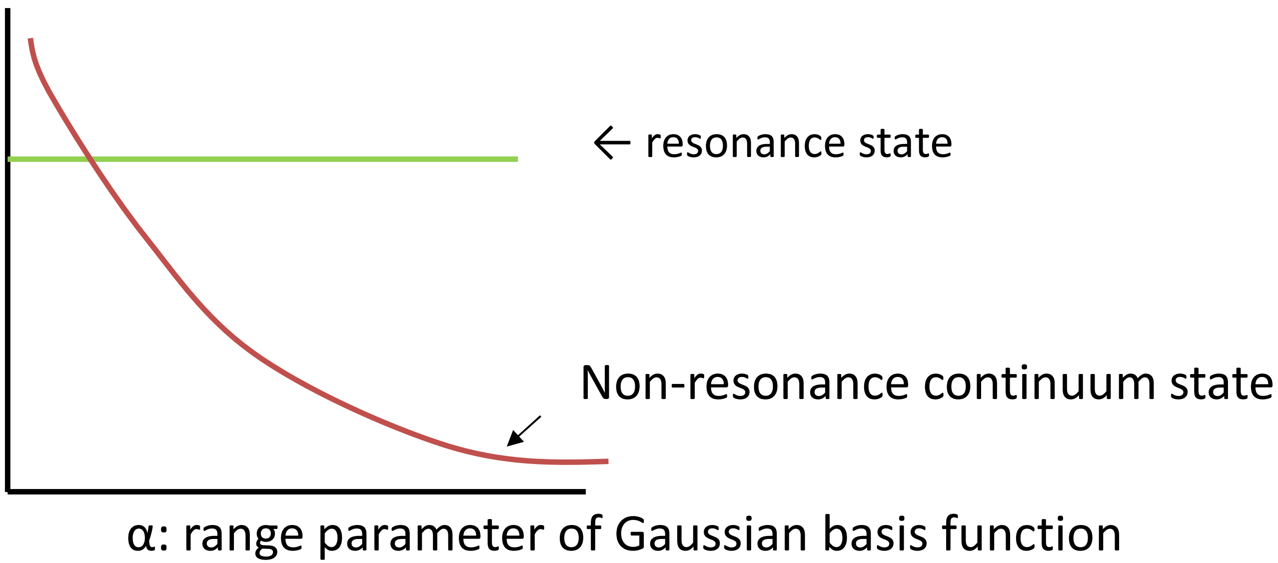
useful method: real scaling method

often used in atomic physics

In this method, we artificially scale the range parameters of our Gaussian basis functions by multiplying a factor α:

$r_n \rightarrow \alpha r_n$ in $r^l \exp^{(-r/r_n)^2}$ for exmple $0.8 < \alpha < 1.5$

and repeat the diagonalization of Hamiltonian for many value of α.



[schematic illustration of the real scaling]

What is the result in our pentaquark calculation?

of nonstoichiometric samples, because they have an

[5] R. F. Brebrick, J. Phys. Chem. Solids 32, 551 (1971).

Resonance state lifetimes from stabilization graphs

Jack Simons[a)]

Chemistry Department, University of Utah, Salt Lake City, Utah 84112
(Received 20 January 1981; accepted 18 May 1981)

The stabilization method (SM) pioneered by Taylor and co-workers[1] has proven to be a valuable tool for estimating the energies of long-lived metastable states of electron–atom, electron–molecule, and atom–diatom complexes. In implementing the SM one searches for eigenvalues arising from a matrix representation of the relevant Hamiltonian H which are "stable" as the basis set used to construct H is varied.

To obtain lifetimes of metastable states, one can choose from among a variety of techniques[2–7] (e.g., phase shift analysis, Feshbach projection "golden rule" formulas, Siegert methods, and complex coordinate scaling methods), many of which use the stabilized *eigenvector* as starting information. Here we demonstrate that one can obtain an *estimate* of the desired lifetime directly from the stabilization graph in a manner which makes a close connection with the complex coordinate rotation method (CRM) for which a satisfactory mathematical basis exists.

The starting point of our development is the observation that both the stable eigenvalue (E_r) and the eigenvalue(s) (E_c) which come from above and cross E_r (see Fig. 1 and Refs. 9–11 and 13) vary in a nearly linear manner (with α) near their avoided crossing points. This observation leads us to propose that the two eigenvalues arising in each such avoided crossing can be thought of as arising from two "uncoupled" states having energies $\epsilon_r(\alpha)=\epsilon+S_r(\alpha-\alpha_c)$ and $\epsilon_c(\alpha)=\epsilon+S_c(\alpha-\alpha_c)$, where S_r and S_c are the *slopes* of the linear parts of the stable and "continuum" eigenvalues, respectively. α_c is the value of α at which these two straight lines would intersect, and ϵ is their common value at $\alpha=\alpha_c$. This modeling of ϵ_r and ϵ_c is simply based upon the *observa-*

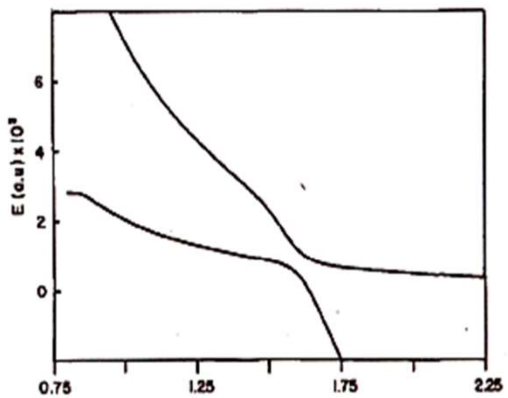
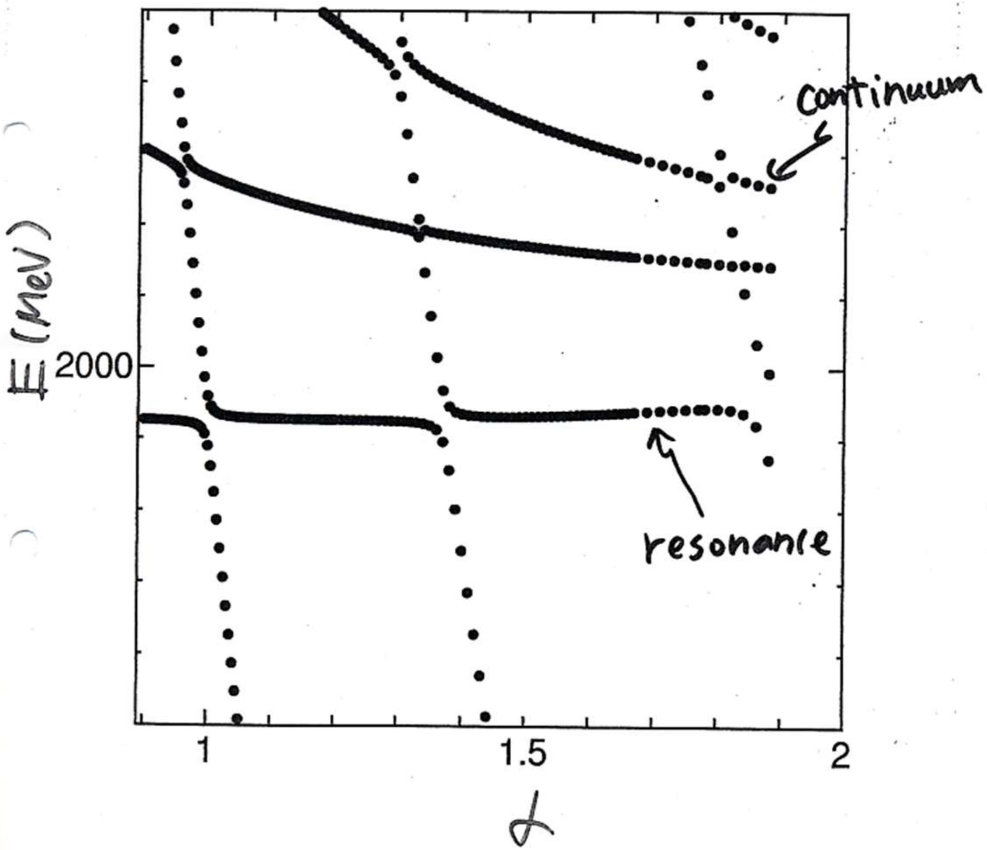


FIG. 1. Stabilization graph for the $^2\pi$ shape resonance state of LiH^- (Ref. 9).

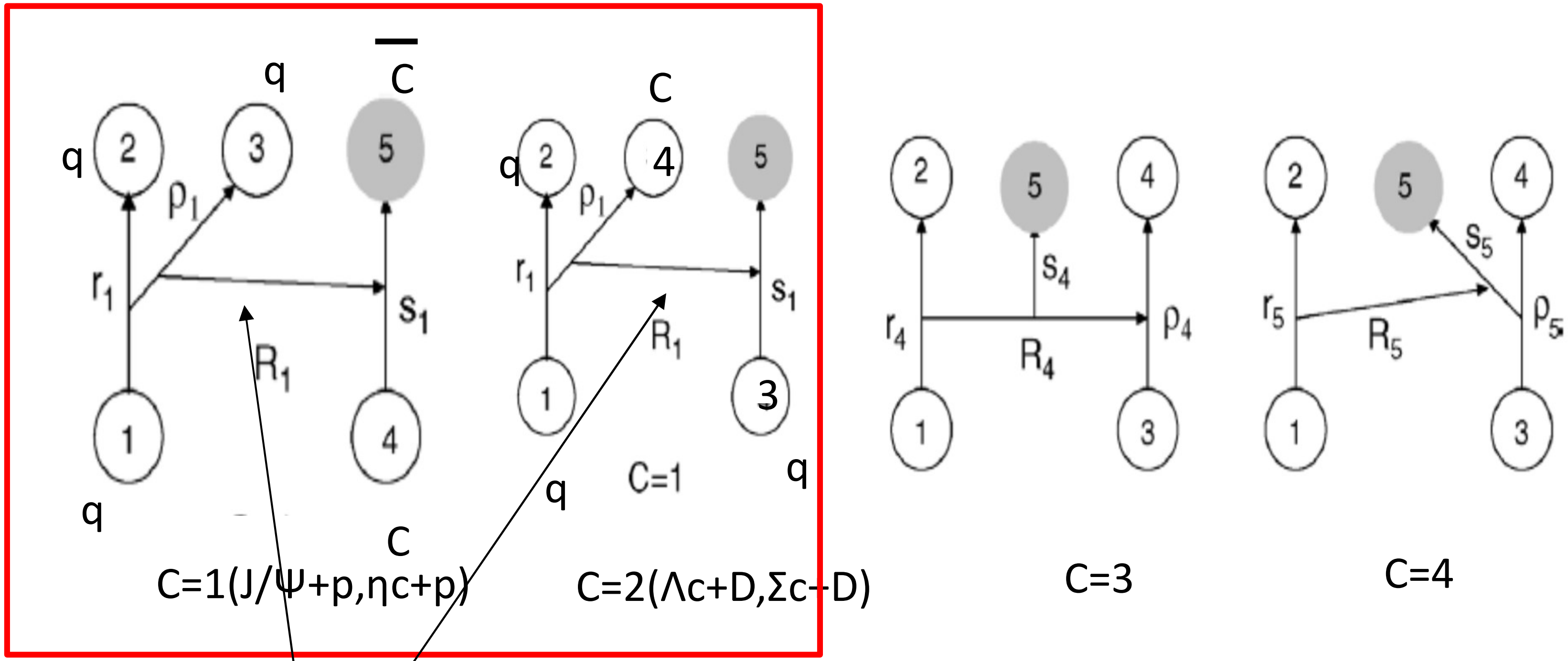
 0021-9606/81/172465-03$01.00

Example of real scaling

Not result of penta quark system



What is the result of our pentquark calculation?



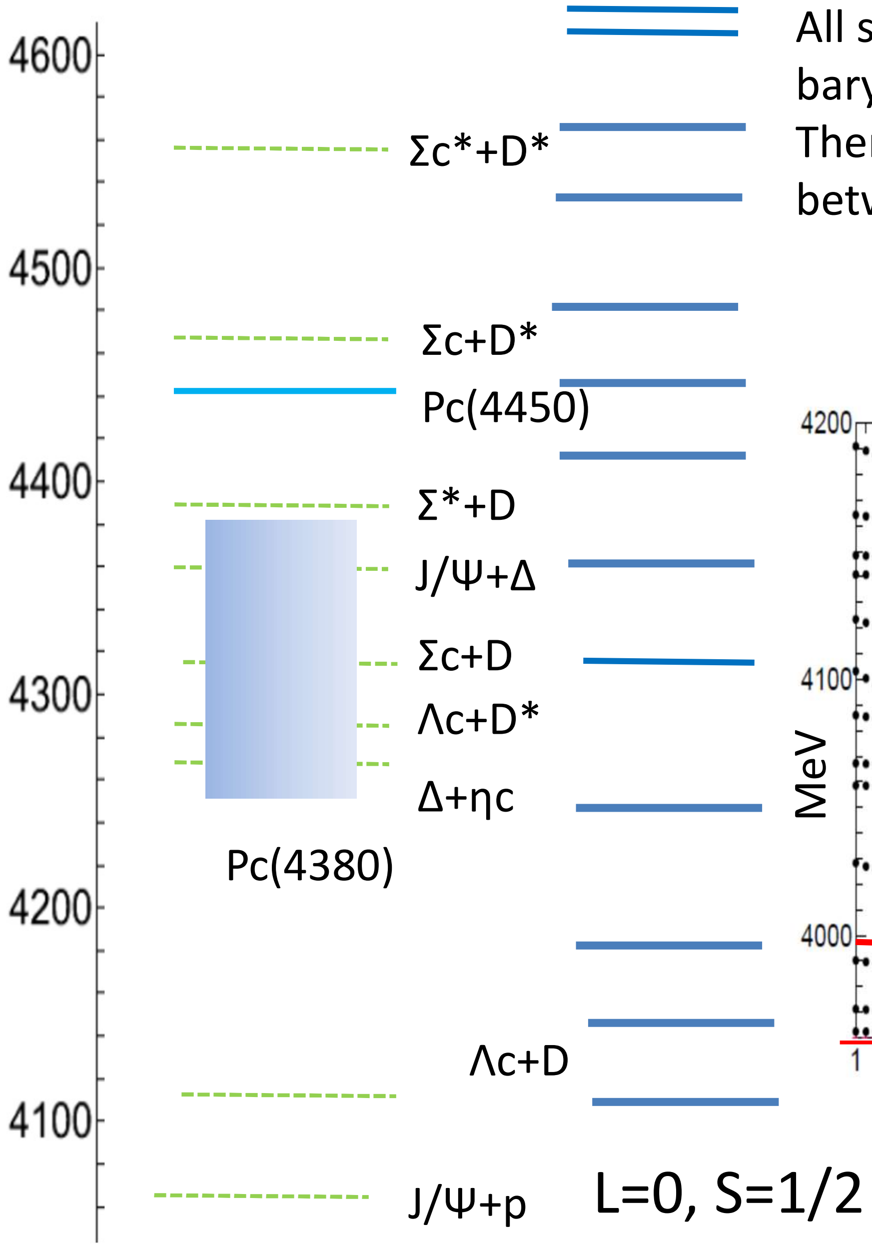
$$\phi_{n_R L_c M}(\mathbf{R}) = R^{L_c} e^{-(R/\bar{R}_{n_R})^2} Y_{L_c M}(\hat{\mathbf{R}})$$

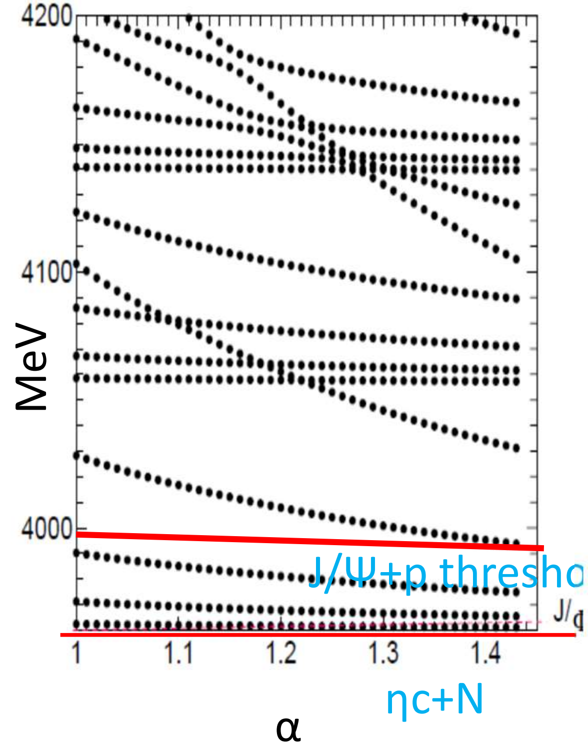
$R_{nR} \Rightarrow \alpha R_{nR}$

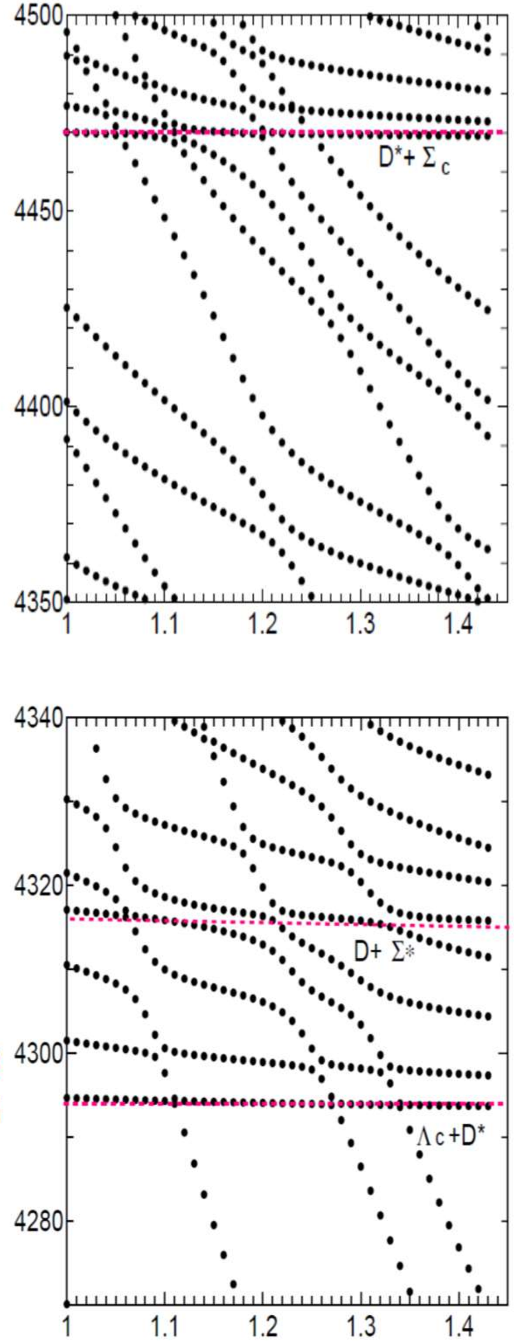
Results before doing the scattering calcu

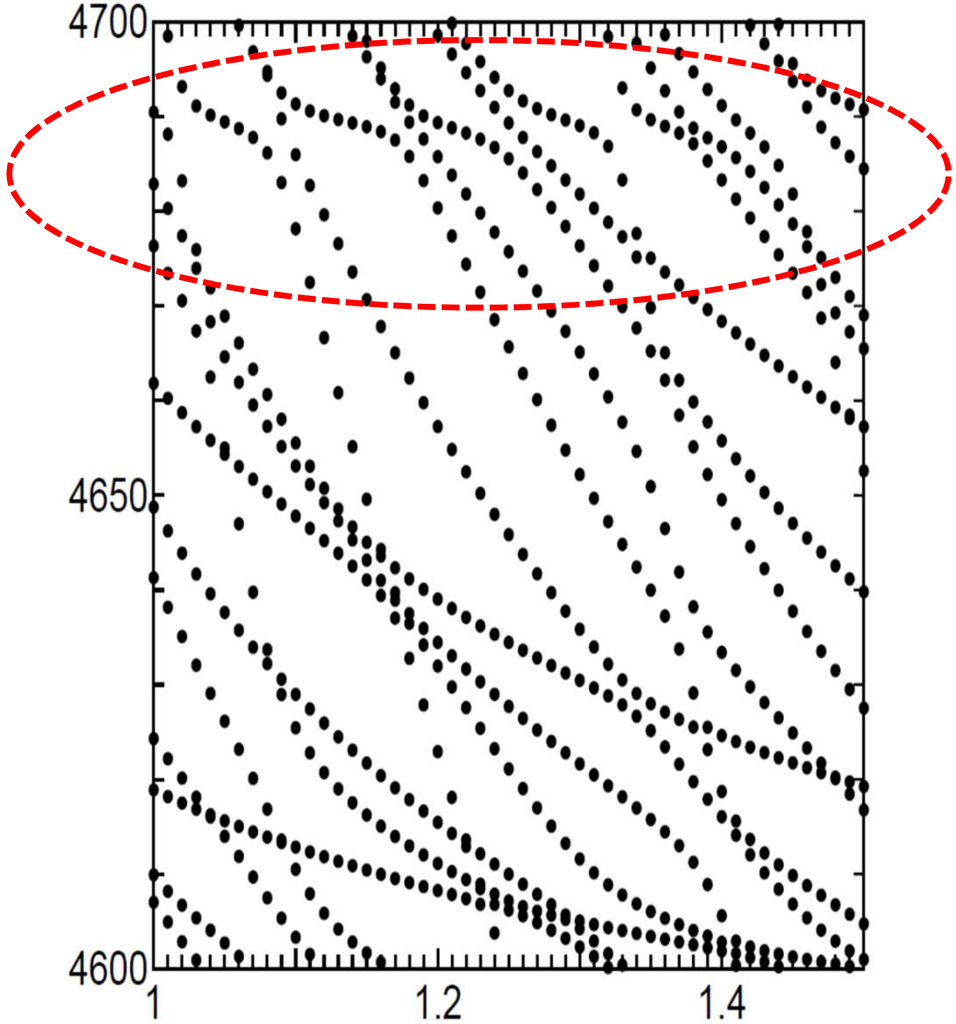
Bound-state approximation

All states are melted into ea
baryon continuum decaying s
Then, there is no resonant st
between 4000 MeV to 4600 M







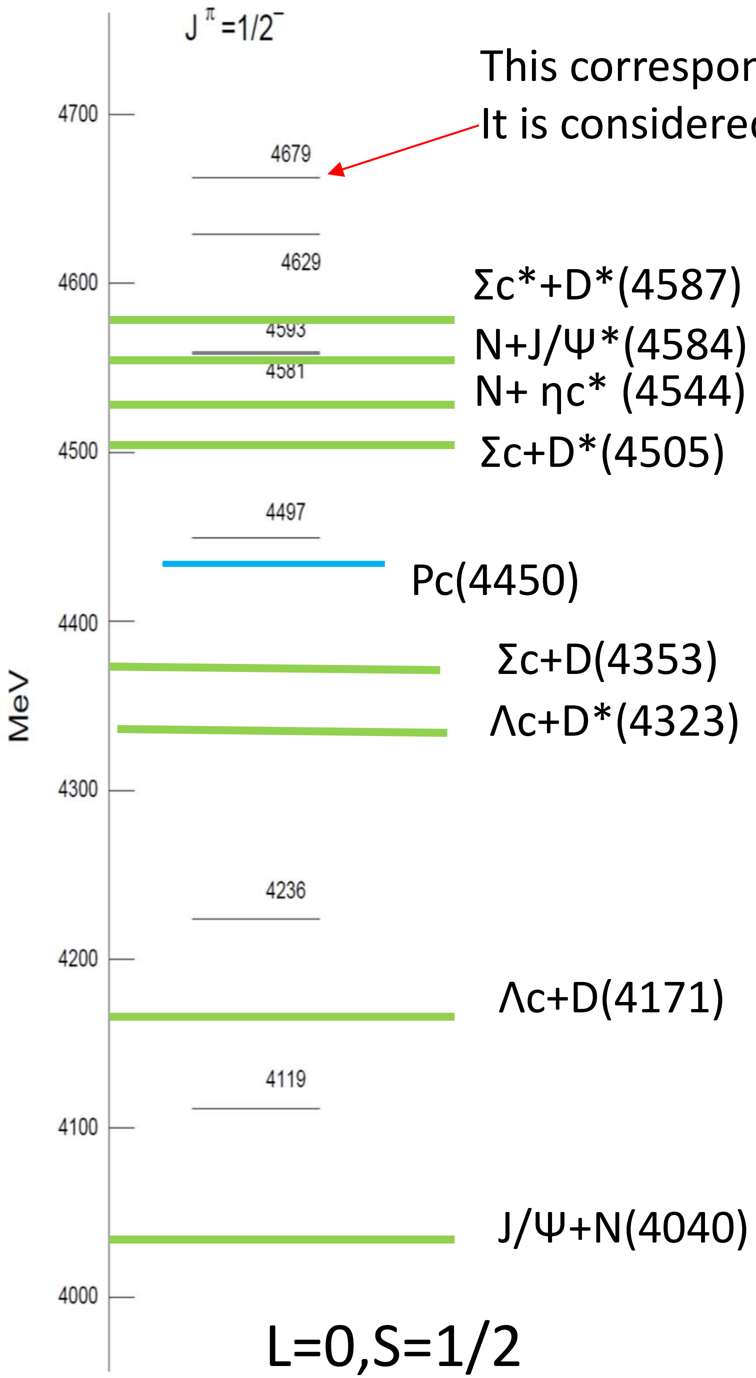


One resonance
at 4690 MeV

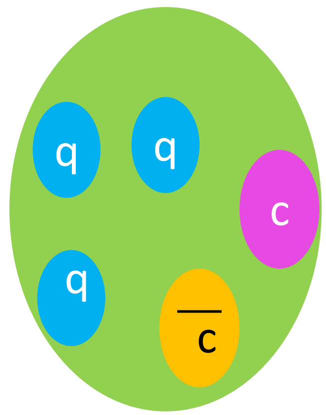
Much higher than the
observed data

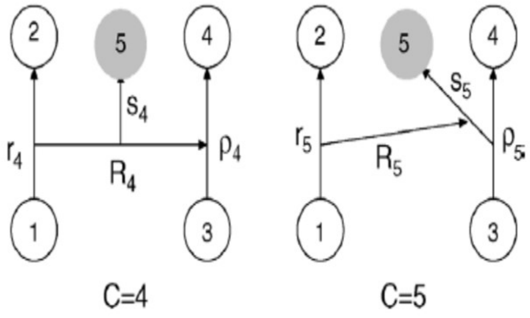
Why we have a resonance
state at such higher energy?

This corresponds to resonant state, like a feshbach resonant state.
It is considered that other states are melted into various thresholc



L=0,S=1/2





Connected channels

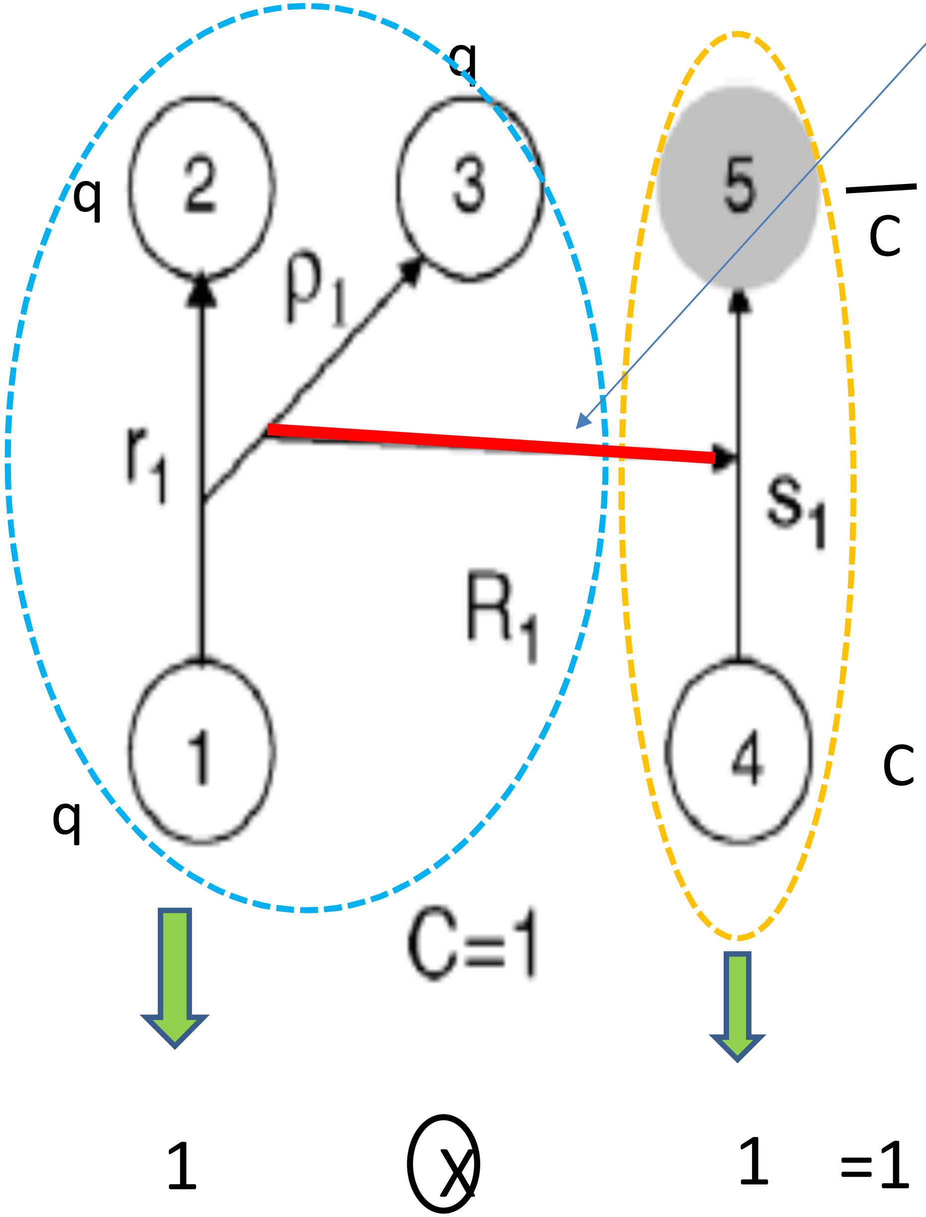
conclusion of penta-quark

・Motivated by the observed Pc(4380) and Pc(4450) systems at LHCb, we calculated energy spectra of qqqcc system using non-relativistic constituent quark model. To obtain resonant states, we also use real scaling method.

In our calculation, we could not obtain at observed energy region. From our calculation, we would suggest that the resonant states observed at LHCb are meson-baryon resonant states which we cannot calculate in our model.

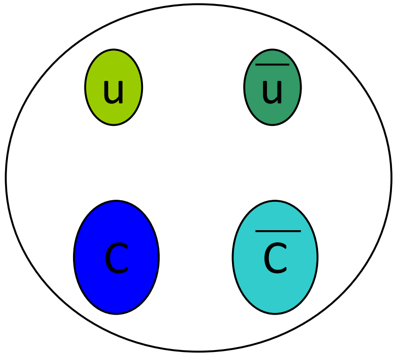
Meson-baryon 共鳴状態は適用できない？

Wavefunction of Color part



この間の
カラーが切れていて、
Short-range partには
相互作用が
働くが、遠方はなし。
これが問題。
Meson-baryonの
共鳴状態がある場合に、
このモデルでは記述できない

We have serious question: whether or not quark model can be applicab to multi-quark system?



今まで、ペンタクオークで、共鳴状態ばかりを使っていた。
ペンタクオークだと問題？共鳴だと問題？

What about bound states of tetra-quark system?

実験はないが、格子QCDによる計算値がある。この計算がクオーク模型の計算と比較してどうなのか？

	$I(J^P)$	[22]	[23]	[24]	[25]
$bb\bar{q}\bar{q}$	$0(1^+)$	-189 ± 13	-143 ± 34	–	-186 ± 15
$bc\bar{q}\bar{q}$	$0(1^+)$	–	–	13 ± 3	–
$cc\bar{q}\bar{q}$	$0(1^+)$	–	-23 ± 11	–	–
$bs\bar{q}\bar{q}$	$0(1^+)$	–	–	16 ± 2	–
$bb\bar{s}\bar{q}$	$\frac{1}{2}(1^+)$	-98 ± 10	-87 ± 32	–	–
$bb\bar{q}\bar{q}$	$1(0^+)$	–	-5 ± 18	–	–
$bc\bar{q}\bar{q}$	$0(0^+)$	–	–	17 ± 3	–
$cc\bar{q}\bar{q}$	$1(0^+)$	–	26 ± 11	–	–
$bs\bar{q}\bar{q}$	$0(0^+)$	–	–	18 ± 2	–

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Physics Letters B

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Stable double-heavy tetraquarks: Spectrum and structure

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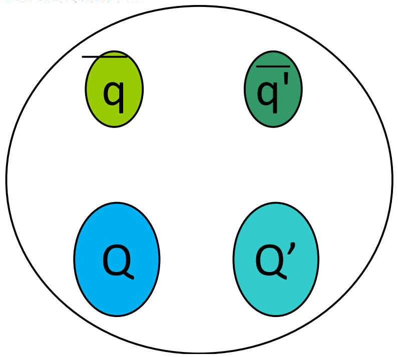
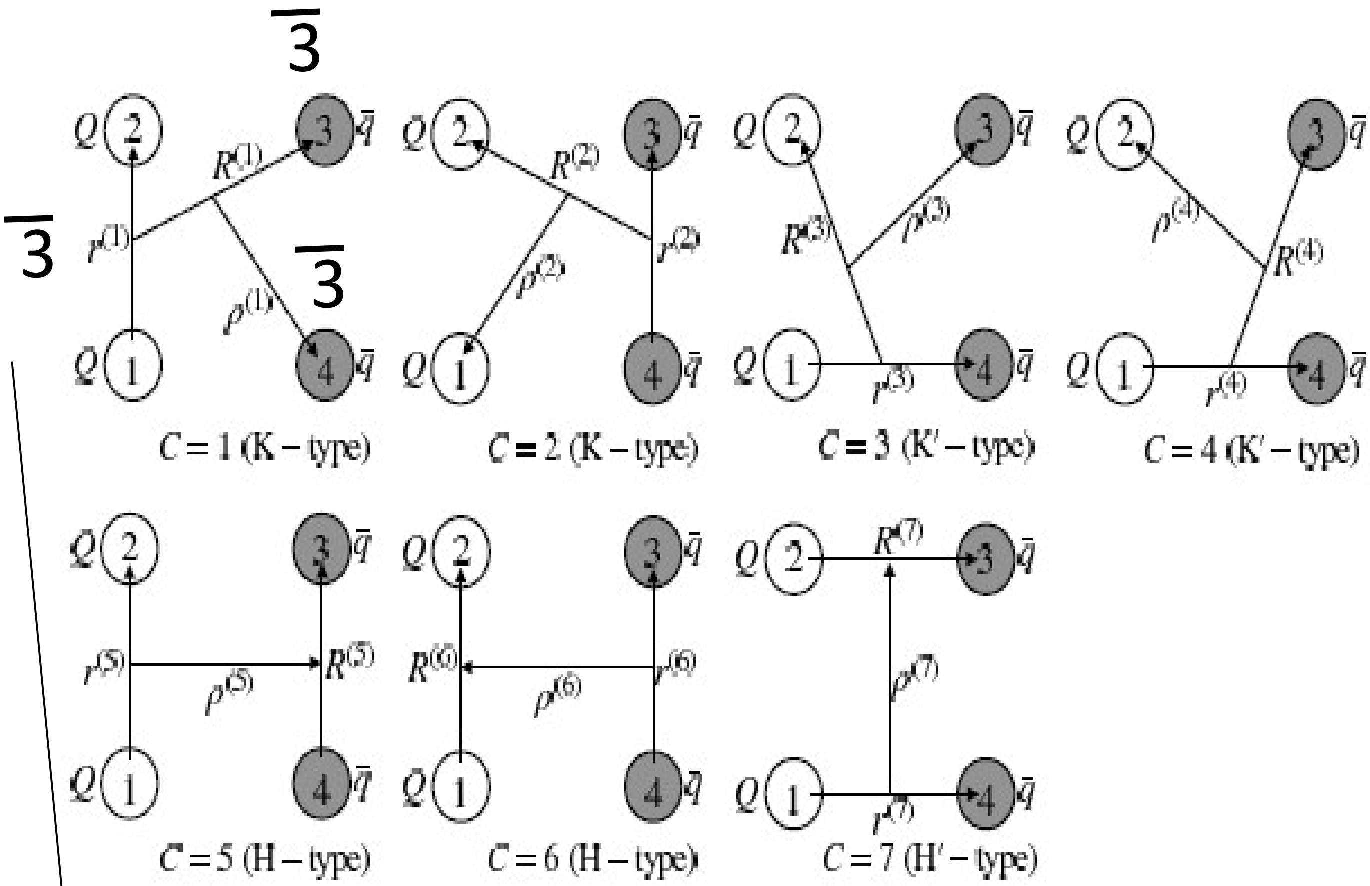
ABSTRACT

Bound states of double-heavy tetraquarks are studied in a constituent quark model. Two bound states are found for isospin and spin-parity $I(J^P)=0(1^+)$ in the $bb\bar{u}\bar{d}$ channel. One is deeply bound and compact made of colored diquarks, while the other is shallow and extended as a BB^* molecule. The former agrees well with lattice QCD results. A systematic decrease in the binding energy is seen by replacing one of the heavy quarks to a lighter one. Altogether we find ten bound states. It is shown for the first time that hadrons with totally different natures emerge from a single Hamiltonian.

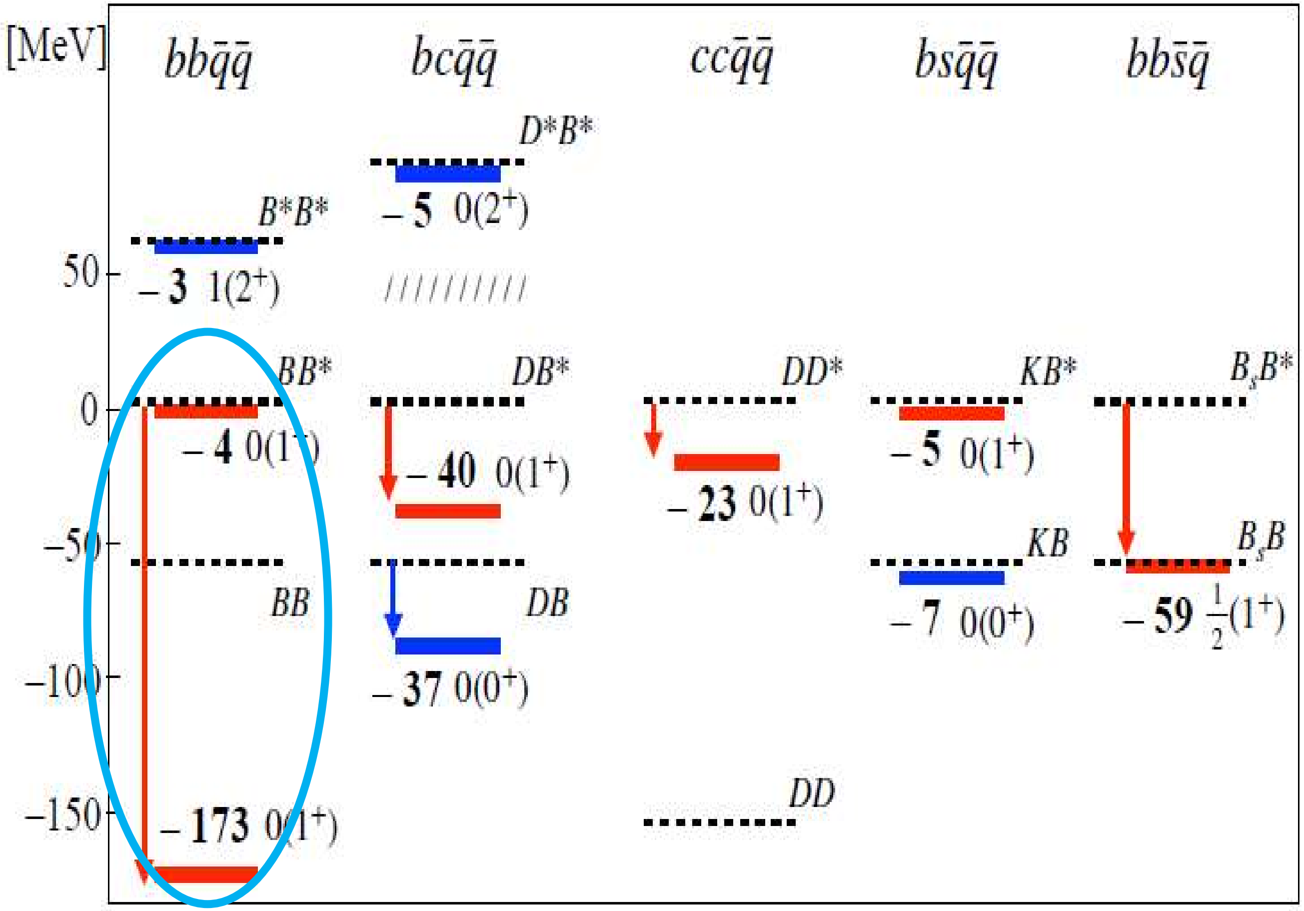




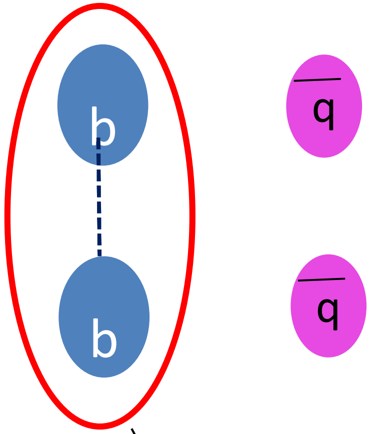
$$\bar{3}\otimes\bar{3}=3 \quad 3\otimes\bar{3}=1$$

We use the same qq potential as in the qqqcc-bar.

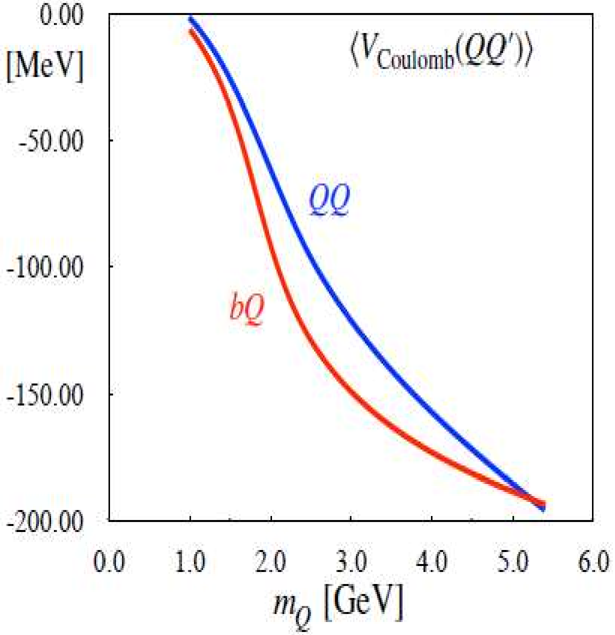
We calculated all of possible states of tetra-quark systems.
Here is energy spectra of bound states.



We have some deeply bound states and weakly bound states.



Mass is very heavy and then distance between bb is so short
Therefore, due to the attraction of Coulomb interaction, 1/r, we will have a chance to make bound states.



Expectation values of coulomb interaction in QQq-barq-bar
and bQq-barq-bar

You can see that as mass of quark is increased, expectation of coulomb interaction becomes larger.

To discuss on validity of our model (quark model), we compare our results with those by LQCD.
You can see that our results are almost consistent with the results by LQCD.

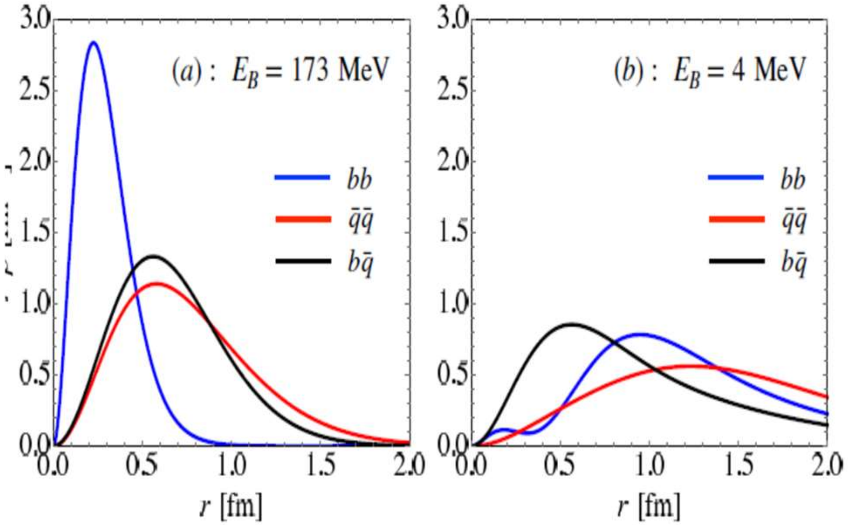
	$I(J^P)$	This work	[22]	[23]	[24]	[25]
$bb\bar{q}\bar{q}$	$0(1^+)$	-173	-189 ± 13	-143 ± 34	–	-186 ± 15
$bc\bar{q}\bar{q}$	$0(1^+)$	-40	–	–	13 ± 3	–
$cc\bar{q}\bar{q}$	$0(1^+)$	-23	–	-23 ± 11	–	–
$bs\bar{q}\bar{q}$	$0(1^+)$	-5	–	–	16 ± 2	–
$bb\bar{s}\bar{q}$	$\frac{1}{2}(1^+)$	-59	-98 ± 10	-87 ± 32	–	–
$bb\bar{q}\bar{q}$	$1(0^+)$	N	–	-5 ± 18	–	–
$bc\bar{q}\bar{q}$	$0(0^+)$	-37	–	–	17 ± 3	–
$cc\bar{q}\bar{q}$	$1(0^+)$	N	–	26 ± 11	–	–
$bs\bar{q}\bar{q}$	$0(0^+)$	-7	–	–	18 ± 2	–

From this fact, in the case of tetra-quark bound state, quark model does work well.
If it is possible to observe these states experimentally,
we can see compact tetra-quark systems.

How is structure of tetra quark system?

$QQ'\bar{q}\bar{q}$	$I(J^P)$	$-E_B$	$R_{QQ'}$	$R_{Q\bar{q}}$	$R_{Q'\bar{q}}$	$R_{\bar{q}\bar{q}}$	$R_{Q\bar{q}-Q'\bar{q}}$
$bb\bar{q}\bar{q}$	$0(1^+)$	-173	0.34	0.84		0.74	0.32
$bb\bar{q}\bar{q}$	$0(1^+)$	-4	1.09	0.93		1.11	1.07
$bc\bar{q}\bar{q}$	$0(1^+)$	-40	0.65	0.79	0.80	0.94	0.61
$cc\bar{q}\bar{q}$	$0(1^+)$	-23	0.83	0.85		1.00	0.75
$bc\bar{q}\bar{q}$	$0(2^+)$	-5	1.72	1.38	1.40	1.93	1.57

TABLE III. Mean distance $R_{qq'}$ [fm] for various tetraquarks. Binding energies E_B are in units of MeV.



Distances in bbq-barq-bar with deeply bound state are much compact than those with weakly bound state.

Summary

For the bound state of tetra-quark system, results with our model are consistent with results by LQCD.
We could say that our model would be reliable for the bound states of tetra-quark system.

Next, what about resonant states of tetra-quark system?
The calculation is on going.

Currently, we can obtain 'compact' resonant state.
So, in the future, we should investigate how we could get meson-meson clustering states with our model and study how the clustering state form.

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^{7}H ground state as a ^{3}H+4n resonance

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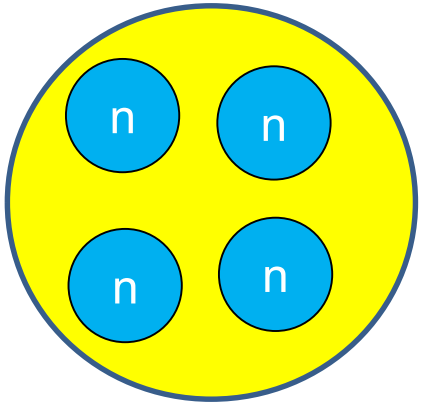
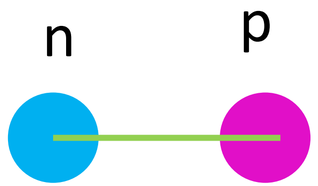


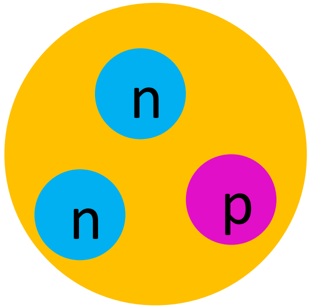
4n

The lightest isotope is Hydrogen (H).

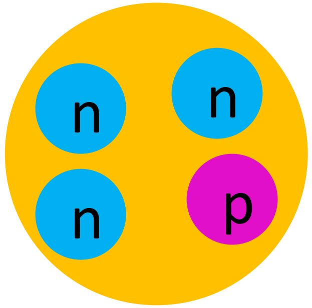
Exp.



^{2}H J=1^{+} -2.22 MeV



^{3}H J=1/2^{+} -8.48 MeV



^{4}H

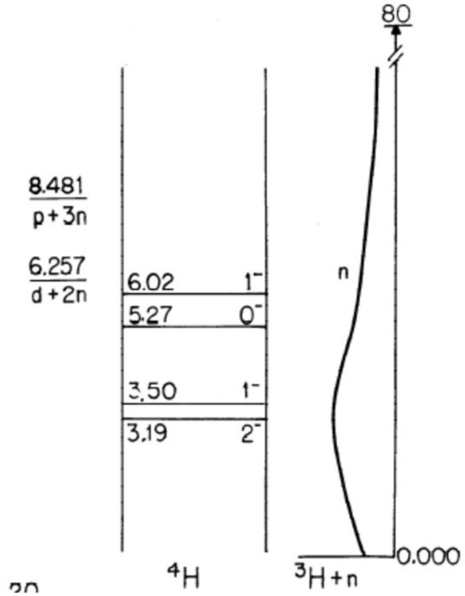


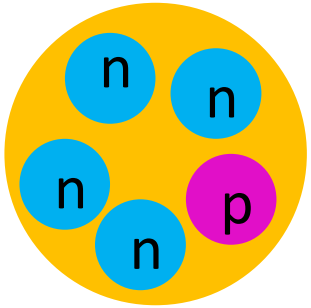
Table 4.1: Energy levels of ^{4}H defined for channel radius $a_n = 4.9$ fm. All energies and widths are in the cm system.

E_x (MeV)	J^π	T	Γ (MeV)	Decay	Reactions
g.s. [a]	2^-	1	5.42	n, ^{3}H	1, 11
0.31	1^-	1	6.73 [b]	n, ^{3}H	11, 12
2.08	0^-	1	8.92	n, ^{3}H	
2.83	1^-	1	12.99 [c]	n, ^{3}H	11, 12

[a] 3.19 MeV above the n + ^{3}H mass.

[b] Primarily $^{3}P_1$.

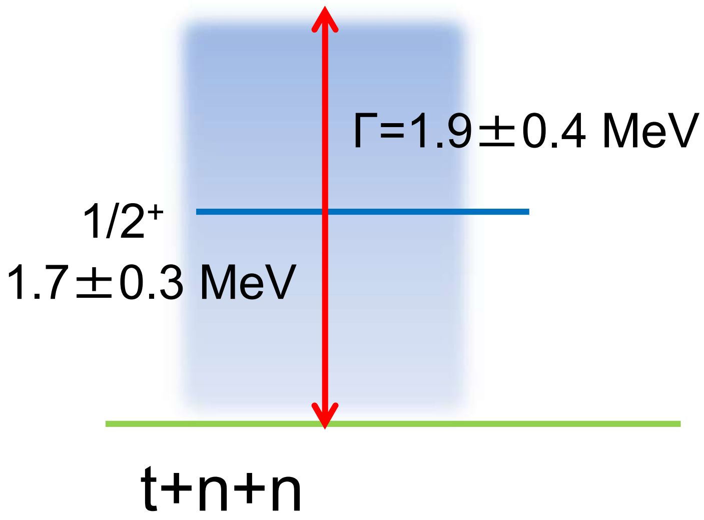
[c] Primarily $^{1}P_1$.



^{5}H

transfer reaction $p(^{6}He, {}^{2}He)^{5}H$

A. A. Korcheninnikov, et al. Phys. Rev. Lett. 87 (2001) 092501.



Superheavy hydrogen

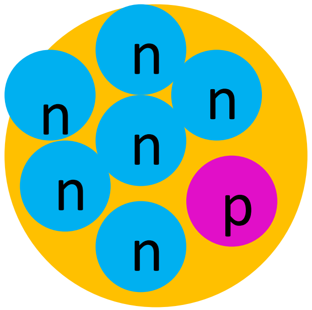
(E_R, Γ_R) (MeV)	
J^{π}	$1/2^{+}$
^{5}H (full)	(1.57, 1.53)
^{5}H ($d = 0$)	(1.55, 1.35)
Theor. [16]	(2.26, 2.93)
Theor. [12]	(2.5–3.0, 3–4)
Theor. [13]	(3.0–3.2, 1–4)
Theor. [15]	(1.59, 2.48)
Exp. [3]	(1.7 ± 0.3, 1.9 ± 0.4)
Exp. [8]	(1.8 ± 0.1, < 0.5)
Exp. [4]	(1.8, 1.3)
Exp. [5]	(2, 2.5)
Exp. [6]	(3, 6)
Exp. [9]	(5.5 ± 0.2, 5.4 ± 0.6)

[3] A.A. Korosheninnikov et al., PRL87 (2001) 092501
[8] S.I. Sidorchuk et al., NPA719 (2003) 13
[4] M.S. Golovkov et al. PRC 72 (2005) 064612
[5] G. M. Ter-Akopian et al., Eur. Phys. J A25 (2005) 315.

Energy of 5H is similar. But decay width is dependent on experiment.

In 2017, we have a new data on ^{5}H.
A. H. Wuosmaa, Phys. Rev. C95, 014310 (2017)
^{6}He (d,^{3}He) ^{5}H

E_r=2.4±0.3 MeV Γ=5.3 ±0.4 MeV



^{7}H

A. A. Korsheninnikov et al., PRL 90, 082501 (2003)
M. Caamano et al., PRL99, 062502(2007)
PRC 78, 044001 (2008)

$E_r=0.57^{+0.42}_{-0.21}$ MeV   from t+4n threshold

$\Gamma=0.09^{+0.94}_{-0.06}$ MeV

^{12}C(^{8}He,^{7}H)^{13}N reaction

If we have narrow decay at lower energy, we could have heavier H-hydrogen isotope such as ^{9}H.

What is limit for H-isotope?   Probably ^{7}H?

Theoretical calculation for ^{5}H and ^{7}H

N. K. Timofeyuk, PRC65, 064306(2002), PRC69 , 034336(2004)
Volkov NN potential, Hyperspherical harmonics method: 5-body and 7-body calculations
^{5}H: about 1 MeV above t+n+n threshold.
^{7}H: about 3MeV above t+4n threshold
She calculated the energies with bound state approximation.
Then, she did not give decay width for these nuclei.

S. Aoyama and N. Itagaki, PRC80,021304 (R)
Volkov NN potential, AMD calculation
^{7}H: 4.2 MeV above t+4n threshold, no calculation for decay width
No report for the energy of ^{5}H

H. H. Li et al., PRC 104, L061306 (2021)
Gamow shell model calculation using Minnesota NN potential.

Energy and decay width of ^{5}H is 1.4 MeV and 0.5 MeV, respectively.
Energy and decay width of ^{7}H is about 2-3MeV and about 0.1 MeV, respectively.

They predicted to have very narrow decay width for ^{5}H and ^{7}H.

Experiment situation:

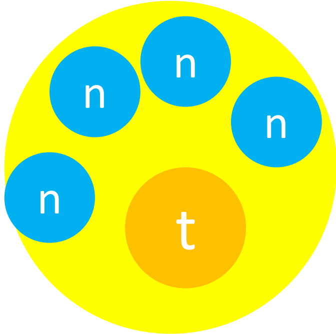
Recently, ^{8}He (p,2p) ^{7}H reaction has been done at RIBF.
RIBF Experimental Proposal NP1512-SAMURAI34.
The analysis is on going.

Then, it is timely to calculate ^{7}H to obtain the energy and width theoretically.

Motivated by this situation, we study ^{7}H structure within the framework of t+4n 5-body problem. We also discuss on the energy and decay width of ^{5}H within t+n+n three-body problem.

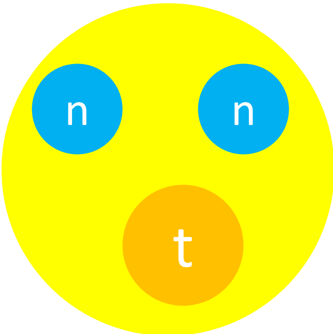
Framework

NN: Minnesota potential (central potential)

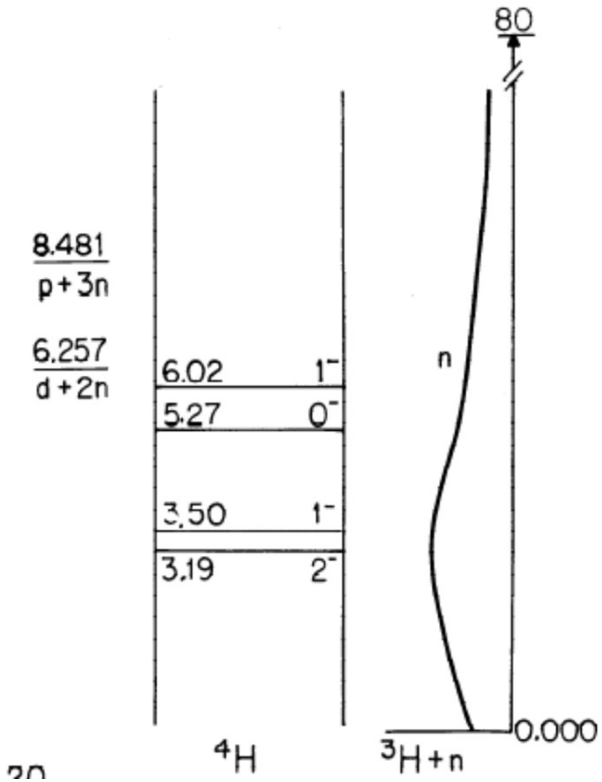


^{7}H=t+4n model

t-n potential => there is a large degree of ambiguity.
Only several data for phase shift of t-n



^{5}H



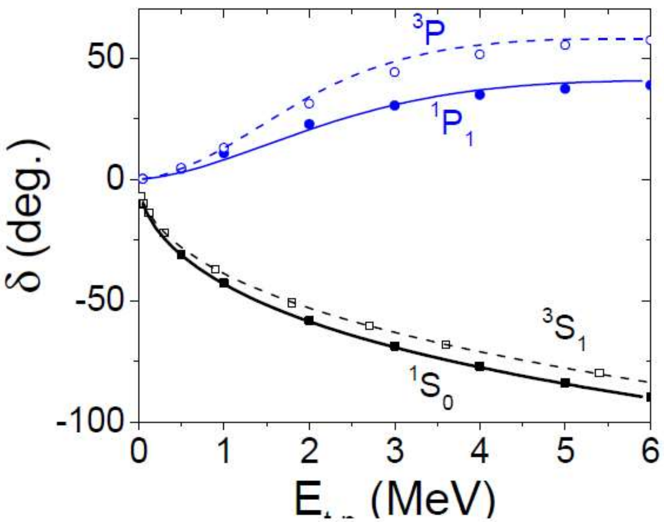
$$V(r,l,s)_{nt} = \delta_{l,0}|\varphi_0\rangle\lambda_\infty\langle\varphi_0| + \sum_{i=1}^{2}\left(\upsilon_i^{(c)} + (-)^l\upsilon_i^{(P)} + \frac{\hat{S}^2}{2}\upsilon_i^{(s)} + (-)^l\frac{\hat{S}^2}{2}\upsilon_i^{(SP)}\right)\exp(-\alpha_i r^2)$$

$$|\varphi_0\rangle = \exp(-a_0 r^2)$$

$$\lambda_\infty = \infty$$

i	1	2
$\alpha_i(fm^{-2})$	0.471241	0.0549825
$\upsilon_i^{(c)}(MeV)$	−41.3619	1.22768
$\upsilon_i^{(P)}(MeV)$	−0.309720	6.89574
$\upsilon_i^{(s)}(MeV)$	−28.2483	−0.972465
$\upsilon_i^{(SP)}(MeV)$	10.3308	−1.25695

$$a_0 = 0.1979068 \quad fm^{-2}$$



Based on four-body calculation with MT I-III

α_i	V_{nt} (1)	4N [12]
$L = 1^-, S = 0$	1.28-2.61 i	0.88(5)-2.20(5) i
$L = 1^-, S = 1$	1.33-1.84 i	1.08(3)-2.03(3) i

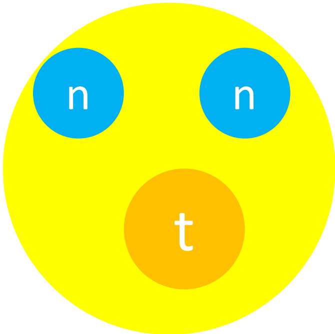
Two-body calculation of t-n is almost consistent with that of 4-body calculation.

+ I introduce a phenomenological
three-body t-n-n force to obtain energy trajectory.

$$V_{tnn}(\rho) = -V_0\, e^{-\frac{\rho^2}{b_3^2}} \qquad \rho^2 = \frac{m_n}{M} r_{nn}^2 + \frac{m_t}{M^2} r_{nt}^2 + \frac{m_t}{M^2} r_{nt}^2 \qquad M = 2m_n + m_t$$

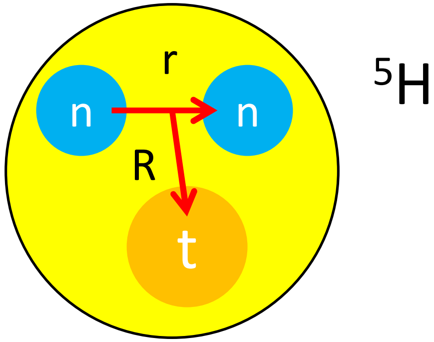
V_0,b_3 : parameters. Fit so as to reproduce the data of ^5H

apply



^5H

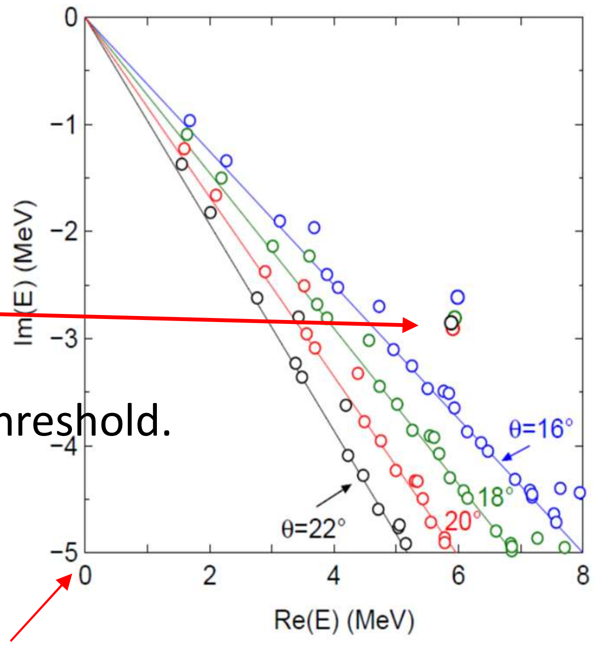
Observed data of ^{5}H is resonant state.



To obtain resonant state of ^{5}H,
we use complex scaling method.

$$r_c \to r_c e^{i\theta},\ R_c \to R_c e^{i\theta},$$

The energy pole is stable
with respect to θ.
Re(E) corresponds to energy
With respect to 4n breakup threshold.
Im(E) corresponds to Γ/2.



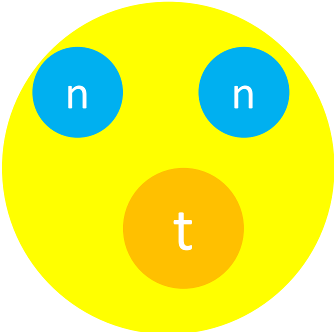
t+n+n breakup threshold

+ I introduce a phenomenological
three-body t-n-n force to obtain energy trajectory.

$$V_{tnn}(\rho) = -V_0\, e^{-\frac{\rho^2}{b_3^2}} \qquad \rho^2 = \frac{m_n}{M} r_{nn}^2 + \frac{m_t}{M^2} r_{nt}^2 + \frac{m_t}{M^2} r_{nt}^2 \qquad M = 2m_n + m_t$$

V_0, b_3 : parameters.

Fit so as to reproduce the data of ^{5}H



^{5}H

Question: Which experimental data of ^{5}H should we fit?

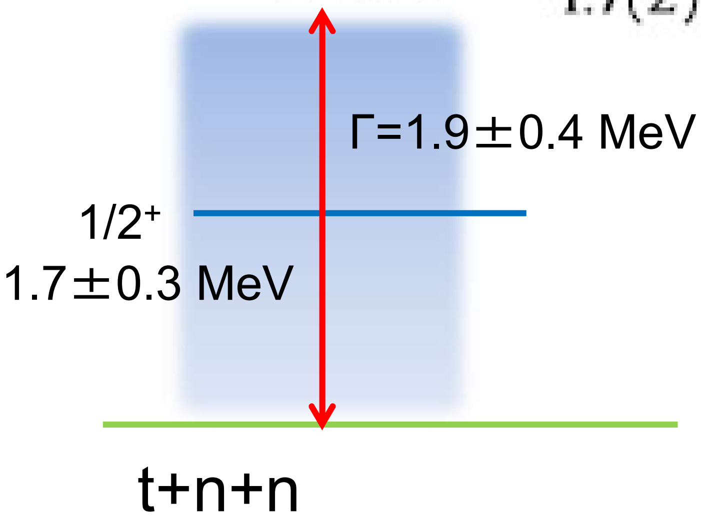
(E_R, Γ_R) (MeV)	
J^π	$1/2^+$
^{5}H (full)	(1.57, 1.53)
^{5}H ($d = 0$)	(1.55, 1.35)
Theor. [16]	(2.26, 2.93)
Theor. [12]	(2.5–3.0, 3–4)
Theor. [13]	(3.0–3.2, 1–4)
Theor. [15]	(1.59, 2.48)
Exp. [3]	$(1.7 \pm 0.3, 1.9 \pm 0.4)$
Exp. [8]	$(1.8 \pm 0.1, < 0.5)$
Exp. [4]	(1.8, 1.3)
Exp. [5]	(2, 2.5)
Exp. [6]	(3, 6)
Exp. [9]	$(5.5 \pm 0.2, 5.4 \pm 0.6)$

[3] A.A. Korosheninnikov et al., PRL87 (2001) 092501
[8] S.I. Sidorchuk et al., NPA719 (2003) 13
[4] M.S. Golovkov et al. PRC 72 (2005) 064612
[5] G. M. Ter-Akopian et al., Eur. Phys. J A25 (2005) 315.

Energy of ^{5}H is similar. But decay width is dependent on experiment.

R. Lazauskas, E. Hiyama, J. Carbonell, PRB 791 335 (2019)
Fadeev-Yakubovsky method calculation of ^{5}H

	J=1/2$^+$	
	E_R	Γ
N3LO $^{(ACCC)}$	1.8(1)	2.4(2)
$(SECS)$	1.9(2)	2.4(2)
INOY $^{(ACCC)}$	1.7(1)	2.4(2)
$(SECS)$	1.8(1)	2.4(2)
MT13 $^{(ACCC)}$	1.4(1)	2.4(2)
$(SECS)$	1.7(2)	2.4(2)



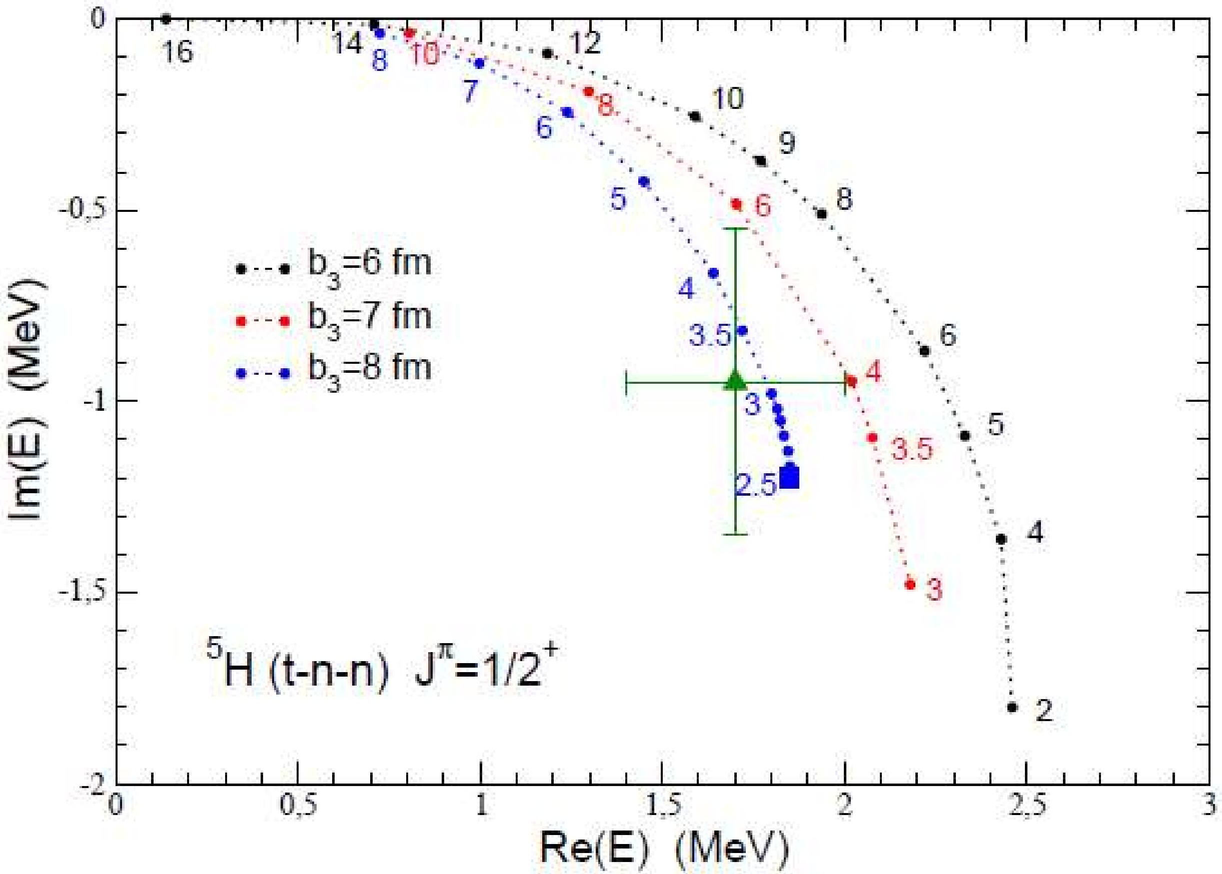
We take this result as 'exp.' data.

Close to the below exp.data

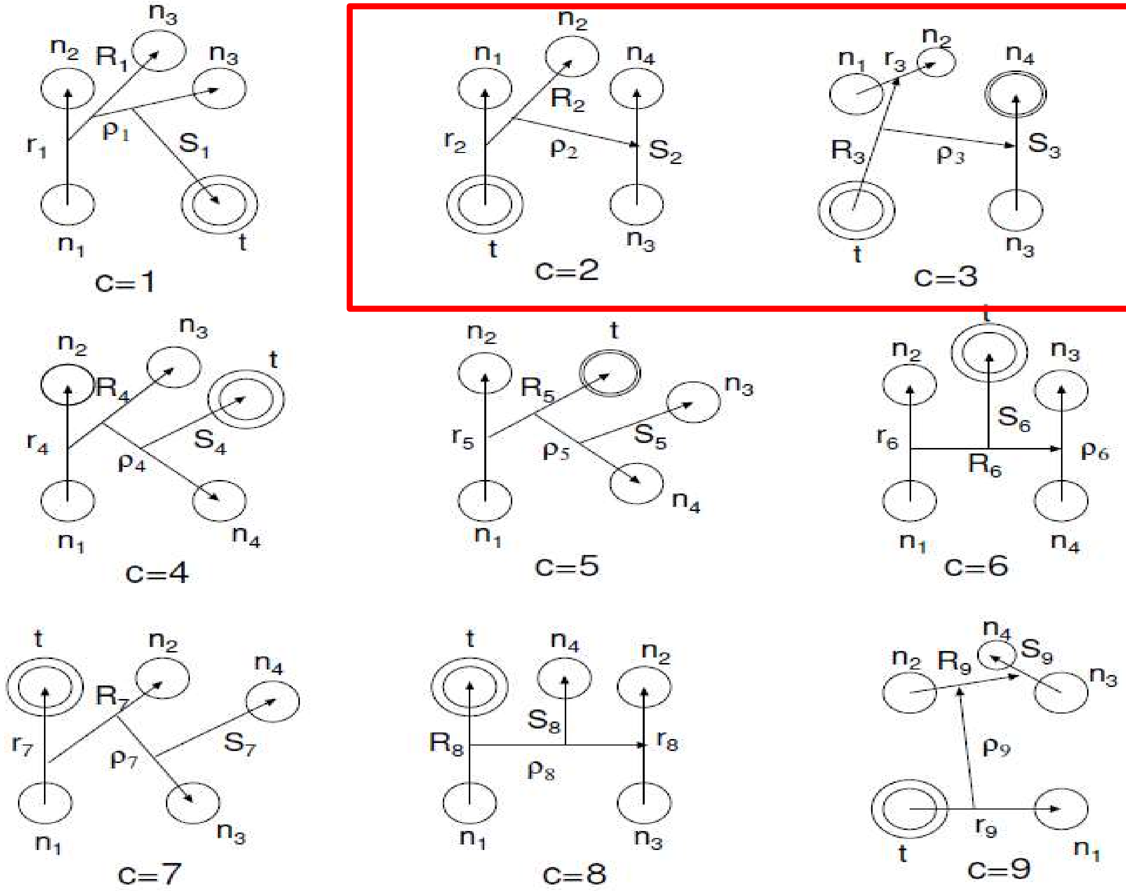
A. A. Korcheninnikov, et al. Phys. Rev. Lett. 87 (2001) 092501.

$$V_{tnn}(\rho) = -V_0\, e^{-\frac{\rho^2}{b_3^2}} \qquad \rho^2 = \frac{m_n}{M} r_{nn}^2 + \frac{m_t}{M^2} r_{nt}^2 + \frac{m_t}{M^2} r_{nt}^2 \qquad M = 2m_n + m_t$$

When b_3=8 fm and V_0=3 to 2.5 MeV, the energy pole of ^{5}H is close to exp. data. If we have this potential parameter, what is energy pole of ^{7}H?



Framework of ^{7}H The Hamiltonian is the same as the case of ^{5}H.



(t+n+n)+nn

Totally 120 Jacobi coordinates

$$
\begin{aligned}
\Psi_{JM}(^7\mathrm{H}) = & \left[\left[[[\eta_{\frac{1}{2}}(n)\eta_{\frac{1}{2}}(n)]_t\eta_{\frac{1}{2}}(n)]_{T_0}\eta_{\frac{1}{2}}(n)\right]_{T_4}\eta_{\frac{1}{2}}(t)\right]_{TT_z} \\
& \times \left[\left[\left[[[\chi_{\frac{1}{2}}(n)\chi_{\frac{1}{2}}(n)]_t\chi_{\frac{1}{2}}(n)]_\Sigma\chi_{\frac{1}{2}}(n)\right]_{S_4}\chi_{\frac{1}{2}}(t)\right]_S\right. \\
& \times \left.[[[\phi_\ell(r_c)\psi_L(R_c)]_\Lambda\phi_\lambda(\rho_c)]_I\phi_\xi(s_c)]_K\right]_{JM}
\end{aligned}
$$

Form of each basis function

5-body spatial function

$$\overbrace{\Big[\,\Big[\,[\phi^{(c)}_{nl}(\mathbf{r}_c)\,\psi^{(c)}_{NL}(\mathbf{R}_c)]_I\,\varphi^{(c)}_{n'l'}(\boldsymbol{\rho}_c)\Big]_K\,\Phi^{(c)}_{N'L'}(\mathbf{S}_c)\,\Big]_L}$$

Gaussian for radial part :

$$\phi_{nlm}(\mathbf{r}) = r^l\, e^{-(r/r_n)^2} Y_{lm}(\widehat{\mathbf{r}})$$

geometric progression for Gaussian ranges :

$$r_n = r_1 a^{n-1} \qquad (n = 1 - n_{\max})$$

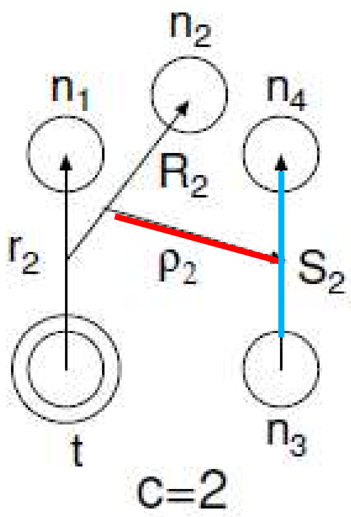
Similarly for the other basis :

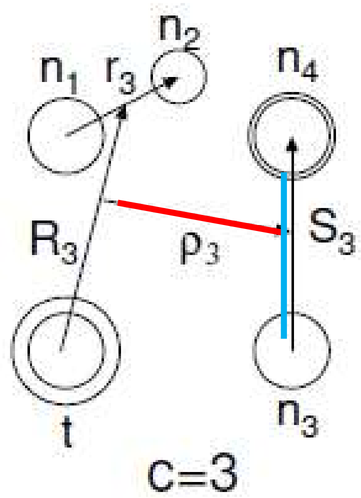
$$\psi^{(c)}_{NLM}(\mathbf{R}_c) \qquad \varphi^{(c)}_{n'l'm'}(\boldsymbol{\rho}_c) \qquad \Phi^{(c)}_{N'L'M'}(\mathbf{S}_c)$$

Use of this type gaussian basis is known to be very suitable for describing simultaneously both the short-range correlations and long-range tail behaviour of few-body systems;

This is precisely shown in $\longrightarrow$

Gaussian Expansion Method (GEM)
(review paper) E. H., Y. Kino and M. Kamimura,
Prog. Part. Nucl. Phys., 51 (2003) 223.





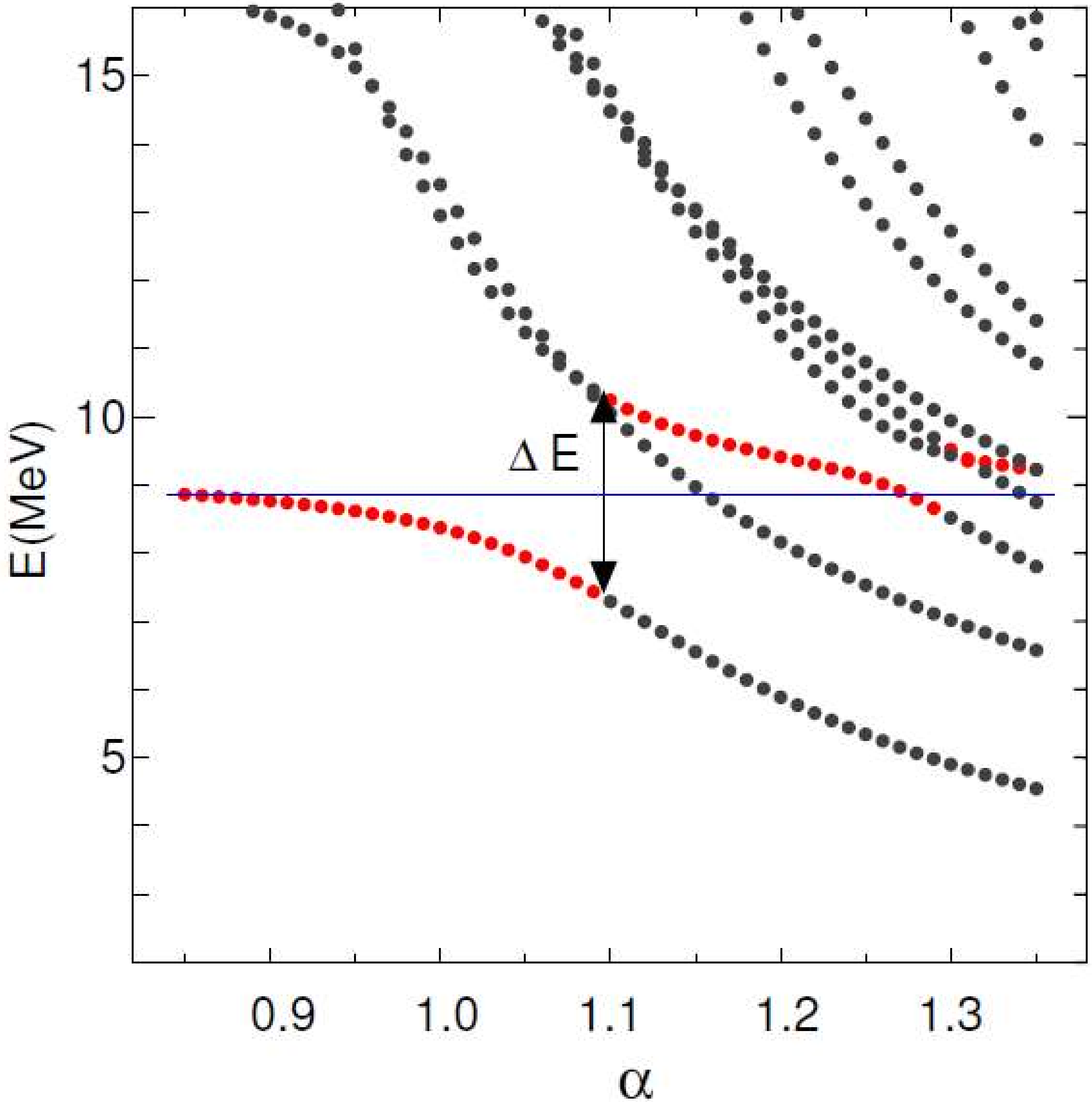
$\rho_n \Rightarrow \alpha\rho_n$

$s_n \Rightarrow \alpha s_n$

Real scaling method

$$V_{tnn}(\rho) = -V_0 \, e^{-\frac{\rho^2}{b_3^2}}$$

b_3=8.0fm V_0=-3 MeV

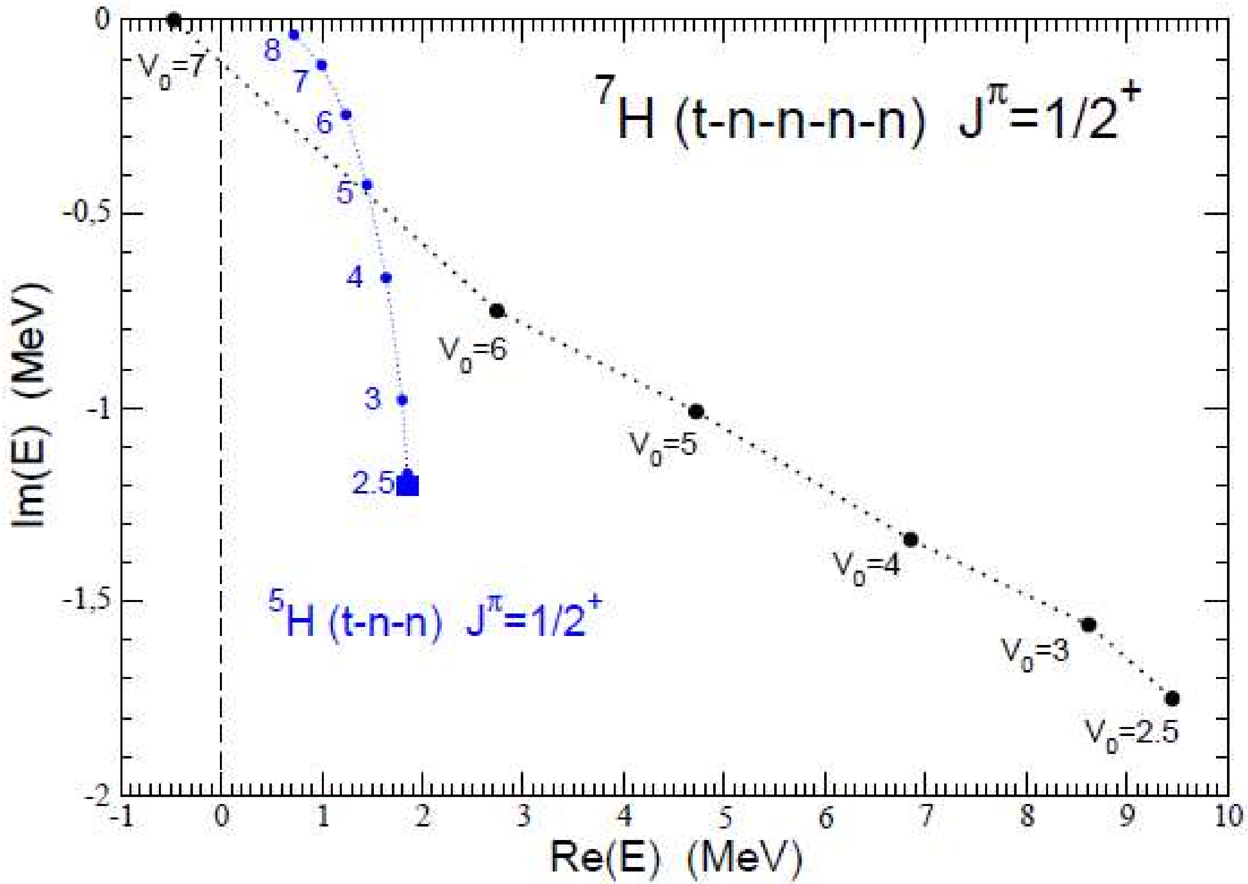


Er～8.8 MeV
Γ～ 3.1 MeV

With respect to
t+4n threshold

5H: close to
Exp. data

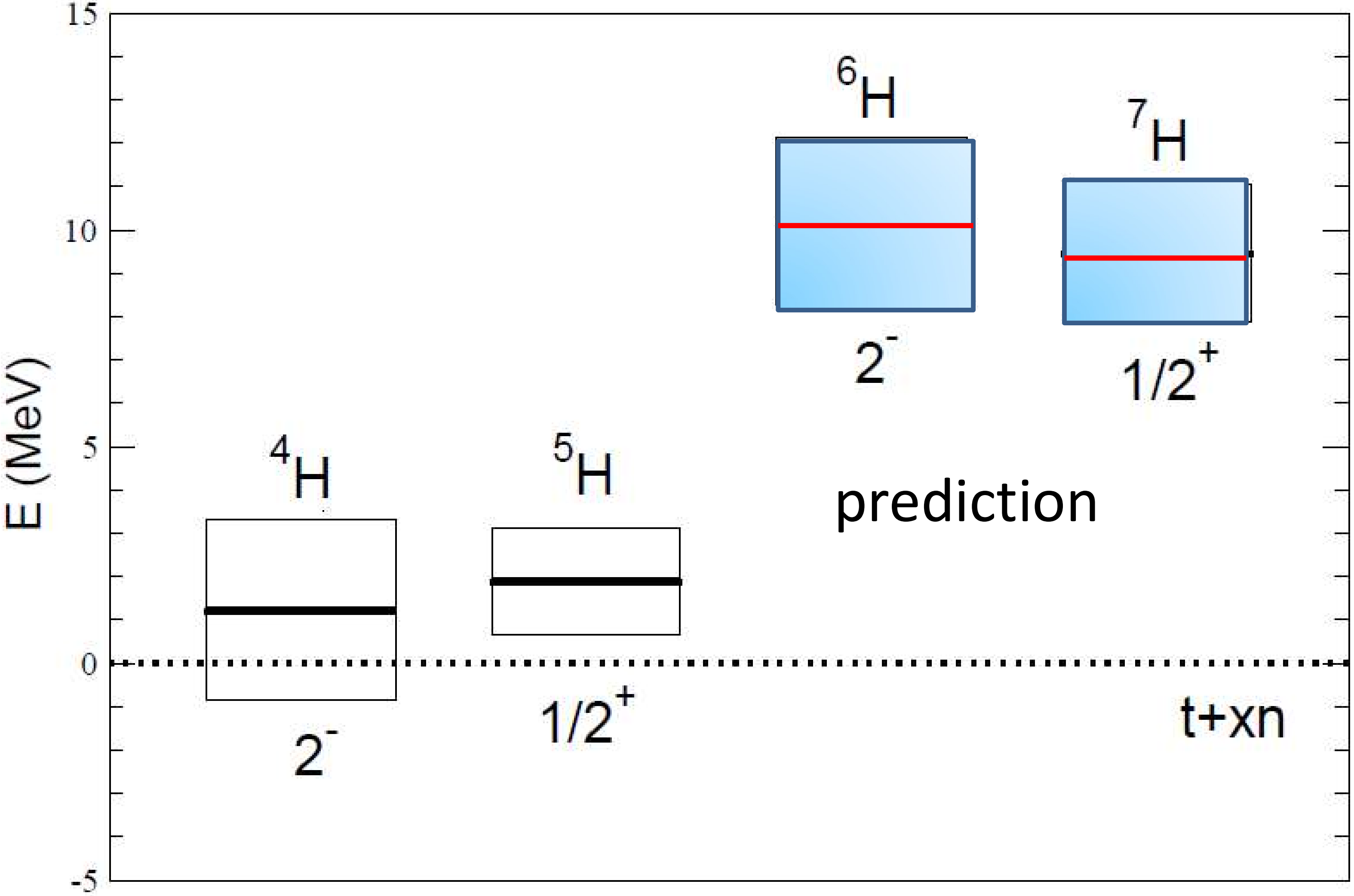
Im (E)=Γ/2



For V_0=2.5, we reproduce the data of ^{5}H accurately.
In this case, the energy pole of ^{7}H, E=9.5 MeV, Γ～3.5 MeV.
Our energy of ^{7}H is much higher and broad decay width.

Summary of H-isotope (according to our calculation)

End of H-isotope



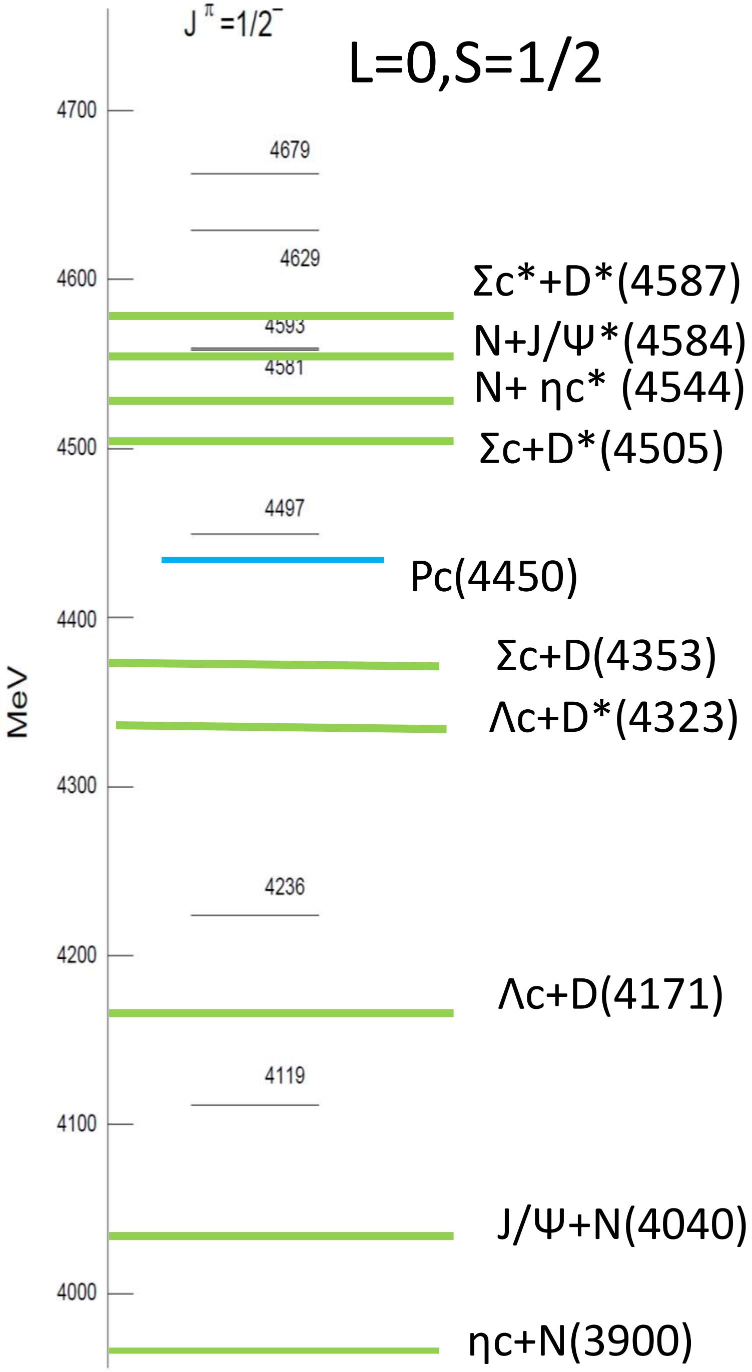
Summary

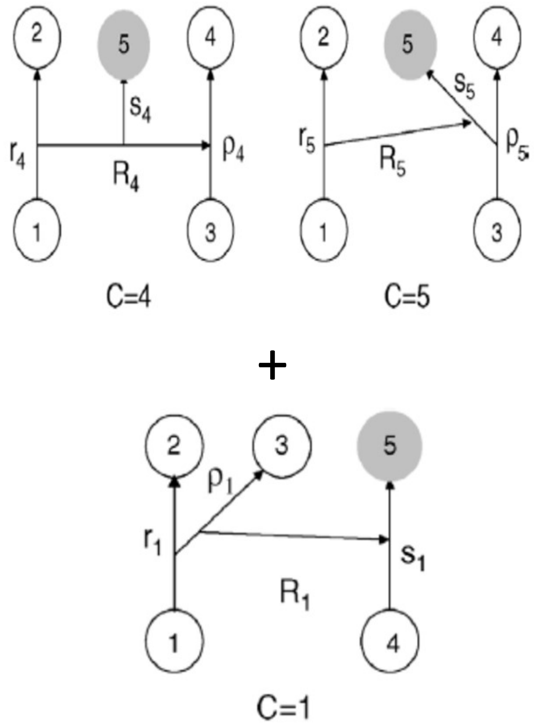
Assuming Er～1.9 MeV and Γ～2.4 MeV for ^{5}H,
Our calculated energy and decay width of ^{7}H are
about Er～ 8 to 9 MeV, and Γ～ 3 MeV.
That is much higher than ^{5}H+n+n threshold,
broad decay width.

^{8}He (p,2p) ^{7}H reaction was done at RIBF,recently.
RIBF Experimental Proposal NP1512-SAMURAI34.
The analysis is on going. =>The result will be reported by Lenain.

I am waiting for future experimental result.

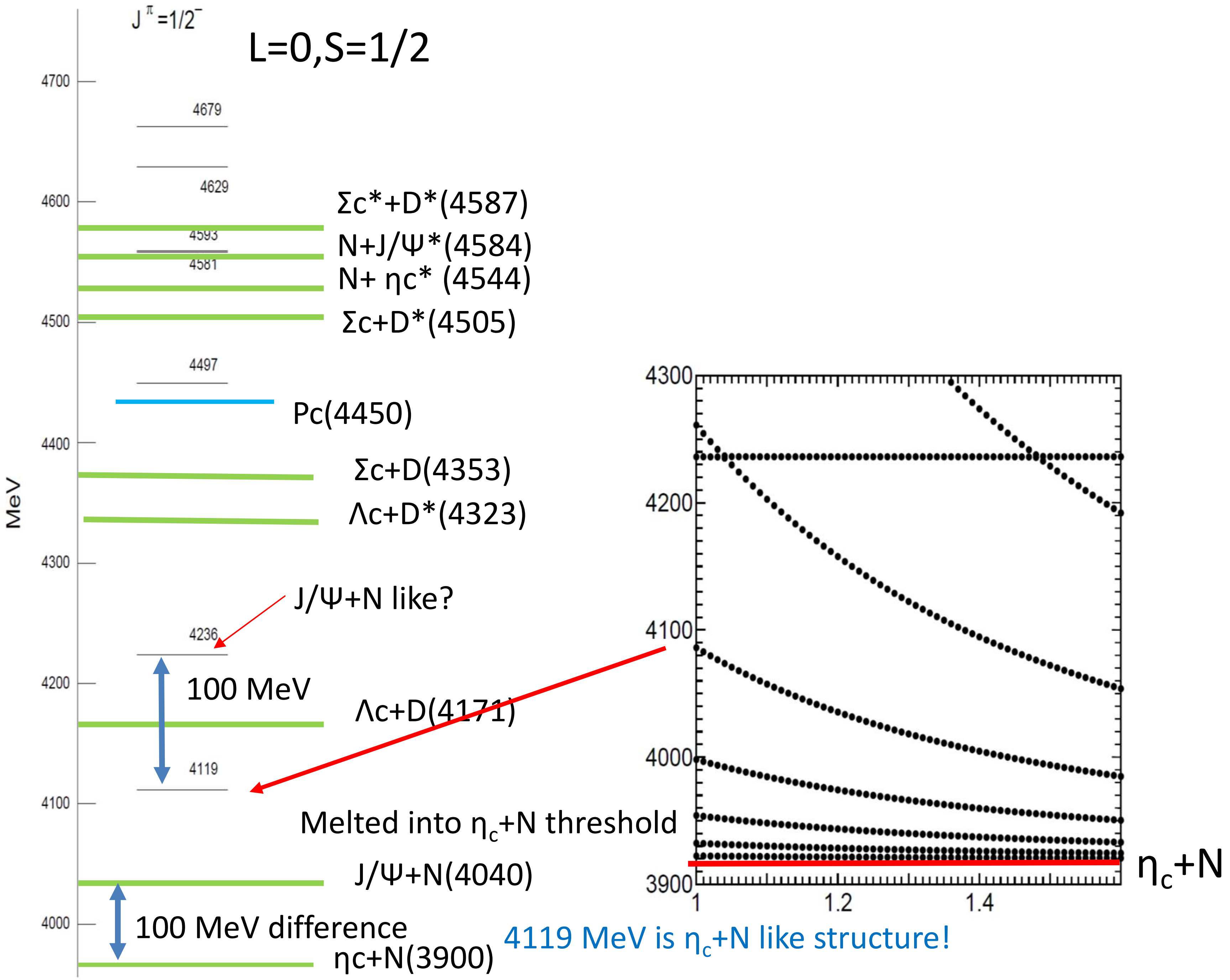
Thank you!



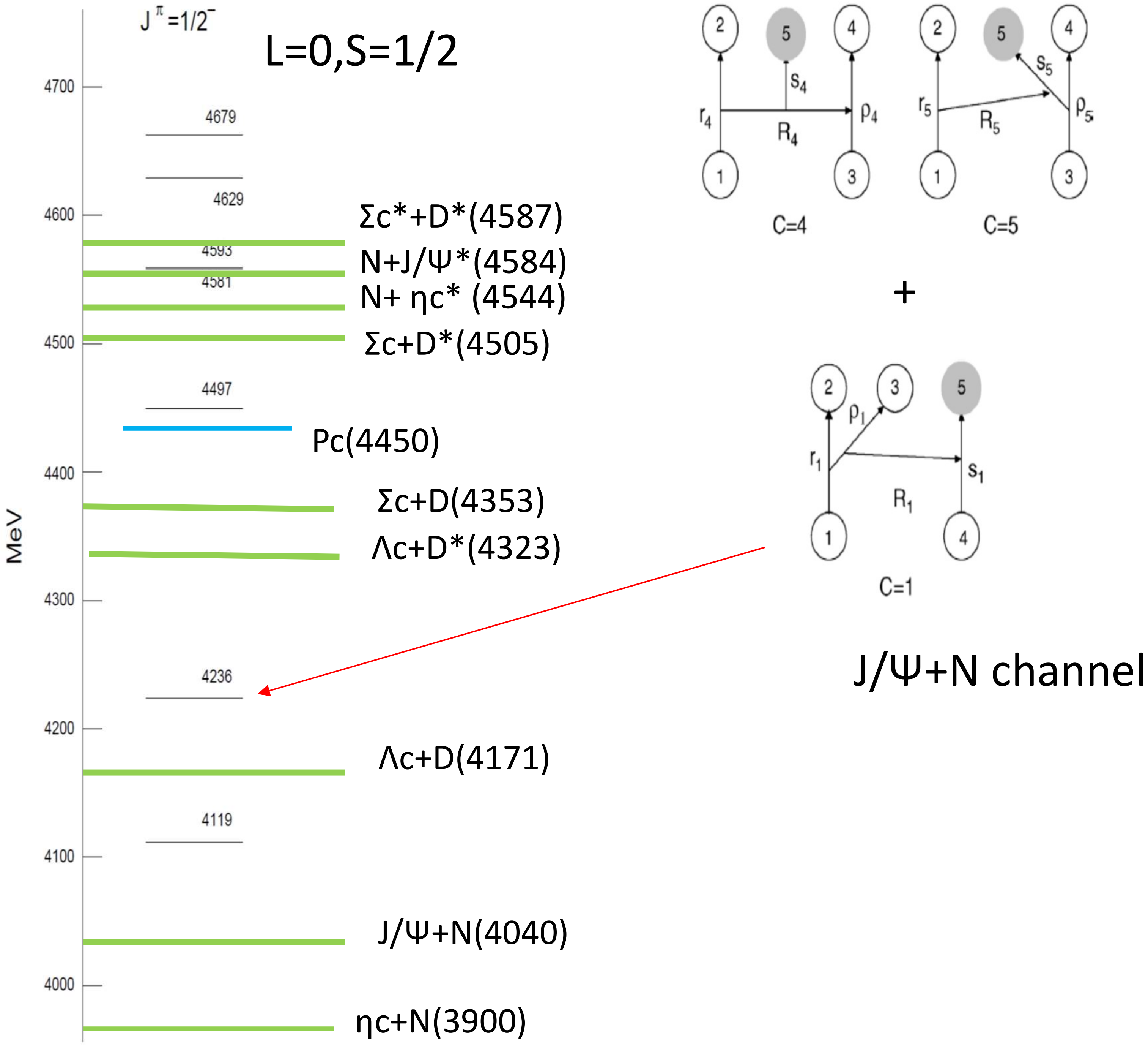


ηc+N channel

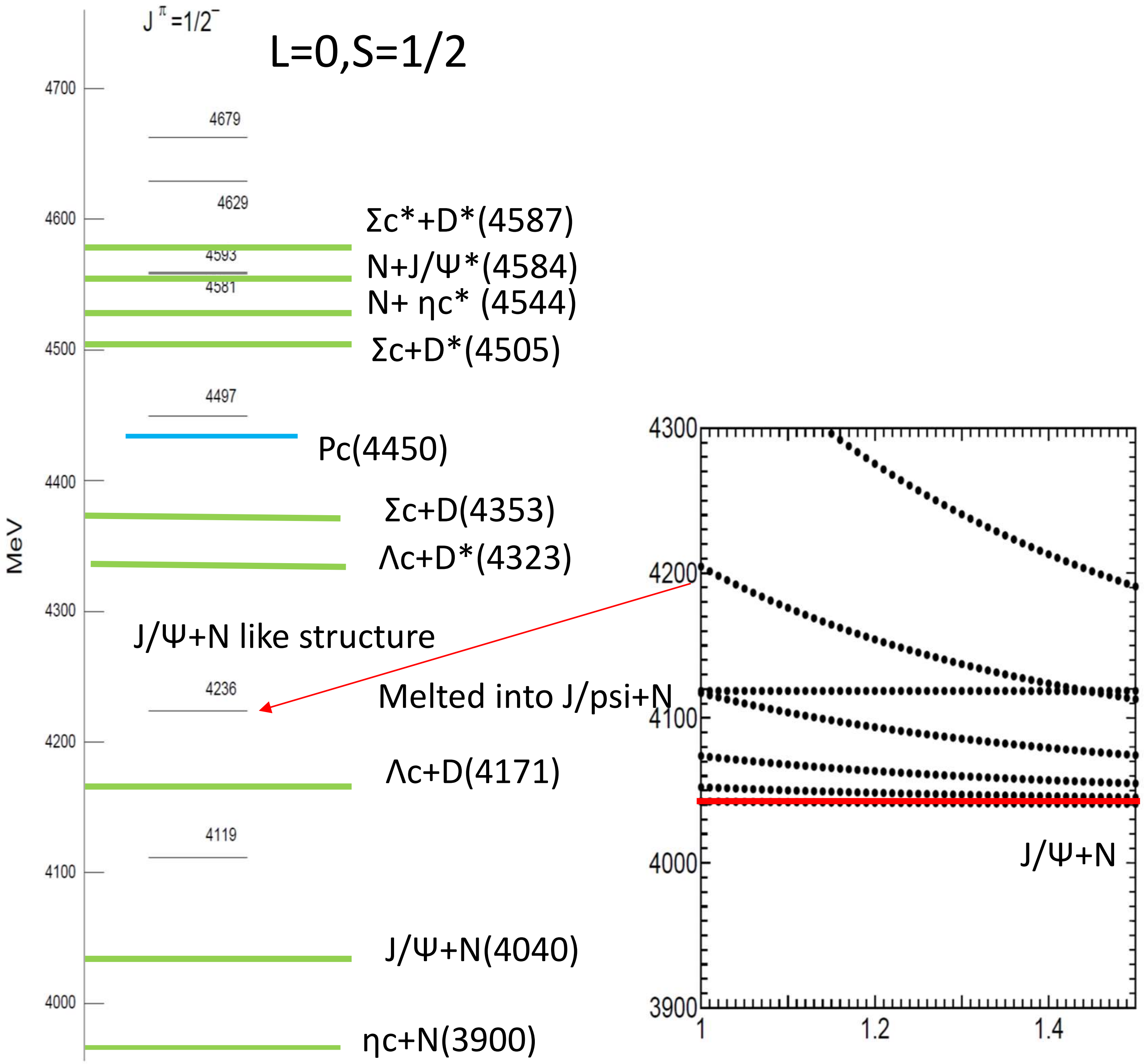
Conjecture: 4119 MeV can be describe as ηc+N like.
However, due the restriction of the configurations, namely, by only C=4 and 5 channels, the mass energy is up than the ηc+N by about 200 MeV.
In order to investigate this conjecture, we solve scattering states including ηc+N channel only with real scaling method. If 4119 MeV is ηc+N like structure, this state should be melted into ηc+N threshold.



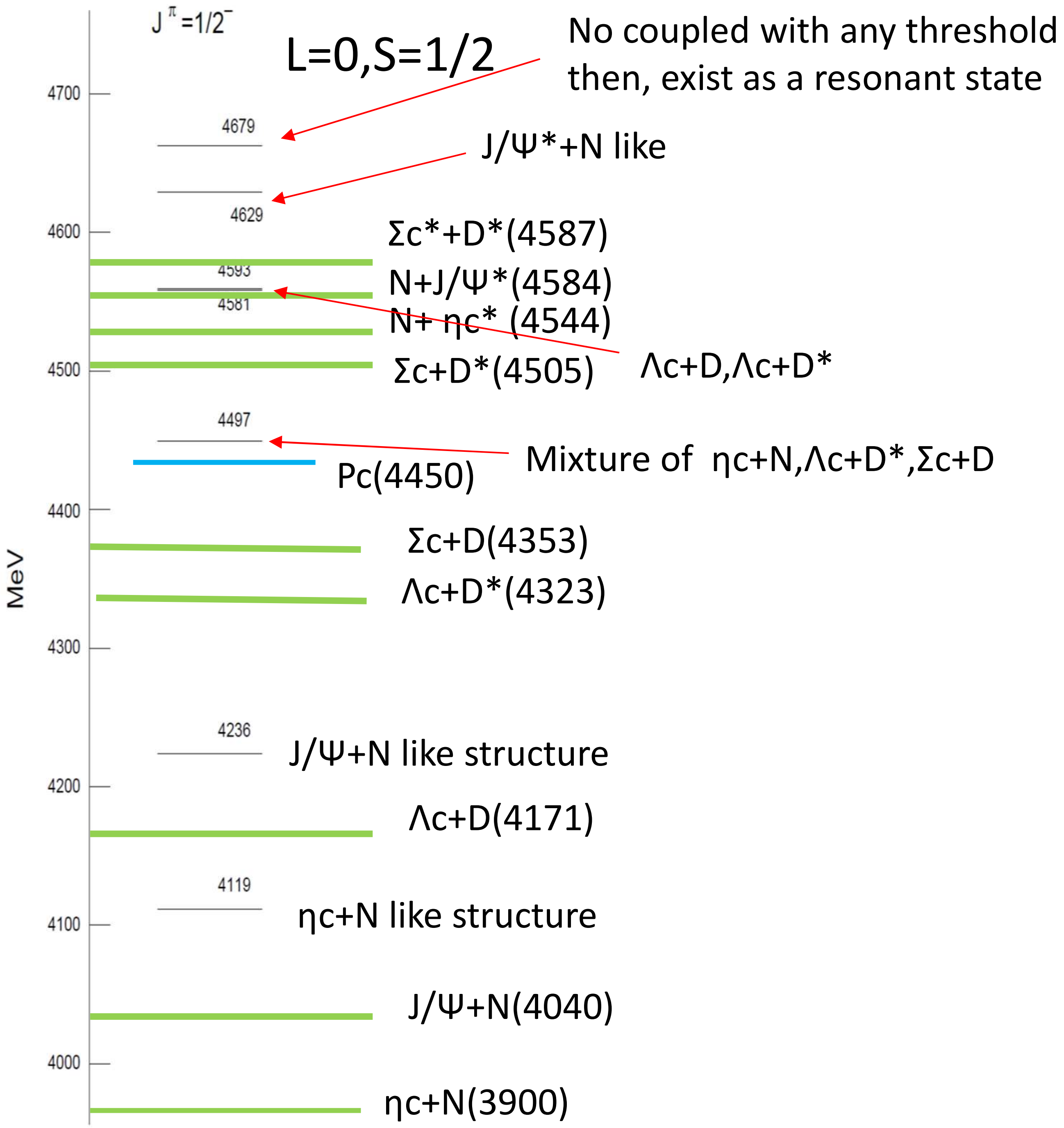
J π =1/2−
L=0,S=1/2
4700
4679
4629
4600
Σc*+D*(4587)
4593
N+J/Ψ*(4584)
4581
N+ ηc* (4544)
4500
Σc+D*(4505)
4497
Pc(4450)
4400
Σc+D(4353)
Λc+D*(4323)
MeV
4300
J/Ψ+N like?
4236
100 MeV
4200
Λc+D(4171)
4119
4100
Melted into ηc+N threshold
J/Ψ+N(4040)
4000
100 MeV difference
ηc+N(3900)
4300
4200
4100
4000
3900
1
1.2
1.4
ηc+N
4119 MeV is ηc+N like structure!



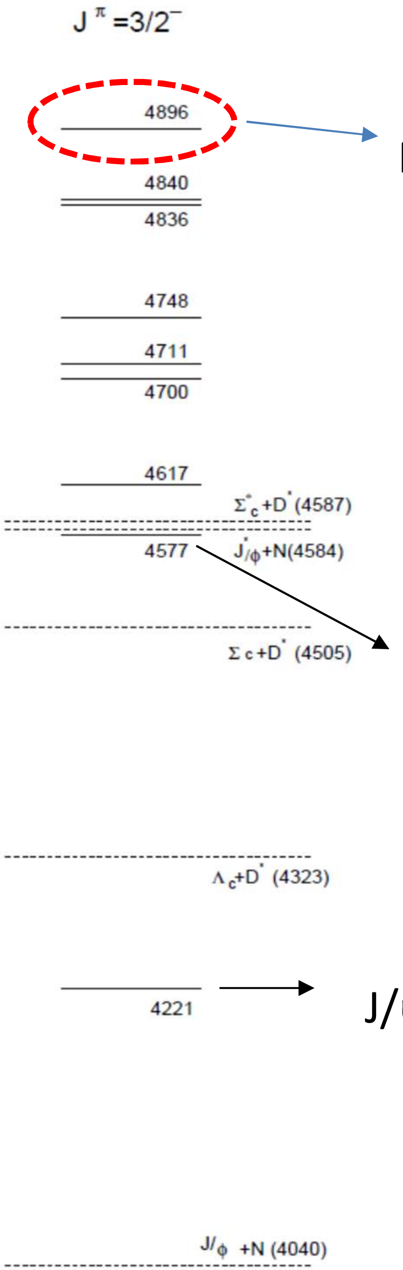
$J^{\pi}=1/2^{-}$
L=0,S=1/2
MeV
4700
4679
4629
4600
4593
4581
4500
4497
4400
4300
4236
4200
4119
4100
4000
Σc*+D*(4587)
N+J/Ψ*(4584)
N+ ηc* (4544)
Σc+D*(4505)
Pc(4450)
Σc+D(4353)
Λc+D*(4323)
Λc+D(4171)
J/Ψ+N(4040)
ηc+N(3900)
C=4
C=5
+
C=1
J/Ψ+N channel



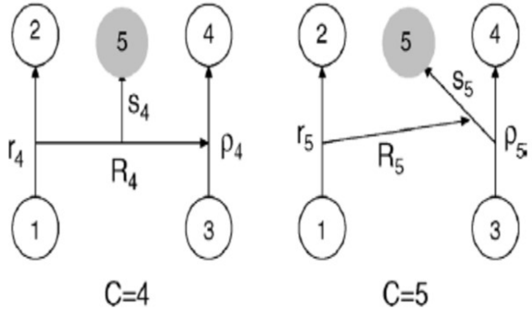
J π =1/2−
L=0,S=1/2
4700
4679
4629
4600
Σc*+D*(4587)
4593
N+J/Ψ*(4584)
4581
N+ ηc* (4544)
4500
Σc+D*(4505)
4497
Pc(4450)
4400
Σc+D(4353)
Λc+D*(4323)
MeV
4300
J/Ψ+N like structure
4236
Melted into J/psi+N
4200
Λc+D(4171)
4119
4100
J/Ψ+N(4040)
4000
ηc+N(3900)
4300
4200
4100
4000
3900
J/Ψ+N
1
1.2
1.4



$J^{\pi}=1/2^{-}$
L=0,S=1/2
No coupled with any threshold then, exist as a resonant state
J/Ψ*+N like
4679
4629
Σc*+D*(4587)
4593
N+J/Ψ*(4584)
4581
N+ ηc* (4544)
Λc+D,Λc+D*
Σc+D*(4505)
4497
Mixture of ηc+N,Λc+D*,Σc+D
Pc(4450)
Σc+D(4353)
Λc+D*(4323)
4236
J/Ψ+N like structure
Λc+D(4171)
4119
ηc+N like structure
J/Ψ+N(4040)
ηc+N(3900)
MeV
4700
4600
4500
4400
4300
4200
4100
4000



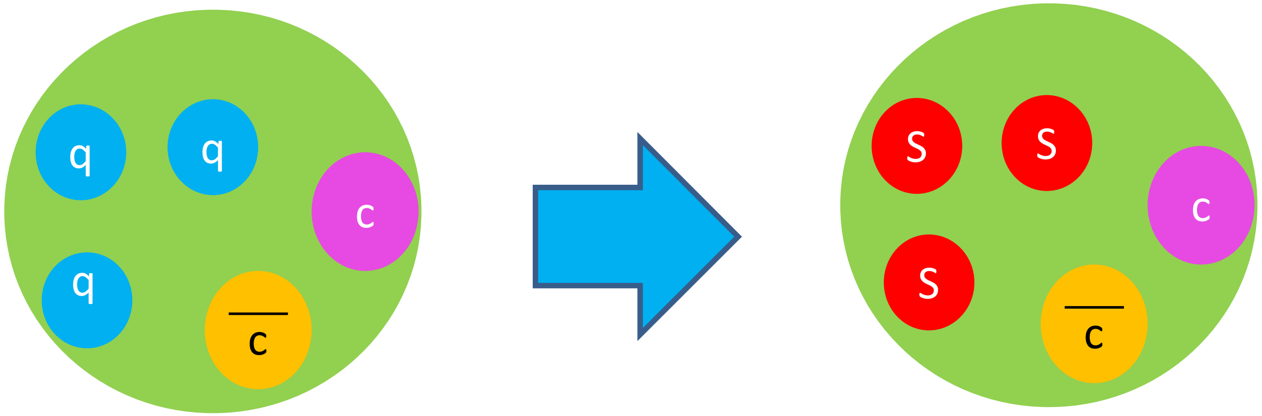
Resonant state=> it is highly
energy region than the observed data.



Λc+D*,J/ψ+N,Σc+D*

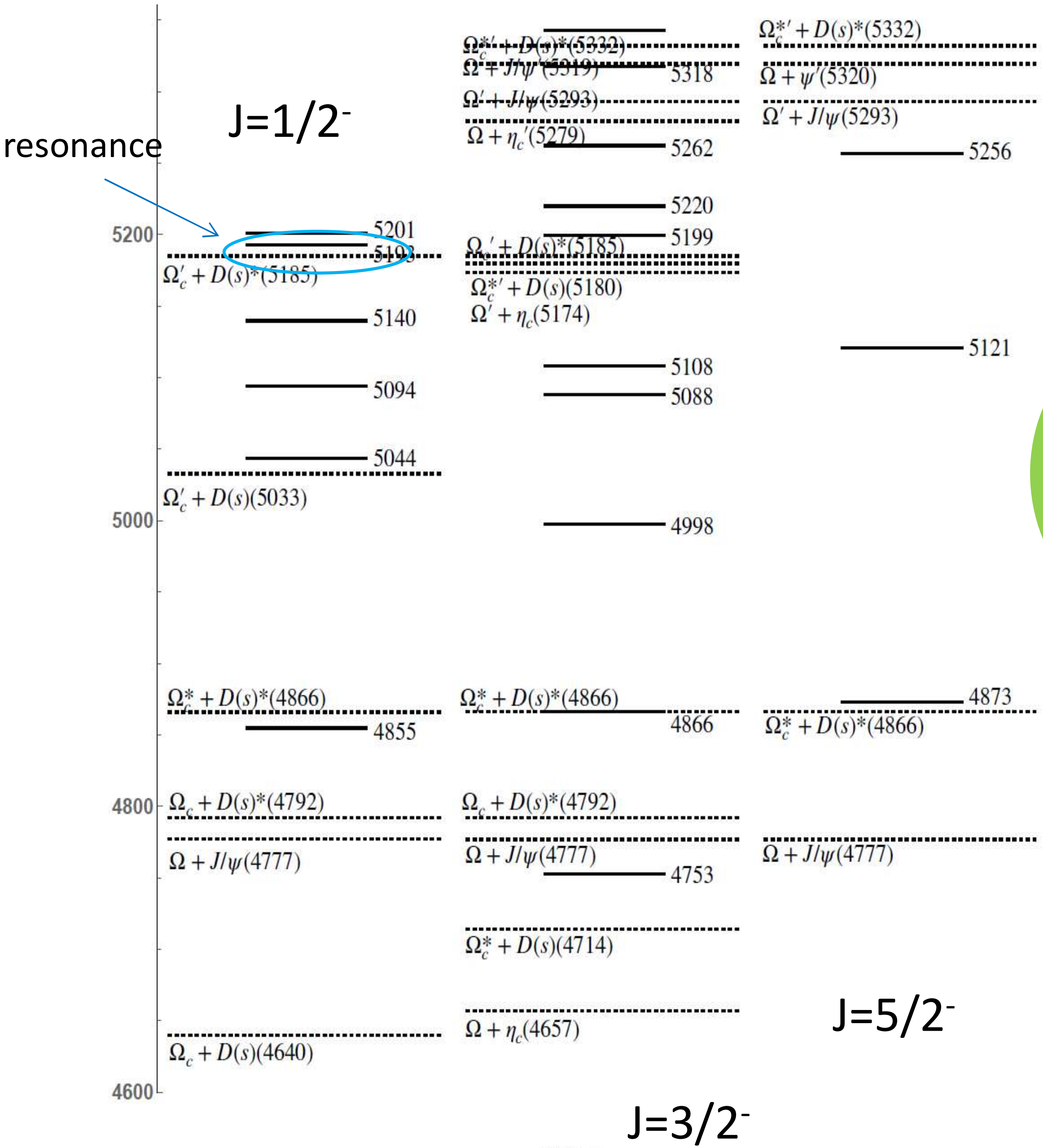
J/ψ+N structure

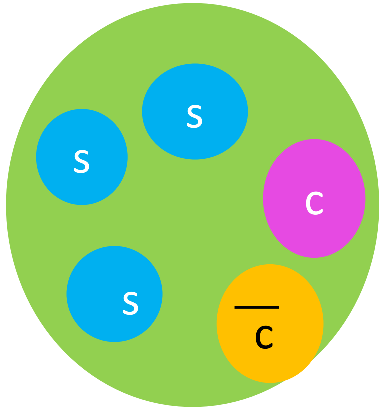
It is difficult to predict the state at such much higher energy region. Because, three-body open channel would be open by around 300 MeV than the lowest threshold. We have not included this open channel in this energy region.



Let us convert q into s-quark!

If compact resonant state can be obtain by less 300 MeV above than the lowest threshold, we can propose to experimentalist to search ssscc-bar state. And later, we can compare the our results with experimental data to check validity of our model.





It is difficult to predict the compact resonant states in sscc-bar system with our model.

From the calculation of penta-quark system,
It is difficult to say that our model is reliable for penta-quark system
Then, we should focus on simpler case, namely,
tetra quark bound state.
We can compare results by LQCD to check validity of our model.