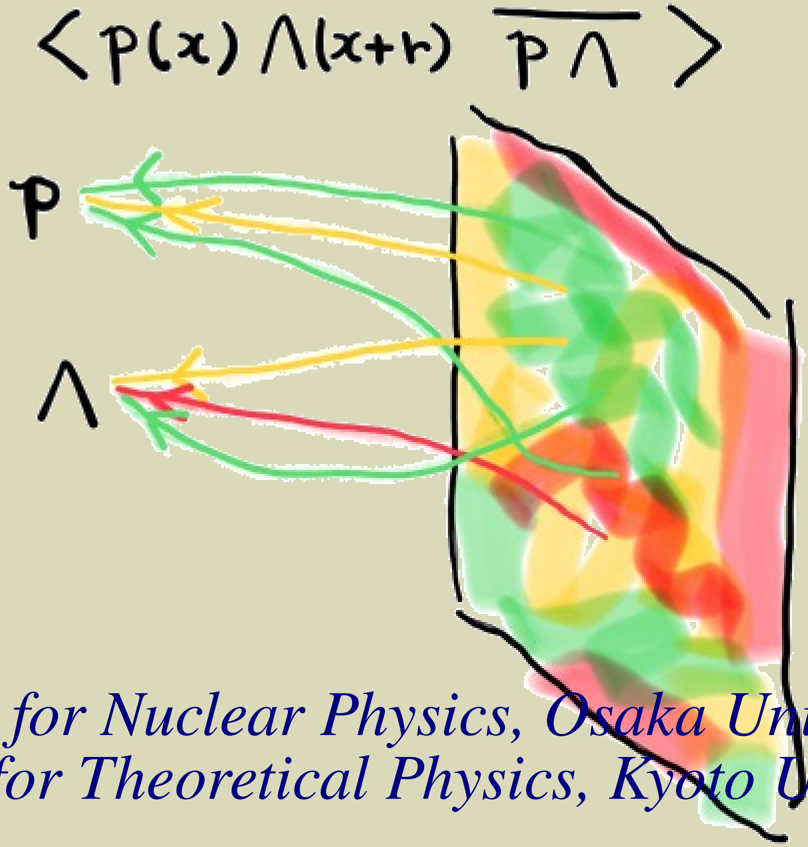
Hyperon-Nucleon potentials from lattice QCD

H. Nemura[1],



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Yukawa Institute for Theoretical Physics, Kyoto University, Japan

arXiv:1510.00903 [hep-lat]
arXiv:1810.04046 [hep-lat]
arXiv:2203.07661 [hep-lat]

Hyperon-Nucleon potentials from lattice QCD

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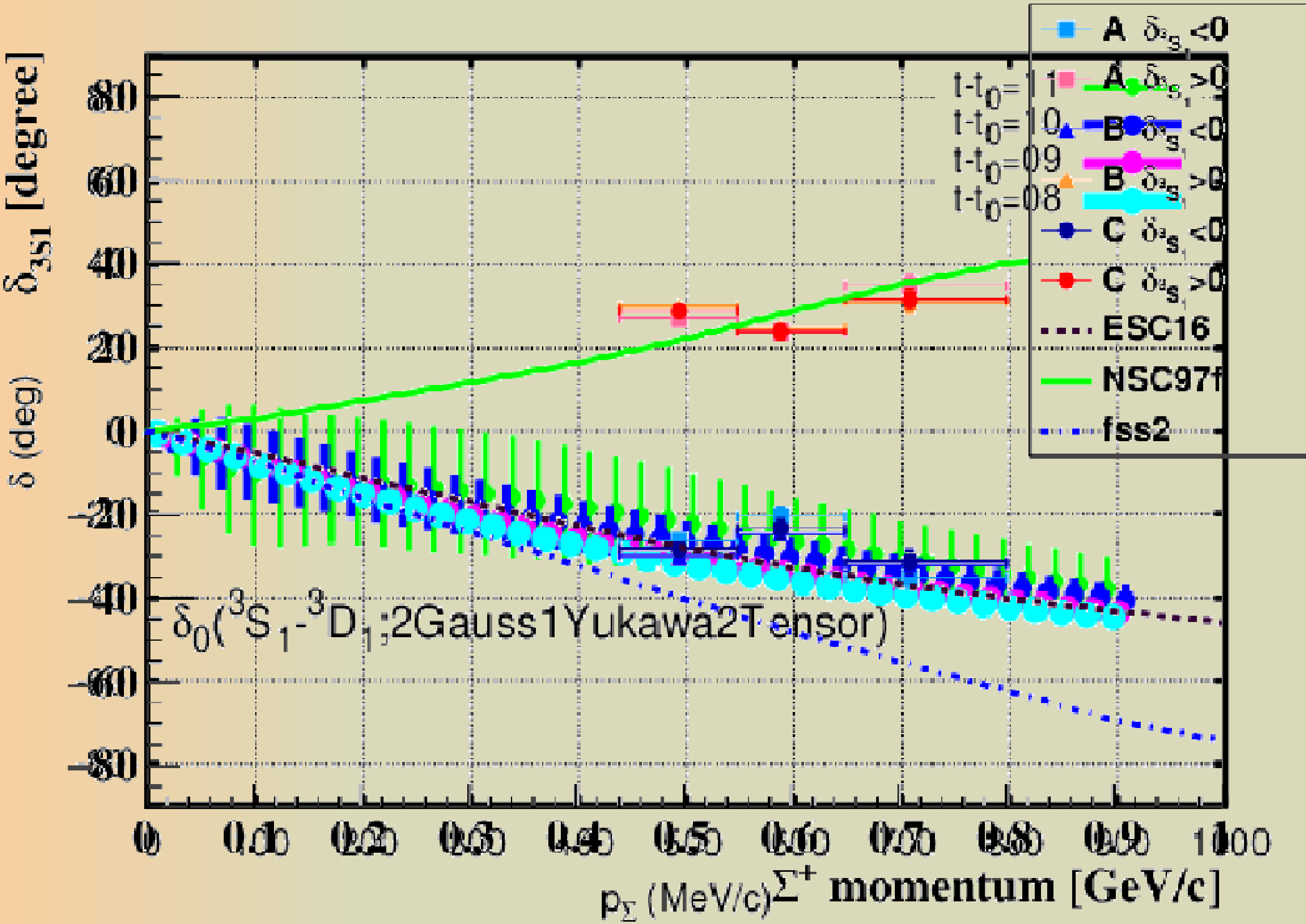
arXiv:1510.00903 [hep-lat]
arXiv:1810.04046 [hep-lat]
arXiv:2203.07661 [hep-lat]

SN(I=3/2) phase shift at almost physical point



arXiv:1810.04046 [hep-lat]

arXiv:2203.08393v3 [nucl-ex]

Plan of research

QCD

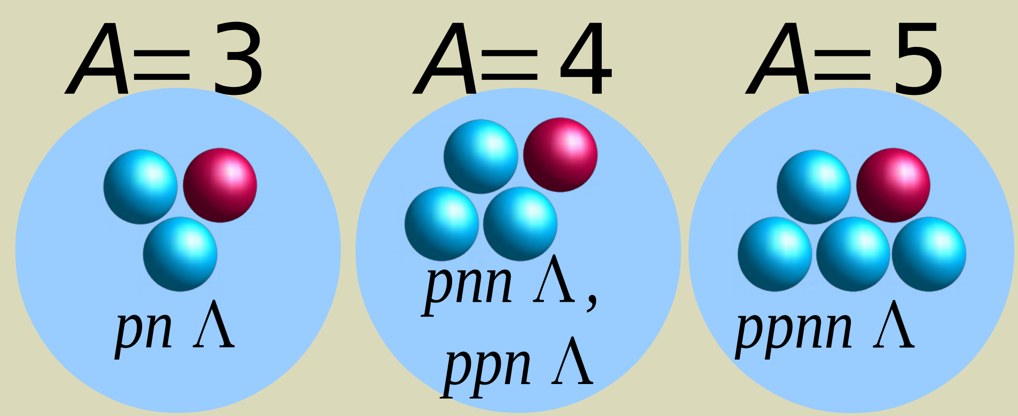
J-PARC,
JLab, GSI, MAMI, ...
YN scattering,
hypernuclei

Baryon interaction

Structure and reaction of (hyper)nuclei

Equation of State (EoS) of nuclear matter

Neutron star and supernova



An improved recipe for NY potential:

- cf. Ishii (HAL QCD), PLB712 (2012) 437.

- Take account of not only the spatial correlation but also the temporal correlation in terms of the R-correlator:

$$-\frac{1}{2\mu}\nabla^2 R(t,\vec{r})+\int d^3r'\, U(\vec{r},\vec{r}')R(t,\vec{r}')=-\frac{\partial}{\partial t}R(t,\vec{r})$$

$$\rightarrow \frac{k^2}{2\mu}R(t,\vec{r})$$

- A general expression of the potential:

$$U(\vec{r},\vec{r}')=V_{NY}(\vec{r},\nabla)\delta(\vec{r}-\vec{r}')$$

$$\begin{aligned} V_{NY} = {} & V_0(r)+V_\sigma(r)(\vec{\sigma}_N\cdot\vec{\sigma}_Y) \\ & +V_T(r)S_{12}+V_{LS}(r)(\vec{L}\cdot\vec{S}_+) \\ & +V_{ALS}(r)(\vec{L}\cdot\vec{S}_-)+O(\nabla^2) \end{aligned}$$

Benchmark test calculation of a four-nucleon bound state,
Phys. Rev. C64, 044001 (2001).

TABLE I. The expectation values $\langle T \rangle$ and $\langle V \rangle$ of kinetic and potential energies, the binding energies E_b in MeV, and the radius in fm.

Method	$\langle T \rangle$	$\langle V \rangle$	E_b	$\sqrt{\langle r^2 \rangle}$
FY	102.39(5)	−128.33(10)	−25.94(5)	1.485(3)
CRCGV	102.30	−128.20	−25.90	1.482
SVM	102.35	−128.27	−25.92	1.486
HH	102.44	−128.34	−25.90(1)	1.483
GFMC	102.3(1.0)	−128.25(1.0)	−25.93(2)	1.490(5)
NCSM	103.35	−129.45	−25.80(20)	1.485
EIHH	100.8(9)	−126.7(9)	−25.944(10)	1.486

Generalization to the various baryon-baryon channels strangeness S=0 to -4 systems

$$\langle pn\overline{pn}\rangle, \tag{4.1}$$

$$\begin{array}{lll}
\langle p\Lambda\overline{p\Lambda}\rangle, & \langle p\Lambda\overline{\Sigma^+ n}\rangle, & \langle p\Lambda\overline{\Sigma^0 p}\rangle, \\
\langle \Sigma^+ n\overline{p\Lambda}\rangle, & \langle \Sigma^+ n\overline{\Sigma^+ n}\rangle, & \langle \Sigma^+ n\overline{\Sigma^0 p}\rangle, \\
\langle \Sigma^0 p\overline{p\Lambda}\rangle, & \langle \Sigma^0 p\overline{\Sigma^+ n}\rangle, & \langle \Sigma^0 p\overline{\Sigma^0 p}\rangle,
\end{array} \tag{4.2}$$

$$\begin{array}{llllll}
\langle \Lambda\Lambda\overline{\Lambda\Lambda}\rangle, & \langle \Lambda\Lambda\overline{p\Xi^-}\rangle, & \langle \Lambda\Lambda\overline{n\Xi^0}\rangle, & \langle \Lambda\Lambda\overline{\Sigma^+\Sigma^-}\rangle, & \langle \Lambda\Lambda\overline{\Sigma^0\Sigma^0}\rangle, & \\
\langle p\Xi^-\overline{\Lambda\Lambda}\rangle, & \langle p\Xi^-\overline{p\Xi^-}\rangle, & \langle p\Xi^-\overline{n\Xi^0}\rangle, & \langle p\Xi^-\overline{\Sigma^+\Sigma^-}\rangle, & \langle p\Xi^-\overline{\Sigma^0\Sigma^0}\rangle, & \langle p\Xi^-\overline{\Sigma^0\Lambda}\rangle, \\
\langle n\Xi^0\overline{\Lambda\Lambda}\rangle, & \langle n\Xi^0\overline{p\Xi^-}\rangle, & \langle n\Xi^0\overline{n\Xi^0}\rangle, & \langle n\Xi^0\overline{\Sigma^+\Sigma^-}\rangle, & \langle n\Xi^0\overline{\Sigma^0\Sigma^0}\rangle, & \langle n\Xi^0\overline{\Sigma^0\Lambda}\rangle, \\
\langle \Sigma^+\Sigma^-\overline{\Lambda\Lambda}\rangle, & \langle \Sigma^+\Sigma^-\overline{p\Xi^-}\rangle, & \langle \Sigma^+\Sigma^-\overline{n\Xi^0}\rangle, & \langle \Sigma^+\Sigma^-\overline{\Sigma^+\Sigma^-}\rangle, & \langle \Sigma^+\Sigma^-\overline{\Sigma^0\Sigma^0}\rangle, & \langle \Sigma^+\Sigma^-\overline{\Sigma^0\Lambda}\rangle, \\
\langle \Sigma^0\Sigma^0\overline{\Lambda\Lambda}\rangle, & \langle \Sigma^0\Sigma^0\overline{p\Xi^-}\rangle, & \langle \Sigma^0\Sigma^0\overline{n\Xi^0}\rangle, & \langle \Sigma^0\Sigma^0\overline{\Sigma^+\Sigma^-}\rangle, & \langle \Sigma^0\Sigma^0\overline{\Sigma^0\Sigma^0}\rangle, & \\
 & \langle \Sigma^0\Lambda\overline{p\Xi^-}\rangle, & \langle \Sigma^0\Lambda\overline{n\Xi^0}\rangle, & \langle \Sigma^0\Lambda\overline{\Sigma^+\Sigma^-}\rangle, & & \langle \Sigma^0\Lambda\overline{\Sigma^0\Lambda}\rangle,
\end{array} \tag{4.3}$$

$$\begin{array}{lll}
\langle \Xi^-\Lambda\overline{\Xi^-\Lambda}\rangle, & \langle \Xi^-\Lambda\overline{\Sigma^-\Xi^0}\rangle, & \langle \Xi^-\Lambda\overline{\Sigma^0\Xi^-}\rangle, \\
\langle \Sigma^-\Xi^0\overline{\Xi^-\Lambda}\rangle, & \langle \Sigma^-\Xi^0\overline{\Sigma^-\Xi^0}\rangle, & \langle \Sigma^-\Xi^0\overline{\Sigma^0\Xi^-}\rangle, \\
\langle \Sigma^0\Xi^-\overline{\Xi^-\Lambda}\rangle, & \langle \Sigma^0\Xi^-\overline{\Sigma^-\Xi^0}\rangle, & \langle \Sigma^0\Xi^-\overline{\Sigma^0\Xi^-}\rangle,
\end{array} \tag{4.4}$$

$$\langle \Xi^-\Xi^0\overline{\Xi^-\Xi^0}\rangle. \tag{4.5}$$

Make better use of the computing resources!

HN, CPC **207**, 91(2016) [arXiv:1510.00903[hep-lat]],
(See also arXiv:1604.08346)

Summary

(I-1) SN and LN potentials (central, tensor) at $(m_\pi, m_K) \approx (145, 525)$MeV.
phase shifts in low energy regions

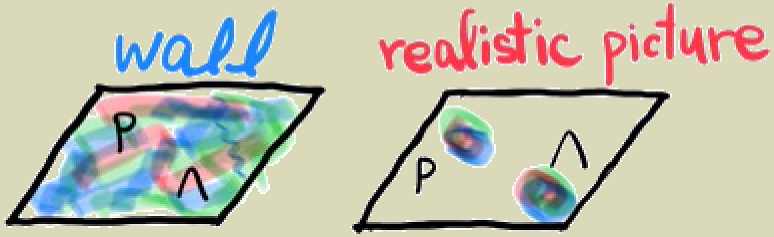
Repulsive SN(I=3/2) 3S1 interaction is consistent with experiment.
Both 1S0 and 3S1 LN interactions are attractive.

(I-2) Effective block algorithm for the various baron-baryon interaction

Comput.Phys.Commun.**207**,91(2016) [arXiv:1510.00903(hep-lat)]

Simultaneous calc. (NN to XiXi) is the point we have to take into account for the comprehensive perspective as well as energy-computing-resource efficiency.

The algorithm will be applied to more wide range problems.



Future work:

(II-1) Physical quantities including the binding energies of few-body problem of light hypernuclei with the lattice YN (and NN) potentials

(II-2) New application of effective baryon block algorithm for the various baron-baryon interaction from NN to ΞΞ.

> Classification of baryon blocks from NN to ΞΞ, which comprises 52 4pt-correlators (2639 diagrams)

> In search of a better approach to conducting lattice nuclear physics.

> Spin-orbit force.

Backup slides

Outline

- Introduction
  - HAL QCD method for baryon-baryon interaction
- Preliminary results of LN-SN potentials at $(m_\pi, m_K) \approx (145, 525)$MeV
  - SN(I=3/2), central and tensor potentials
    - Repulsive (3S1) and attractive (1S0) phase shifts
  - Single channel analysis for LN ==> central and tensor potentials
    - Phase shifts at low energy region below the SN threshold
  - LN-SN(I=1/2), central and tensor potentials
- Effective block algorithm for various baryon-baryon channels, CPC**207**,91(2016)[1510.00903]
  - New application of the algorithm
- Summary

格子ＱＣＤによるポテンシャル導出の手順(超簡略版)

(1) 4点相関関数を計算する。

$$F^{\langle B_1 B_2 \overline{B_3 B_4}\rangle}_{\alpha\beta,JM}(\vec{r},t-t_0) = \sum_{\vec{X}} \left\langle 0 \left| B_{1,\alpha}(\vec{X}+\vec{r},t) B_{2,\beta}(\vec{X},t) \overline{\mathscr{J}^{(J,M)}_{B_3 B_4}}(t_0) \right| 0 \right\rangle, \quad (2.3)$$

(2) 時間依存法を使うためにしきい値だけ時間相関をずらす

$$R^{\langle B_1 B_2 \overline{B_3 B_4}\rangle}_{\alpha\beta,JM}(\vec{r},t-t_0) = \mathrm{e}^{(m_{B_1}+m_{B_2})(t-t_0)} F^{\langle B_1 B_2 \overline{B_3 B_4}\rangle}_{\alpha\beta,JM}(\vec{r},t-t_0)$$

$$= \sum_n A_n \sum_{\vec{X}} \left\langle 0 \left| B_{1,\alpha}(\vec{X}+\vec{r},0) B_{2,\beta}(\vec{X},0) \right| E_n \right\rangle \mathrm{e}^{-(E_n-m_{B_1}-m_{B_2})(t-t_0)} + O(\mathrm{e}^{-(E_{\mathrm{th}}-m_{B_1}-m_{B_2})(t-t_0)}) \quad (2.4)$$

(3) チャネルごとにしきい値が異なるので、それを考慮した時間依存型Schroedinger方程式からポテンシャルを求める

$$\left(\frac{\nabla^2}{2\mu_\lambda} - \frac{\partial}{\partial t}\right) R_{\lambda\varepsilon}(\vec{r},t) \simeq V^{(\mathrm{LO})}_{\lambda\lambda'}(\vec{r}) \theta_{\lambda\lambda'} R_{\lambda'\varepsilon}(\vec{r},t), \text{ with } \theta_{\lambda\lambda'} = \mathrm{e}^{(m_{B_1}+m_{B_2}-m_{B'_1}-m_{B'_2})(t-t_0)}.$$

(※) “moderately large imaginary time” で計算を行う

(※※) 2種類の励起状態を区別している

[1]The potential is obtained from the NBS wave function at moderately large imaginary time; it would be $t-t_0 \gg 1/m_\pi \sim 1.4$ fm. In addition, no single state saturation between the ground state and the excited states with respect to the relative motion, e.g., $t-t_0 \gg (\Delta E)^{-1} = \left((2\pi)^2/(2\mu(La)^2)\right)^{-1} \simeq 8.0$ fm, is required for the HAL QCD method[13].

Effective block algorithm for various baryon-baryon correlators

HN, CPC207,91(2016), arXiv:1510.00903(hep-lat)

Numerical cost (# of iterative operations) in this algorithm

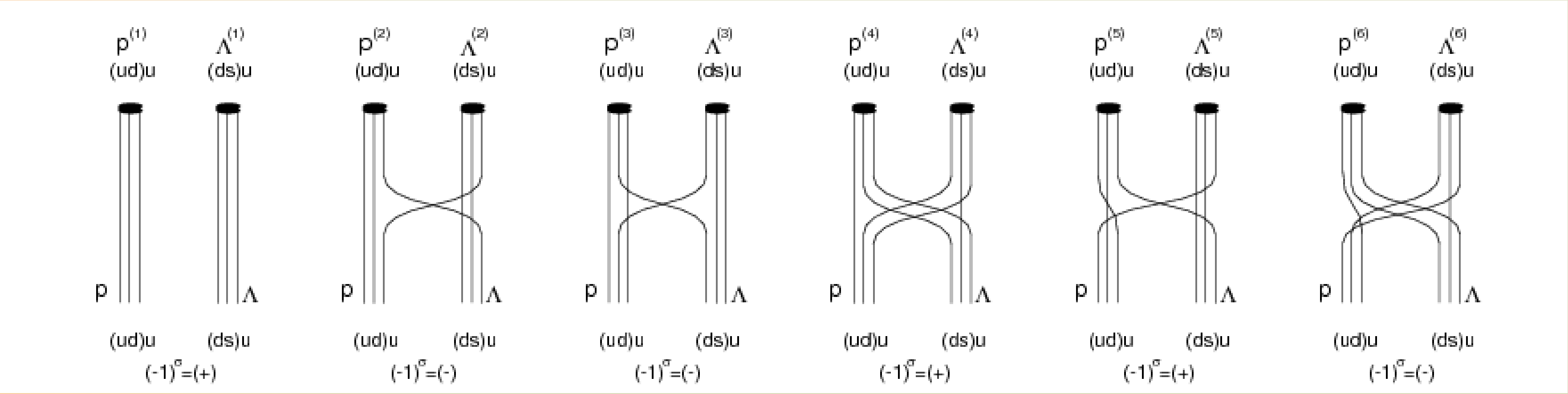
$$1+N_c^{\,2}+N_c^{\,2}N_\alpha^{\,2}+N_c^{\,2}N_\alpha^{\,2}+N_c^{\,2}N_\alpha+N_c^{\,2}N_\alpha=370$$

In an intermediate step:

$$(N_c!\ N_\alpha)^B\times N_u!\ N_d!\ N_s!\ \times 2^{N_\Lambda+N_{\Sigma^0}-B}=3456$$

In a naïve approach:

$$(N_c!\ N_\alpha)^{2B}\times N_u!\ N_d!\ N_s!\ =3,981,312$$



Generalization to the various baryon-baryon channels strangeness S=0 to -4 systems

$$\langle p n \overline{p n} \rangle, \tag{4.1}$$

$$\begin{aligned}
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\langle p\Xi^- \overline{\Lambda\Lambda} \rangle, & \langle p\Xi^- \overline{p\Xi^-} \rangle, & \langle p\Xi^- \overline{n\Xi^0} \rangle, & \langle p\Xi^- \overline{\Sigma^+\Sigma^-} \rangle, & \langle p\Xi^- \overline{\Sigma^0\Sigma^0} \rangle, & \langle p\Xi^- \overline{\Sigma^0\Lambda} \rangle, \\
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\langle \Sigma^0\Sigma^0 \overline{\Lambda\Lambda} \rangle, & \langle \Sigma^0\Sigma^0 \overline{p\Xi^-} \rangle, & \langle \Sigma^0\Sigma^0 \overline{n\Xi^0} \rangle, & \langle \Sigma^0\Sigma^0 \overline{\Sigma^+\Sigma^-} \rangle, & \langle \Sigma^0\Sigma^0 \overline{\Sigma^0\Sigma^0} \rangle, & \\
 & \langle \Sigma^0\Lambda \overline{p\Xi^-} \rangle, & \langle \Sigma^0\Lambda \overline{n\Xi^0} \rangle, & \langle \Sigma^0\Lambda \overline{\Sigma^+\Sigma^-} \rangle, & & \langle \Sigma^0\Lambda \overline{\Sigma^0\Lambda} \rangle,
\end{array} \tag{4.3}$$

$$\begin{aligned}
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$$\langle \Xi^-\Xi^0 \overline{\Xi^-\Xi^0} \rangle. \tag{4.5}$$

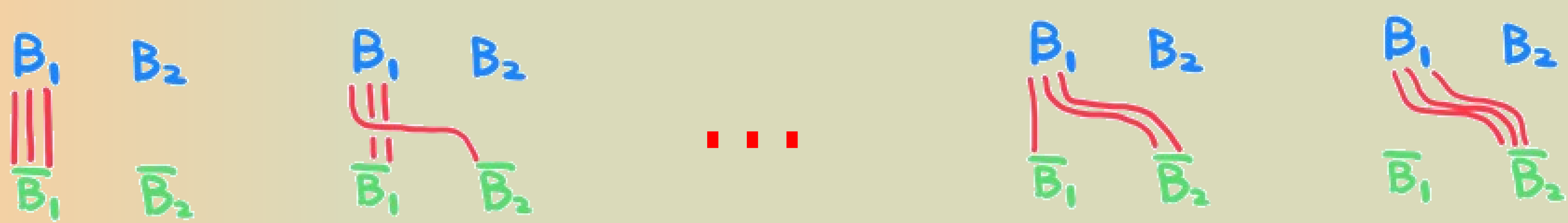
Make better use of the computing resources!

HN, CPC **207**, 91(2016) [arXiv:1510.00903[hep-lat]],
(See also arXiv:1604.08346)

Classification of baryon blocks in the effective block algorithm

- The number of declared blocks in terms of quark propagation form, i.e., from [111] to [222], in the simultaneous calculation of 4pt correlators from NN to ΞΞ

- Proton: 18+ 0+ 31+ 0+106+ 16+121+ 12 = 304
- Σ^+: 3+ 0+ 10+ 0+ 52+ 3+ 55+ 1 = 124
- Ξ^0: 16+ 19+ 0+ 0+118+102+ 29+ 14 = 298
- Λ(dsu): 242+318+436+408+290+266+376+248 = 2584
- Λ(sud): 94+164+102+132+130+164+102+ 96 = 984
- Λ(uds): 94+102+130+102+164+132+164+ 96 = 984

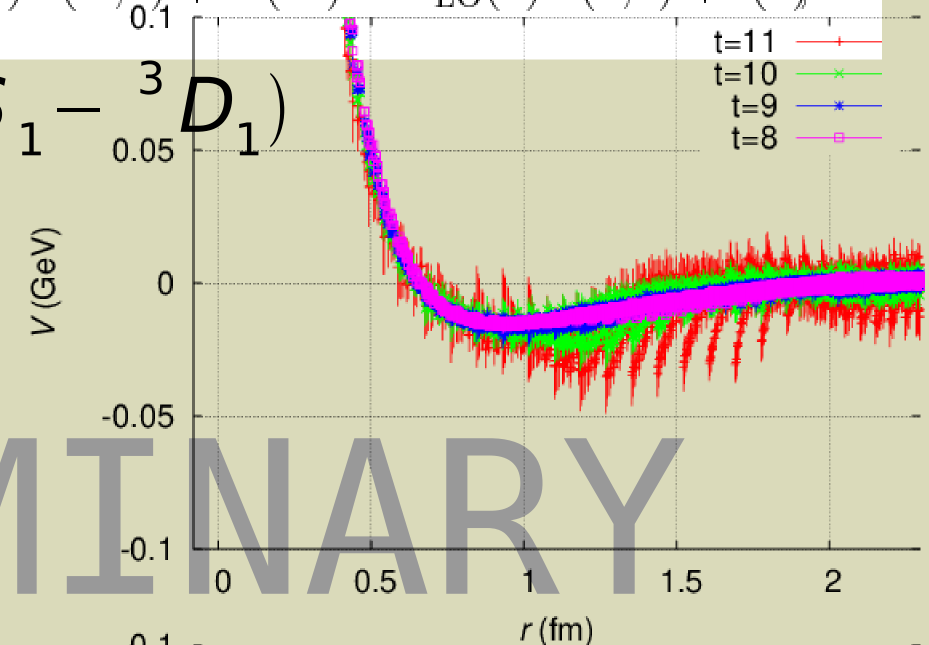


Preliminary result of LN potential at the $(m_\pi, m_K) \approx (145, 525)$ MeV

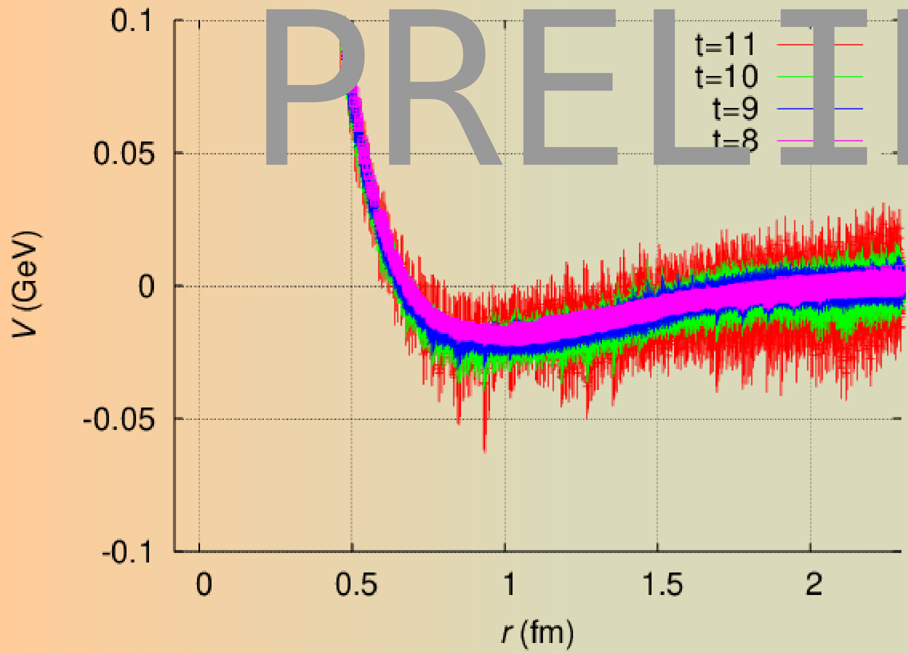
$$\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t}\right) R(\vec{r},t) = \int d^3r'\, U(\vec{r},\vec{r}')R(\vec{r}',t) + O(k^4) = V_{\mathrm{LO}}(\vec{r})R(\vec{r},t) + \cdots \quad (8)$$

ΛN

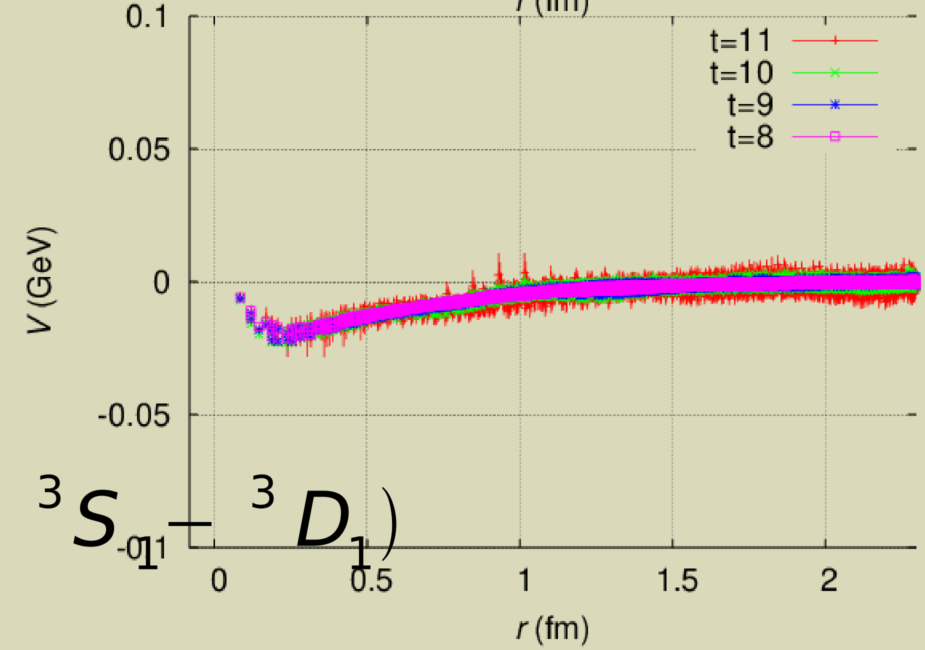
$V_C(\,^3S_1 - \,^3D_1)$



PRELIMINARY

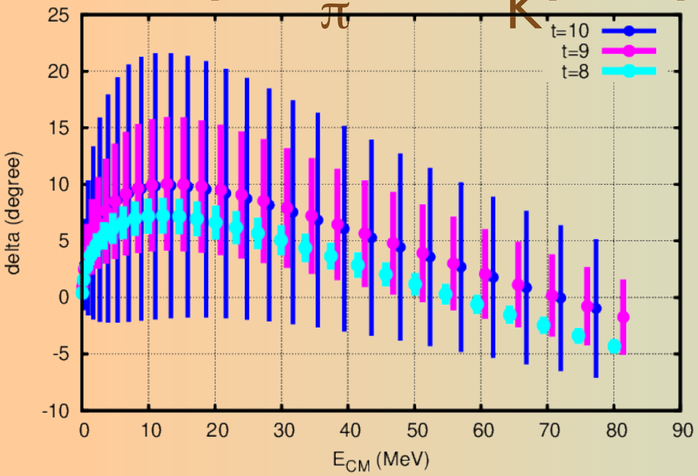


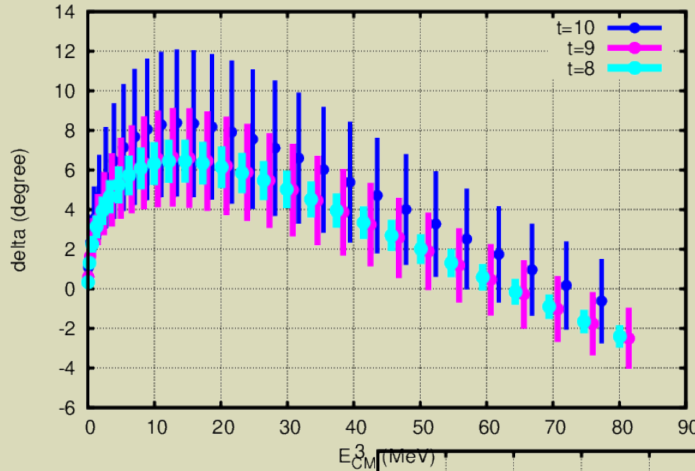
$V_C(\,^1S_0)$



$V_T(\,^3S_1 - \,^3D_1)$

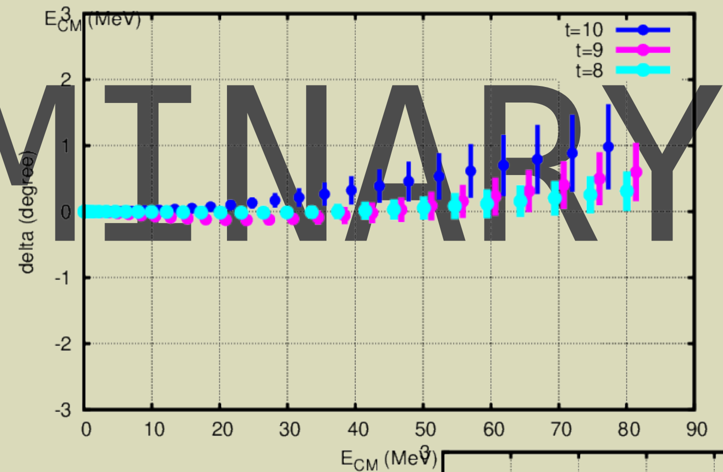
Preliminary results of the LN phase shift at (m_π, m_K)≈(145, 525)MeV

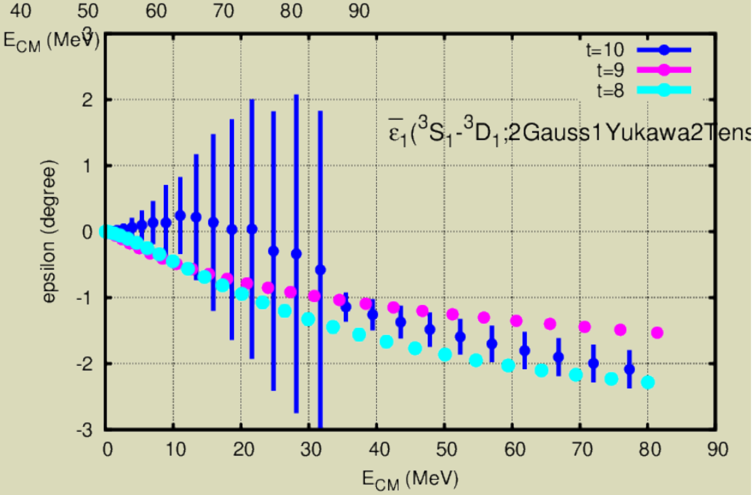


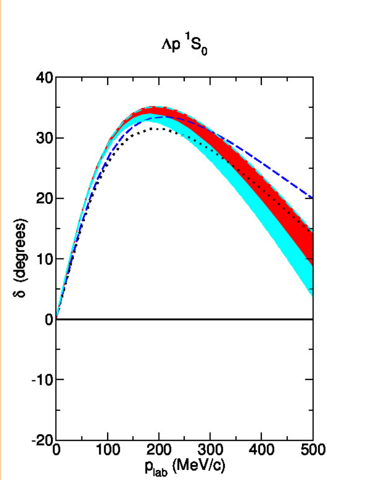


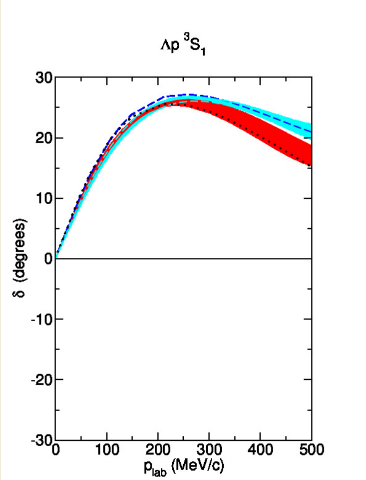
ΛN

PRELIMINARY









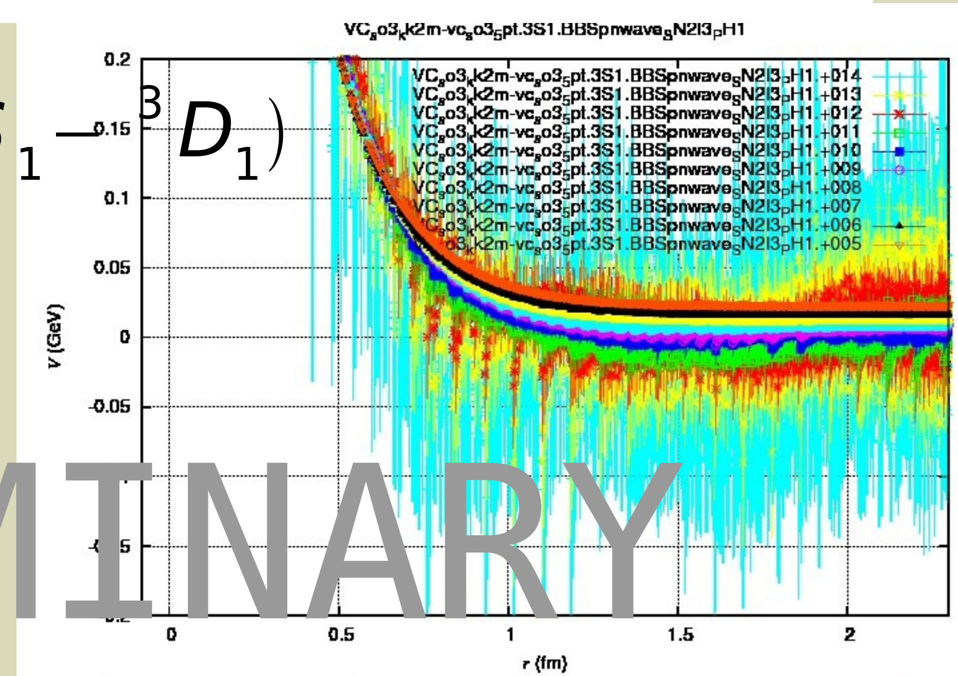
Haidenbauer et al., Eur.Phys.J.A(2020)56:91.

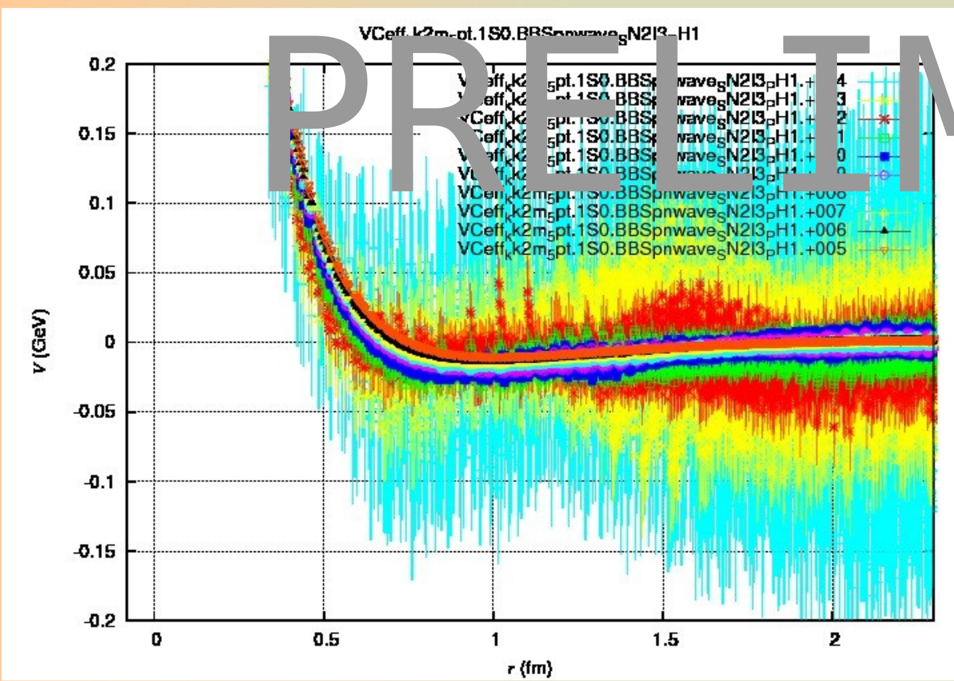
Very preliminary result of LN potential at the physical point

$$\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t}\right) R(\vec{r}, t) \;=\; \int d^3r'\, U(\vec{r}, \vec{r}')R(\vec{r}', t) + O(k^4) = V_{\rm LO}(\vec{r})R(\vec{r}, t) + \cdot (8)$$

$\Sigma N(I = 3/2)$

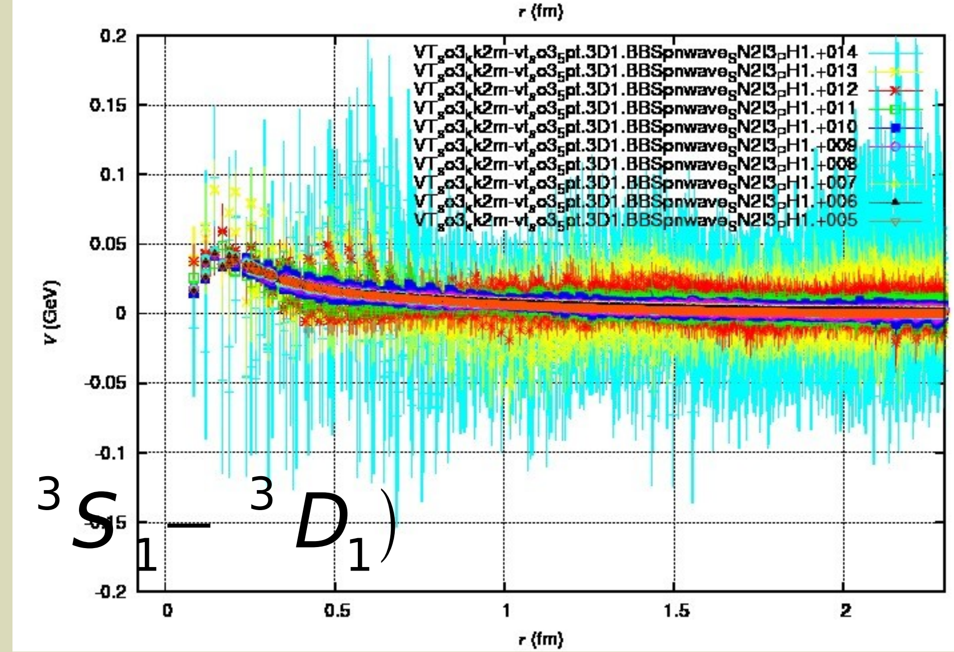
$V_C(\,^3S_1 - \,^3D_1)$





PRELIMINARY

$V_C(\,^1S_0)$

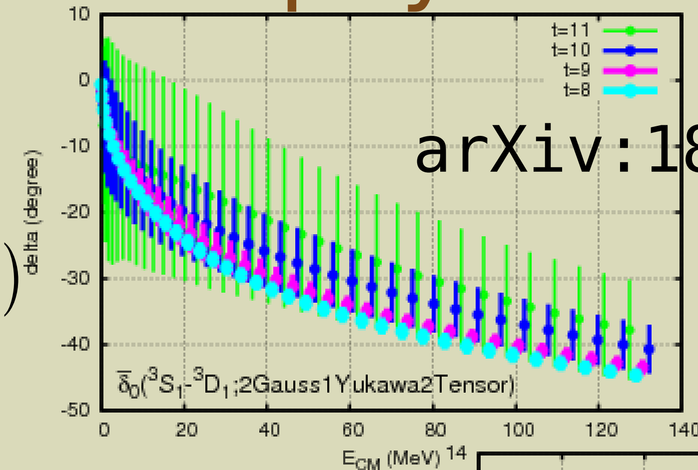


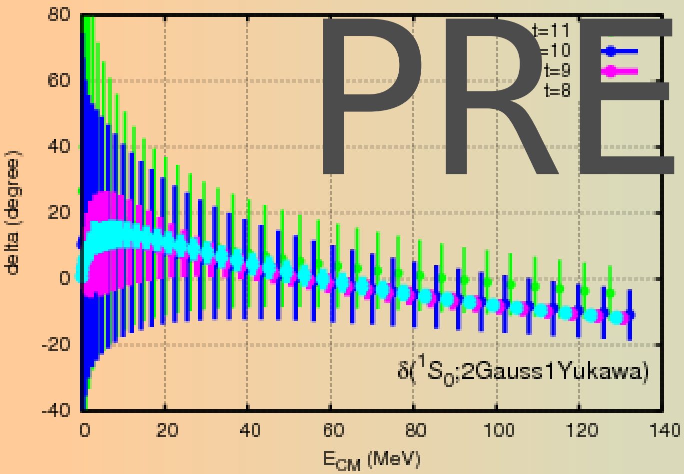
$V_T(\,^3S_1 - \,^3D_1)$

Very preliminary results of the SN(I=3/2) phase shift at the physical point

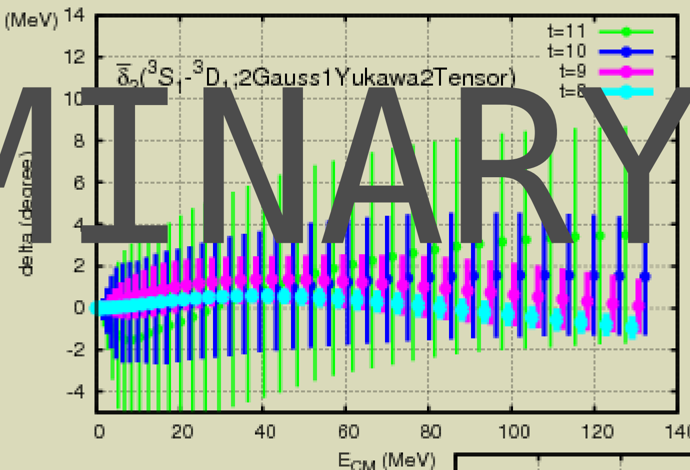
arXiv:1810.04046 [hep-lat]

$\Sigma N(I=3/2)$

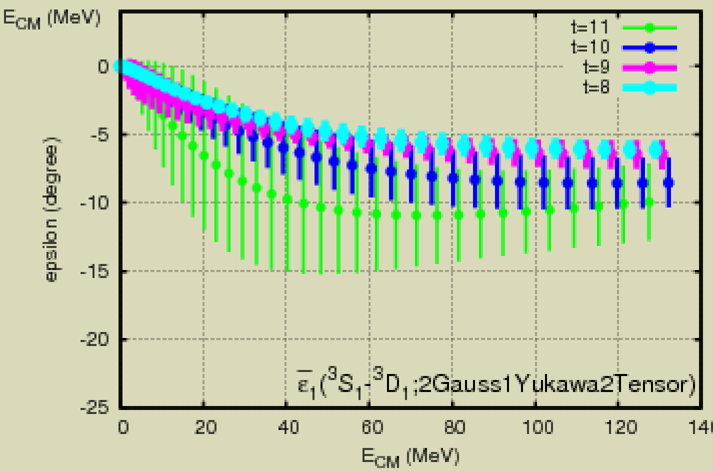




PRELIMINARY



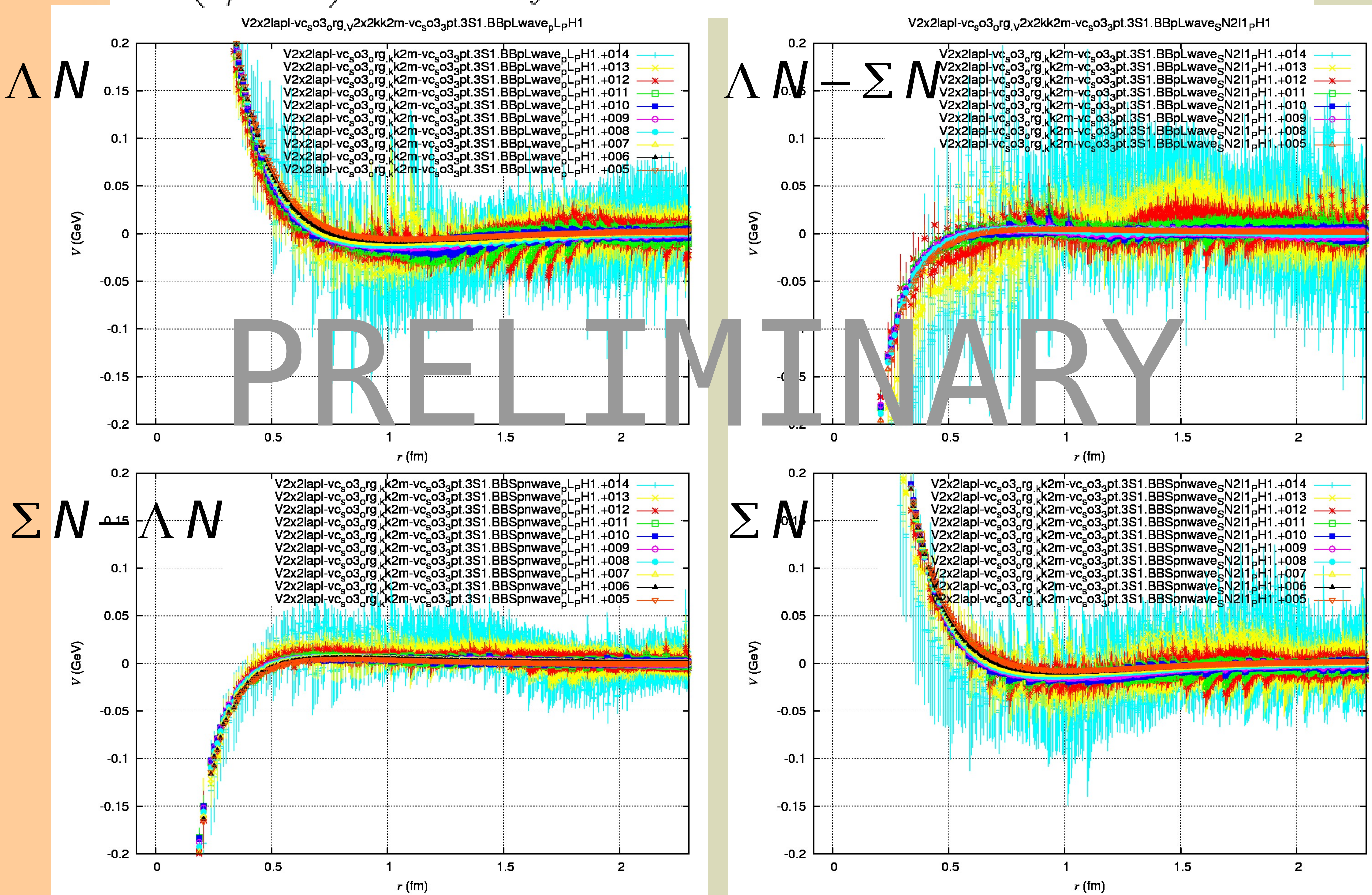
More or less qualitatively similar to (recent) phenomenological approaches:
Fujiwara, et al., PRC54(1996)2180,
Arisaka, et al., PTP104(2000)995,
Haidenbauer et al., NPA915(2013)24.



Very preliminary result of LN potential at the physical point

$V_C(\,^3S_1 - \,^3D_1)$

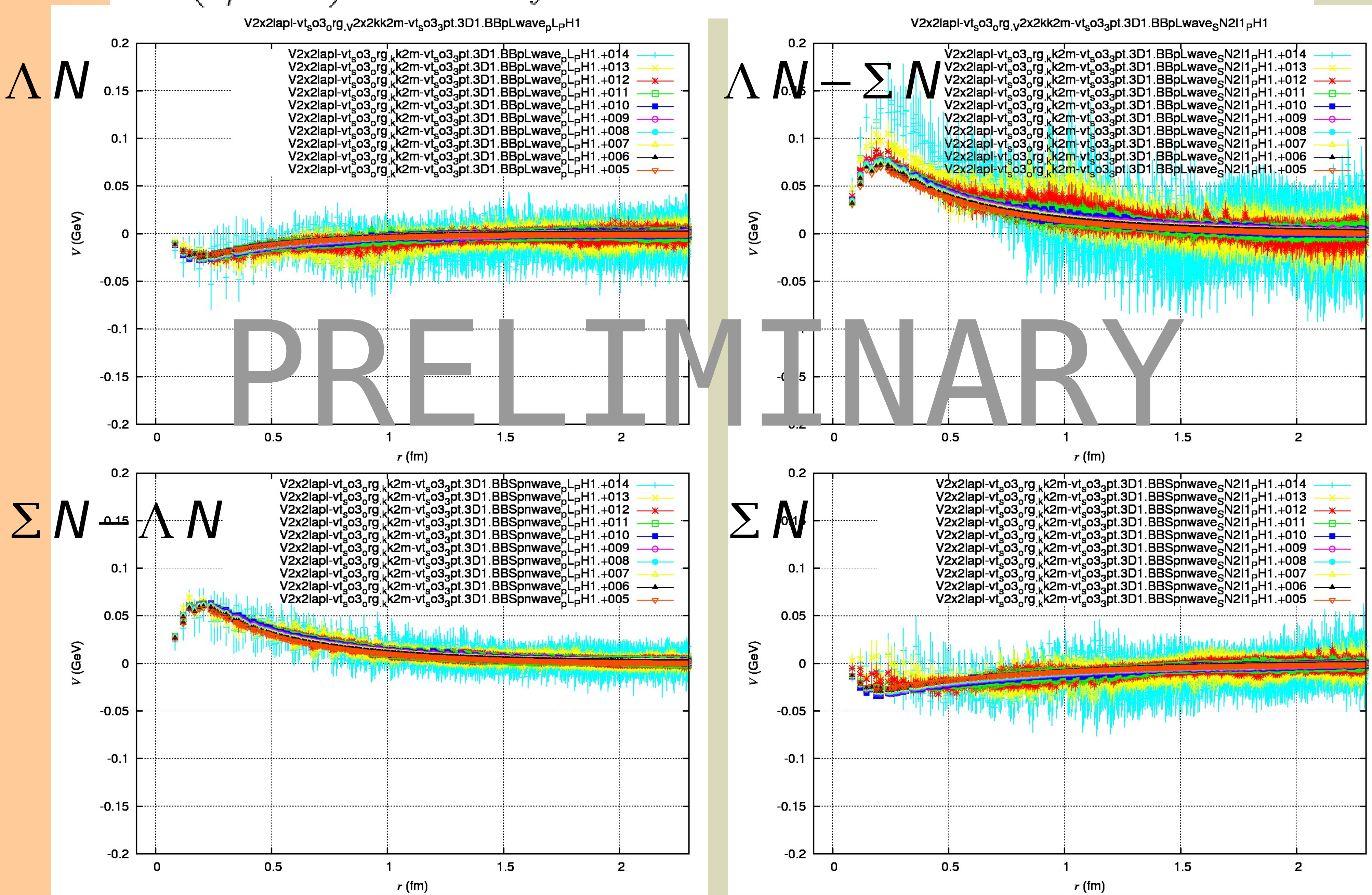
$$\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t}\right) R(\vec{r},t) \;=\; \int d^3r'\, U(\vec{r},\vec{r}')R(\vec{r}',t) + O(k^4) = V_{\rm LO}(\vec{r})R(\vec{r},t) + \cdots (8)$$



Very preliminary result of LN potential at the physical point

$V_T({}^3S_1 - {}^3D_1)$

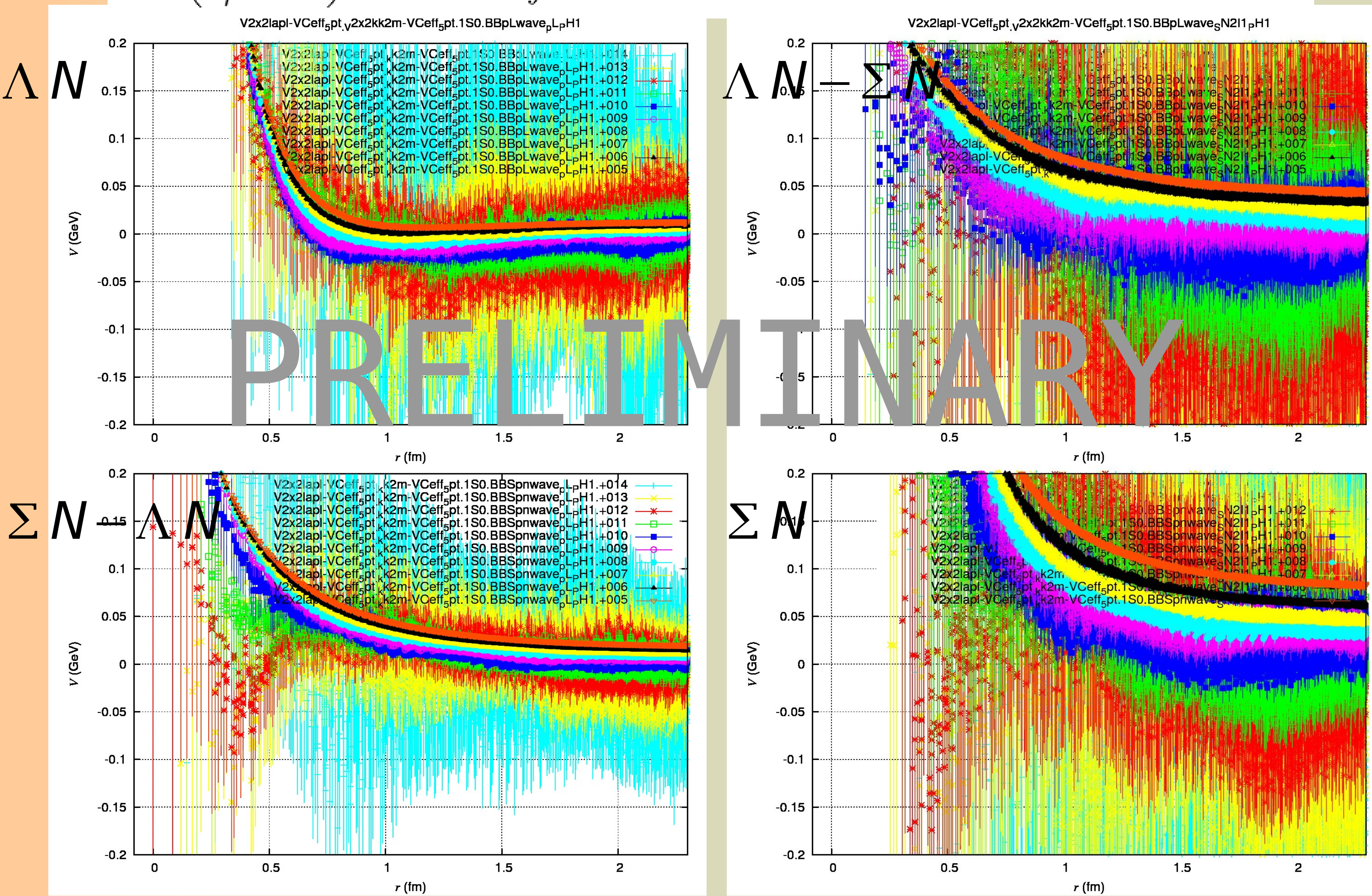
$$\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t}\right) R(\vec{r},t) \;=\; \int d^3r'\, U(\vec{r},\vec{r}')R(\vec{r}',t) + O(k^4) = V_{\rm LO}(\vec{r})R(\vec{r},t) + \cdots (8)$$



Very preliminary result of LN potential at the physical point

$V_C(^1S_0)$

$$\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t}\right) R(\vec{r},t) = \int d^3r' \, U(\vec{r},\vec{r}') R(\vec{r}',t) + O(k^4) = V_{\mathrm{LO}}(\vec{r}) R(\vec{r},t) + \cdots (8)$$



Almost physical point lattice QCD calculation using N_F=2+1 clover fermion + Iwasaki gauge action

- APE-Stout smearing (r=0.1, n_{stout}=6)
- Non-perturbatively O(a) improved Wilson Clover action at β=1.82 on $96^3 \times 96$ lattice
- $1/a$ = 2.3 GeV (a = 0.085 fm)
- Volume: $96^4 \rightarrow (8\text{fm})^4$
- m_p=145MeV, m_K=525MeV
- DDHMC(ud) and UVPHMC(s) with preconditioning
- K.-I.Ishikawa, et al., PoS LAT2015, 075; arXiv:1511.09222 [hep-lat].

- NBS wf is measured using wall quark source with Coulomb gauge fixing, spatial PBD and temporal DBC; #stat=207configs x 4rotation x 96src