The spatial and time structure of the odd-frequency Cooper pair wave-function

2023/2/10 14:50-15:05　第8回クラスター階層領域研究会

Keio univ.

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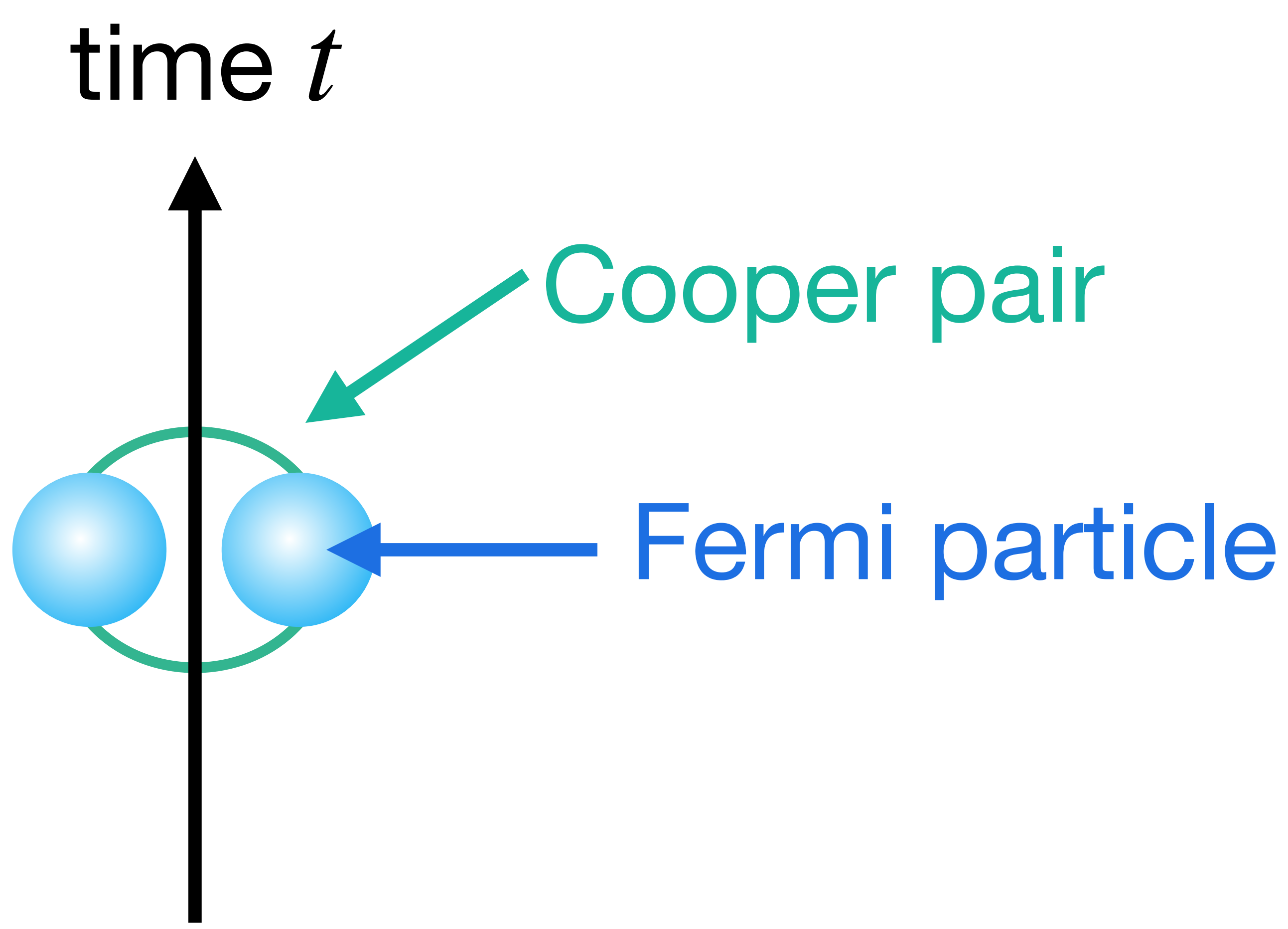
Outline

- Introduction
  What is odd-frequency superfluidity?
- Formalism
- Results
  T_c : superfluid phase transition temperature
  $F^{\mathrm{T}}(\boldsymbol{r}, t)$: pair wave function (anomalous Green's function)
- Summary

Classification of Fermi superfluid

even-frequency superfluidity



$$F(t_1, t_2) = +F(t_2, t_1)$$

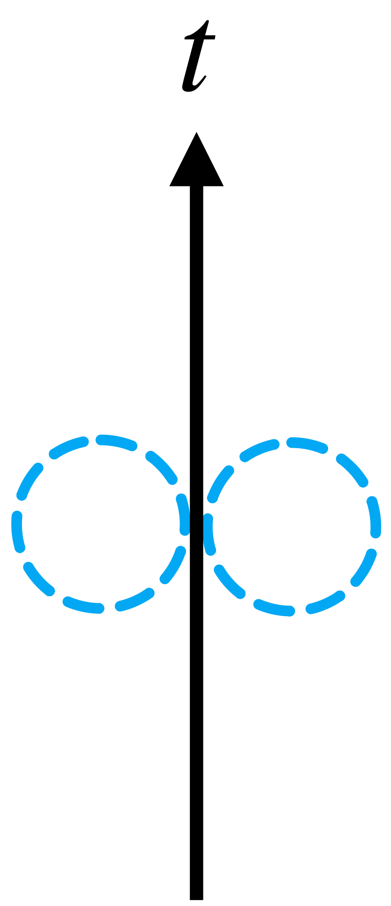
$$F(-\omega) = +F(\omega)$$

It is possible to form a pair with the particles at the same time.

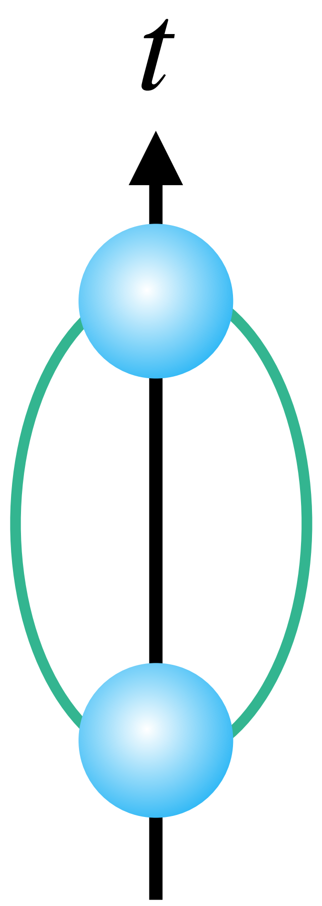
odd-frequency superfluidity

$$F(t_1, t_2) = -F(t_2, t_1)$$

$$F(-\omega) = -F(\omega)$$



A pair with particles
at the same time is prohibited.



A pair with particles at different
times only exists.

Ex. of Odd-frequency Fermi superfluid

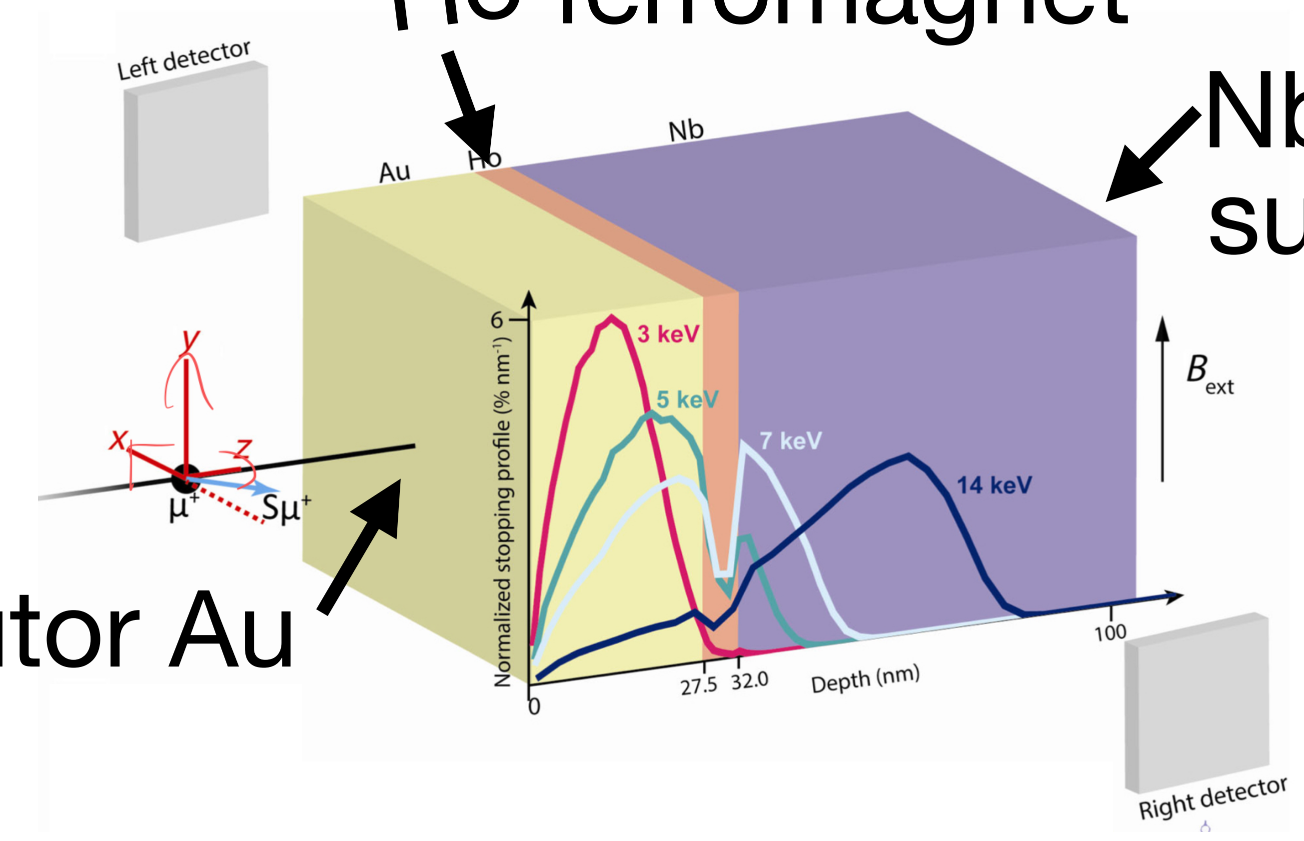
Junction system

A. Di Bernardo et al Phys. Rev. X. **5** (2015) 041021

Ho ferromagnet

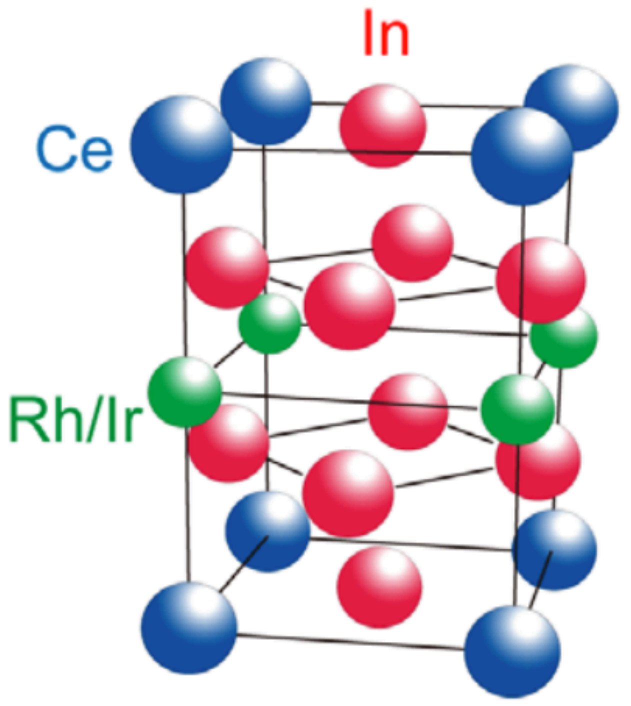
Nb
superconductor

Normal condutor Au

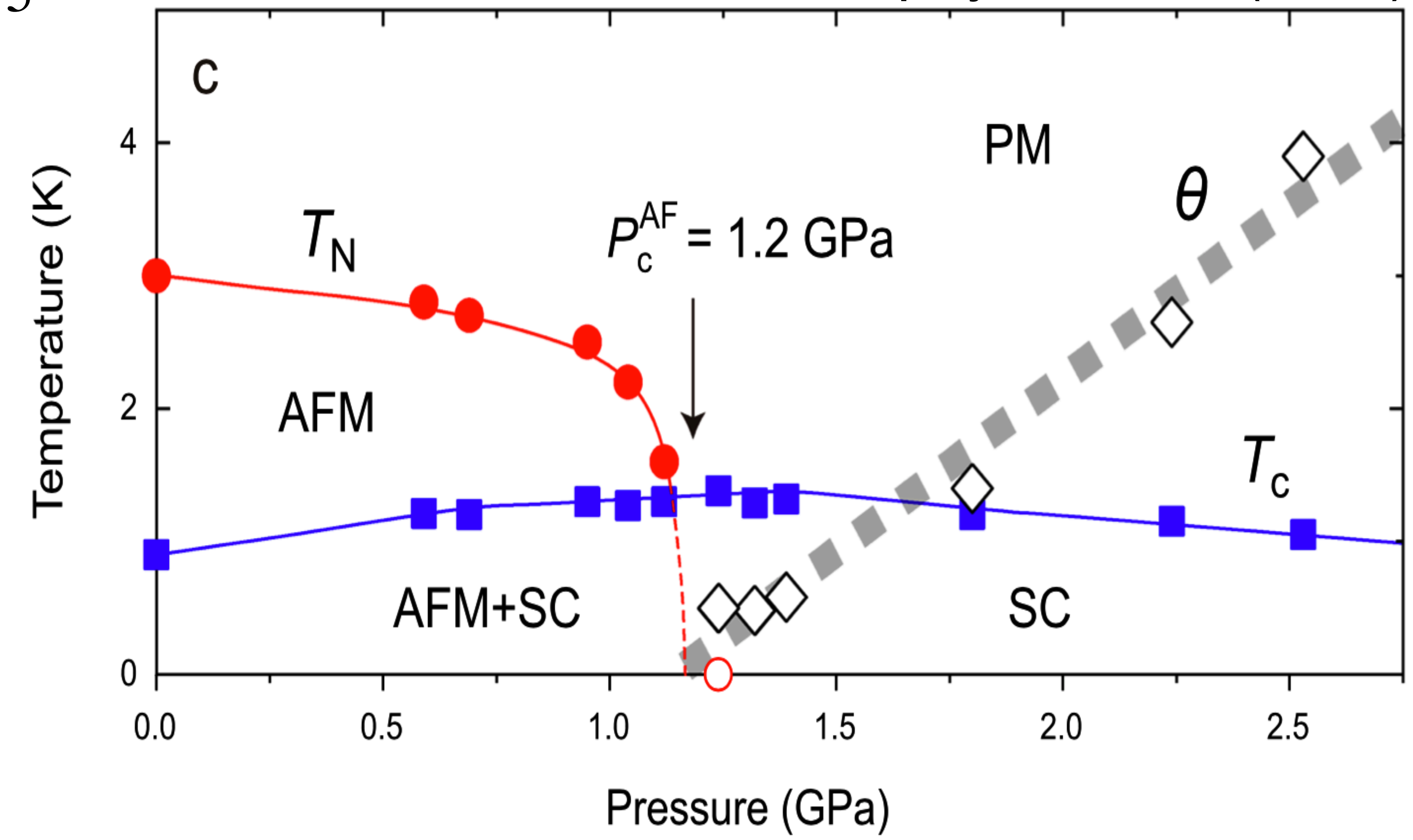


Bulk system

$CeRh_{0.5}Ir_{0.5}In_5$

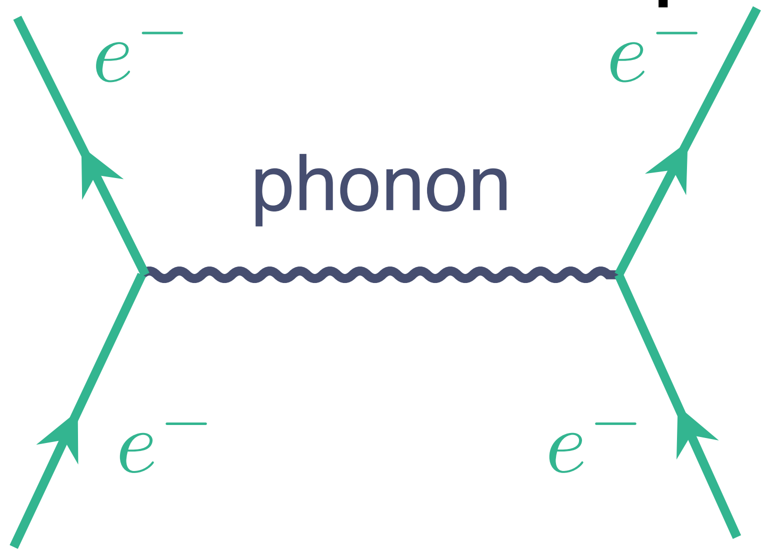


S. Kawasaki et. al., commun. phys. **3**, 148 (2020)



odd-frequency p-wave singlet

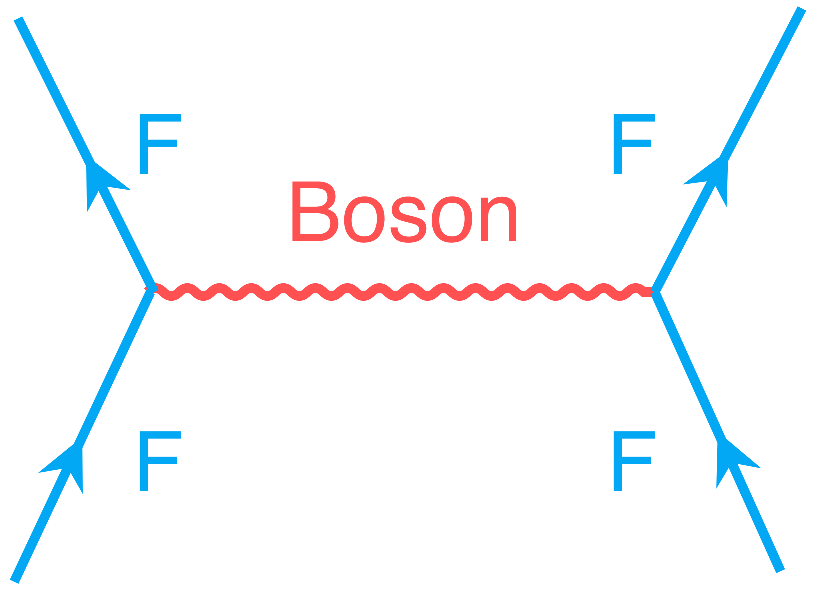
Electron and optical phonon system



H. Kusunose, Y. Fuseya ,and K. Miyake
J. Phys. Soc. Jpn. 80 (2011) 044711

odd-frequency s-wave triplet

Mixture of bosonic and fermionic cold atoms



R. M. Kalas, et. al.
Phys. Rev. B. 78 (2008) 184513

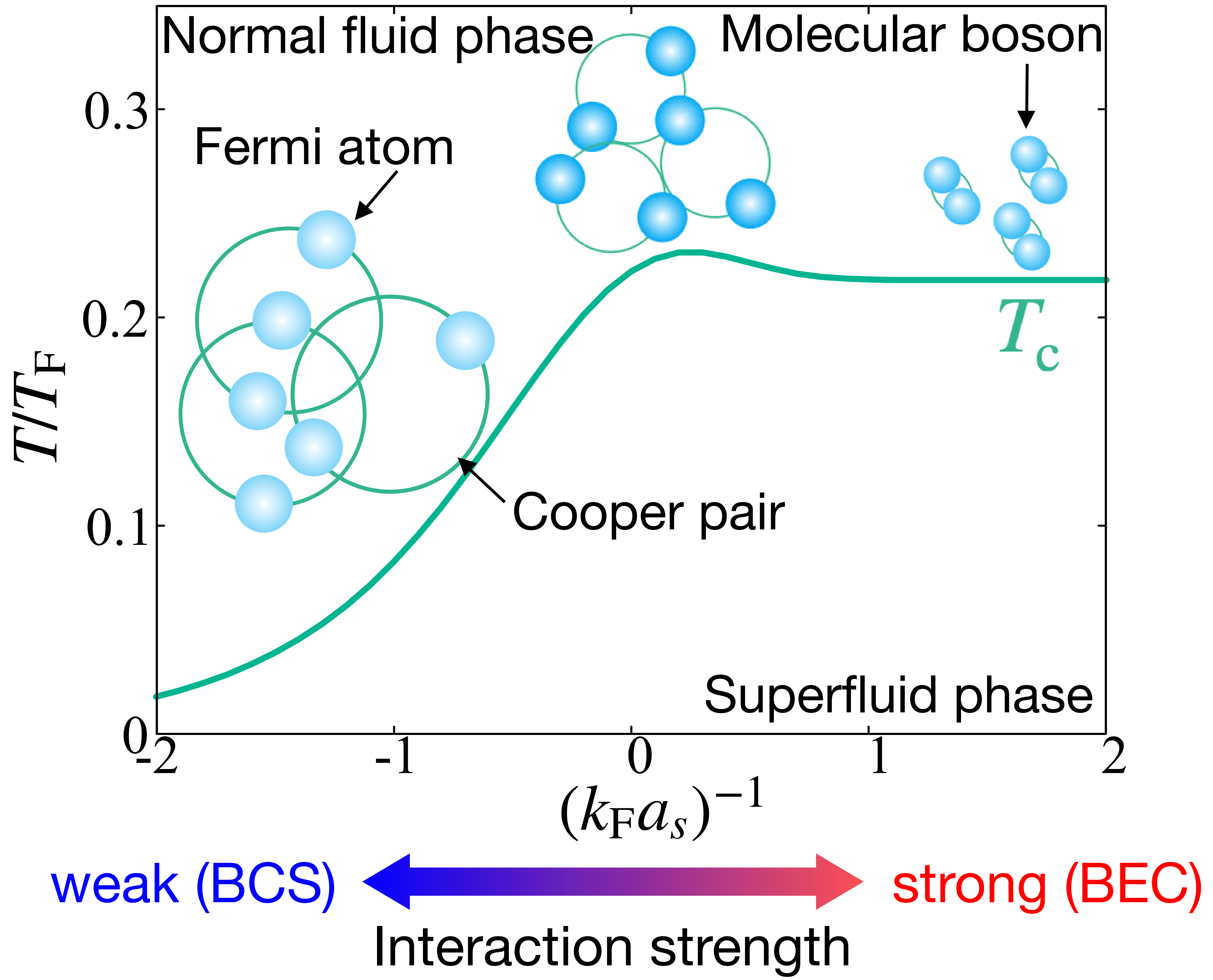
even-frequency p-wave

odd-frequency s-wave

Motivation

even-frequency case

BCS-BEC crossover

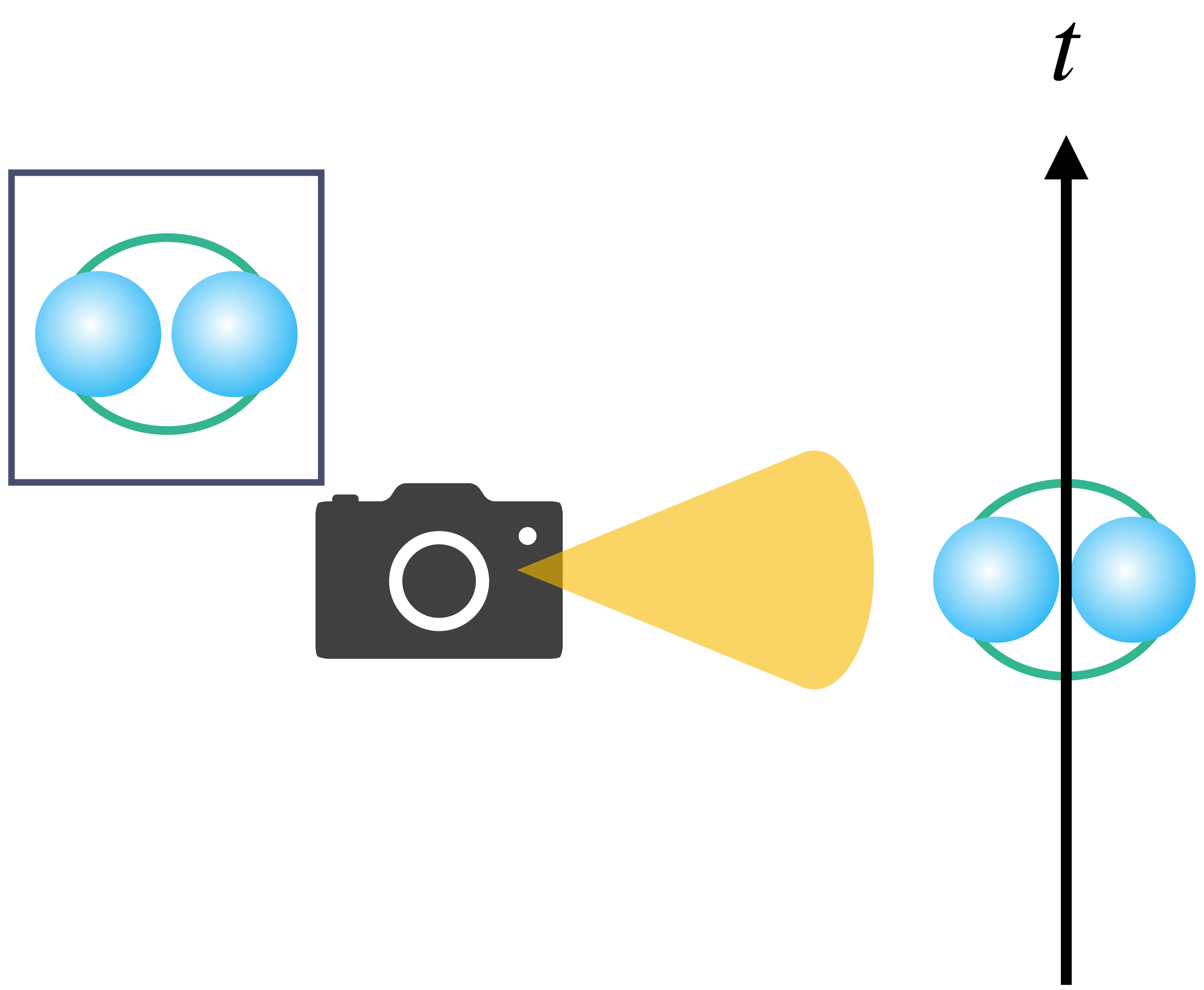


Molecular formation by Fermi particles at different times

Can Cooper pairs formed by two Fermi particles at **different times** be regarded as **boson**?

even-frequency superfluidity

"Molecules" formed by particles at the same time



When you take a snapshot...
There is two fermions in the picture.

→boson

odd-frequency superfluidity

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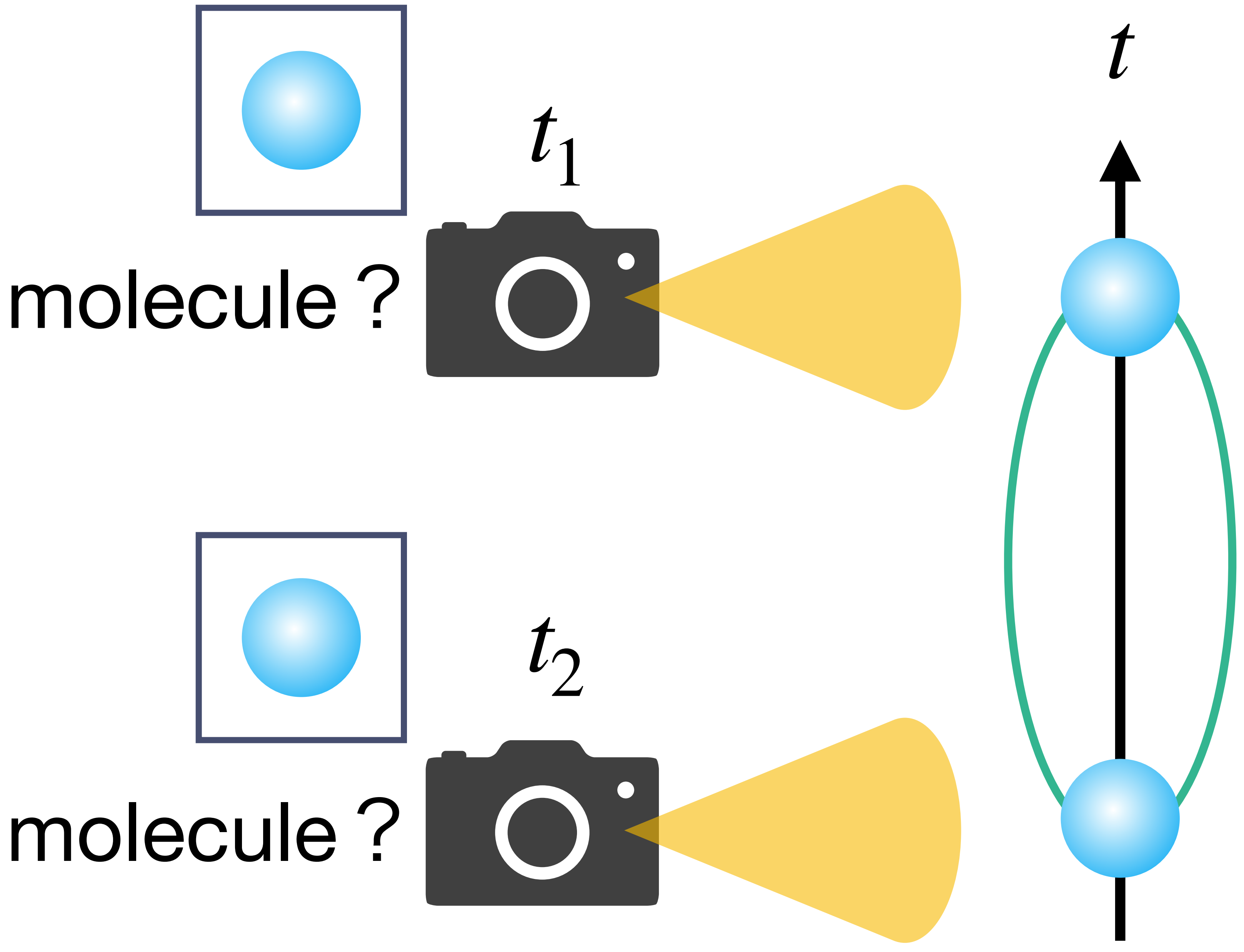

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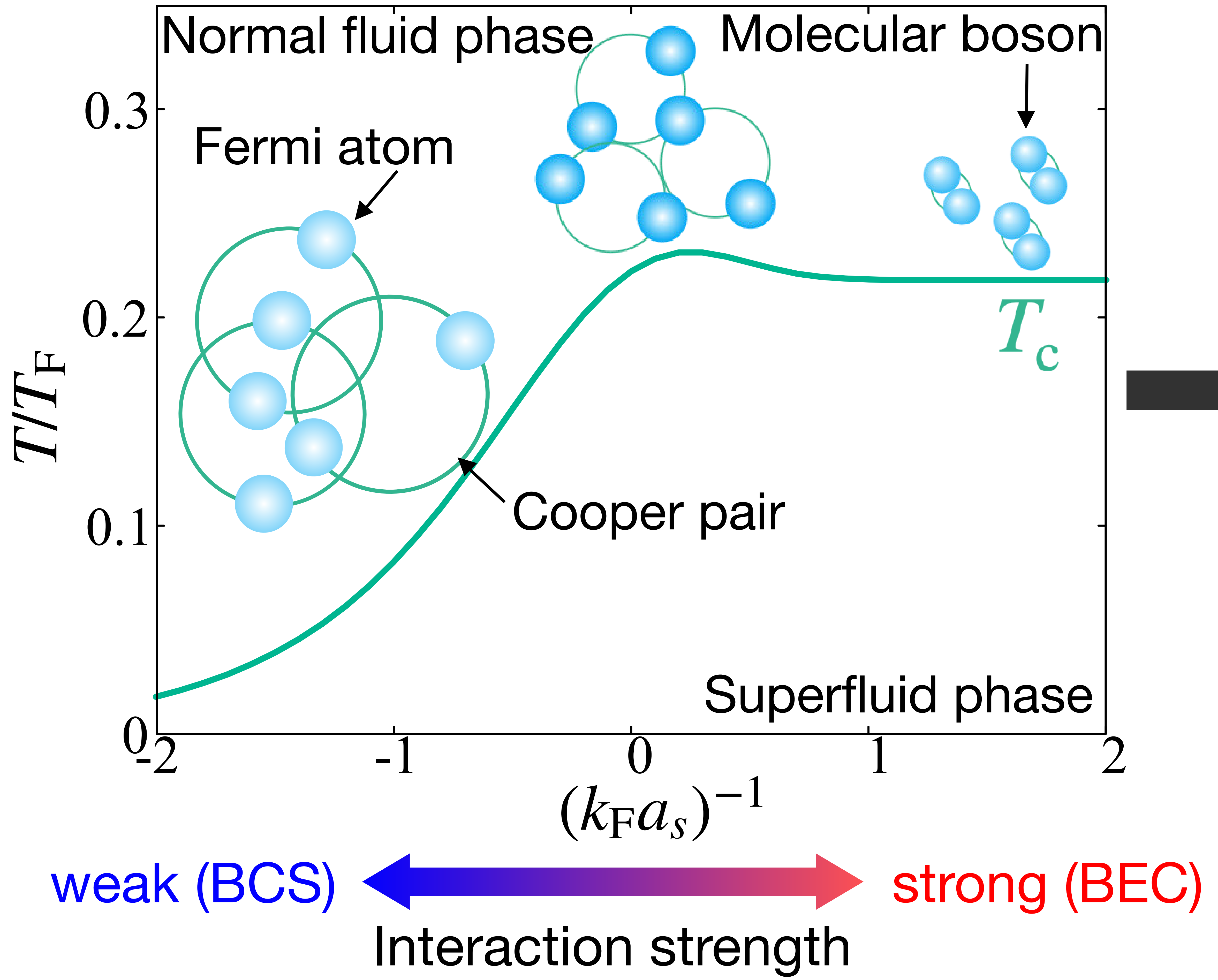
→boson?

Motivation

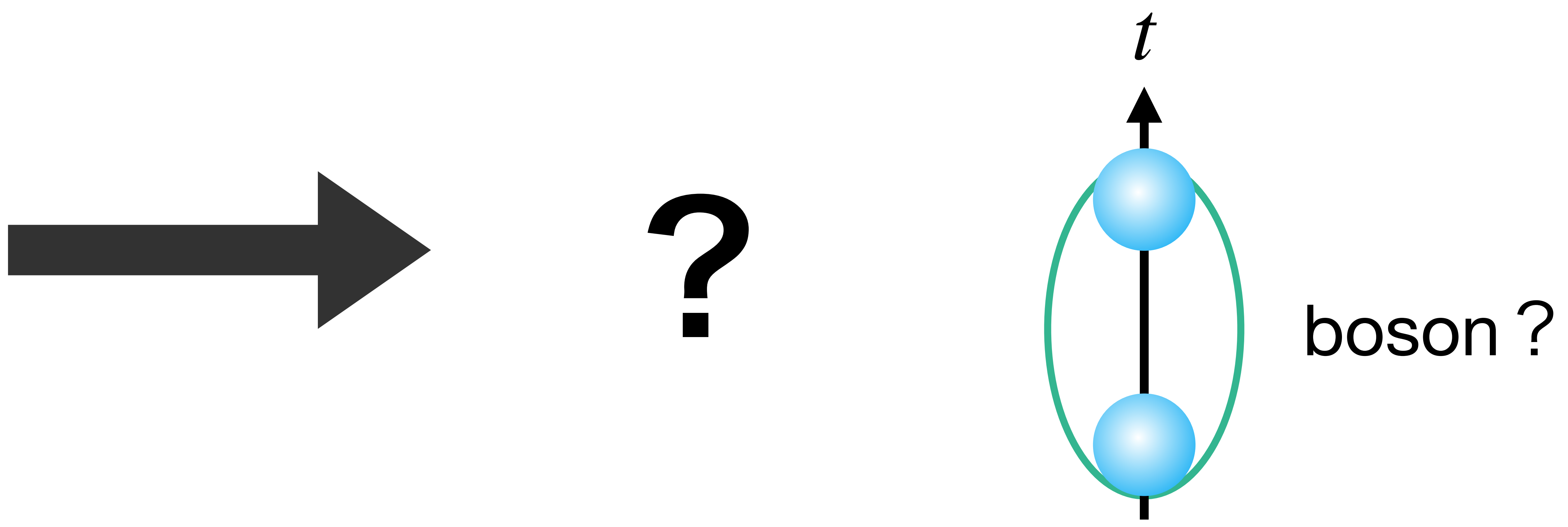
even-frequency case

BCS-BEC crossover



odd-frequency case

It is not obvious whether the BCS-BEC crossover occurs.



Goal

We construct a strong-coupling theory for **odd-frequency superfluidity** and investigate

- superfluid phase transition temperature T_c
- Cooper pair wave-function $F(r, t)$

to clarify

"BCS-BEC crossover can occur and the molecule formed by two fermions at ***different times*** *can be viewed as a* ***boson****?"*

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Model

Action (1 component Fermi Gas) $S = S_0 + S_1$

$$S_0 = \sum_{\boldsymbol{p},\omega_n} \bar{\psi}_{\boldsymbol{p},\omega_n}(-i\omega_n + \xi_{\boldsymbol{p}})\psi_{\boldsymbol{p},\omega_n} \qquad \xi_p = \frac{p^2}{2m} - \mu$$

$$S_1 = \frac{1}{2\beta} \sum_{\boldsymbol{p_1},\omega_{n_1},\boldsymbol{p_2},\omega_{n_2},\boldsymbol{q},\nu_n} {\color{red}V_{\boldsymbol{p_1},\omega_{n_1},\boldsymbol{p_2},\omega_{n_2}}} \bar{\psi}_{\boldsymbol{p_2}+\frac{\boldsymbol{q}}{2},\omega_{n_2}+\nu_n} \bar{\psi}_{-\boldsymbol{p_2}+\frac{\boldsymbol{q}}{2},-\omega_{n_2}} \psi_{-\boldsymbol{p_1}+\frac{\boldsymbol{q}}{2},-\omega_{n_1}} \psi_{\boldsymbol{p_1}+\frac{\boldsymbol{q}}{2},\omega_{n_1}+\nu_n}$$

Strongly retarded attractive interaction (depending on frequency)

$$V_{\boldsymbol{p_1},\omega_{n_1},\boldsymbol{p_2},\omega_{n_2}} = -U\gamma_{\omega_{n_1}}\gamma_{\omega_{n_2}}$$

Assumed frequency dependence: **separable** form

$$\gamma_{\omega_n} = \frac{\omega_n}{\sqrt{\omega_n^2 + \omega_0^2}}$$

Odd in ω_n → odd-frequency superfluidity

U **: strength of the contact interaction**

$$\frac{1}{U} = -\frac{m}{4\pi a_s} + \sum_{p<p_{\max}} \frac{1}{2\epsilon_p}$$

a_s **: s-wave scattering length**

Small U Large

Weak Coupling (BCS) Strong Coupling (BEC)

$-\infty$ $(k_F a_s)^{-1}$ $+\infty$

Strong-coupling theory

$$Z = \int D\bar{\psi} D\psi e^{-S(\bar{\psi},\psi)} \quad \blacktriangleright \quad Z = \int D\bar{\psi} D\psi \int D\bar{\Delta} D\Delta e^{-S(\bar{\psi},\psi,\bar{\Delta},\Delta)} \quad \blacktriangleright \quad Z = \int D\bar{\Delta} D\Delta e^{-S_{\text{eff}}(\Delta,\bar{\Delta})}$$

Hubbard-Stratonovich Trans. Integral out ψ

$\Delta(\omega)$ at $T = 0$: BCS-Leggett theory

Z is replaced by saddle point value:

$$Z = \int D\bar{\Delta} D\Delta e^{-S_{\text{eff}}(\Delta,\bar{\Delta})} \longrightarrow Z_{\text{MF}} \simeq e^{-S_{\text{eff}}(\Delta,\bar{\Delta})}$$

Free energy

$$\Omega_{\text{MF}} = -\frac{1}{\beta} \ln Z_{\text{MF}}$$

Gap equation

$$\frac{\partial \Omega_{\text{MF}}}{\partial \bar{\Delta}} = 0$$

Particle num. eq. $\longrightarrow$ Δ_{MF}, μ

$$N = -\frac{\partial \Omega_{\text{MF}}}{\partial \mu}$$

T_c : NSR theory

$$Z = \int D\bar{\Delta} D\Delta e^{-S_{\text{eff}}(\Delta,\bar{\Delta})}$$

$$\downarrow \quad \Delta_q = \Delta_{\text{MF}} + \underline{\eta_q}$$

$$Z \simeq \underline{Z_{\text{MF}}} \times \underline{Z_{\text{NSR}}}$$

saddle point fluctuation

$$Z_{\text{MF}} \simeq e^{-S_{\text{eff}}(\Delta,\bar{\Delta})} \qquad Z_{\text{NSR}} = \int D\bar{\eta} D\eta e^{-\frac{1}{2}\sum_q \bar{\eta}_q \Gamma_q^{-1} \eta_q}$$

Gap equation

$$\left. \frac{\partial \Omega_{\text{MF}}(\Delta)}{\partial \bar{\Delta}} \right|_{\Delta=0} = 0 \longrightarrow T_c, \ \mu$$

Particle num. eq.

$$N = N_{\text{MF}} + \underline{N_{\text{NSR}}}$$ fluctuation

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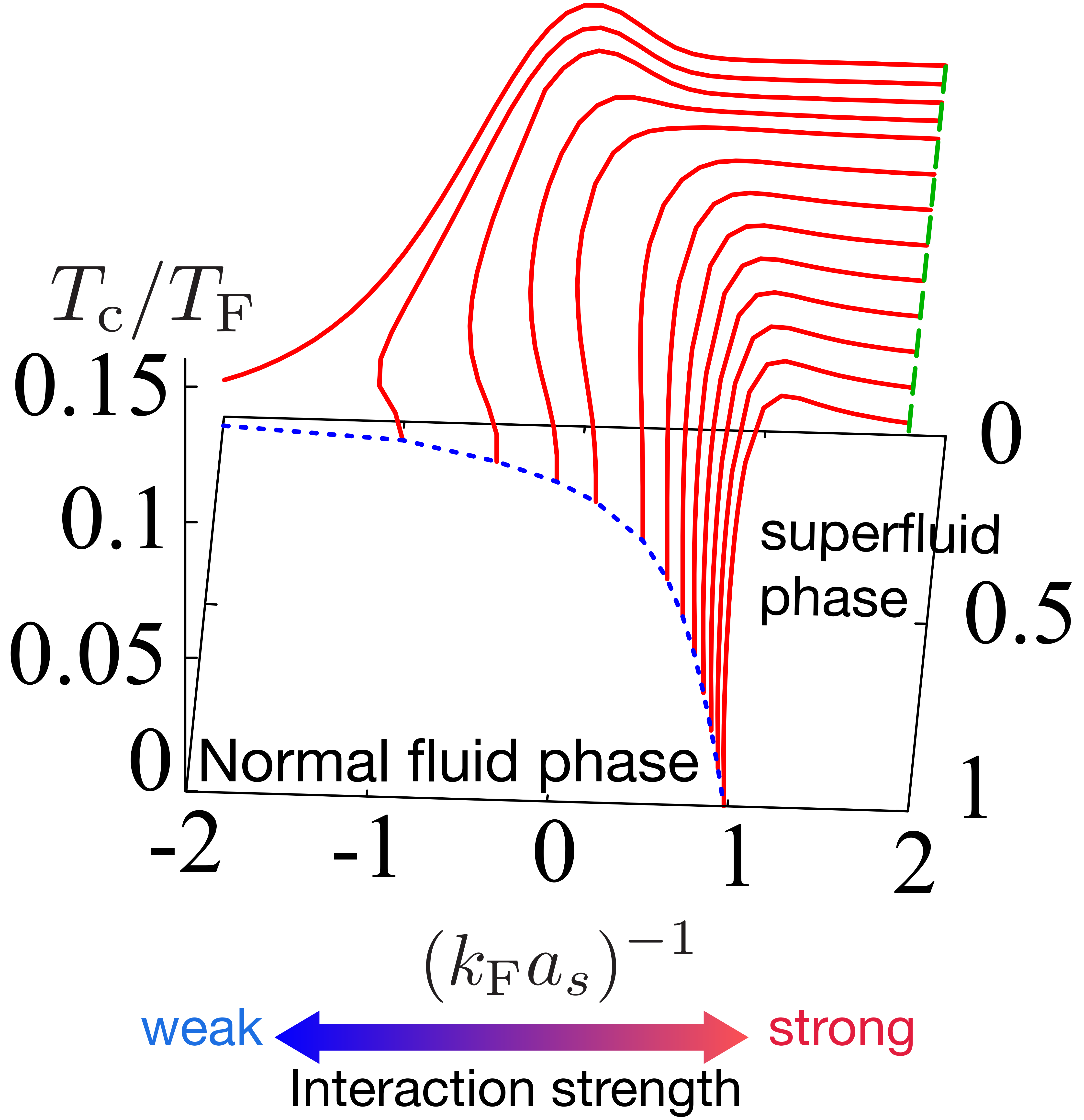
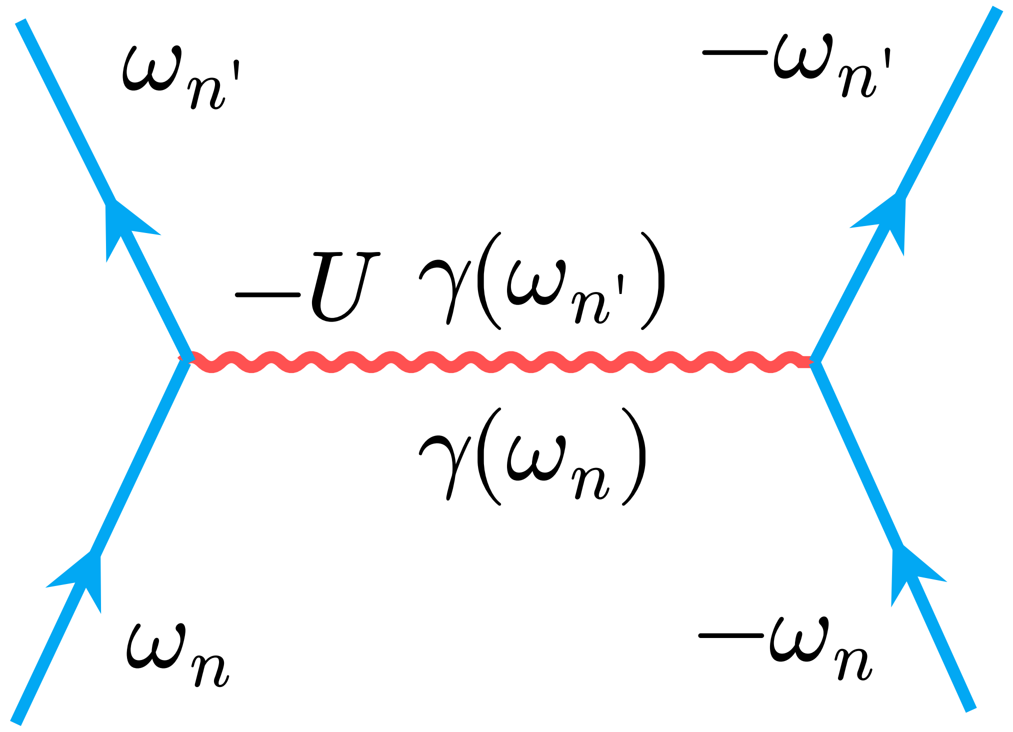
Superfluid phase transition temperature T_c

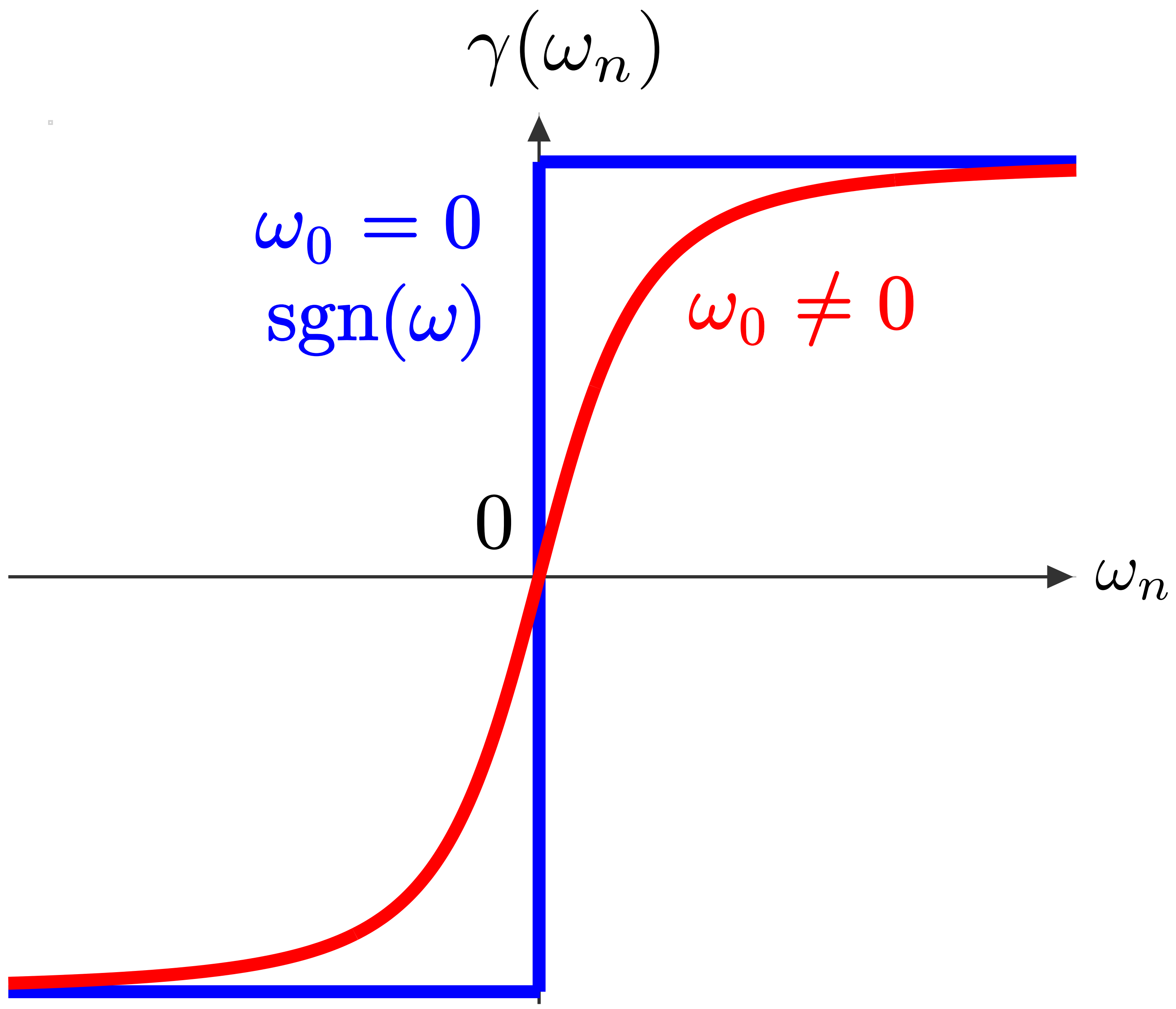


Frequency dependence

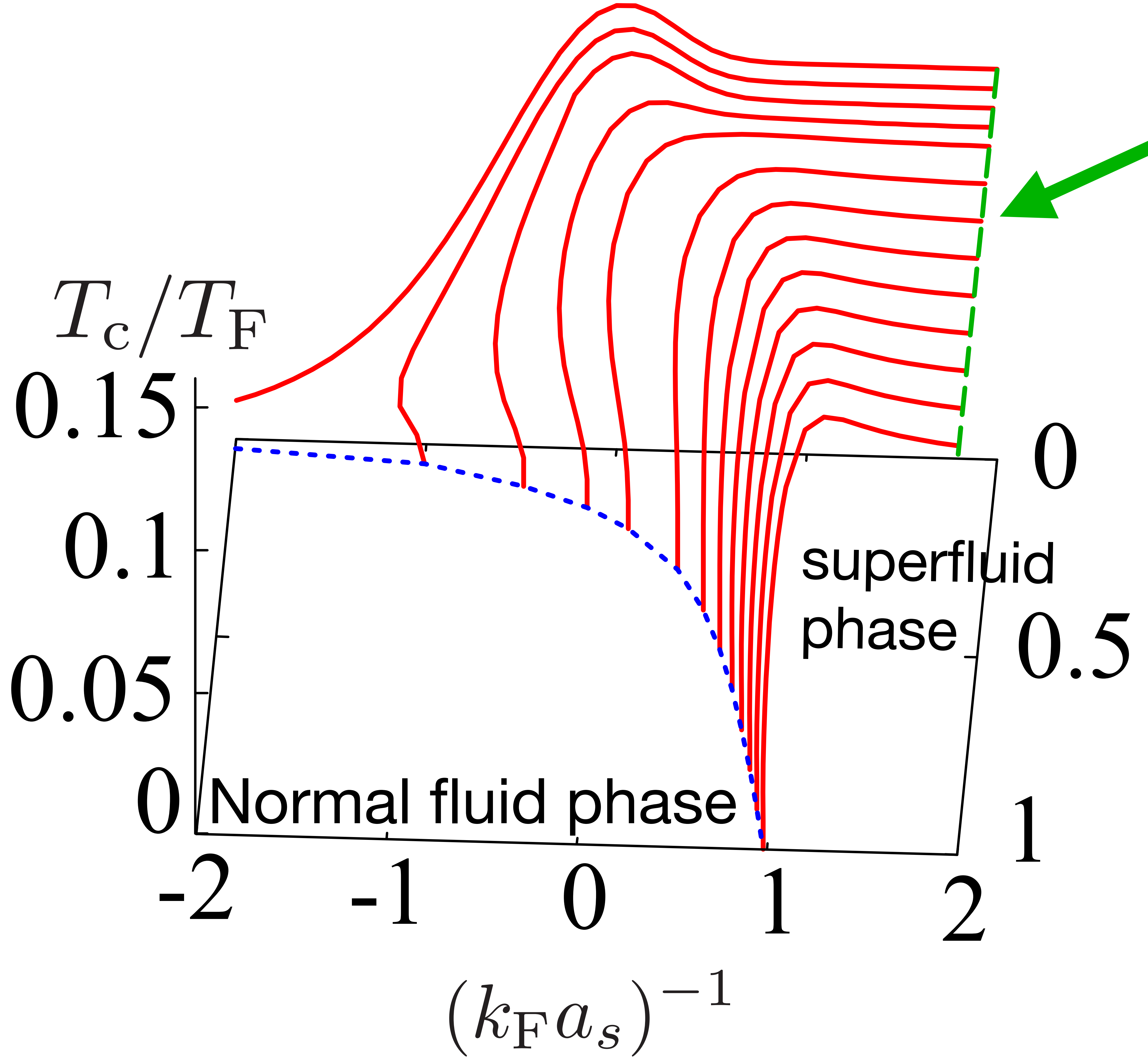
$$V_\mathrm{odd}(\omega_n,\omega_{n'}) = -U\gamma(\omega_n)\gamma(\omega_{n'})$$

$$\gamma_{\omega_n} = \frac{\omega_n}{\sqrt{\omega_n^2+\omega_0^2}}$$





Superfluid phase transition temperature T_{c}



weak ⟷ strong

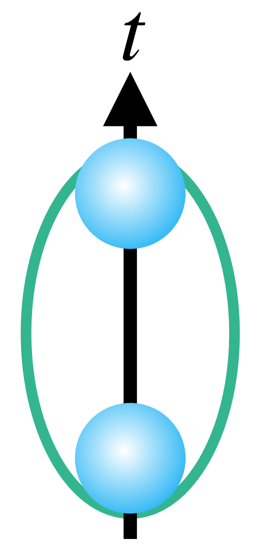
Interaction strength

$T_c = \underline{0.137 T_{\mathrm{F}}}$

$N = \dfrac{k_{\mathrm{F}}}{6\pi^3}$: Total num. of atom

BEC transition temperature of $N/2$ molecules

Molecule formed by two Fermi particles at **different times** can be regarded as **boson** in the strong coupling limit



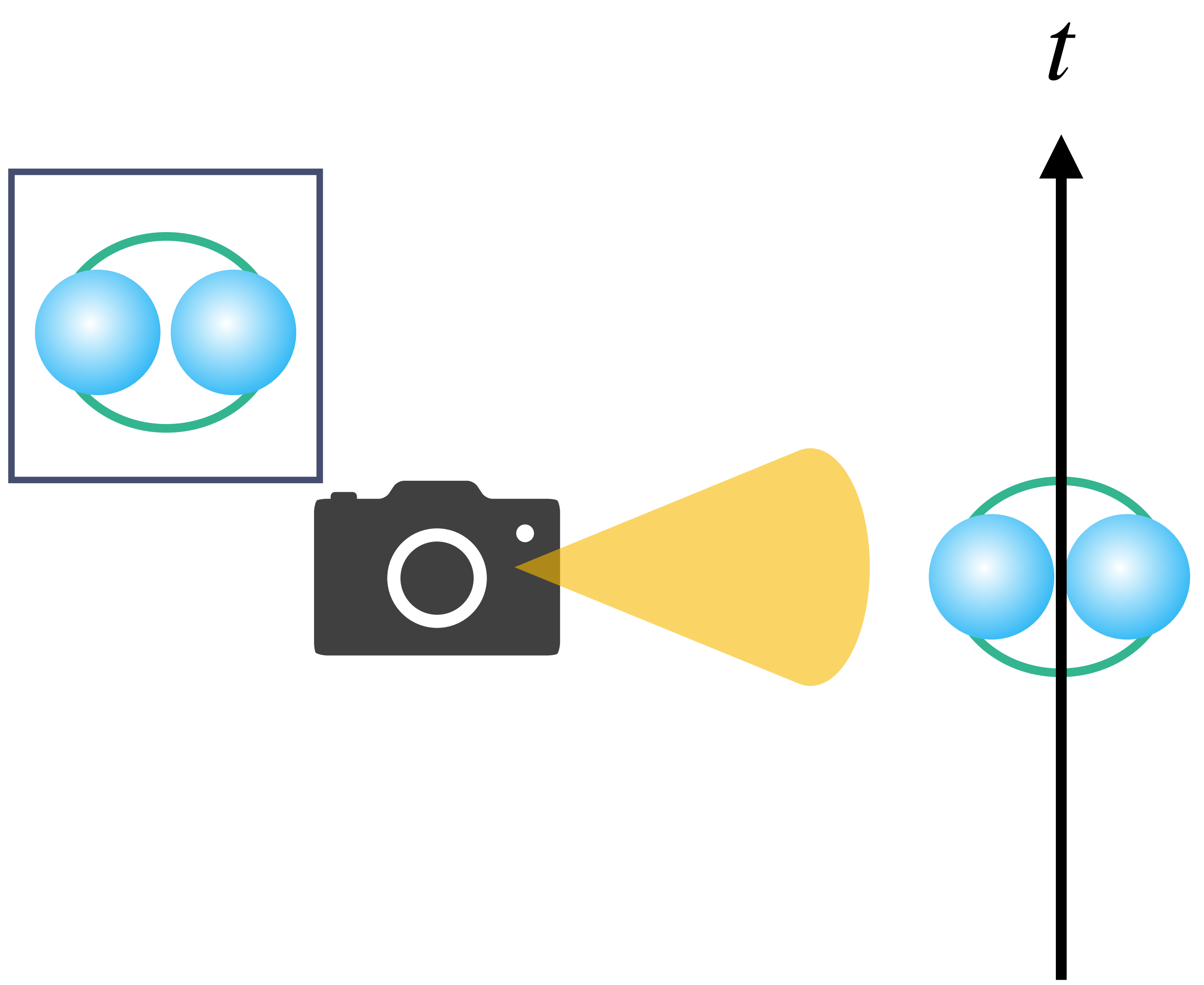
Molecular formation by Fermi particles at different times

Can Cooper pairs formed by two Fermi particles at **different times** be regarded as **boson**?

Yes

even-frequency superfluidity

"Molecules" formed by particles at the same time



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→boson

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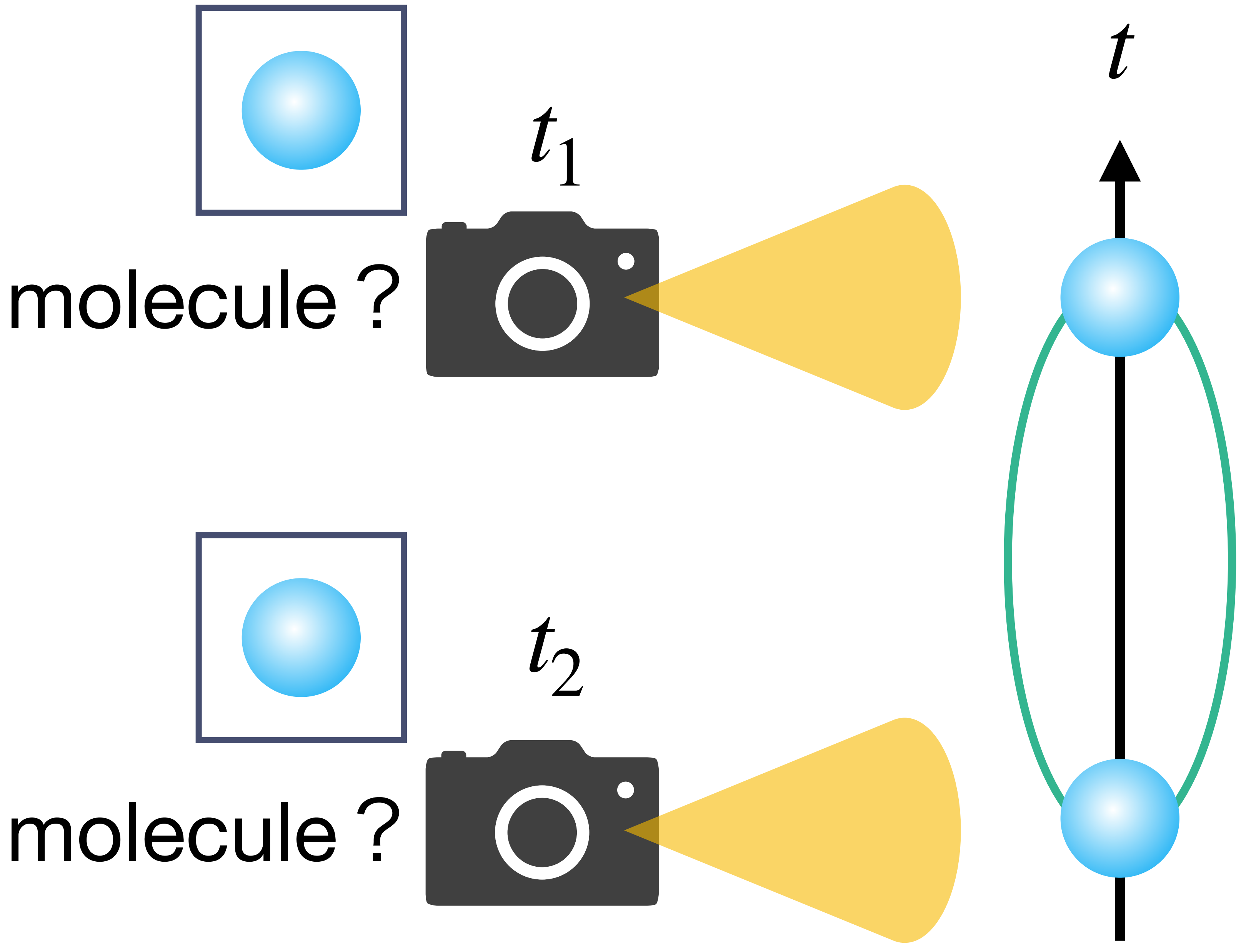

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→boson?

Cooper pair wave-function

Anomalous Green's function

$$F^{\mathrm{T}}(\boldsymbol{r},t) = -i\langle T\psi(\boldsymbol{r},t)\psi(0,0)\rangle$$

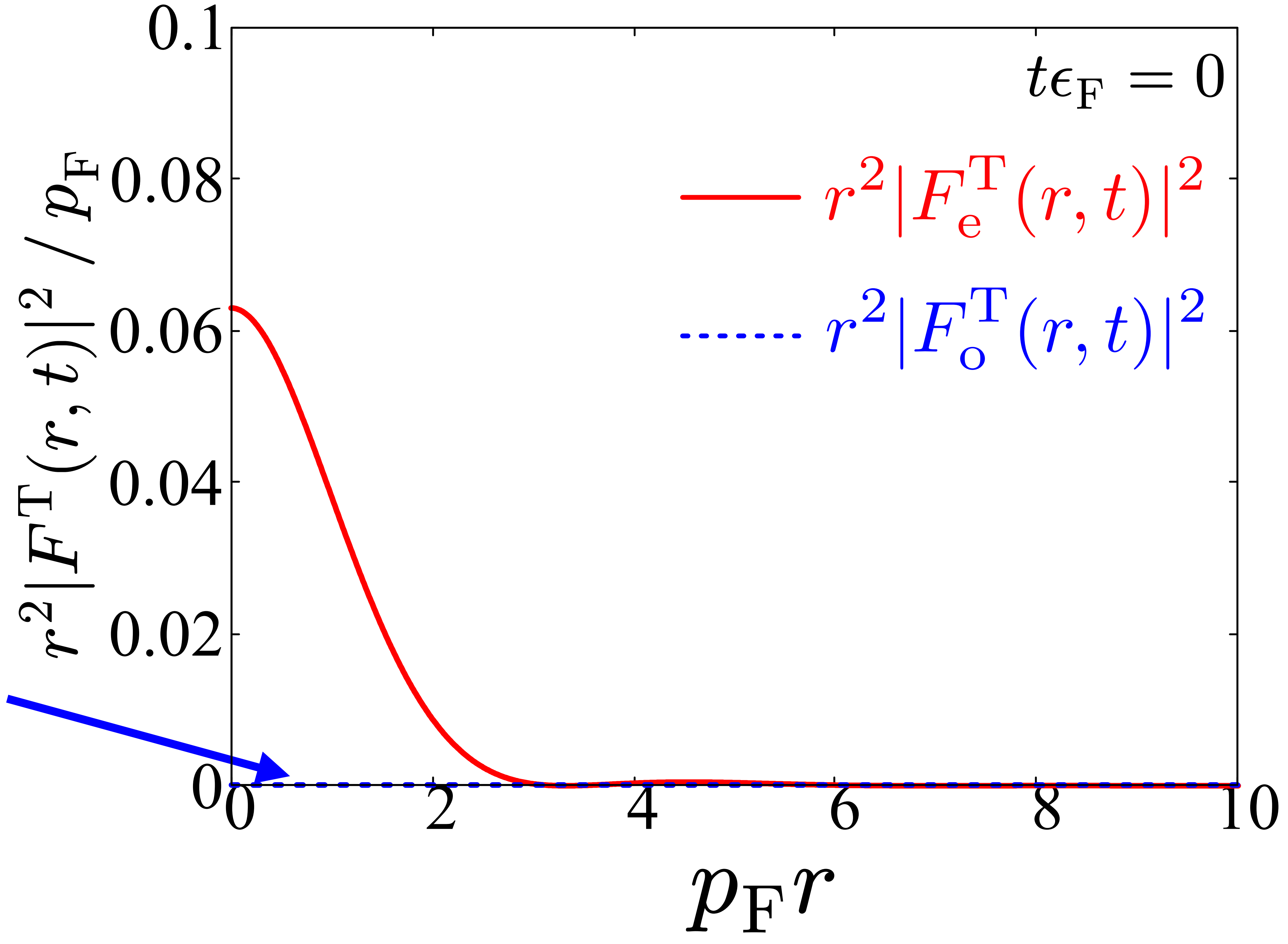
T : Time ordering operator

ψ : Fermion annihilation operator

can be viewed as the wave function of particle 1 seen from particle 2 in a Cooper pair.

$F_{\mathrm{e}}^{\mathrm{T}}(r,t)$: even-frequency $F_{\mathrm{o}}^{\mathrm{T}}(r,t)$: odd-frequency $\omega_0 = 0$

$(k_{\mathrm{F}}a_s)^{-1} = 0$



$F_{\mathrm{o}}^{\mathrm{T}}(r,t)$ odd function of *t*

$F_{\mathrm{o}}^{\mathrm{T}}(r,t=0) = 0$

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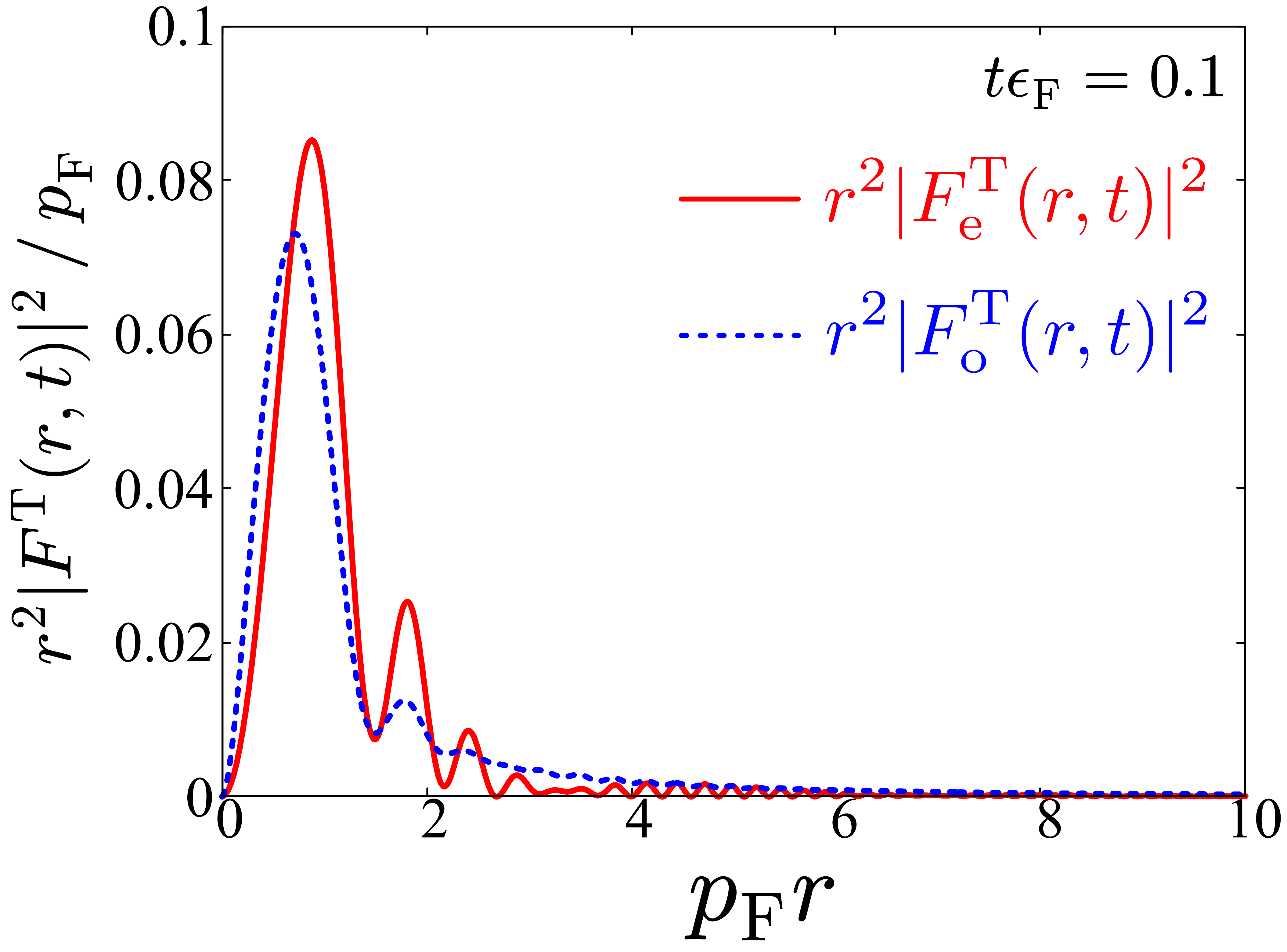

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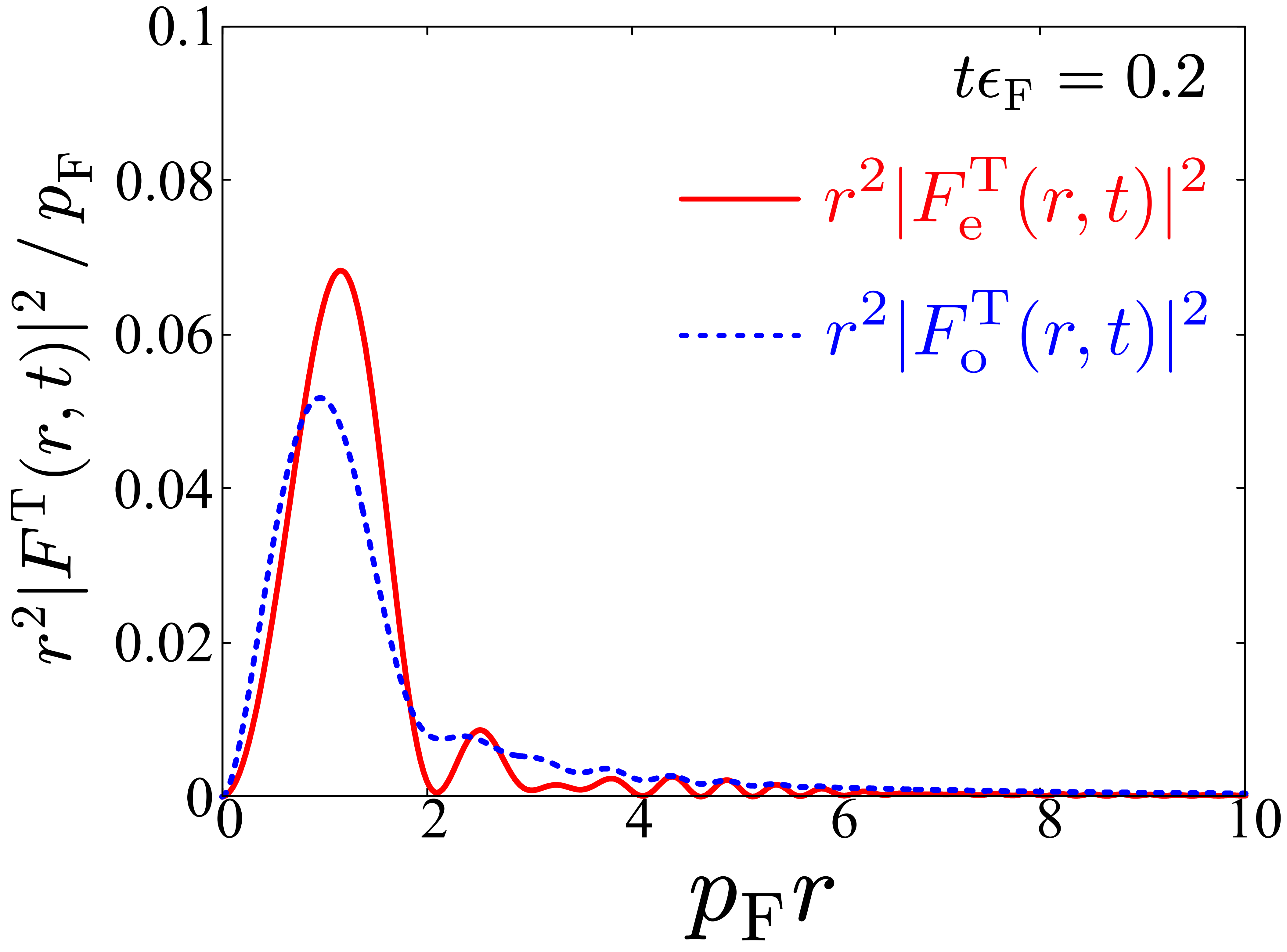

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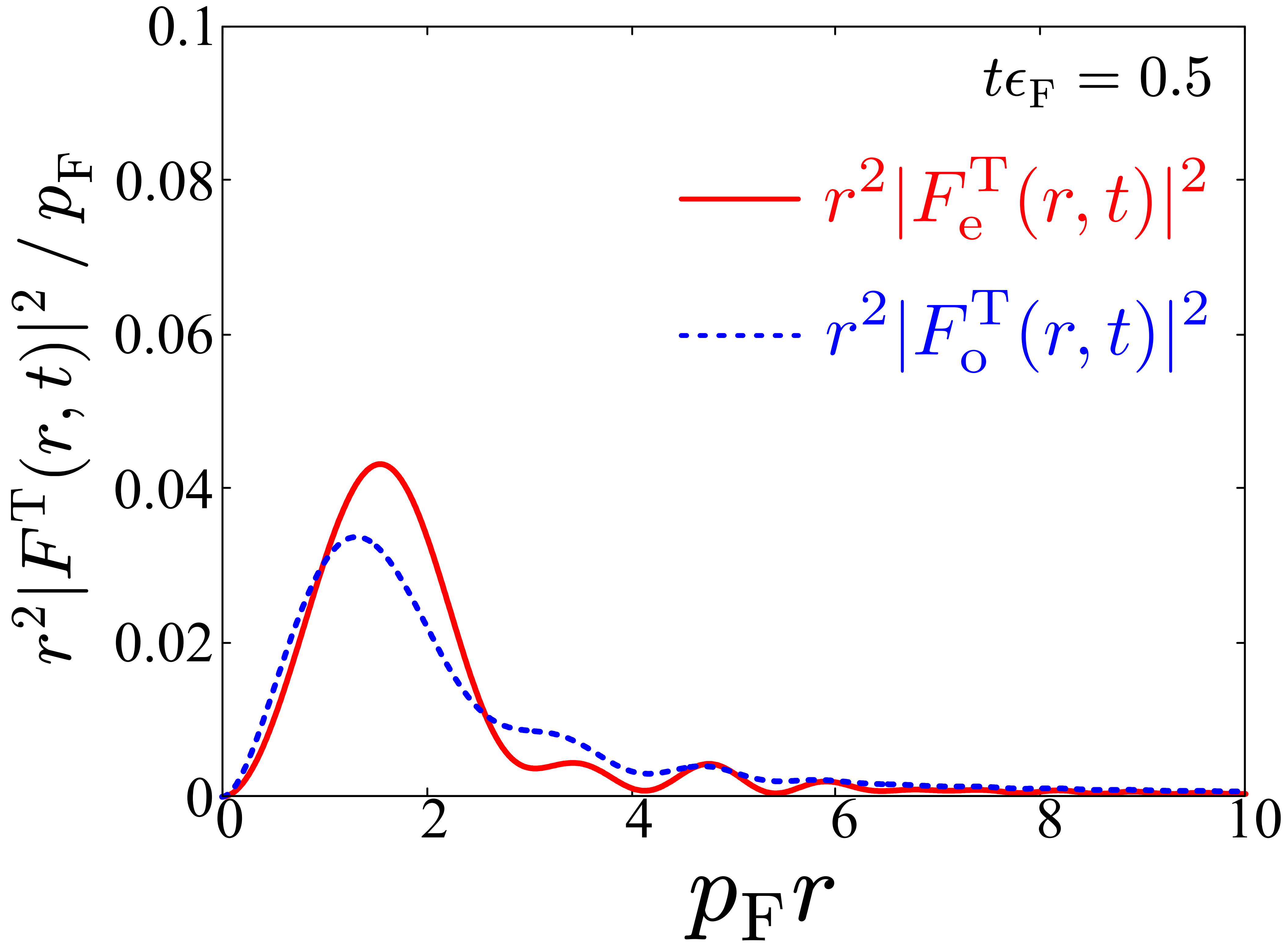

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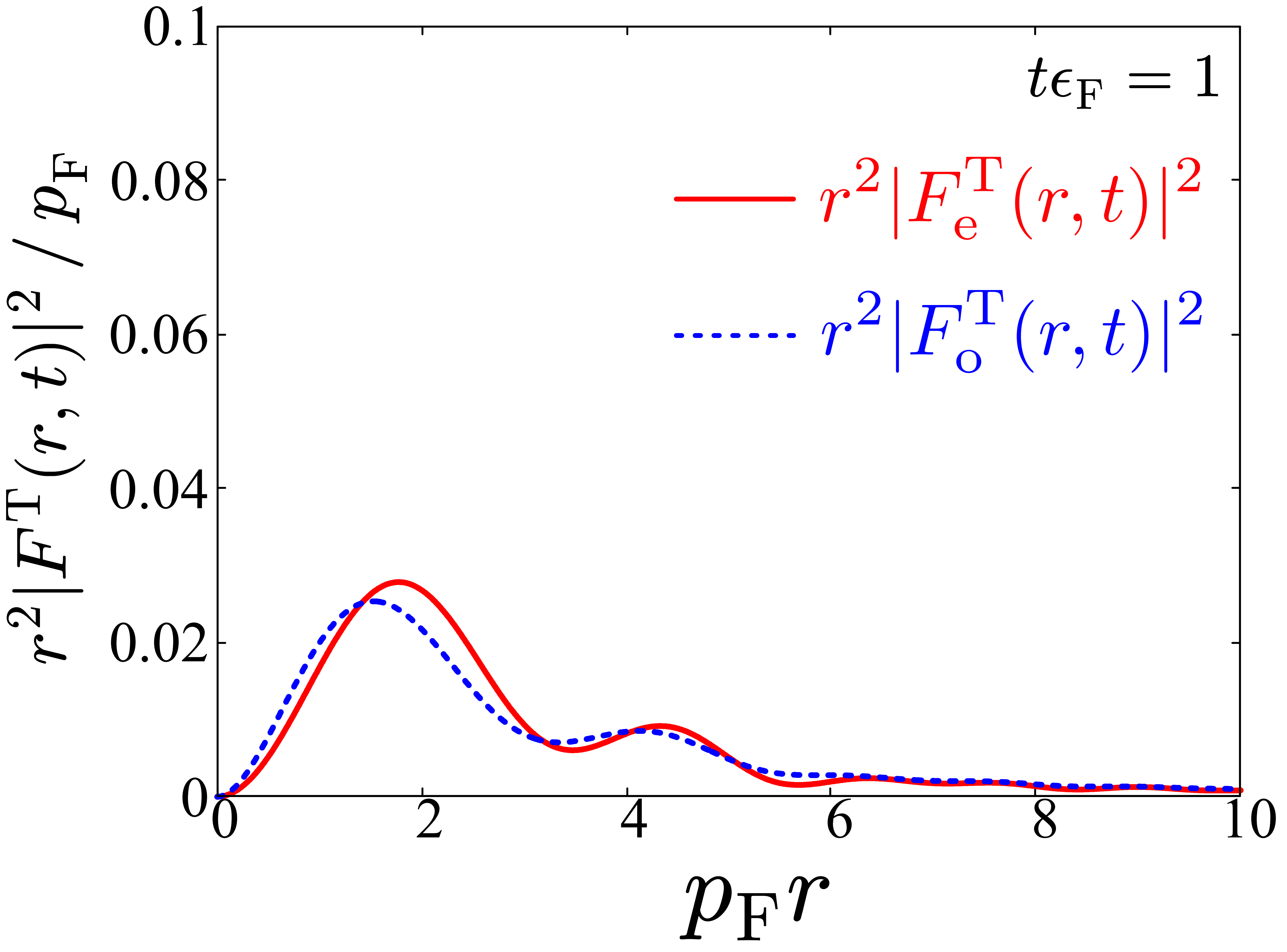

Cooper pair wave-function

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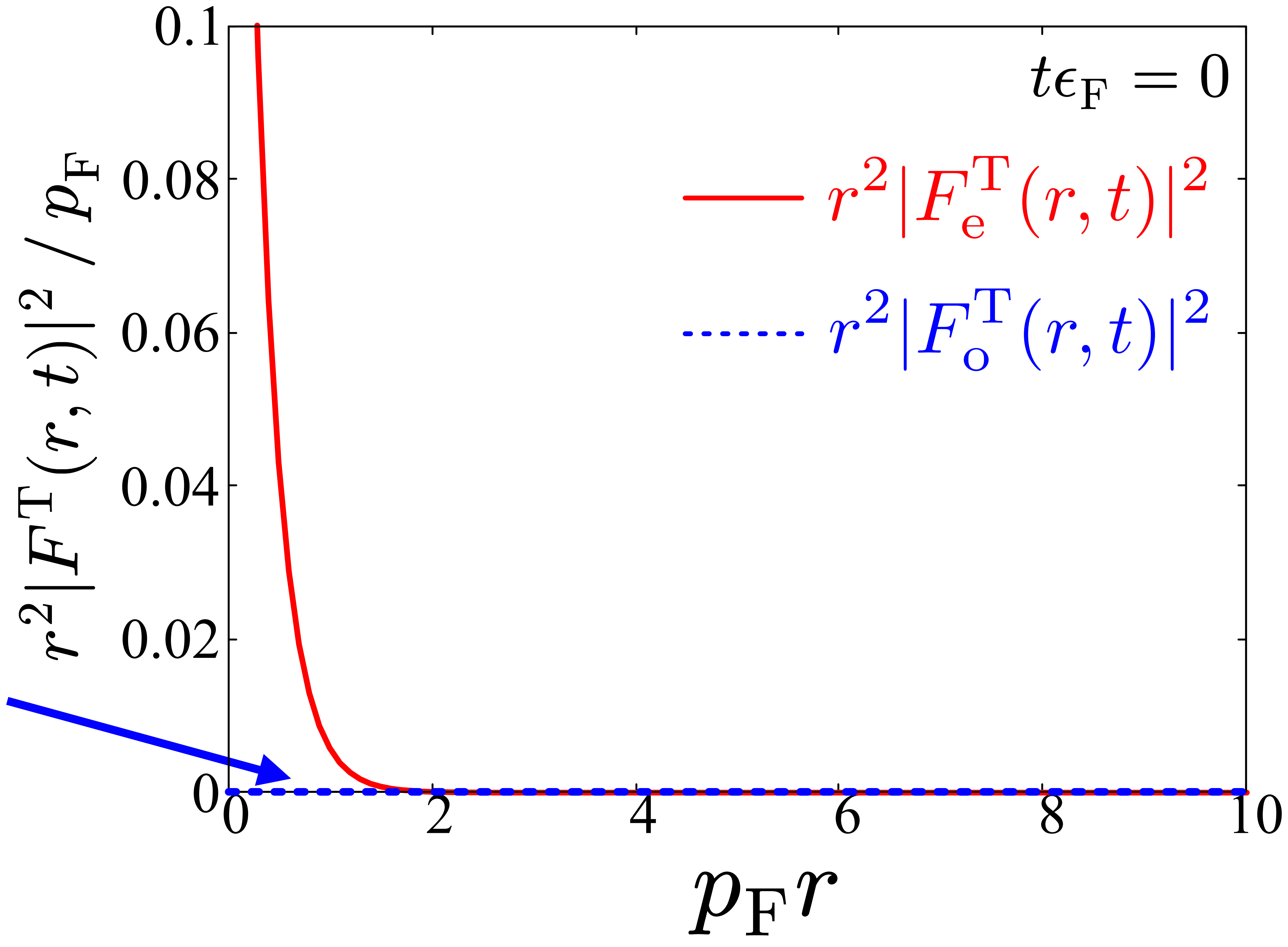
$F_{\mathrm{e}}^{\mathrm{T}}(r,t)$: even-frequency $F_{\mathrm{o}}^{\mathrm{T}}(r,t)$: odd-frequency $\omega_0 = 0$

$(k_{\mathrm{F}}a_s)^{-1} = 2$



$F_{\mathrm{o}}^{\mathrm{T}}(r,t)$ is odd function of t

$F_{\mathrm{o}}^{\mathrm{T}}(r,t=0) = 0$

Cooper pair wave-function

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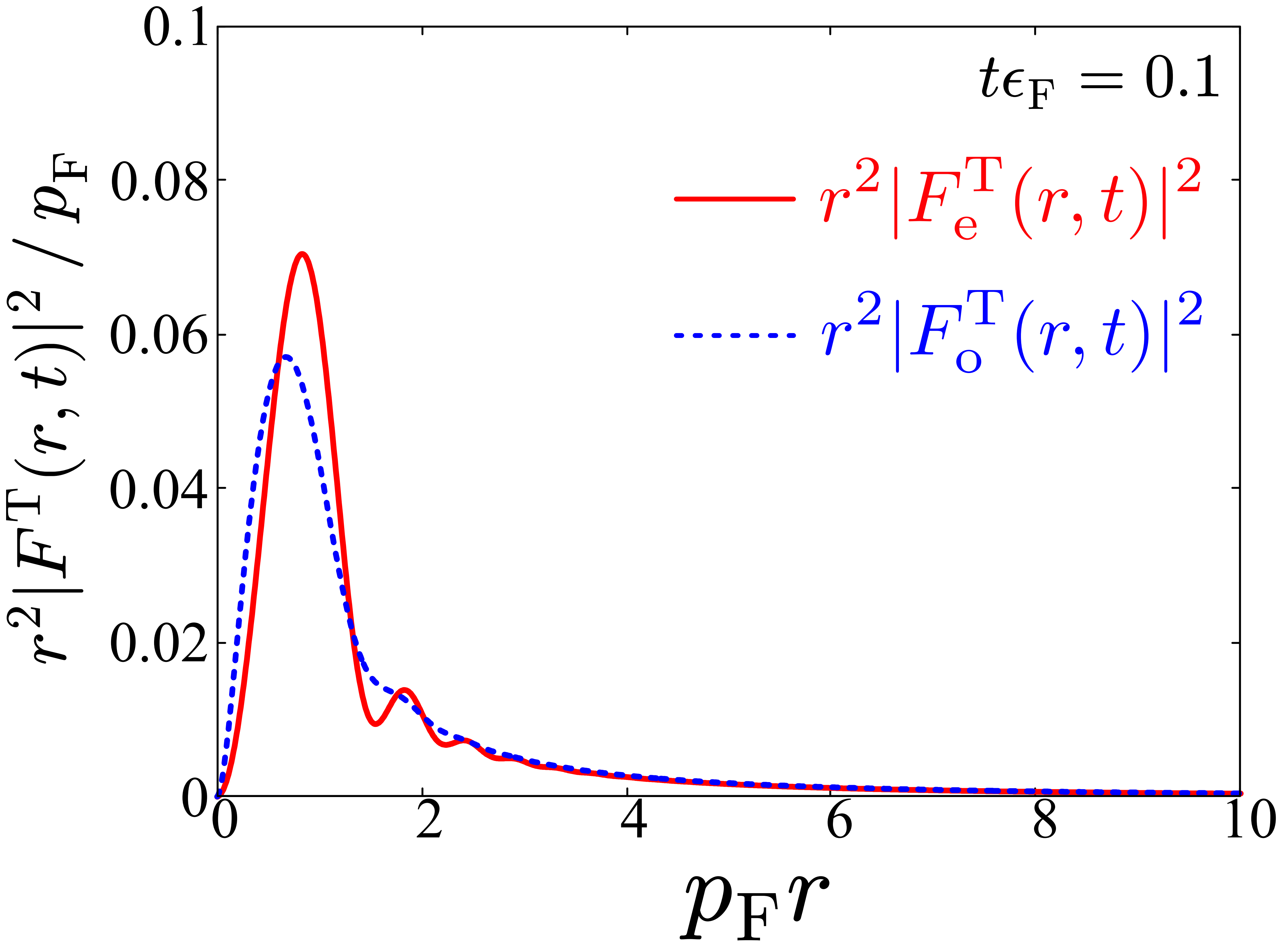

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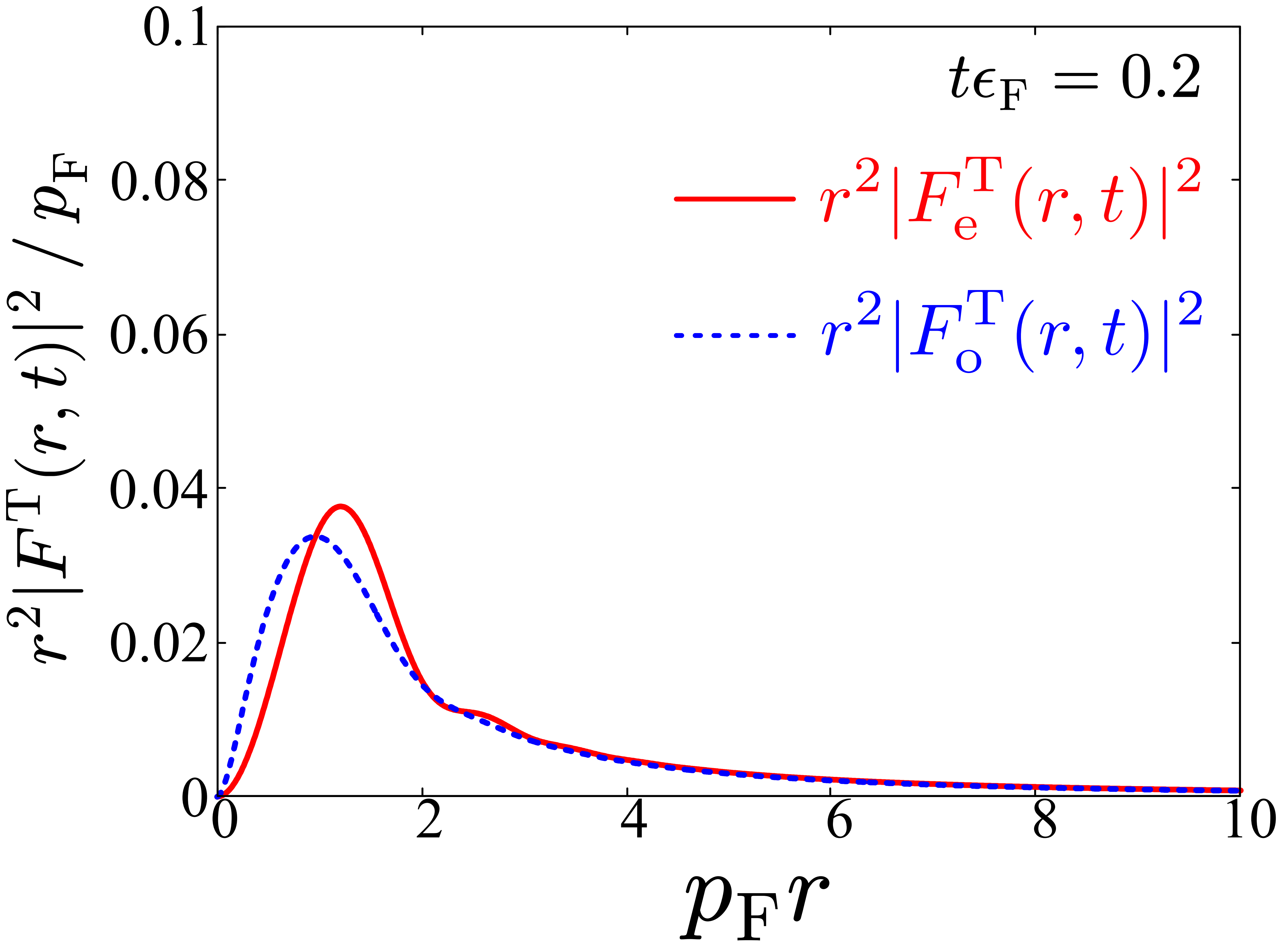

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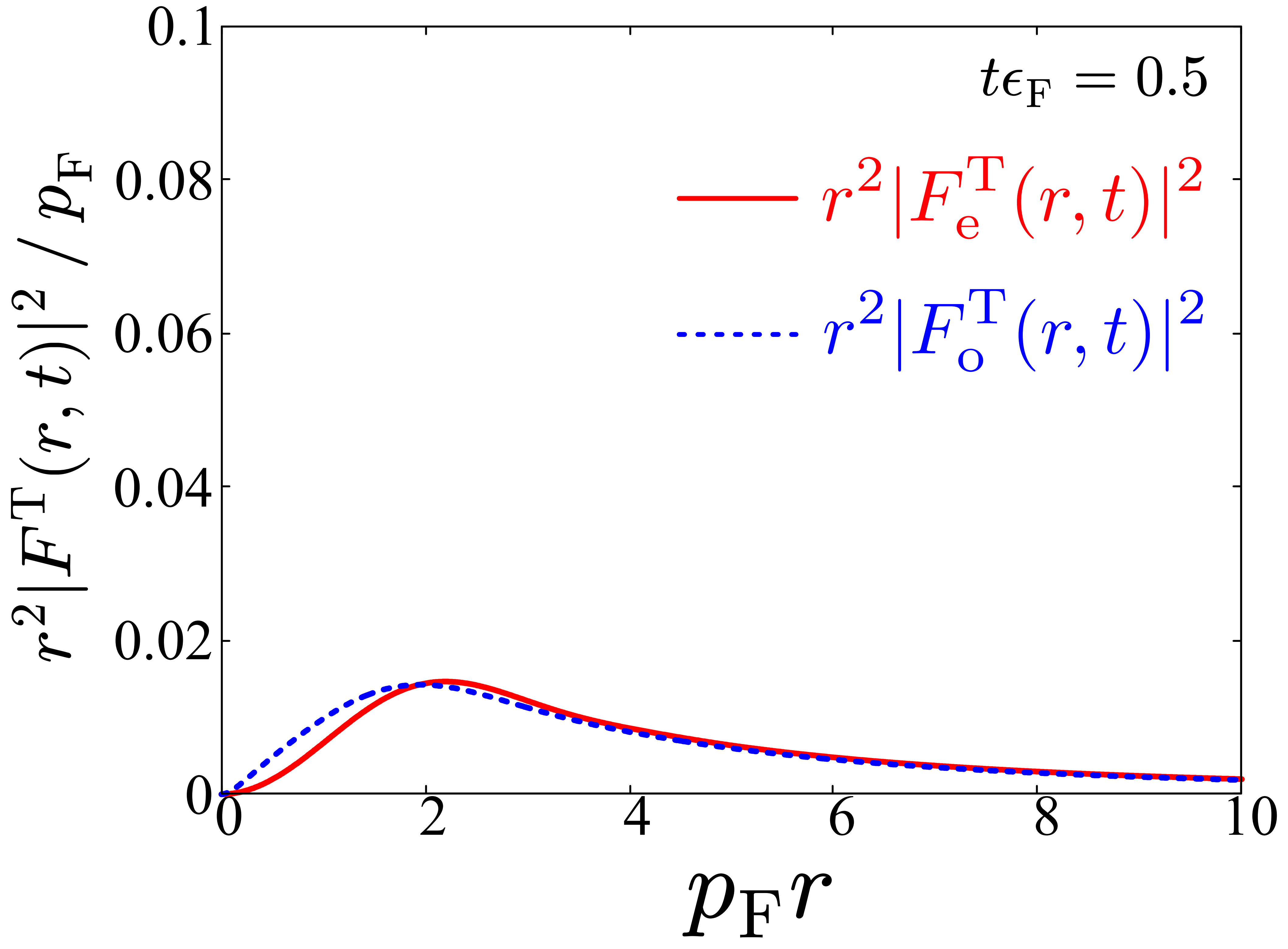

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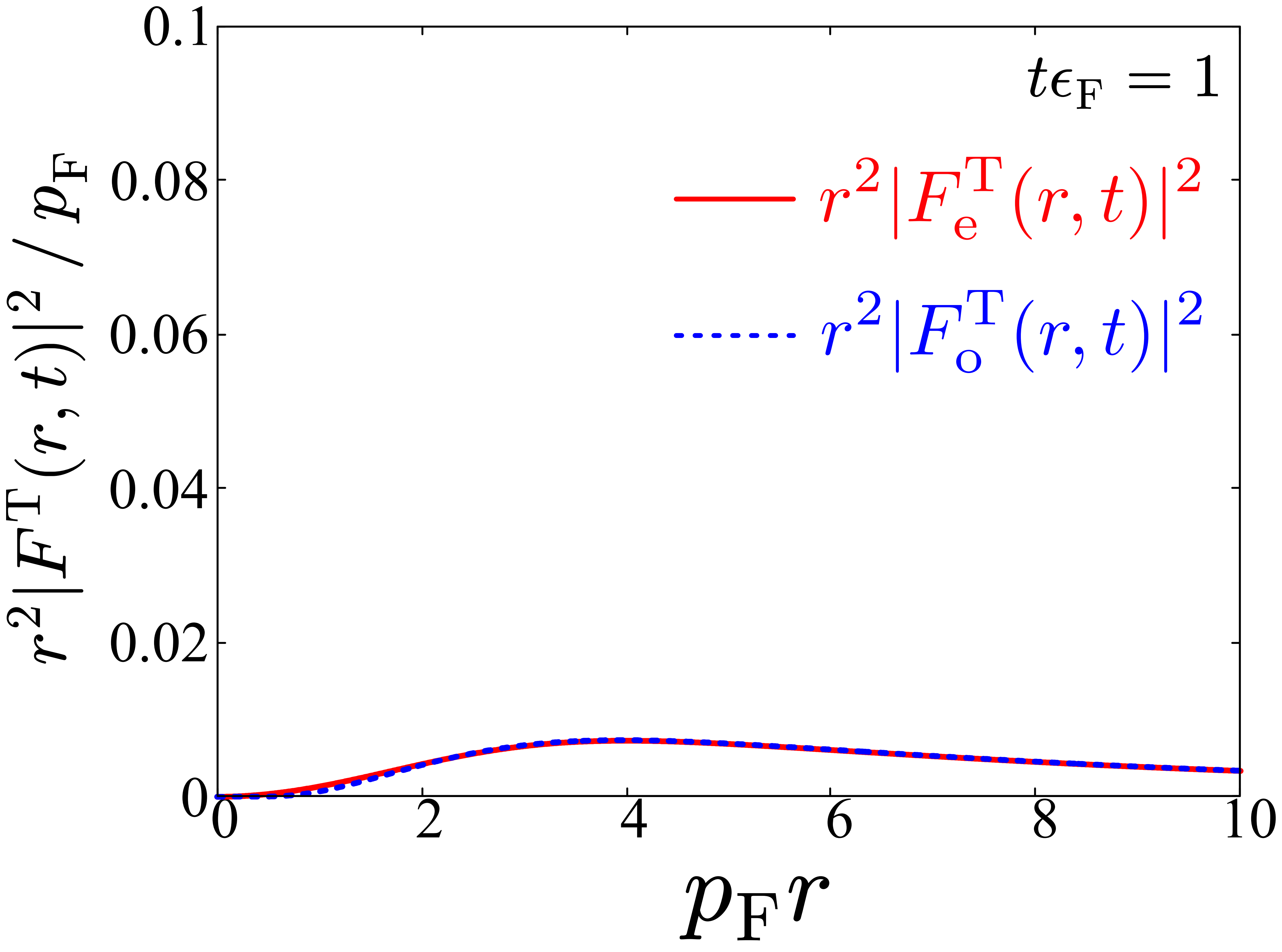
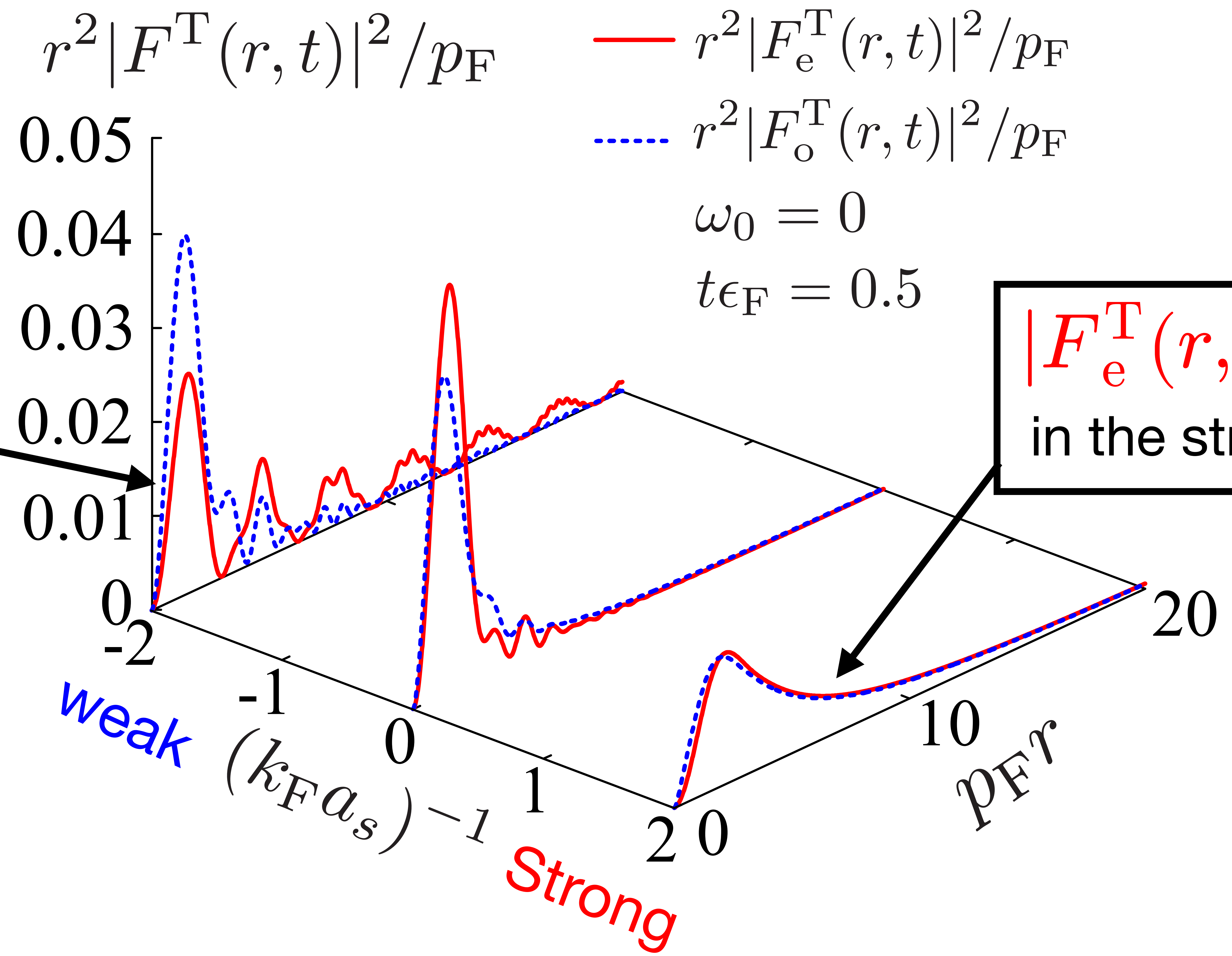


$(k_{\mathrm{F}}a_s)^{-1} = 2$

$F^{\mathrm{T}}(r,t)$: Cooper pair wave-function

$|F_{\mathrm{e}}^{\mathrm{T}}(r,t)| \neq |F_{\mathrm{o}}^{\mathrm{T}}(r,t)|$
in the weak-coupling region

$F_{\mathrm{even}}^{\mathrm{T}}(r,t)$: spatially spreads
||
Size of the Cooper pair is large



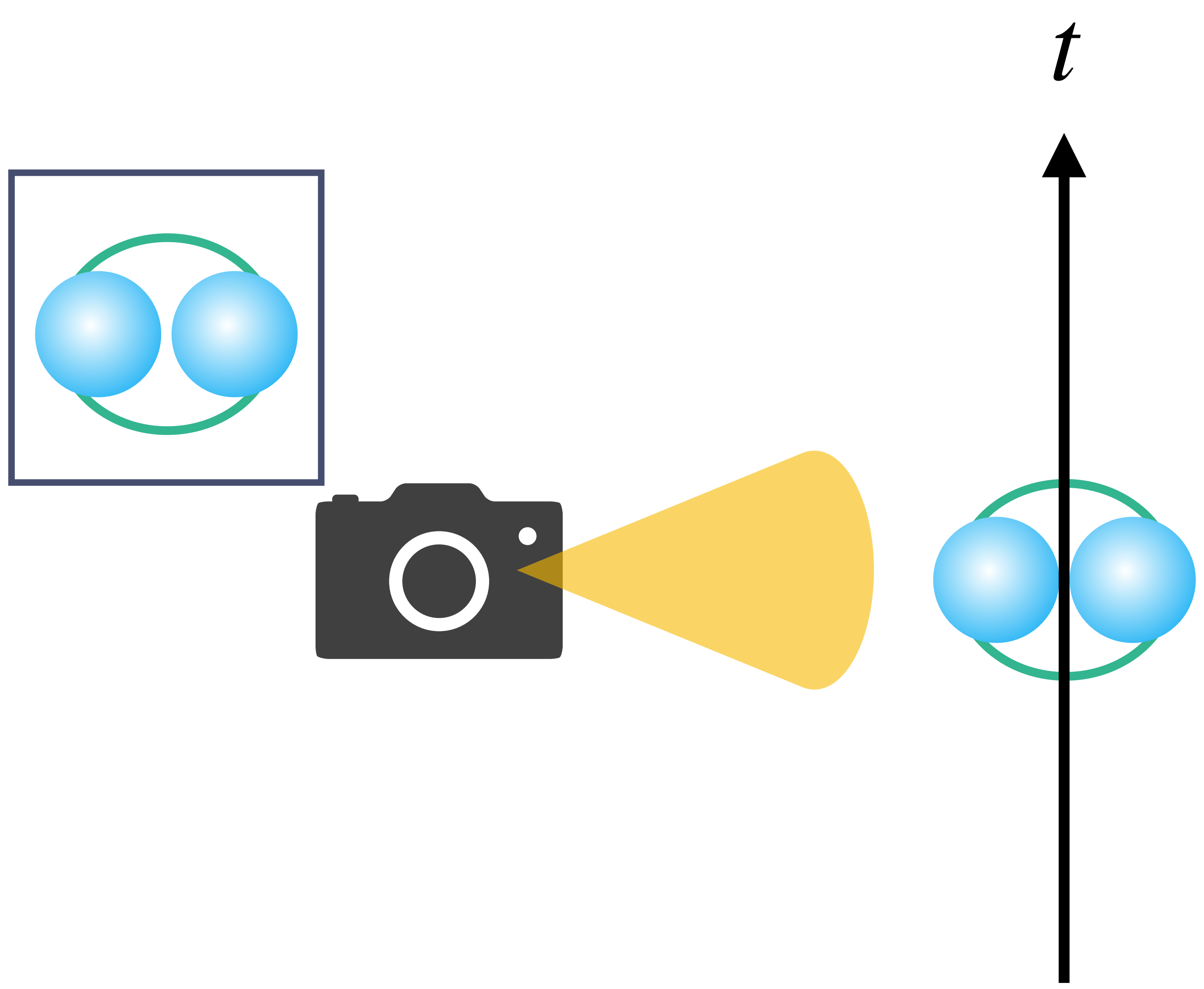
$|F_{\mathrm{e}}^{\mathrm{T}}(r,t)| = |F_{\mathrm{o}}^{\mathrm{T}}(r,t)|$
in the strong coupling region

Conclusion

Cooper pairs formed by two Fermi particles at **different times** can be regarded as **boson.**

even-frequency superfluidity

"Molecules" formed by particles at the same time



When you take a snapshot...
There is one molecule in the picture.

→boson

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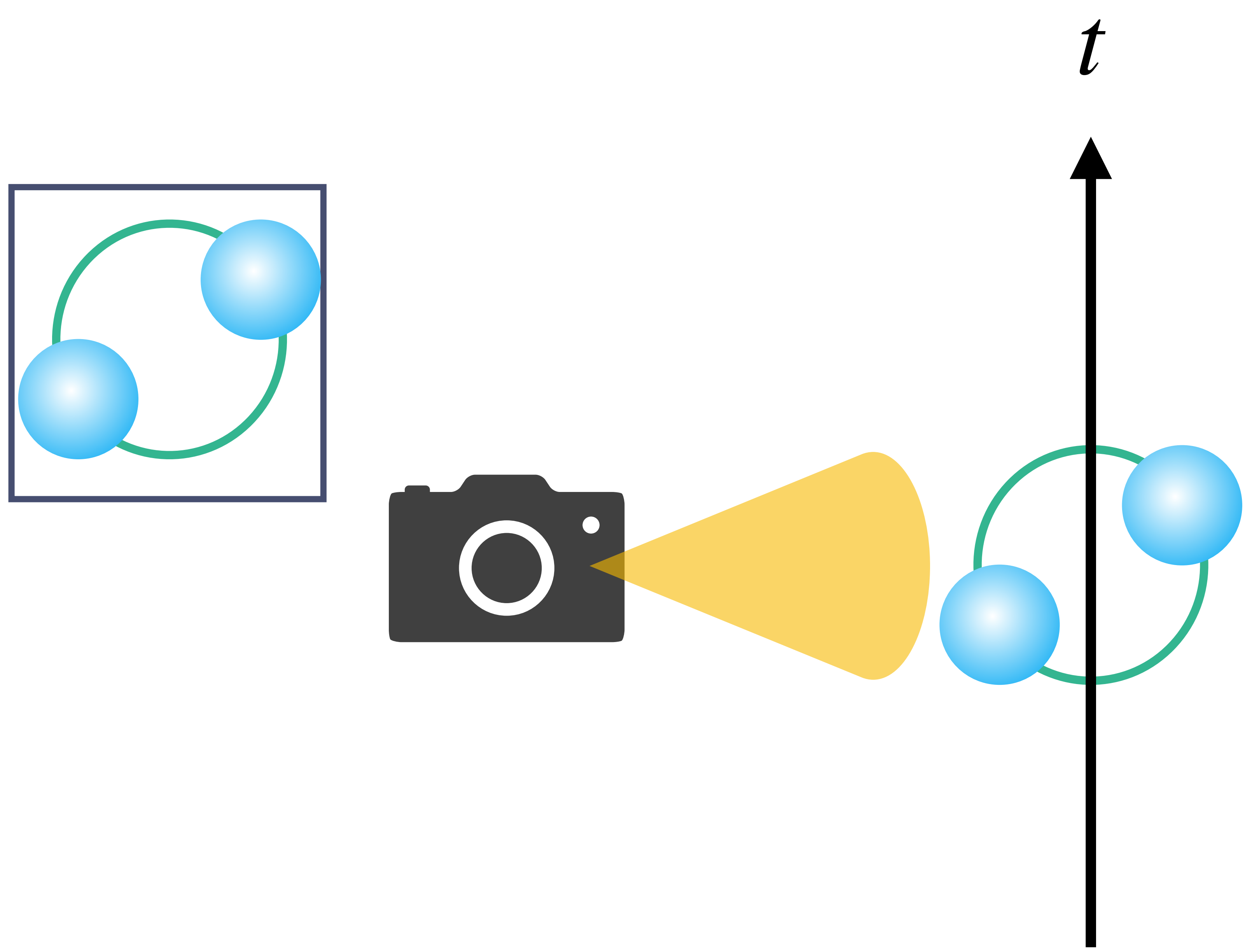

Two Fermi particles in the same position
with a slight time difference

→boson

Summary

We construct a strong-coupling theory
for odd-frequencysuperfluidity.

✤T_c : superfluid phase transition temperature

T_c can be understood as a
"BEC temperature of tightly bound molecular boson"
in the strong-coupling region.

✤$F^\mathrm{T}(r, t)$: anomalous Green's function
which is related to the Cooper pair wave function.

$F^\mathrm{T}(r, t)$ in the odd-frequency case coincides with
that in the even-frequency case in the strong coupling limit
when $t \neq 0$.