Cluster of Quarks in Hadrons

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第８回クラスター階層領域研究会

大阪大学（オンライン）

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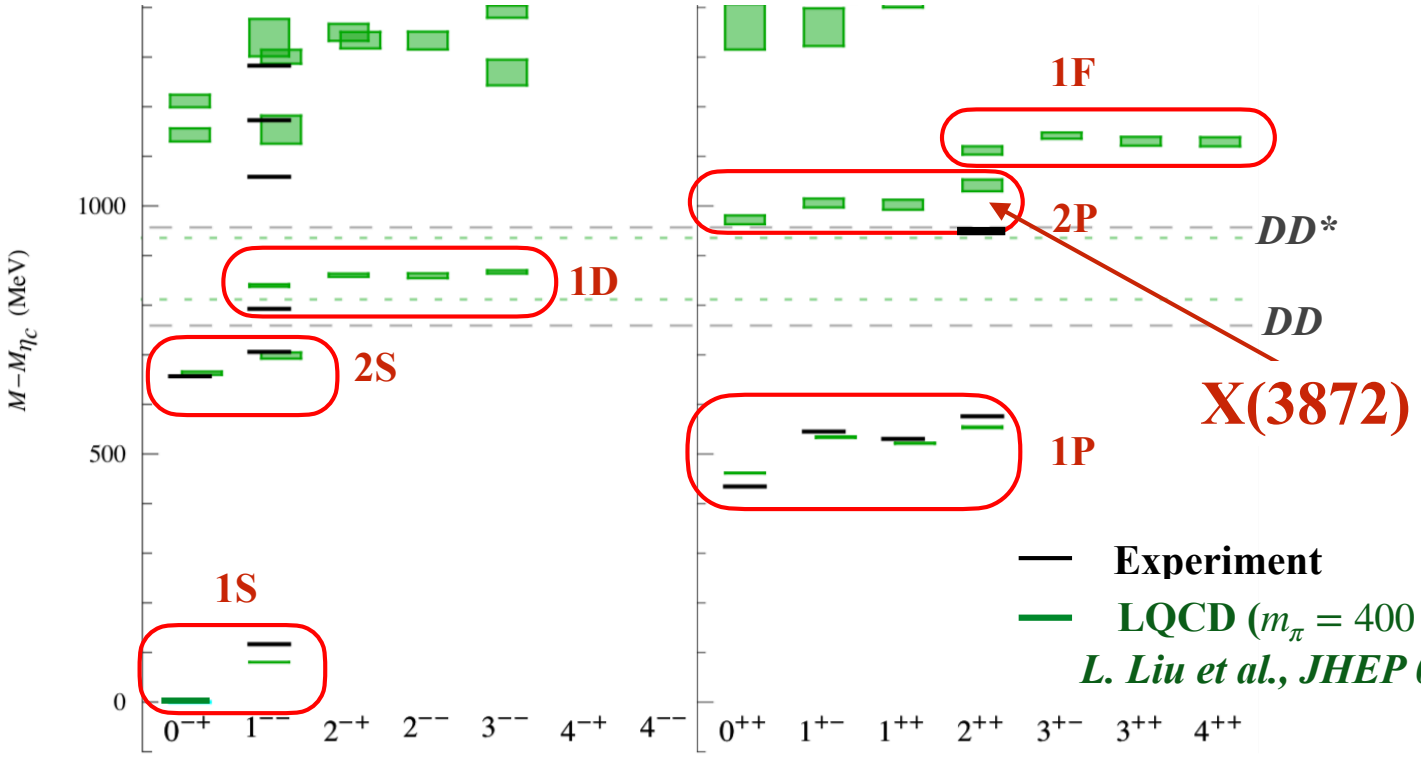
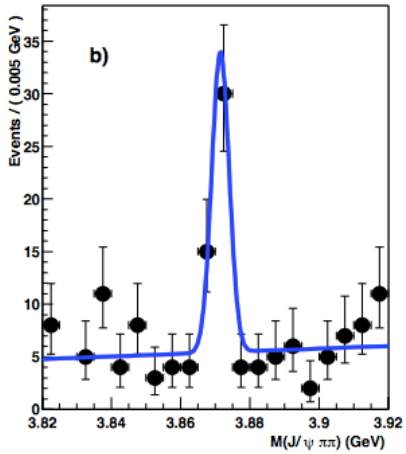
New Hadrons

- **Charmonium spectrum**





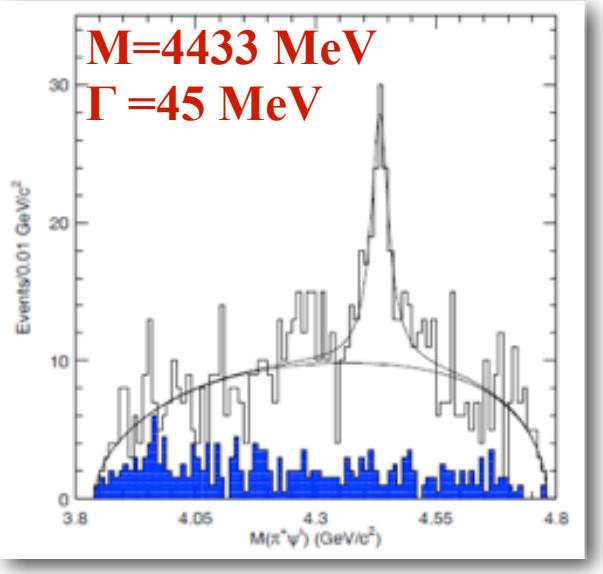
Belle, PRL 91 (2003)

- **X(3872) : $c\bar{c}$ and $c\bar{c}(u\bar{u} + d\bar{d}) = D\bar{D}^*$ are mixed.**
- **Emerging a new hierarchy by adding a pair of $q\bar{q}$.**

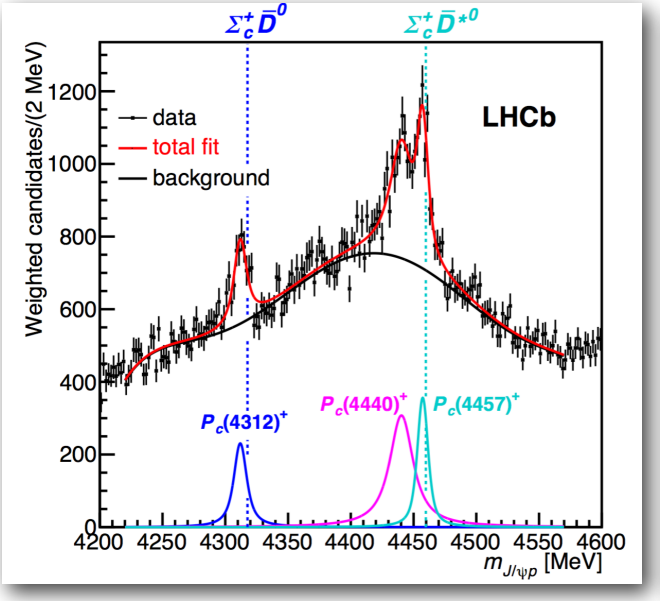
New Hadrons

- **Numerous multi-quark resonances are observed!**

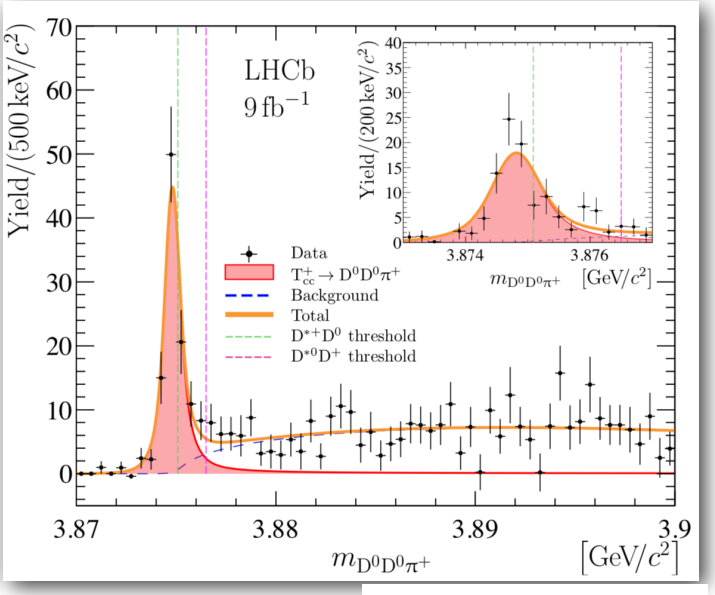
$Z_c^+(4430)$ *Belle*
PRL 100 (2008) 142001

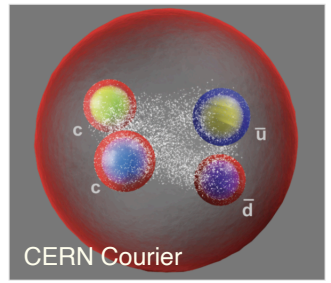


P_c (4312) (4440) (4457) *LHCb*
PRL 115 (2015), PRL 122 (2019)



T_{cc} *LHCb*
Nature Phys., 18 (2022)





- **Clear effect of the hadronic thresholds.**
- **But maybe more ingredients in deeper level.**
- ***It is difficult to manage transitions between Fock spaces.***

House of composite hadrons

- **1st floor: QCD quarks and gluons (ex. Lattice QCD)**

  They do not give intuitive picture for

  – quantum numbers of the low-lying states

  – why multi-quark states are rare?

- **2nd floor: Quasi-Particles**

  (Effective Degrees of Freedom)

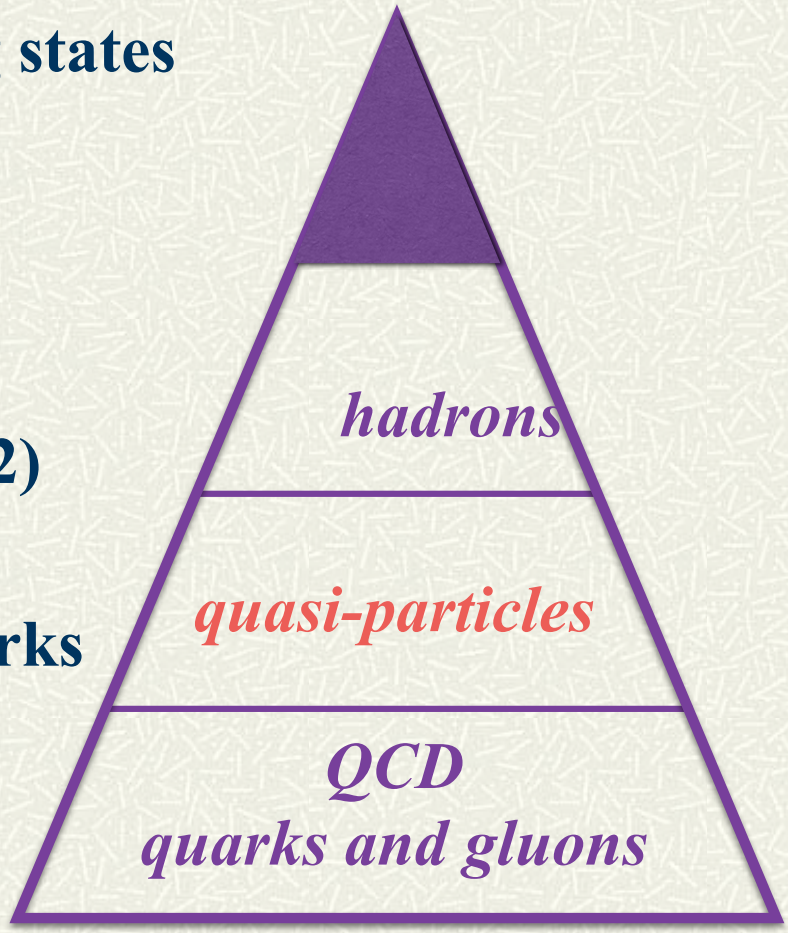
- **Colored QP (201) and White QP (202)**

  colored QP in compact hadrons

  heavy baryons, tetraquarks

  white QP in molecular hadrons

  $\Lambda(1405) = N\bar{K}$



Room 201: colored quasiparticle (CQP)

- **Constituent quark** with spin 1/2, color 3 and flavor

  Mass ~ 330 MeV (u,d), 550 MeV (s), 1500 MeV (c)

- **Diquark**

  diquark correlation, clusters in hadrons

  [ud] (0^+, I=0) : good diquark, mass ~ 500-600 MeV

- **Constituent gluon**

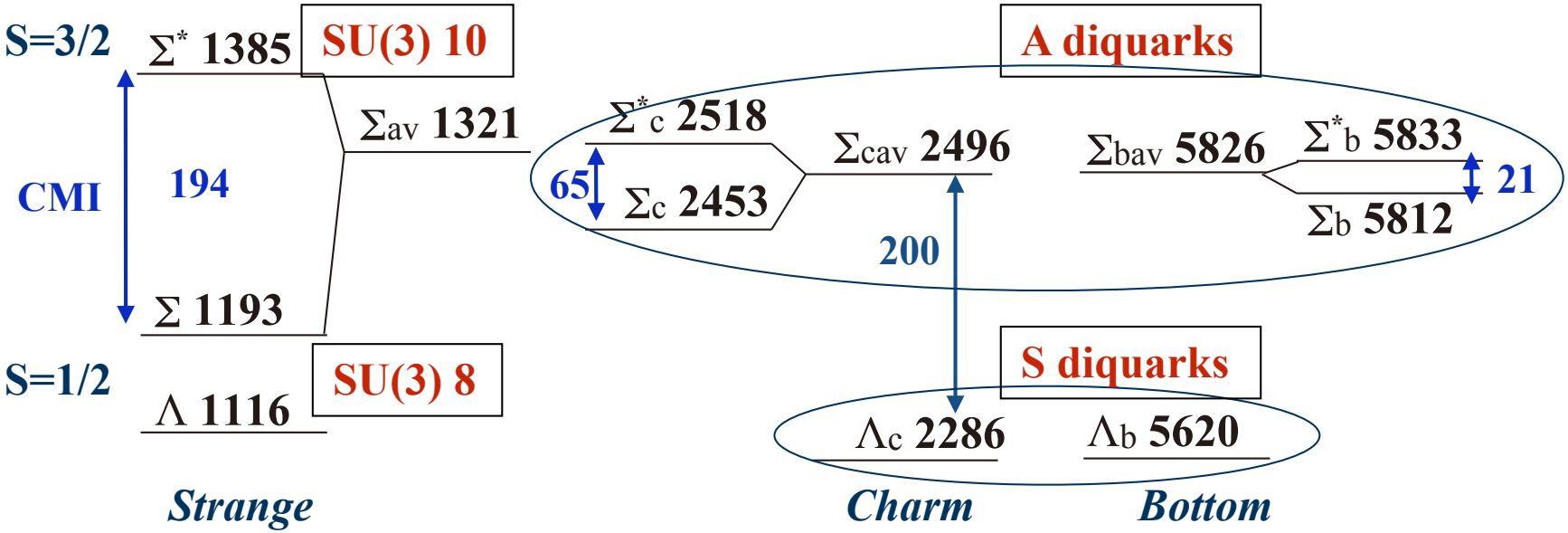
  explicit gluon degrees of freedom: exotic mesons ($J^{\pi}=1^{-+}$, 0^{+-})

  Mass ~ 1 GeV (string vibration modes)

Diquarks

Makoto Oka (ASRC, JAEA & Nishina Center, RIKEN)

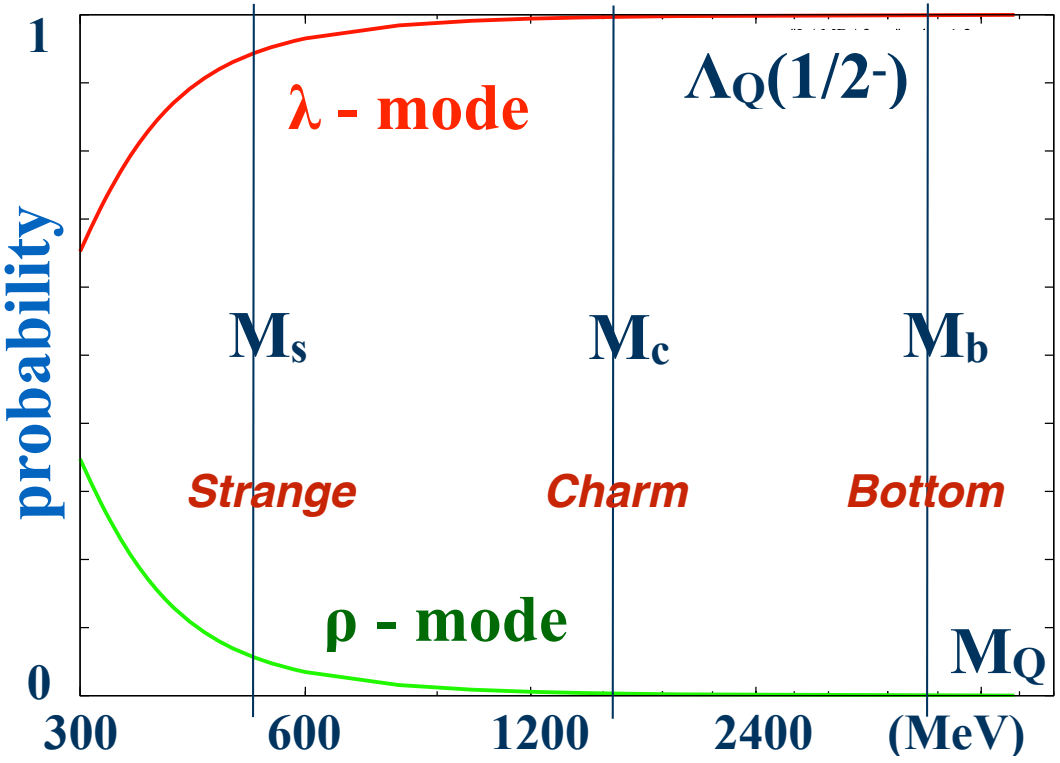
Diquarks in Heavy Baryons

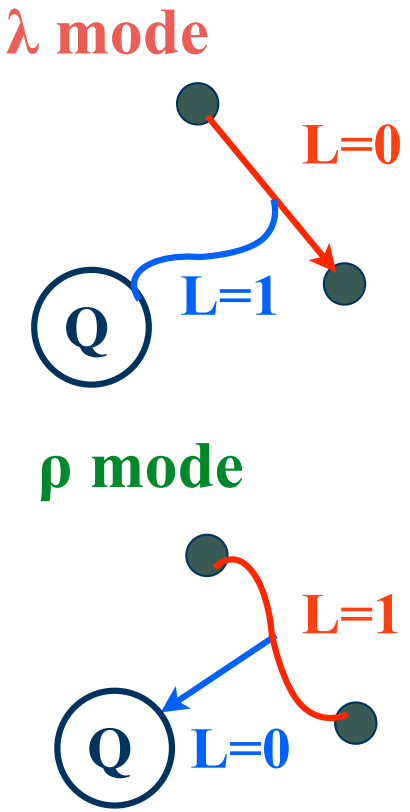


- In the strange sector, the spin-spin force splits SU(3) 8 and 10.
- In the heavier sectors, the heavy-quark spin symmetry suppresses $\Sigma_Q(1/2)$ - $\Sigma_Q(3/2)$ splitting.
- Diquark structure will appear clearly in charm and bottom baryons, but not so in the non-strange and strange sectors.

P-wave excited states: from s to c/b

- Probabilities of λ and ρ modes *v.s.* heavy quark mass in the lowest P-wave $\Lambda_Q(1/2^-)$ state





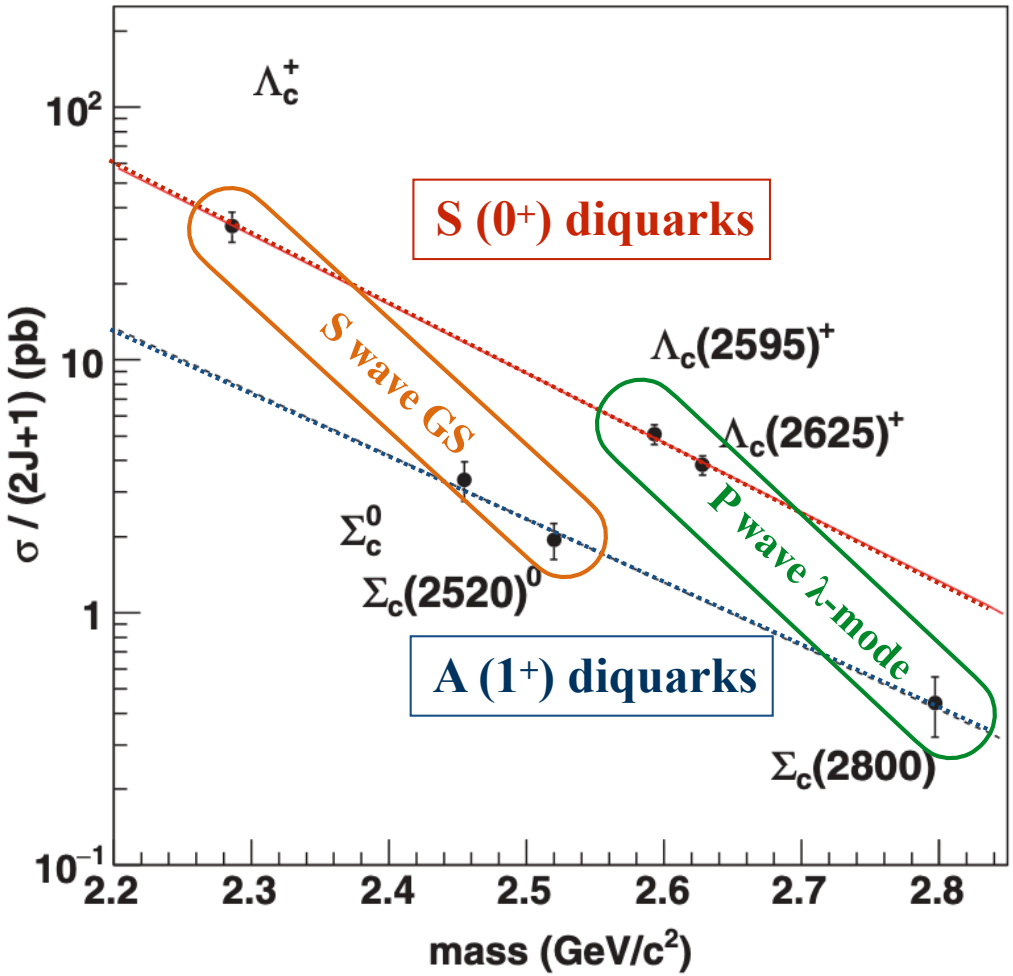
Quark model calculation by Yoshida, et al., PRD 92, 114029 (2015)

Diquarks in Heavy Baryons

Production cross sections of hyperons and charmed baryons from e^+e^- annihilation near $\sqrt{s} = 10.52$ GeV

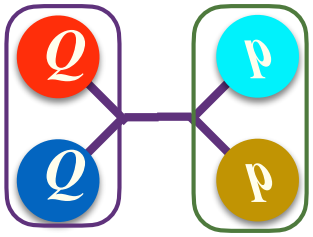
Belle Collaboration, Phys. Rev. D97, 072005 (2018)

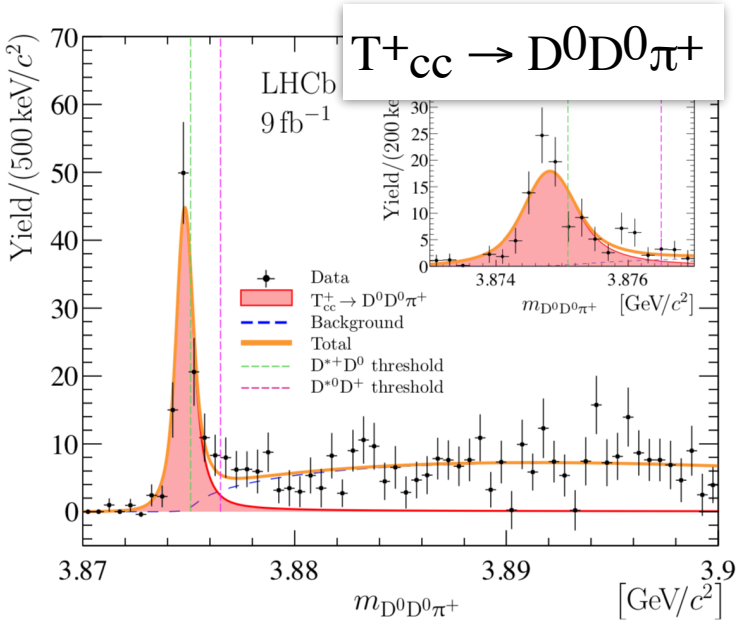
- The production cross sections are consistent with $\sigma(0^+) \sim 3\,\sigma(1^+)$, which suggests a strong scalar diquark correlation.
- The cross sections are consistent with the λ mode excitations for $\Lambda_c(2595)$, $\Lambda_c(2625)$ and $\Sigma_c(2800)$, and supports the quark model.



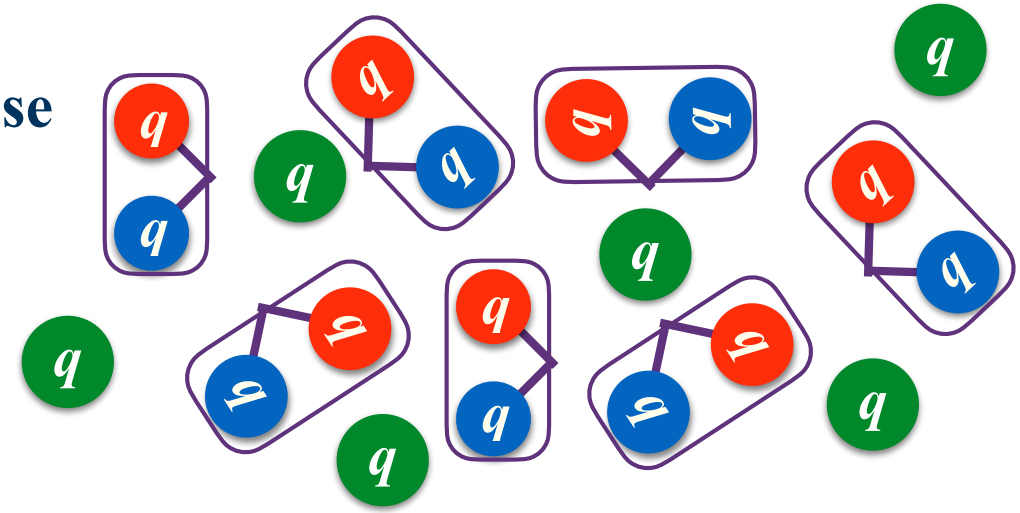
Diquarks in exotic hadrons/matter

- **Doubly heavy tetraquark bound states**
  $T_{QQ} = QQ\bar{q}\bar{q}$





- **Diquark condensate**
  in dense hadronic matter
  => color-superconducting phase



Chiral Effective Theory of Diquarks

Makoto Oka (ASRC, JAEA & Nishina Center, RIKEN)

Chiral Effective Theory of Diquarks

- **Goal: to explore properties of *light diquarks* under SU(3)×SU(3) chiral symmetry and answer questions such as**

  What are the chiral partners of diquarks and their implications to hadron spectroscopy?

  How do we observe the chiral properties of diquarks?

  What are the roles of $U(1)_A$ anomaly in diquark interactions?

  How do diquarks decay strongly?

  How do diquarks behave in matter, where chiral symmetry is partially restored?

- ***Chiral effective Lagrangian based on the linear representation of diquarks (and S+PS mesons)***

Chiral Effective Theory of Diquarks

M. Harada, Y.R. Liu, M.O., K. Suzuki, "*Chiral effective theory of diquarks and $U_A(1)$ anomaly*", Phys. Rev. D 101, 054038 (2020)

Y. Kim, E. Hiyama, M.O., K. Suzuki, "*Spectrum of singly heavy baryons from a chiral effective theory of diquarks*", Phys. Rev. D 102, 014004 (2020)

Y. Kawakami, M. Harada, M.O., K. Suzuki, "*Suppression of decay widths in singly heavy baryons induced by the $U_A(1)$ anomaly*", Phys. Rev. D 102, 114004 (2020)

Y. Kim, Y.R. Liu, M.O., K. Suzuki, "*Heavy baryon spectrum with chiral multiplets of scalar and vector diquarks*", Phys. Rev. D 104, 054012 (2021)

Y. Kim, M.O., K. Suzuki, "*Doubly heavy tetraquarks in a chiral-diquark picture*", Phys. Rev. D 105, 074021 (2022)

Y. Kim, M.O., D. Suenaga, K. Suzuki, "*Strong decays of singly heavy baryons from a chiral effective theory of diquarks*", ArXiv: 2212.08338

Scalar/Pseudoscalar Diquarks

- **The effective Lagrangian with $SU(3)_R \times SU(3)_L$ symmetry**

M. Harada, Y.R. Liu, M.O., K. Suzuki, PR D101, 054038 (2020)

$$\mathcal{L} = \mathcal{D}_\mu d_{R,i} \left(\mathcal{D}^\mu d_{R,i}\right)^\dagger + \mathcal{D}_\mu d_{L,i} \left(\mathcal{D}^\mu d_{L,i}\right)^\dagger$$

$$-m_0^2 (d_{R,i} d_{R,i}^\dagger + d_{L,i} d_{L,i}^\dagger)$$ **Chiral symmetric mass**

$$-\frac{m_1^2}{f} (d_{R,i} \Sigma_{ij}^\dagger d_{L,j}^\dagger + d_{L,i} \Sigma_{ij} d_{R,j}^\dagger)$$ **$U_A(1)$ anomalous mass**

$$-\frac{m_2^2}{2f^2} \epsilon_{ijk} \epsilon_{\ell mn} (d_{R,k} \Sigma_{\ell i} \Sigma_{mj} d_{L,n}^\dagger + d_{L,k} \Sigma_{\ell i}^\dagger \Sigma_{mj}^\dagger d_{R,n}^\dagger)$$ **SCSB mass**

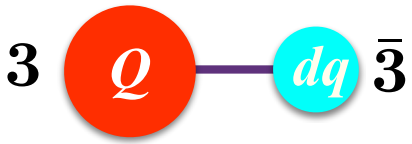
$$+\frac{1}{4} \mathrm{Tr} \left[\partial^\mu \Sigma^\dagger \partial_\mu \Sigma\right] + V(\Sigma).$$

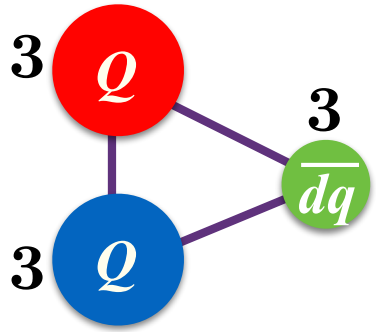
$$\Sigma_{ij} \equiv \sigma_{ij} + i\pi_{ij}$$ **Scalar and PS nonets**

Diquark-Heavy-Quark model

- **Single-Heavy-Baryon with a *Q-dq* potential:**

$$V(r) = -\frac{\alpha}{r} + \lambda r + C,$$





- **Double-Heavy-Tetraquarks with *Q-Q* and *Q-dq*^{bar} potentials**

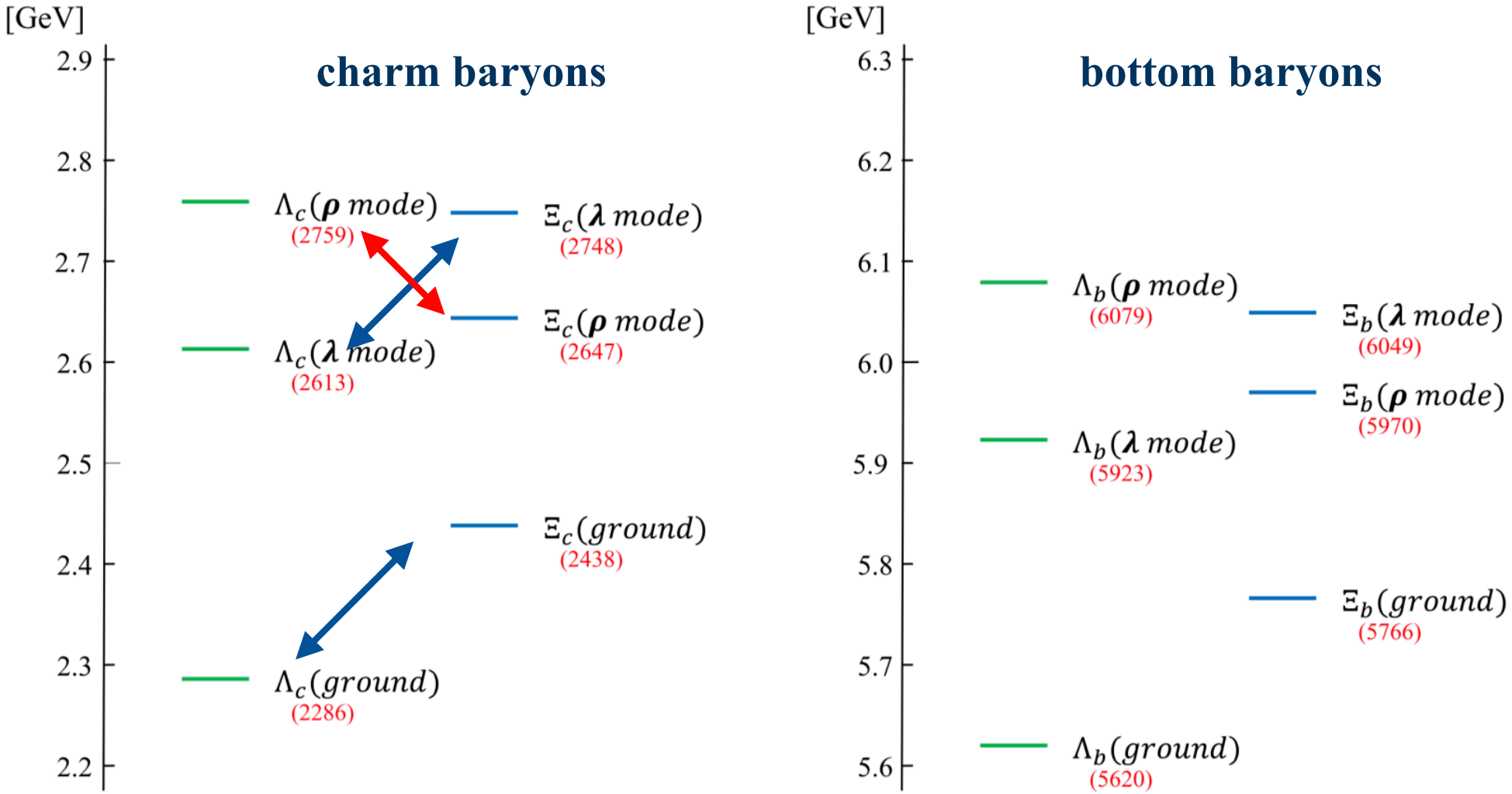
α	$\lambda(\mathrm{GeV}^2)$	$C_c(\mathrm{GeV})$	$C_b(\mathrm{GeV})$	$M_c(\mathrm{GeV})$	$M_b(\mathrm{GeV})$
$(2/3)\times 90/\mu$	0.165	−0.58418362	−0.58829590	1.750	5.112

B. Silvestre-Brac, C. Semay, Z. Phys. C 59, 457 (1993)
T. Yoshida, E. Hiyama, A. Hosaka, M. Oka, K. Sadato, PR D 92, 114029 (2015)

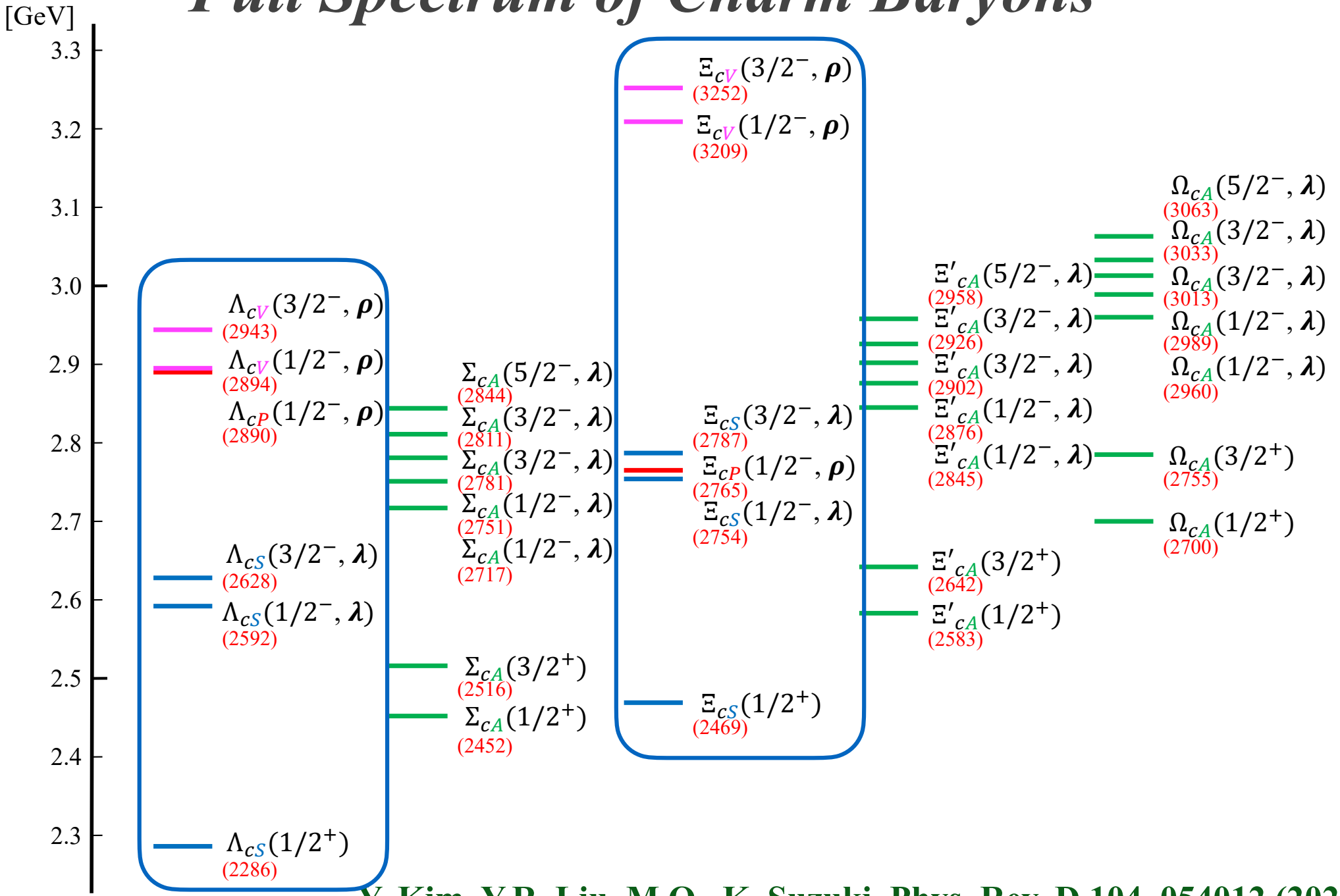
$$M_{(ud)}(0^+) = 725 \text{ MeV} \qquad M_{(ud)}(0^-) = 1265 \text{ MeV}$$
$$M_{(us)}(0^+) = 906 \text{ MeV} \qquad M_{(us)}(0^-) = 1142 \text{ MeV}$$
$$M_{(qq)}(1^+) = 974 \text{ MeV} \qquad M_{(qq)}(1^-) = 1447 \text{ MeV}$$
$$M_{(qs)}(1^+) = 1116 \text{ MeV} \qquad M_{(ss)}(1^+) = 1242 \text{ MeV}$$

Inverse mass hierarchy for Baryons

Y. Kim, E. Hiyama, M. O., K. Suzuki, Phys. Rev. D 102, 014004 (2020)



Full Spectrum of Charm Baryons



Y. Kim, Y.R. Liu, M.O., K. Suzuki, Phys. Rev. D 104, 054012 (2021)

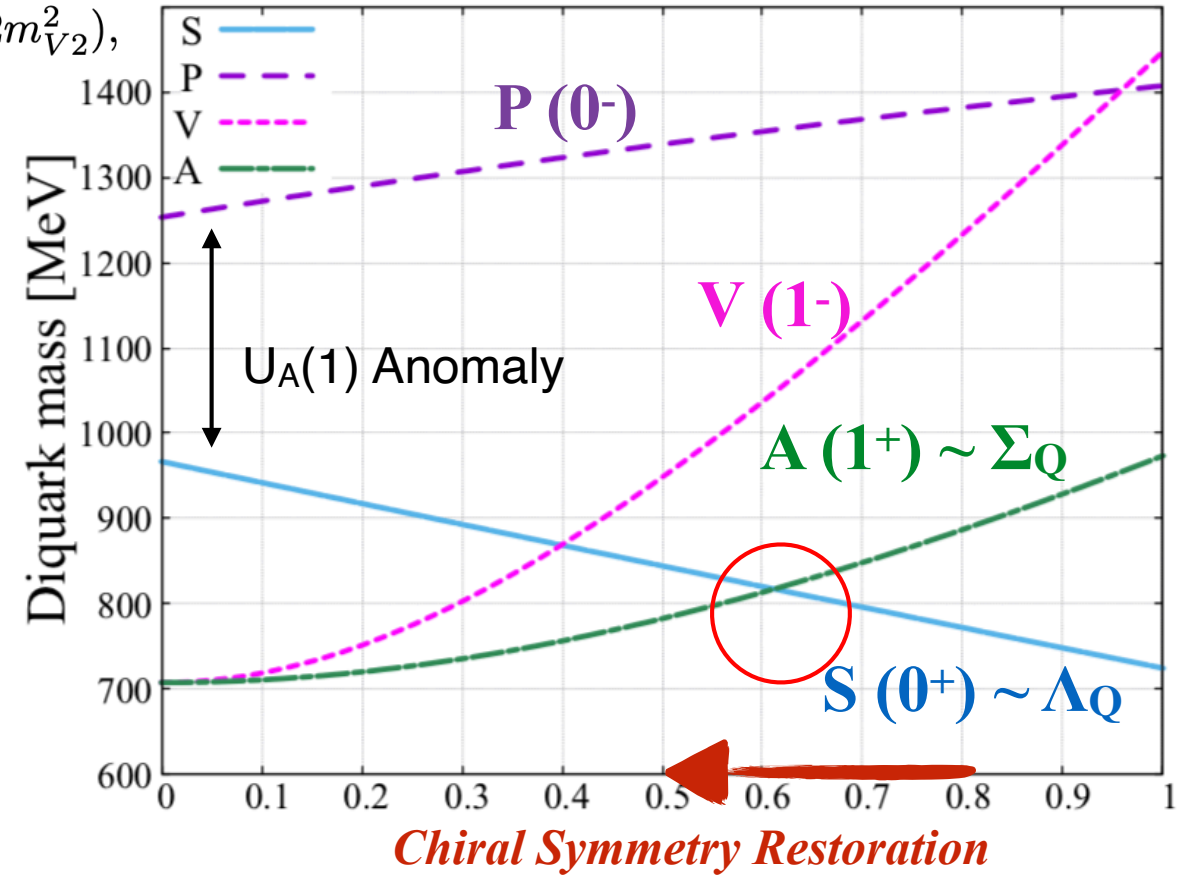
Chiral symmetry vs Diquarks

- **Masses of the 0+ and 1+ diquarks under chiral restoration**

$$m_{[ud]} = \sqrt{m_{S0}^2 - (x+\alpha-1)m_{S1}^2 - x^2 m_{S2}^2},$$
$$m_{\{uu/ud/dd\}} = \sqrt{m_{V0}^2 + x^2(m_{V1}^2 + 2m_{V2}^2)},$$

$$m_{S0}^2 = (1031\ \mathrm{MeV})^2$$
$$m_{S1}^2 = (606\ \mathrm{MeV})^2$$
$$m_{S2}^2 = -(274\ \mathrm{MeV})^2$$

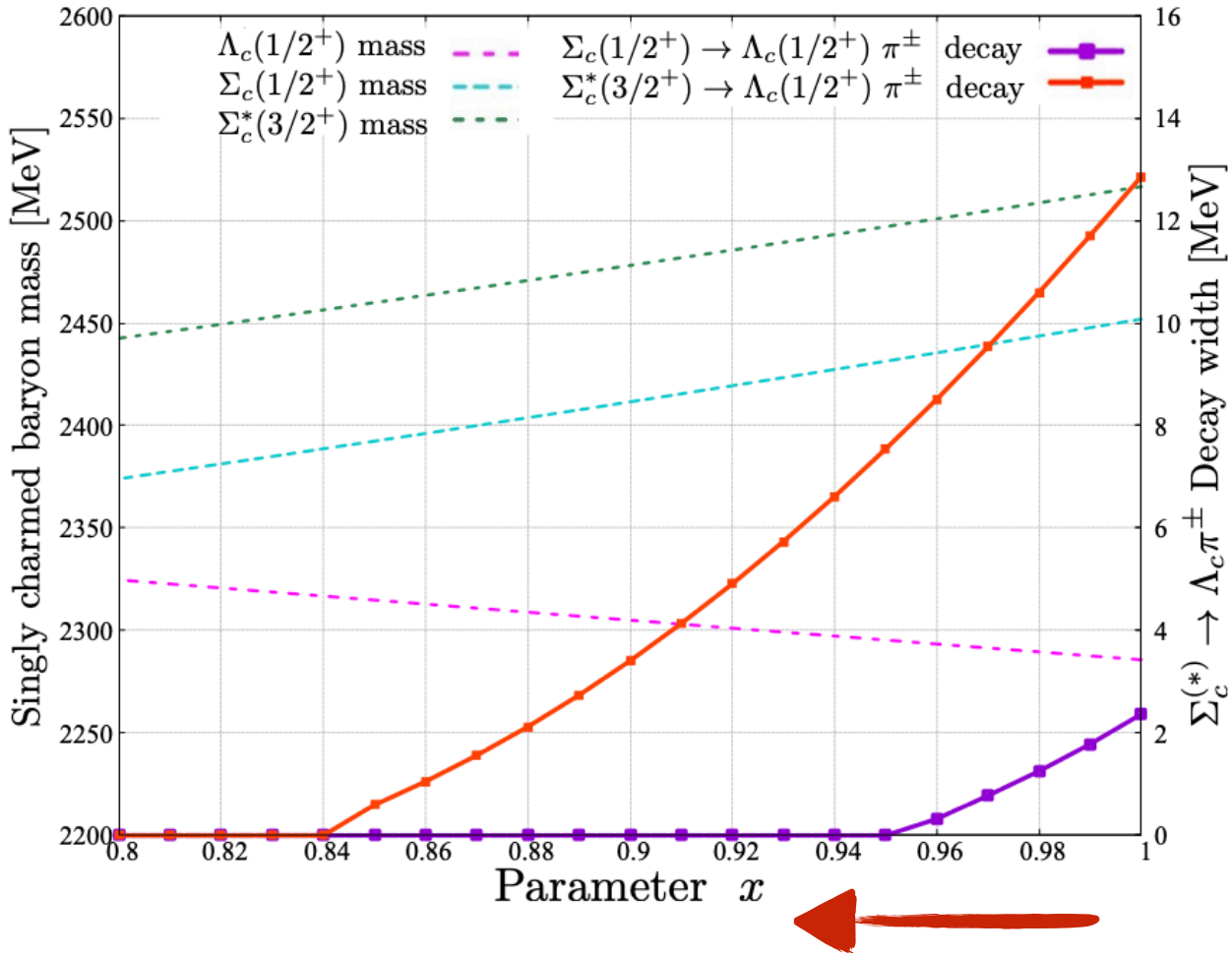
$$m_{V0}^2 = (708\ \mathrm{MeV})^2$$
$$m_{V1}^2 = -(760\ \mathrm{MeV})^2$$
$$m_{V2}^2 = (714\ \mathrm{MeV})^2$$



Y. Kim, Y.R. Liu, M.O., K. Suzuki, Phys. Rev. D 104, 054012 (2021)

Decays of SHB in matter

- **Decays of Σ_c, Σ_c^* baryons under chiral symmetry restoration**



Chiral Symmetry Restoration

Doubly Heavy Tetraquarks

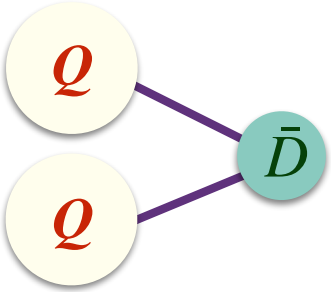
- **The same model is applied to DHTQ**

Y. Kim, M.O., K. Suzuki, Phys. Rev. D 105, 074021 (2022)

- **Potential between a heavy quark and a (anti)diquark**

$$V_0(r) = -\frac{\alpha}{r} + \lambda r + C$$

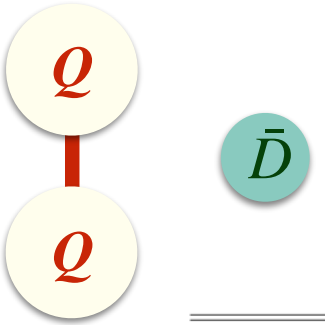
$$V_{ss}(r) = (\boldsymbol{s}_i \cdot \boldsymbol{s}_j)\frac{\kappa}{M_i M_j}\frac{\Lambda^2}{r}\exp(-\Lambda r)$$



α	$\mathbf{1/2} \times 60/\mu$
λ (GeV^2)	$\mathbf{1/2} \times 0.165$
C_c (GeV)	$\mathbf{1/2} \times (-0.83116597)$
C_b (GeV)	$\mathbf{1/2} \times (-0.81898703)$
κ_c	$\mathbf{1/2} \times 0.8586$
κ_b	$\mathbf{1/2} \times 0.6635$
Λ (fm^{-1})	3.5

- **Potential between heavy quarks (a la Semay-Silvestra Brac)**

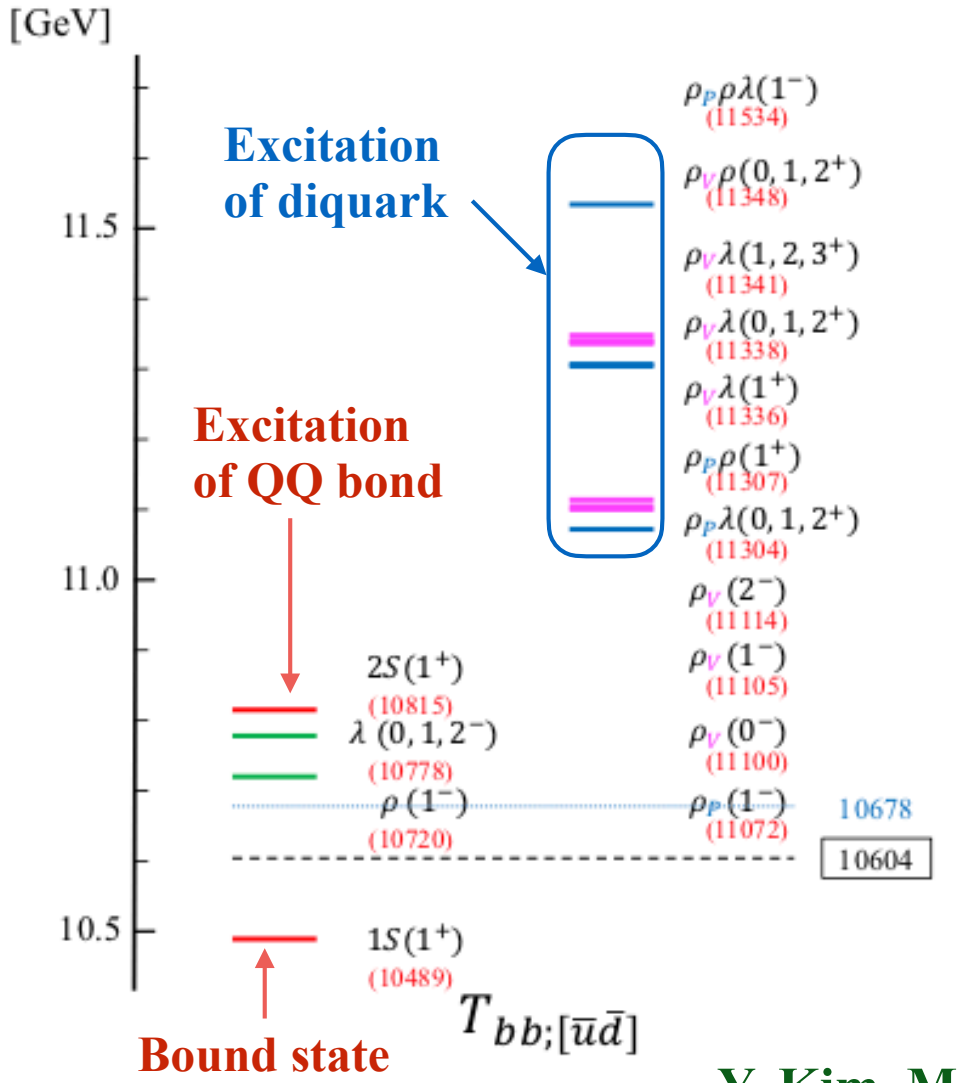
$$V(r) = -\frac{\alpha}{r} + \lambda r^p + C + (\boldsymbol{s}_i \cdot \boldsymbol{s}_j)\frac{8\kappa}{3m_i m_j\sqrt{\pi}}\frac{e^{-r^2/r_0^2}}{r_0^3}$$

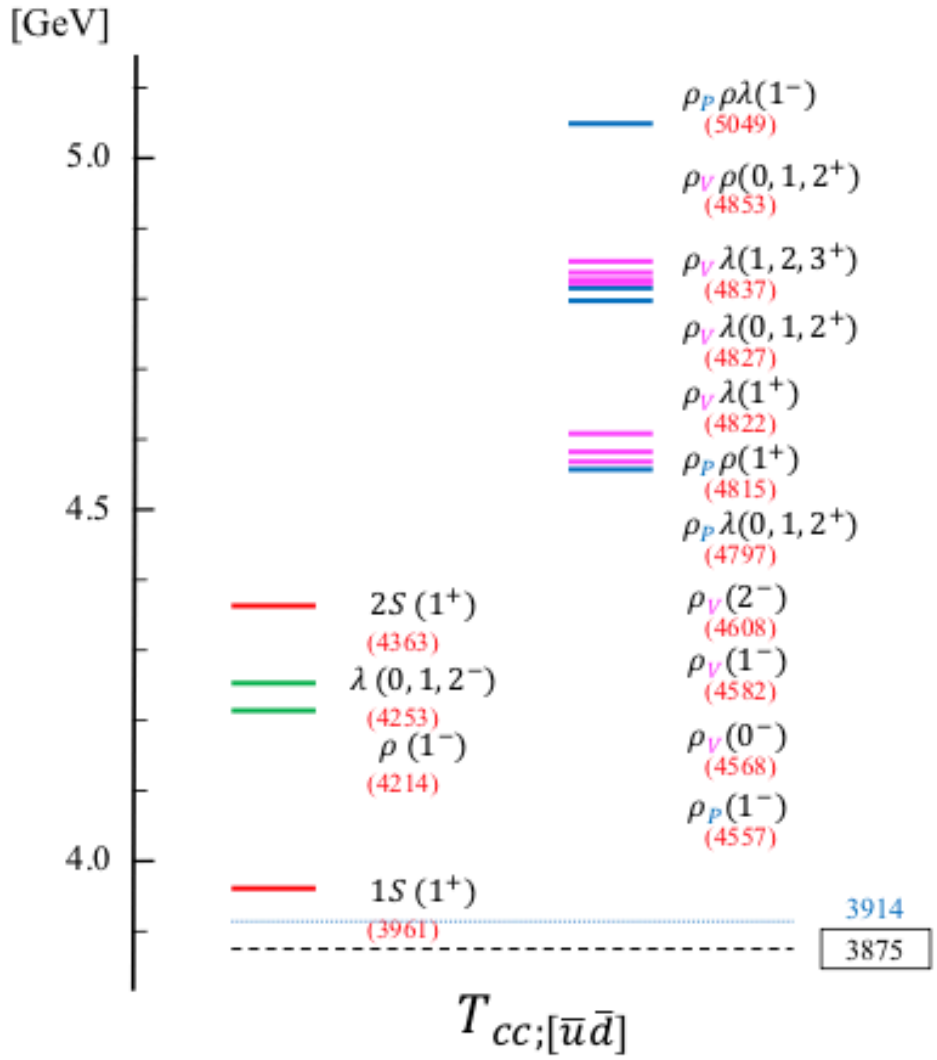


α	$\mathbf{1/2} \times 0.4242$
λ (GeV^{p+1})	$\mathbf{1/2} \times 0.3898$
C (GeV)	$\mathbf{1/2} \times (-1.1313)$
κ	$\mathbf{1/2} \times 1.8025$
A(GeV^{B-1})	1.5296
B	0.3263

Doubly Heavy Tetraquarks

- T_{QQ} states with I=0 (Flavor 3)



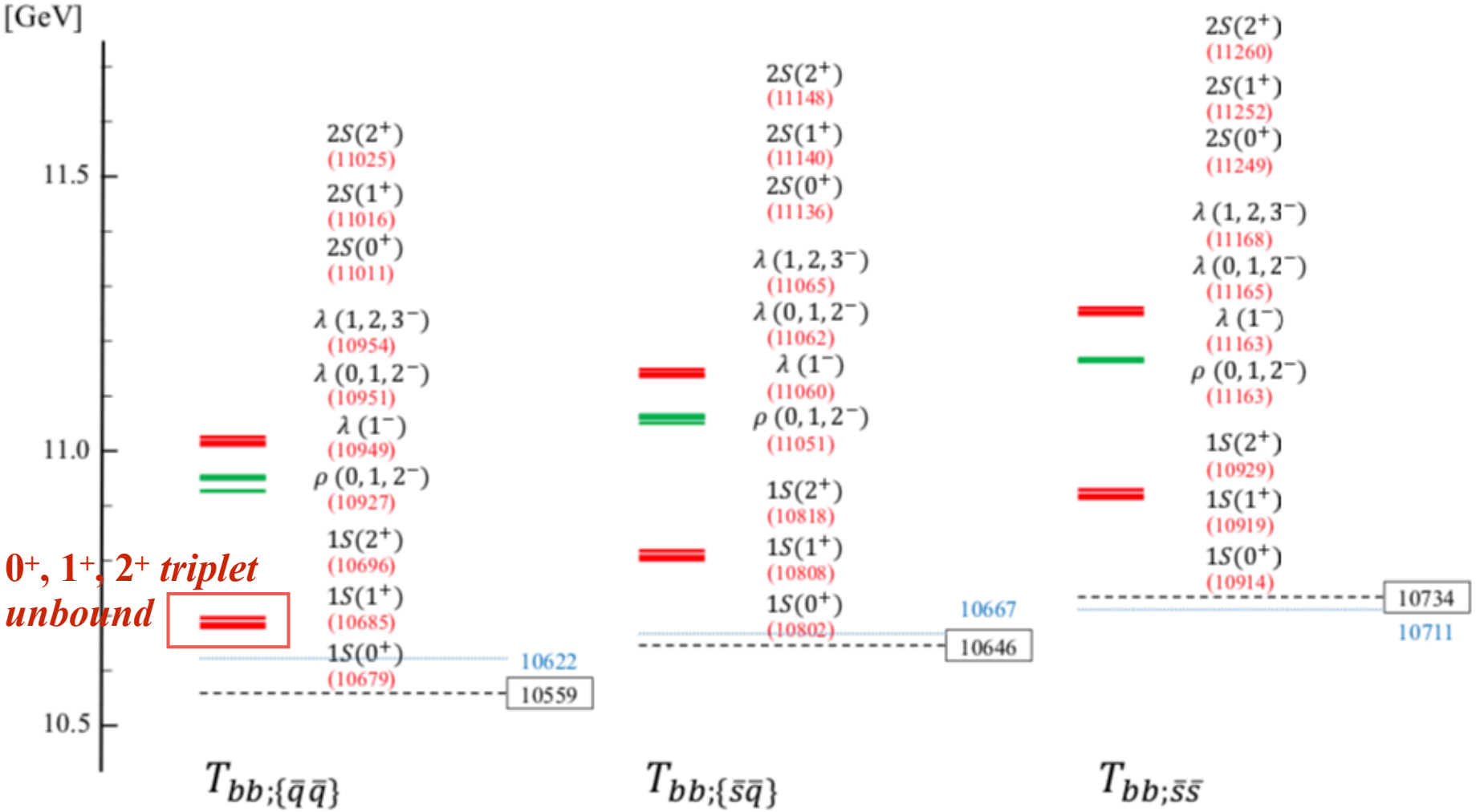


Y. Kim, M.O., K. Suzuki, Phys. Rev. D 105, 074021 (2022)

Doubly Heavy Tetraquarks

Y. Kim, M.O., K. Suzuki, Phys. Rev. D 105, 074021 (2022)

- Flavor SU(3) 6^{bar} (I=1) states



Comparison: masses of 1S ground states with prior researches

[1] Q. Meng, E. Hiyama, A. Hosaka, M. Oka, P. Gubler, and K.U. Can, T.T. Takahashi, and H.S. Zong,
"Stable double-heavy tetraquarks: spectrum and structure", Phys. Lett. B, 814.136095. (2021).

[2] Eric Braaten, Li-Ping He, and Abhishek Mohapatra,
"Masses of doubly heavy tetraquarks with error bars," Phys. Rev. D 103, 016001 (2021).

[3] E.J. Eichten and C. Quigg,
"Heavy-quark symmetry implies stable heavy tetraquark mesons $QQ\bar{q}\bar{q}$", Phys. Rev. Lett. 119, 202002 (2017).

[4] A. Francis, R. J. Hudspith, R. Lewis and K. Maltman,
"Lattice prediction for Deeply Bound Doubly Heavy Tetraquarks", Phys. Rev. Lett. 118, 142001 (2017).

[5] P. Junnarkar, N. Mathur and M. Padmanath
"A study of doubly heavy tetraquarks in Lattice QCD", Phys. Rev. D 99, 034507 (2019).

※Unit: MeV

Particle (1^+)	$T_{bb;[\bar{u}\bar{d}]}$		$T_{bb;[\bar{d}\bar{s}]([\bar{s}\bar{u}])}$		$T_{cc;[\bar{u}\bar{d}]}$		$T_{cc;[\bar{d}\bar{s}]([\bar{s}\bar{u}])}$	
(Threshold)	**10604**		**10692**		**3876**		**3977**	
Ref. [1]	10444	(-160)	10625	(-67)	3865	(-11)	. . .	
Ref. [2]	10471	(-133)	10644	(-48)	3947	(+71)	4124	(+147)
Ref. [3]	10482	(-122)	10643	(-49)	3978	(+102)	4156	(+179)
Ref. [4]	10415	(-189)	10594	(-98)	. . .		. . .	
Ref. [5]	10461	(-143)	10605	(-87)	3853	(-23)	3969	(-8)
This work	**10489**	**(-115)**	**10664**	**(-28)**	**3961**	**(+85)**	**4141**	**(+164)**

Y. Kim, M.O., K. Suzuki, Phys. Rev. D 105, 074021 (2022)

Summary

- Diquark is a non-trivial colored cluster in hadrons, which is clearly seen as a component in single heavy baryons.
- Chiral properties of light diquarks have been studied in the context of heavy baryon spectroscopy.
- $U_A(1)$ anomaly seems to play significant roles.
- Chiral restoration may give notable effects on the spectra and decays of heavy baryons.

- Future problems:
  roles of diquarks around the hadronic thresholds
  roles of diquarks and the other quark clusters in exotic hadrons