Hierarchical Structure of Potential by Particle Transfer

（粒子交換によるポテンシャルの階層構造）

Transversal study from Atom-Molecular to Quark-Gluon Systems by the GPT Potential

Shinsho Oryu (尾立晋祥）

Collaborators Yasuhisa Hiratsuka (平塚靖久）
Takashi Watanabe （渡邊　隆）

Tokyo University of Science, Noda Chiba

- 量子クラスターで読み解く物質の階層構造
- 第8回領域研究会吹田キャンパス接合化学研究所荒田記念会館
- 令和５年２月９日（木）～１１日（土）

Journal of Physics Communications

PAPER

Generation of long- and short-range potentials from atom-molecules to quark-gluon systems by the GPT potential

Shinsho Oryu

Department of Physics, Faculty of Science and Technology, Tokyo University of Science, Noda, Chiba 278-8510, Japan

E-mail: oryu@rs.noda.tus.ac.jp

Keywords: generation of Efimov's potential, long-range nuclear potential, general particle transfer potential, long-range three-body force potential, a new Faddeev treatment

I. Coulomb Scattering (L-S equation): EM force

$$t(p,p'z) = V^C(p,p') + \int_0^\infty V^C(p,p'')G_0(z;p'')t(p'',p';z)p''^2\,dp''$$

integral kernel diverges at $p = p'' = k;\ z = k^2/2m$

$$V^C(p,p') = \frac{ZZ'e^2}{(p-p')^2} \to \infty\,; \qquad G_0(z;p'') = \frac{2m}{k^2 - p''^2 + i\varepsilon} \to \infty$$

$$\mathcal{F}\left\{V^C(p,p')\right\} = V(r) = \frac{ZZ'e^2}{r}$$

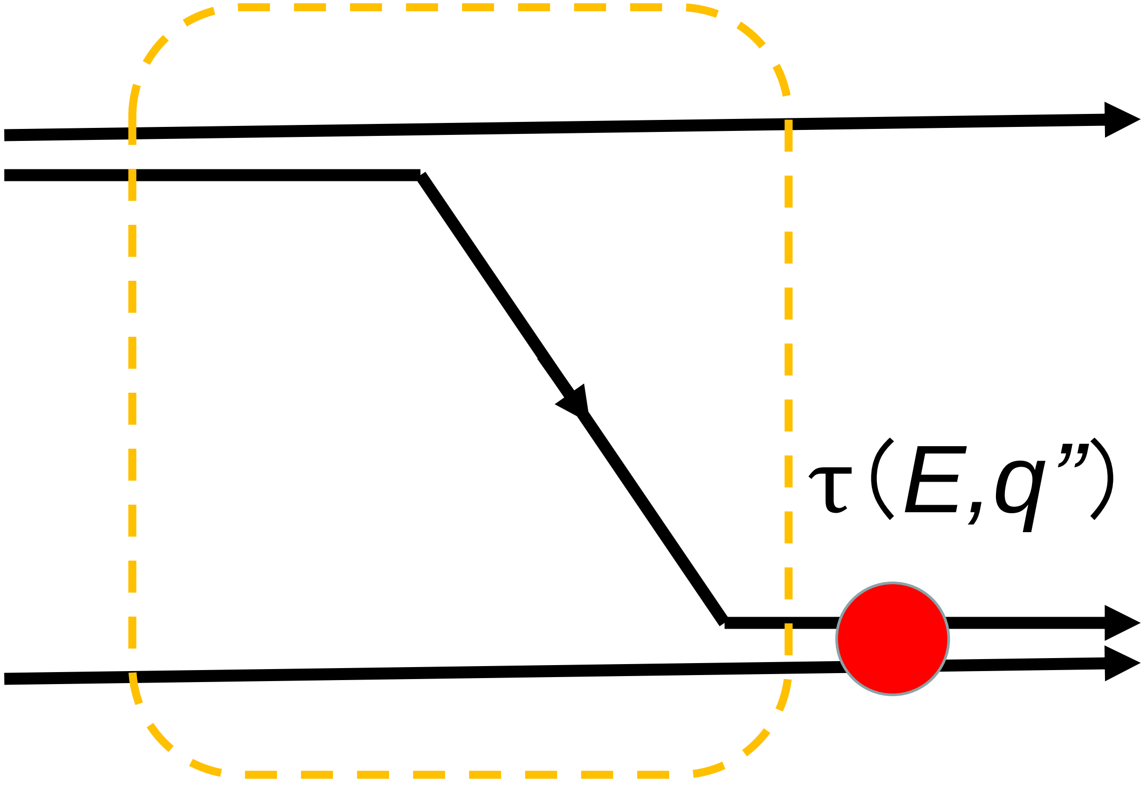
II. Three-body equation (AGS-equation): Nuclear force

$$X(q,q';E) = Z(q,q';E) + \int_0^\infty Z(q,q'';E)\tau(E;q'')X(q'',q';E)q''^2\,dq''$$

$$Z(q,q'';E) \to \infty \qquad\qquad \tau(E;q'') \to \infty$$

at $E = q'' = p'' = 0$ $\qquad\qquad \propto 1/(E+\varepsilon_B-q''^2/2\mu)$

Kernel of AGS（Alt-Grassberger-Sandhas） equation



General particle transfer Feynman diagram

Alt, Grassberger, Sandhas, 1968,
Reduction of the three-particle collision problem to multi-channel two-particle Lippmann-Schwinger equation, Nucl. Phys. B2, 167-80

b) Born term of the quasi two-body equation:

$$Z_{\alpha\beta}(q,q';E)=\frac{g_{\alpha}(p)g_{\beta}(p')(1-\delta_{\alpha\beta})\mathcal{R}_{\alpha\beta}}{(E+\varepsilon_{B})-q^{2}/2\mu}\rightarrow\infty$$

at $p=p'=0,\ E_{cm}=(E+\varepsilon_B)=0,\ q=0$

(Q2T：quasi two-body threshold)

$$\tau(E,q)\propto 1 / [E+\varepsilon_B) - q^2/2\mu] \rightarrow \infty$$

Divergence problem is independent of the two-body binding energy: $\varepsilon_B\ (\neq 0)$

Introduction of the GPT potential:

$$C_{\alpha\beta} \equiv -2\mu g_\alpha(0) g_\beta(0)(1-\delta_{\alpha\beta})\mathcal{R}_{\alpha\beta} \qquad -1 \leq \mathcal{R}_{\alpha\beta} \leq 1$$

For $E_{cm} < 0$

& $0 < C_{\alpha\beta}$; with $\sigma^2 = 2\mu |E + \varepsilon_B| = 2\mu |E_{cm}|$,

$$Z_{\alpha\beta} \to \frac{g_\alpha(0) g_\beta(0)(1-\delta_{\alpha\beta})\mathcal{R}_{\alpha\beta}}{-|E_{cm}| - q^2/2\mu} = -\frac{C_{\alpha\beta}}{q^2+\sigma^2} \to \infty$$

at Q2T

$$\mathcal{F}\left\{-\frac{C_{\alpha\beta}}{q^2+\sigma^2}\right\} = V(r) = -V_0 \boxed{\frac{e^{-\sigma r}}{r}}$$

Let us smoothing the divergence : $1/(q^2+\sigma^2)\to\infty$

at $\sigma=\sqrt{2\mu|E_{cm}|}=0$ & $q=0$ (or Q2T):

we adopt a statistical average

with respect of σ a =1/m_π

$$P=\frac{\sigma^{n-2}e^{-a\sigma}}{\rho}$$

GPT-potential

$$with\quad \rho=\int_0^\infty \sigma^{n-2}e^{-a\sigma}d\sigma=\frac{\Gamma(n-1)}{a^{n-1}}$$

$$\mathcal{E}\left\{\frac{e^{-\sigma r}}{r}\right\}\equiv\frac{1}{\rho}\int_0^\infty \sigma^{n-2}e^{-a\sigma}\left\{\frac{e^{-\sigma r}}{r}\right\}d\sigma=\boxed{\frac{a^{n-1}}{r(r+a)^{n-1}}}$$

Euler Integral of the 2nd kind

$$V(r) = \frac{V_0 a^{n-1}}{r(r+a)^{n-1}};$$

GPT-Potential

$$0 < n: \quad \frac{V_0 a^{n-1}}{r(r+a)^{n-1}} = \begin{cases} r \ll a : V(r) \to V_0 \dfrac{e^{-(n-1)r/a}}{r} = V_0 \dfrac{e^{-\mu r}}{r} \\ a << r \quad : V(r) \to V_0 \dfrac{a^{n-1}}{r^n} \end{cases}$$

$$\text{index:} \quad (n-1)/a = \mu \quad \Rightarrow \quad n = a\mu + 1 = \mu/m_\pi + 1$$

$$-|n| = n \leq 0: \quad V_0 \frac{(r+a)^{|n|+1}}{a^{|n|+1} r} = \begin{cases} r << a \quad : V(r) \to V_0 \dfrac{1}{r} \\ a << r \quad : V(r) \to V_0 \dfrac{r^{|n|}}{a^{|n|+1}} \end{cases}$$

Confinement potential

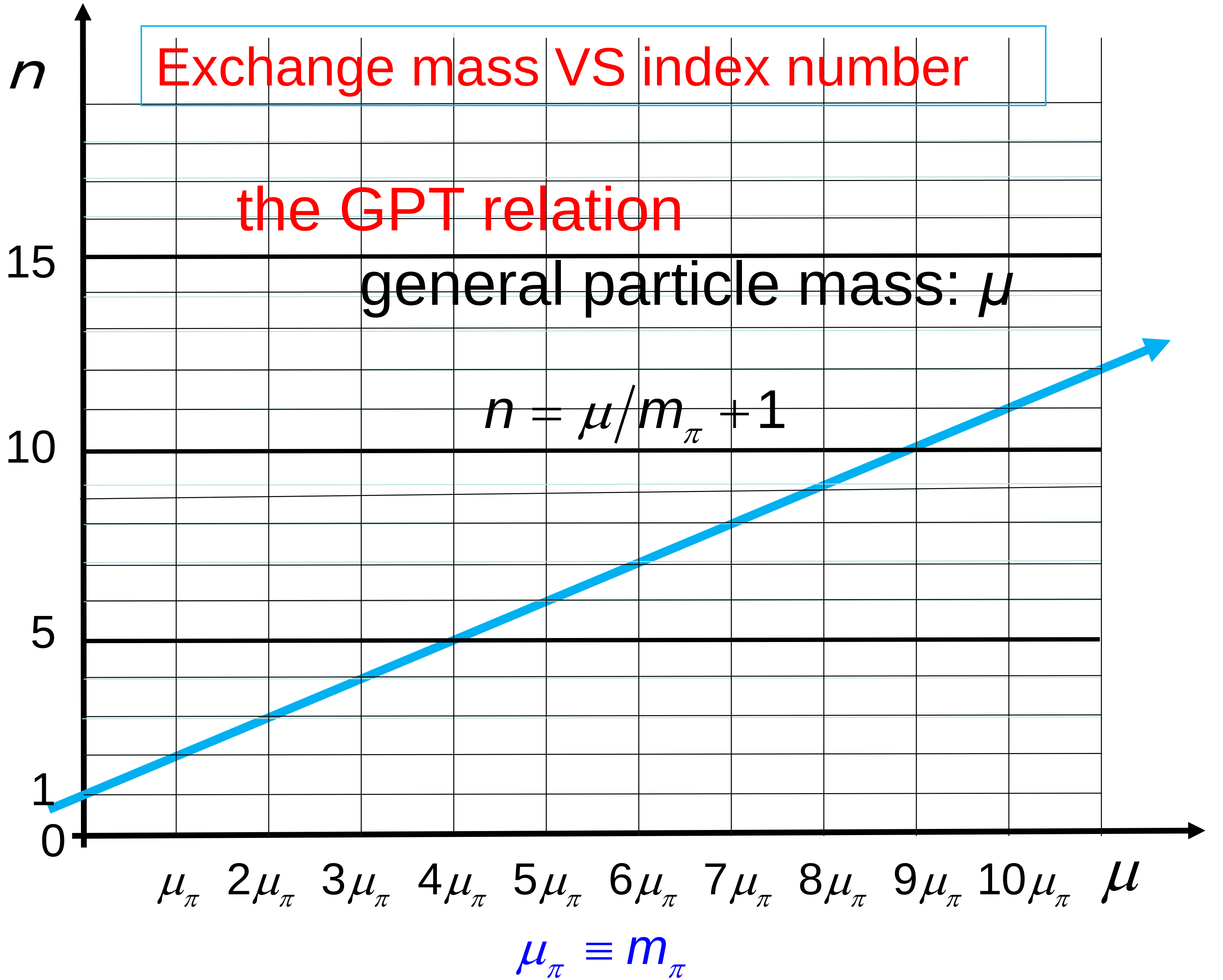
GPT-potential: below the 3BT with parametes α, n.

V_0 , V_0' are potential depths. $0 < n$ gives pion exchange

γ	n *index*	$r \ll \alpha$	GPT-potential	$\alpha \ll r$
-1	1	V_0/r	$\Omega(r)/r$	V_0'/r
$-1/2$	2	$V_0 e^{-(r/\alpha)}/r$	$\Omega(r)\alpha/\left[r(r+\alpha)\right]$	$V_0'\alpha/r^2$
0	3	$V_0 e^{-(2r/\alpha)}/r$	$\Omega(r)\alpha^2/\left[r(r+\alpha)^2\right]$	$V_0'\alpha^2/r^3$
$1/2$	4	$V_0 e^{-(3r/\alpha)}/r$	$\Omega(r)\alpha^3/\left[r(r+\alpha)^3\right]$	$V_0'\alpha^3/r^4$
1	5	$V_0 e^{-(4r/\alpha)}/r$	$\Omega(r)\alpha^4/\left[r(r+\alpha)^4\right]$	$V_0'\alpha^4/r^5$
$3/2$	6	$V_0 e^{-(5r/\alpha)}/r$	$\Omega(r)\alpha^5/\left[r(r+\alpha)^5\right]$	$V_0'\alpha^5/r^6$
2	7	$V_0 e^{-(6r/a)}/r$	$\Omega(r)\alpha^6/\left[r(r+\alpha)^6\right]$	$V_0'\alpha^6/r^7$
...	...	...	...	...
...	...	...	...	...
$(n-3)/2$	n	$V_0 e^{-(n-1)r/\alpha}/r$	$\Omega(r)\alpha^{n-1}/\left[r(r+\alpha)^{n-1}\right]$	$V_0'\alpha^{n-1}/r^n$

GPT-potential: with parametes $\boldsymbol{a}$ and n. V_0, V_0' the potential depths. $n \leqslant 0 \Leftrightarrow$ gluon exchange (q - q int.)

$\|n\|$	$r \ll \boldsymbol{a}$	GPT-potential	$\boldsymbol{a} \ll r$
0	V_0/r	$\Omega(r)\left[(r+\boldsymbol{a})/\boldsymbol{a}r\right]$	$V_0'/\boldsymbol{a}$
1	V_0/r	$\Omega(r)\left[(r+\boldsymbol{a})^2/\boldsymbol{a}^2 r\right]$	$V_0' r/\boldsymbol{a}^2$
2	V_0/r	$\Omega(r)\left[(r+\boldsymbol{a})^3/\boldsymbol{a}^3 r\right]$	$V_0' r^2/\boldsymbol{a}^3$
3	V_0/r	$\Omega(r)\left[(r+\boldsymbol{a})^4/\boldsymbol{a}^4 r\right]$	$V_0' r^3/\boldsymbol{a}^4$
4	V_0/r	$\Omega(r)\left[(r+\boldsymbol{a})^5/\boldsymbol{a}^5 r\right]$	$V_0' r^4/\boldsymbol{a}^5$
5	V_0/r	$\Omega(r)\left[(r+\boldsymbol{a})^6/\boldsymbol{a}^6 r\right]$	$V_0' r^5/\boldsymbol{a}^6$
...	...	...	...
...	...	...	...
$\|n\|$	V_0/r	$\Omega(r)\left[(r+\boldsymbol{a})^{\|n\|+1}/\boldsymbol{a}^{\|n\|+1} r\right]$	$V_0' r^{\|n\|}/\boldsymbol{a}^{\|n\|+1}$


Exchange mass VS index number
the GPT relation
general particle mass: μ
$n = \mu / m_\pi + 1$
n
15
10
5
1
0
μ_π $2\mu_\pi$ $3\mu_\pi$ $4\mu_\pi$ $5\mu_\pi$ $6\mu_\pi$ $7\mu_\pi$ $8\mu_\pi$ $9\mu_\pi$ $10\mu_\pi$ μ
$\mu_\pi \equiv m_\pi$

The second order perturbation formula for the N-N interaction is given by using momentum conservation at vertexes for the nucleon and meson momenta $\boldsymbol{q}_1$, $\boldsymbol{q}_2$ and $\boldsymbol{k}$, by using the meson energy $\omega_k = \sqrt{k^2 + m_\pi^2}$, and the initial two-nucleon energy E_0,

$$W_2 = \sum_{\boldsymbol{k}} \left\{ \frac{< 0|H'_{N_2 \cdot \pi}|m >< m|H'_{N_1 \cdot \pi}|0 >}{E_0 - [E'(q_1, q_2) + \omega_k]} + (1 \leftrightarrow 2) \right\}, \quad (1)$$

The historical method adopted a *static* approximation to introduction the OPEP with $0 = E_0 \approx E'(q_1, q_2)$ assuming $\Delta = m_\pi / M_N = 0.14703$ is small.
We neglect the nucleon-recoil effect by the meson creation and annihilation. Therefore, the static approximation of Eq.(4) becomes

$$W_2 \approx \sum_{\boldsymbol{k}} \left\{ \frac{< 0|H'_{N_2 \cdot \pi}|m >< m|H'_{N_1 \cdot \pi}|0 >}{-\omega_k} + (1 \leftrightarrow 2) \right\}. \quad (2)$$

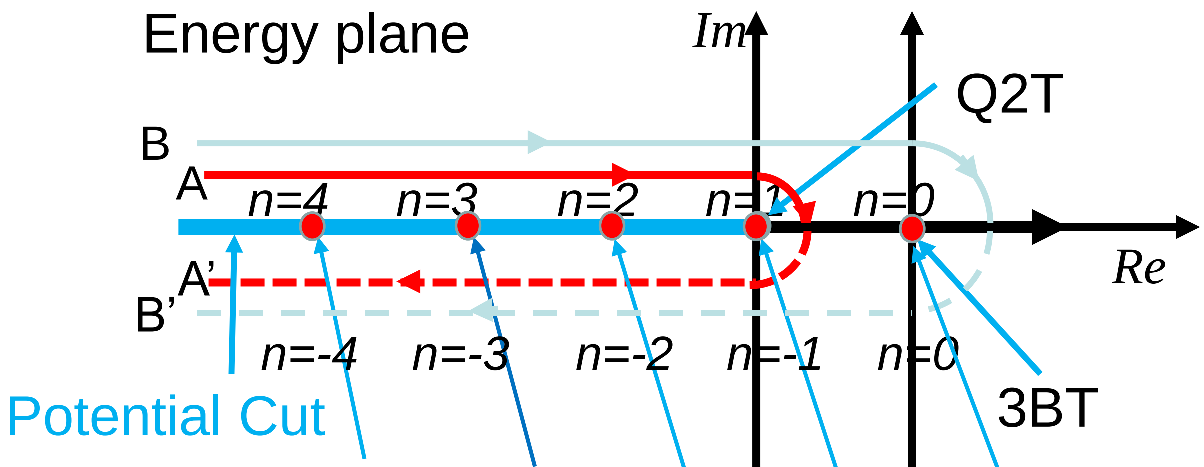
裳華房　物理学選書9　武田皢　宮沢弘成　共著「素粒子物理学」
236p[核力]:
…中間状態のエネルギーはω_kだけ初めの状態のエネルギーより大きいので（陽子は中間子に比べて非常に重いので中間子を放出してもほとんど静止しており，そのエネルギーは変わらないと近似してよい）…………静的近似をすれば（１）式は（２）式とおいてよい。

This equation gives the Yukawa potential or the OPEP by the formula:

$$
\begin{aligned}
V^{\pi}(\boldsymbol{r}) &= \Omega^{\pi}(\boldsymbol{r};2)\frac{e^{-m_{\pi}r}}{r}, && (3)\\
\Omega^{\pi}(\boldsymbol{r};2) &= \frac{f^2}{3}\left\{\sigma_{\mathbf{1}}\cdot\sigma_{\mathbf{2}}+\left(1+\frac{3}{m_{\pi}r}+\frac{3}{m_{\pi}^2r^2}\right)S_{12}\right\}\\
S_{12} &= 3\frac{(\sigma_{\mathbf{1}}\cdot\boldsymbol{r})(\sigma_{\mathbf{2}}\cdot\boldsymbol{r})}{r^2}-\sigma_1\cdot\sigma_2. && (4)
\end{aligned}
$$

Ⅰ. Three-Body Problem in hadron systems

GPT-potential on the Riemann sheet



$$\frac{V_0 a^{n-1}}{r(r+a)^{n-1}} \cdots\cdots \frac{V_0 a^3}{r(r+a)^3} \quad \frac{V_0 a^2}{r(r+a)^2} \quad \frac{V_0 a}{r(r+a)} \quad \frac{V_0}{r}$$

$$\frac{V_0'(r+a)^{|n|+1}}{a^{|n|+1} r} \cdots \frac{V_0'(r+a)^5}{a^5 r} \quad \frac{V_0'(r+a)^4}{a^4 r} \quad \frac{V_0'(r+a)^3}{a^3 r} \quad \frac{V_0'(r+a)^2}{a^2 r} \quad \frac{V_0'(r+a)}{a r}$$

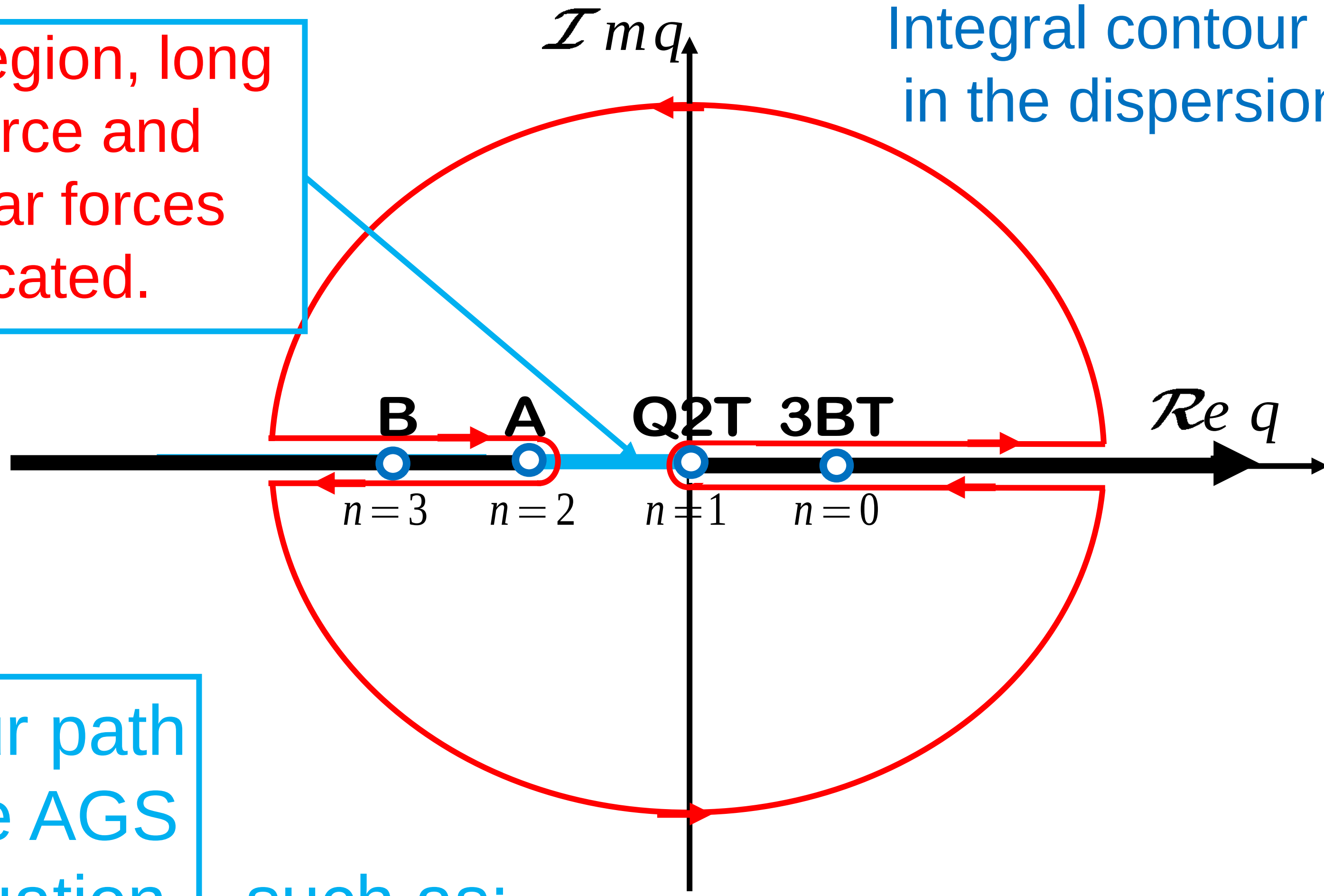
Potential depth and the range between singular points *n* are depend on the exchange particles

The right hand cur (from n=1（Q2T）to +infinity (black line). The left hand cut (from n=1 to –infinity) light blue line. The 3-body break up cut (from n=0(3BT) to +infinity) Integral contour (small red line on the Riemann sheet) goes down to the unphysical 2nd Riemann sheet (small red dotted line) . The negative indexes n<0 belong to the unphysical 2nd sheet. The GPT-potential corresponds a quark-quark interaction which has a confinement potential. Furthermore, particles on unphysical sheet could not be observed.

Lefthand-cut is started from Q2T to -infinity. The pinching singularity occurs, then the equation can not be solved. In order to solve the equation, the long range force is truncated between n=1~n=2.

In this region, long range force and non linear forces are truncated.

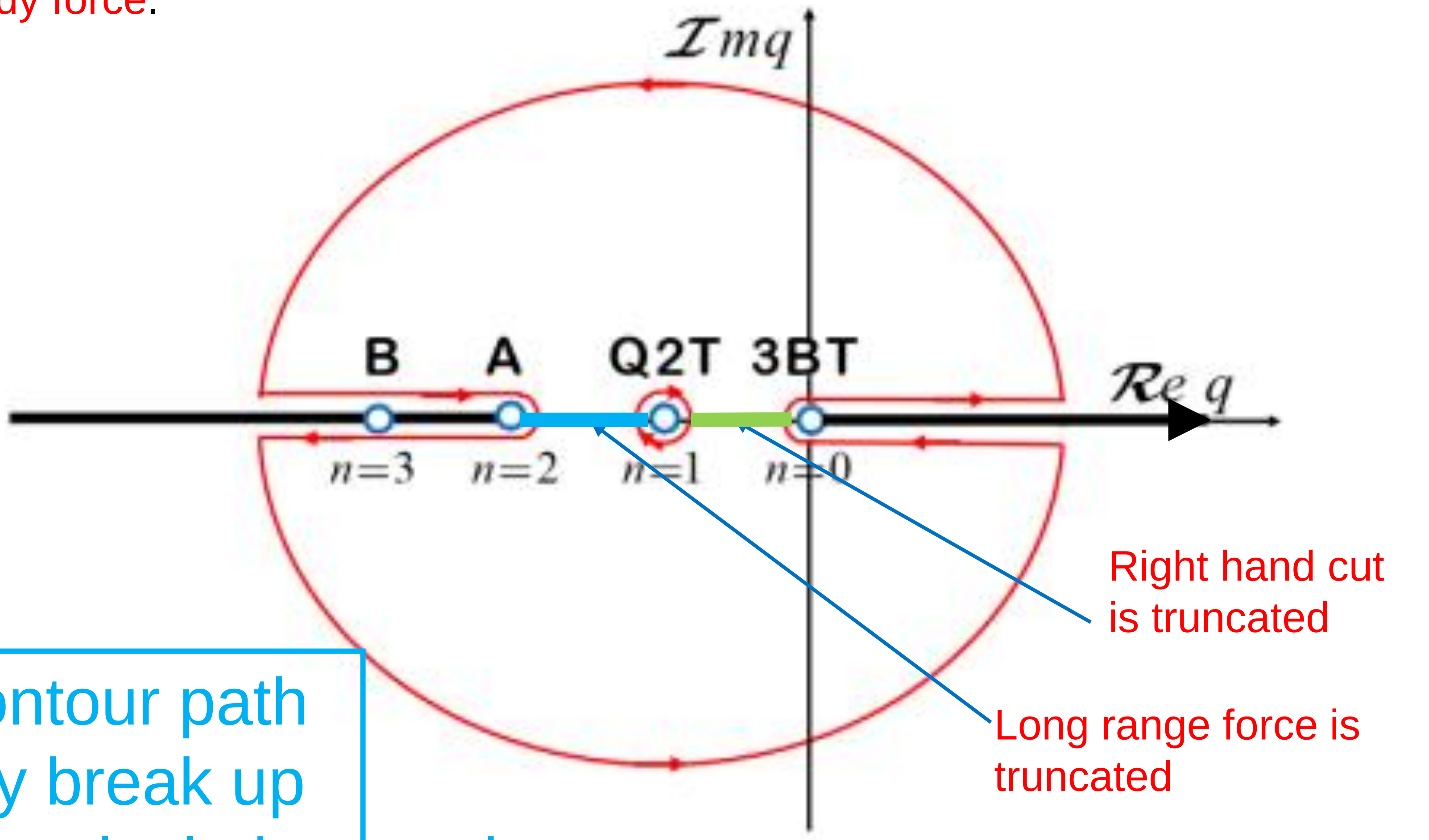
Integral contour in the dispersion theory



Contour path of the AGS equation such as;

Alt, Grassberger, Sandhas, 1968,
Reduction of the three-particle collision problem to multi-channel two-particle Lippmann-Schwinger equation, Nucl. Phys. B2, 167-80

In order to calculate the break-up scattering, Cahill-Sloan adopted the righthand cut from the 3BT to +infinity and they defined a moving pole for Q2T. The long range force was also truncated. This method derive wrong scattering length, cross section and three-body force.



Usual contour path of 3-body break up Faddeev calculation, such as

Cahill-Sloan 1971, Theory of Neutron-Deuteron Break-up at 14.4MeV, Nucl. Phys. S165, 161-179.

Cahill-Sloan 1971,

Theory of Neutron-Deuteron Break-up at 14.4MeV, Nucl. Phys. S165, 161-179.

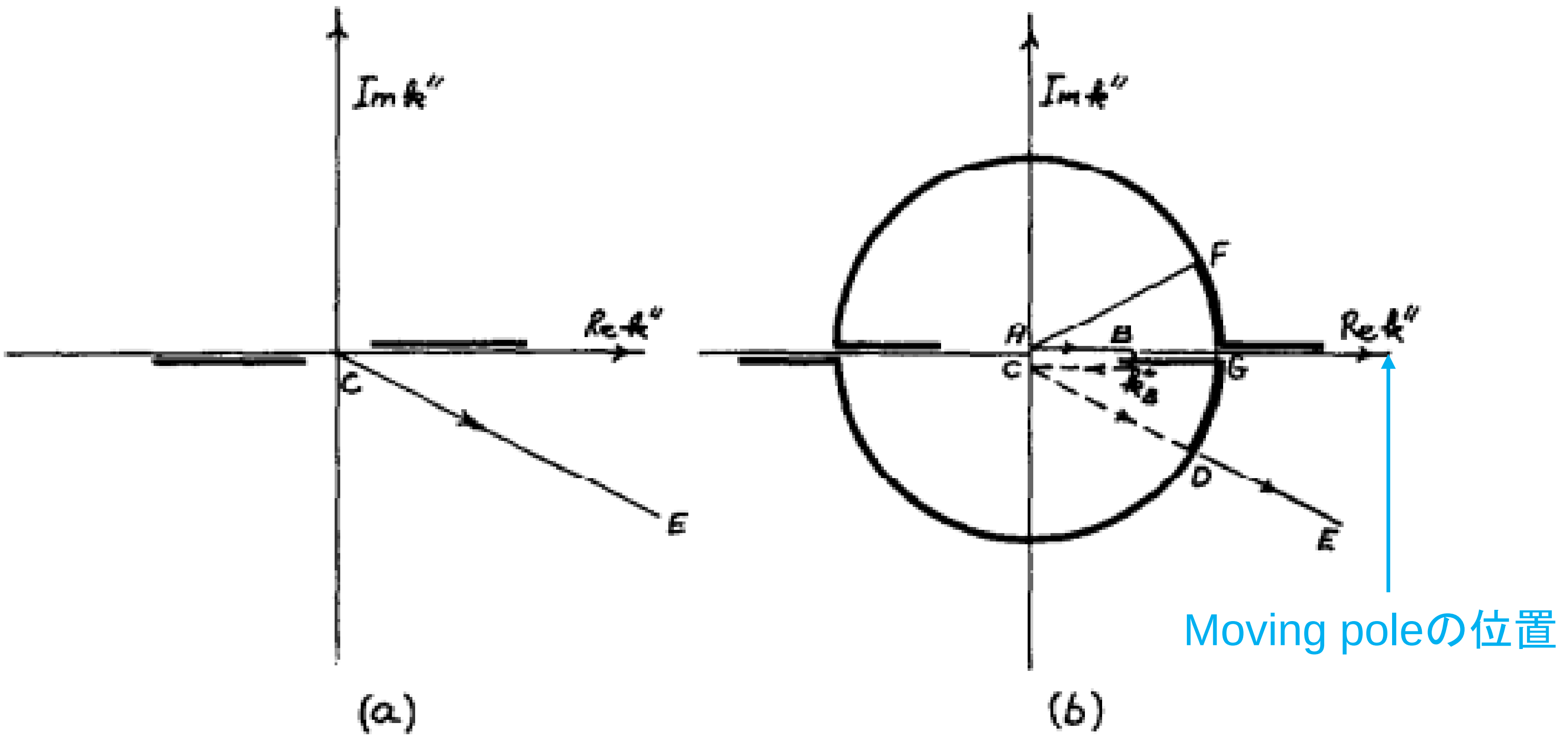


Fig. 2. Integration contours in k'' plane. The dashed line refers to the contour on the next sheet reached through the branch cuts, which are shown by thick lines.

II Atom-molecular systems:

A) $n=1$ by the GPT relation:

$$n=\frac{\mu}{m_e}+1 \quad =1........, \quad \mu=0$$

$$V(r)=V_0/r^n=V_0/r \quad \text{causes an ionic bond.}$$

B) $n\geq 2:$ $V(r)=V_0/r^n$ causes a covalent bond,

C) $\mu=5m_e$ $n=5+1=6$ $V(r)=V_0/r^n=V_0/r^6$

$\mu=6m_e$ $n=6+1=7$ $V(r)=V_0/r^n=V_0/r^7$

$5m_e$ and $6m_e$ transfer generate the Van der Waals potentials, although they were traditionally introduced by some sophisticated methods.

These are simply given by the GPT method !,

III Long- & Short Range Three-Body Force Potential

Let us define a three-body force by a kind of continued fraction.

3-Body Force Potential:

(3-identical)

$$V \propto \frac{1}{b_0 + (V_{12}{}^{-1} + V_{23}{}^{-1} + V_{31}{}^{-1})}$$

3-Body Force Potential

for channel-1 $\Rightarrow$ channel-2

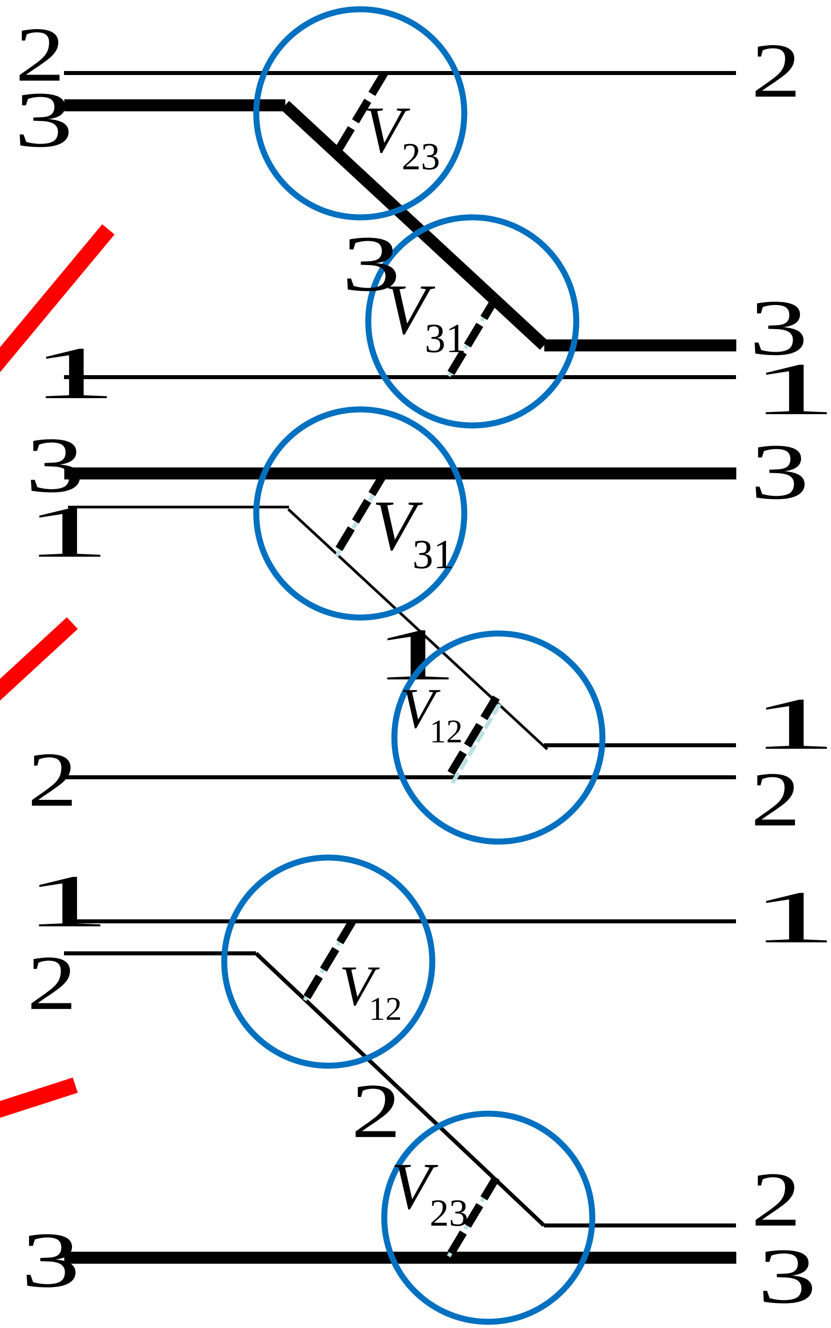
$$V_{\mathbf{3BF}}^{12} = \frac{1}{b_0 + (V_{23}{}^{-1} + V_{31}{}^{-1})}$$

for channel-2 $\Rightarrow$ channel-3

$$V_{\mathbf{3BF}}^{23} = \frac{1}{b_0 + (V_{31}{}^{-1} + V_{12}{}^{-1})}$$

for channel-3 $\Rightarrow$ channel-1

$$V_{\mathbf{3BF}}^{31} = \frac{1}{b_0 + (V_{12}{}^{-1} + V_{23}{}^{-1})}$$



.

$$V^{3BLF} = \frac{1}{1+V_{12}^{-1}+V_{23}^{-1}+V_{31}^{-1}} \Rightarrow \left[1+\sum_{i\neq j=1}^{3}\left\{V_{ij0}a_{ij}/r_{ij}^{\ 2}\right\}^{-1}\right]^{-1}$$

$$= \frac{1}{1+r_{12}^2/(V_0 a)_{12}+r_{23}^2/(V_0 a)_{23}+r_{31}^2/(V_0 a)_{31}} \equiv \frac{1}{1+\alpha_{12}r_{12}^2+\alpha_{23}r_{23}^2+\alpha_{31}r_{31}^2}$$

$$V^{3BSF} = \frac{1}{1+V_{12}^{-1}+V_{23}^{-1}+V_{31}^{-1}} \Rightarrow \left[1+\sum_{i\neq j=1}^{3}\left\{W_{ij0}e^{-b_{ij\leq}r_{ij}^{\ 2}}\right\}^{-1}\right]^{-1}$$

$$\approx W_0 \exp\left[-\frac{b_{12}r_{12}^{\ 2}+b_{23}r_{23}^{\ 2}+b_{31}r_{31}^{\ 2}}{a_t^{\ 2}}\right]$$

Three-body nonlinear long and short range forces could be joined by the GPT potential.

The linear interactions are satisfied only for the intermediate region, but in a very long and a short range regions nonlinear forces could be required.

Summary

１）A Coulomb scattering problem by the L-S equation can not be solved by the pinching singularity at the threshold. By the same way, the three-body AGS equation has a pinching singularity at the Q2T.   Both singularities come from a long range property of the interaction.

クーロン散乱の運動量表示では，Ｌｉｐｐｍａｎｎ－Ｓｃｈｗｉｎｇｅｒ方程式の閾値における pinching-singularity（狭窄発散）のため解けない。一方，ハドロン系の３体散乱においても，３体AGS方程式は準２体閾値(Q2T)で，同様の発散を 生ずる。

両者とも発散は長距離ポテンシャルに起因している。

２）By smoothing the singularity of the AGS equation, a general particle transfer (GPT) potential $a^{n-1}/[r(r+a)^{n-1}]$ is obtained, where the Euler integral of the second kind was done with the weight function: $\sigma^{n-2}\, e^{-a\sigma}$ (by a=1/m_π).

この発散をπ中間子の到達距離まで smoothingをする。a=1/m_π とし, $\sigma^{n-2}\, e^{-a\sigma}$ の重みを掛けた積分(オイラー変換) をするとポテンシャルは　　$a^{n-1}/[r(r+a)^{n-1}]$　：GPT-potential

３）We found a relation between the index number and the masses of the general particles:　$n=\mu/m_{\pi}+1$
交換粒子の数 n とその質量 μ の関係は　$n=\mu/m_{\pi}+1$

４）In the atom-molecular systems, the basic mass is an electron.　Therefore, the above index number and the mass relation is given by using the electron mass instead of the pion mass in the hadron system: ：

$$n=\mu/m_e+1$$

n=1 corresponds to μ=0, which is the photon exchange ionic bond.　1<n means covalent bond, n=6,7 are the Van der Waals potentials.

電子交換に由来する原子分子系:　$n=\mu/m_e+1$

- n=1　光子交換　イオン結合
- n=2　電子1個交換　共有結合
- n=6,7　電子5，6個交換　Van der Waals力

5）If we take negative *n*, GPT potential becomes a confinement potential: $[r(r+a)]^{|n|+1} / (a^{|n|+1} r)$. Poles of the confinement potential appear in the second Riemann sheet or the unphysical plane, therefore the generated particles can not be observed.

n を負にとると，閉じ込めポテンシャル:

$[r(r+a)]^{|n|+1} / (a^{|n|+1} r)$ が現れる。

この時，ポテンシャルの極が非物理的リーマン面に現れ，

生成粒子は測定にかからない？

6）In the hadron three-body problems, the generation of the mesons, and the quark degree's of freedom lead four-, five-, six- body problems. Therefore, a linear three-body problem by the Faddeev equation is only allowed in a special region, and another many body effects is taken into account to a nonlinear three-body force (3BF) potentials.

ハドロン3体問題でもメソンの生成，クオーク自由度の発生により４，５，６….体問題となり，線形３体問題として取り扱える範囲は限られる。 非線形部分は３体力として取り込まれる。

7) A part of the truncated long range potential is represented by a long range 3BF potential, while the truncated short range potential is given by a short range 3BF potential. Historical 3BF potentials（NLO, N2LO, N3LO, N4LO…）are relatively isotropic, however the GPT-based 3BF potential has an angular dependence.

切断されたポテンシャルの長距離部分は長距離3体力として短距離力は短距離3体力としてFaddeev方程式に繰り込む。GPT-3体力は，通常角度依存性を持ち従来の3体力（NLO, N2LO, N3LO, N4LO…）を補正することになる。最小パラメータの3体力（引力）で，中性子過剰核の記述も可能？

８）A new method of three-body Faddeev equation is integrated from a Q2T, while traditional one was done from the 3BT.

３体Faddeev方程式の

新しい解法： Q2Tから積分し，短距離・長距離3体力を繰り込む。

従来の方法：3BTから積分。これはBorromean系しか通用しない？

幾つかの基本的な物理量の再現すら出来ていない？

Thanks for attention！