Near-threshold hadron scattering using effective field theory

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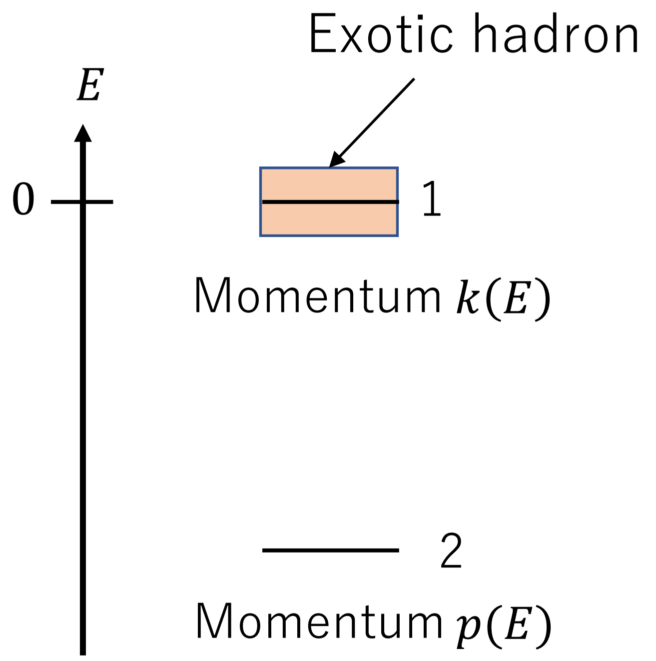
Background

Exotic hadrons ⇨ $T_{cc}, X(3872), f_0(980), a_0, P_c, Z_c$

Internal structure ⇔ Scattering lengths a and effective range r

For near-threshold exotic hadrons, channel couplings are important.



Unstable exotic hadron near the threshold of channel 1

⇨ Flatté amplitude has been used[1].

Scattering lengths a_F and effective range r_F have been determined by the Flatté amplitude[2].

a and r in more general framework?

[1] R. Aaij et al. [LHCb], Phys. Rev. D 102, no.9, 092005 (2020)

[2] V. Baru et al., Eur. Phys. J. A, 23, 523-533 (2005)

Flatté amplitude

The Flatté amplitude

$$f^F = h(E)\begin{pmatrix} g_1^2 & g_1 g_2 \\ g_1 g_2 & g_2^2 \end{pmatrix}$$

The Flatté parameters

g_1, g_2 : Real coupling constants

E_{BW} : Bare energy

The Flatté amplitude has the threshold effect.

$$h(E) = -\frac{1}{2}\frac{1}{E - E_{BW} + i\, g_2^2 p(E)/2 + i g_1^2\, k(E)/2}$$

f_{11}^F, f_{22}^F can be written as the effective range expansion in k.

$$f_{11}^F, f_{22}^F \propto \left(-\frac{1}{a_F} + \frac{1}{2} r_F k^2 - ik + O(k^4)\right)^{-1}$$

a_F : Scattering lengths

r_F : Effective range

a_F and r_F

We consider the region near the threshold 1(region Ⅱ and Ⅲ).

1→1 scattering does not occur in region Ⅱ.

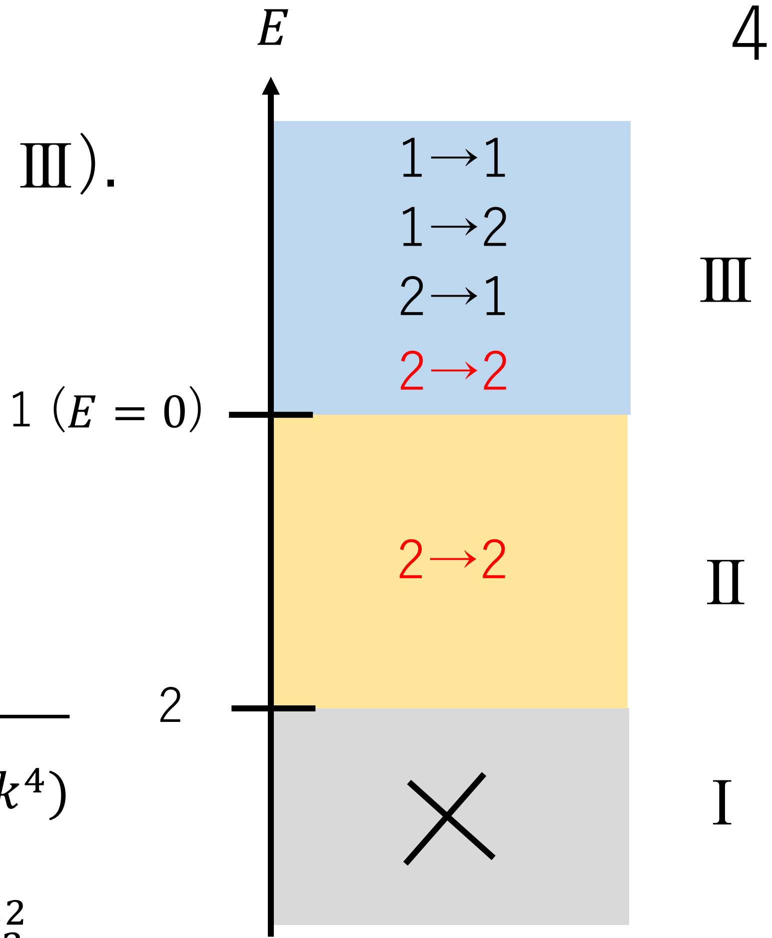
2→2 scattering occurs in both region Ⅱ and Ⅲ.

⇨ a_F, r_F are determined from f_{22}^F (ex. [3][1] for $X(3872)$).

$$f_{22}^F = \frac{\frac{g_2^2}{g_1^2}}{\left(\frac{2E_{BW}}{g_1^2} - i\frac{g_2^2}{g_1^2}p_0\right) - \left(\frac{2}{m_k g_1^2} + i\frac{g_2^2}{2p_0 g_1^2}\right)k^2 - ik + O(k^4)}$$

$$a_F = -\frac{g_1^2}{2E_{BW} - ig_2^2 p_0} \qquad r_F = -\frac{4}{m_k g_1^2} + i\frac{g_2^2}{p_0 g_1^2}$$

Scattering length　　　Effective range



[3] A. Esposito et al., Phys. Rev. D 105 (2022),
[1] R. Aaij et al. [LHCb], Phys. Rev. D102, no.9, 092005 (2020)

EFT amplitude

As more general framework, we consider the effective field theory(EFT).

The effective field theory(EFT)
- Nonrelativistic
- Contact interaction
- Two channels

The scattering amplitude derived from EFT[4]

$$f^{EFT}(E) = \left\{ \frac{1}{a_{12}^2} - \left(\frac{1}{a_{22}} + ip(E) \right) \left(\frac{1}{a_{11}} + ik(E) \right) \right\}^{-1} \begin{pmatrix} \left(\frac{1}{a_{22}} + ip(E) \right) & \frac{1}{a_{12}} \\ \frac{1}{a_{12}} & \left(\frac{1}{a_{11}} + ik(E) \right) \end{pmatrix}$$

a_{11}, a_{12}, a_{22} : Three EFT parameters

f^{EFT} satisfies the optical theorem with channel couplings.

[4] T.D. Cohen et al. , Phys. Lett. B 588 (2004)

f^{EFT} components

Effective range expansion for f^{EFT}

$$f_{11}^{EFT} = \frac{1}{\underbrace{\left(\frac{1}{\frac{a_{12}^2}{a_{22}} + ip_0 a_{12}^2} - \frac{1}{a_{11}}\right)}_{= -\frac{1}{a_{EFT}}} - \underbrace{\frac{i}{2\left(\frac{a_{12}}{a_{22}} + ip_0 a_{12}\right)^2 p_0}}_{= \frac{r_{EFT}}{2}} k^2 + O(k^4) - ik}$$

f_{11}^{EFT} can be written as the effective range expansion in k.

$$f_{22}^{EFT} = \frac{\left(\frac{a_{12}}{a_{11}}\right)^2}{\left(\frac{1}{a_{11}} - \frac{a_{12}^2}{a_{11}^2 a_{22}} - i\frac{p_0 a_{12}^2}{a_{11}^2}\right) - \left(a_{11} + i\frac{a_{12}^2}{2p_0 a_{11}^2}\right) k^2 - ik + \underline{O(k^3)}}$$

f_{22}^{EFT} cannot be written as the effective range expansion in k.

The correct scattering length and effective range must be defined by f_{11}.

Analytic comparison

We discuss the validity of a_F and r_F determined in f_{22}^F in Flatté amplitude

a_{EFT} and r_{EFT} in EFT amplitude

$$a_{EFT} = \frac{a_{11}a_{12}^2(1+ip_0a_{22})}{a_{12}^2(1+ip_0a_{22}) - a_{11}a_{22}} \qquad r_{EFT} = -\frac{i}{p_0}\left\{\frac{a_{22}}{a_{12}(1+ip_0a_{22})}\right\}^2$$

a_F and r_F in Flatté amplitude with EFT parameters a_{11}, a_{12}, a_{22}

$$a_F = \frac{a_{11}^2 a_{12}}{a_{12}^2(1+ip_0a_{22}) - a_{11}a_{22}} \qquad r_F = -2a_{11} - i\frac{a_{12}^2}{p_0 a_{11}^2}$$

a_F and r_F are analytically different from a_{EFT} and r_{EFT}.

Flatté parameters

Black lines represent the cross sections by the mock data.

$f^F(g_1^2, g_2^2, E_{BW})$ ⇨ $f^F(R, \alpha)$ (near the threshold)[2]

$\sigma_{11}^F(R, \alpha)$ $(E > 0)$ (1)

$\sigma_{22}^F(R, \alpha)$ $(E > 0)$ (2)

$\sigma_{22}^F(R, \alpha)$ $(E < 0)$ (3) $(\kappa = i|k|)$

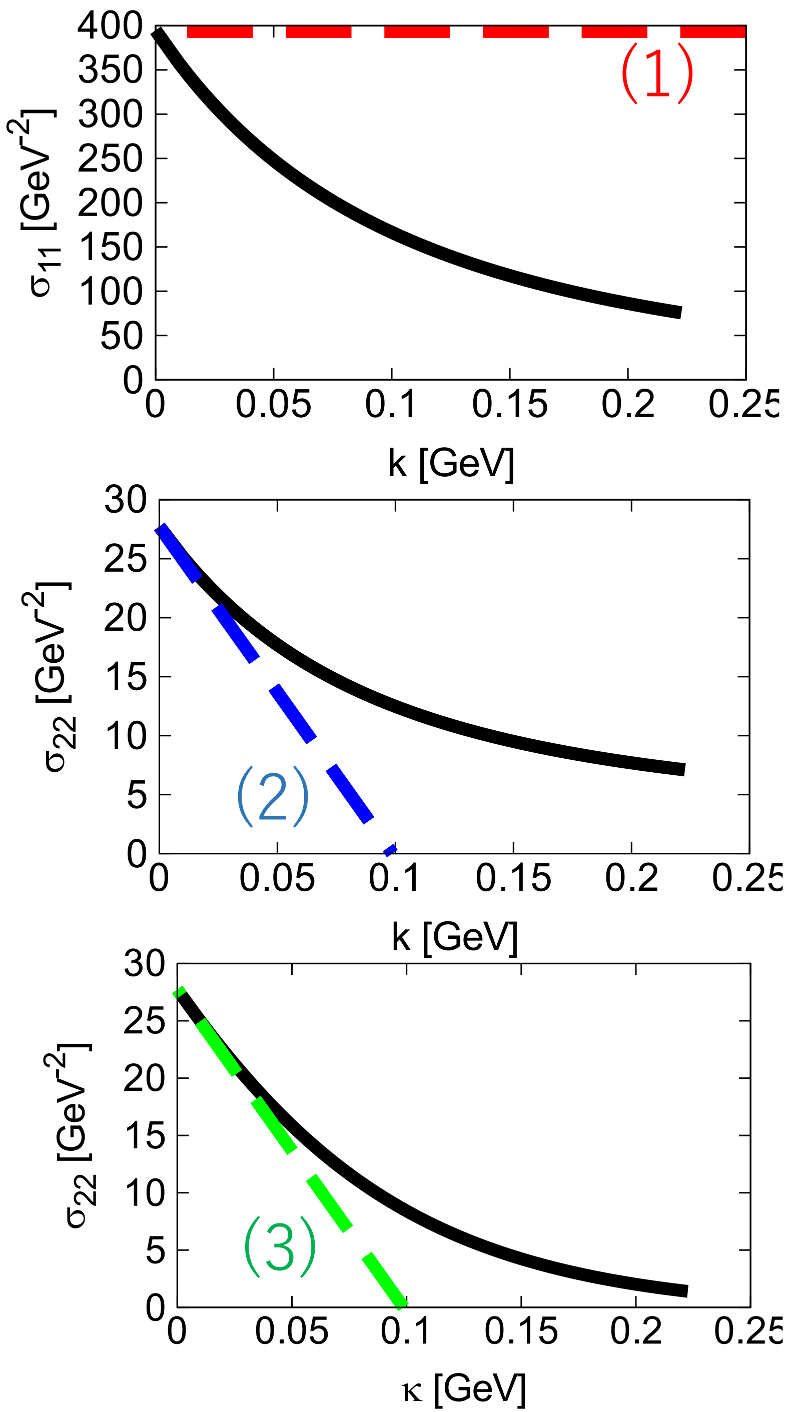
$$\alpha = \frac{2E_{BW}}{g_2^2 p_0}, \quad R = \frac{g_1^2}{g_2^2}$$

The parameters(R, α) are determined by slopes of cross section (2), (3) [2].

⇨ Calculated σ_{11}^F, does not reproduce the mock data.

The cross section σ^F is written by only two parameters.

σ_{11}^F and σ_{22}^F cannot be described simultaneously.



[2] V. Baru et al., Eur. Phys. J. A, 23, 523-533 (2005)

EFT parameters

The EFT cross section σ^{EFT} is written by three parameters(a_{11}, a_{12}, a_{22}) near the threshold.

$$\sigma_{11}^{EFT}(a_{11}, a_{12}, a_{22}) \qquad (E > 0) \qquad (1)$$

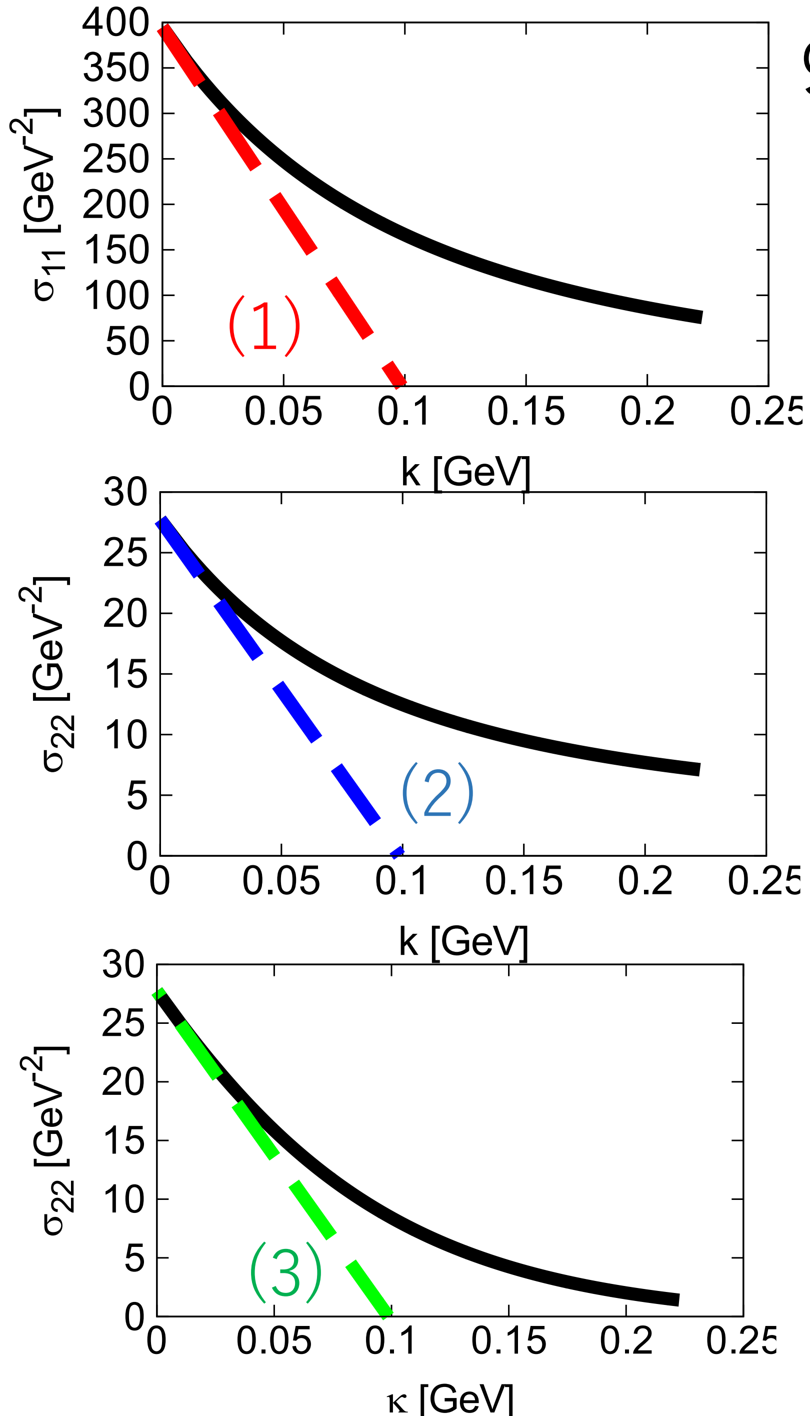
$$\sigma_{22}^{EFT}(a_{11}, a_{12}, a_{22}) \qquad (E > 0) \qquad (2)$$

$$\sigma_{22}^{EFT}(a_{11}, a_{12}, a_{22}) \qquad (E < 0) \qquad (3) \qquad (\kappa = i|k|)$$

The parameters(a_{11}, a_{12}, a_{22}) are determined by slopes of cross section (1), (2), (3).

The cross section σ^{EFT} is written by three parameters.

σ_{11}^{EFT} and σ_{22}^{EFT} can be described simultaneously.



Summary

- We compare a_F, r_F with a_{EFT}, r_{EFT} analytically.

⇨ a_F and r_F are different from a_{EFT} and r_{EFT} analytically.

- We determine the parameters from slopes of the cross section.

⇨ Flatté : The cross section σ^F is written by only two parameters.
σ_{11}^F and σ_{22}^F cannot be described simultaneously.

EFT : The cross section σ^{EFT} is written by three parameters.
σ_{11}^{EFT} and σ_{22}^{EFT} can be described simultaneously.

We must use the scattering amplitude derived from EFT satisfying the optical theorem, for analysis of the experimental data.

Pole position

Using parameters Ref.[6]

Flatté pole potision

$$k_p^{F(1)} = -98.7 - 98.7i \text{ [MeV]}$$

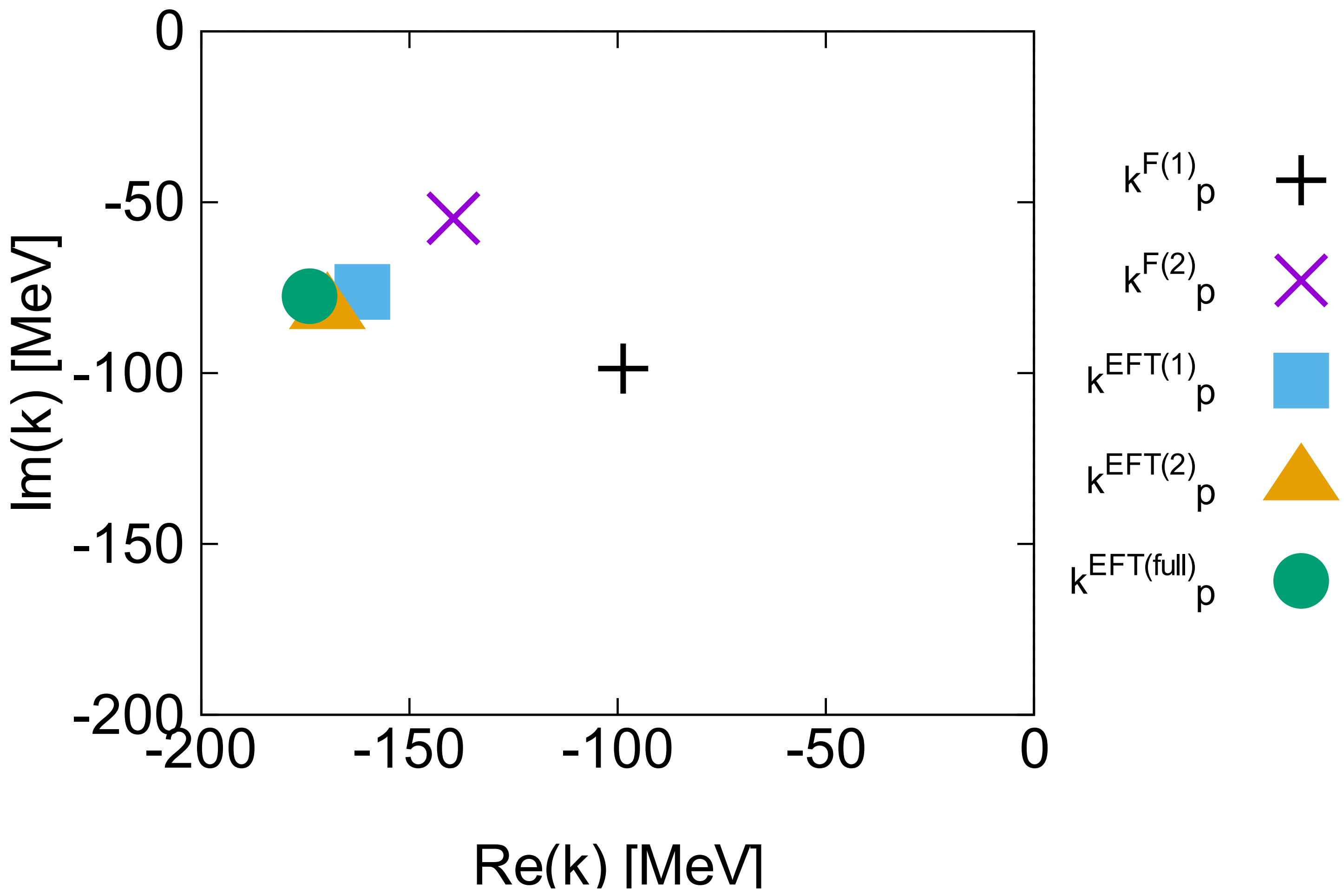
$$k_p^{F(2)} = -139 - 54.7i \text{ [MeV]}$$

EFT pole potision

$$k_p^{EFT(1)} = -161 - 76.2i \text{ [MeV]}$$

$$k_p^{EFT(2)} = -170 - 81.4i \text{ [MeV]}$$

$$k_p^{EFT(full)} = -174 - 77.5i \text{ [MeV]}$$

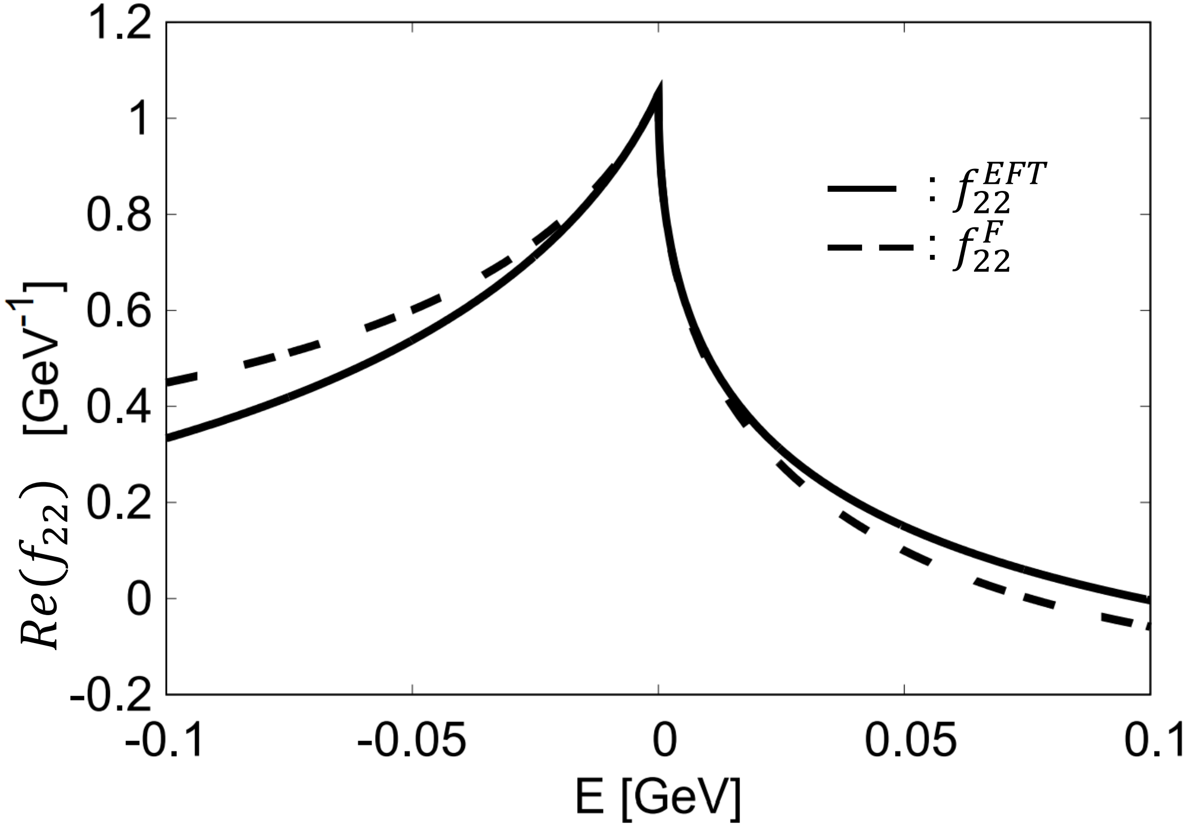


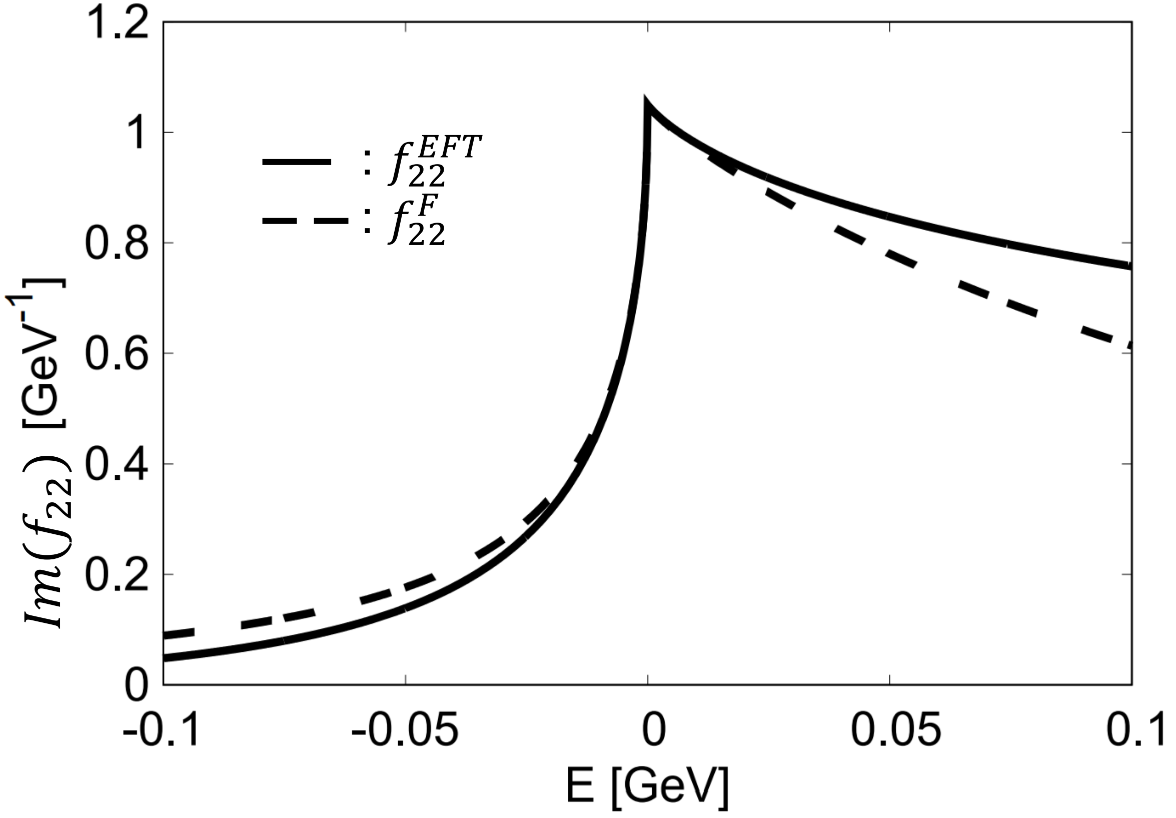
[6] R.R. Akhametshin et al., Phys. Lett B 462, 380 (1999)

Backup

$$g_1^2 = \frac{2}{m_1 a_{11}} \qquad g_2^2 = \frac{2a_{12}^2}{m_1 a_{11}^3} \qquad E_{BW} = \frac{1}{m_1}\left(\frac{1}{a_{11}^2} - \frac{a_{12}^2}{a_{11}^3 a_{22}}\right)$$

Parameter : Ref[6]





[6] R.R. Akhametshin et al., Phys. Lett B 462, 380 (1999)

Determination of Flatté parameters

$$\sigma_{22}^F = 4\pi\left|f_{22}^F\right|^2$$

$$f_{22}^F = \frac{\frac{g_2^2}{g_1^2}}{\left(\frac{2E_{BW}}{g_1^2} - i\frac{g_2^2}{g_1^2}p_0\right) - \left(\frac{2}{m_1 g_1^2} + i\frac{g_2^2}{2p_0 g_1^2}\right)k^2 - ik + O(k^4)}$$

Second reading order

$$f_{22}^F = \frac{\frac{g_2^2}{g_1^2}}{\left(\frac{2E_{BW}}{g_2^2 p_0}\frac{g_2^2 p_0}{g_1^2} - i\frac{g_2^2}{g_1^2}p_0\right) - ik} \qquad (E > 0)$$

$$f_{22}^F = \frac{\frac{g_2^2}{g_1^2}}{\left(\frac{2E_{BW}}{g_2^2 p_0}\frac{g_2^2 p_0}{g_1^2} - i\frac{g_2^2}{g_1^2}p_0\right) + \kappa} \qquad (E < 0)$$

$$f_{22}^F = \frac{\frac{1}{R}}{\left(\alpha\frac{p_0}{R} - i\frac{1}{R}p_0\right) - ik}$$

$$f_{22}^F = \frac{\frac{1}{R}}{\left(\alpha\frac{p_0}{R} - i\frac{1}{R}p_0\right) + \kappa}$$

$$\sigma_{22}^F = 4\pi\frac{\frac{1}{R}}{\left(\alpha\frac{p_0}{R} - i\frac{1}{R}p_0\right) - ik} \times \frac{\frac{1}{R}}{\left(\alpha\frac{p_0}{R} + i\frac{1}{R}p_0\right) + ik}$$

$$\sigma_{22}^F = 4\pi\frac{\frac{1}{R}}{\left(\alpha\frac{p_0}{R} - i\frac{1}{R}p_0\right) + \kappa} \times \frac{\frac{1}{R}}{\left(\alpha\frac{p_0}{R} + i\frac{1}{R}p_0\right) + \kappa}$$

$$\sigma_{22}^{F} = 4\pi \frac{1}{R} \frac{1}{(\alpha - i)\frac{p_0}{R} - ik} \times \frac{1}{R} \frac{1}{(\alpha + i)\frac{p_0}{R} + ik}$$

$$\sigma_{22}^{F} = 4\pi \frac{1}{(\alpha - i)p_0 - iRk} \times \frac{1}{(\alpha + i)p_0 + iRk}$$

$$\sigma_{22}^{F} = \frac{4\pi}{p_0^2} \frac{1}{(\alpha^2 + 1) + \frac{2Rk}{p_0}}$$

$$\sigma_{22}^{F} = \frac{4\pi}{p_0^2} \frac{1}{(\alpha^2 + 1)} \left(\frac{1}{1 + \frac{2Rk}{(\alpha^2 + 1)p_0}} \right)$$

$$\sigma_{22}^{F} = \frac{4\pi}{p_0^2} \frac{1}{(\alpha^2 + 1)} \left(1 - \frac{2Rk}{(\alpha^2 + 1)p_0} \right)$$

$$\frac{4\pi}{p_0^2} \left(\frac{-2R}{(\alpha^2 + 1)^2 p_0} \right)$$

$$\alpha = \frac{2E_{BW}}{g_2^2 p_0}$$

$$R = \frac{g_1^2}{g_2^2}$$

$$\sigma_{22}^{F} = 4\pi \frac{\frac{1}{R}}{\left(\alpha \frac{p_0}{R} - i \frac{1}{R} p_0\right) + \kappa} \times \frac{\frac{1}{R}}{\left(\alpha \frac{p_0}{R} + i \frac{1}{R} p_0\right) + \kappa}$$

$$\sigma_{22}^{F} = 4\pi \frac{1}{(\alpha - i)p_0 + R\kappa} \times \frac{1}{(\alpha + i)p_0 + R\kappa}$$

$$\sigma_{22}^{F} = \frac{4\pi}{p_0^2} \frac{1}{(\alpha^2 + 1) + \frac{2\alpha R\kappa}{p_0}}$$

$$\sigma_{22}^{F} = \frac{4\pi}{p_0^2} \frac{1}{(\alpha^2 + 1)} \left(\frac{1}{1 + \frac{2\alpha R\kappa}{(\alpha^2 + 1)p_0}} \right)$$

$$\sigma_{22}^{F} = \frac{4\pi}{p_0^2} \frac{1}{(\alpha^2 + 1)} \left(1 - \frac{2\alpha R\kappa}{(\alpha^2 + 1)p_0} \right)$$

$$\frac{4\pi}{p_0^2} \frac{1}{(\alpha^2 + 1)} \left(\frac{-2\alpha R\kappa}{(\alpha^2 + 1)p_0} \right)$$

$$f_{11}^{F} = \frac{1}{\left(\frac{2E_{BW}}{g_1^2} - i\frac{g_2^2}{g_1^2}p_0\right) - \left(\frac{2}{m_1 g_1^2} + i\frac{g_2^2}{2p_0 g_1^2}\right)k^2 - ik + O(k^4)}$$

$$f_{11}^{F} = \frac{1}{\left(\frac{2E_{BW}}{g_1^2} - i\frac{g_2^2}{g_1^2}p_0\right) - ik}$$

$$f_{11}^{F} = \frac{1}{\left(\alpha\frac{p_0}{R} - i\frac{1}{R}p_0\right) - ik}$$

$$f_{11}^{F} = Rf_{22}^{F}$$

$$\sigma_{11}^{F} = R^2\left|f_{22}^{F}\right|^2$$

$$\sigma_{11}^{F} = \frac{4\pi}{R^2 p_0^2}\frac{1}{(\alpha^2+1)}\left(1 - \frac{2Rk}{(\alpha^2+1)p_0}\right)$$

$$\frac{4\pi}{Rp_0^2}\left(\frac{-2}{(\alpha^2+1)^2 p_0}\right)$$

EFT

$$f_{11}^{EFT} = \frac{1}{\left(\frac{1}{\frac{a_{12}^2}{a_{22}} + ip_0 a_{12}^2} - \frac{1}{a_{11}}\right) - ik}$$

$$b_{11} = \frac{1}{a_{11}} \qquad b_{12} = \frac{1}{a_{12}} \qquad b_{22} = \frac{1}{a_{22}}$$

$$f_{11}^{EFT} = \frac{1}{\left(\frac{b_{12}}{b_{22} + ip_0} - b_{11}\right) - ik}$$

$$f_{11}^{EFT} = \frac{1}{\left(\frac{b_{12}^2 b_{22}}{b_{22}^2 + p_0^2} - b_{11}\right) + i\left(\frac{-b_{12}^2 p_0}{b_{22}^2 + ip_0^2}\right) - ik}$$

$$c_1 = \frac{b_{12}^2 b_{22}}{b_{22}^2 + p_0^2} - b_{11} \qquad c_2 = \frac{-b_{12}^2 p_0}{b_{22}^2 + p_0^2}$$

$$f_{11}^{EFT} = \frac{1}{c_1 + ic_2 - ik}$$

$$\sigma_{11}^{EFT} = 4\pi \times \frac{1}{c_1 + ic_2 - ik} \times \frac{1}{c_1 - ic_2 + ik}$$

$$\sigma_{11}^{EFT} = \frac{4\pi}{c_1^2 + c_2^2 - 2c_2 k}$$

$$\sigma_{11}^{EFT} = \frac{4\pi}{c_1^2 + c_2^2 - 2c_2 k}$$

$$\sigma_{11}^{EFT} = \frac{4\pi}{c_1^2 + c_2^2}\left(\frac{1}{1 - \frac{2c_2}{c_1^2 + c_2^2}k}\right)$$

$$\sigma_{11}^{EFT} \cong \frac{4\pi}{c_1^2 + c_2^2}\left(1 + \frac{2c_2}{c_1^2 + c_2^2}k\right)$$

$$\frac{8\pi c_2}{(c_1^2 + c_2^2)^2} \qquad (E > 0)$$

$$f_{22}^{EFT} = \frac{\left(\frac{a_{12}}{a_{11}}\right)^2}{\left(\frac{1}{a_{11}} - \frac{a_{12}^2}{a_{11}^2 a_{22}} - i\frac{p_0 a_{12}^2}{a_{11}^2}\right) - ik} \quad (E > 0)$$

$$f_{22}^{EFT} = \frac{1}{\left(\frac{a_{11}}{a_{12}^2} - \frac{1}{a_{22}} - ip_0\right) - i\left(\frac{a_{11}}{a_{12}}\right)^2 k}$$

$$f_{22}^{EFT} = \frac{1}{\left(\frac{b_{12}^2}{b_{11}} - b_{22} - ip_0\right) - i\left(\frac{b_{12}}{b_{11}}\right)^2 k}$$

$$f_{22}^{EFT} = \frac{1}{(c_3 - ip_0) - ic_4 k}$$

$$\sigma_{22}^{EFT} = \frac{4\pi}{c_3^2 + p_0^2 + 2p_0 c_4 k}$$

$$\sigma_{22}^{EFT} = \frac{4\pi}{c_3^2 + p_0^2}\left(\frac{1}{1 + \frac{2p_0 c_4}{c_3^2 + p_0^2}k}\right)$$

$$\sigma_{22}^{EFT} = \frac{4\pi}{c_3^2 + p_0^2}\left(1 - \frac{2p_0 c_4}{c_3^2 + p_0^2}k\right)$$

$$\frac{8\pi p_0 c_4}{(c_3^2 + p_0^2)^2}$$

$$c_3 = \frac{b_{12}^2}{b_{11}} - b_{22}$$

$$c_4 = \left(\frac{b_{12}}{b_{11}}\right)^2$$

$$f_{22}^{EFT} = \frac{\left(\frac{a_{12}}{a_{11}}\right)^2}{\left(\frac{1}{a_{11}} - \frac{a_{12}^2}{a_{11}^2 a_{22}} - i\frac{p_0 a_{12}^2}{a_{11}^2}\right) + \kappa} \quad (E < 0)$$

$$f_{22}^{EFT} = \frac{1}{\left(\frac{a_{11}}{a_{12}^2} - \frac{1}{a_{22}} - ip_0\right) + \left(\frac{a_{11}}{a_{12}}\right)^2 \kappa}$$

$$f_{22}^{EFT} = \frac{1}{\left(\frac{b_{12}^2}{b_{11}} - b_{22} - ip_0\right) + \left(\frac{b_{12}}{b_{11}}\right)^2 \kappa}$$

$$f_{22}^{EFT} = \frac{1}{(c_3 - ip_0) + c_4 \kappa}$$

$$\sigma_{22}^{EFT} = \frac{4\pi}{c_3^2 + p_0^2 + 2c_3 c_4 \kappa}$$

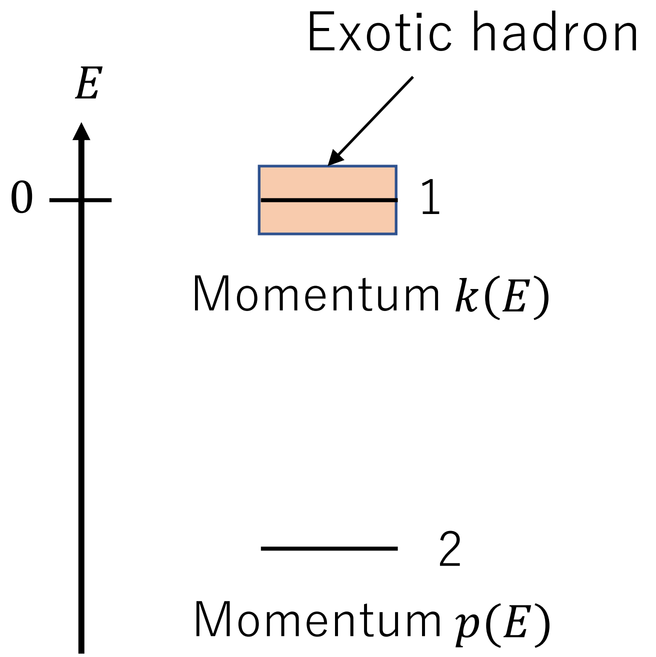
$$\sigma_{22}^{EFT} = \frac{4\pi}{c_3^2 + p_0^2}\left(\frac{1}{1 + \frac{2c_3 c_4}{c_3^2 + p_0^2}\kappa}\right)$$

$$\sigma_{22}^{EFT} = \frac{4\pi}{c_3^2 + p_0^2}\left(1 - \frac{2c_3 c_4}{c_3^2 + p_0^2}\kappa\right)$$

$$\frac{8\pi c_3 c_4}{(c_3^2 + p_0^2)^2}$$

General form

We consider the two-channel scattering.



- Conservation of probability

⇨ Optical theorem with channel couplings

⇩

$$f^{-1} = \begin{pmatrix} M_{11}(E) - ik & M_{12}(E) \\ M_{21}(E) & M_{22}(E) - ip \end{pmatrix}$$

M_{nm}: Analytic functions of E
k, p: Momentum

Flatté amplitude : $\det(f^F) = \det\begin{pmatrix} g_1^2 & g_1 g_2 \\ g_1 g_2 & g_2^2 \end{pmatrix} = 0$

⇨ **Flatté amplitude does not satisfy the optical theorem.**

Analytic comparison

We compare a_F, r_F with a_{EFT}, r_{EFT}, using EFT parameters.

Matching of f_{22} at small k.

$$f_{22}^F = \frac{\frac{g_2^2}{g_1^2}}{\left(\frac{2E_{BW}}{g_1^2} - i\frac{g_2^2}{g_1^2}p_0\right) - \left(\frac{2}{m_1 g_1^2} + i\frac{g_2^2}{2p_0 g_1^2}\right)k^2 - ik + O(k^4)} \qquad f_{22}^{EFT} = \frac{\left(\frac{a_{12}}{a_{11}}\right)^2}{\left(\frac{1}{a_{11}} - \frac{a_{12}^2}{a_{11}^2 a_{22}} - i\frac{p_0 a_{12}^2}{a_{11}^2}\right) - \left(a_{11} + i\frac{a_{12}^2}{2p_0 a_{11}^2}\right)k^2 - ik + O(k^3)}$$

⇩

$$g_1^2 = \frac{2}{m_1 a_{11}} \qquad g_2^2 = \frac{2a_{12}^2}{m_1 a_{11}^3} \qquad E_{BW} = \frac{1}{m_1}\left(\frac{1}{a_{11}^2} - \frac{a_{12}^2}{a_{11}^3 a_{22}}\right)$$

a_F and r_F can be expressed by the EFT parameter a_{11}, a_{12}, a_{22}.

$$a_F(g_1^2, g_2^2, E_{BW}) \qquad \Rightarrow \qquad a_F(a_{11}, a_{12}, a_{22})$$
$$r_F(g_1^2, g_2^2, E_{BW}) \qquad \Rightarrow \qquad r_F(a_{11}, a_{12}, a_{22})$$

Pole position

We compare the pole potion of f^{EFT} with that of f^{F}.

The EFT scattering amplitude

$$f^{EFT}(E) = \underbrace{\left\{\frac{1}{a_{12}^2} - \left(\frac{1}{a_{22}} + ip(E)\right)\left(\frac{1}{a_{11}} + ik(E)\right)\right\}}_{=0}^{-1} \begin{pmatrix} \left(\frac{1}{a_{22}} + ip(E)\right) & \frac{1}{a_{12}} \\ \frac{1}{a_{12}} & \left(\frac{1}{a_{11}} + ik(E)\right) \end{pmatrix}$$

a_{11}, a_{12}, a_{22}; EFT parameters

The Flatté scattering amplitude

$$f^{F}(E) = \underbrace{\left\{2E_{BW} - E - ig_1^2 k(E) - ig_2^2 p(E)\right\}}_{=0}^{-1} \begin{pmatrix} g_1^2 & g_1 g_2 \\ g_1 g_2 & g_2^2 \end{pmatrix}$$

E_{BW}, g_1^2, g_2^2; Flatté parameters

$k(E)$; channel 1 momentum

$p(E)$; channel 2 momentum

Flatté parameters from near-threshold data

Create mock data of the cross section from EFT amplitude which satisfies the optical theorem.

$$\sigma = \int f f^* d\Omega = 4\pi |f|^2$$

Fit the cross section near the threshold($E \cong 0$) by the Flatté amplitude.

$1/f_{22}^F$ up to order k^1 can be written by only two parameters R, α[2].

$$f_{22}^F = \frac{\frac{g_2^2}{g_1^2}}{\left(\frac{2E_{BW}}{g_1^2} - i\frac{g_2^2}{g_1^2}p_0\right) - ik} = \frac{\frac{1}{R}}{\left(\alpha\frac{p_0}{R} - i\frac{1}{R}p_0\right) - ik} \qquad \alpha = \frac{2E_{BW}}{g_2^2 p_0}$$

We find $1/f_{11}^F$ up to k^1 can also be written by only two parameters R, α.

$$R = \frac{g_1^2}{g_2^2}$$

$$f_{11}^F = \frac{1}{\left(\frac{2E_{BW}}{g_1^2} - i\frac{g_2^2}{g_1^2}p_0\right) - ik} = \frac{1}{\left(\alpha\frac{p_0}{R} - i\frac{1}{R}p_0\right) - ik}$$

$f^F(g_1^2, g_2^2, E_{BW})$ three parameters ⇨ $f^F(R, \alpha)$ two parameters(near the threshold)

[2] V. Baru et al. Eur. Phys. J. A, 23, 523-533 (2005)

Flatté parameters

We expand cross section up to liner in momentum.

$$\sigma_{11}^{F}(R,\alpha) = \frac{4\pi}{R^2 p_0^2}\frac{1}{(\alpha^2+1)}\left(1-\frac{2Rk}{(\alpha^2+1)p_0}\right) \qquad (E>0)$$

Below the threshold, the momentum k becomes pure imaginary

$$\sigma_{22}^{F}(R,\alpha) = \frac{4\pi}{p_0^2}\frac{1}{(\alpha^2+1)}\left(1-\frac{2Rk}{(\alpha^2+1)p_0}\right) \qquad (E>0)$$

$$\sigma_{22}^{F}(R,\alpha) = \frac{4\pi}{p_0^2}\frac{1}{(\alpha^2+1)}\left(1-\frac{2\alpha R\kappa}{(\alpha^2+1)p_0}\right) \qquad (E<0) \qquad (\kappa = i|k|)$$

The slopes $\boldsymbol{\sigma^F}$ are determined by two parameters$(\boldsymbol{R},\boldsymbol{\alpha})$ near the threshold[2].

[2] V. Baru et al. Eur. Phys. J. A, 23, 523-533 (2005)

EFT parameters from near-threshold data

Next, we also determine the EFT parameters a_{11}, a_{12}, a_{22}

$1/f_{22}^{EFT}$ up to order k^1

$$f_{22}^{EFT} = \frac{\left(\frac{a_{12}}{a_{11}}\right)^2}{\left(\frac{1}{a_{11}} - \frac{a_{12}^2}{a_{11}^2 a_{22}} - i\frac{p_0 a_{12}^2}{a_{11}^2}\right) - ik} = \frac{1}{(c_3 - ip_0) - ic_4 k} \qquad c_3 = \frac{a_{11}}{a_{12}^2} - \frac{1}{a_{22}}, \qquad c_4 = \left(\frac{a_{11}}{a_{12}}\right)^2$$

f_{22}^{EFT} is written by two parameters c_3, c_4.

$1/f_{11}^{EFT}$ up to order k^1

$$f_{11}^{EFT} = \frac{1}{\left(\frac{1}{\frac{a_{12}^2}{a_{22}} + ip_0 a_{12}^2} - \frac{1}{a_{11}}\right) - ik} = \frac{1}{c_1 + ic_2 - ik} \qquad c_1 = \frac{\frac{1}{a_{12}^2 a_{22}}}{\frac{1}{a_{22}^2} + p_0^2} - \frac{1}{a_{11}}, \qquad c_2 = \frac{-\frac{1}{a_{12}^2} p_0}{\frac{1}{a_{22}^2} + p_0^2}$$

f_{11}^{EFT} is written by different two parameters c_1, c_2.

$f^{EFT}(a_{11}, a_{12}, a_{22})$ three parameters(near the threshold)

EFT parameters

We expand cross section up to liner in momentum.

$$c_1 = \frac{\frac{1}{a_{12}^2 a_{22}}}{\frac{1}{a_{22}^2} + p_0^2} - \frac{1}{a_{11}},$$

$$\sigma_{11}^{EFT} = \frac{4\pi}{c_1^2 + c_2^2}\left(1 + \frac{2c_2}{c_1^2 + c_2^2} k\right) \qquad (E > 0)$$

$$c_2 = \frac{-\frac{1}{a_{12}^2} p_0}{\frac{1}{a_{22}^2} + p_0^2}$$

Below the threshold, the momentum k becomes pure imaginary

$$\sigma_{22}^{EFT} = \frac{4\pi}{c_3^2 + p_0^2}\left(1 - \frac{2p_0 c_4}{c_3^2 + p_0^2} k\right) \qquad (E > 0)$$

$$\sigma_{22}^{EFT} = \frac{4\pi}{c_3^2 + p_0^2}\left(1 - \frac{2c_3 c_4}{c_3^2 + p_0^2} \kappa\right) \qquad (E < 0) \qquad (\kappa = i|k|)$$

$$c_3 = \frac{a_{11}}{a_{12}^2} - \frac{1}{a_{22}}, \qquad c_4 = \left(\frac{a_{11}}{a_{12}}\right)^2$$

The EFT cross section σ^{EFT} depends on three parameters(a_{11}, a_{12}, a_{22}) near the threshold.

Numerical comparison

Application to the $\pi\pi$-$K\bar{K}$ system with $f_0(980)$ for quantitative comparison

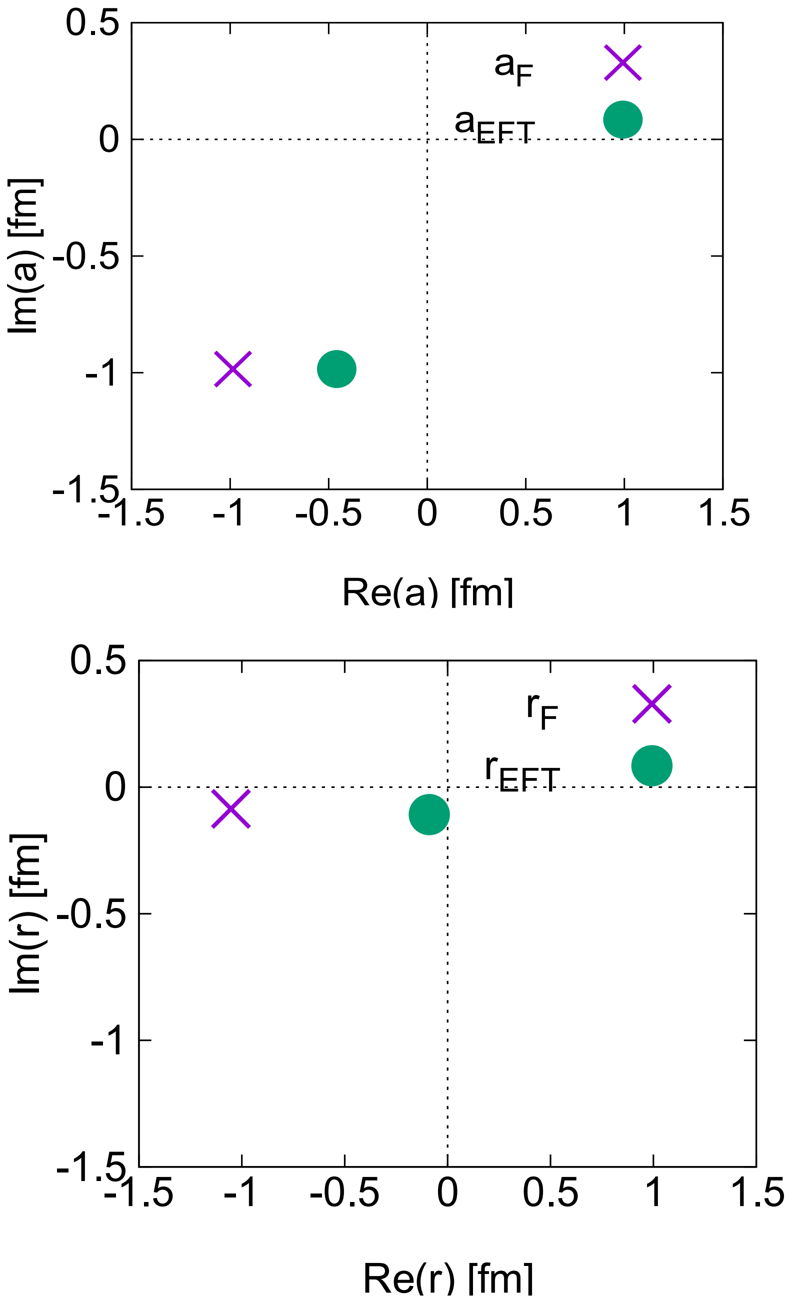
EFT parameters a_{11}, a_{12}, a_{22} corresponding to Ref.[6]

$a_{11} = 0.53$ [fm], $a_{12} = 0.24$ [fm], $a_{22} = 0.15$ [fm]

⇩

$a_F = -0.98 - 0.98i$[fm] $r_F = -1.05 - 0.08i$[fm]

$a_{EFT} = -0.45 - 0.98i$[fm] $r_{EFT} = -0.09 - 0.10i$[fm]



a_F and r_F are quantitatively different from a_{EFT} and r_{EFT} in the physical system.

[6] R.R. Akhametshin et al., Phys. Lett B 462, 380 (1999)

Pole position

$\pi\pi$-$K\bar{K}$ system with $f_0(980)$

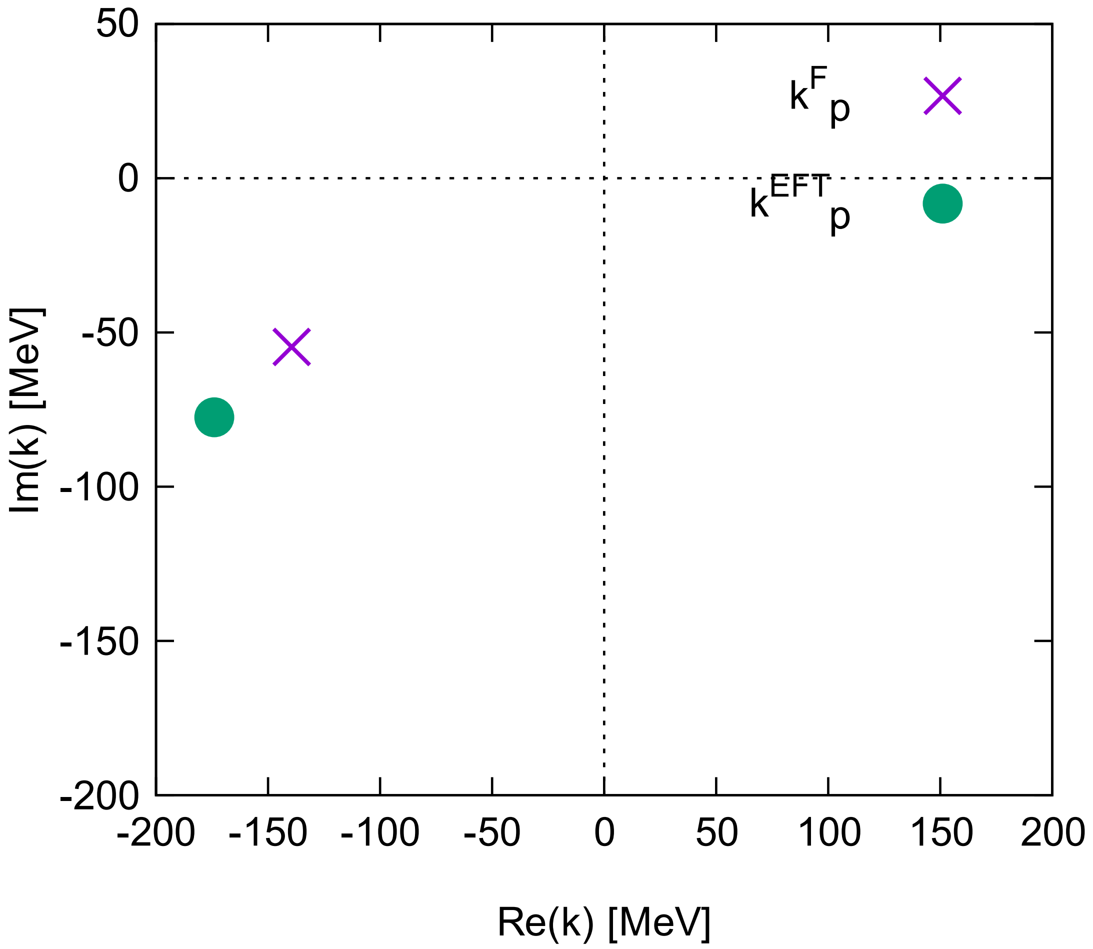
EFT parameters a_{11}, a_{12}, a_{22} corresponding to Ref.[6]

$$a_{11} = 0.53\ [\mathrm{fm}],\ a_{12} = 0.24\ [\mathrm{fm}],\ a_{22} = 0.15\ [\mathrm{fm}]$$

Numerically solve pole condition

Flatté pole : $k_p^F = -139 - 55i$ [MeV]

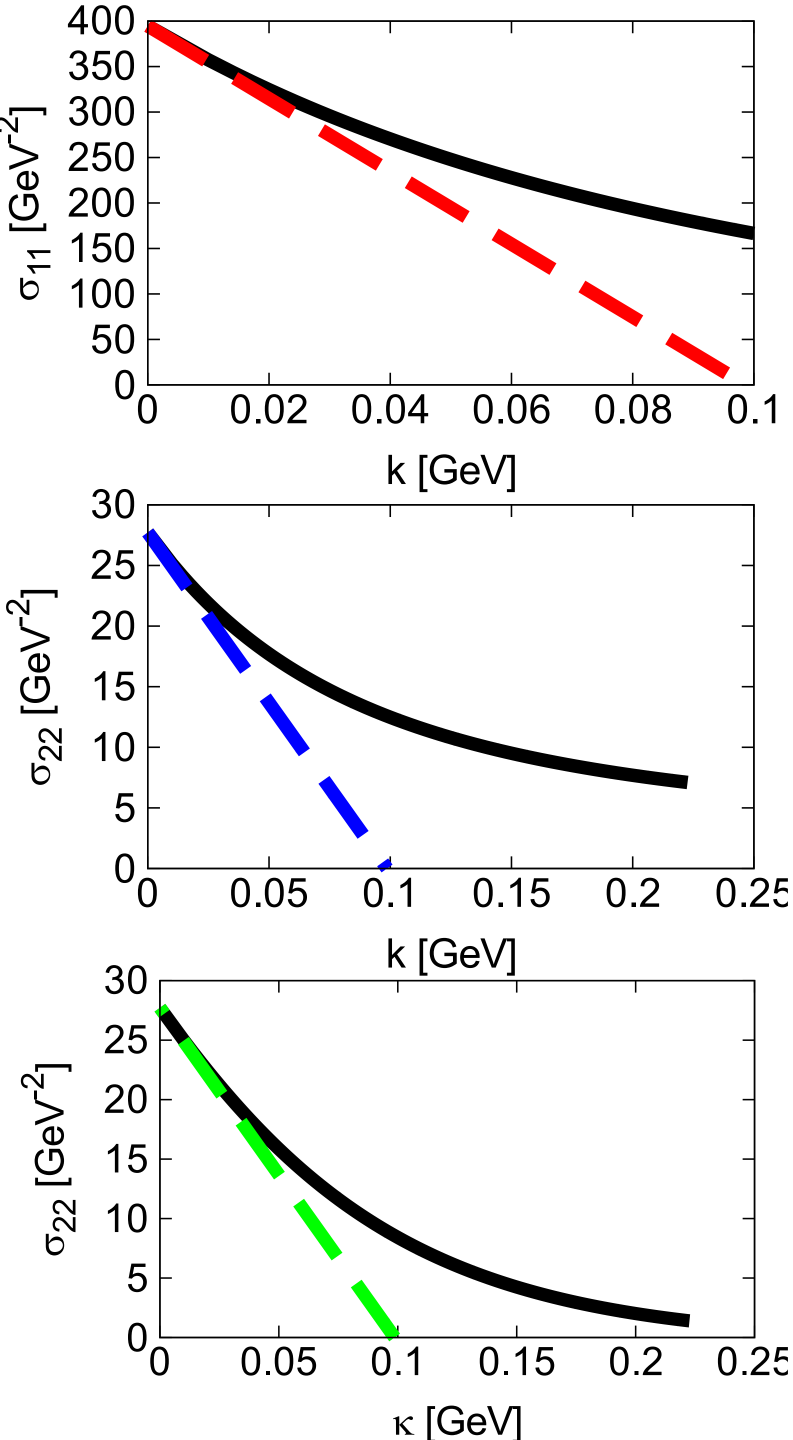
EFT pole : $k_p^{EFT} = -174 - 78i$ [MeV]



The Flatté pole position is different from the EFT pole position.

[6] R.R. Akhametshin et al., Phys. Lett B 462, 380 (1999)

Procedure



Three slopes ⇒ a_{11}, a_{12}, a_{22}

⇓

$$a_{EFT} = \frac{a_{11}a_{12}^2(1+ip_0a_{22})}{a_{12}^2(1+ip_0a_{22}) - a_{11}a_{22}}$$

$$r_{EFT} = -\frac{i}{p_0}\left\{\frac{a_{22}}{a_{12}(1+ip_0a_{22})}\right\}^2$$

a_{EFT} can be determined by only near threshold data.

r_{EFT} suffers from the higher order contributions.