

Baryon-baryon interactions by the HAL QCD method on lattice

土居孝寛 (Takahiro Doi in RCNP, Osaka Univ.)

And HAL QCD collaboration.

S. Aoki, K. Murakami (YITP),

T. Aoyama (KEK)

T. Doi, T. Hatsuda, T. Sugiura, E. Itou, L. Yan (RIKEN)

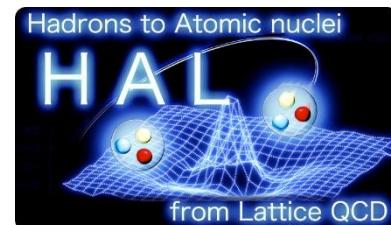
L. Yan (Peking Univ., RIKEN)

F. Etminan (Univ. of Birjand)

N. Ishii , T. M. Doi, K. Murano, H. Nemura (RCNP)

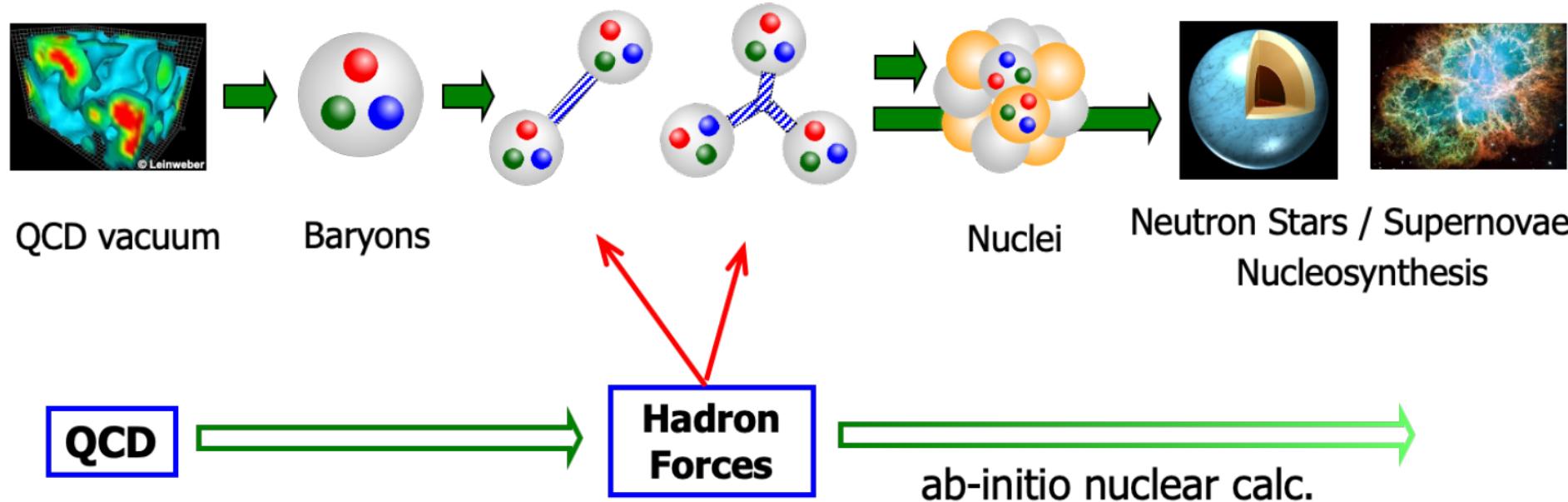
Y. Ikeda, K. Sasaki (Osaka Univ.)

T. Inoue (Nihon Univ.)



HAL QCD collaborationの目標

QCDの第一原理計算からハドロン間力を定量的に得る

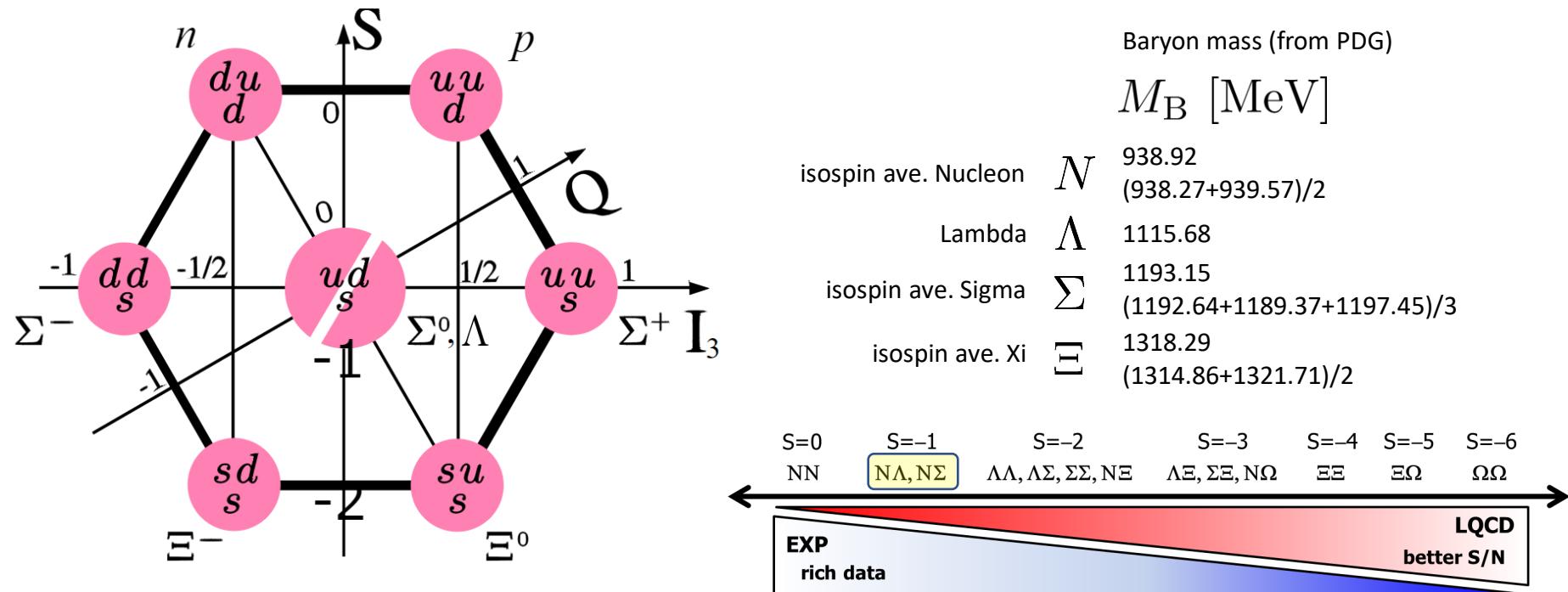


QCD作用のシミュレーションする事でハドロンの物理現象を定量的に理解
一つ上のマクロ階層である量子多体計算のインプット・実験との比較へ

Target in this study:

Baryon-Baryon interactions in S=-1 channel

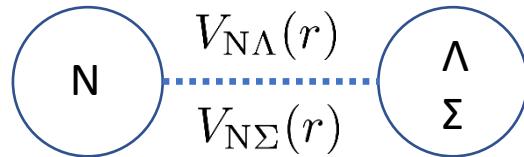
👉 S=-1: NΛ and NΣ (coupled) channel potentials



On physical-point lattice QCD configuration !

富岳でゲージ配位を生成し、富岳でバリオン間相互作用を計算した。

NΛ-NΣ ポテンシャル



◎重要性

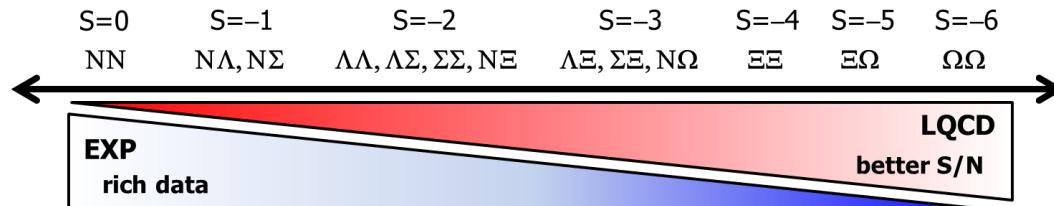
- ・原子核物理学(核子のみ)からストレンジネス核物理(核子+ハイペロン)に移行するため必須。
- ・NNの場合(核子間相互作用)と同量の実験はできないので、格子QCDから決めるることは非常に重要。
- ・ Σp 散乱実験から $N\Lambda$ 相互作用、 $N\Sigma$ 相互作用を決める実験(at J-PARC)との直接比較ができる第一原理計算はHAL potentialのみ

◎応用

- ・ハイパー核分光に直結
- ・中性子星の内部構造などをミクロな視点から理解するために応用できる

◎難しい

- ・シグナルが悪く、精密計算が難しい

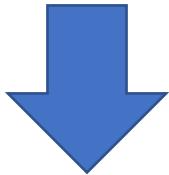


Outline

- 富岳におけるゲージ配位生成(結果のみ)
- $N\Lambda-N\Sigma$ potential
- 今後の展望



nearly physical point



physical point

π 137 [MeV]

K 502 [MeV]

K-conf.

Nf=2+1, Iwasaki gauge + clover fermion action

beta=1.82 ($1/a \simeq 2.3$ GeV)

$96^4 \leftrightarrow (8.1\text{fm})^4$

$(\kappa_{u,d}, \kappa_s) = (0.126117, 0.124790)$

$m_\pi \simeq 146\text{MeV}$, $m_K \simeq 525$ MeV

F-conf.

Nf=2+1, Iwasaki gauge + clover fermion action

beta=1.82 ($1/a \simeq 2.3$ GeV)

$96^4 \leftrightarrow (8.1\text{fm})^4$

$(\kappa_{u,d}, \kappa_s) = (0.126117, 0.124902)$

total independent conf=1600conf.

(320 conf. \times 5 run = 1600 conf.)

N 940 [MeV]

本当の物理点 (Particle Data Group 2020)

$m_{\pi^+} \simeq 139.57\text{MeV}$, $m_{\pi^0} \simeq 134.98\text{MeV}$

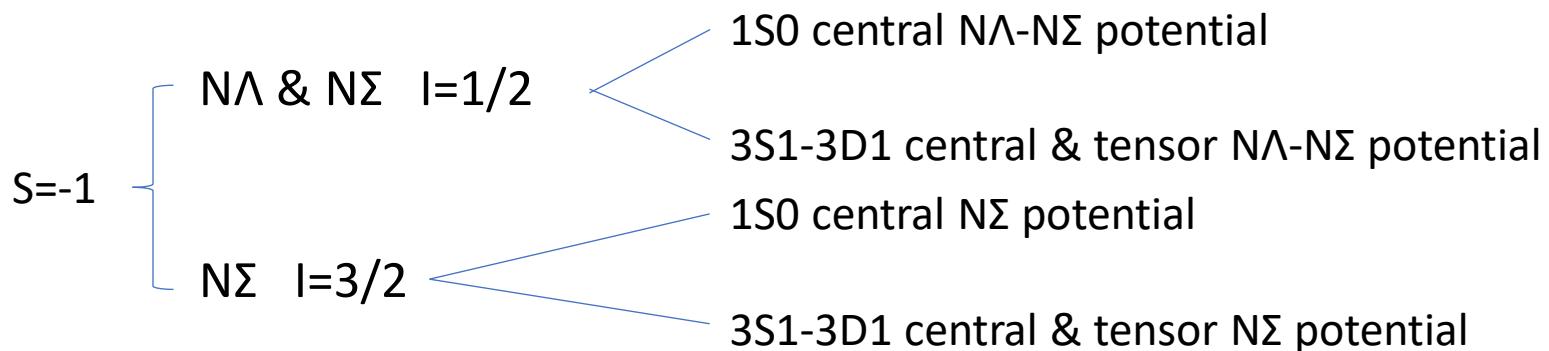
Isospin averaged pion mass $m_\pi \simeq 138.0\text{MeV}$

$m_{K^+} \simeq 493.68\text{MeV}$, $m_{K^0} \simeq 497.61\text{MeV}$

Isospin averaged Kaon mass $m_K \simeq 495.6\text{MeV}$

Outline

- ・富岳におけるゲージ配位生成(結果のみ)
- ・**NΛ-NΣ potential**
- ・今後の展望

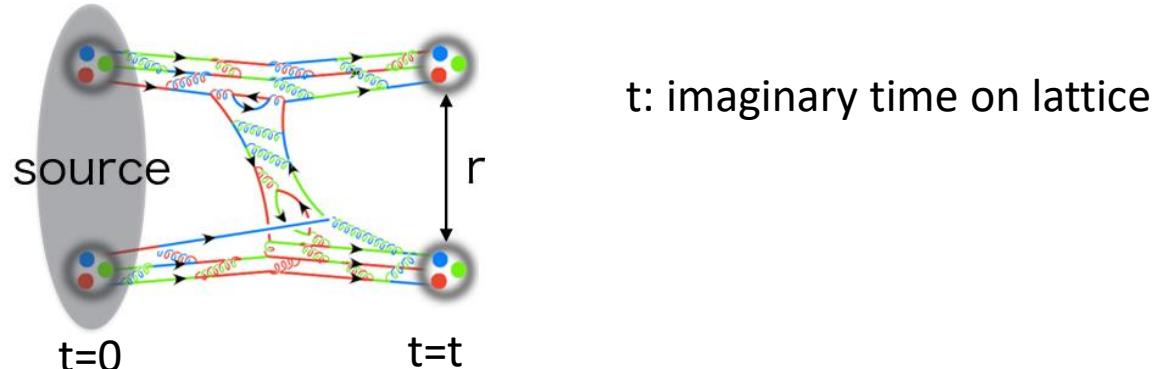


HAL QCD method

Ishii, Aoki & Hatsuda, Phys. Rev. Lett. 99 (2007) 022001
 Ishii+ [HAL QCD Coll.], Phys. Lett. B712 (2012) 437

In the case of NN potential

$$G_{NN}(\mathbf{r}, t) = \langle 0 | N(\mathbf{r}, t) N(\mathbf{0}, t) | \overline{J_{\text{src}}(t=0)} | 0 \rangle$$



Nambu-Bethe-Salpeter(NBS) wave function with relative momentum k is obtained at infinite t

$$G_{NN} \rightarrow \psi(\mathbf{r}) = \langle 0 | N(\mathbf{r}, t) N(\mathbf{0}, t) | N(k) N(-k); W \rangle$$

$$t \rightarrow \infty$$

$$\psi(\mathbf{r}) \simeq A_l \frac{\sin(kr - l\pi/2 + \delta_l(k))}{kr}, \quad r > R$$

R: interaction range

- We can extract **scattering phase shift** from NBS wave function.
- NN potential can be calculated so that Schrödinger eq. with NBS function.

(time-dependent) HAL QCD method

Ishii+ [HAL QCD Coll.], Phys. Lett. B712 (2012) 437

In the case of NN potential

$$G_{NN}(\mathbf{r}, t) = \langle 0 | N(\mathbf{r}, t) N(\mathbf{0}, t) | \overline{J_{\text{src}}(t=0)} | 0 \rangle$$

$$\begin{aligned} R(\mathbf{r}, t) &\equiv G_{NN}(\mathbf{r}, t)/G_N(t)^2 && \text{Many states contributes} \\ &= \sum_i A_{W_i} \psi_{W_i}(\mathbf{r}) e^{-(W_i - 2m)t} && i: \text{each energy eigen state} \end{aligned}$$

Under inelastic threshold, all excited scattering states share the same $U(\mathbf{r}, \mathbf{r}')$:

$$(\nabla^2 + \underline{k_{W_i}}) \psi_{W_i}(\mathbf{r}) = m \int d\mathbf{r}' U(\mathbf{r}, \mathbf{r}') \underline{\psi_{W_i}(\mathbf{r}')}$$

- All equations ($i=0, 1, 2, 3, \dots$ up to elastic threshold) can be combined as

$$\left(-\frac{\partial}{\partial t} + \frac{1}{4m} \frac{\partial^2}{\partial t^2} + \frac{\nabla^2}{m} \right) R(\mathbf{r}, t) = \int d\mathbf{r}' U(\mathbf{r}, \mathbf{r}') R(\mathbf{r}', t)$$

- Local potential is obtained by derivative expansion

$$U(\mathbf{r}, \mathbf{r}') = V_C(r) + V_T(r) S_{12} + V_{LS}(r) \mathbf{L} \cdot \mathbf{S} + \dots$$

LO

LO

NLO

Partial wave($L=0,2$) decomposition on the lattice

Method 1. A_1^+ projection of cubic group

$$R^{A_1^+}(\mathbf{r}) \equiv \frac{1}{48} \sum_{g \in O_h} R(g^{-1}\mathbf{r})$$

: This has dominant contribution from $L=0$ and small contribution from $L=4,6,\dots$



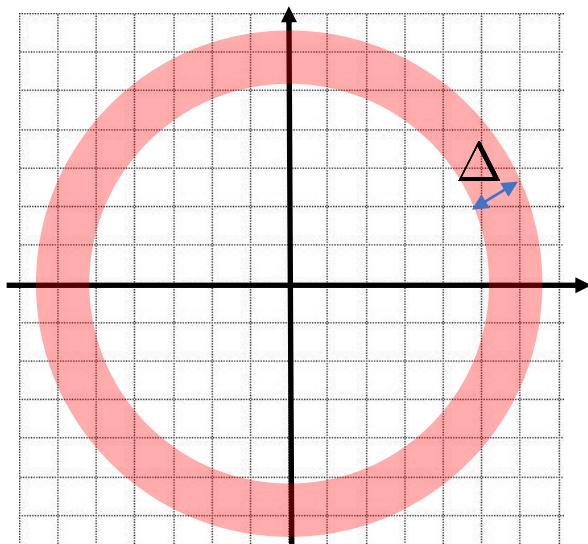
$$\text{S-wave } R_S(\mathbf{r}) = R^{A_1^+}(\mathbf{r})$$

$$\text{D-wave } R_D(\mathbf{r}) = R(\mathbf{r}) - R^{A_1^+}(\mathbf{r})$$

M. Luscher, Nucl. Phys. B 354 (1991), 531.
Aoki, Hatsuda, Ishii, PTEP 123 (2010).

Method 2. Misner's method

C. W. Misner, Class. Quant. Grav. 21 (2004) S243.
T. Miyamoto et al., Phys. Rev. D 101 (2020) 074514.



$$\text{Use } R(\mathbf{r}) = \sum_{n,l,m} c_{nlm}^\Delta G_n^\Delta(r) Y_{lm}(\theta, \phi)$$

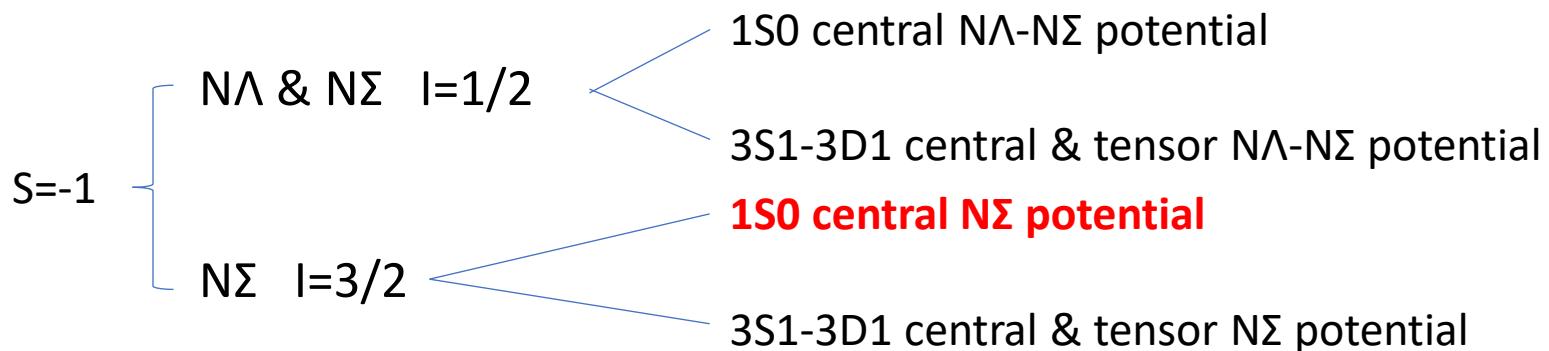
new basis function in \mathbf{r}

$$\text{instead of } R(\mathbf{r}) = \sum_{l,m} g_{lm}(r) Y_{lm}(\theta, \phi)$$

sophisticated partial wave decomposition on the lattice

Outline

- ・富岳におけるゲージ配位生成(結果のみ)
- ・**NΛ-NΣ potential**
- ・今後の展望



N Σ potential

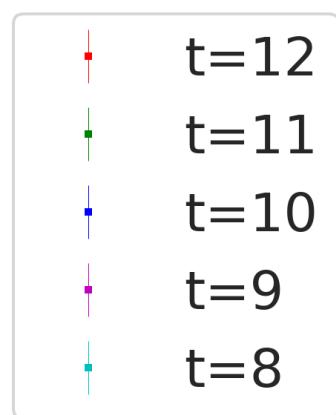
1S0, I=3/2

central

binsize=80

Nconf=800

w/ Misner

 $V(r)$ [MeV]

40

20

0

-20

-40

0

1

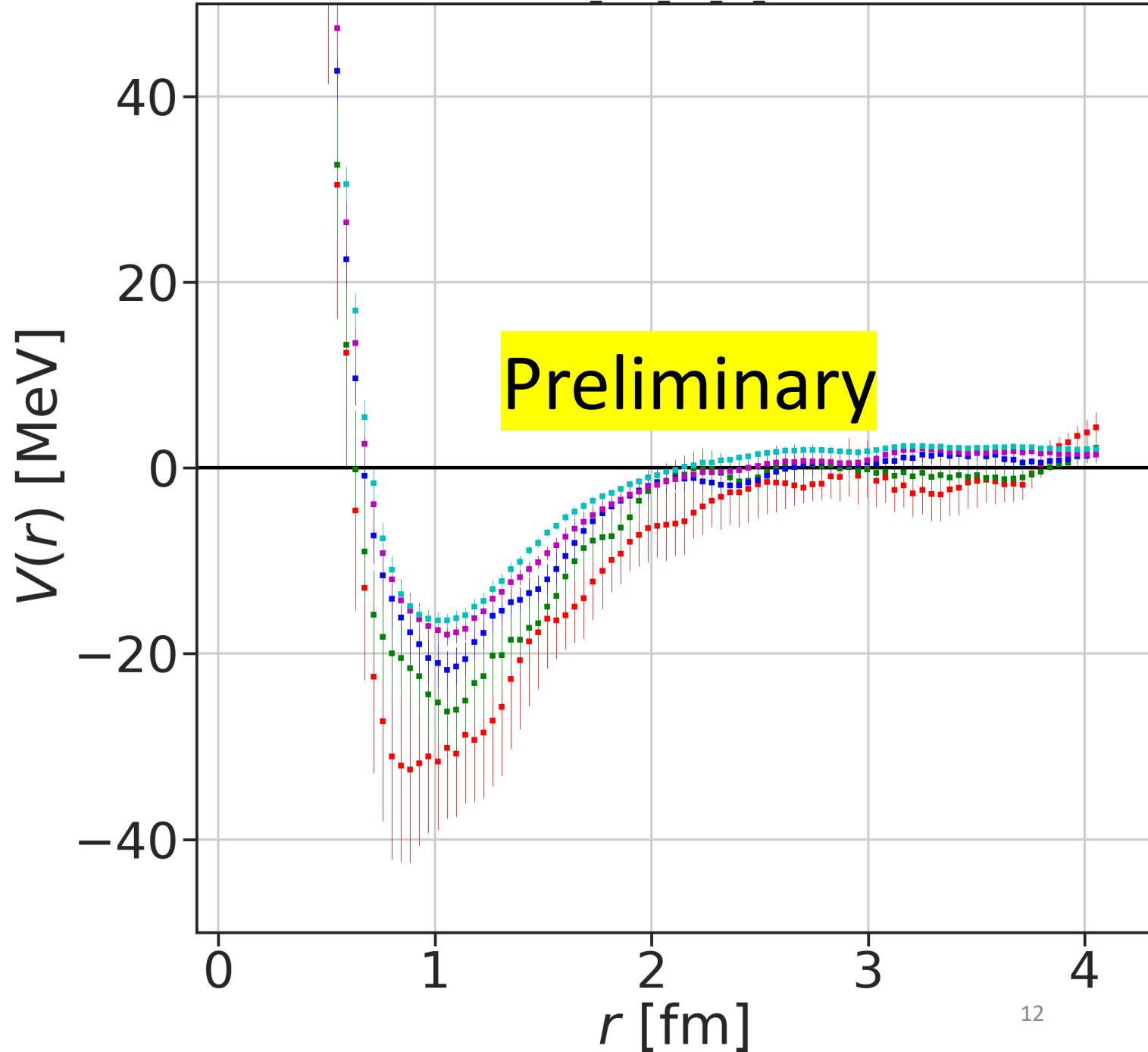
2

3

12

 r [fm]

Preliminary



NΣ potential

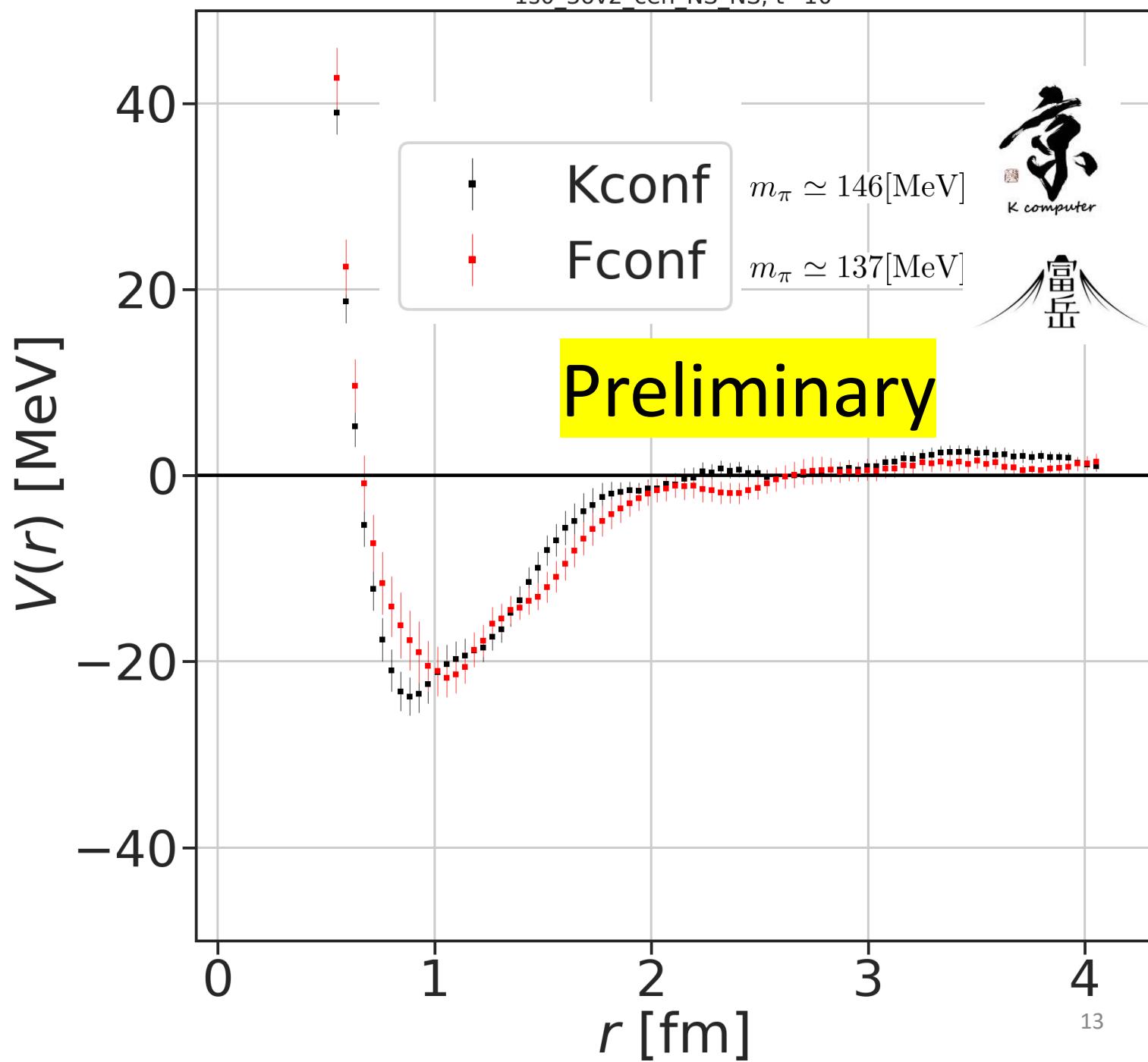
1S0, I=3/2

central

binsize=80

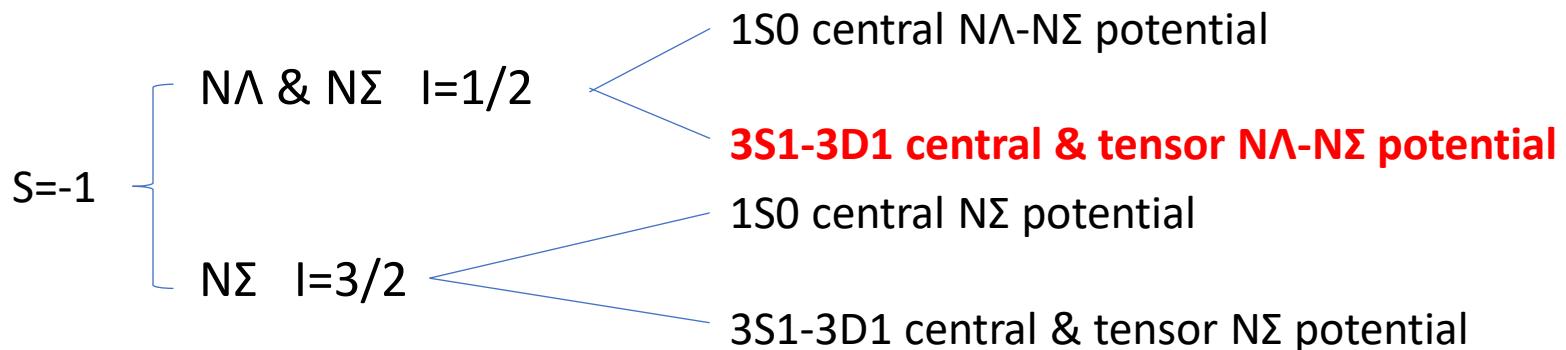
Nconf=800

w/ Misner

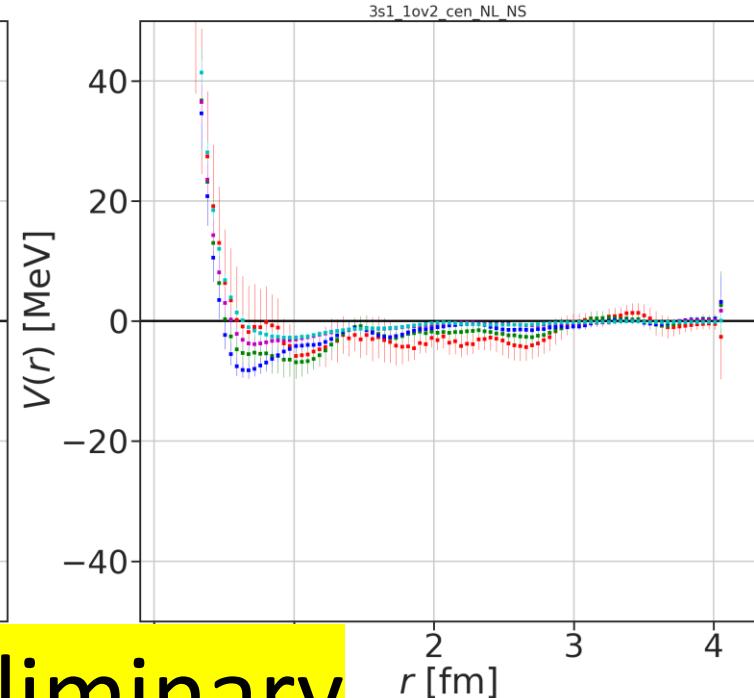
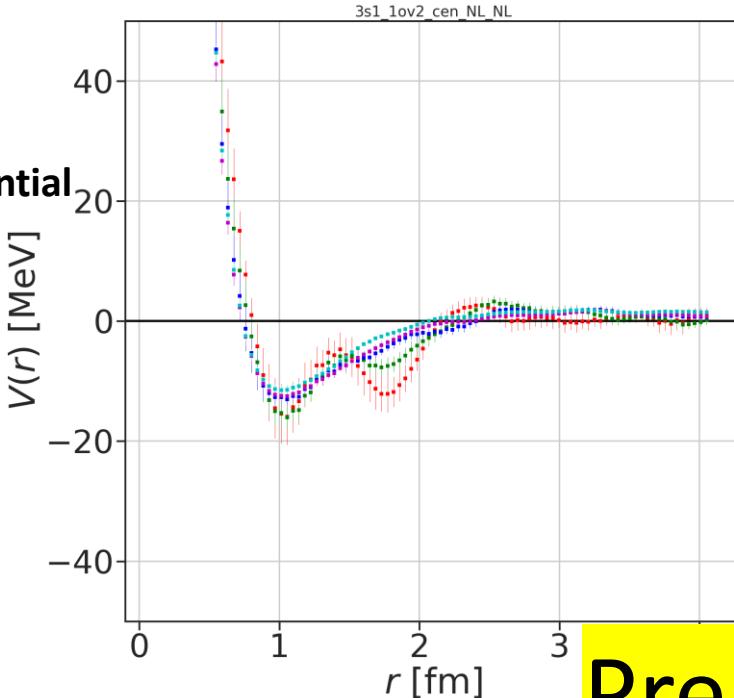


Outline

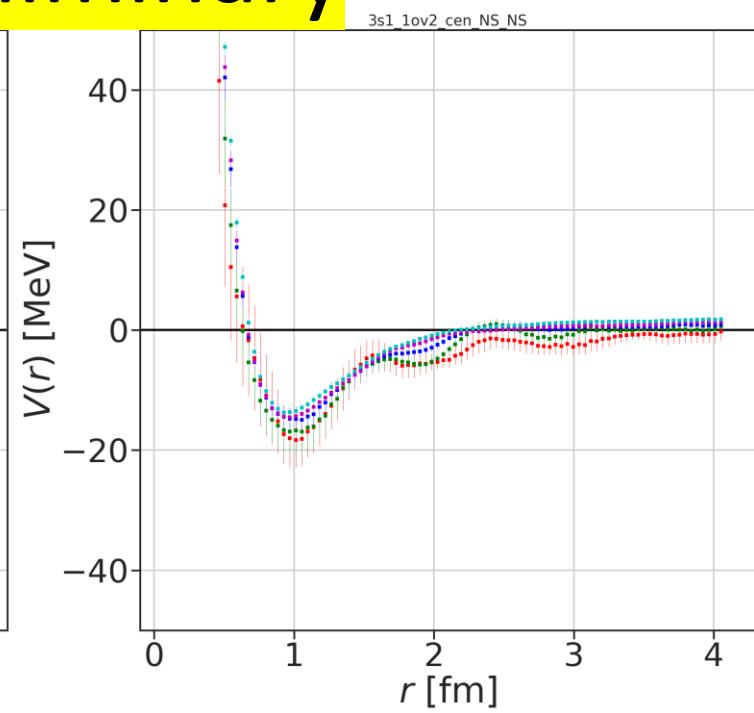
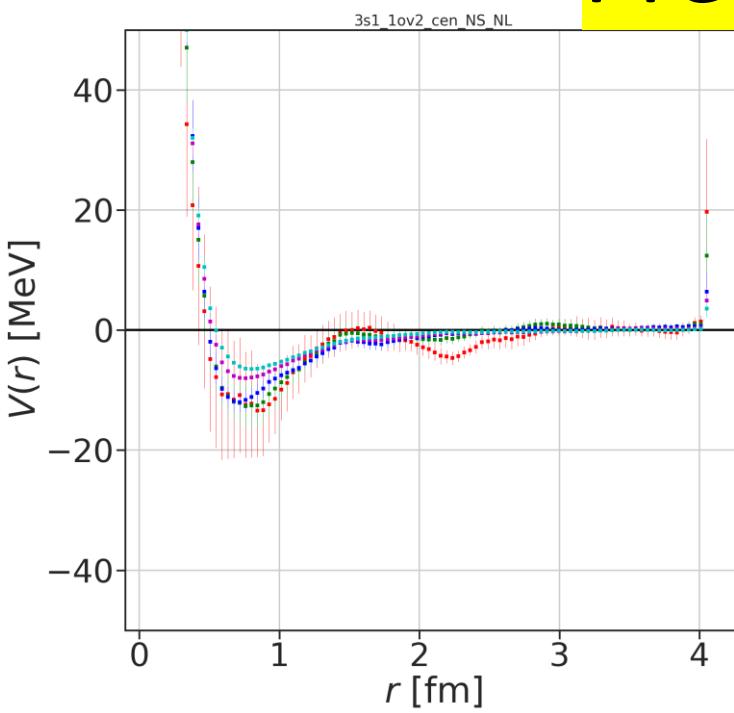
- 富岳におけるゲージ配位生成(結果のみ)
- hadron mass
- **NΛ-NΣ potential**
- 今後の展望



$\Lambda - \Sigma$
 coupled channel potential
 $3S1, l=1/2$
 central
 binsize=80
 Nconf=800
 w/ Misner

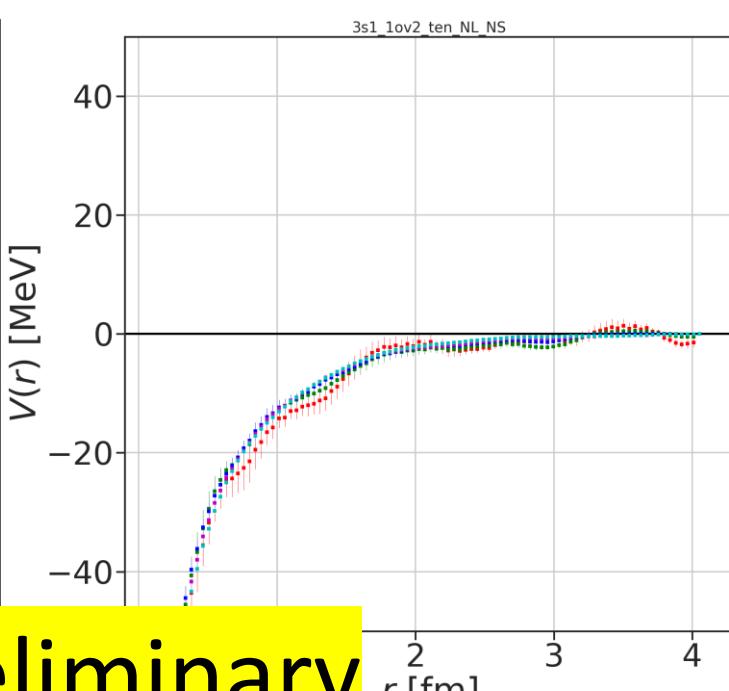
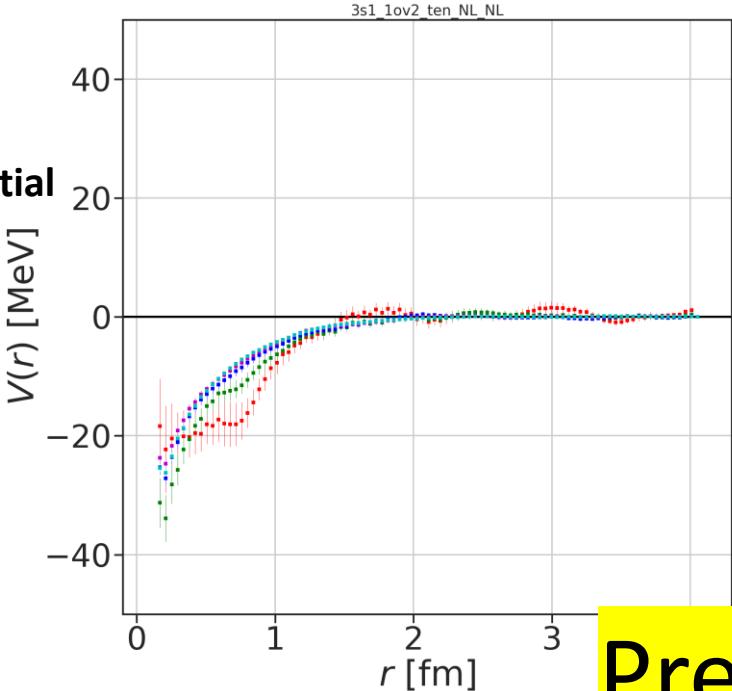


Preliminary



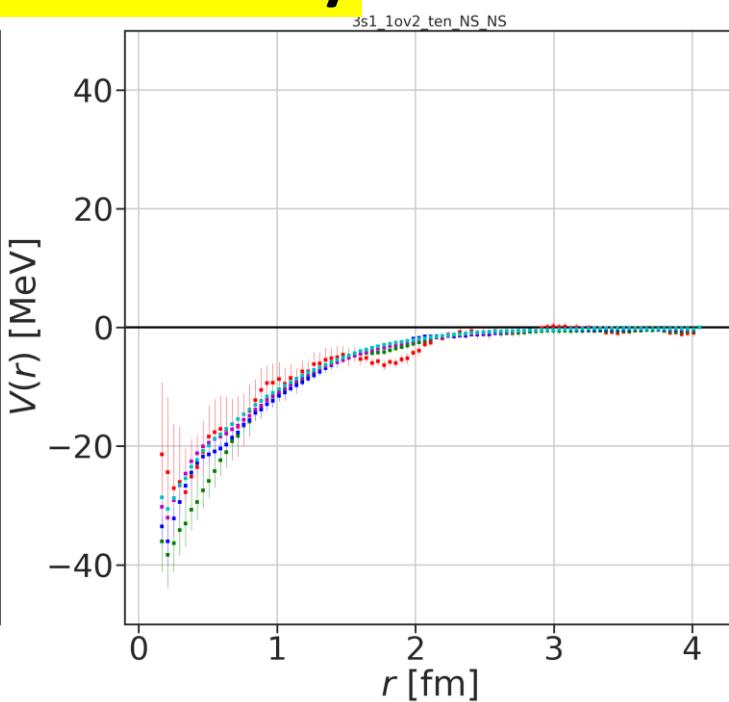
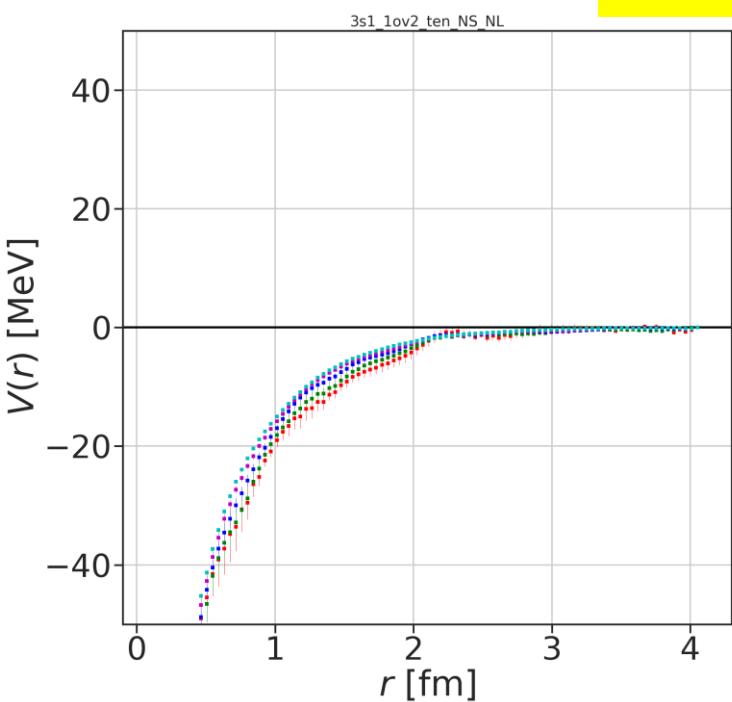
\bullet t=12
 \square t=11
 \blacksquare t=10
 \blacksquare t=9
 \blacksquare t=8

$\Lambda - \Sigma$
coupled channel potential
3S1, $l=1/2$
tensor
binsize=80
Nconf=800
w/ Misner



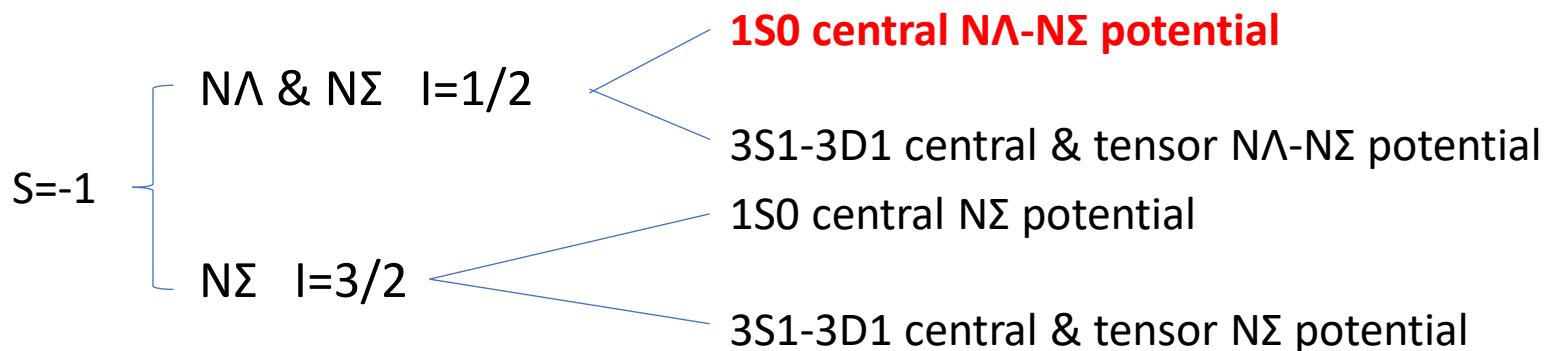
Preliminary

$t=12$
 $t=11$
 $t=10$
 $t=9$
 $t=8$

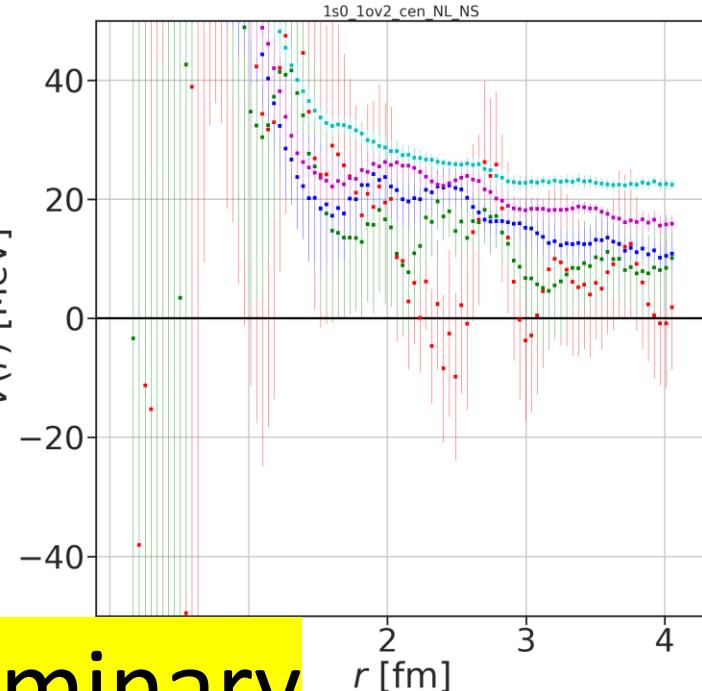
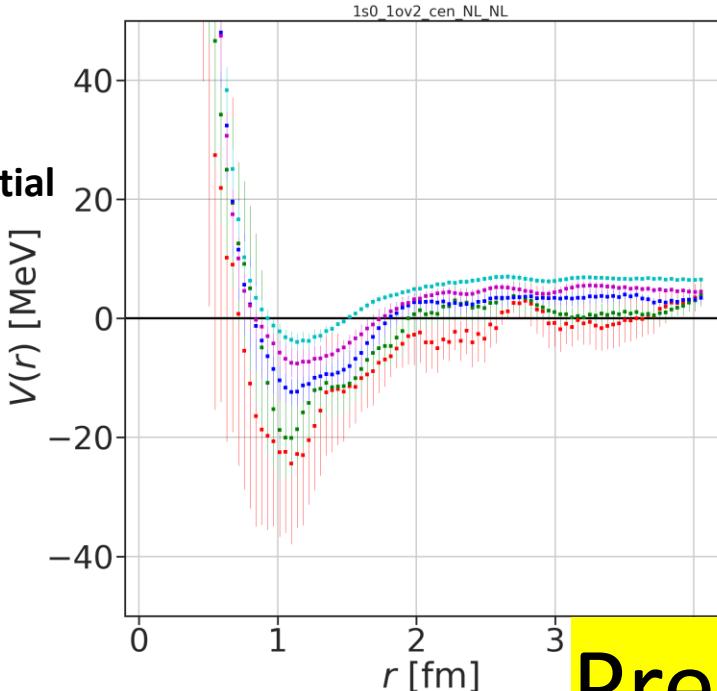


Outline

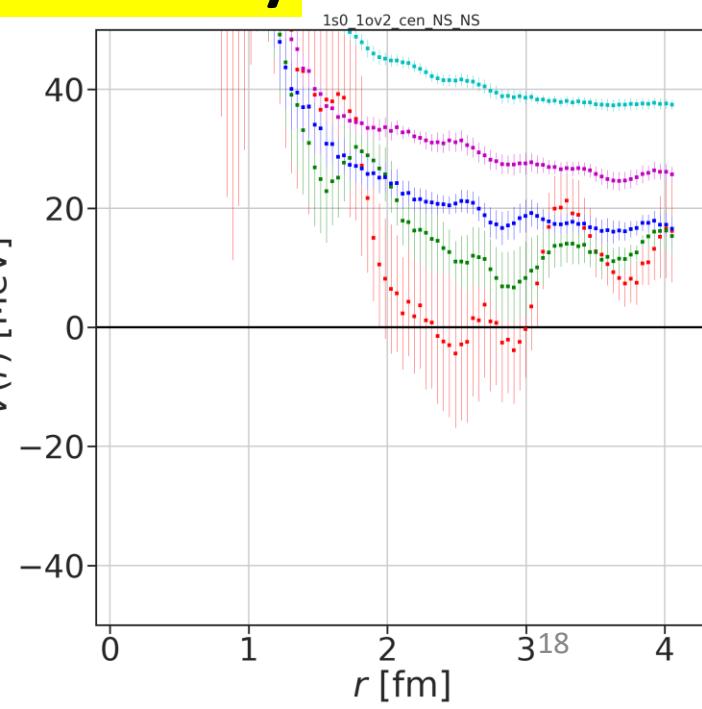
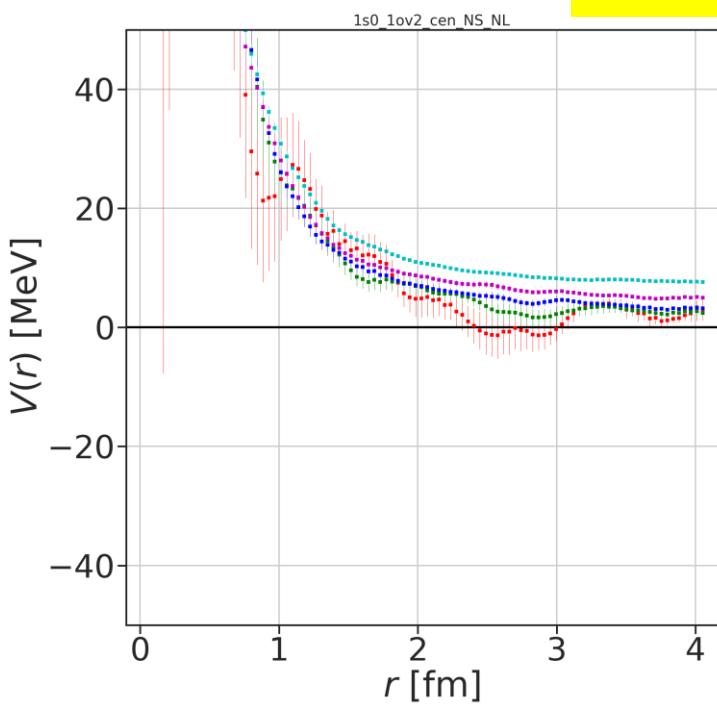
- 富岳におけるゲージ配位生成(結果のみ)
- hadron mass
- **NΛ-NΣ potential**
- 今後の展望



$N\Lambda - N\Sigma$
coupled channel potential
1S0, $I=1/2$
central
binsize=80
Nconf=800
w/ Misner



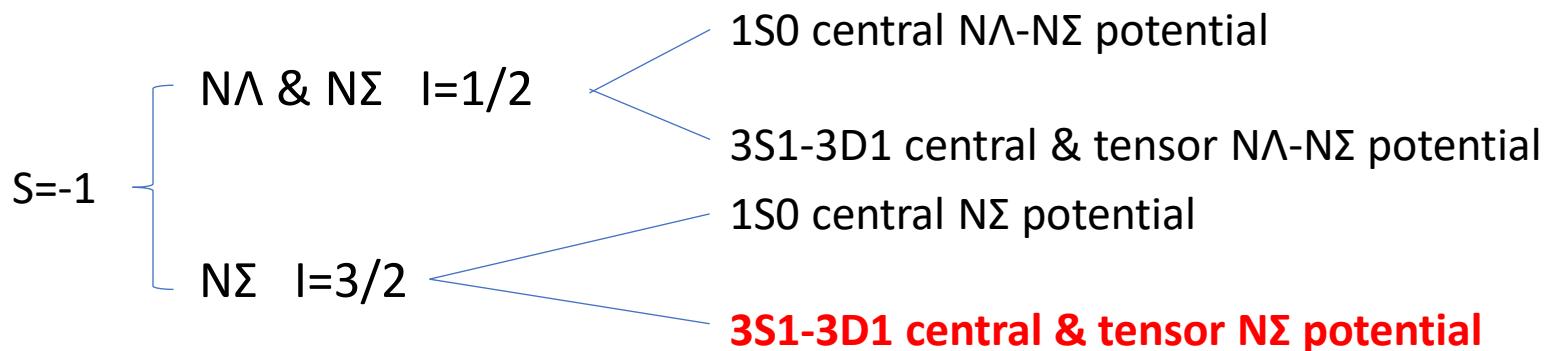
Preliminary



| | |
|---|--------|
| • | $t=12$ |
| ■ | $t=11$ |
| ■ | $t=10$ |
| ■ | $t=9$ |
| ■ | $t=8$ |

Outline

- 富岳におけるゲージ配位生成(結果のみ)
- hadron mass
- **NΛ-NΣ potential**
- 今後の展望



$N\Sigma$ potential

3S1, $I=3/2$

central

binsize=80

Nconf=800

w/ Misner

- t=12
- t=11
- t=10
- t=9
- t=8

$V(r)$ [MeV]

40

20

0

-20

-40

0

1

2

3

4

r [fm]

Preliminary

N Σ potential

3S1, I=3/2

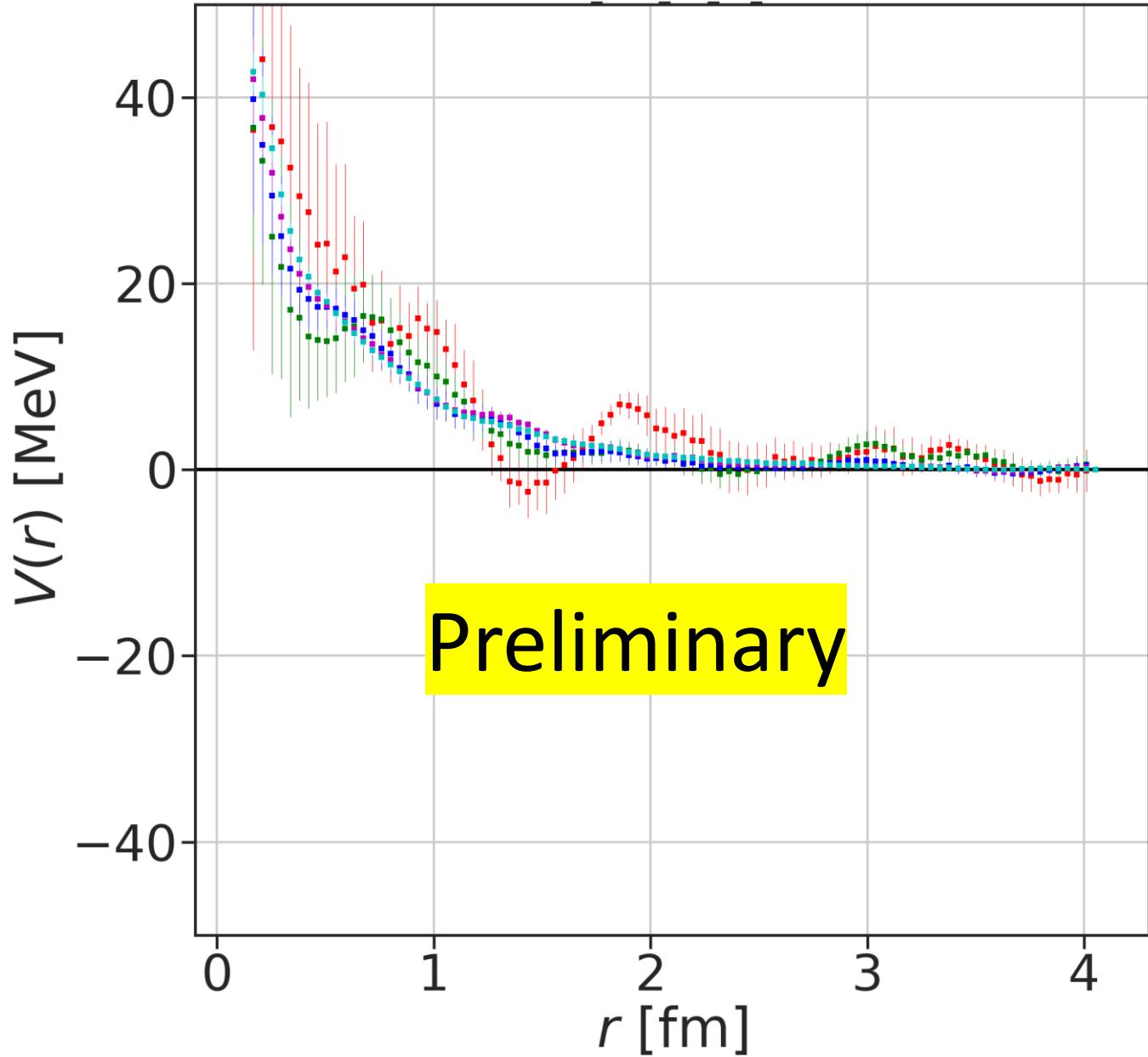
tensor

binsize=80

Nconf=800

w/ Misner

- t=12
- t=11
- t=10
- t=9
- t=8



attractive

| flavor multiplet | baryon pair (isospin) |
|---------------------------------|--|
| spin 27 | $\{NN\}(I=1), \{N\Sigma\}(I=3/2), \{\Sigma\Sigma\}(I=2),$ $\{\Sigma\Xi\}(I=3/2), \{\Xi\Xi\}(I=1)$ |
| 1S0 8_s | none |
| 1 | none |
| 3S1 10* | $[NN](I=0), [\Sigma\Xi](I=3/2)$ |
| 10 | $[N\Sigma](I=3/2), [\Xi\Xi](I=0)$ |
| 8_a | $[N\Xi](I=0)$ repulsive |

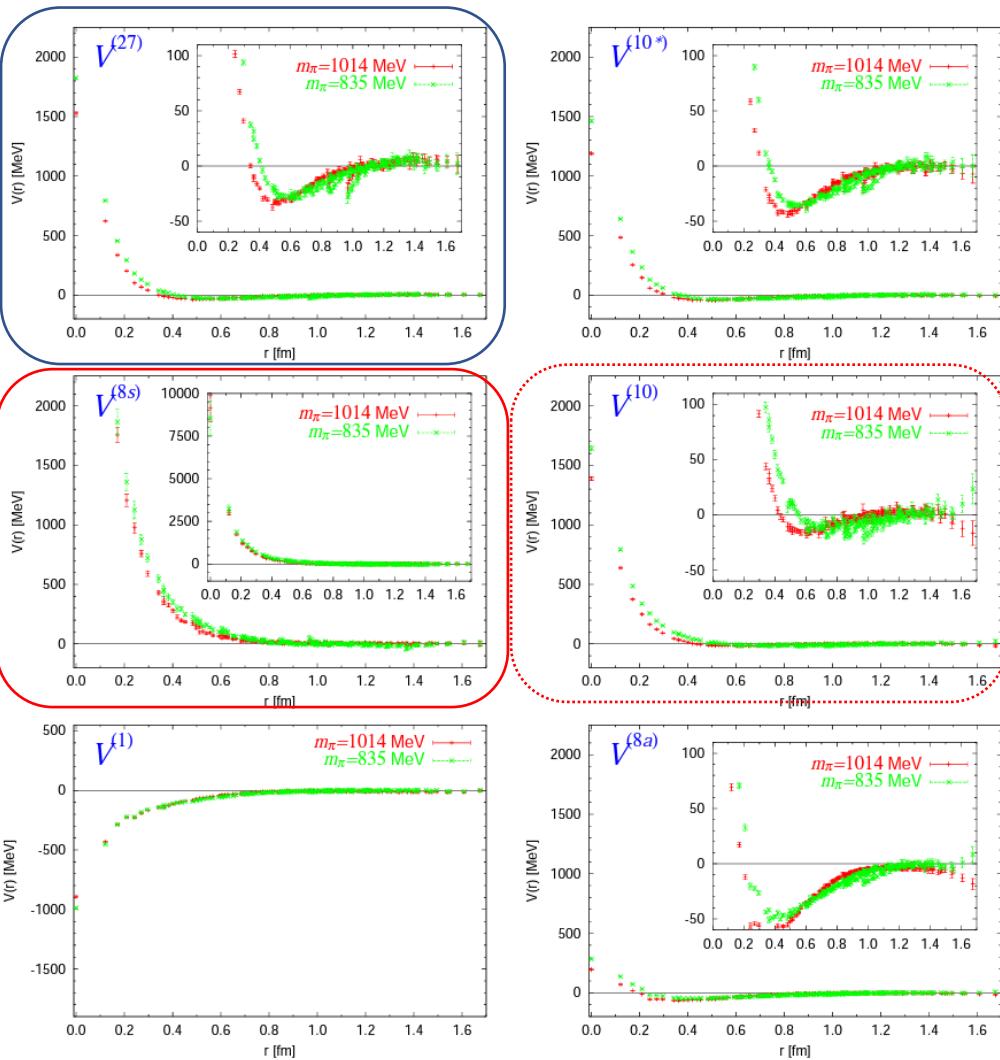
$S = -1, I = 1/2, {}^1S_0$ sector.

repulsive

$$\begin{pmatrix} \langle NA| \\ \langle N\Sigma| \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{9}{10}} & -\sqrt{\frac{1}{10}} \\ \sqrt{\frac{1}{10}} & \sqrt{\frac{9}{10}} \end{pmatrix} \begin{pmatrix} \langle 27| \\ \langle 8_s| \end{pmatrix}$$

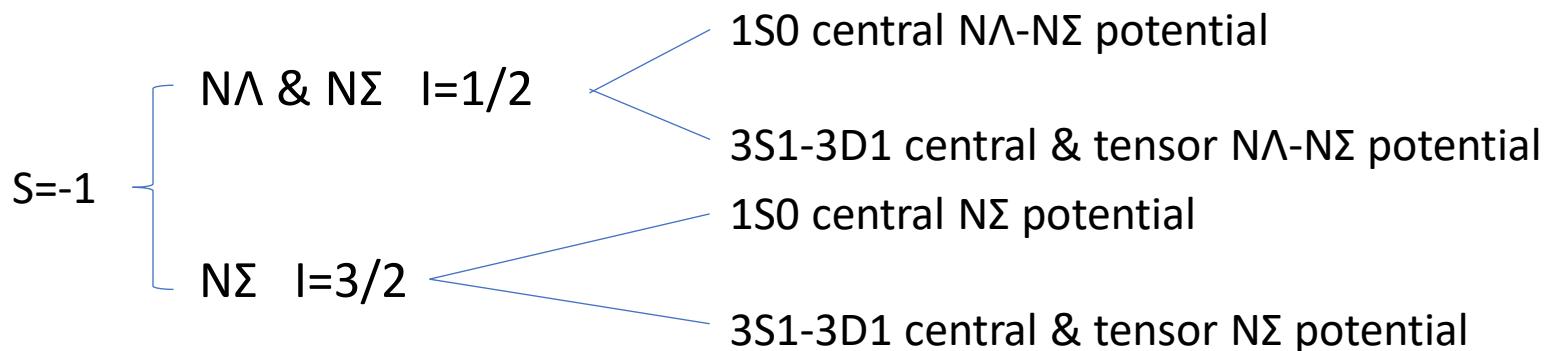
$S = -1, I = 1/2, {}^3S_1$ sector.

$$\begin{pmatrix} \langle NA| \\ \langle N\Sigma| \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{1}{2}} & -\sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \end{pmatrix} \begin{pmatrix} \langle 10^*| \\ \langle 8_a| \end{pmatrix}$$



Outline

- 富岳におけるゲージ配位生成(結果のみ)
- hadron mass
- $N\Lambda$ - $N\Sigma$ potential
- 今後の展望



signalだけ取り出したい

$$G_{N\Lambda}(\mathbf{r}, t) = \langle 0 | N(\mathbf{r}, t) \Lambda(\mathbf{0}, t) | \overline{J_{\text{src}}(t=0)} | 0 \rangle$$

$$\begin{aligned} R(\mathbf{r}, t) &\equiv \frac{G_{N\Lambda}(\mathbf{r}, t)}{G_N(t)G_\Lambda(t)} && \text{Many states contributes} \\ &= \sum_i A_{W_i} \psi_{W_i}(\mathbf{r}) e^{-(W_i - m_N - m_\Lambda)t} && i: \text{each energy eigen state} \end{aligned}$$

$$R(\mathbf{r}, t) = R^{\text{signal}}(\mathbf{r}, t) + R^{\text{inelastic}}(\mathbf{r}, t) \quad (R^{\text{inelastic}}(\mathbf{r}, t) \rightarrow 0(t \rightarrow \infty))$$

格子QCD計算で得られるのは左辺で、知りたいのは右辺第一項

tを大きくする以外で右辺第二項を引きたい。

現在試行錯誤中。。。。

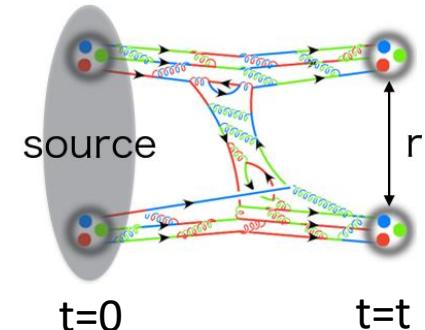
近似的にinelastic contaminationを取り除く

1体バリオンの相関関数のinelastic contaminationを考える

$$G_B(t) = \sum_{\mathbf{r}} \langle 0 | B(\mathbf{r}, t) | \overline{J_{\text{src}}(t=0)} | 0 \rangle$$

$$G_B^{\text{ela}}(t) \equiv A_B e^{-m_B t} \quad \text{Fitted function}$$

$$G_B^{\text{inel}}(t) \equiv G_B(t) - G_B^{\text{ela}}(t)$$



1体バリオン相関関数のinelastic contaminationを使って、

2体バリオン相関関数(NBS wave function)のinelastic contaminationを推定する

$$G_{N\Lambda}^{\text{inel}}(t) = G_N^{\text{ela}}(t)G_\Lambda^{\text{inel}}(t) + G_N^{\text{inel}}(t)G_\Lambda^{\text{ela}}(t) + G_N^{\text{inel}}(t)G_\Lambda^{\text{inel}}(t)$$

| | |
|-----------|-----------|
| Nucleon | Lambda |
| 2pt corr. | 2pt corr. |

improved 4点関数を使ってポテンシャルの計算をする

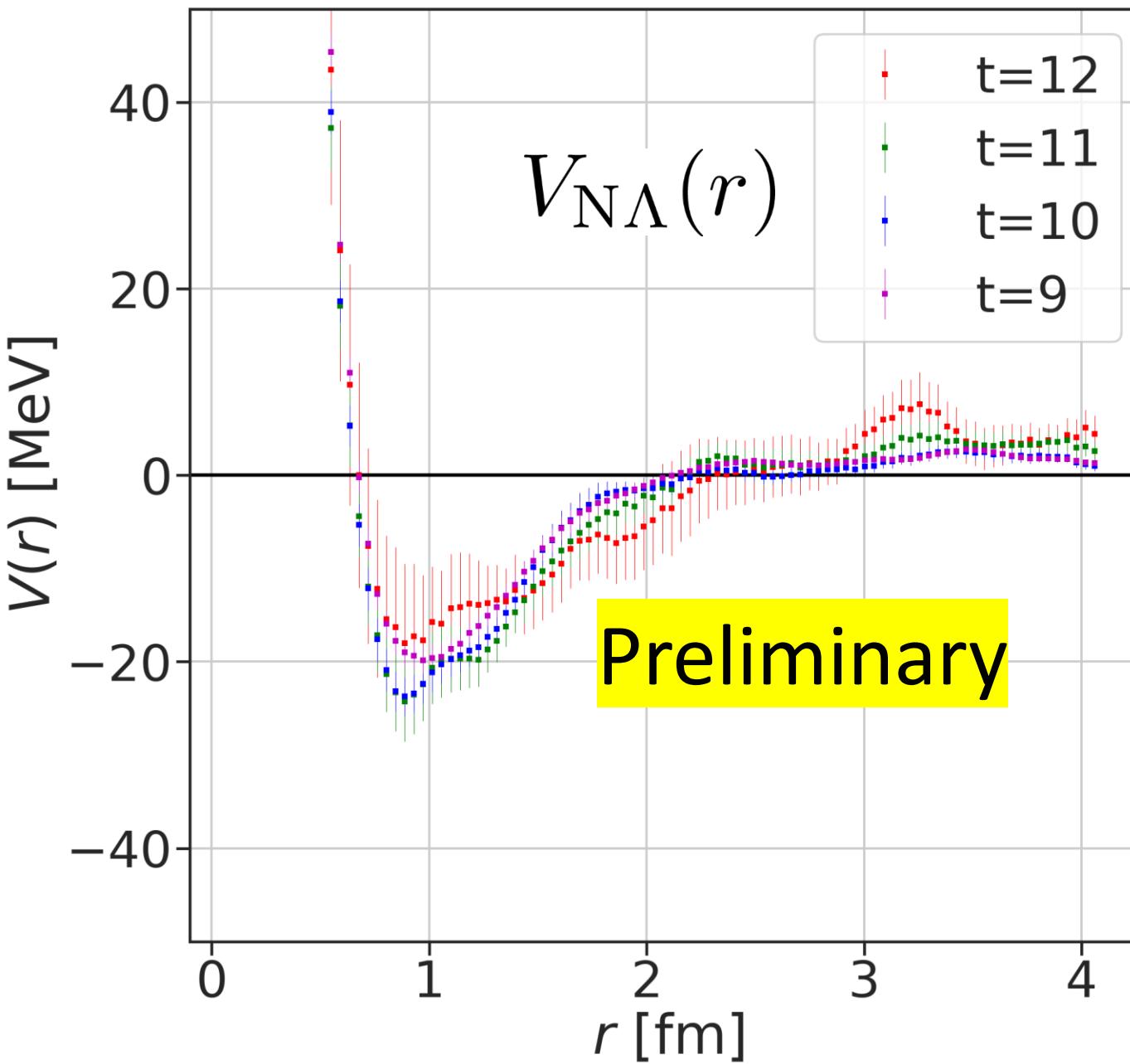
$$G_{N\Lambda}(\mathbf{r}, t) \rightarrow G_{N\Lambda}(\mathbf{r}, t) - \alpha G_{N\Lambda}^{\text{inel}}(t)$$

free gauge configurationの時は

$$G_{N\Lambda}(\mathbf{r}, t) = \frac{1}{4L^3} G_N(t) G_\Lambda(t) \quad \alpha = \frac{1}{4V}$$

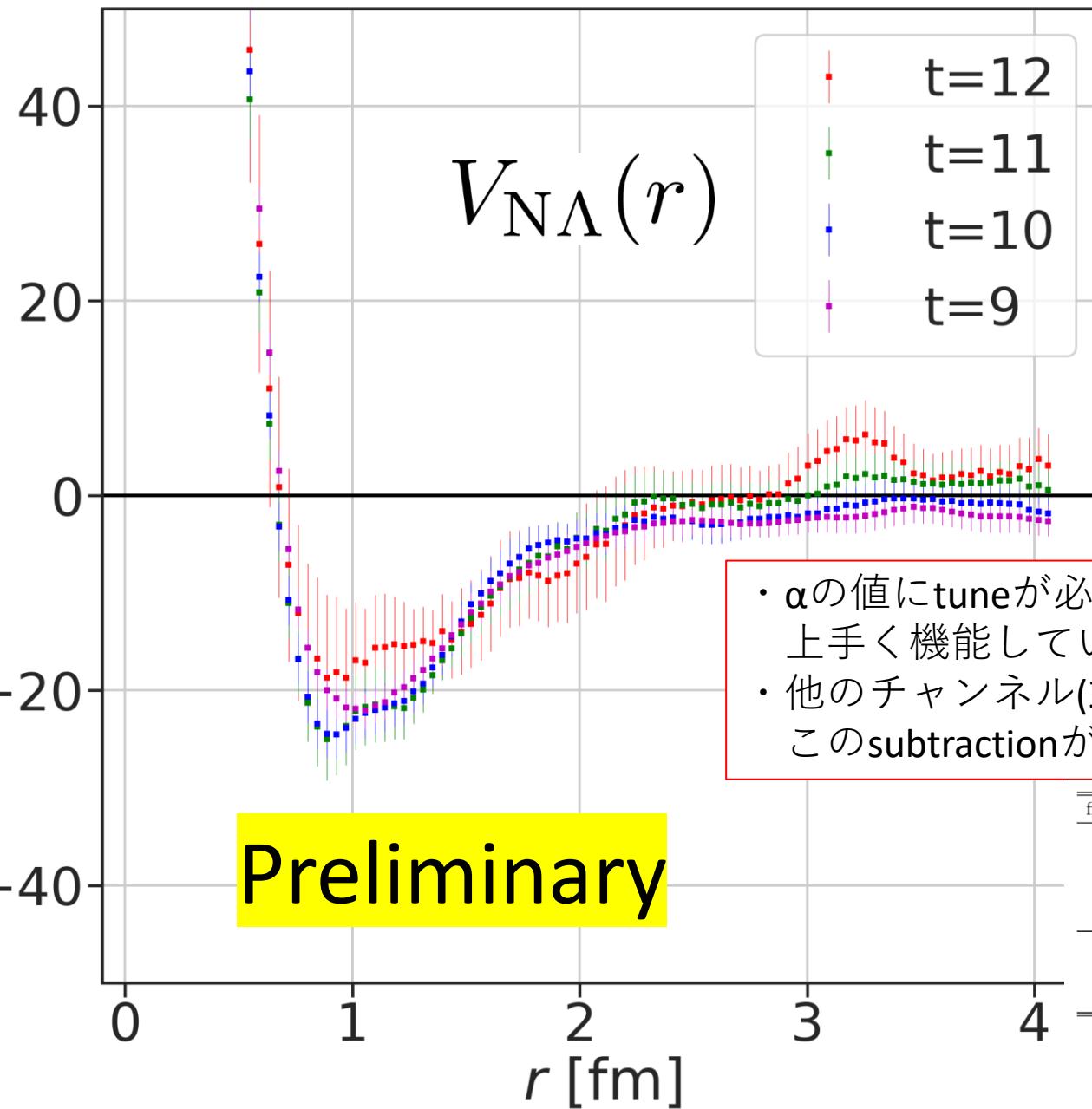
original results

$$R(\mathbf{r}, t) = \frac{G_{N\Lambda}(\mathbf{r}, t)}{G_N(t)G_\Lambda(t)}$$



近似的にinelastic contaminationを取り除く

$V(r)$ [MeV]



$$\tilde{R}(\mathbf{r}, t) = \frac{G_{N\Lambda}(\mathbf{r}, t) - \alpha G_{N\Lambda}^{\text{inel}}(t)}{G_N^{\text{ela}}(t)G_\Lambda^{\text{ela}}(t)}$$

$$= R(\mathbf{r}, t) - \alpha R^{\text{inel}}(t)$$

$$\alpha = \frac{1}{4V}$$

$$R(\mathbf{r}, t) \equiv \frac{G_{N\Lambda}(\mathbf{r}, t)}{G_N^{\text{ela}}(t)G_\Lambda^{\text{ela}}(t)}$$

| flavor multiplet | baryon pair (isospin) |
|------------------|---|
| 27 | {NN}(I=1), {NΣ}(I=3/2), {ΣΣ}(I=2), {ΣΞ}(I=3/2), {ΞΞ}(I=1) |
| 8 _s | none |
| 1 | none |
| 10* | [NN](I=0), [ΣΞ](I=3/2) |
| 10 | [NΣ](I=3/2), [ΞΞ](I=0) |
| 8 _a | [NΞ](I=0) |

repulsive

Summary and Outlook

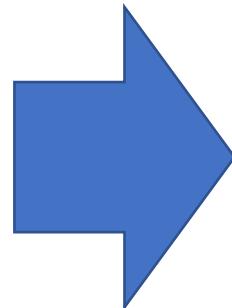
◎物理点ゲージ配位生成

$$1/a \simeq 2339[\text{MeV}]$$

$$m_\pi \simeq 137[\text{MeV}]$$

$$m_K \simeq 502[\text{MeV}]$$

$$m_N \simeq 940[\text{MeV}]$$

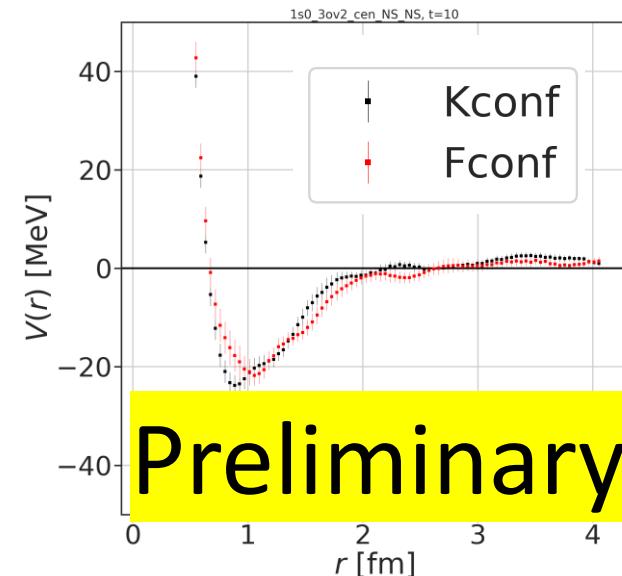


今後は

- topological charge測定
- 連続極限

が生成できた。(1600 configurations)

◎ハドロン間相互作用



- 物理点ハドロン間相互作用が得られた。
- 軽いクォークの質量依存性が見られた。
- シグナルが悪いチャンネルは inelastic excited stateからの影響を取り除く努力が必要。



他のチャンネルで手法の有用性を確立して $N\Lambda$ - $N\Sigma$ potential 等に応用。