# 3-body force & beyond in 1D cold atoms

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第８回クラスター階層領域研究会

Feb.9 - Feb.11 (2023) @ Osaka

# Plan of this talk



## 1. 3-body force naturally arises in 1D

- **Thermal conductivity**

  T. Tanaka & Y. Nishida, Phys. Rev. E 106, 064104 (2022)
  "Thermal conductivity of a weakly interacting Bose gas in quasi-one-dimension"

- **Cluster formation**

  Y. Nishida, Phys. Rev. A 97, 061603(R) (2018)
  "Universal bound states of 1-dimensional bosons with 2- and 3-body attractions"

## 2. Artificial control of 3-body force & beyond

- **Quantum droplet**

  Y. Sekino & Y. Nishida, Phys. Rev. A 97, 011602(R) (2018)
  "Quantum droplet of one-dimensional bosons with a three-body attraction"

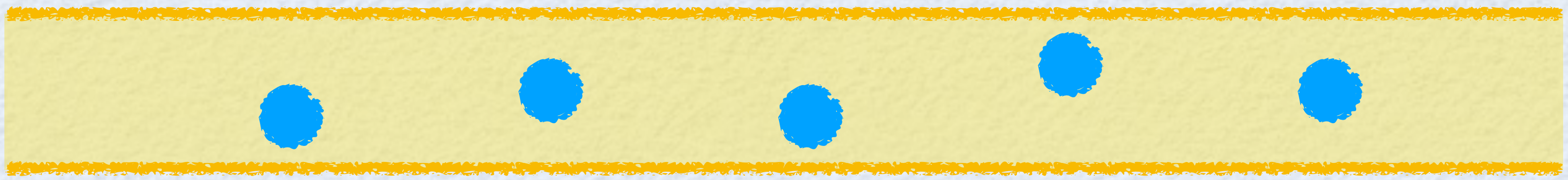
- **Efimov effect**

  Y. Nishida & D. T. Son, Phys. Rev. A 82, 043606 (2010)
  "Universal four-component Fermi gas in one dimension"

# 3-body force in 1D

# Bosons in quasi-1D

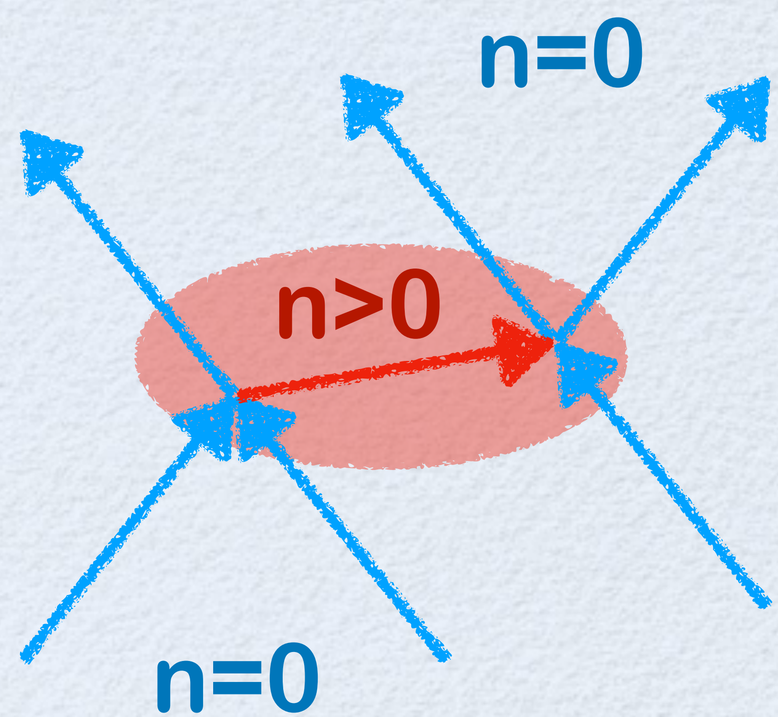
## Confining 3D bosons into a tight 1D waveguide



$$\hat{\mathcal{H}}_{3\mathrm{D}} = \hat{\Phi}^\dagger \left( -\frac{\nabla^2}{2m} + \frac{x^2+y^2}{2ml_\perp^4} \right) \hat{\Phi} + \frac{2\pi a_{3\mathrm{D}}}{m} (\hat{\Phi}^\dagger \hat{\Phi})^2$$

$$T, \mu \ll \hbar\omega_\perp, \quad |a_{3\mathrm{D}}| \ll l_\perp$$

$$\hat{\mathcal{H}}_{1\mathrm{D}} = -\hat{\phi}^\dagger \frac{\partial_x^2}{2m} \hat{\phi} + \frac{g_2}{2} (\hat{\phi}^\dagger \hat{\phi})^2$$

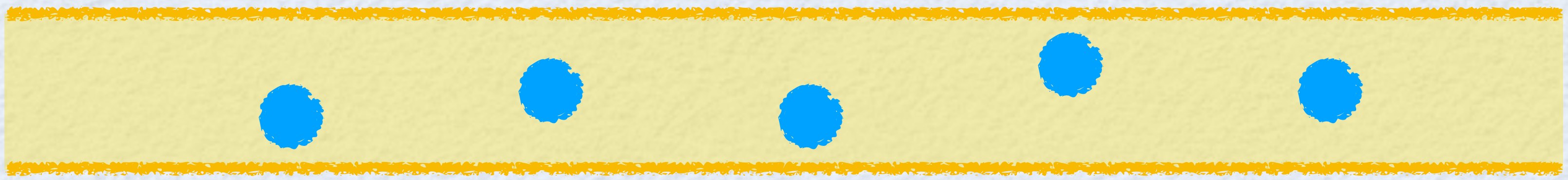



**effective 3-body interaction**

I. E. Mazets & J. Schmiedmayer, NJP (2010)

# Bosons in quasi-1D

## Confining 3D bosons into a tight 1D waveguide



$$\hat{\mathcal{H}}_{3D} = \hat{\Phi}^\dagger \left( -\frac{\nabla^2}{2m} + \frac{x^2 + y^2}{2ml_\perp^4} \right) \hat{\Phi} + \frac{2\pi a_{3D}}{m} (\hat{\Phi}^\dagger \hat{\Phi})^2$$

$$T, \mu \ll \hbar\omega_\perp, \quad |a_{3D}| \ll l_\perp$$

$$\hat{\mathcal{H}}_{1D} = -\hat{\phi}^\dagger \frac{\partial_x^2}{2m} \hat{\phi} + \frac{g_2}{2} (\hat{\phi}^\dagger \hat{\phi})^2 + \frac{g_3}{6} (\hat{\phi}^\dagger \hat{\phi})^3 + \cdots$$

with $g_2 = 2\frac{a_{3D}}{ml_\perp^2}, \quad g_3 = -12 \ln(4/3) \frac{a_{3D}^2}{ml_\perp^2}$

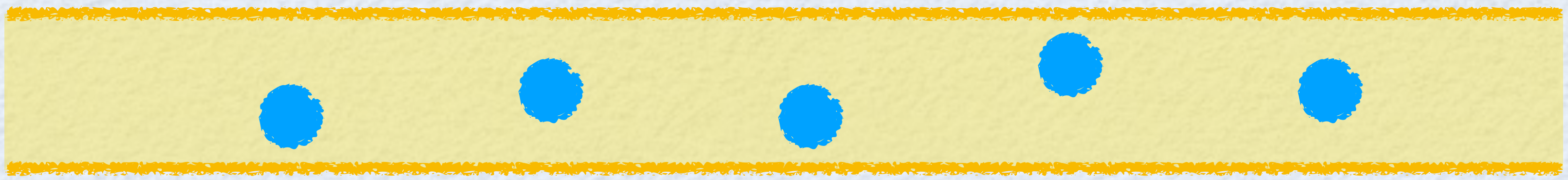
**effective 3-body interaction**

I. E. Mazets & J. Schmiedmayer, NJP (2010)

# Bosons in quasi-1D



## Confining 3D bosons into a tight 1D waveguide



$$\hat{\mathcal{H}}_{3\mathrm{D}} = \hat{\Phi}^\dagger \left( -\frac{\nabla^2}{2m} + \frac{x^2+y^2}{2ml_\perp^4} \right) \hat{\Phi} + \frac{2\pi a_{3\mathrm{D}}}{m} (\hat{\Phi}^\dagger \hat{\Phi})^2$$

$$T, \mu \ll \hbar\omega_\perp, \quad |a_{3\mathrm{D}}| \ll l_\perp$$

$$\hat{\mathcal{H}}_{1\mathrm{D}} = -\hat{\phi}^\dagger \frac{\partial_x^2}{2m} \hat{\phi} + \frac{g_2}{2} (\hat{\phi}^\dagger \hat{\phi})^2 + \frac{g_3}{6} (\hat{\phi}^\dagger \hat{\phi})^3 + \cdots$$

integrability preserving

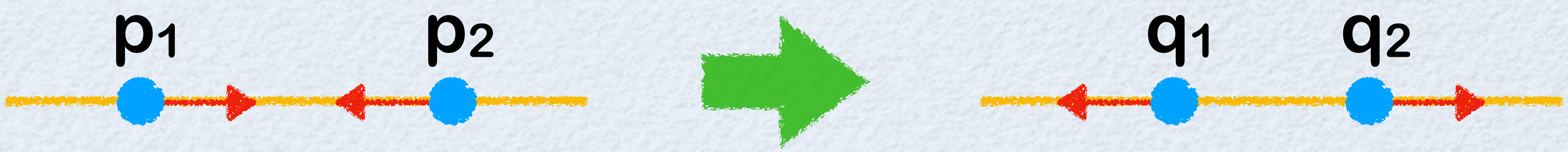
integrability breaking

# Bosons in quasi-1D



$$\hat{\mathcal{H}}_{\mathrm{1D}} = -\hat{\phi}^\dagger \frac{\partial_x^2}{2m}\hat{\phi} + \frac{g_2}{2}(\hat{\phi}^\dagger\hat{\phi})^2 + \frac{g_3}{6}(\hat{\phi}^\dagger\hat{\phi})^3 + \cdots$$

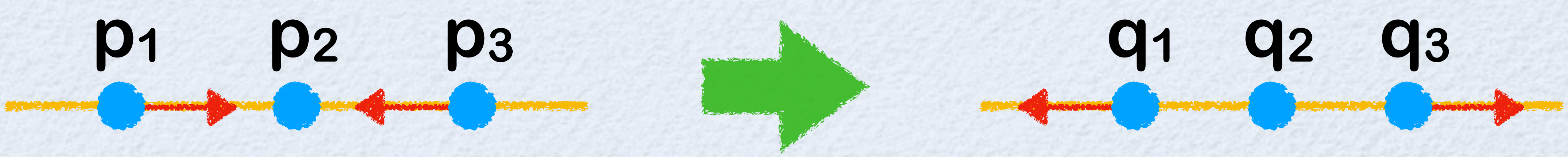
2-body scattering ⇒ $\{p_1, p_2\} = \{q_1, q_2\}$




cannot change momentum distribution

due to energy and momentum conservations

3-body scattering ⇒ $\{p_1, p_2, p_3\} \neq \{q_1, q_2, q_3\}$



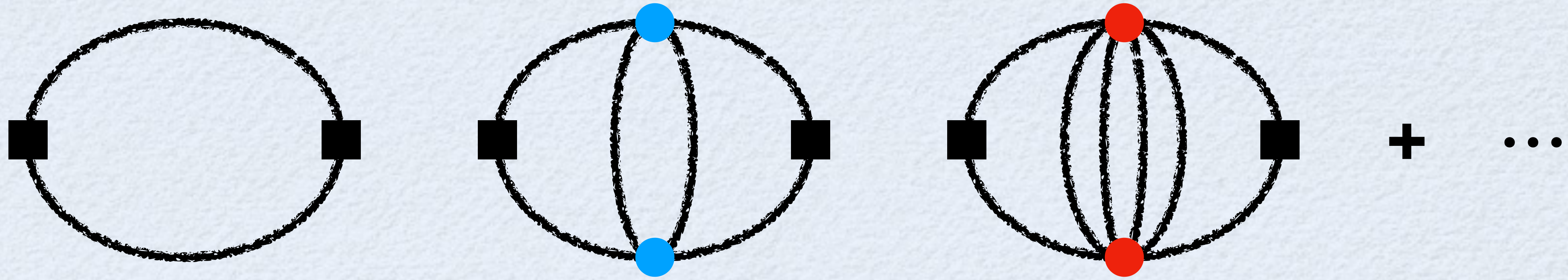

can change momentum distribution

# Thermal conductivity



$$\hat{\mathcal{H}}_{1\mathrm{D}} = -\hat{\phi}^\dagger \frac{\partial_x^2}{2m}\hat{\phi} + \frac{g_2}{2}(\hat{\phi}^\dagger\hat{\phi})^2 + \frac{g_3}{6}(\hat{\phi}^\dagger\hat{\phi})^3 + \cdots$$

Thermal conductivity at weak coupling is evaluated by Kubo formula + resummation




$$\kappa = \frac{\mathcal{N}}{m^3 g_3^2}\tilde{\kappa}(\tilde{T}) = \frac{(l_\perp/a_{3\mathrm{D}})^4\mathcal{N}}{[12\ln(4/3)]^2 m}\tilde{\kappa}(\tilde{T})$$

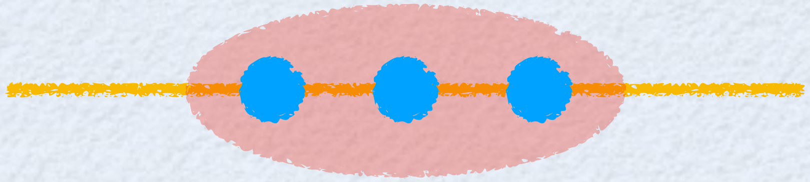
2-body coupling cancels out and 3-body coupling dominates

T. Tanaka & Y. N. PRE (2022)

# Cluster formation



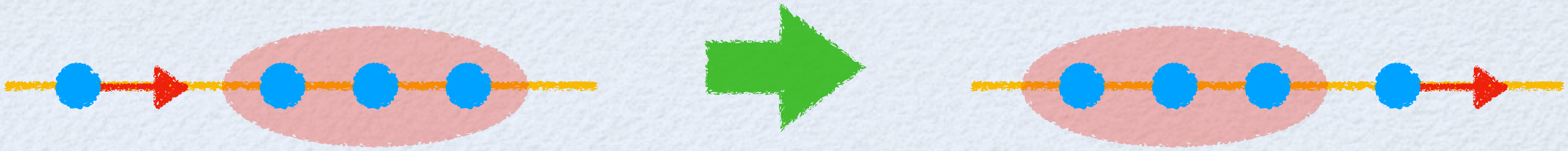
N bosons with attractive $g_2$ form a cluster



$$E_N = -\frac{N(N^2-1)}{24} m g_2^2$$

J. B. McGuire, J. Math. Phys. (1964)

N-body cluster + extra boson



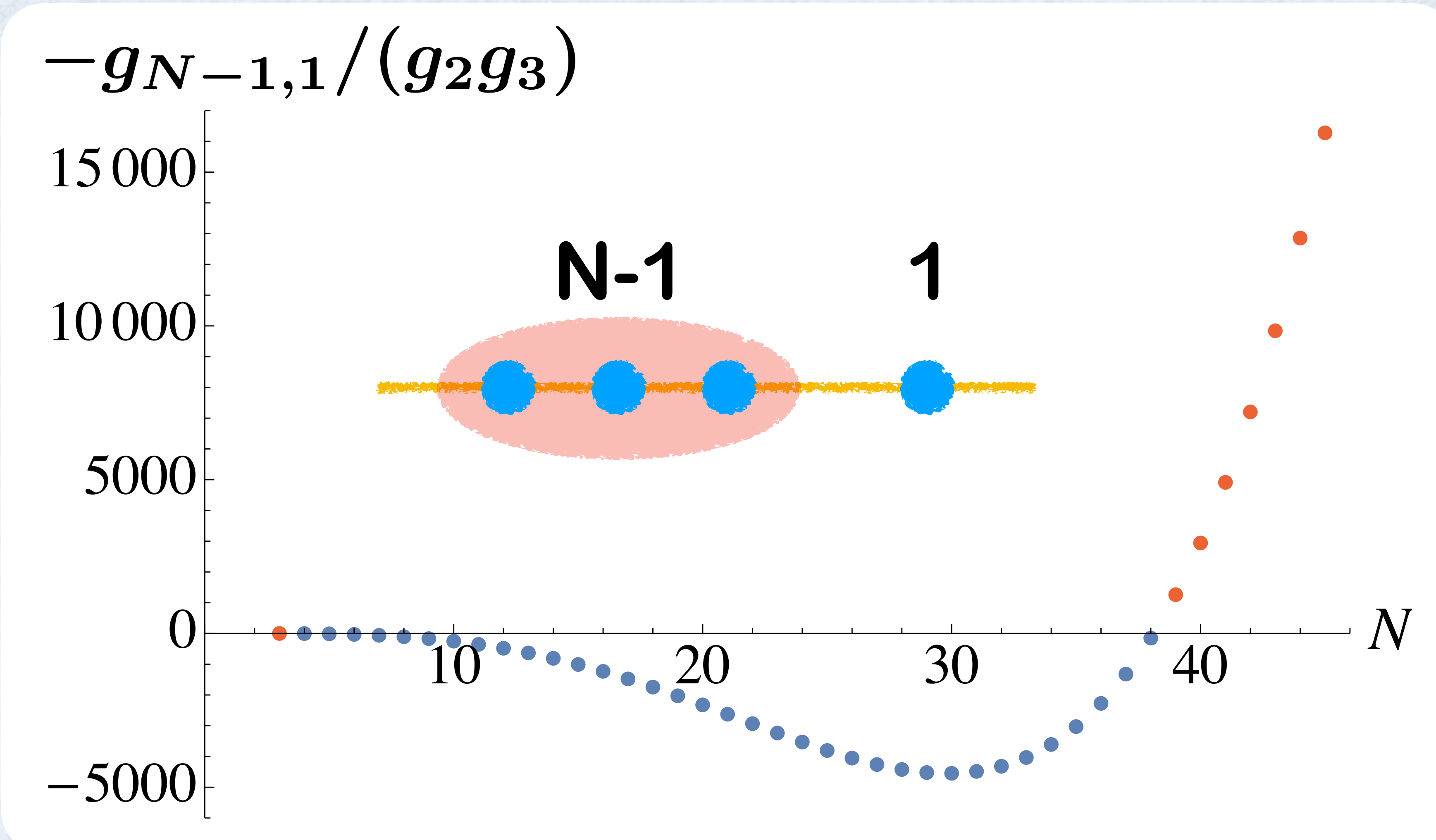
No interaction

Interaction between cluster & boson is also dominated by 3-body coupling

# Cluster formation



Cluster-boson coupling $g_{N-1,1}$ induced by $g_3$




Y. N., PRA (2018)

- Coupling is repulsive for N=4,5,...,38
- Coupling is attractive for N=3 & 39,40,...

New N-body cluster formation by 3-body force

# Artificial control of 3-body force

# Tunable 2-body & 3-body force

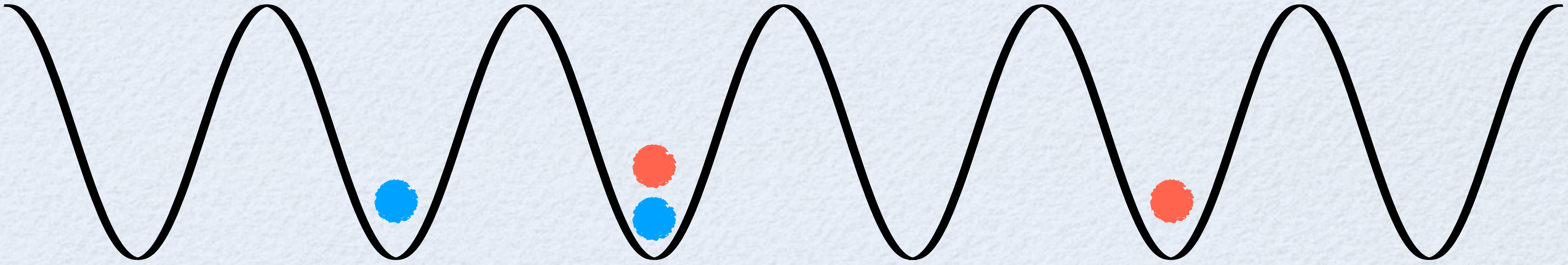


## Coupled 2-component bosons in an optical lattice

$$\hat{H} = -t \sum_{\langle i,j \rangle} \sum_{\sigma} \hat{b}^\dagger_{\sigma i} \hat{b}_{\sigma j} + \sum_i \sum_{\sigma,\tau} \frac{g_{\sigma\tau}}{2} \hat{b}^\dagger_{\sigma i} \hat{b}^\dagger_{\tau i} \hat{b}_{\tau i} \hat{b}_{\sigma i}$$

$$- \frac{\Delta}{2} \sum_i (\hat{b}^\dagger_{\uparrow i} \hat{b}_{\uparrow i} - \hat{b}^\dagger_{\downarrow i} \hat{b}_{\downarrow i}) - \frac{\Omega}{2} \sum_i (\hat{b}^\dagger_{\uparrow i} \hat{b}_{\downarrow i} + \hat{b}^\dagger_{\downarrow i} \hat{b}_{\uparrow i})$$

D. S. Petrov
PRA (2014)



# Tunable 2-body & 3-body force

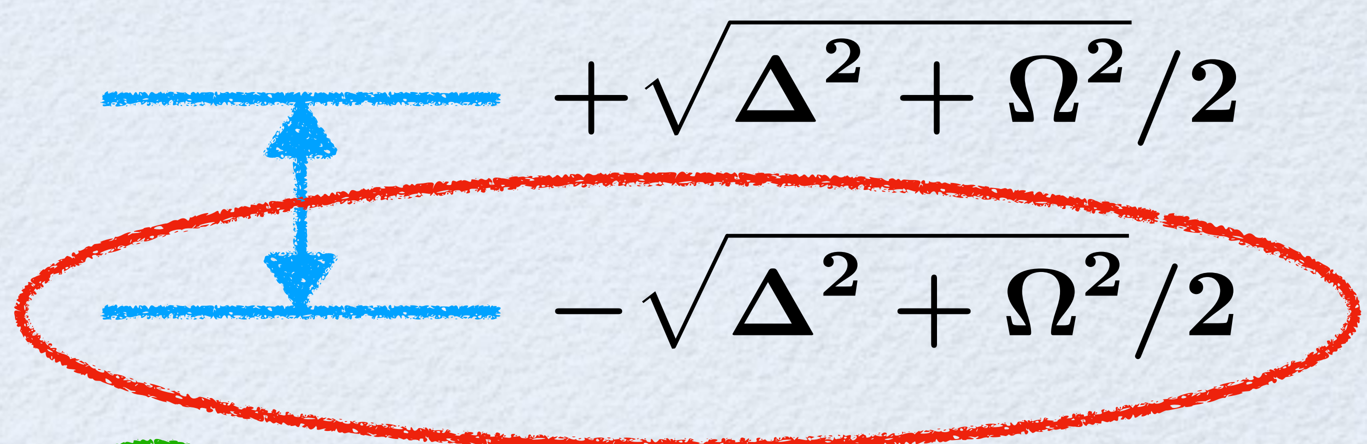
## Coupled 2-component bosons in an optical lattice

$$\hat{H} = -t\sum_{\langle i,j\rangle}\sum_{\sigma}\hat{b}^{\dagger}_{\sigma i}\hat{b}_{\sigma j} + \sum_{i}\sum_{\sigma,\tau}\frac{g_{\sigma\tau}}{2}\hat{b}^{\dagger}_{\sigma i}\hat{b}^{\dagger}_{\tau i}\hat{b}_{\tau i}\hat{b}_{\sigma i}$$

$$-\frac{\Delta}{2}\sum_{i}(\hat{b}^{\dagger}_{\uparrow i}\hat{b}_{\uparrow i} - \hat{b}^{\dagger}_{\downarrow i}\hat{b}_{\downarrow i}) - \frac{\Omega}{2}\sum_{i}(\hat{b}^{\dagger}_{\uparrow i}\hat{b}_{\downarrow i} + \hat{b}^{\dagger}_{\downarrow i}\hat{b}_{\uparrow i})$$

D. S. Petrov
PRA (2014)

$$T, \mu \ll \sqrt{\Delta^2 + \Omega^2}$$



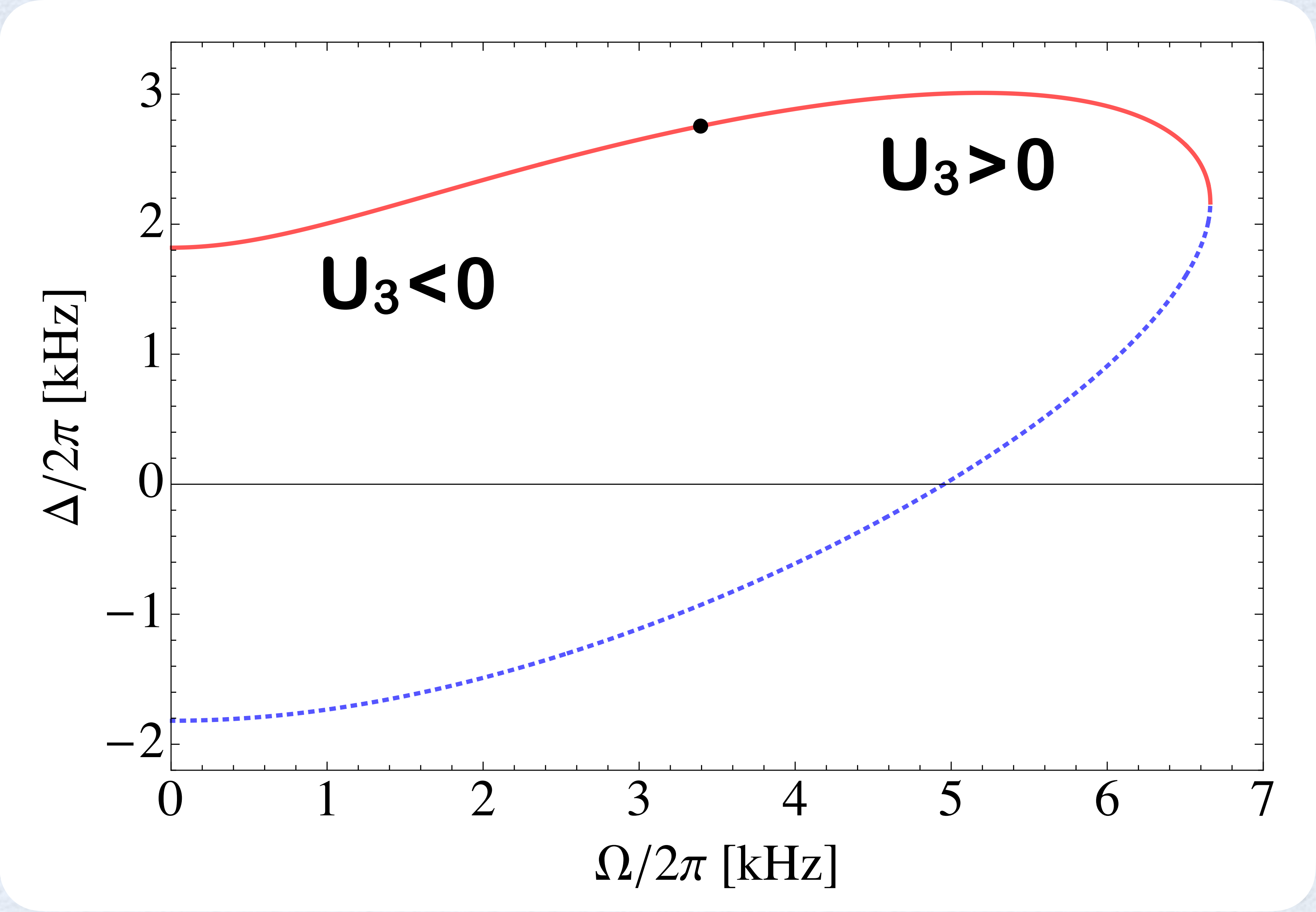

$$\hat{H}_{\text{eff}} = -t\sum_{\langle i,j\rangle}\hat{b}^{\dagger}_{i}\hat{b}_{j} + \sum_{i}\sum_{N=1}^{\infty}\frac{U_N}{N!}(\hat{b}^{\dagger}_{i}b_{i})^{N}$$

**Effective N-body coupling depends on $\Delta$, $\Omega$, $g_{\sigma\tau}$**

# Tunable 2-body & 3-body force



**Independent control** of 2-body & 3-body coupling is possible by tuning $\Delta$, $\Omega$




Y. Sekino & Y. N.
PRA (2018)

Line of $U_2=0$ for $^{39}$K @ B=58G

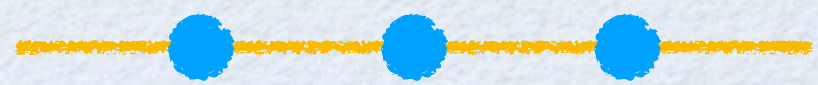
# Tunable 2-body & 3-body force

## 1D bosons with N-body interaction ONLY

$$\hat{\mathcal{H}} = -\hat{\phi}^\dagger \frac{\partial_x^2}{2m}\hat{\phi} + \frac{g_N}{N!}(\hat{\phi}^\dagger\hat{\phi})^N$$

## N-body problem with N-body interaction in 1D

⇒ N-1 relative coordinates



$$\hat{H}_N = -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial r_1^2} + \cdots + \frac{\partial^2}{\partial r_{N-1}^2}\right) + g_N \delta^{N-1}(\vec{r})$$

## 2-body problem with 2-body interaction in D dimensions

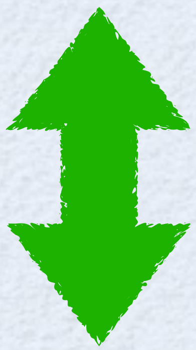
⇒ D relative coordinates

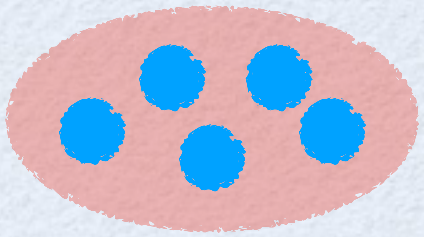
**N-body interaction in 1D "realizes" 2-body interaction in (N-1) dimensions**

# Quantum droplet

**N ≫ 1 bosons with 2-body attraction in 2D**

**form a quantum droplet with** $E_N \sim \exp(2.15N)$

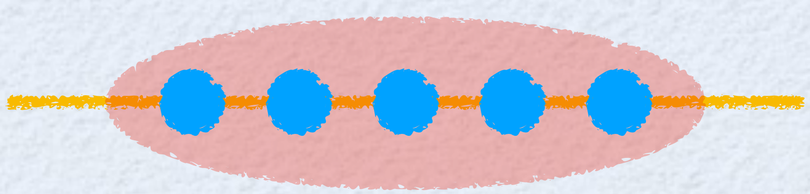




H.-W. Hammer & D. T. Son, PRL (2004)

**N ≫ 1 bosons with 3-body attraction in 1D**

**form a quantum droplet with** $E_N \sim \exp(1.47N^2)$



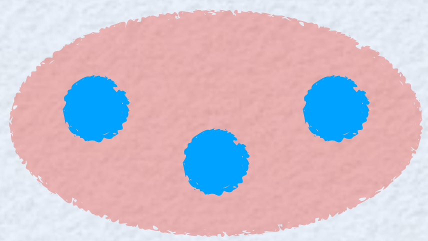
Y. Sekino & Y. N., PRA (2018)

# Efimov effect



3 bosons at 2-body resonance in 3D

show Efimov effect with $E_n \sim (22.7)^{-2n}$



V. Efimov, PLB (1970)

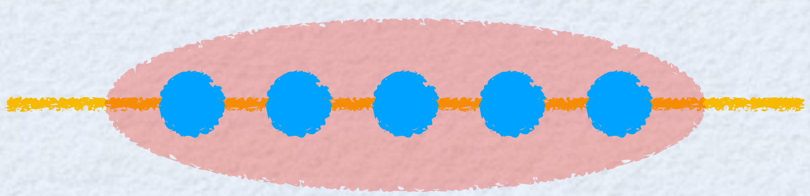
Efimov effect is usually possible only in 3D

E. Nielsen, et al., Phys. Rep. (2001)

5 bosons at 4-body resonance in 1D

show Efimov effect with $E_n \sim (12.4)^{-2n}$

D. T. Son & Y. N., PRA (2010)



Unusual realization of Efimov effect in 1D !

# ~~Plan~~ Summary of this talk



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