

**HIGH-MOMENTUM CORRELATED  
NUCLEONS  
AND  
TENSOR BLOCKING EFFECTS  
DETERMINE THE SHELL STRUCTURE  
IN NEUTRON-RICH NUCLEI**

ISAO TANIHATA  
IRCNPC, BEIHANG UNIVERSITY  
AND  
RCNP, OSAKA UNIVERSITY



# New Shell Structures observed in Neutron Rich Nuclei

Discoveries in these 30 years

- “Doubly magic”  $^{10}\text{He}$  and  $^{28}\text{O}$  are not bound.
- New magic numbers  $N=6, 14, 16, 32, 34$  emerge.
- Traditional magic numbers  $N=8$  and  $N=20$  disappear.
- The neutron dripline for O isotopes is  $N=16$  but extend much more in F isotopes.
- The ground states of  $^{11}\text{Li}$  and  $^{11}\text{Be}$  are mainly  $2s_{1/2}$  wave but the main component of  $^{12}\text{Be}$  is  $1d_{5/2}$ .

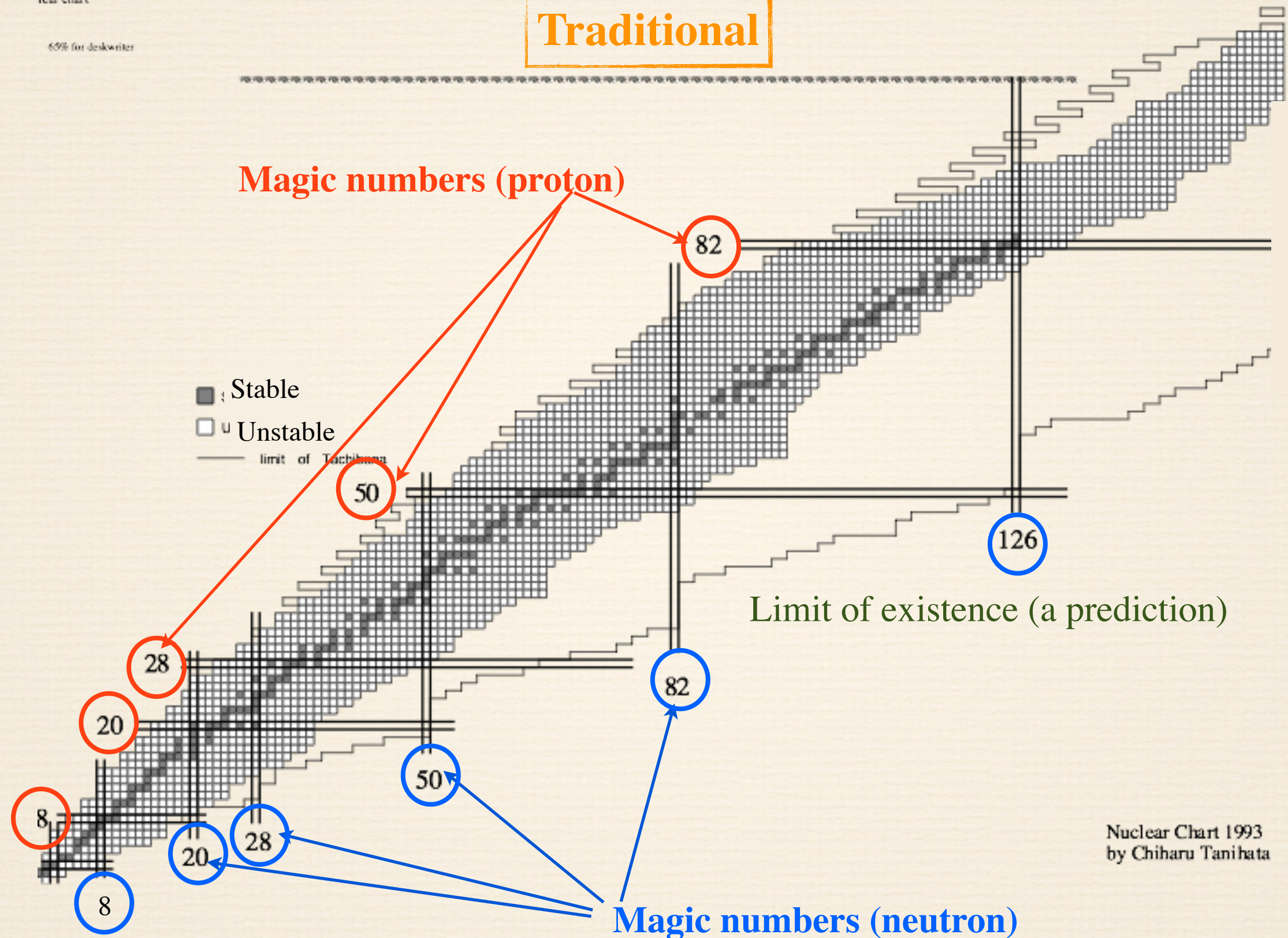


# Nuclear Chart and Magic Numbers

lear chart

60% for desktop

Traditional



Nuclear Chart 1993  
by Chiharu Tanihata

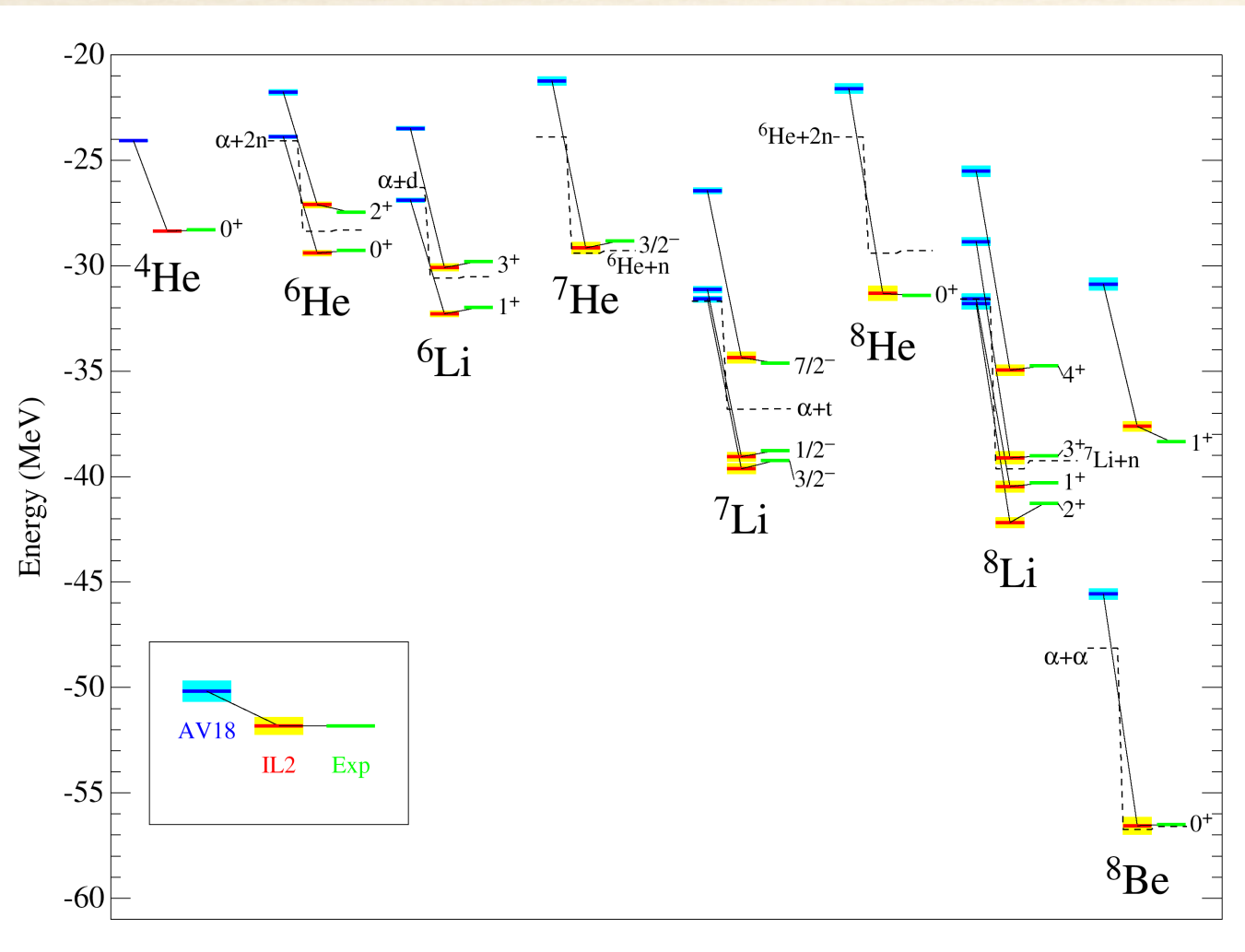
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# The most advanced, An ab-initio calculations of nuclei from two-body interactions

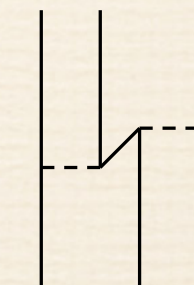
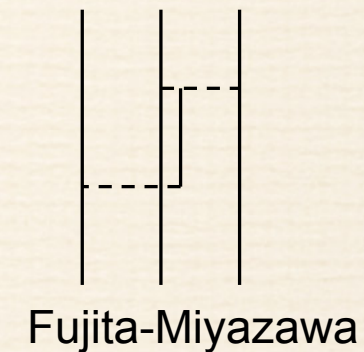
Variational Calculation of few body systems with NN interactions



$$\Psi = \phi(r_{12})\phi(r_{23})\dots\phi(r_{ij})$$

Tensor interactions are very important,  
also need 3-body interactions

$V_{\text{NNN}}$



C. Pieper and R. B. Wiringa, Annu. Rev. Nucl. Part. Sci.51(2001)



# Conclusion:

## Pion exchange interactions are important

$$\frac{\langle \Psi | V_{\pi} | \Psi \rangle}{\langle \Psi | V_{NN} | \Psi \rangle} \sim 80\%$$

- 80% of attraction comes from pions.
  - *R. B. Wiringa: Ann. Rev. Nucl. Part. Sci. 51(2001)*

Argonne GFMC

(Green Function Monte Carlo)

# Pion exchange interaction

$$\vec{\sigma} \cdot \vec{q} \left| -\frac{\pi}{0^-} \right| \vec{\sigma} \cdot \vec{q}$$

pion is a pseudo-scalar (0-) meson

$$\vec{\sigma}_1 \cdot \vec{q} \frac{1}{m_\pi^2 + q^2} \vec{\sigma}_2 \cdot \vec{q}$$

$$S_{12}(\hat{q}) = \sqrt{\frac{24\pi}{5}} [Y_2(\hat{q}) [\sigma_1 \sigma_2]_2]_0$$

$$= \frac{1}{3} \frac{q^2}{m_\pi^2 + q^2} S_{12}(\hat{q}) + \frac{1}{3} \left( 1 - \frac{m_\pi^2}{m_\pi^2 + q^2} \right) \vec{\sigma}_1 \cdot \vec{\sigma}_2$$

Tensor

Spin-spin

prefer high momentum

prefer low momentum



# Present days nuclear models

## *Interactions*

- Nuclear interactions from QCD.
- Empirical interactions from nucleon-nucleon data.
- Effective Field theory
- Phenomenological interaction (Skyrme....)

## *Nuclear models*

- Ab-initio models
- The shell model
- Mean field models (non-relativistic, relativistic)
- “Ab-initio” type models (no-core shell model...)



# Present days nuclear models

## *Interactions*

**Non-of them treat tensor forces appropriately. (except 1)**

**(Inclusion of the interaction itself or treatment of high-momentum nucleons.)**

**We need a theory to treat explicitly the tensor interactions!**

2. The s

3. Mean

4. “Ab-

**Ab-initio calculations can not be extended  
to nuclei of  $A > 12$ !**



# Importance of tensor interactions (known facts)

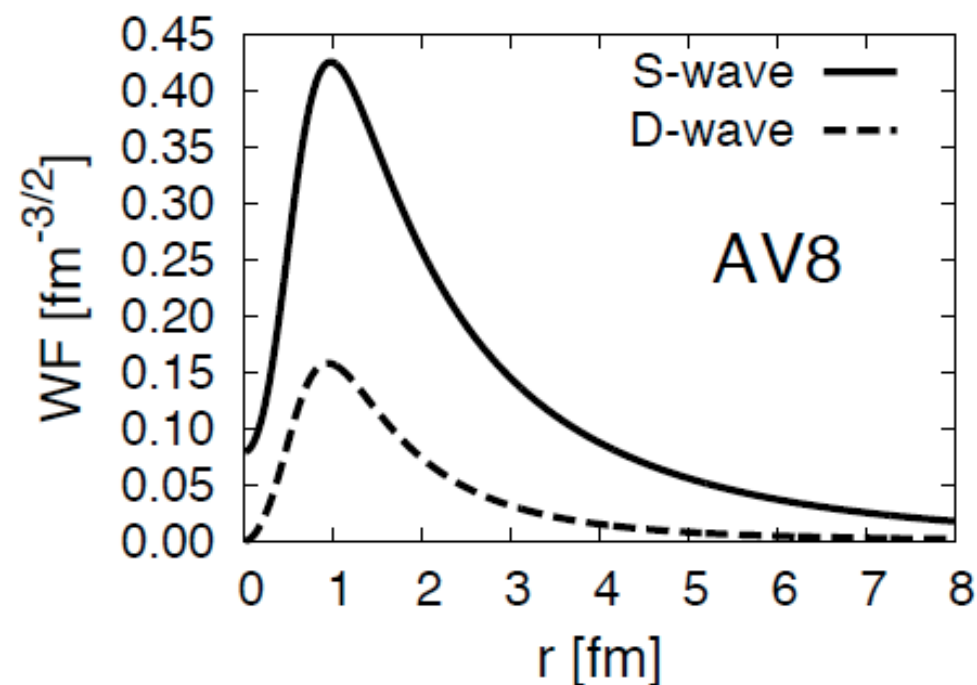
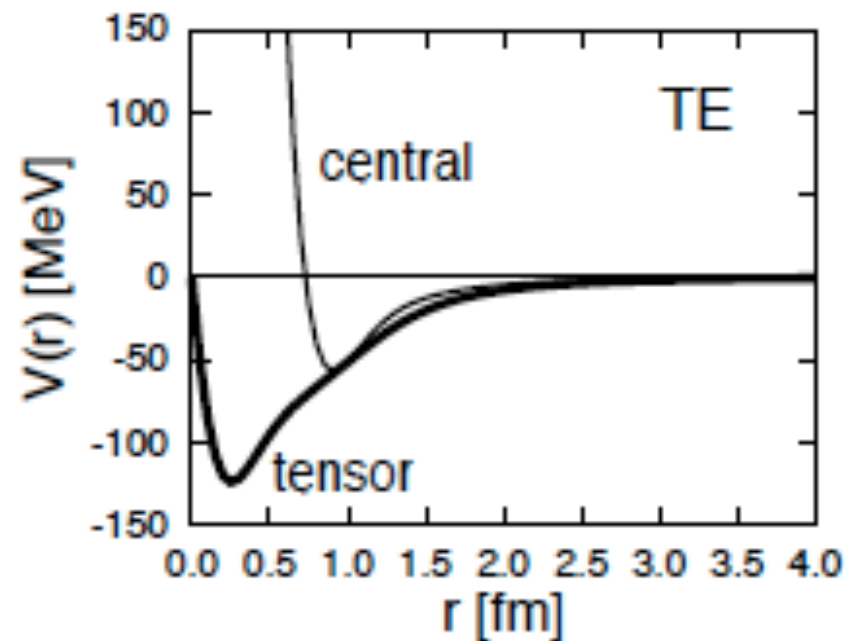
- Deuteron
- Saturation properties of the nuclear matter.
- As an origin of spin-orbit ( $l \cdot s$ ) interactions.
- Magnetic moments of doubly-closed  $\pm 1$  nuclei



# Deuteron = a Nucleon Bound State

$S=1$  and  $L=0$  or 2

Binding of deuteron ( $1^+$ )



Energy	-2.24 [MeV]
--------	-------------

Kinetic	19.88
(SS)	11.31
(DD)	8.57

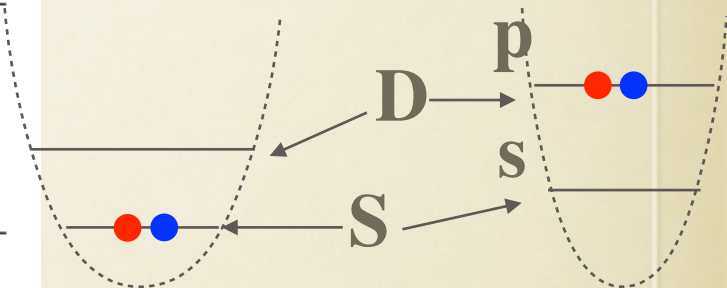
Central	-4.46
(SS)	-3.96
(DD)	-0.50

Tensor	-16.64
(SD)	-18.93
(DD)	2.29

LS	-1.02
----	-------

P(D)	5.78 [%]
------	----------

Radius	1.96 [fm]
(SS)	2.00 [fm]
(DD)	1.22 [fm]



shell model式

**D wave has a large relative momentum**



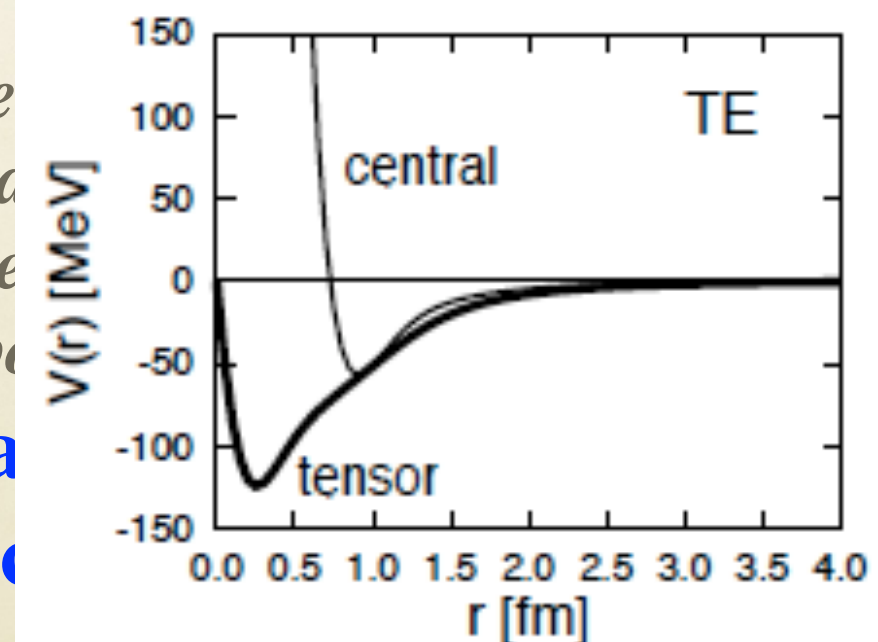
# Why nuclear matter density saturate?

- Nuclear matter
  - $E/A=16 \text{ MeV}$ ,  $\rho=1.6 \text{ nucleons/fm}^3$ .  $P_F \sim 1.2 \text{ fm}^{-1}$ .
  - average nucleon distance  $\sim 1.6 \text{ fm}$ .

**The repulsive core of the central interaction is not the main cause!**

- *The practical saturation problem then re-saturation occurs at the right density and we have seen above, the saturation in the three factors, in decreasing order of importance:*

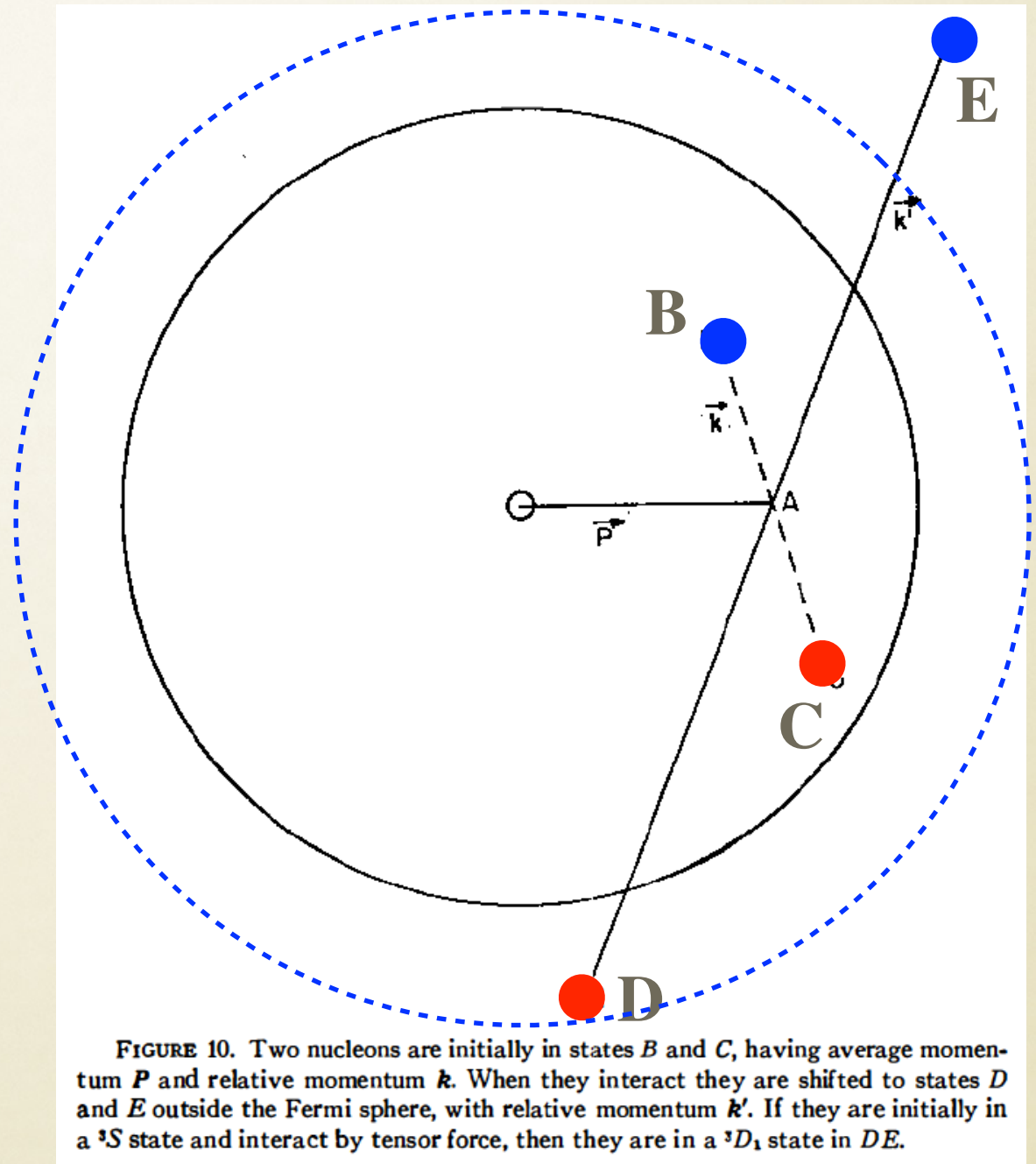
**(a) tensor force; (b) exchange character of the force and (c) repulsive core**





# Excitation to D state and Blocking

- Two nucleons are initially in states  $B$  and  $C$ . When they interact they are shifted to state  $D$  and  $E$  outside the Fermi sphere. If they are initially in a  $^3S$  state and interaction with tensor force, then they are in a  $^3D_1$  state in  $DE$ .
- This one of the important mechanism to gain binding energy by the tensor force.
- **When the density comes higher, Fermi sphere comes larger. A part of the open space for this excitation is blocked and thus the energy gain, unless otherwise obtained, is lost. This is the mechanism of Saturation by blocking of the tensor interaction.**





# Tensor force as an origin of $I\cdot S$ interactions

- T. Terasawa, Progr. Theo. Phys. vol.23, 87 (1960).
- A. Arima and T. Terasawa, Progr. Theo. Phys. vol.23, 115 (1960).
- We will see the mechanism later



# Magnetic moments of Doubly closed shell $\pm 1$ nuclei

- Please refer:

K. Shimizu, M. Ichimura, A. Arima, Nucl. Phys. A,  
226, p282 (1974)



# Explicit treatment of Tensor interactions in nuclei

- **Tensor optimized shell model**
  - *T. Myo, Y. Kikuchi, K. Kato, H. Toki, K. Ikeda, Progr. Theo. Phys. 119, 561 (2008).*
  - *T. Myo, H. Toki, and K. Ikeda, Progr. Theo. Phys. 121, 511 (2009).*
- **Binding of  $^4\text{He}$ .**
- **Saturation properties of the nuclei. (energy and density)**
- **Two-particle excitations and high-momentum nucleons.**



# Tensor Optimized Shell Model (TOSM)

*Myo, Toki, Ikeda, Kato, Sugimoto, PTP 117 (2006)*

0p-0h + 2p-2h

$$\Phi(^4\text{He}) = \sum_i C_i \psi_i(\{b_\alpha\}) = C_1 (0s)^4 + C_2 (0s)^2(\overline{0p}_{1/2})^2 + \dots$$

size parameter:  $b_{0s} \neq b_{\overline{0p}}$

**It is not a perturbation model!**

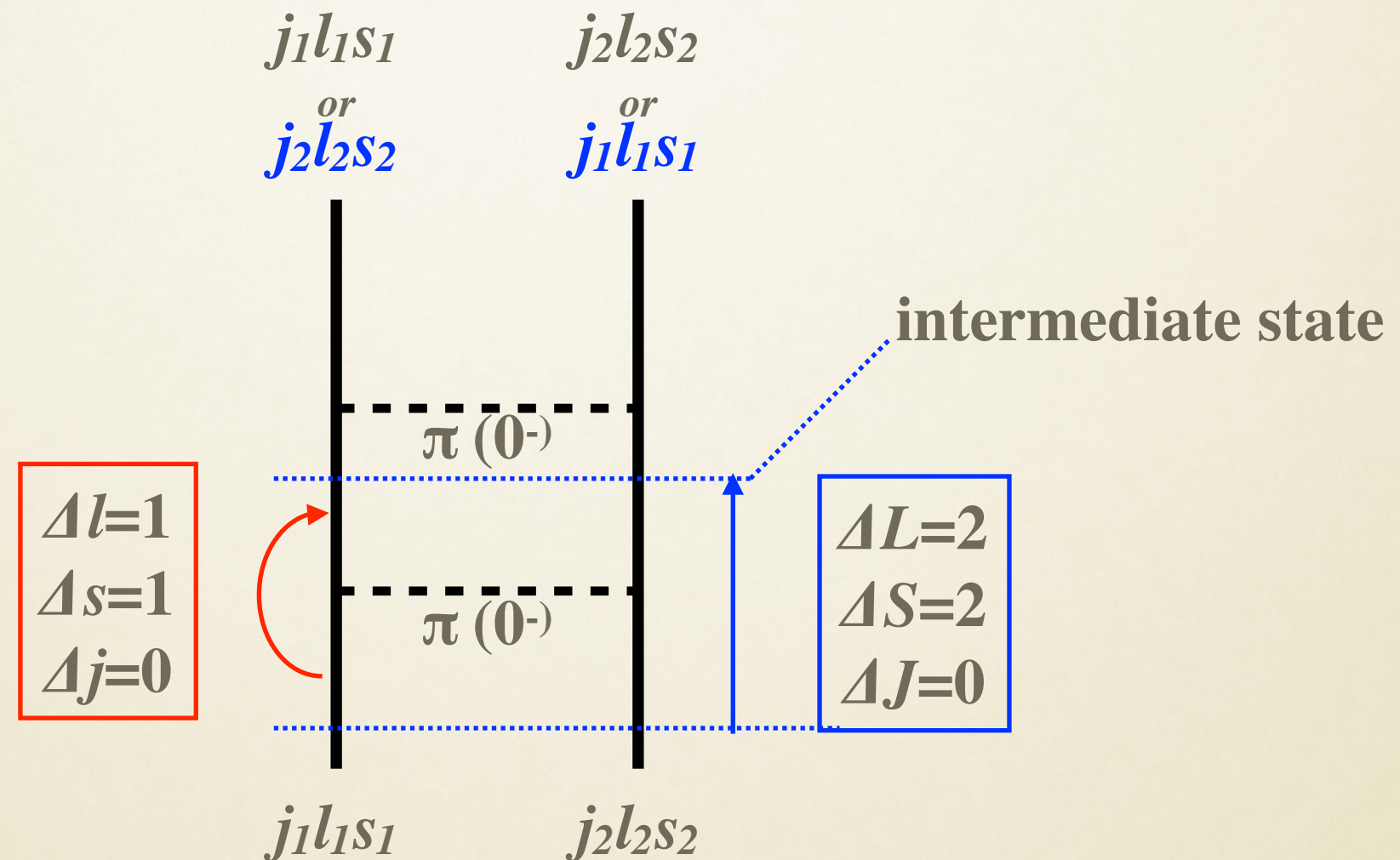
Energy variation

$$H = \sum_{i=1}^A t_i - T_G + \sum_{i < j}^A v_{ij}, \quad v_{ij} = v_{ij}^C + v_{ij}^T + v_{ij}^{LS} + v_{ij}^{Cmb}$$

$$\delta \frac{\langle \Phi | H | \Phi \rangle}{\langle \Phi | \Phi \rangle} = 0 \quad \Rightarrow \quad \frac{\partial \langle H - E \rangle}{\partial b_\alpha} = 0, \quad \frac{\partial \langle H - E \rangle}{\partial C_i} = 0.$$



# Selection rule of the tensor interaction

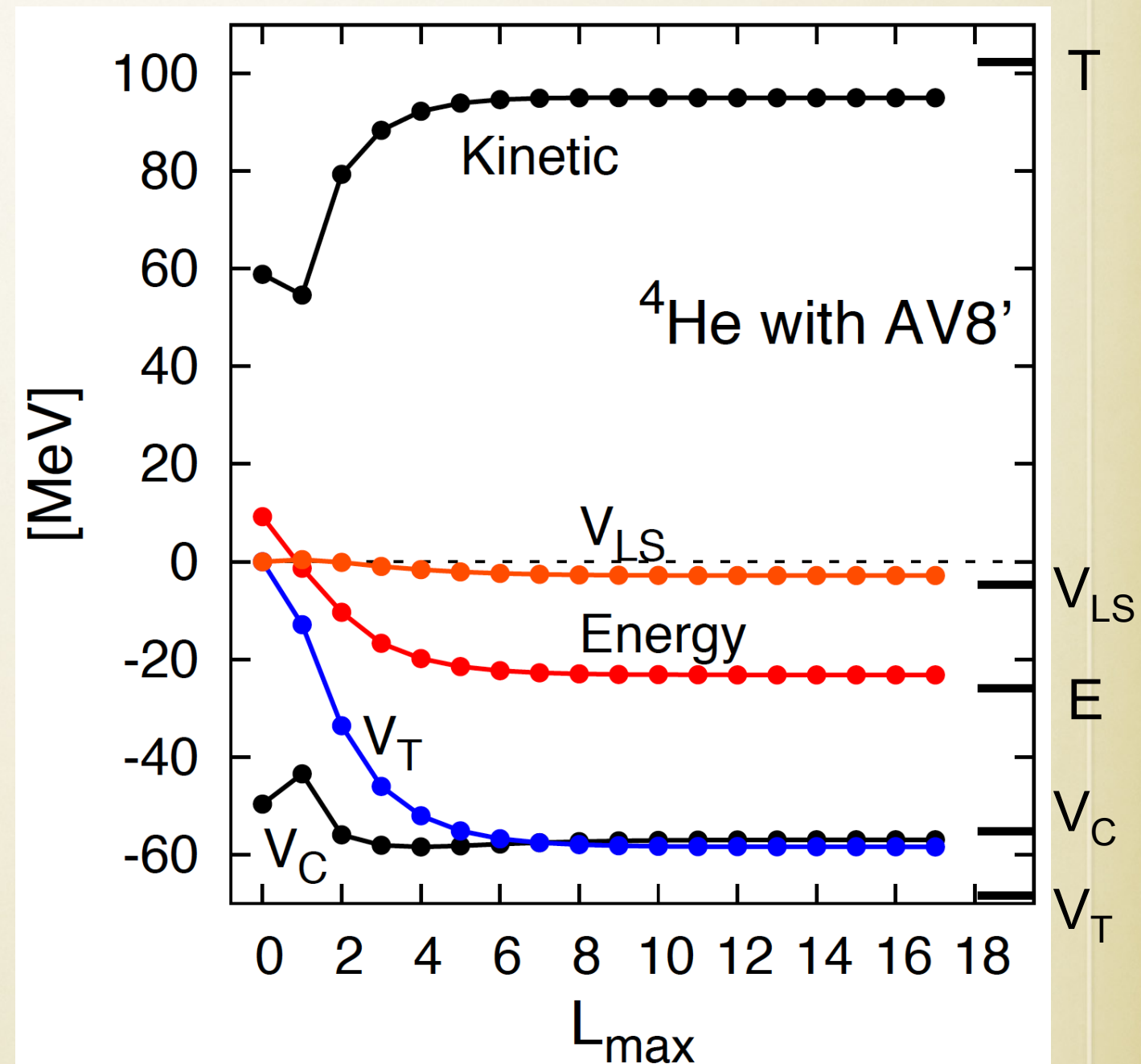




# Tensor interaction in nucleus

## $^4\text{He}$

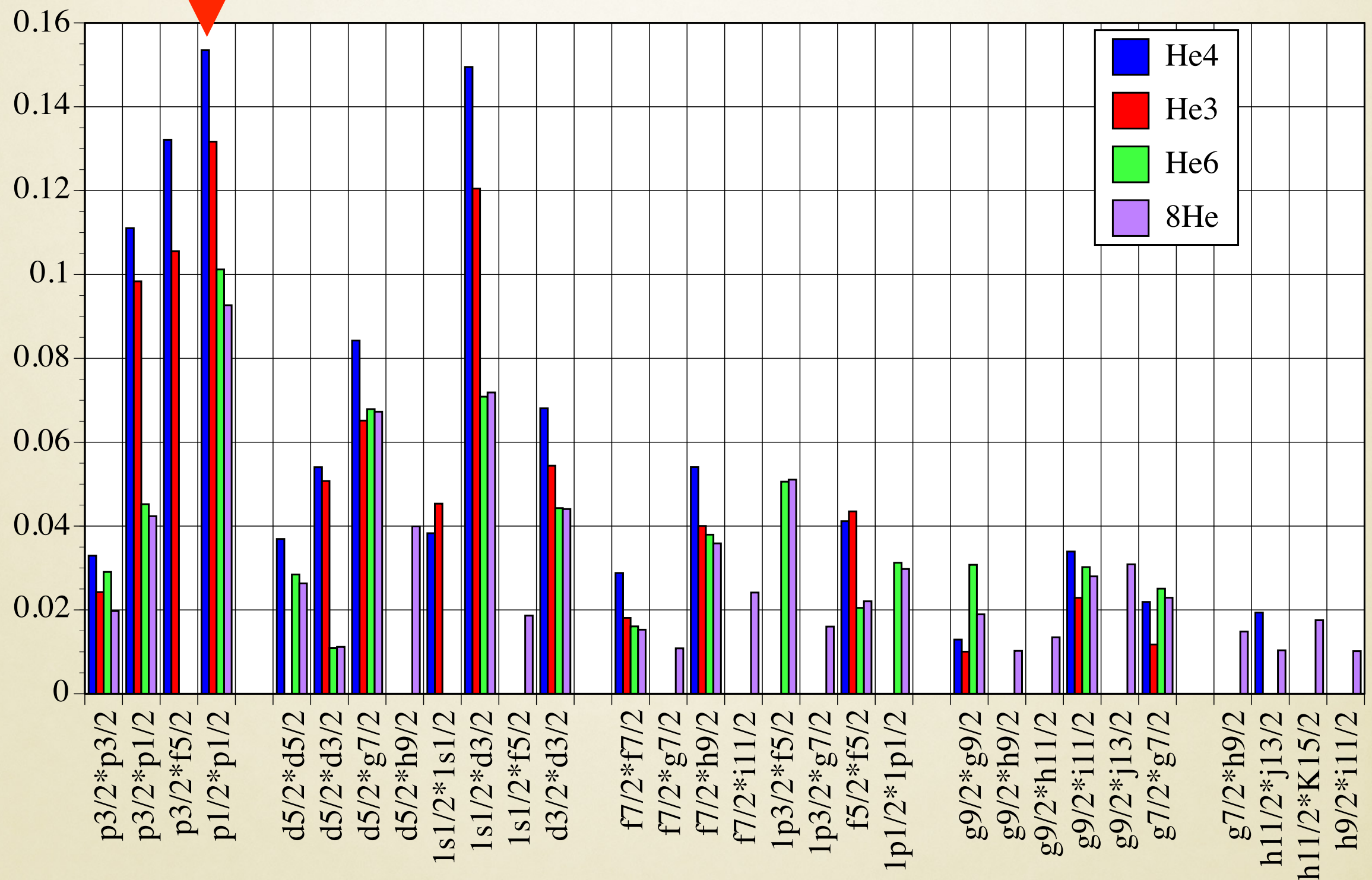
- $V_T$  contribute from higher  $l$  orbitals and convergence is slow.
- 2p-2h excitations of p-n pair under  $\Delta S=2, \Delta L=2$  provide tensor energies.
- Tensor interactions give  $\sim 60$  MeV of potential energy.
- **Remember**  $L=1$  excitation already gives 8 MeV of potential energy.



Tensor Optimized Shell Model by T. Myo, H. Toki and K. Ikeda, Progr. Theor. Phys. **121** 511 (2009)

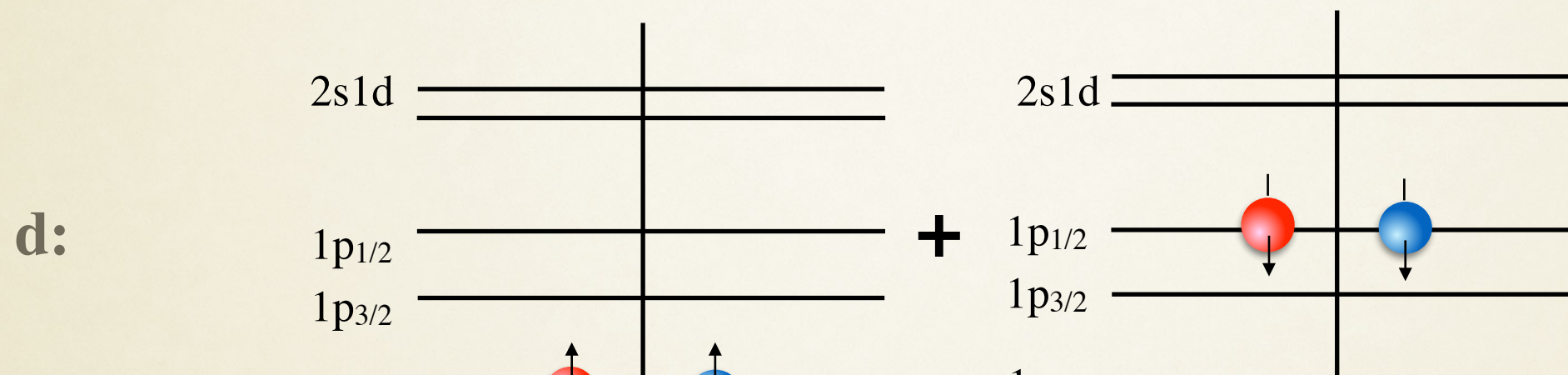


# Mixed 2p-2h configurations in He isotopes

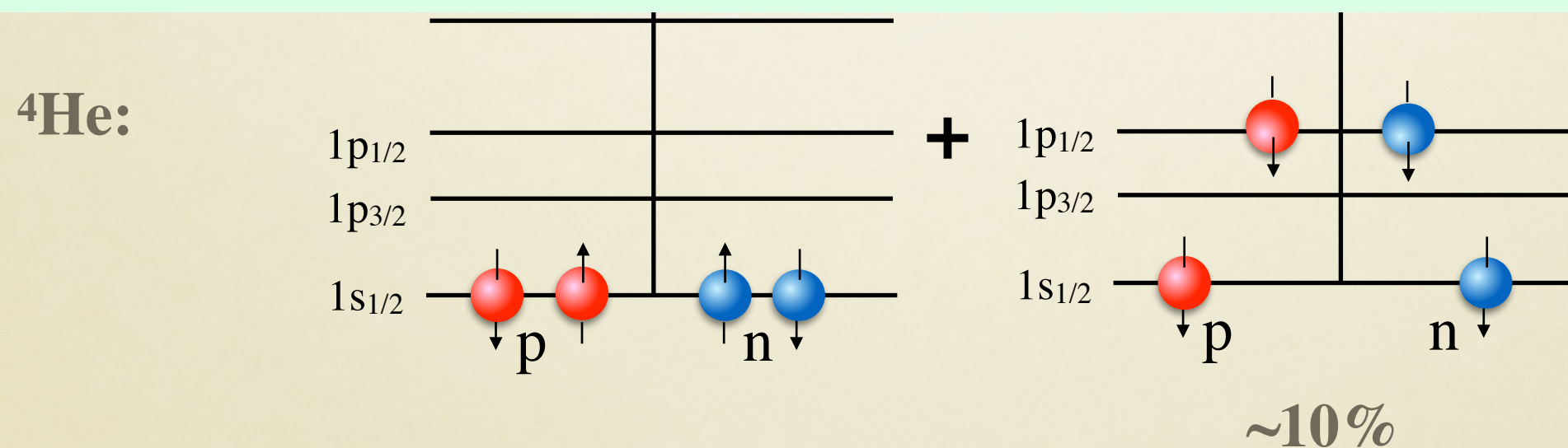




# The most important mixing



**Highest spin orbital ( $j>$ ) in a major shell is not used for the tensor interaction.  
An example is  $1p_{3/2}$  orbital in  $^4\text{He}$  and deuteron.**

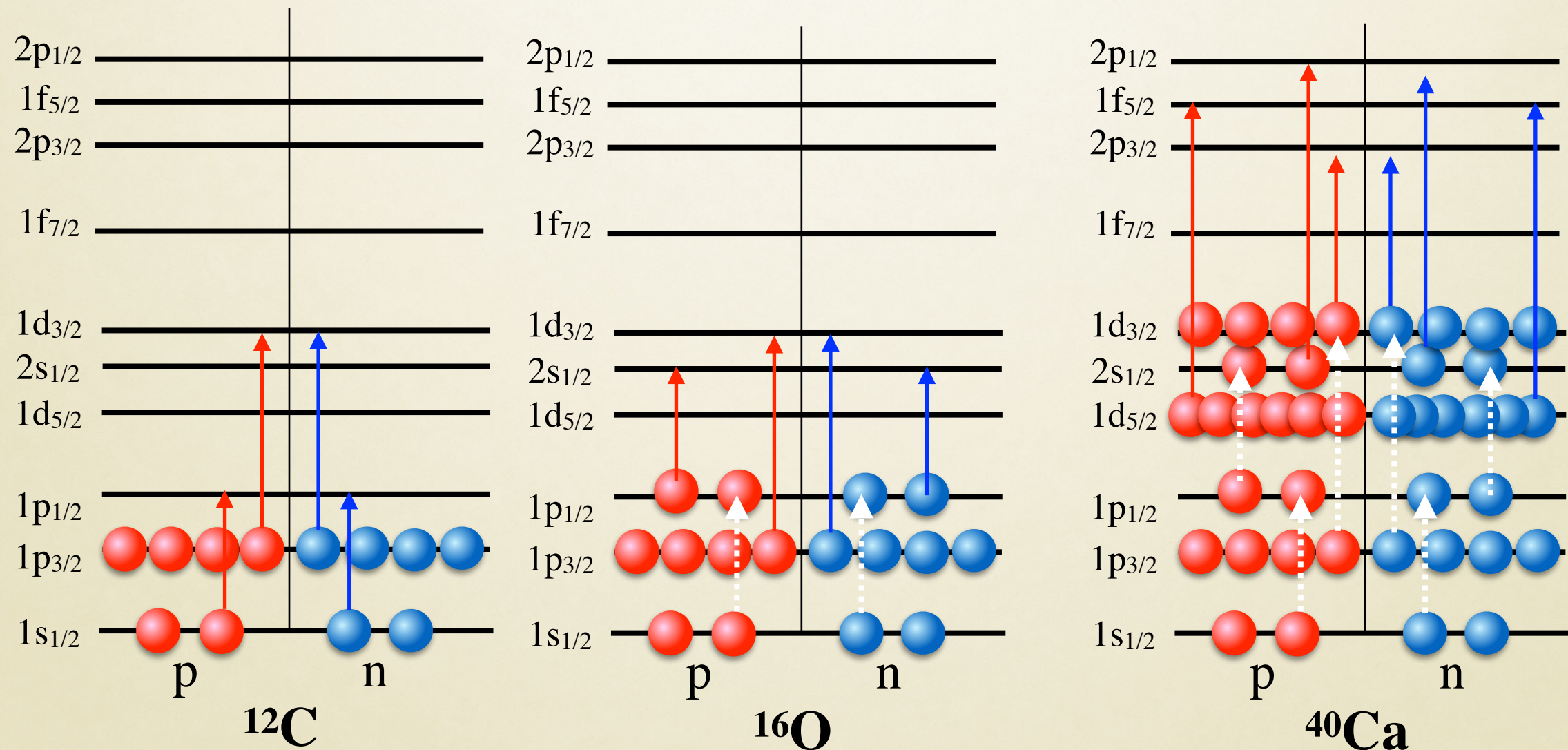


**In more general p-n pairs from  $(nlj)^2$  configuration to  $(n+1l+1j)^2$  or  $(n+1l-1j)^2$**



# Nuclear Saturation by Tensor Blocking

Blocking and Opening occurs simultaneously and keep the binding per nucleon to be almost constant.

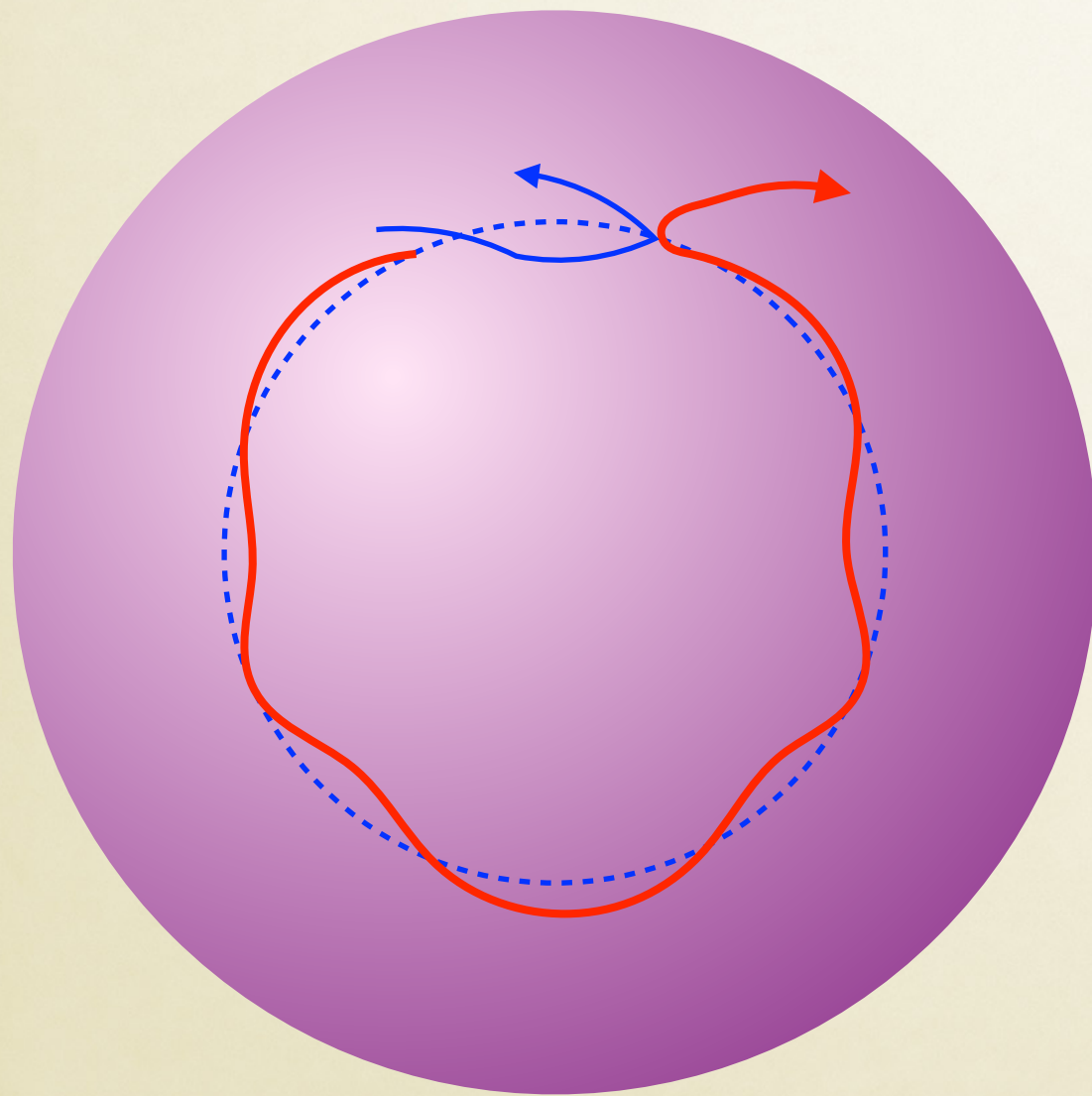


$^{12}\text{C}$  do not suffer blocking

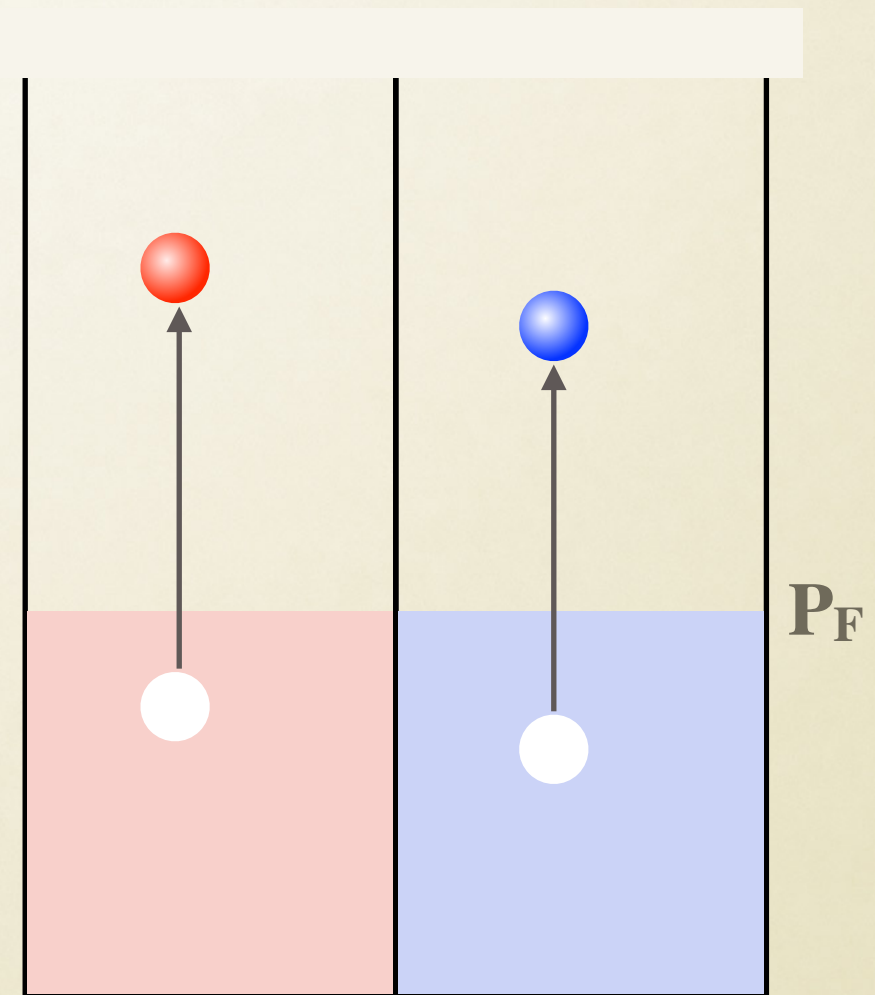
$^{16}\text{O}$  suffer blocking from  $1s_{1/2}$  to  $1p_{1/2}$ .  
But open new excitation from  $1p_{1/2}$  to  $2s_{1/2}$ .



- and, 2p-2h excitations by tensor interactions make nucleons with much higher momentum than usual Fermi motion.



Coordinate space



Momentum space

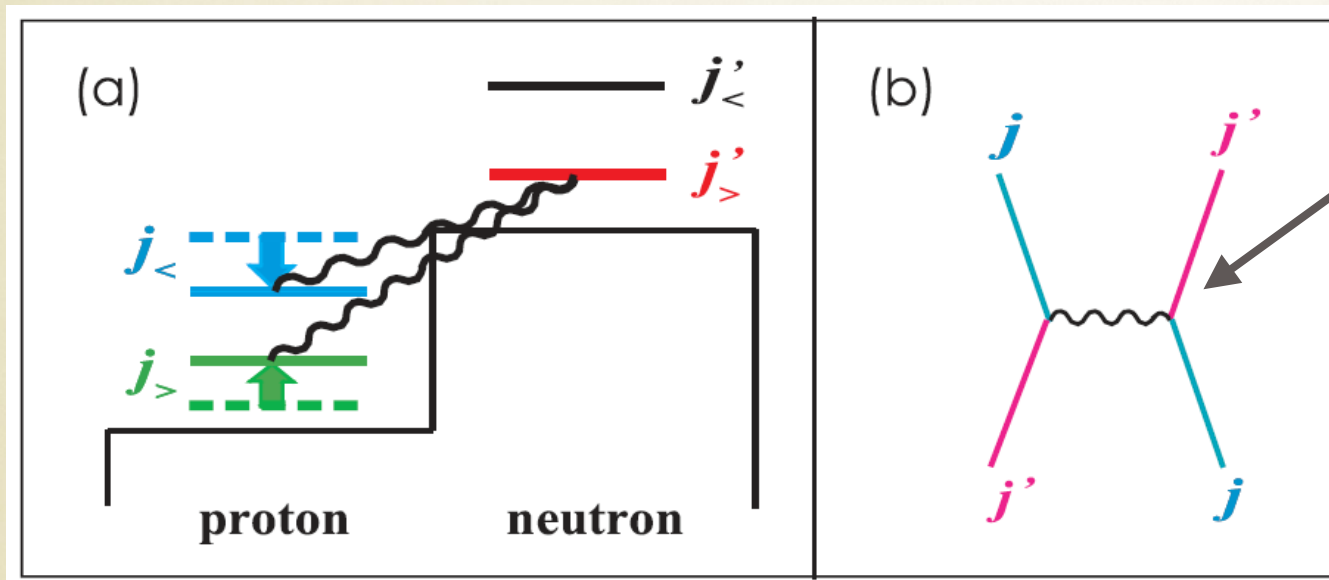


# Inclusion of tensor interaction into shell model

## (first order perturbation) $\langle \Psi | V_T | \Psi \rangle$

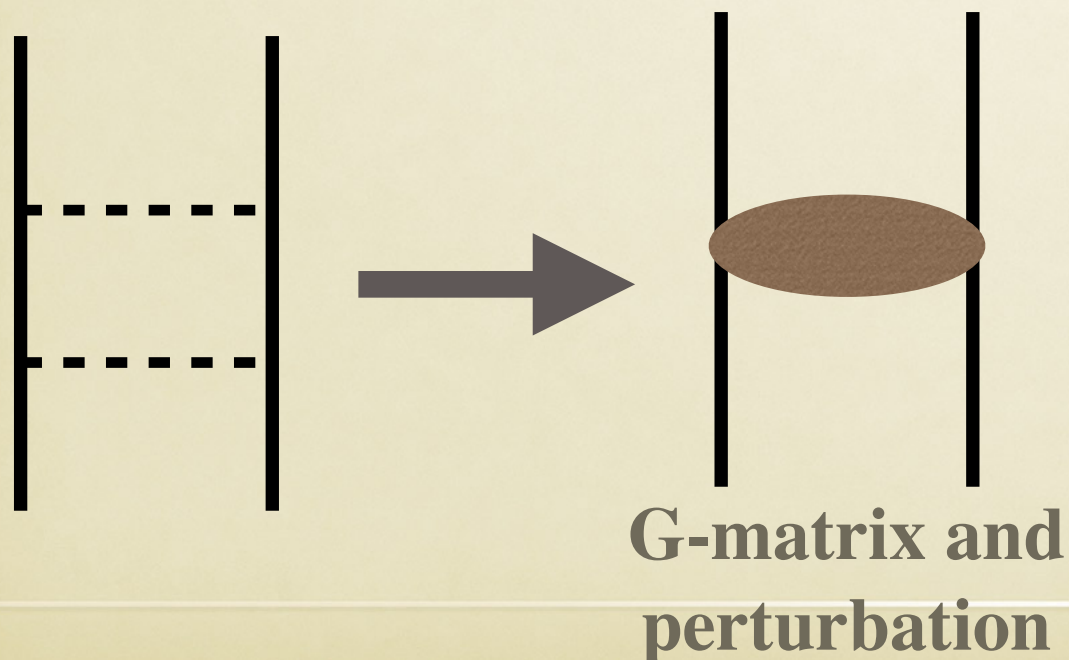
### Monopole interaction

- T. Otsuka et al., Phys. Rev. Letters 95, 232502 (2005).



**Only exchange terms contribute.  
(First order perturbation)**

**It does not contribute to doubly  
closed shell nuclei.**



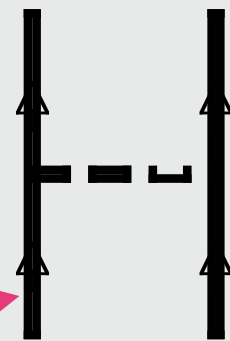


# DIFFERENCE BETWEEN SHELL MODEL TREATMENT AND REALITY

pion exchange:  $\propto S_{12} \frac{q^2}{m^2 + q^2}$

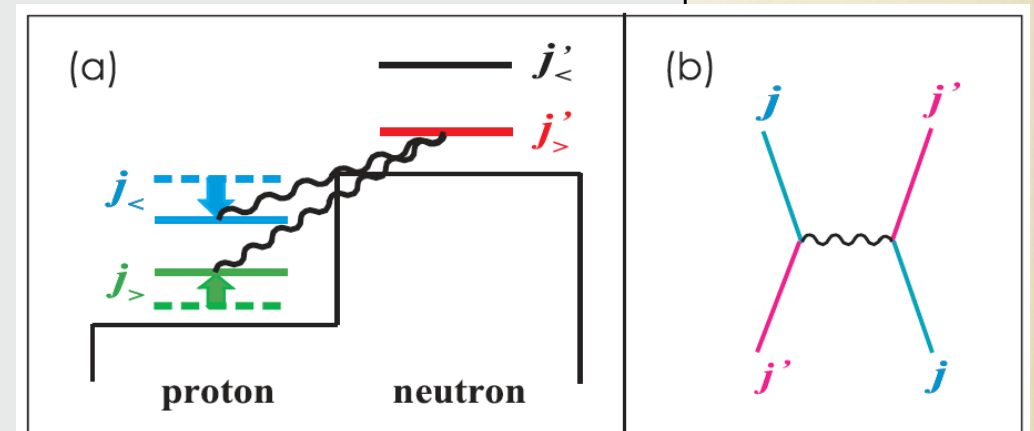
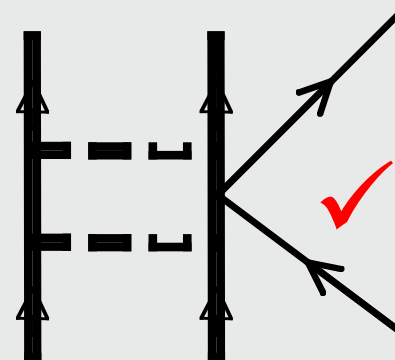
## Shell Model Approach

One pion

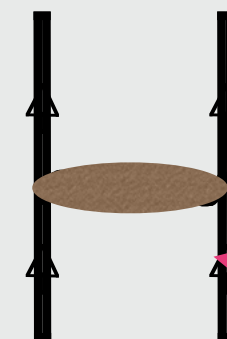


Model wave function without high momentum

Two pions



G matrix



Model wave function without high momentum

✓ High Momentum Component




# Breathe deeply

- **Pion exchange provides the Central and the Tensor Forces.**
- **The central force prefer a low momentum.**
- **The tensor force prefer a high momentum.**
- **Attraction by the tensor force is gained by a transition of a  $pn$  pair to higher orbitals under selection rule  $\Delta S=2, \Delta L=2$ . (in shell model language)**
- **Tensor interaction is blocked when nucleon occupy higher orbitals and thus nuclear saturation occurs.**



# High-momentum nucleons in ground states of a nucleus?

- Recently confirmed by (p, n) and (p, pd) reactions of  $^{16}\text{O}$  at RCNP.  
see
  - H.-J. Ong et al., Phys. Lett. B 725, 277-281 (2013)
  - S. Terashima et. al., Phys. Rev. Lett. 121, 242501 (2018)
- Also observed in quasi-free electron scattering.
  - (e, e'pn)



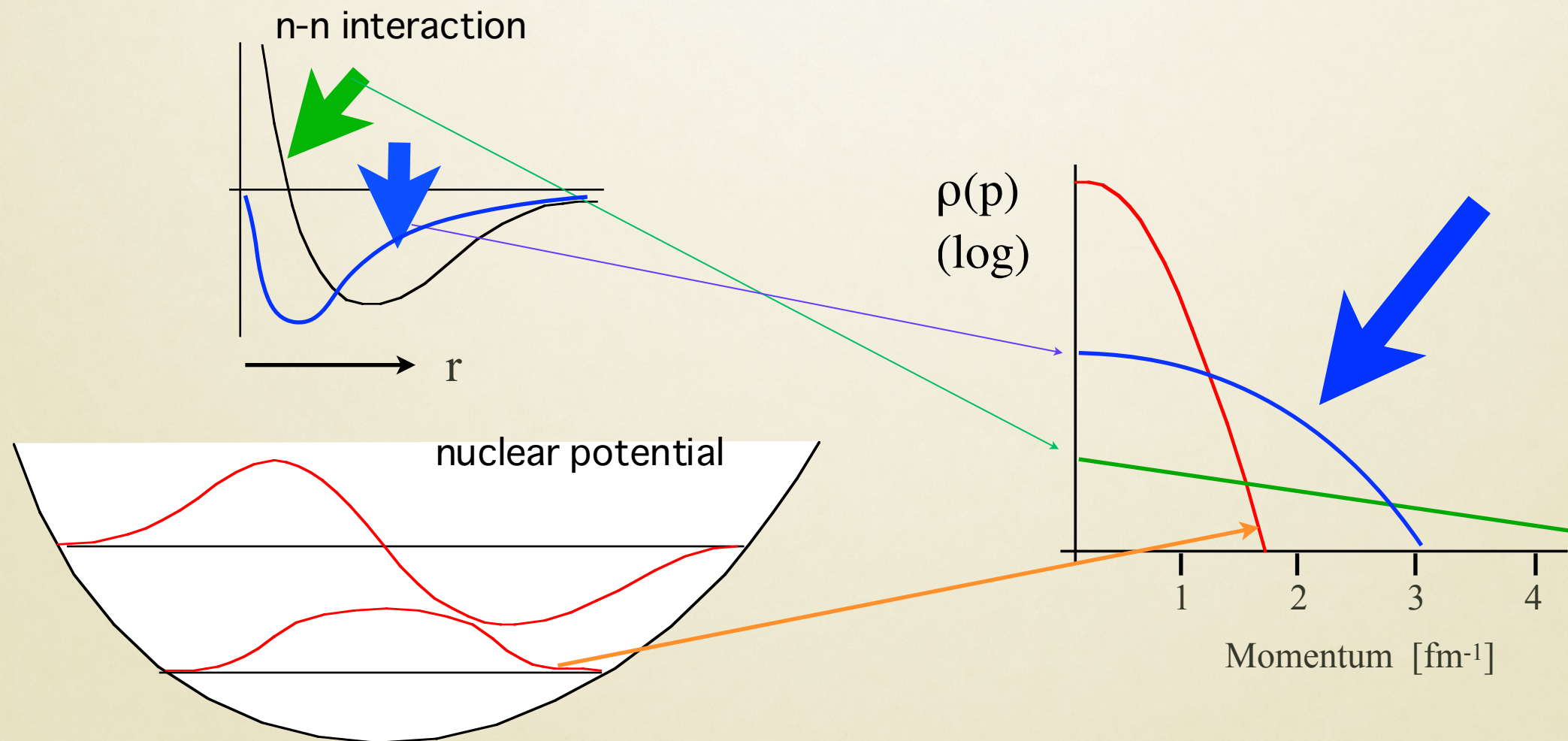
This paper tells, for the first time,  
a high-momentum nucleon pair is  $S=1$  and  $T=0$ ,  
consistent with tensor correlated pair.



# Tensor forces as high momentum

## WAVE FUNCTIONS

- Range of tensor force is about the distance of pion exchange, and thus gives momentum component specific to it.

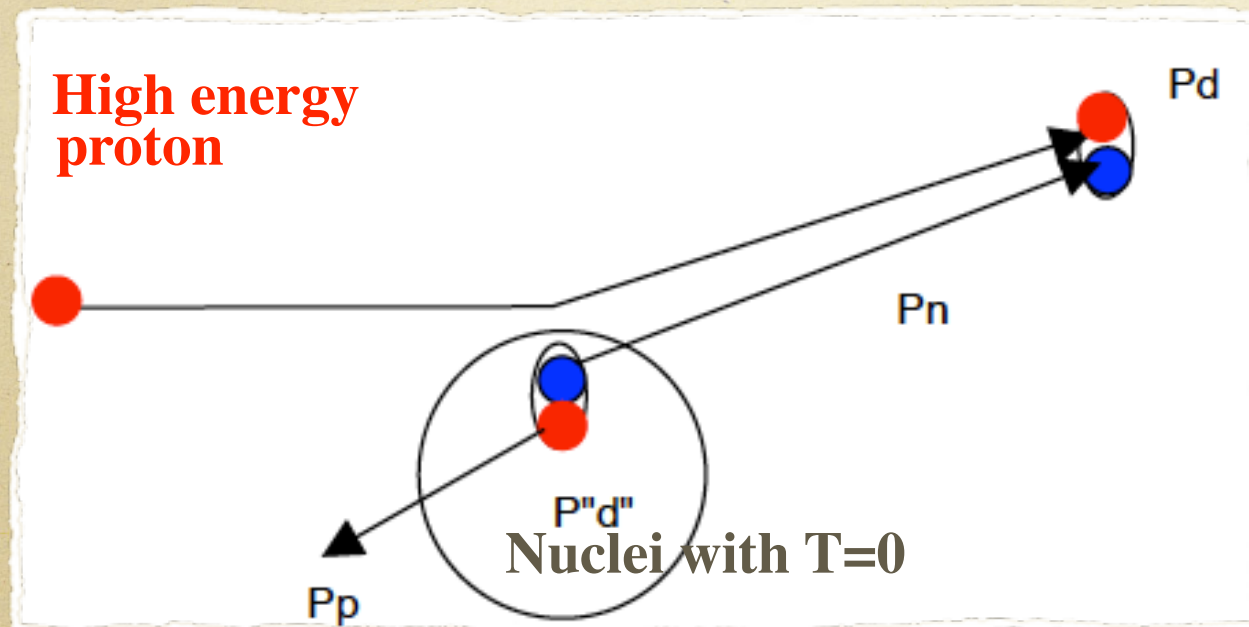




# $^{16}\text{O}(p,pd)^{14}\text{N}$

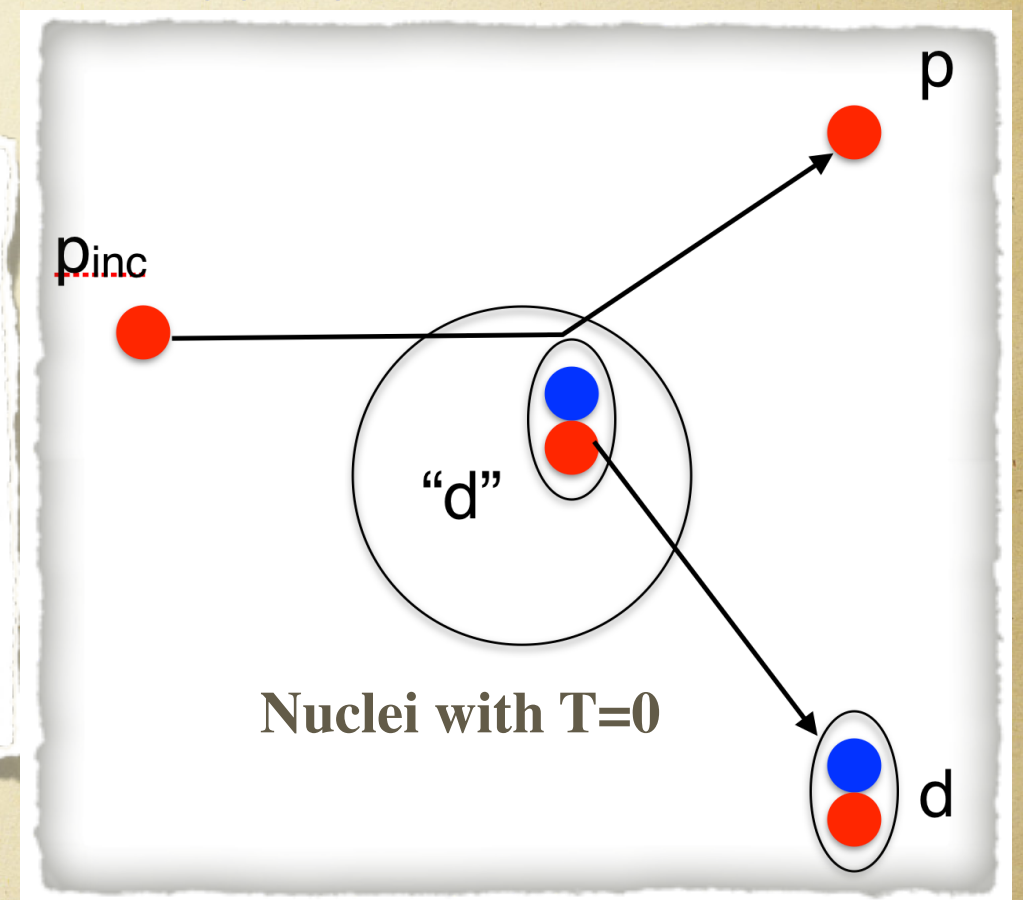
- A measurement of correlated pn pairs in nuclei with large relative momenta.

**Pick up**



$T$  of residual nuclei =  $T$  of "d"

**Knock out**

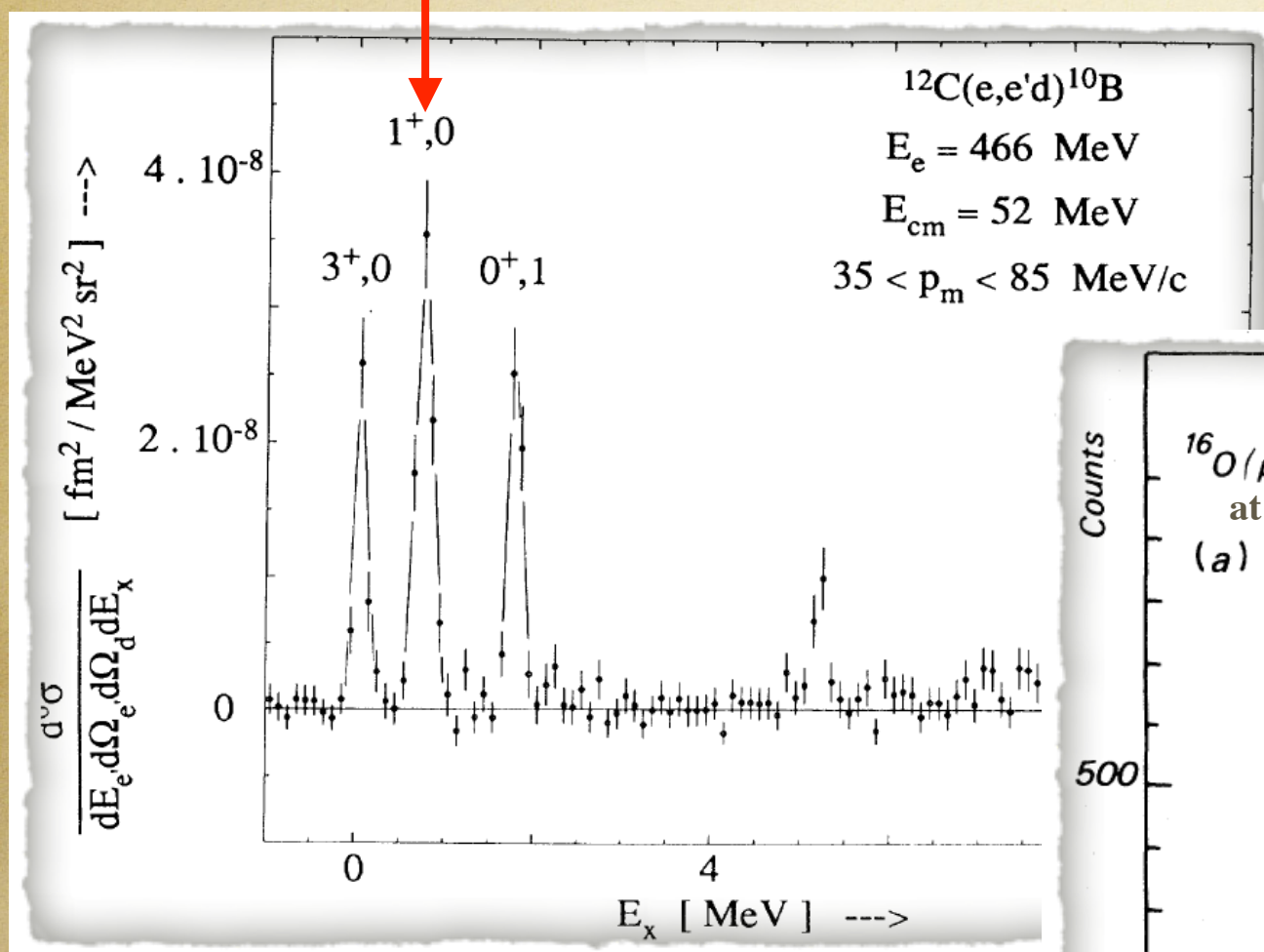


$T$  of residual nuclei = 0 or 1  
: independent from  $T$  of "d"

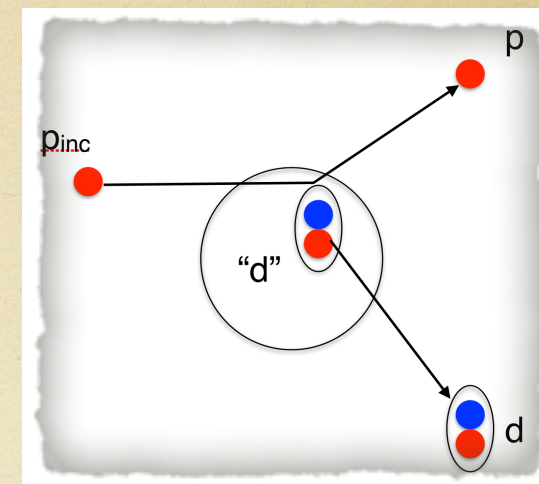


# Knock out data

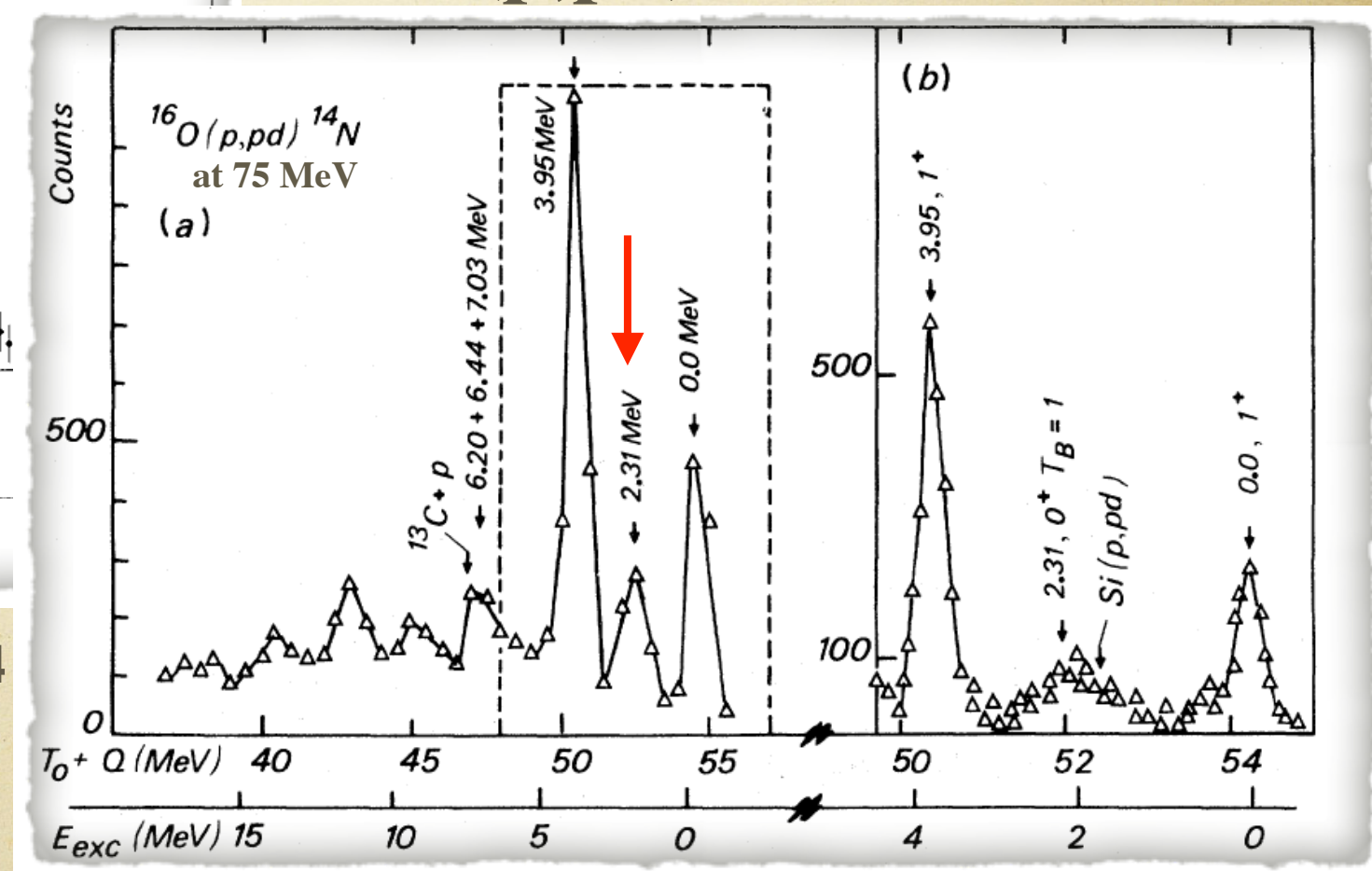
$^{12}\text{C}(e,e'd)^{10}\text{B}$  @466MeV



E. Ent et al., Phys. Rev. Lett. 62 (1989) 24



$^{16}\text{O}(p,pd)^{14}\text{N}$  at 75 MeV



J. Y. Grossiord et al., Phys. Rev. C 15 (1977) 843.



# $^{16}\text{O}(p, pd)^{14}\text{N}$ reaction at a large momentum transfer

$E_p=400$  MeV

Terashima et al. Phys. Rev. Lett. 121, 242501 (2018)

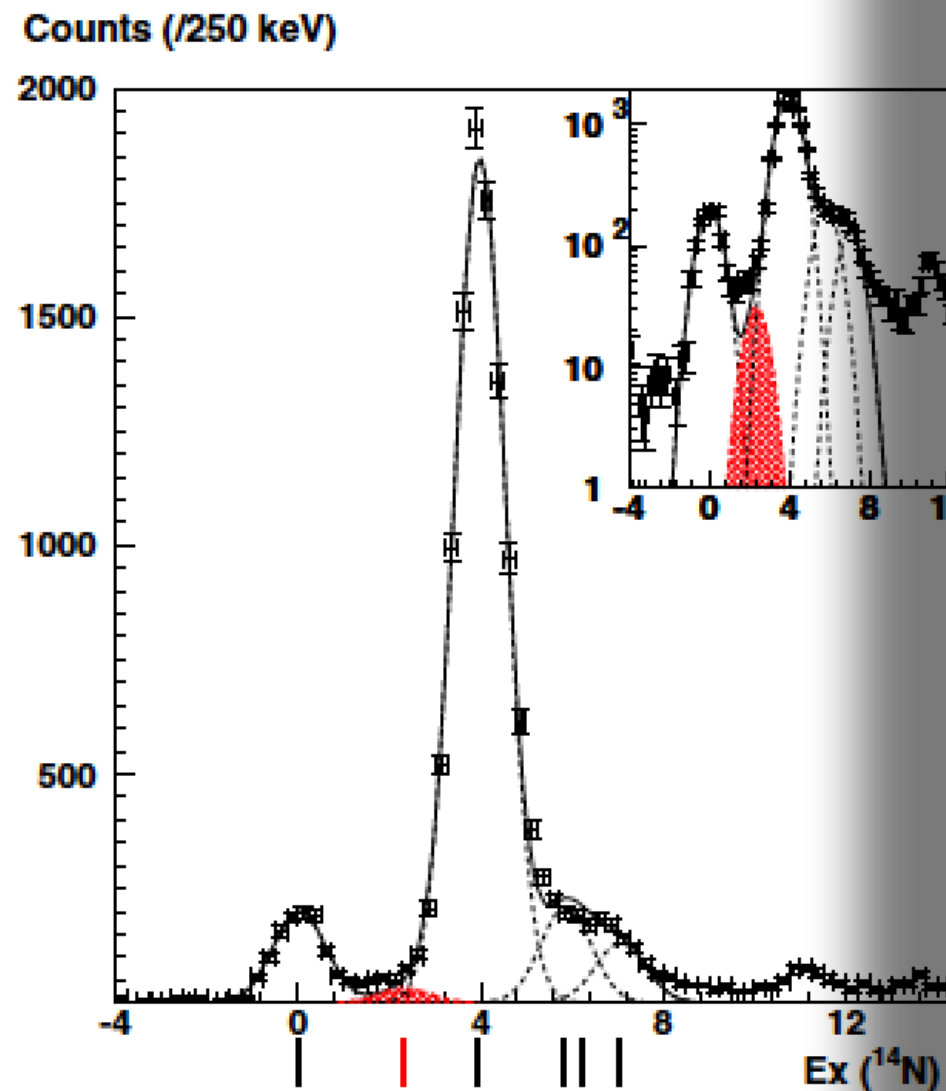
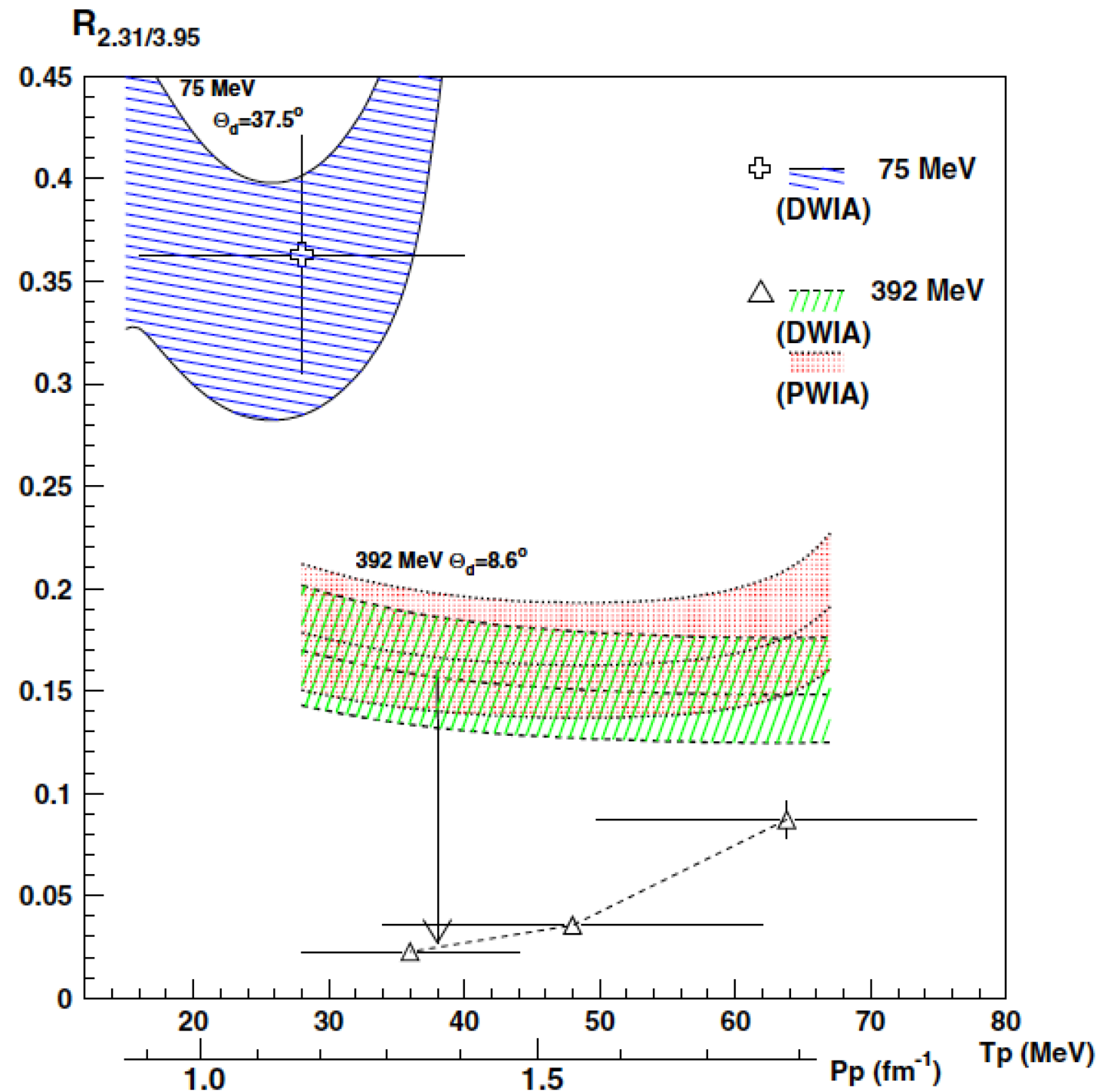


FIG. 2. The excitation energy spectrum of  $^{16}\text{O}$  ( $\theta_d = 8.6^\circ/\theta_p = 138.4^\circ$ ) with the total and individual fits shown by the solid and dashed lines, respectively.





# So

- High-momentum pair is there in ground states of nuclei.
- High-momentum pn pair is important to provide a large amount of binding in nuclei though the tensor interaction.
- But no nuclear model include such nucleons explicitly.

**In general this effect is considered just to cut the shell model space restricting only low momentum nucleons.**

**Missing strength of 20%**

**That is not true!**

**Ground state properties are strongly affected by High-Momentum Nucleons**



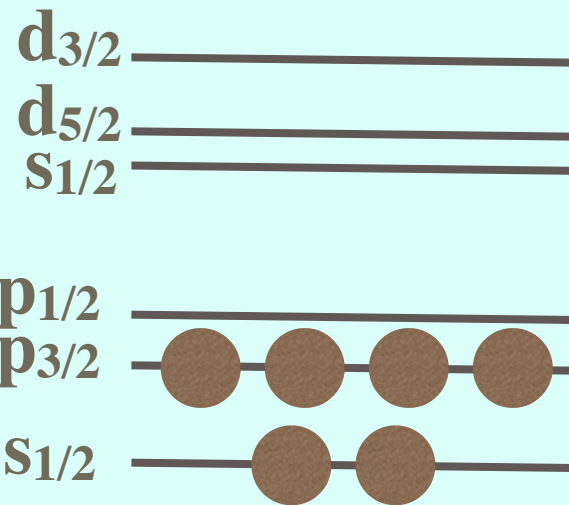
**and an effect of high-momentum component explains  
structure of a halo nucleus  $^{11}\text{Li}$ !**



# s- and p- waves mixing in $^{11}\text{Li}$

## Usual orbitals

- Momentum transfer
- Equilibrium
- Beta-decay
- 30-40%
- two-neutron



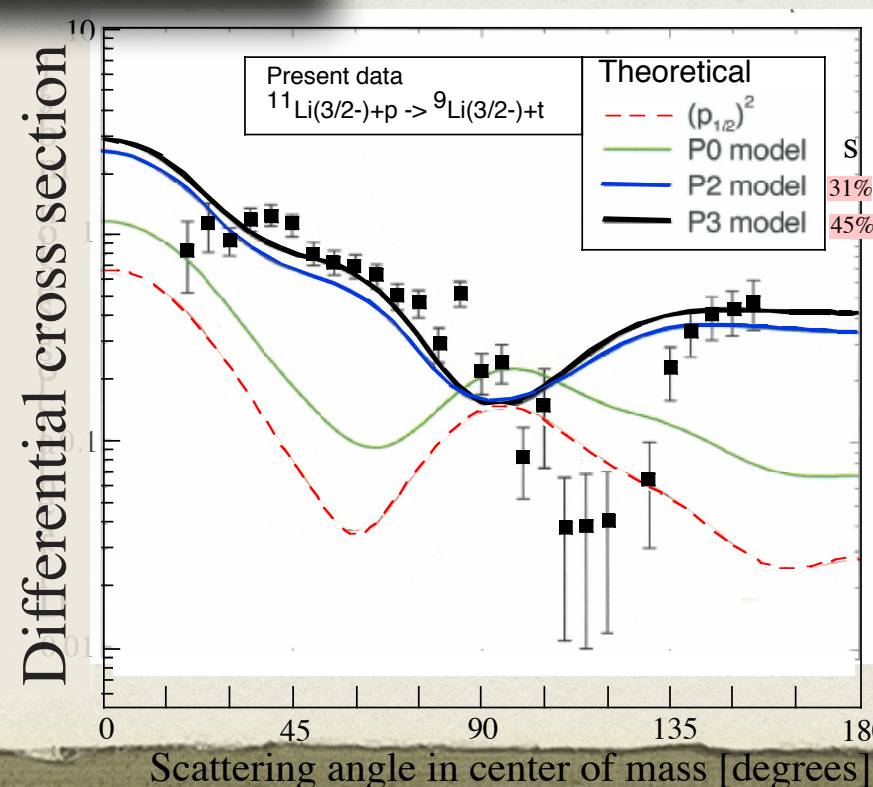
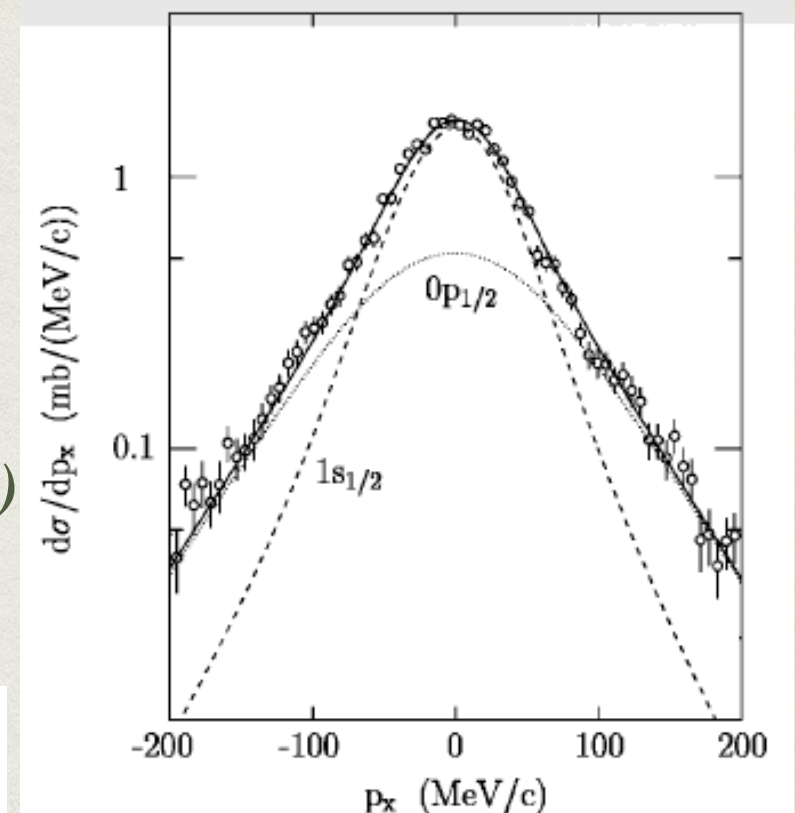
fragments  $^{10}\text{Li}$

(mon 1999)

ent of  $p_{1/2}$  (Borge 1997)

ion

- ( $^{11}\text{Li}+p \rightarrow ^9\text{Li}+t$ )
- 31-45%  $s_{1/2}$  and  $p_{1/2}$
- (2008)



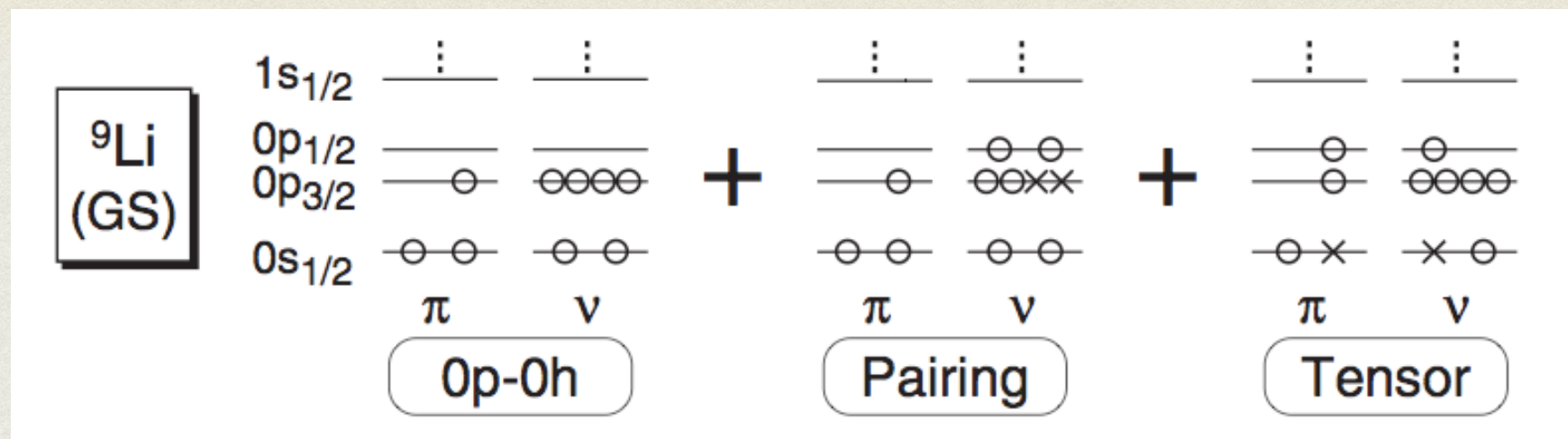
41 % s

35 % s

I. Tanihata et al., Phys. Rev. Lett  
100 (2008) 192502..



# Mixing of $s_{1/2}$ and $p_{1/2}$ in $^{11}\text{Li}$



T. Myo, K. Kato, H. Toki, K. Ikeda, Phys. Rev. **76** (2007) 024305.



# TOSM

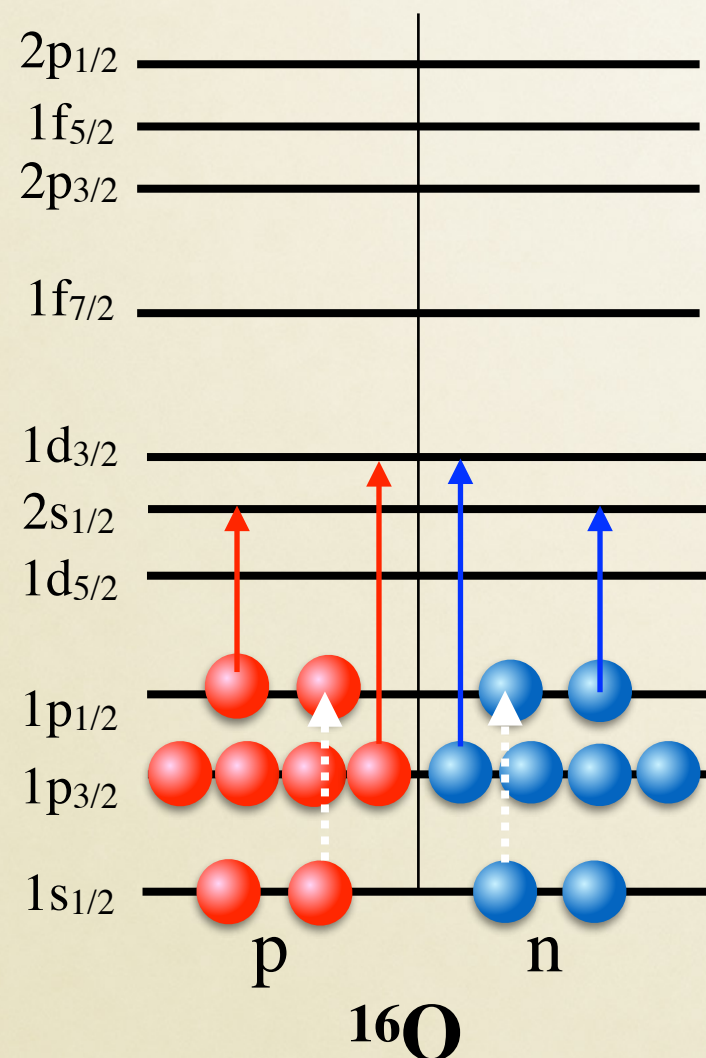
*Myo, Toki, Ikeda, Kato, Sugimoto, PTP 117 (2006)*

- It provides good account of ground and low excited states up to Be isotopes.
- It explains, for the first time, the s-p mixing in halo  $^{11}\text{Li}$  nucleus.
- However it needs extensive computer power and it is difficult to extend the calculations above Be presently.
- But now we know that the **tensor blocking** is important to explain the structure of neutron rich nuclei.



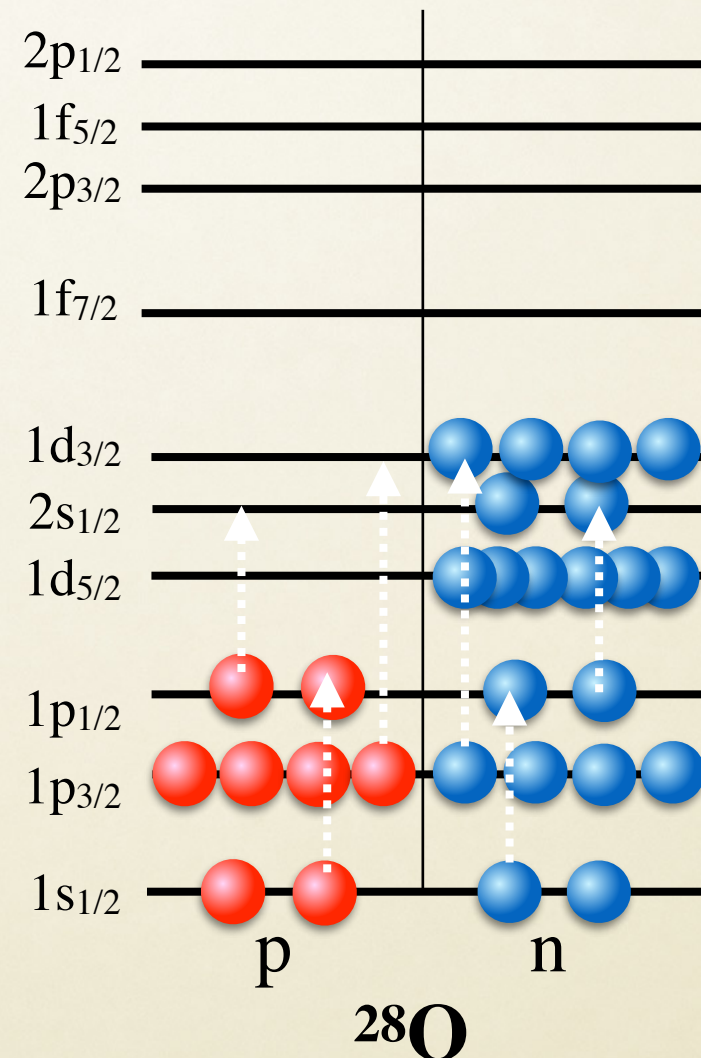
# What is the difference between stable and neutron rich nuclei?

## Symmetric nuclei



Blocking and Opening occur simultaneously.

## Neutron rich nuclei



Only tensor blocking occurs.



# Tensor Blocking Shell Model



- Use spirit of TOSM (include 2p-2h excitation explicitly so that tensor force is treated well).
- Treat only  $\Delta l=1$  orbital separately. All light nuclei so far fills only up to  $\Delta l=1$  orbitals.
- Higher excitations  $\Delta l \geq 2$  is treated in Q-space and consider only to contribute to the mean field potential.



# Tensor Blocking Shell Model

Relation to the shell model

$$H = \sum T^i + \sum v_c^{ij} + \sum v_T^{ij}$$

residual interactions and treated them as perturbation.

$$H = \sum T^i + V_{sh} + \sum v_{sh}^{ij}$$

We write tensor part explicitly, then

$$V = H - \sum T^i = V_C + V_T + \sum v_c^{ij} + \sum v_T^{ij}$$

$$V_{sh} = V_C + V_T$$

$$\sum v_{sh}^{ij} = \sum v_c^{ij} + \sum v_T^{ij}$$

In the shell model tensor blocking is not treated then so we separate blocking part, (Blocking occurs only in  $\Delta l=1$  transition)

$$\begin{aligned} V_T &= V_{T1} - V_{T1}^0 + (V_{T1}^0 + \sum_{l=2...} V_{Tl}) \\ &= V_{T1} - V_{T1}^0 + V_T^0 \end{aligned}$$

$$V = V_{sh} + (V_{T1} - V_{T1}^0) + \sum \bar{v}_{sh}^{ij}$$

$$(V_{T1} - V_{T1}^0)$$

The difference of the potential energy between an open shell and the occupied shell.

*If you include high-momentum correlated nucleons, you can not forget this term.*

$V_C + V_T^0$  is exactly the shell model potential ( $V_{sh}$ ) at the closed shell.

-> We can use (Woods-Saxon potential + Blocking)



# Tensor Blocking Shell Model

$$H = T + V_C + V_T$$

$$\Psi = \psi_{sh} + \psi_{2p-2h}$$

$\psi_{sh}$  only low momentum  
 $\psi_{2p-2h}$  includes high-momentum

Potential energy

**Otsuka's monopole term**

$$\langle \Psi | V_C + V_T | \Psi \rangle = \langle \psi_{sh} | V_C | \psi_{sh} \rangle + \langle \psi_{sh} | V_T | \psi_{sh} \rangle + 2 \langle \psi_{sh} | V_T | \psi_{2p-2h} \rangle + \langle \psi_{2p-2h} | V | \psi_{2p-2h} \rangle$$

$$\langle \psi_{sh} | V_T | \psi_{2p-2h} \rangle = \langle \psi_{sh} | V_T | \psi_{2p-2h}^{\Delta l=1} \rangle + \langle \psi_{sh} | V_T | \psi_{2p-2h}^{\Delta l \geq 2} \rangle$$

$\Delta l=1$  gives 5~8 MeV additional energy in the binding.

So we treat all other term as a mean field potential given by phenomenologically accepted Woods-Saxon potential.

Usual shell model

$$H_{sh} = \sum_{i,j} v_{ij} \\ = V_{mf} + \sum_{i,j} \bar{v}_{i,j}$$

mean field potential  
and  
residual interactions

$V_{mf}$ : includes tensor  
 $\bar{v}_{i,j}$  : does not!



# **Take a depth breathe for the last sections**

- **We start from single-particle orbitals obtained from a classical Woods-Saxon potential.**
- **Then we follow the change of binding energy by the change of tensor interactions when nucleons are added to a new orbital.**
- **The magic number is where energy gap to next nucleus is large.**



# Now it's become very simple

- Only two ingredients,

mean field

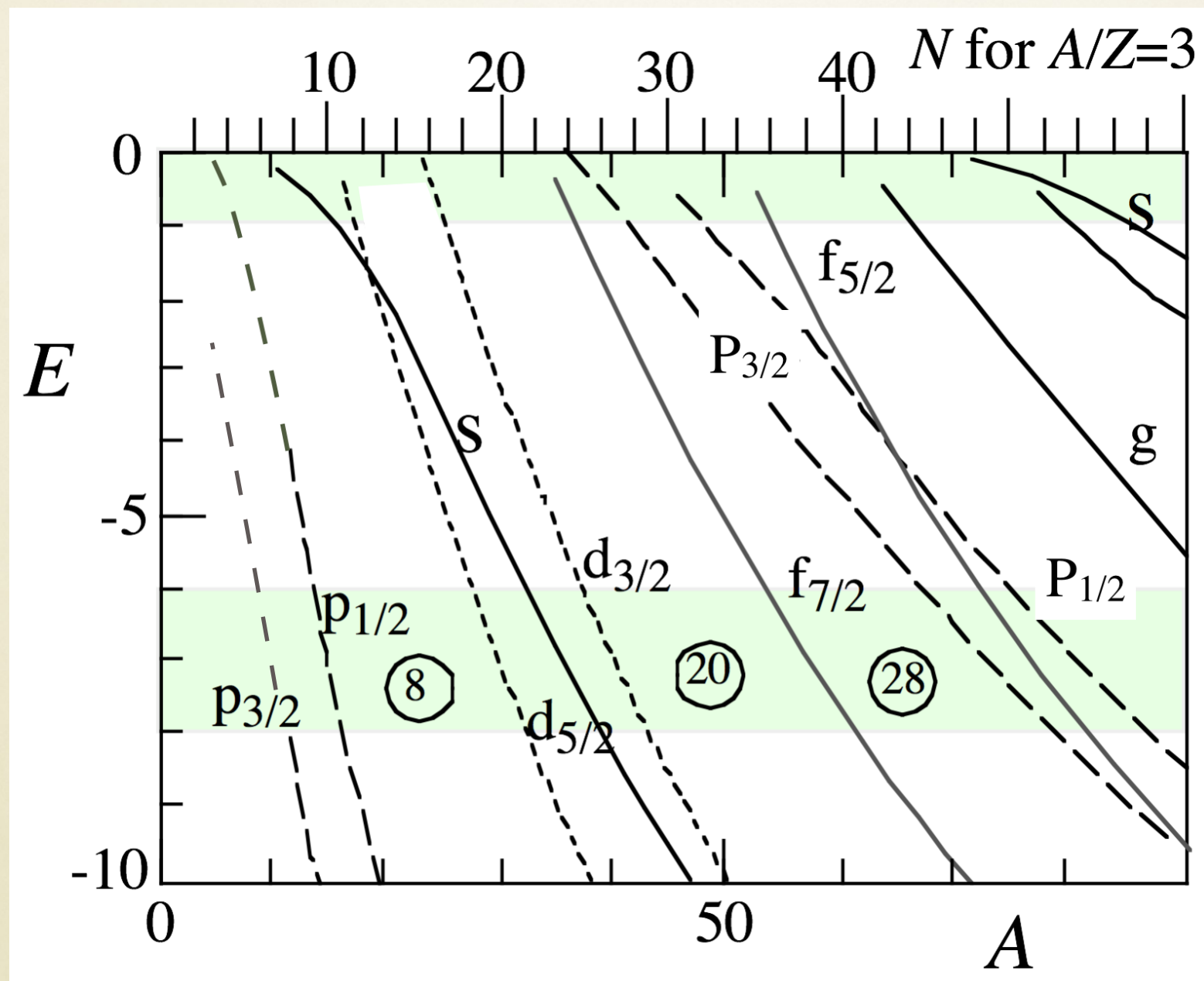
and

$\Delta l=1$  tensor.



# Orbitals in W-S potential

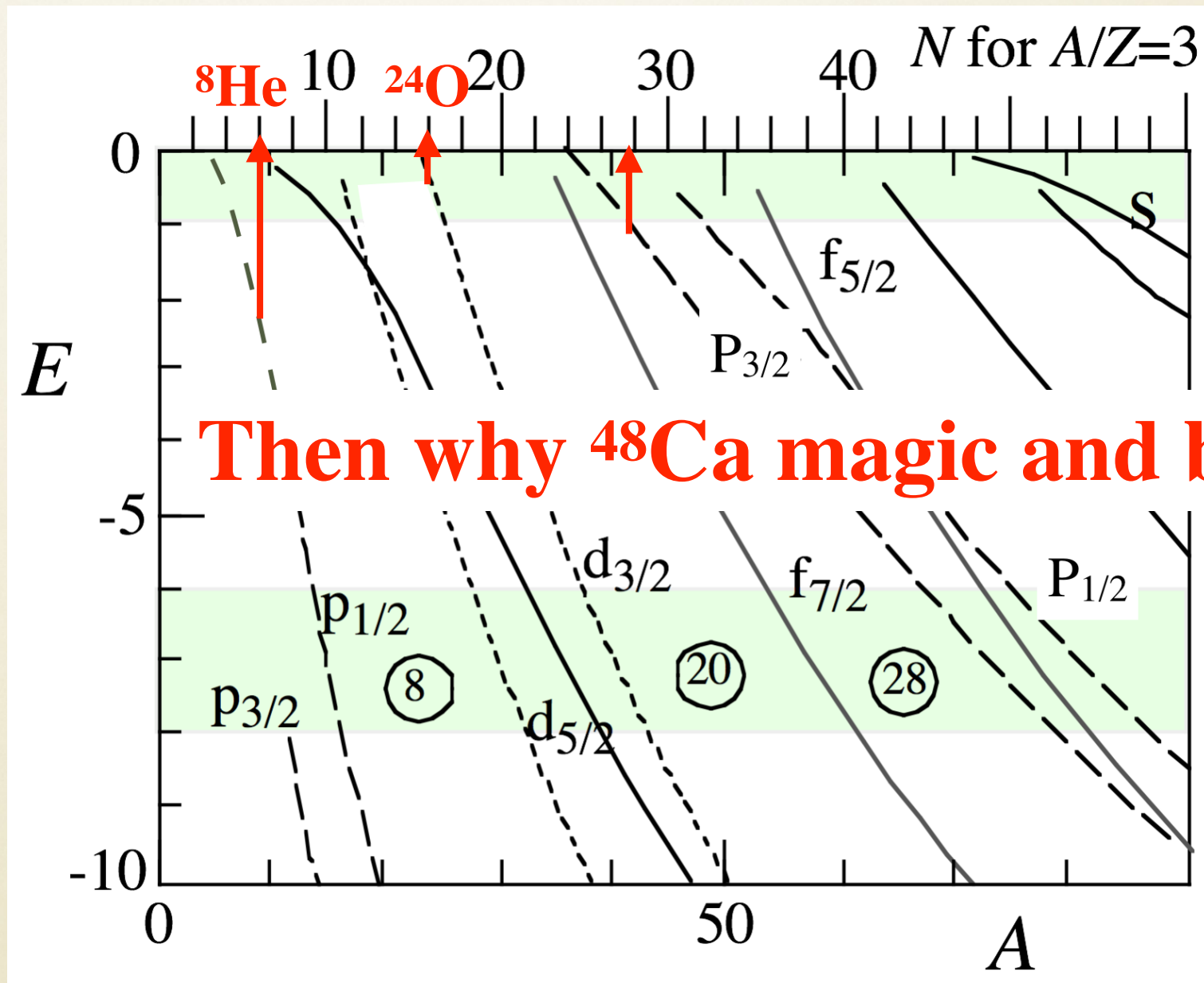
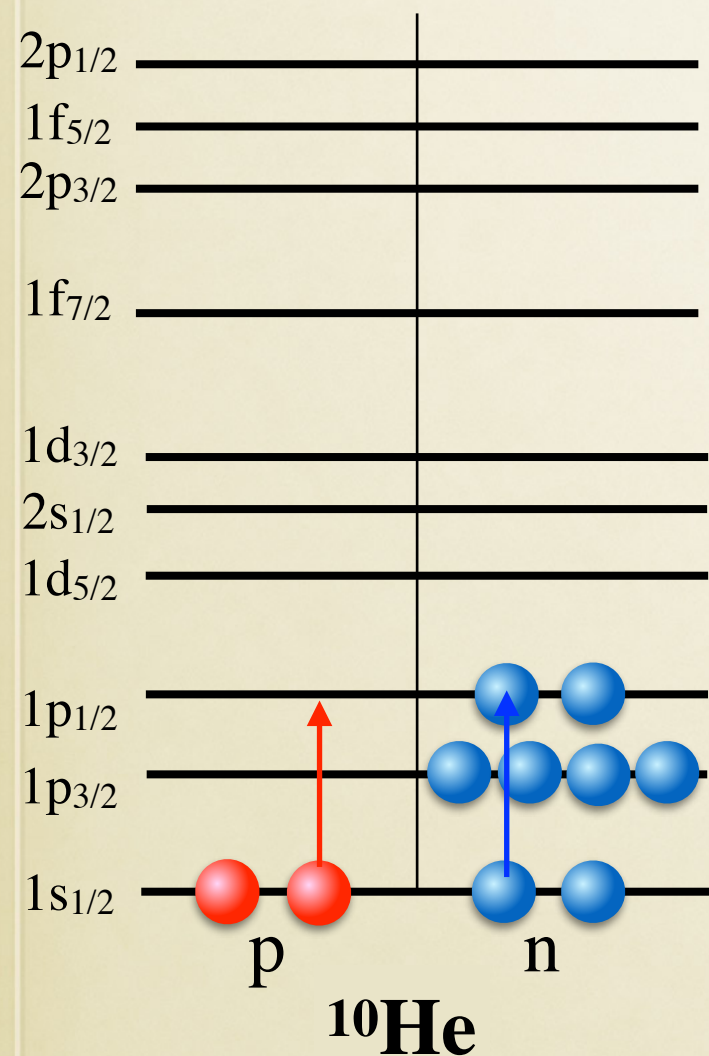
**A blocking of 2p-2h excitation looses  $>5$  MeV**



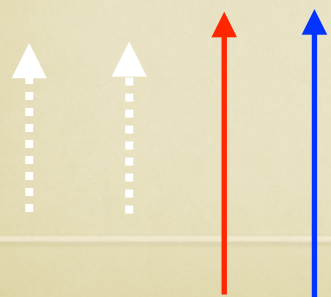
Woods-Saxon potential parameters are from the book of Bohr and Mottelson.  
Calculations are made for  $A/Z=3$  nuclei.



# Why doubly magic $^{10}\text{He}$ and $^{28}\text{O}$ are not bound?

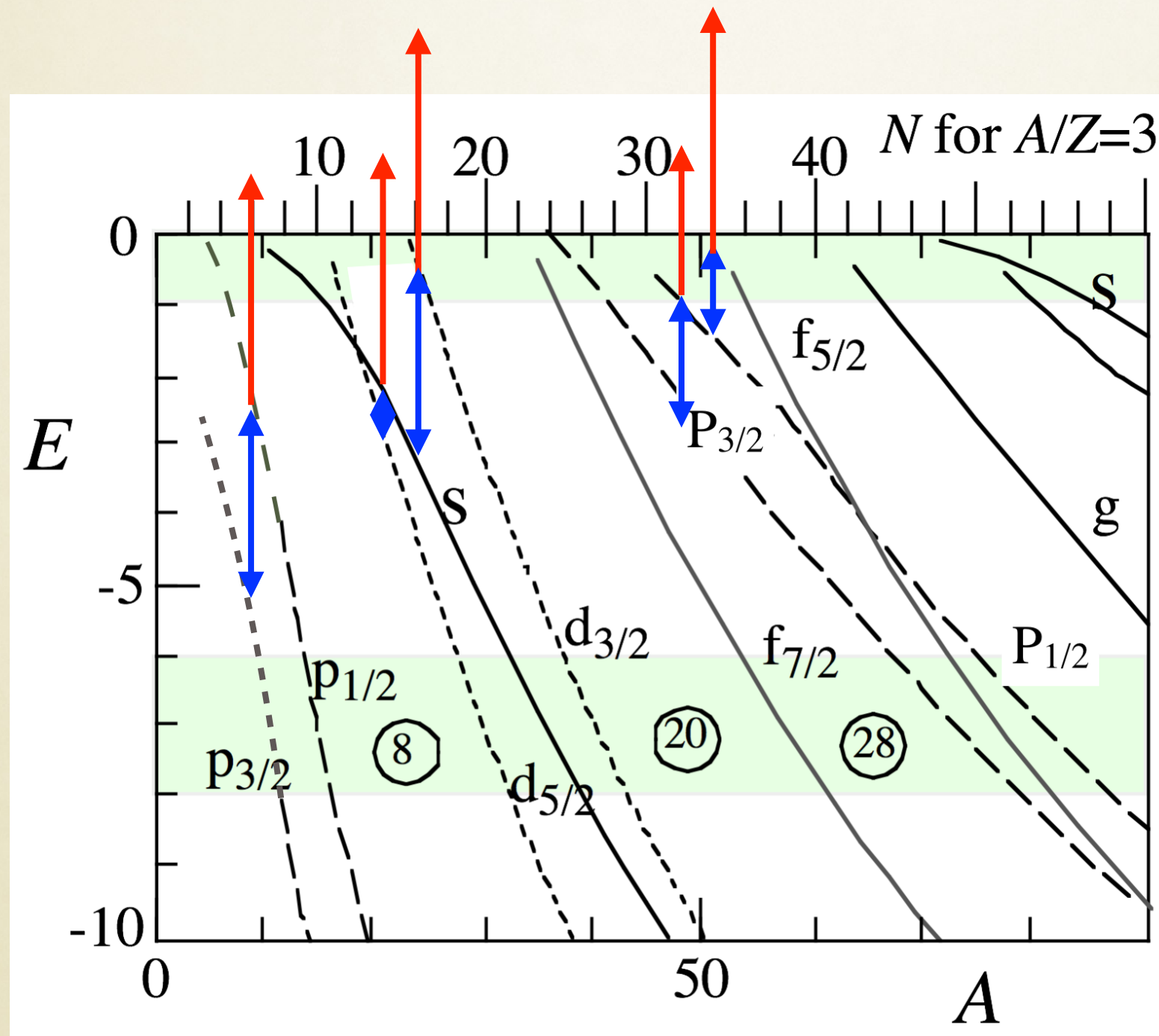


Then why  $^{48}\text{Ca}$  magic and bound?





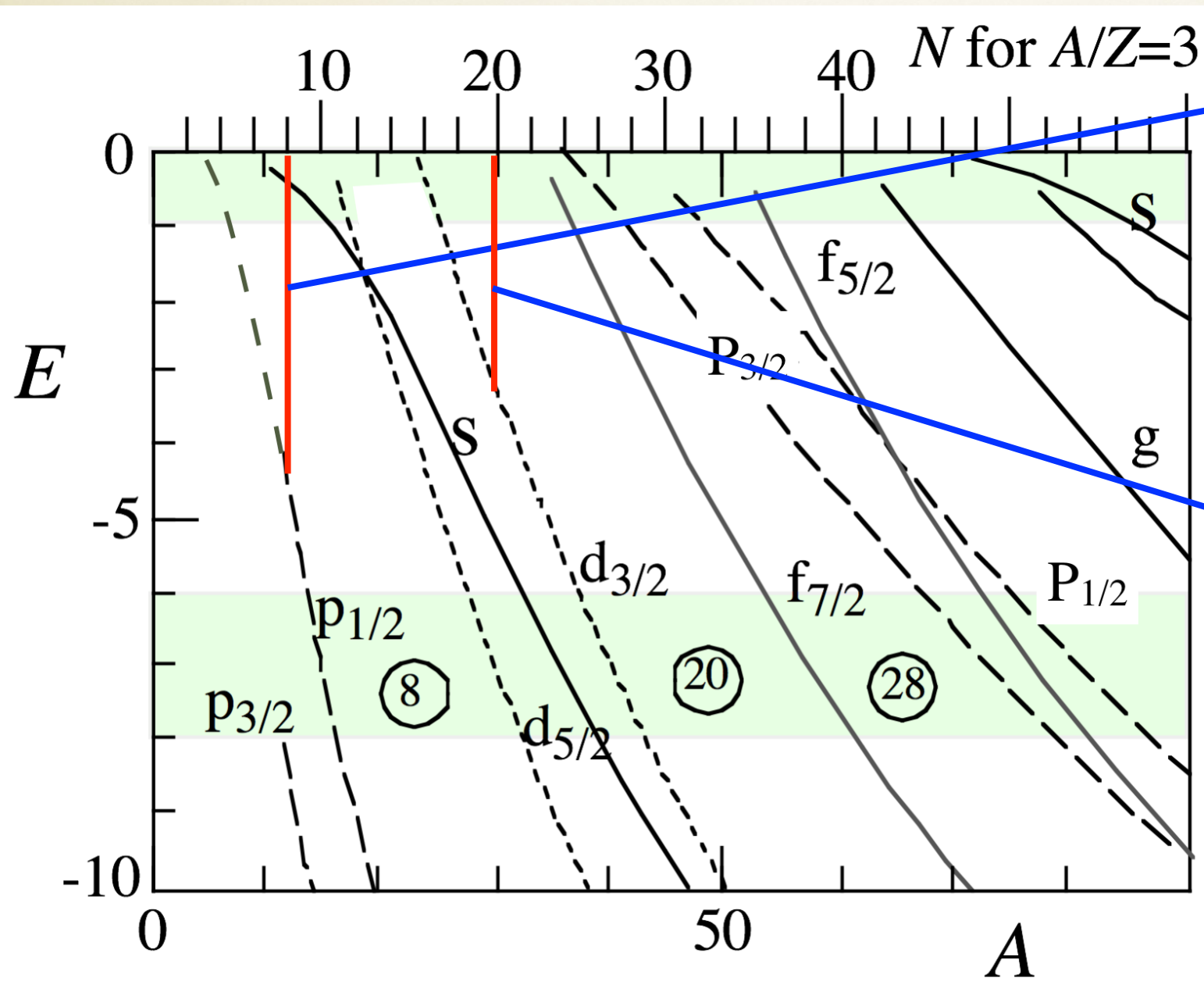
# How are new magic numbers $N=6, 14, 16, 32, 34$ made?



energy gaps become more than factor of two larger due to the tensor blocking.



# Why magic numbers $N=8$ and $N=20$ disappear in neutron-rich nuclei?



Originally a large gap but the tensor blocking effectively bring  $p_{1/2}$  much loosely bound and mixes with sd-shell. Blocking does not occur for  $s_{1/2}$  until proton fills  $p_{1/2}$ .

Originally the energy gap is larger than  $\sim 4$  MeV but the tensor blocking effectively bring  $d_{3/2}$  much loosely bound and mixes with fp-shell. For loosely bound nuclei not only  $f_{7/2}$  but also  $p_{3/2}$  comes closer.  $f_{7/2}$  has no blocking effect and  $p_{3/2}$  do not until proton fills  $d_{3/2}$ .



# Why the neutron drip line suddenly extend very much in F isotopes?

- Tensor opening occurs in F and binding energy is back to normal.



# Summary

- Importance of the tensor interaction is reviewed.
- Effects of recently observed high-momentum pn pair are considered in relation to the nuclear structures.
- Importance of the tensor blocking, that is significant in neutron rich nuclei, are discussed.
- A new model of nuclei “Tensor Blocking Shell Model” is introduced and used to examine new behaviors of neutron rich nuclei.

# Conclusion

- All new magic numbers appeared in neutron rich nuclei are consistently explained.
- Disappearance of traditional magic numbers and non binding of  $^8\text{He}$  and  $^{28}\text{O}$  are explained.
- Sudden extension of dripline in F is understood.
- A peculiar change of GS configurations is understood.



# Some related matters

- Neutron dripline may be closer than present predictions.
- Next magic number in neutron rich nuclei is 56 or 58.
- Magic numbers in neutron rich region affect the R-process.
- Effect of tensor blocking near the stability line may give new views to residual interactions.
- Relation to the  $l \cdot s$  interactions



# $l \cdot s$ splitting in $^5\text{He}$

Progress of Theoretical Physics, Vol. 23, No. 1, January 1960

## Spin-Orbit Splitting and Tensor Force. I\*

Tokuo TERASAWA\*\*

*The Institute for Solid State Physics, University of Tokyo, Tokyo*

It has been shown that about **a half of the experimental values of the doublet splittings in  $^5\text{He}$  and  $^{15}\text{N}$  can be derived**, using the meson-theoretic potential or the Serber one, **through the second order effect of perturbation theory**. The splitting energies have been shown to be mainly ascribed to the facts that **(1) the tensor force is strong and, especially, is stronger in the triplet even states than in the triplet odd states, and (2) the deformation of the closed shell core induced by the tensor force between the core-nucleons is restricted so as to satisfy the Pauli principle with the outside nucleon.**

Fig. 1

(IIIb)



# Collaborators

- **H. Toki, RCNP Osaka Univ., Osaka, Japan**
- **S. Terashima, IRCNPC Beihang Univ., Beijing, China**
- **H.-J. Ong, RCNP Osaka Univ., Osaka Japan**

**Thank you for your attention**



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**Like a change from the geocentric system  
to heliocentric system!**

*Thank you for your kind attention*