What is the eikonal approximation? Why and when is it useful for describing nuclear collisions and selected nuclear reactions?

Tokyo Institute of Technology
Ookayama, 20th January 2011

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Session (learning) aims:

To discuss <u>approximate</u> solutions of the Schrodinger equation for states of **two, three** or more bodies at ‘high energies’ by introducing the eikonal (forward-scattering dominated) approximation of the reaction dynamics.

To bring out the importance of the eikonal S-matrix, a function if the impact parameter of the projectile, or of a component of the projectile, in this formulation of the reaction and scattering of the interacting systems.

To gain an impression of how accurate the eikonal approximation, is as a function of the projectile energy, and to see how one can describe both point particle and composite projectile scattering using these methods.

There are many good direct reactions texts:

Direct nuclear reaction theories (Wiley, Interscience monographs and texts in physics and astronomy, v. 25) Norman Austern

Direct Nuclear Reactions (Oxford University Press, International Series of Monographs on Physics, 856 pages) G R Satchler

Introduction to the Quantum Theory of Scattering (Academic, Pure and Applied Physics, Vol 26, 398 pages) L S Rodberg, R M Thaler

Direct Nuclear Reactions (World Scientific Publishing, 396 pages) Norman K. Glendenning

Introduction to Nuclear Reactions (Taylor & Francis, Graduate Student Series in Physics, 515 pages) C A Bertulani, P Danielewicz

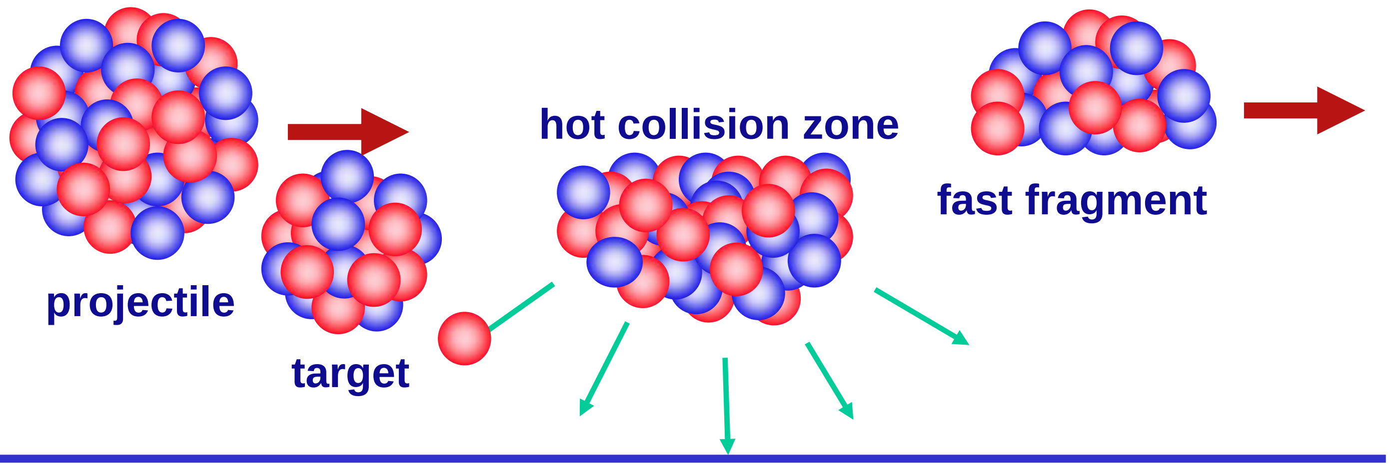
Theoretical Nuclear Physics: Nuclear Reactions (Wiley Classics Library, 1938 pages) Herman Feshbach

Introduction to Nuclear Reactions (Oxford University Press, 332 pages) G R Satchler

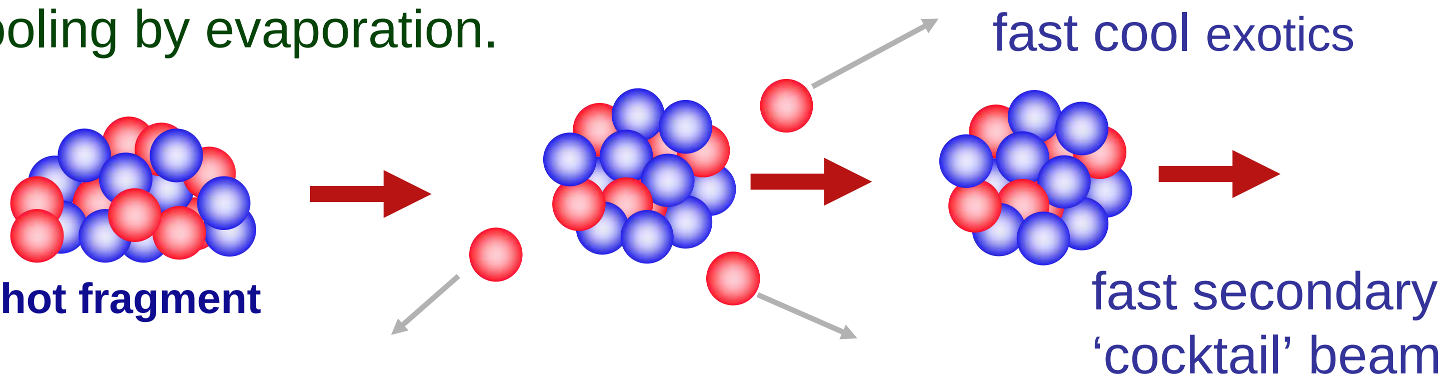
Nuclear Reactions for Astrophysics (Cambridge University Press, 2010) Ian Thompson and Filomena Nunes

Exotic nuclei production - projectile fragmentation

Random removal of protons and neutrons from heavy projectile in peripheral collisions at high energy - 100 MeV per nucleon or more

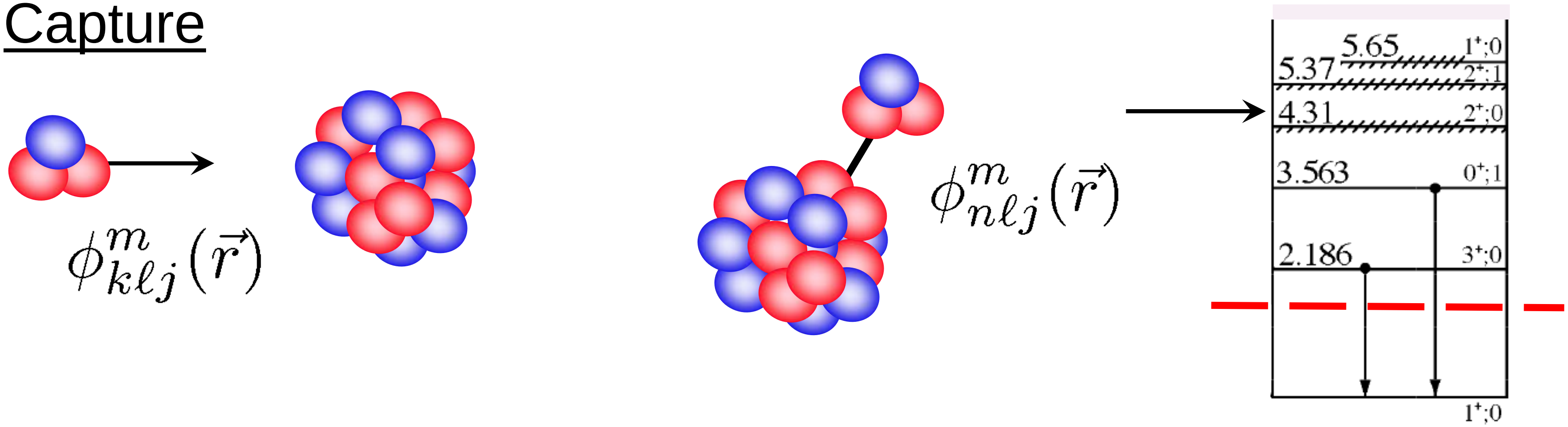


Cooling by evaporation.

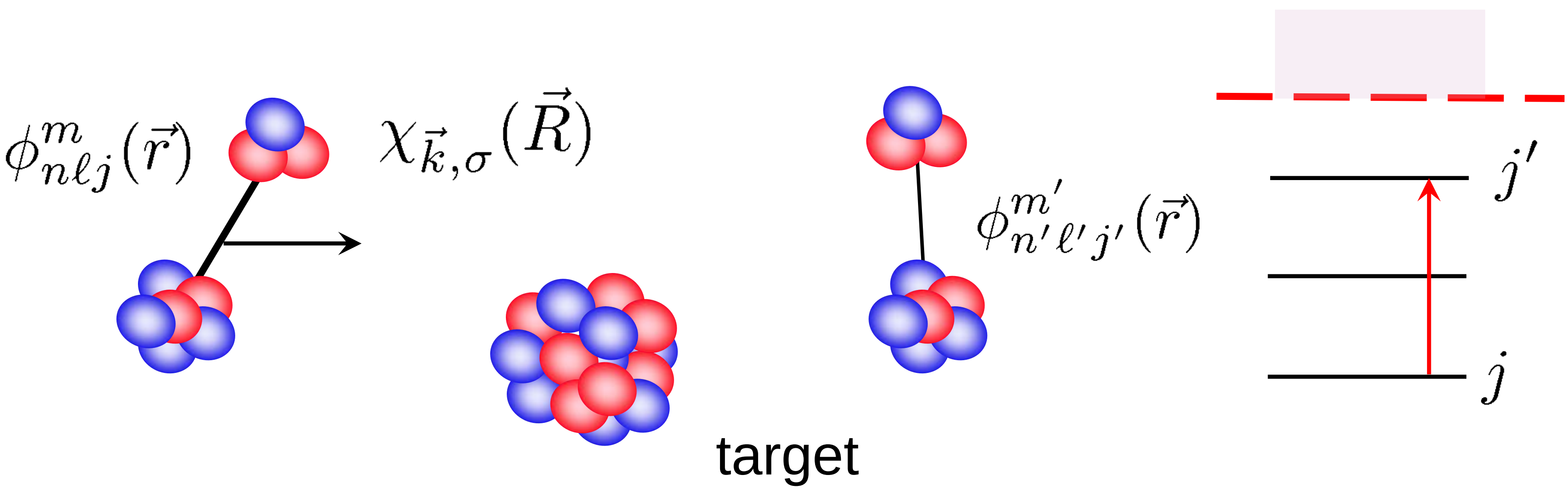


Direct reactions – types and characteristics

Capture



Inelastic excitations (bound to bound states) DWBA

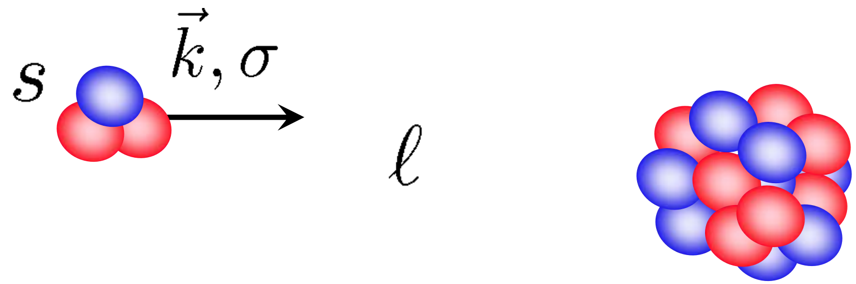


Direct reactions – requirements

<u>Description of wave functions for scattering</u> of nucleons or clusters from a heavier target and/or at higher energies: (a) high nuclear level density and broad overlapping resonances, (b) many open reaction channels, inelasticity and absorption. <u>Use a complex (absorptive) optical model potential</u> – from theory or simply fitted to elastic scattering data for the system and the energy of interest.

<u>Distorted waves</u>:

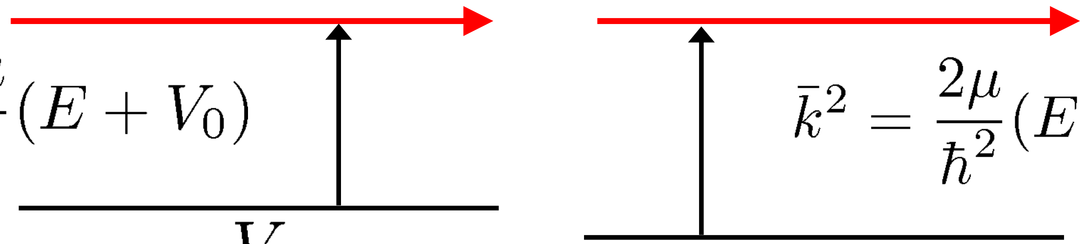
$$\chi_{\vec{k},\sigma}(\vec{r})$$



$$U(r) = V_C(r) + V(R) \boxed{+\, iW(r)} + V_{so}(r)\vec{\ell}\cdot\vec{s}$$

Optical potentials – the role of the imaginary part

$$\psi(x) = e^{ikx} \qquad \bar{\psi}(x) = e^{i\bar{k}x}$$



$$k^2 = \frac{2\mu}{\hbar^2}(E + V_0) \qquad \bar{k}^2 = \frac{2\mu}{\hbar^2}(E + V_0 + iW_0)$$

$$E \qquad -V_0 \qquad -V_0 - iW_0$$

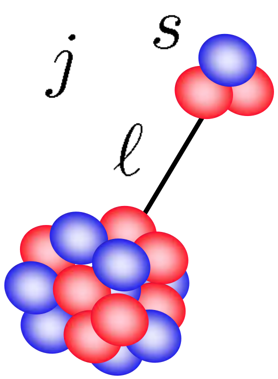
$$\bar{k}^2 = \frac{2\mu}{\hbar^2}(E + V_0 + iW_0) = \frac{2\mu}{\hbar^2}(E + V_0)\left[1 + \frac{iW_0}{E + V_0}\right]$$

$$\boxed{\bar{k} = k\left[1 + \frac{iW_0}{E + V_0}\right]^{1/2} \approx k\left[1 + \frac{iW_0}{2(E + V_0)}\right], \quad W_0 \ll E, V_0}$$

$$\text{So, for } W_0 > 0, \quad \bar{k} = k + ik_i/2, \quad k_i = kW_0/(E + V_0) > 0,$$

$$\boxed{\bar{\psi}(x) = e^{i\bar{k}x} = e^{ikx}e^{-\frac{1}{2}k_i x}, \qquad |\bar{\psi}(x)|^2 = e^{-k_i x}}$$

The Schrodinger equation (1)



So, using usual notation

$$\left(-\frac{\hbar^2}{2\mu}\nabla_r^2 + U(r) - E_{cm}\right)\phi_{\ell j}^m(\vec{r}) = 0, \quad \mu = \frac{m_c m_v}{m_c + m_v}$$

and defining $\phi_{\ell j}^m(\vec{r}) = \sum_{\lambda\sigma}(\ell\lambda s\sigma|jm)\frac{u_{\ell j}(r)}{r}Y_\ell^\lambda(\hat{r})\chi_s^\sigma$

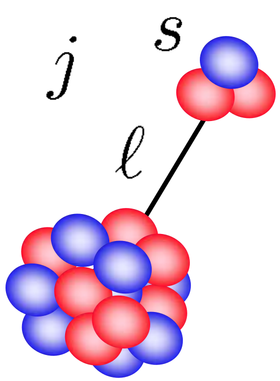
$$\boxed{\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} + \frac{2\mu}{\hbar^2}[E_{cm} - U_{\ell j}(r)]\right)u_{\ell j}(r) = 0}$$

bound states $E_{cm} < 0$ scattering states $E_{cm} > 0$

With $U(r) = V_C(r) + V(r)\boxed{+\,iW(r)} + V_{so}(r)\vec{\ell}\cdot\vec{s}$

$$\boxed{\begin{aligned} U_{\ell j}(r) \quad &= \quad V_C(r) + V(r)\boxed{+\,iW(r)} \\ &+ \quad V_{so}(r)[j(j+1) - \ell(\ell+1) - s(s+1)]/2 \end{aligned}}$$

The Schrodinger equation (2)



Must solve

$$\boxed{\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} + \frac{2\mu}{\hbar^2}[E_{cm} - U_{\ell j}(r)]\right) u_{\ell j}(r) = 0}$$

<u>bound states</u> $E_{cm} < 0 \quad \kappa_b = \sqrt{\frac{2\mu|E_{cm}|}{\hbar^2}}$

$$\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} - \frac{2\mu}{\hbar^2}U_{\ell j}(r) - \kappa_b^2\right)\boxed{u_{n\ell j}(r)} = 0$$

Discrete spectrum

<u>scattering states</u> $E_{cm} > 0 \quad k = \sqrt{\frac{2\mu E_{cm}}{\hbar^2}}$

$$\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} - \frac{2\mu}{\hbar^2}U_{\ell j}(r) + k^2\right)\boxed{u_{k\ell j}(r)} = 0$$

Continuous spectrum

Large r: The phase shift and partial wave S-matrix

Scattering states

$$E_{cm} > 0 \quad k = \sqrt{\frac{2\mu E_{cm}}{\hbar^2}}$$

$$\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} - \frac{2\mu}{\hbar^2}U_{\ell j}(r) + k^2\right) u_{k\ell j}(r) = 0$$

and beyond the range of the nuclear forces, then

$$\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} - \frac{2\eta k}{r} + k^2\right) u_{k\ell j}(r) = 0, \quad \eta = \frac{\mu Z_c Z_v e^2}{\hbar k}$$

$F_\ell(\eta, kr)$, $G_\ell(\eta, kr)$ regular and irregular Coulomb functions

$$\boxed{\begin{aligned} u_{k\ell j}(r) \quad &\to \quad e^{i\delta_{\ell j}}\left[\cos\delta_{\ell j} F_\ell(\eta, kr) + \sin\delta_{\ell j} G_\ell(\eta, kr)\right] \\ &\to \quad (i/2)[H_\ell^{(-)}(\eta, kr) - S_{\ell j} H_\ell^{(+)}(\eta, kr)] \end{aligned}}$$

$$H_\ell^{(\pm)}(\eta, kr) = G_\ell(\eta, kr) \pm iF_\ell(\eta, kr)$$

Phase shift and partial wave S-matrix: Recall

$$u_{k\ell j}(r) \quad \rightarrow \quad e^{i\delta_{\ell j}} \underbrace{[\cos\delta_{\ell j} F_\ell(\eta, kr) + \sin\delta_{\ell j} G_\ell(\eta, kr)]}$$

If *U(r)* is real, the phase shifts $\delta_{\ell j}$ are real, and […] also

$$u_{k\ell j}(r) \quad \rightarrow \quad (i/2)[\underline{H_\ell^{(-)}}(\eta, kr) - S_{\ell j}\underline{H_\ell^{(+)}}(\eta, kr)]$$

$$S_{\ell j} = e^{2i\delta_{\ell j}}$$

Ingoing waves — outgoing waves

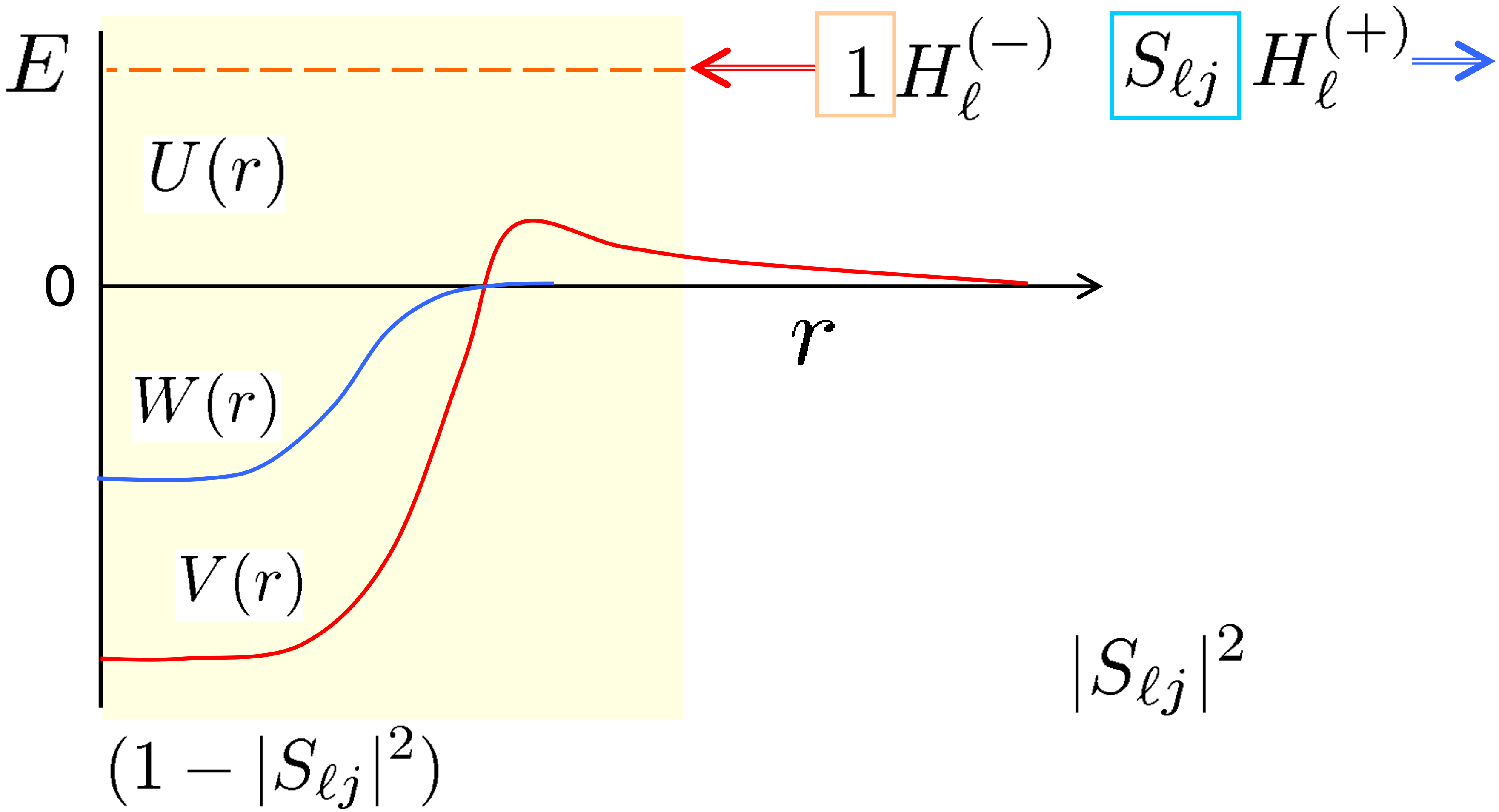
$|S_{\ell j}|^2$ survival probability in the scattering

$(1 - |S_{\ell j}|^2)$ absorption probability in the scattering

Having calculate the phase shifts and the partial wave S-matrix elements we can then compute all scattering observables for this energy and potential (but later).

S-matrix - ingoing and outgoing waves amplitudes

$$u_{k\ell j}(r) \rightarrow (i/2)[\boxed{1}\, H_\ell^{(-)} - \boxed{S_{\ell j}} H_\ell^{(+)}]$$



Eikonal approximation: for point particles (1)

Approximate (semi-classical) scattering solution of

$$\left(-\frac{\hbar^2}{2\mu}\nabla_r^2 + U(r) - E_{cm}\right)\chi^+_{\vec{k}}(\vec{r}) = 0, \qquad \mu = \frac{m_c m_v}{m_c + m_v}$$

$$\left(\nabla_r^2 - \frac{2\mu}{\hbar^2}U(r) + k^2\right)\chi^+_{\vec{k}}(\vec{r}) = 0$$

valid when $\boxed{|U|/E \ll 1, \quad ka \gg 1}$ → small wavelength high energy

Key steps are: (1) the distorted wave function is written

$$\chi^+_{\vec{k}}(\vec{r}) = \exp(i\vec{k}\cdot\vec{r})\,\boxed{\omega(\vec{r})}$$

all effects due to *U(r)*, modulation function

(2) Substituting this product form in the Schrodinger Eq.

$$\left[2i\vec{k}\cdot\nabla\omega(\vec{r}) - \frac{2\mu}{\hbar^2}U(r)\omega(\vec{r}) + \nabla^2\omega(\vec{r})\right]\exp(i\vec{k}\cdot\vec{r}) = 0$$

Eikonal approximation: point neutral particles (2)

$$\left[2i\vec{k}\cdot\nabla\omega(\vec{r}) - \frac{2\mu}{\hbar^2}U(r)\omega(\vec{r}) + \cancel{\nabla^2\omega(\vec{r})}\right]\exp(i\vec{k}\cdot\vec{r}) = 0$$

The conditions $|U|/E \ll 1, \quad ka \gg 1$ → imply that

$$\boxed{2\vec{k}\cdot\nabla\omega(\vec{r}) \gg \nabla^2\omega(\vec{r})}$$

Slow spatial variation cf. k

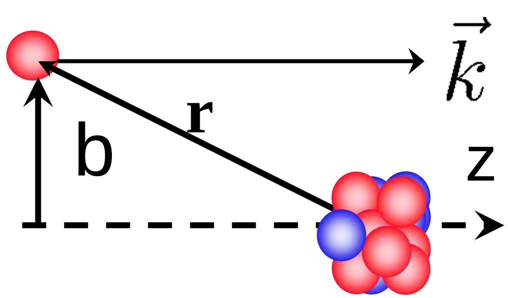
and choosing the z-axis in the beam direction $\vec{k}$

$$\frac{d\omega}{dz} \approx -\frac{i\mu}{\hbar^2 k}U(r)\omega(\vec{r})$$

with solution

phase that develops with z

$$\boxed{\omega(\vec{r}) = \exp\left[-\frac{i\mu}{\hbar^2 k}\int_{-\infty}^{z} U(r)dz'\right]}$$



1D integral over a straight line path through U at the impact parameter b

Eikonal approximation: point neutral particles (3)

$$\chi^+_{\vec{k}}(\vec{r}) = \exp(i\vec{k}\cdot\vec{r})\ \omega(\vec{r}) \approx \exp(i\vec{k}\cdot\vec{r})\ \boxed{\exp\left[-\frac{i\mu}{\hbar^2 k}\int_{-\infty}^{z} U(r)dz'\right]}$$

So, after the interaction and as $z \to \infty$

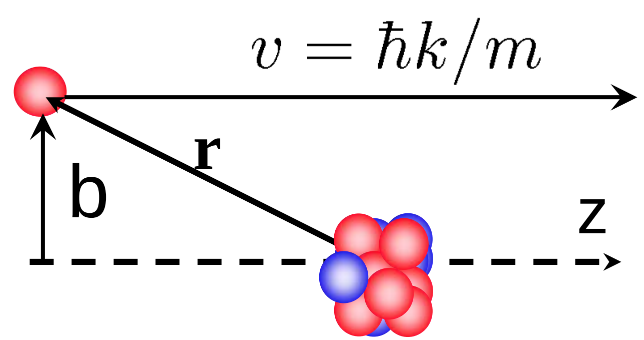
$$\chi^+_{\vec{k}}(\vec{r}) \to \exp(i\vec{k}\cdot\vec{r})\ \exp\left[-\frac{i\mu}{\hbar^2 k}\int_{-\infty}^{\infty} U(r)dz'\right] = S(b)\exp(i\vec{k}\cdot\vec{r})$$

$$\boxed{\chi^+_{\vec{k}}(\vec{r}) \to S(b)\exp(i\vec{k}\cdot\vec{r})}$$

S(b) is amplitude of the forward going outgoing waves from the scattering at impact parameter b

Eikonal approximation to the S-matrix S(b)

$$\boxed{S(b) = \exp\left[-\frac{i}{\hbar v}\int_{-\infty}^{\infty} U(r)dz'\right]}$$

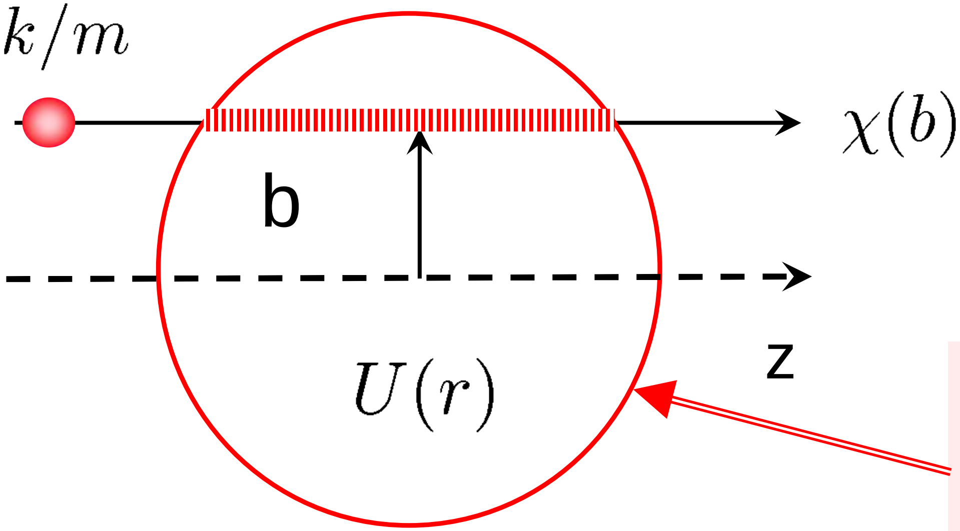


Moreover, the structure of the theory generalises simply to few-body projectiles

Eikonal approximation: point particles - summary

$$\chi^{+}_{\vec{k}}(\vec{r}) = \exp(i\vec{k}\cdot\vec{r})\ \exp\left[-\frac{i\mu}{\hbar^2 k}\int_{-\infty}^{z} U(r)dz'\right]$$

$$v = \hbar k/m$$



$$\chi(b) = -\frac{1}{\hbar v}\int_{-\infty}^{\infty} U(r)dz$$

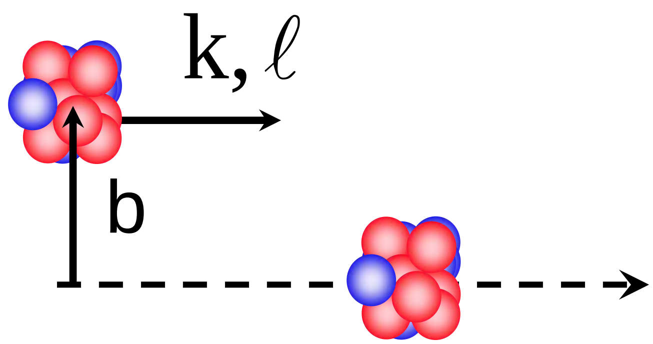
limit of range of finite ranged potential

$$\chi^{+}_{\vec{k}}(\vec{r}) \rightarrow S(b)\exp(i\vec{k}\cdot\vec{r})$$

$$\boxed{S(b) = \exp\left[i\chi(b)\right] = \exp\left[-\frac{i}{\hbar v}\int_{-\infty}^{\infty} U(r)dz'\right]}$$

Semi-classical models for the S-matrix - S(b)

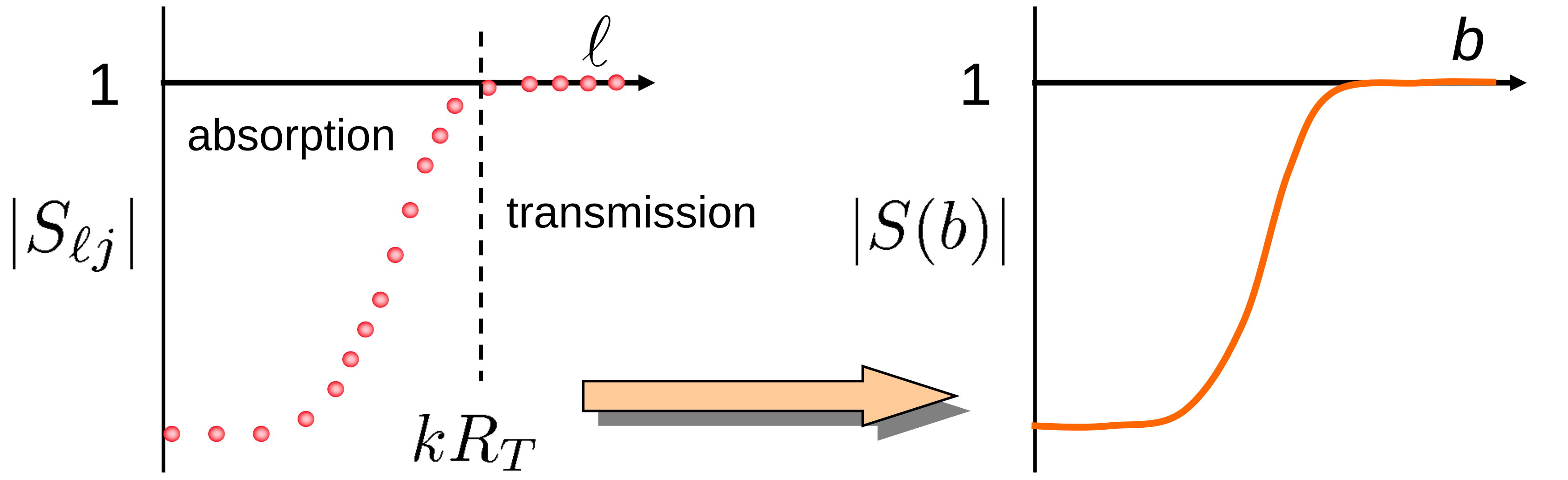
b=impact parameter



for high energy/or large mass,
semi-classical ideas are good

kb $\simeq \ell$, actually $\Rightarrow \ell$+1/2

$$L^2 = \ell(\ell+1) = (\ell+1/2)^2 - \cancel{1/4} \approx (kb)^2$$



Point particle scattering – cross sections

All cross sections, etc. can be computed from the S-matrix, in either the partial wave or the eikonal (impact parameter) representation, for example (spinless case):

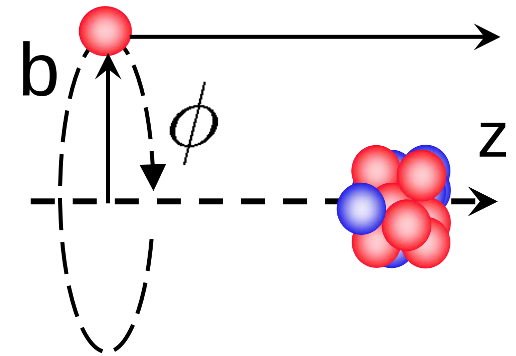
$$\sigma_{el} = \frac{\pi}{k^2}\sum_{\ell=0}^{\infty}(2\ell+1)|1-S_\ell|^2 \approx \int d^2\vec{b}\,|1-S(b)|^2$$

$$\sigma_R = \frac{\pi}{k^2}\sum_{\ell=0}^{\infty}(2\ell+1)(1-|S_\ell|^2) \approx \int d^2\vec{b}\,(1-|S(b)|^2)$$

$$\sigma_{tot} = \sigma_{el}+\sigma_R = 2\int d^2\vec{b}\,[1-\mathrm{Re}.S(b)] \quad \text{etc.}$$

and where (cylindrical coordinates)

$$\int d^2\vec{b} \equiv \int_0^\infty b\,db \int_0^{2\pi} d\phi = 2\pi\int_0^\infty b\,db$$

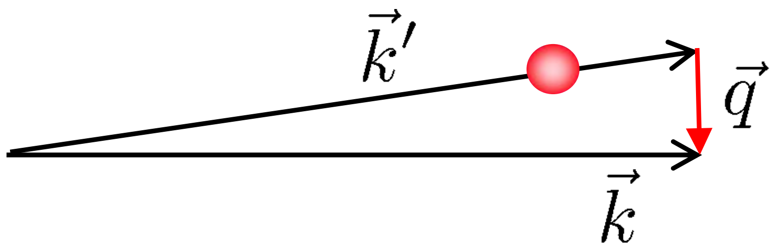


Point particle – the differential cross section

Using the standard result from scattering theory, the elastic scattering amplitude is

$$\begin{aligned} f(\theta) &= -\frac{\mu}{2\pi\hbar^2}\int d\vec{r}\,\exp(-i\vec{k}'\cdot\vec{r})\,U(r)\,\chi^{+}_{\vec{k}}(\vec{r}) \\ &= -\frac{\mu}{2\pi\hbar^2}\int d\vec{r}\,\exp(-i\vec{k}'\cdot\vec{r})\,U(r)\,\exp(i\vec{k}\cdot\vec{r})\,\omega(\vec{r}) \\ &= -\frac{\mu}{2\pi\hbar^2}\int d\vec{r}\,\exp(i\vec{q}\cdot\vec{r})\,U(r)\,\omega(\vec{r}) \end{aligned}$$

with $\vec{q} = \vec{k} - \vec{k}',\ q = 2k\sin(\theta/2)$ is the momentum transfer.
Consistent with the earlier high energy (forward scattering) approximation



$$\boxed{\begin{aligned} \vec{q}\cdot\vec{r} &\approx \vec{q}\cdot\vec{b} \\ \vec{q}\cdot\vec{k} &\approx 0 \end{aligned}}$$

Point particles – the differential cross section

So, the elastic scattering amplitude

$$f(\theta) = -\frac{\mu}{2\pi\hbar^2}\int d\vec{r}\,\exp(i\vec{q}\cdot\vec{r})\,U(r)\,\omega(\vec{r})$$

is approximated by

$$f_{eik}(\theta) = -\frac{ik}{2\pi}\int d^2\vec{b}\,\exp(i\vec{q}\cdot\vec{b})\int_{-\infty}^{\infty}\frac{d\omega}{dz}dz$$

$$\begin{cases} \dfrac{d\omega}{dz} = -\dfrac{i\mu}{\hbar^2 k}U(r)\omega(\vec{r}) \\ U(r)\omega(\vec{r}) = \dfrac{i\hbar^2 k}{\mu}\dfrac{d\omega}{dz} \end{cases}$$

Performing the z- and azimuthal ϕ integrals

$$\boxed{f_{eik}(\theta) = -ik\int_0^{\infty} b\,db\,J_0(qb)\,[S(b)-1]}$$

$$\boxed{S(b) = \exp\left[i\chi(b)\right] = \exp\left[-\frac{i}{\hbar v}\int_{-\infty}^{\infty}U(r)dz'\right]}$$

$J_0(qb)$

Bessel function

Point particle – the Coulomb interaction

Treatment of the Coulomb interaction (as in partial wave analysis) requires a little care. Problem is, eikonal phase integral due to Coulomb potential diverges logarithmically.

$$\chi_C(b) = -\frac{1}{\hbar v}\int_{-a}^{+a} V_C(r)\,dz$$

Must 'screen' the potential at some large screening radius

$$f_{eik}(\theta) = e^{i\chi_a}\left[f_{pt}(\theta) - ik\underbrace{\int_0^\infty b\,db\,J_0(qb)\,e^{i\chi_{pt}}\left[\bar{S}(b)-1\right]}_{\text{nuclear scattering in the presence of Coulomb}}\right]$$

overall unobservable screening phase

usual Coulomb (Rutherford) point charge amplitude

nuclear scattering in the presence of Coulomb

$$\bar{\chi}(b) = \chi_N(b) + \chi_\rho(b) - \chi_{pt}(b)$$

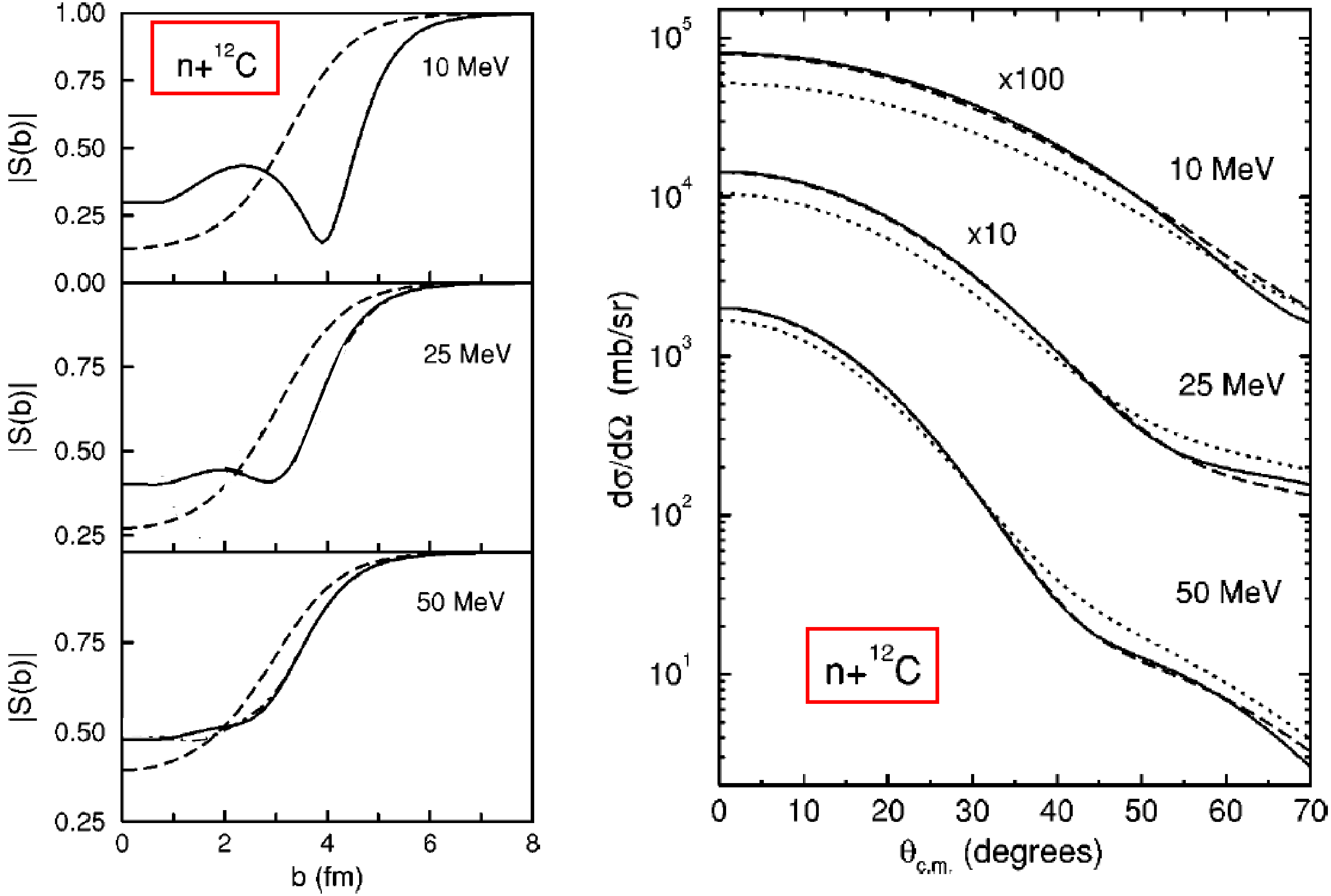
nuclear phase

Due to finite charge distribution

$$\chi_{pt}(b) = 2\eta\ln(kb)$$

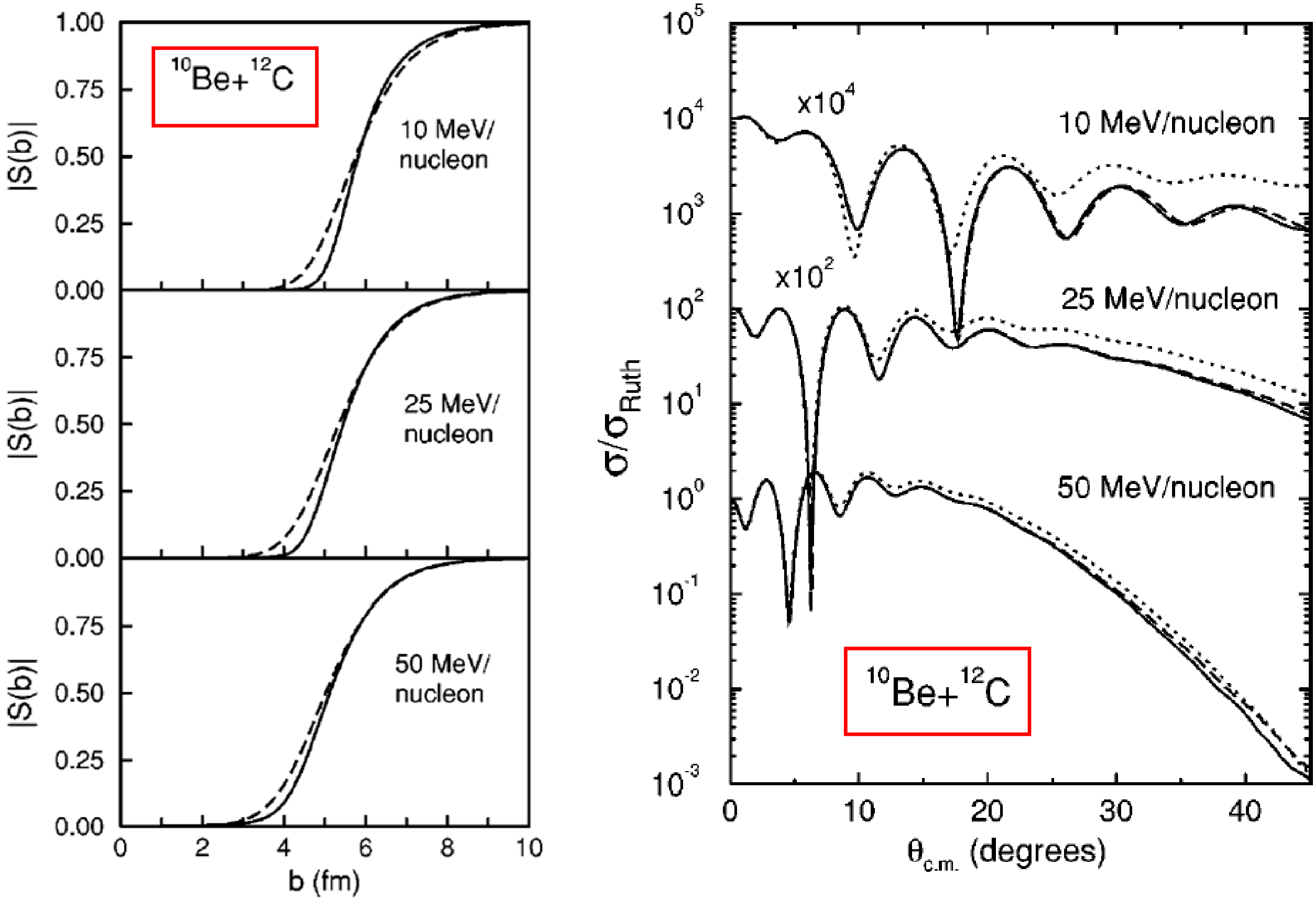
See e.g. J.M. Brooke, J.S. Al-Khalili, and J.A. Tostevin PRC **59** 1560

Accuracy of the eikonal S(b) and cross sections



J.M. Brooke, J.S. Al-Khalili, and J.A. Tostevin PRC **59** 1560

Accuracy of the eikonal S(b) and cross sections



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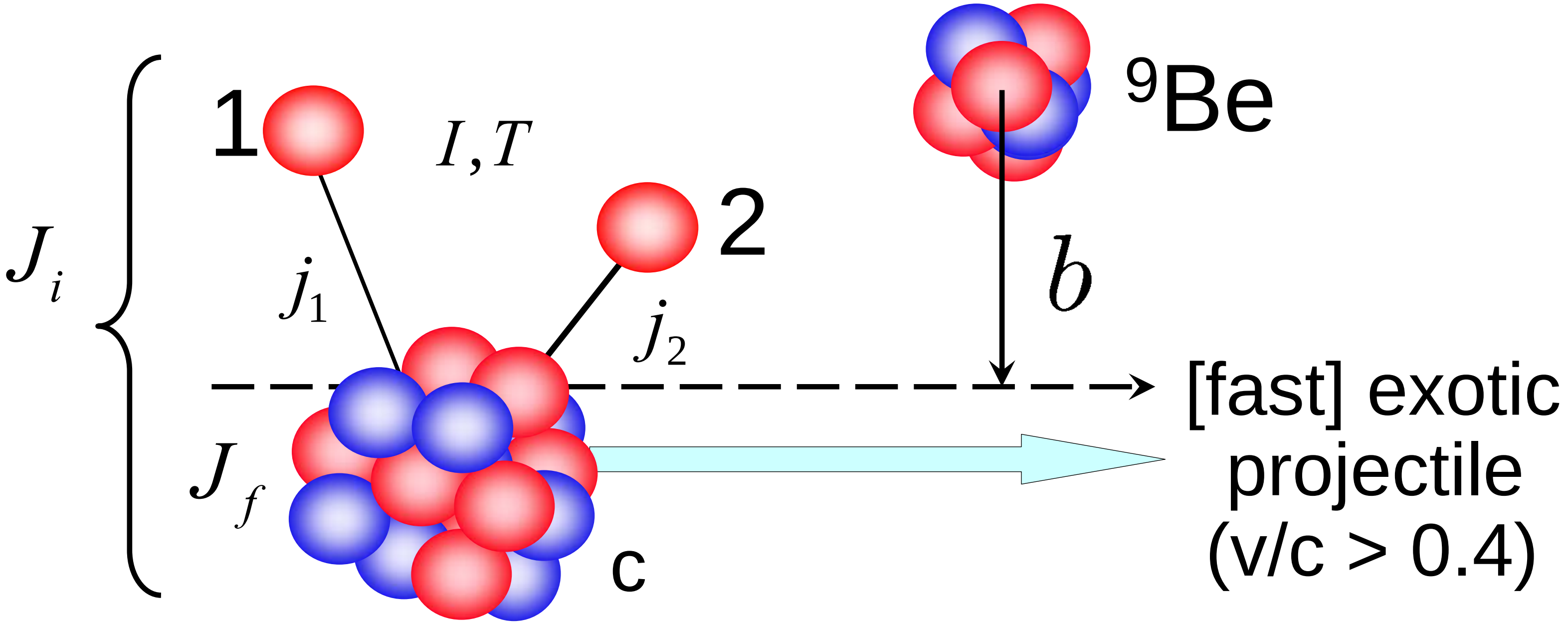
Non-point particles: such as in knockout reactions

One such experimental option is one or two-nucleon removal – at ~100 MeV/nucleon



Experiments do not measure target final states. Final state of core c measured – using decay gamma rays.

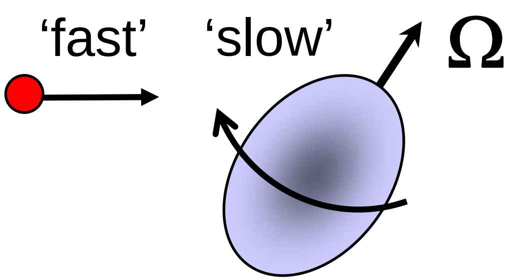
How can we describe and what can we learn from these?

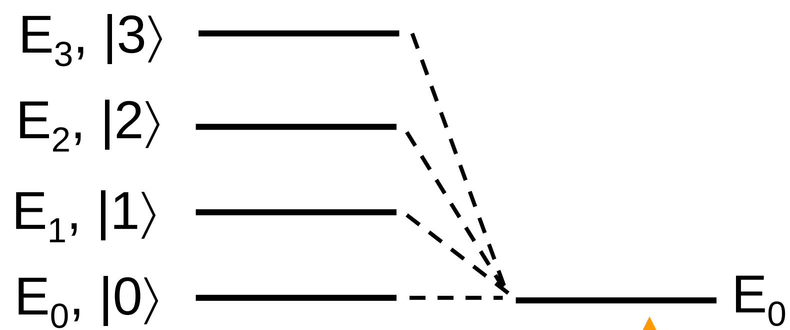
P.G. Hansen and J.A. Tostevin, Ann Rev Nucl Part Sci **53** (2003) 219

Adiabatic (sudden) approximations in physics

Identify high energy/fast and low energy/slow degrees of freedom

Fast neutron scattering from a rotational nucleus



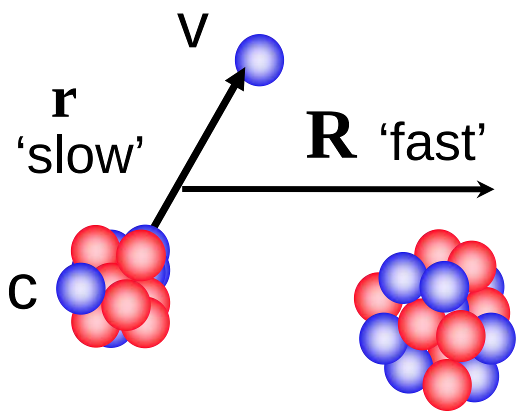


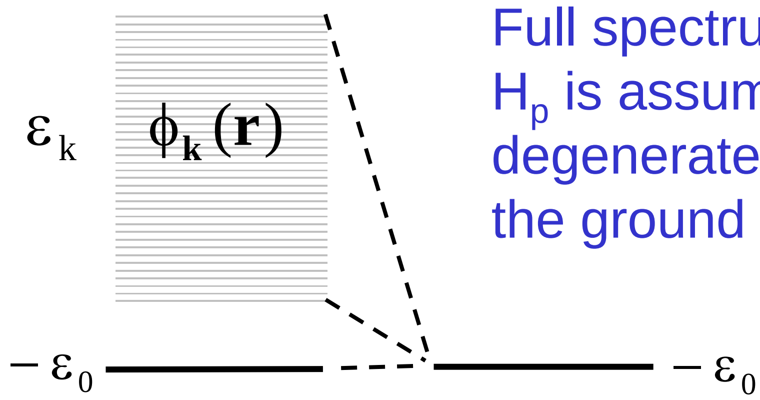
Fix Ω, calculate scattering amplitude $f(\theta, \Omega)$ for each (fixed) Ω.

moment of inertia $\rightarrow \infty$ and rotational spectrum is assumed degenerate

Transition amplitudes $f_{\alpha\beta}(\theta) = \langle \beta \,|\, f(\theta, \Omega) \,|\, \alpha \rangle_\Omega$

Few-body projectiles – the adiabatic model





Full spectrum of H_p is assumed degenerate with the ground state

Freeze internal co-ordinate $\mathbf{r}$ then scatter c+v from target and compute $f(\theta,\mathbf{r})$ for all required <u>fixed</u> values of $\mathbf{r}$

Physical amplitude for breakup to state $\phi_{\mathbf{k}}(\mathbf{r})$ is then,

$$f_k(\theta) = \langle \phi_k \,|\, f(\theta, \mathbf{r}) \,|\, \phi_0 \rangle_{\mathbf{r}}$$

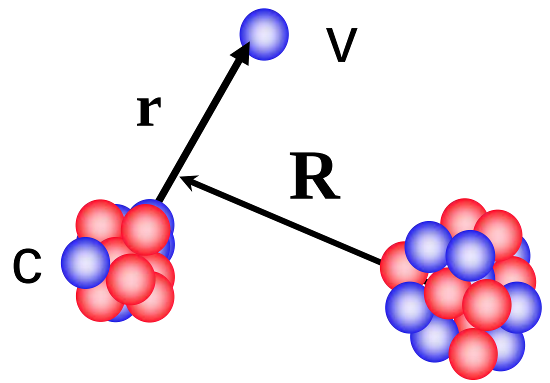
Achieved by replacing $H_p \rightarrow -\varepsilon_0$ in Schrödinger equation

Adiabatic approximation - time perspective

The time-dependent equation is

$$H\Psi(\mathbf{r},\mathbf{R},t) = i\hbar\frac{\partial\Psi}{\partial t}$$

and can be written



$$\Psi(\mathbf{r},\mathbf{R},t) = \Lambda\,\Phi(\mathbf{r}(t),\mathbf{R}), \quad \mathbf{r}(t) = \Lambda^{+}\,\mathbf{r}\,\Lambda$$

$\Lambda = \exp\{-i(H_p + \varepsilon_0)t/\hbar\}$ and where

$$[T_R + U(\mathbf{r}(t),\mathbf{R}) - \varepsilon_0]\Phi(\mathbf{r}(t),\mathbf{R}) = i\hbar\frac{\partial\Phi}{\partial t}$$

Adiabatic equation

$$[T_R + U(\mathbf{r},\mathbf{R})]\Phi(\mathbf{r},\mathbf{R}) = \left(E + \varepsilon_0\right)\Phi(\mathbf{r},\mathbf{R})$$

Adiabatic step <u>assumes</u>

$\mathbf{r}(t) \approx \mathbf{r}(0)=\mathbf{r}$=fixed or Λ=1 for the collision time t_{coll}

requires

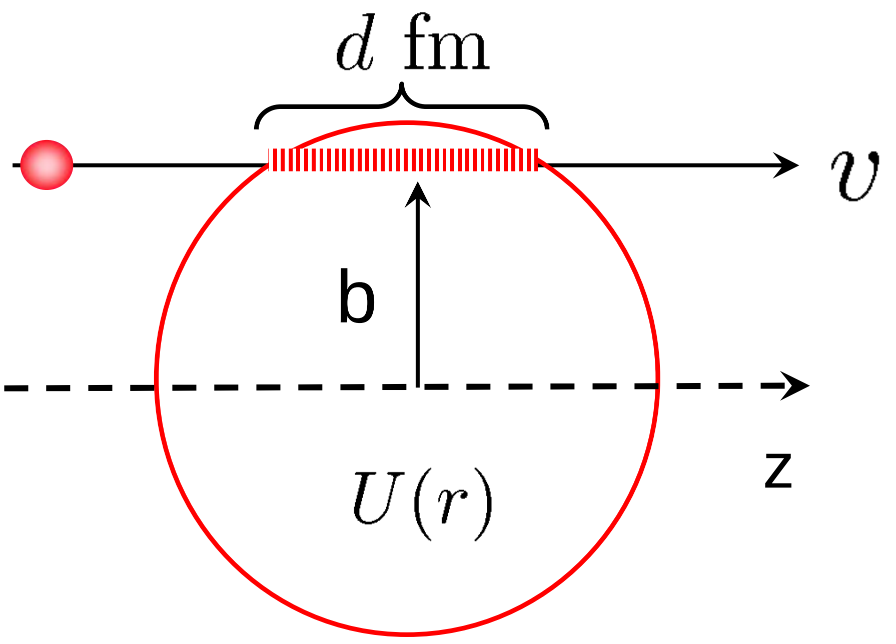
$$(H_p + \varepsilon_0)t_{coll}/\hbar << 1$$

Reaction timescales – surface grazing collisions

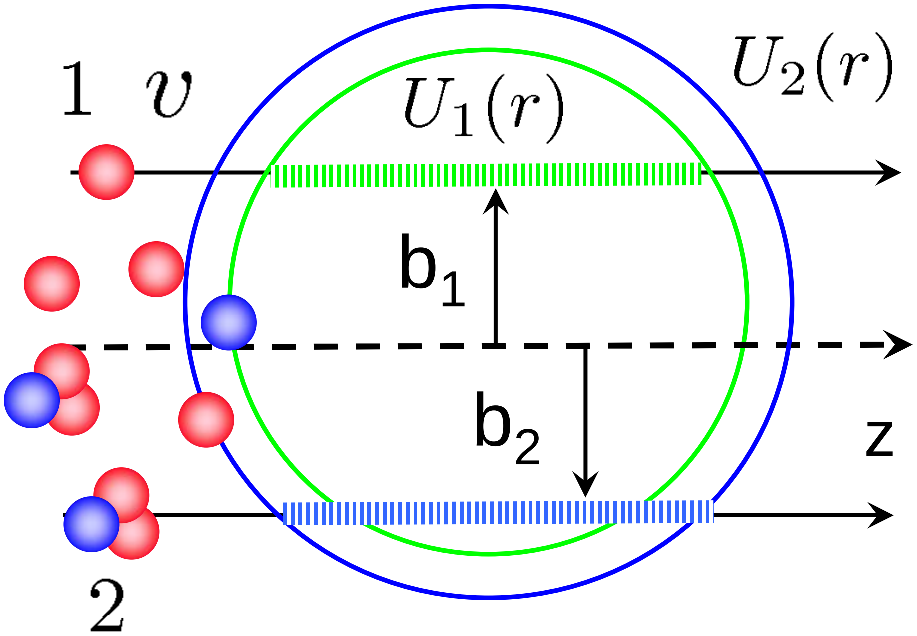
For 100 and 250 MeV/u incident energy:

$$\gamma = 1.1, \quad v/c = 0.42, \qquad \gamma = 1.25, \quad v/c = 0.6,$$

$$\Delta t = 7.9 \times d \times 10^{-24} s, \qquad \Delta t = 5.6 \times d \times 10^{-24} s$$



Adiabatic approximation: composite projectile



$$\chi_i(b) = -\frac{1}{\hbar v}\int_{-\infty}^{\infty} U_i(r)dz$$

Total interaction energy

$$U(r_1,\ldots) = \sum_i U_i(r_i)$$

$$S_i(b_i) = \exp\left[i\chi_i(b_i)\right] = \exp\left[-\frac{i}{\hbar v}\int_{-\infty}^{\infty} U_i(r_i)dz'\right]$$

$$\chi(b_1,\ldots) = -\frac{1}{\hbar v}\int_{-\infty}^{\infty}\sum_i U_i(r_i)dz$$

with composite systems: get products of the S-matrices

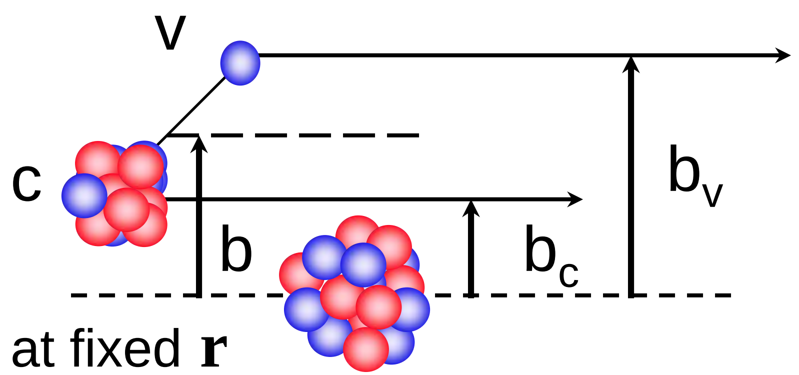
$$\boxed{\exp[i\chi(b_1,\ldots)] = \prod_i S_i(b_i)}$$

Few-body eikonal model amplitudes

So, after the collision, as $Z\rightarrow\infty$ $\quad \omega(\mathbf{r},\mathbf{R}) = S_c(b_c)\,S_v(b_v)$

$$\Psi_{\mathbf{K}}^{\mathrm{Eik}}(\mathbf{r},\mathbf{R}) \rightarrow e^{i\mathbf{K}\cdot\mathbf{R}}\, S_c(b_c)\,S_v(b_v)\,\phi_0(\mathbf{r})$$

with S_c and S_v the eikonal approximations to the S-matrices for the independent scattering of c and v from the target - the dynamics



at fixed $\mathbf{r}$

adiabatic

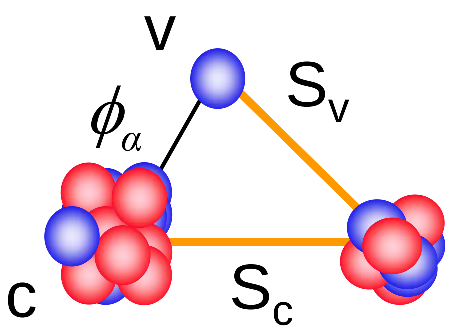
So, elastic amplitude (S-matrix) for the scattering of the projectile at an impact parameter b - i.e. The amplitude that it emerges in state $\phi_0(\mathbf{r})$ is

$$S_p(b) = \langle \phi_0 \,|\, \underbrace{S_c(b_c)\,S_v(b_v)} \,|\, \phi_0 \rangle_{\mathbf{r}}$$

averaged over position probabilities of c and v $\leftarrow$ amplitude that c,v survive interaction with b_c and b_v

Eikonal theory - dynamics and structure

Independent scattering information of c and v from target



$$S_{\alpha\beta}(b) = \langle \phi_\beta \,|\, \overbrace{S_c(b_c)\, S_v(b_v)}^{\text{scattering}} \,|\, \phi_\alpha \rangle$$

structure

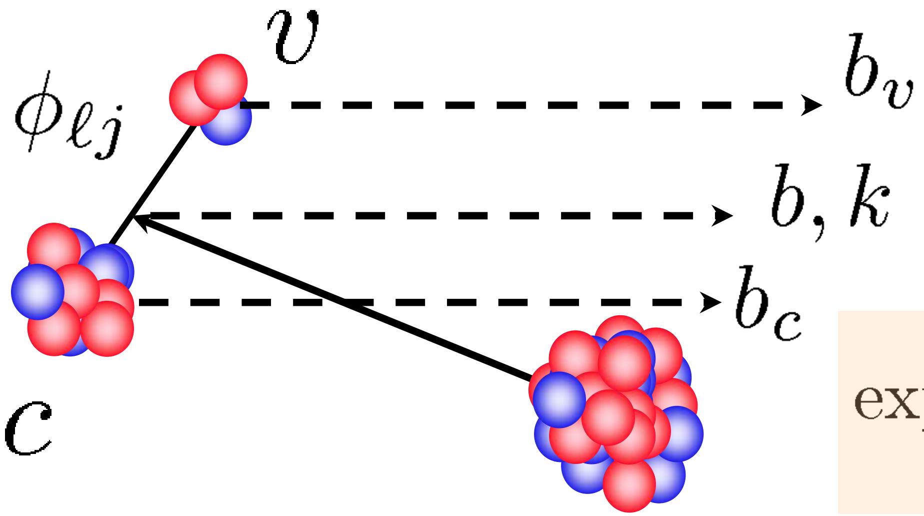
Use the best available few- or many-body wave functions

More generally,

$$S_{\alpha\beta}(b) = \langle \varphi_\beta \,|\, S_1(b_1)\, S_2(b_2) \ldots\ldots\; S_n(b_n) \,|\, \varphi_\alpha \rangle$$

for any choice of 1,2 ,3, ….. n clusters for which a most realistic wave function φ is available

Eikonal approach – generalisation to composites



Total interaction energy

$$U(r_1, \ldots) = \sum_{i=c,v} U_i(r_i)$$

$$\exp[i\chi(b_1, \ldots)] = \prod_{i=c,v} S_i(b_i)$$

$$S_p(b) = \langle \phi_{\ell j} | S_c(b_c) S_v(b_v) | \phi_{\ell j} \rangle_{\vec{r}}$$

You can now calculate bound states (**`bound`**) and eikonal S-matrices (**`eikonal_s`**) and can calculate this composite S-matrix (using **`knockout`**). The elastic scattering of c, v or the composite can then be calculated (using **`glauber`**). So you can now calculate the elastic scattering of the neutron, ^{10}Be, and the composite halo system ^{11}Be ?

Absorptive cross sections - target excitation

Since our effective interactions are complex all our S(b) include the effects of absorption due to inelastic channels → $\boxed{|S(b)|^2 \leq 1}$

$$\sigma_{abs} = \sigma_R - \sigma_{diff} = \int d\mathbf{b}\ \langle \phi_0 | \underbrace{1 - |S_c S_v|^2}_{} | \phi_0 \rangle$$

$$\begin{cases} |S_v|^2 (1 - |S_c|^2) + & \text{v survives, c absorbed} \\ |S_c|^2 (1 - |S_v|^2) + & \rightarrow \boxed{\text{v absorbed, c survives}} \\ (1 - |S_c|^2)(1 - |S_v|^2) & \text{v absorbed, c absorbed} \end{cases}$$

stripping of v from projectile exciting the target. c scatters at most elastically with the target

$$\boxed{\sigma_{strip} = \int d\mathbf{b}\ \langle \phi_0 | |S_c|^2 (1 - |S_v|^2) | \phi_0 \rangle}$$

Related equations exist for the differential cross sections, etc.

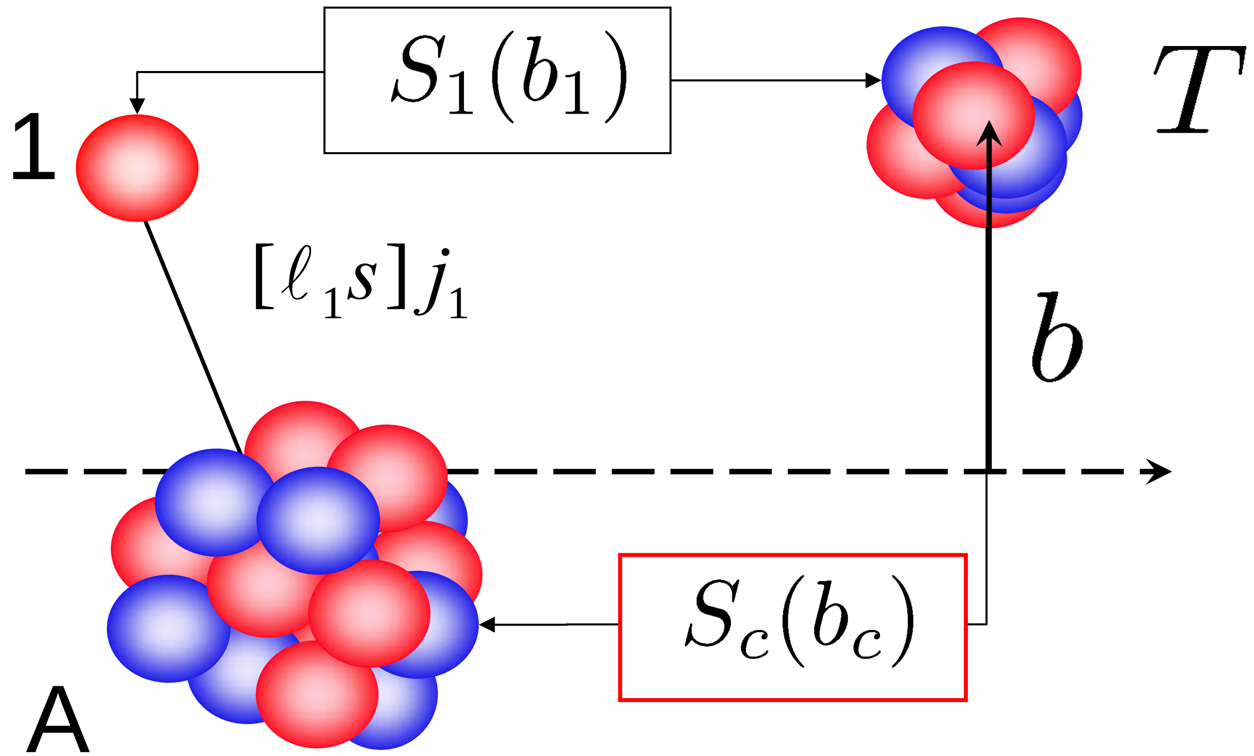
And for two-nucleon removal

$$\sigma_{abs} \quad \rightarrow \quad 1 - |S_c|^2 |S_1|^2 |S_2|^2$$

$$\left.\begin{aligned}
1 \quad &= \quad [\, \boxed{|S_c|^2} + \cancel{(1 - |S_c|^2)} \,] \\
&\times \quad [\, |S_1|^2 + (1 - |S_1|^2) \,] \\
&\times \quad [\, |S_2|^2 + (1 - |S_2|^2) \,]
\end{aligned}\right\} \begin{array}{l} \underline{\text{core survival}} \\ \text{and nucleon} \\ \text{“ removal ”} \end{array}$$

$$\begin{aligned}
\sigma_{abs}^{\mathrm{KO}} \quad \rightarrow \quad & |S_c|^2 (1 - |S_1|^2)(1 - |S_2|^2) \quad \text{2N stripping} \\
+ \quad & \left.\begin{aligned} & |S_c|^2 \, |S_1|^2 (1 - |S_2|^2) \\ + \; & |S_c|^2 \, (1 - |S_1|^2) |S_2|^2 \end{aligned}\right\} \begin{array}{l} \text{1N stripped} \\ \text{1N diffracted} \end{array}
\end{aligned}$$

Stripping of a nucleon – nucleon 'absorbed'



$$\sigma_{\text{strip}} = \int d\mathbf{b}\ \langle \phi_0 \,||\, \mathrm{S_C}\,|^2\,(1 - |\,\mathrm{S_1}\,|^2)\,|\,\phi_0 \rangle$$

Diffractive dissociation of composite systems

The total cross section for removal of the valence particle from the projectile due to the break-up (or diffractive dissociation) mechanism is the break-up amplitude, summed over all final continuum states

$$\sigma_{\rm diff} = \int d\mathbf{k} \int d\mathbf{b}\, |\langle \phi_{\mathbf{k}} \,|\, S_c(b_c)\, S_v(b_v)\, |\, \phi_0 \rangle|^2$$

but, using completeness of the break-up states

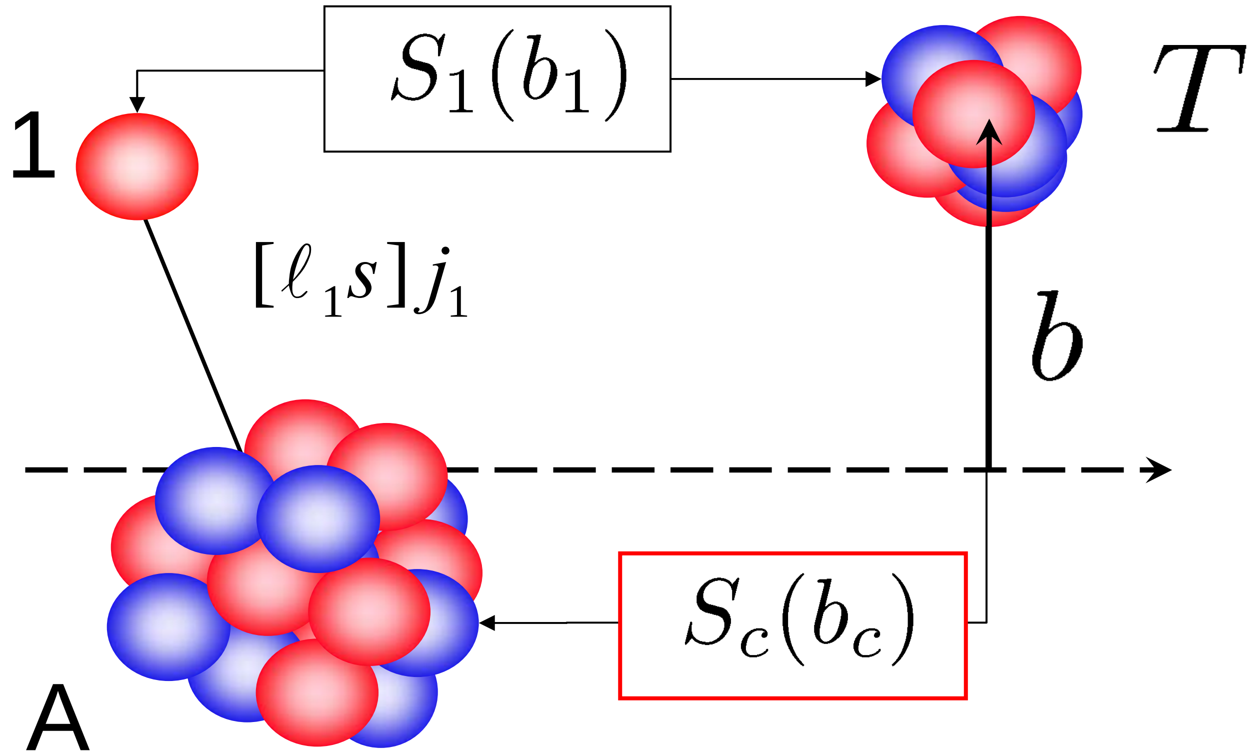
$$\int d\mathbf{k}\, |\phi_{\mathbf{k}}\rangle\langle\phi_{\mathbf{k}}| = 1 - |\phi_0\rangle\langle\phi_0| \boxed{- |\phi_1\rangle\langle\phi_1| \;......}$$

If > 1 bound state

can (for a weakly bound system with a single bound state) be expressed in terms of only the projectile ground state wave function as:

$$\sigma_{\rm diff} = \int d\mathbf{b} \left\{ \langle\phi_0\,||S_c\, S_v|^2\,|\phi_0\rangle - |\langle\phi_0\,|S_c\, S_v\,|\phi_0\rangle|^2 \right\}$$

Diffractive (breakup) removal of a nucleon



$$\sigma_{\mathrm{diff}} = \int \mathrm{d}\mathbf{b} \left\{ \langle \phi_0 \,||\, \mathrm{S_c}\, \mathrm{S_v} \,|^2 \,|\, \phi_0 \rangle - |\, \langle \phi_0 \,|\, \mathrm{S_c}\, \mathrm{S_v} \,|\, \phi_0 \rangle \,|^2 \right\}$$

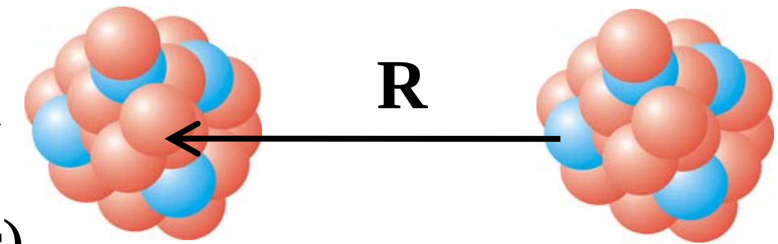
Core-target effective interactions – for $S_c(b_c)$

Double folding

$$U_{AB}(R) = \int d\mathbf{r}_1 \int d\mathbf{r}_2 \; \rho_A(r_1)\, \rho_B(r_2)\, t_{NN}(\mathbf{R} + \mathbf{r}_2 - \mathbf{r}_1)$$

U_{AB}

A



B

$\rho_A(r)$ $\rho_B(r)$

At higher energies – for nucleus-nucleus or nucleon-nucleus systems – first order term of multiple scattering expansion

$$t_{NN}(r) = \left[-\frac{\hbar v}{2} \sigma_{NN} (i + \alpha_{NN}) \right] f(r), \quad \int d\vec{r} f(r) = 1$$

e.g. $f(r) = \delta(r)$

nucleon-nucleon cross section

$$f(r) = (\sqrt{\pi} t)^{-3} \exp(-r^2/t^2)$$

resulting in a COMPLEX nucleus-nucleus potential

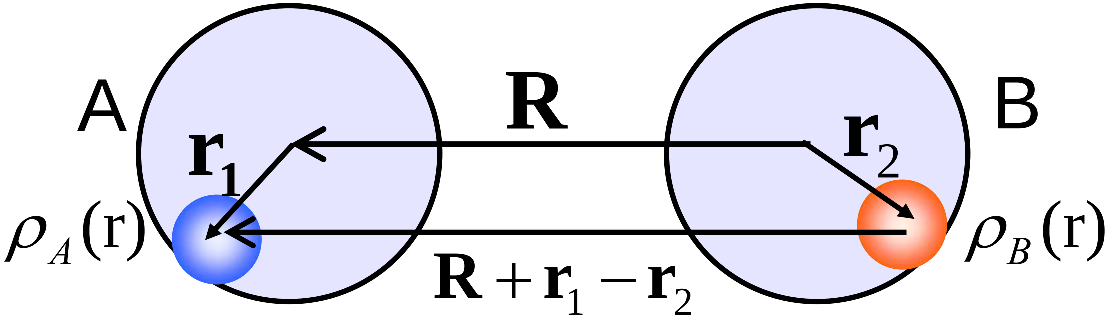
M.E. Brandan and G.R. Satchler, Phys. Rep. **285** (1997) 143-243.

Effective interactions – Folding models

Double folding

$$U_{AB}(R) = \int d\mathbf{r}_1 \int d\mathbf{r}_2 \; \rho_A(r_1)\, \rho_B(r_2)\, v_{NN}(\mathbf{R} + \mathbf{r}_1 - \mathbf{r}_2)$$

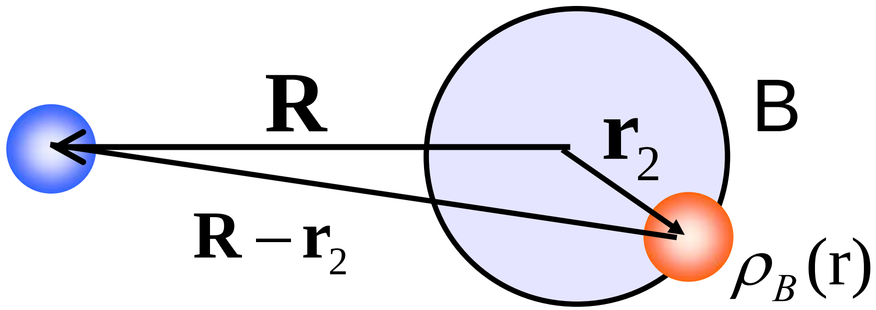
$$U_{AB}$$



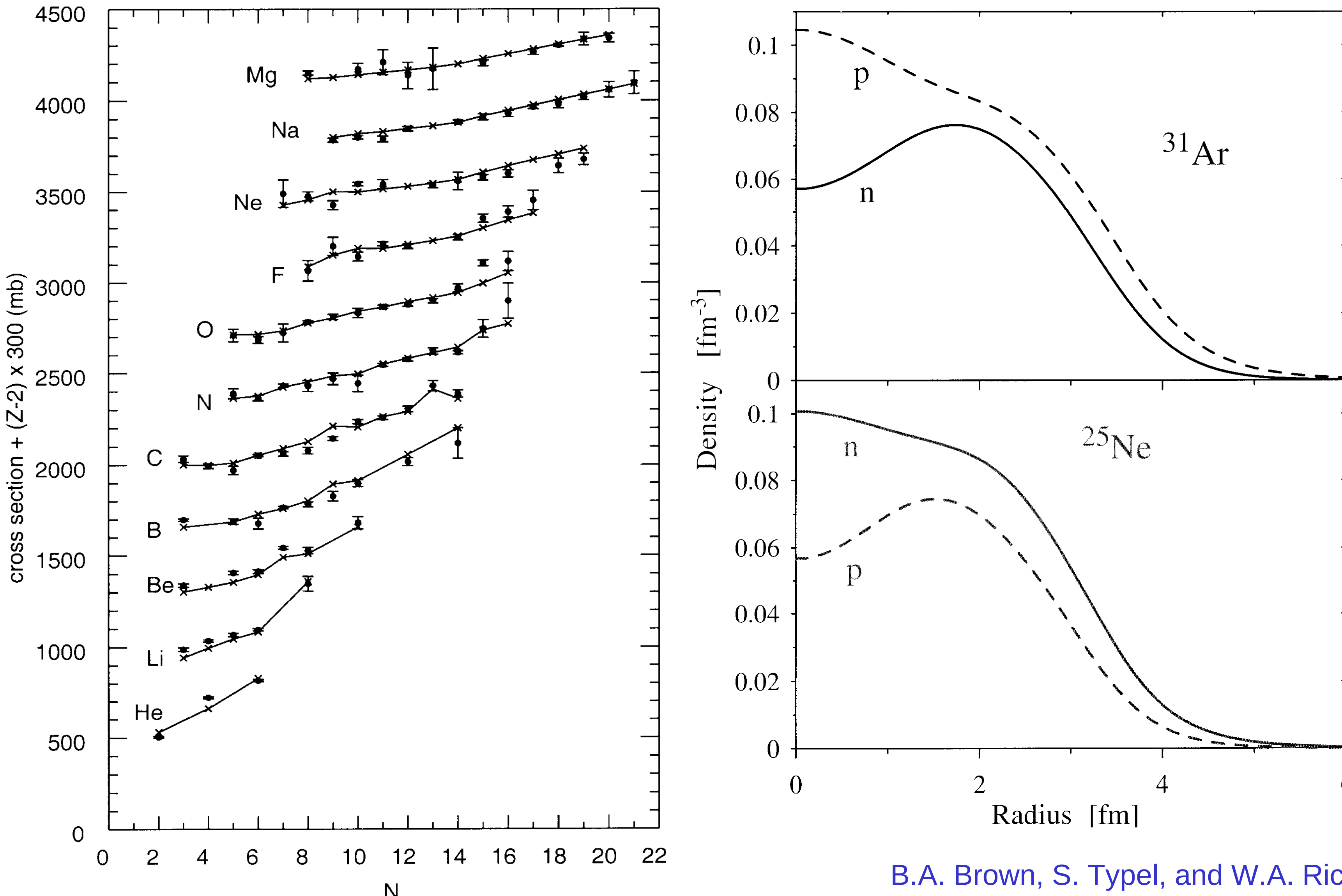
Single folding

$$U_B(R) = \int d\mathbf{r}_2 \; \rho_B(r_2)\, v_{NN}(\mathbf{R} - \mathbf{r}_2)$$

$$U_B$$



Sizes - Skyrme Hartree-Fock radii and densities



B.A. Brown, S. Typel, and W.A. Richter, Phys. Rev. **C65** (2002) 014612